Calculus AB—Exam 2

Section I, Part A

Time: 55 minutes
Number of questions: 28

NO CALCULATOR MAY BE USED IN THIS PART OF THE EXAMINATION.

Directions: Solve each of the following problems. After examining the form of the choices, decide which is the best of the choices given.

In this test: Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

1. $\int_1^3 (3x^2 - 4x)\,dx =$ $\left[x^3 - 2x^2\right]_1^3 \to (3^3 - 2\cdot 3^2) - (1-2)$

   (A) 8 (B) 9 (C) 10

   (D) 12 (E) 62

   $27 - 18 + 1$

   $9 + 1$

   10

$f(x) = x(4x-1)^{1/2}$

$\frac{4x}{2\sqrt{4x-1}} \leftarrow \frac{4x}{2\sqrt{4x-1}}$

$\frac{2x}{\sqrt{4x-1}}$

2. If $f(x) = x\sqrt{4x-1}$, then $f'(x)$ is

   (A) $\dfrac{6x-1}{\sqrt{4x-1}}$. (B) $\dfrac{2x}{\sqrt{4x-1}}$. (C) $\dfrac{1}{\sqrt{4x-1}}$.

   (D) $\dfrac{-6x+2}{\sqrt{4x-1}}$. (E) $\dfrac{9x-2}{2\sqrt{4x-1}}$.

3. If $\int_a^b g(x)\,dx = 4a + b$, then $\int_a^b (g(x) + 7)\,dx =$

   (A) $8b - 11a$. (B) $8b + 11a$. (C) $8b - 3a$.

   (D) $7b - 7a$. (E) $4a + b + 7$.

$f'(x) = -5x^4 + 1 + \frac{1}{2x}$
$f'(-1) = -5(-1)^4 + 1 + \frac{1}{2(-1)}$
$= -5 + 1 - \frac{1}{2}$

4. If $f(x) = -x^5 + x + \frac{1}{x^2}$, then $f'(-1) =$

(A) 8. (B) 2. (C) −2.
(D) −3. (E) −8.

5. $y = 5x^4 - 24x^3 + 24x^2 + 17$ is concave down for $y' = 20x^3 - 72x^2 + 48x$

(A) $x < 0$. (B) $x > 0$. $f(0) = 0$ 288

(C) $x < -2$ or $x > -\frac{2}{5}$. (D) $x < \frac{2}{5}$ or $x > 2$. $f'(-1) = -20 - 72 - 48$
$= \text{down}$

(E) $\frac{2}{5} < x < 2$. $f'(-2) = -160 - 280 - 96$
$= \text{down (L)}$

$f'(1) = 20 - 72 + 48$
$= \text{down (S)}$

$f(2) = 160 - 280 + 96$
$= \text{down (S)}$

6. $\frac{1}{3}\int e^{t/3}\, dt = \frac{1}{3}\int e^{t/3}\, dt \rightarrow \frac{1}{3} \cdot e^{t/3}$

(A) $e^t + C$ (B) $3e^{t/3} + C$ (C) $e^{t/3} + C$
(D) $\frac{1}{3}e^{t/3} + C$ (E) $e^{-2/3t} + C$

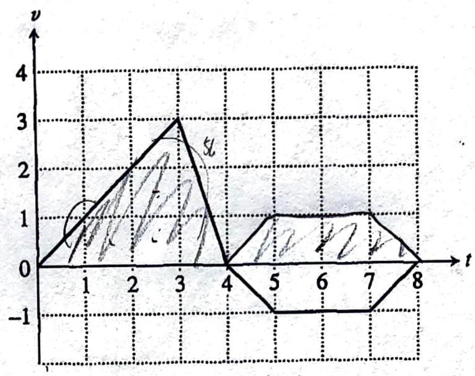
$\cancel{u = x^2}$
$\cancel{du = 2x}$

$u = x^2$
$du = 2x\, dx$

7. $\frac{d}{dx}\cos^3(x^2) = 3\cos^2(x^2) \rightarrow 6x\cos^2(x^2)$

(A) $6x \cos^2 x^2$ (B) $\sin^3 x^2$
(C) $6x \sin x^2 \cos^2 x^2$ (D) $-3 \sin x^2 \cos^2 x^2$
(E) $-6x \sin x^2 \cos^2 x^2$

Questions 8–9 refer to the following situation.



A spider begins to crawl up a vertical blade of grass at time $t = 0$. The velocity v of the spider at time t, $0 \leq t \leq 8$, is given by the function whose graph is shown.

8. At what value of t does the spider change direction?

(A) 3 (B) 4 (C) 5

(D) 7 (E) 8

9. What is the total distance the spider traveled from $t = 0$ to $t = 8$?

(A) 3 (B) 8 (C) 9

(D) 10 (E) 15

10. An equation of the line tangent to the graph of $y = \cos 3x$ at $x = \pi/6$ is

$\cos(3(\pi/6))$

0

(A) $y = 3\left(x - \frac{\pi}{6}\right)$ (B) $y = -\left(x - \frac{\pi}{6}\right)$

(C) $y = -3\left(x - \frac{\pi}{6}\right)$ (D) $y - 1 = -\left(x - \frac{\pi}{6}\right)$

(E) $y - 1 = -2\left(x - \frac{\pi}{6}\right)$

$y' = -3\sin(3x)$

$y' = 3(1)$

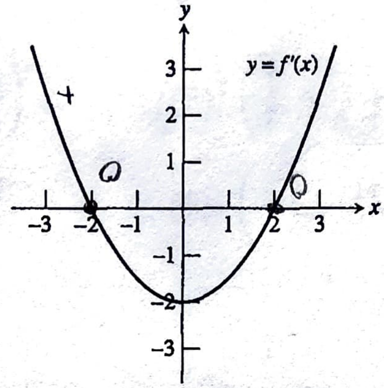
$-3(x - \pi/6)$

$0 = -3(\pi/6) + b$

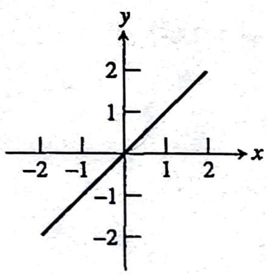
$-\pi/2 = b$

$y = -3x + \pi/2$

11. The graph of the derivative of f is shown in the figure below. Which of the following could be the graph of f?



(A)



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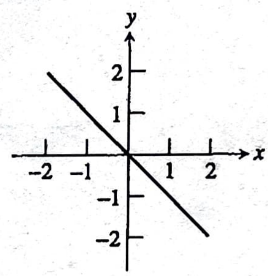

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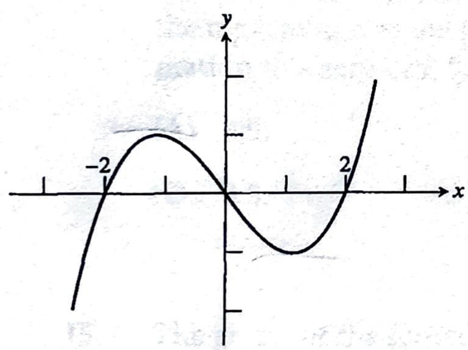

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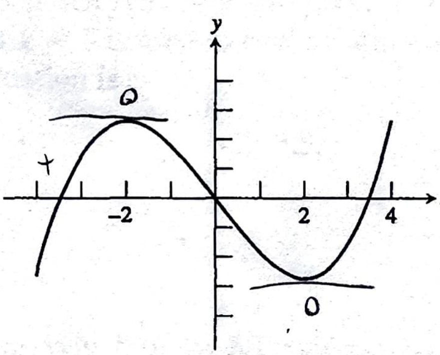
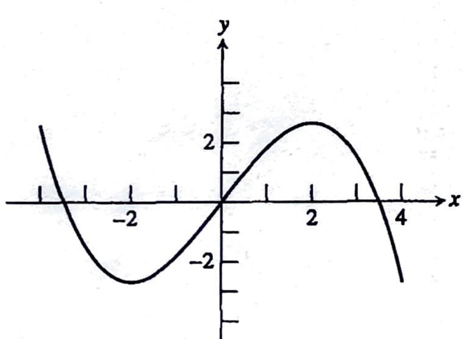


(E)



$y' = x$

$y = \frac{1}{2}\left(\frac{1}{2}\right)^2 - \frac{3}{2}$

$y = \frac{1}{8} - \frac{3}{2} \to \frac{12}{8}$

$y = -\frac{11}{8}$

12. At what point on the graph of $y = \frac{1}{2}x^2 - \frac{3}{2}$ is the tangent line parallel to the line $4x - 8y = 5$? $\to y = \frac{1}{2}x - \frac{5}{8}$

(A) $\left(\frac{1}{2}, -\frac{3}{8}\right)$ (B) $\left(\frac{1}{2}, -\frac{11}{8}\right)$ (C) $\left(2, \frac{3}{8}\right)$

(D) $\left(2, \frac{1}{2}\right)$ (E) $\left(-\frac{1}{2}, -\frac{11}{8}\right)$

13. Let f be a function defined for all real numbers x. If $f'(x) = \frac{|9 - x^2|}{x - 3}$, then f is decreasing on the interval

$f'(3) = \frac{|9 - (3)^2|}{(3) - 3} = 0$

(A) $(-\infty, 3)$. (B) $(-\infty, \infty)$. (C) $(-3, 6)$.

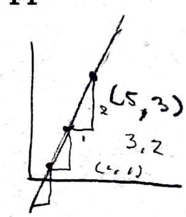
(D) $(-3, \infty)$. (E) $(3, \infty)$.

$f'(4) = \frac{|9 - 4^2|}{4 - 3} = 7$

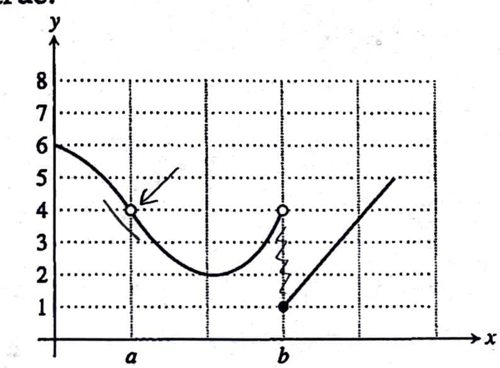
14. Let f be a differentiable function such that $f(5) = 3$ and $f'(5) = 2$. If the tangent line to the graph of f at $x = 5$ is used to find an approximation to a zero of f, that approximation is

(A) 6.5. (B) 4.3. (C) 3.5.

(D) 0.5. (E) 0.3.



15. The graph of the function f is shown. Which of the following statements about f is true?



(A) $\lim_{x \to a} f(x) = \lim_{x \to b} f(x)$ (B) $\lim_{x \to a} f(x) = 4$

(C) $\lim_{x \to b} f(x) = 4$ (D) $\lim_{x \to b} f(x) = 1$

(E) $\lim_{x \to a} f(x)$ does not exist.

$y = x^2 + 2$

$9 = x^2$

$\sqrt{9} = x$

$x = 3, -3$

16. The area of the region enclosed by the graph of $y = x^2 + 2$ and the line $y = 11$ is

(A) 18. (B) 30. (C) 36. (D) 72. (E) 27π.

$\int_{-3}^{3} (11 - (x^2 + 2))\,dx \rightarrow \left[11x - \left(\frac{x^3}{3} + 2x\right)\right]_{-3}^{3}$

$11(3) - \left(\frac{3^3}{3} + 2(3)\right)$

$33 - (9 + 6)$

18

$11(-3) - \left(\frac{-3^3}{3} + 2(-3)\right)$

$-33 + (9 - 6)$

-18

$18 - (-18) = 36$

? 17. If $x^2 = 25 - y^2$, what is the value of $\dfrac{d^2y}{dx^2}$ at the point (3, 4)?

$x^2 + y^2 = 25$

$y^2 = -x^2 + 25$

$y = \sqrt{-x^2 + 25}$

(A) $\dfrac{-25}{64}$ (B) $\dfrac{-7}{64}$ (C) $\dfrac{7}{64}$

(D) $\dfrac{25}{64}$ (E) $\dfrac{4}{3}$

? 18. $\displaystyle\int_{\pi/4}^{\pi/2} \frac{-e^{\cot x}}{\sin^2 x}\,dx =$

(A) $-e$ (B) $1 - e$ (C) -1

(D) $e - 1$ (E) $1 + e$

19. If $f(x) = \ln|1 - x^2|$, then $f'(x) =$

$1 - x^2 \rightarrow -2x$

$\ln|u| \rightarrow \frac{1}{u}$

$\frac{-2x}{1-x^2}$

(A) $\dfrac{-2|x|}{1 - x^2}$. (B) $\dfrac{-2x}{1 - x^2}$.

(C) $\dfrac{1}{1 - x^2}$. (D) $\left|\dfrac{-2x}{1 - x^2}\right|$.

(E) $\dfrac{-2x}{|1 - x^2|}$.

$-\cos(x) \rightarrow -\cos(-2) + -\cos(4)$

20. The average value of $f(x) = -\sin x$ on the interval $[-2, 4]$ is

$\frac{\cos(2) - \cos(4)}{2}$

(A) $\dfrac{\cos 4 + \cos 2}{6}$. (B) $\dfrac{\cos 2 - \cos 4}{2}$.

(C) $\dfrac{\cos 4 + \cos 2}{2}$. (D) $\dfrac{\cos 4 - \cos 2}{2}$.

(E) $\dfrac{\cos 4 - \cos 2}{6}$.

21. Evaluate $\lim_{x \to 1} \frac{\ln x}{3x}$ $\frac{\ln(0.999)}{3(0.999)} \approx 0$ $\frac{\ln(1.0001)}{3(1.0001)} \approx 0$

(A) 0   (B) $\frac{3}{e}$ (C) e

(D) 3   (E) The limit does not exist.

22. What are all values of x for which the function f defined by $f(x) = (x^2 - 15)e^{-x}$ is increasing? $f'(x) = -e^{x}(x^2 - 2x - 15)$

(A) There are no such values of x.   ~~$f'(-3) = -e^{3}((-3)^2 - 6 - 15)$~~

(B) $x < -3$ or $x > 5$

(C) $-5 < x < 3$

(D) $-3 < x < 5$ $(x-5)(x+3)$

(E) All values of x $x = 5, -3$

23. If the region enclosed by the y-axis, the line $y = 2$, and the curve $y = \sqrt[3]{x}$ is revolved about the y-axis, the volume of the solid generated is $\pi\int_0^2 (y^3)^2 dy \rightarrow \pi\left[\frac{y^7}{7}\right]_0^2 \rightarrow \pi\left(\frac{2^7}{7}\right) = \frac{128}{7}\pi$

(A) π.   (B) 4π.   (C) 8π.

(D) $\frac{64\pi}{7}$.   (E) $\frac{128\pi}{7}$.

24. The expression $\frac{1}{30}\left(\sin\frac{1}{30} + \sin\frac{2}{30} + \sin\frac{3}{30} + \cdots + \sin\frac{30}{30}\right)$ is a Riemann sum approximation for

(A) $\int_0^1 \sin\frac{x}{30}\,dx$.   (B) $\int_0^1 \sin x\,dx$.

(C) $\frac{1}{30}\int_0^1 \sin\frac{x}{30}\,dx$.   (D) $\frac{1}{30}\int_0^1 \sin x\,dx$.

(E) $\frac{1}{30}\int_0^{30} \sin x\,dx$.

25. $\int x \sin x^2\,dx =$

(A) $-\frac{1}{2}\cos x^2 + C$ (B) $\frac{1}{2}\cos x^2 + C$

(C) $-x^2 \cos x^2 + C$ (D) $x^2 \cos x^2 + C$

(E) $\frac{1}{2}x^2 \cos\frac{x^2}{3} + C$

26. Let $f(x) = \lim_{h \to 0} \frac{(x+h)^2 - x^2}{h}$. For what value of x does $f(x) = 4$?

(A) -2 (B) -1 $g(x) = x^2$ (C) 1

(D) 2 (E) 4 $f(x)' = g'(x)$

$g'(x) = 2x$

27. Let $f(x) = \begin{cases} 3x^2 - 5, & x \le 1 \\ 6x + 2, & x > 1 \end{cases}$. Which of the following are true statements about this function?

$3(1)^2 - 5 = -2$

$6 + 2 = 8$

I. $\lim_{x \to 1} f(x)$ exists.

II. $\lim_{x \to 1} f'(x)$ exists.

$f'(x) = \begin{cases} 6x, & x \le 1 \\ 6, & x > 1 \end{cases}$

III. $f'(1)$ exists.

(A) None (B) II only (C) III only

(D) II and III (E) I, II, and III

28. Let $g(x) = \cancel{\frac{d}{dx} \int_0^x} \sqrt{t^2 + 9}\, dt$. What is $g(-4)$?

(A) -5 (B) -3 (C) 3

(D) 4 (E) 5

$\sqrt{-4^2 + 9}$

$\sqrt{16+9} = \sqrt{25} = 5$

✧ **End of Part A of Section I** ✧

Calculus AB—Exam 2
Section I, Part B

Time: 50 minutes
Number of questions: 17

A GRAPHING CALCULATOR IS REQUIRED FOR SOME QUESTIONS IN THIS PART OF THE EXAMINATION.

Directions: Solve each of the following problems. After examining the form of the choices, decide which is the best of the choices given.

In this test:

1. The exact numerical value of the correct answer does not always appear among the choices given. When this happens, select from among the choices the number that best approximates the exact numerical value.
2. Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

29. If $f(x) = \dfrac{e^{3x}}{3x}$, then $f'(x) =$

(A) 1.
(B) e^{3x}.
(C) $\dfrac{e^{3x}(1 - 3x)}{3x^2}$.
(D) $\dfrac{e^{3x}(3x + 1)}{3x^2}$.
(E) $\dfrac{e^{3x}(3x - 1)}{3x^2}$.

30. The graph of the function $y = \frac{1}{3}x^3 - x^2 - 5x + 3 \sin x$ changes concavity at $x =$

(A) 3.29.
(B) 2.21.
(C) 1.34. ?
(D) 0.41.
(E) −0.39.

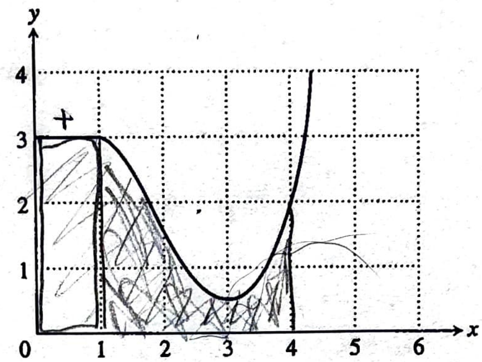
$y' = x^2 - 2x - 5$

$y'' = 2x - 2$

$0 = 2x - 2$

$2 = 2x \quad x = 1$

31. The graph of f is shown. If $\int_1^4 f(x)\,dx = 3.8$ and $F'(x) = f(x)$, then $F(4) - F(0) =$



(A) 0.8.
(B) 2.8.
(C) 4.8.
(D) 6.8.
(E) 8.4.

32. Let f be a function such that $\lim_{h\to 0} \frac{f(7+h)-f(7)}{h} = 4$. Which of the following must be true?

I. f is continuous at $x = 7$.

II. f is differentiable at $x = 7$.

III. The derivative of f is continuous at $x = 7$.

(A) I only
(B) II only
(C) I and II only
(D) I and III only
(E) II and III only

33. Let f be the function given by $f(x) = 5e^{3x^3}$. For what positive value of a is the slope of the line tangent to the graph of f at $(a, f(a))$ equal to 6?

(A) 0.142
(B) 0.344
(C) 0.393
(D) 0.595
(E) 0.714

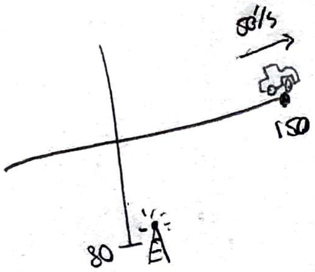
$f'(x) = 45e^{3x^3}x^2$

$6 = 45e^{3x^3}x^2$

$6/45 = e^{3x^3}x^2$

$2/15 = e^{3x^3}x^2$

34. Two roads cross at right angles, one running north/south and the other east/west. Eighty feet south of the intersection is an old radio tower. A car traveling at 50 feet per second passes through the intersection heading east. At how many feet per second is the car moving away from the radio tower 3 seconds after it passes through the intersection?



(A) 43.65 (B) 44.12 (C) 44.59

(D) 56.67 (E) 81.76

$(-1.5)^3(3(-1.5)+6) = -5.0625$

35. If $y = 3x + 6$, what is the minimum value of x^3y? $\rightarrow x^3(3x+6)$

(A) -10.125 (B) -5.0625 (C) -1.5

(D) 0 (E) 1.5

$3x^4 + 6x^3$

$12x^3 + 18x^2$

$0 = -1.5$

36. What is the area of the region in the first quadrant enclosed by the graphs of $y = \sin x$, $y = 2 - x$, and the x-axis?

(A) 0.552 (B) 0.951 (C) 1.106

(D) 1.600 (E) 2.152

$\sin(x) = 2 - x$

$\sin(x) + x = 2 \rightarrow x = 1.1061$

$\int_0^{1.1061} (2-x) - (\sin(x))\,dx + \int_{1.1061}^{2} (\sin(x)) - (2-x)\,dx \approx 1.51$

37. The base of a solid S is the region enclosed by the graph of $y = \sqrt{\ln(x - 1)}$, the line $x = 2e$, and the x-axis. If the cross sections of S perpendicular to the x-axis are squares, then the volume of S is

(A) 1.587. (B) 2.173. (C) 3.185.

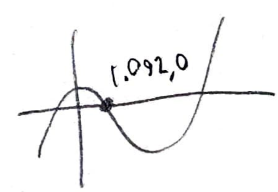
(D) 3.501. (E) 6.347.

$\int_2^{2e} (\sqrt{\ln(x-1)})^2\,dx \rightarrow \int_2^{2e} \ln(x-1)\,dx \rightarrow 3.18$

38. If the derivative of f is given by $f'(x) = 2e^x - 5x^2$, at which of the following values of x does f have a relative maximum value?

(A) -0.494 (B) 0.259 (C) 1.092

(D) 2.543 (E) 3.310



39. Let $f(x) = \sqrt{2x}$. If the rate of change of f at $x = c$ is four times its rate of change at $x = 1$, then $c =$

$f(1) = \sqrt{2}$

(A) $\frac{1}{16}$. (B) $\frac{1}{2\sqrt{2}}$. (C) $\frac{1}{\sqrt{2}}$.

$4\sqrt{2} = \sqrt{2x}$

(D) 1. (E) 32.

$\sqrt{4\sqrt{2}} = 2x \rightarrow \frac{\sqrt{4\sqrt{2}}}{2} = x$

$f(x) = (2x)^{1/2}$

$f'(x) = \frac{1}{\sqrt{2x}} \rightarrow \frac{4}{\sqrt{2}} = \frac{1}{\sqrt{2x}}$

$x = 1/16$

40. At time $t \geq 0$, the acceleration of a particle that is moving along the x-axis is $a(t) = t + 2\sin t$. At $t = 0$, the velocity of the particle is -4. For what value of t will the velocity of the particle be zero?

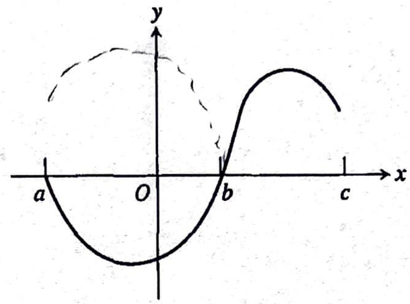
(A) 0 (B) 1.20 (C) 1.78

(D) 2.31 (E) 3.87

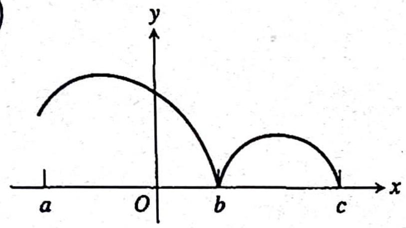
$\int t + 2\sin(t)\, dt = \frac{1}{2}(t^2 - 4\cos(t)) = 0$

$t = 1.202$

41. Let $f(x) = \int_a^x h(t)\,dt$, where h has the graph shown below. Which of the following could be the graph of f?



(A)



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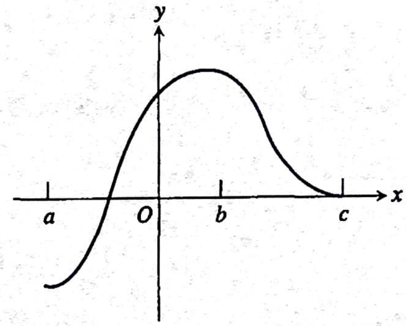

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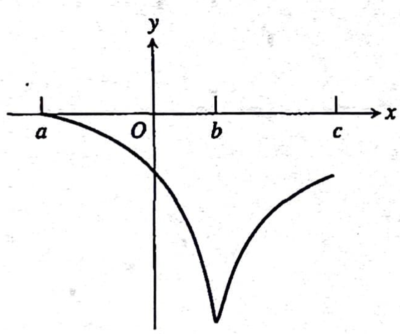
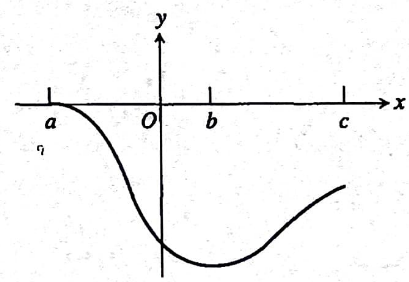
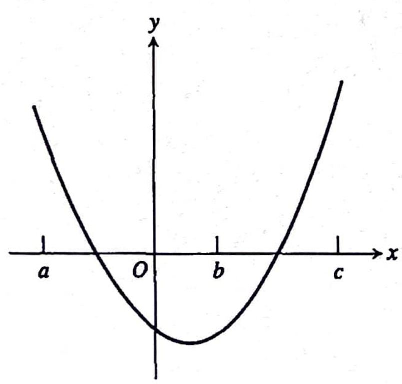


(D)



(E)



42. A continuous function $f(x)$ has the values shown in the table. What is the value of a trapezoidal approximation of $\int_0^3 f(x)\,dx$ using six equal subintervals?

x	0	0.5	1.0	1.5	2.0	2.5	3.0
$f(x)$	8	5	4	3	3	5	8

$(6.5 + 4.5 + 3.5 + 3 + 4 + 6.5) \cdot 0.5 = 14$

(A) 9 (B) 14 (C) 18

(D) 28 (E) 56

43. Which of the following are antiderivatives of $f(x) = 4 \sin x \cos x$?

I. $F(x) = -\cos 2x$

II. $F(x) = 2 \sin^2 x$

III. $F(x) = -2 \cos^2 x$

(A) I only (B) II only (C) III only

(D) I and II (E) I, II, and III

44. Let f be a function such that $f''(x) < 0$ for all x in the closed interval $[3, 4]$, with selected values shown in the table. Which of the following must be true about $f'(3.3)$?

x	3.2	3.3	3.4	3.5
$f(x)$	2.48	2.68	2.86	3.03

0.2 0.18

(A) $f'(3.3) < 0$ (B) $0 < f'(3.3) < 1.6$

(C) $1.6 < f'(3.3) < 1.8$ (D) $1.8 < f'(3.3) < 2.0$

(E) $f'(3.3) > 2.0$

45. If the function f is defined by $f(x) = \int_0^x -\sin t^2 \, dt$ on the closed interval $-1 \leq x \leq 3$, then f has a local maximum at $x =$

(A) -1.084.

(B) 0.

(C) 1.772.

(D) 2.171.

(E) 2.507.