Ch 7 # 22, 24, 27, 28, 31, 32

22. An isotope of neptunium has a half life of 65 minutes. If the decay of Np-240 is modeled by the differential equation dy/dt = -ky, where t is measured in minutes, what is the decay constant k?

65 min = ln(2) / k
ln(2) / 65 min = k
k = 0.01066

24. A colony of bacteria is grown under ideal conditions in a laboratory so that the population increases exponentially with time. At the end of 3 hours there are 10,000 bacteria. At the end of 5 hours there are 40,000 bacteria. How many bacteria were present initially?

$10k = y_0e^{(k * 3)}$
$40k = y_0e^{(k * 5)}$

27. $y = y_0e^{(kt)}$; (0, 2), (2, 5)
    $2 = y_0e^{(k * 0)}$
    $2 = y_0$

$5 = y_0e^{(k * 2)}$
$5 = 2e^{(k * 2)}$
$5/2 = e^{(k * 2)}$
ln(5/2) = k * 2
ln(5/2)/2 = k

$y = 2e^{(k * 0.458145)}$

28. $y = y_0e^{(kt)}$; (-3, 3), (0, 1.1)
    $1.1 = y_0e^{(k * 0)}$
    $1.1 = y_0$

$3 = y_0e^{(k * -3)}$
$3 = 1.1e^{(k * -3)}$
$3/1.1 = e^{(k * -3)}$
ln(3/1.1) = k * -3
ln(3/1.1)/-3 = k

$y = 1.1e^{(k * -0.334434)}$

31. Suppose that a cup of soup cooled from 90°C to 60°C in 10 min in a room whose temperature was 20°C. Use Newton's law of cooling to answer the following questions.

$T(t) = T_s + (T_0 – T_s) e^{-kt}$
$T(t) = 20°C + (90°C – 20°C) e^{-kt}$
$T(t) = 20°C + 70°C e^{-kt}$

$60°C = 20°C + 70°C e^{(-k * 10)}$
$40°C = 70°C e^{(-k * 10)}$
$4/7 = e^{(-k * 10)}$
ln(4/7) = -10k
k = ln(7/4) / 10
k = 0.055962

a. How much longer would it take the soup to cool to 35°C?
27.527 min (solved with calculator)

b. Instead of being left to stand in the room, the cup of 90°C soup is put into a freezer whose temperature is -15°C. How long will it take the soup to cool from 90°C to 35°C?
13.258 min (solved with calculator)

32. The temperature of an ingot of silver is 60°C above room temperature right now. Twenty minutes ago, it was 70°C above room temperature. How far above room temperature will the silver be:

60°C = 25°C + 45°C e^(-k * 20)
35°C = 45°C e^(-k * 20)
7/9 = e^(-k * 20)
ln(7/9) = -20k
k = ln(9/7) / 20
k = 0.012566

a. 15 minutes from now
53.987°C

b. 2 hours from now
32.748°C

c. When will the silver be 10°C above room temperature?
119.694 min