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Lagrangian with the 14 extra upper/lower bound constraints for each variable (the ϵ 's are constant):
P(T) is the unconstrained original probability score.

$$L(T, \lambda) = P(T) + \lambda_1(t_x - \epsilon_0) + \lambda_2(-t_x - \epsilon_0) + \lambda_3(t_y - \epsilon_0) + \lambda_4(-t_y - \epsilon_0) + \lambda_5(t_z - \epsilon_0) + \lambda_6(-t_z - \epsilon_0) + \lambda_7(q_1 - \epsilon_1) + \lambda_8(-q_1 - \epsilon_1) + \lambda_9(q_2 - \epsilon_2) + \lambda_{10}(-q_2 - \epsilon_2) + \lambda_{11}(q_3 - \epsilon_2) + \lambda_{12}(-q_3 - \epsilon_2) + \lambda_{13}(q_4 - \epsilon_2) + \lambda_{14}(-q_4 - \epsilon_2)$$

Gradient with respect to the 21-D parameter space (where each of the $\frac{dP}{dx}$ terms signifies the partial derivative in the unconstrained problem:

$$\nabla P = \left[\frac{dP}{dt_x} + \lambda_1 - \lambda_2, \frac{dP}{dt_y} + \lambda_1 - \lambda_2, \frac{dP}{dt_z} + \lambda_1 - \lambda_2, \frac{dP}{dq_1} + \lambda_1 - \lambda_2, \frac{dP}{dq_2} + \lambda_1 - \lambda_2, \frac{dP}{dq_3} + \lambda_1 - \lambda_2, \frac{dP}{dq_4} + \lambda_1 - \lambda_2, \right. \\ \left. t_x - \epsilon_0, -t_x - \epsilon_0, t_y - \epsilon_0, -t_y - \epsilon_0, t_z - \epsilon_0, -t_z - \epsilon_0, q_1 - \epsilon_1, -q_1 - \epsilon_1, q_2 - \epsilon_2, -q_2 - \epsilon_2, q_3 - \epsilon_2, -q_3 - \epsilon_2, q_4 - \epsilon_2, -q_4 - \epsilon_2 \right]$$

And the new parameter space is:

$$T = [t_x, t_y, t_z, q_1, q_2, q_3, q_4, \lambda_1, \lambda_2, \lambda_3, \lambda_4, \lambda_5, \lambda_6, \lambda_7, \lambda_8, \lambda_9, \lambda_{10}, \lambda_{11}, \lambda_{12}, \lambda_{13}, \lambda_{14}]$$