Ch 6.1.2 # 7-12, 16, 18

7.

```
     LRAM    MRAM    RRAM
```

n = 10: 1.32 1.32 1.32
n = 50: 1.3328 1.3328 1.3328
n = 150: 1.3332 1.3332 1.3332
n = 500: 1.3333 1.3333 1.3333

8. The area of region R is likely 1 1/3

9. f(x) = x^2 - x + 3, a = 0, b = 3
   RRAM: 13.5090045
   LRAM: 13.4910045
   MRAM: 13.5000045
   estimate: 13.5

10. f(x) = 1/x, a = 1, b = 3
    RRAM: 1.097945918
    LRAM: 1.099279252
    MRAM: 1.098612585
    estimate: 1.1

11. f(x) = e^(-x^2), a = 0, b = 2
    RRAM: 0.881099682
    LRAM: 0.8830630507
    MRAM: 0.8820813663
    estimate: 0.882

12. f(x) = sin(x), a = 0, b = pi
    RRAM: 0.0861930554
    LRAM: 0.0860208846
    MRAM: 0.08610697
    estimate: 0.0861

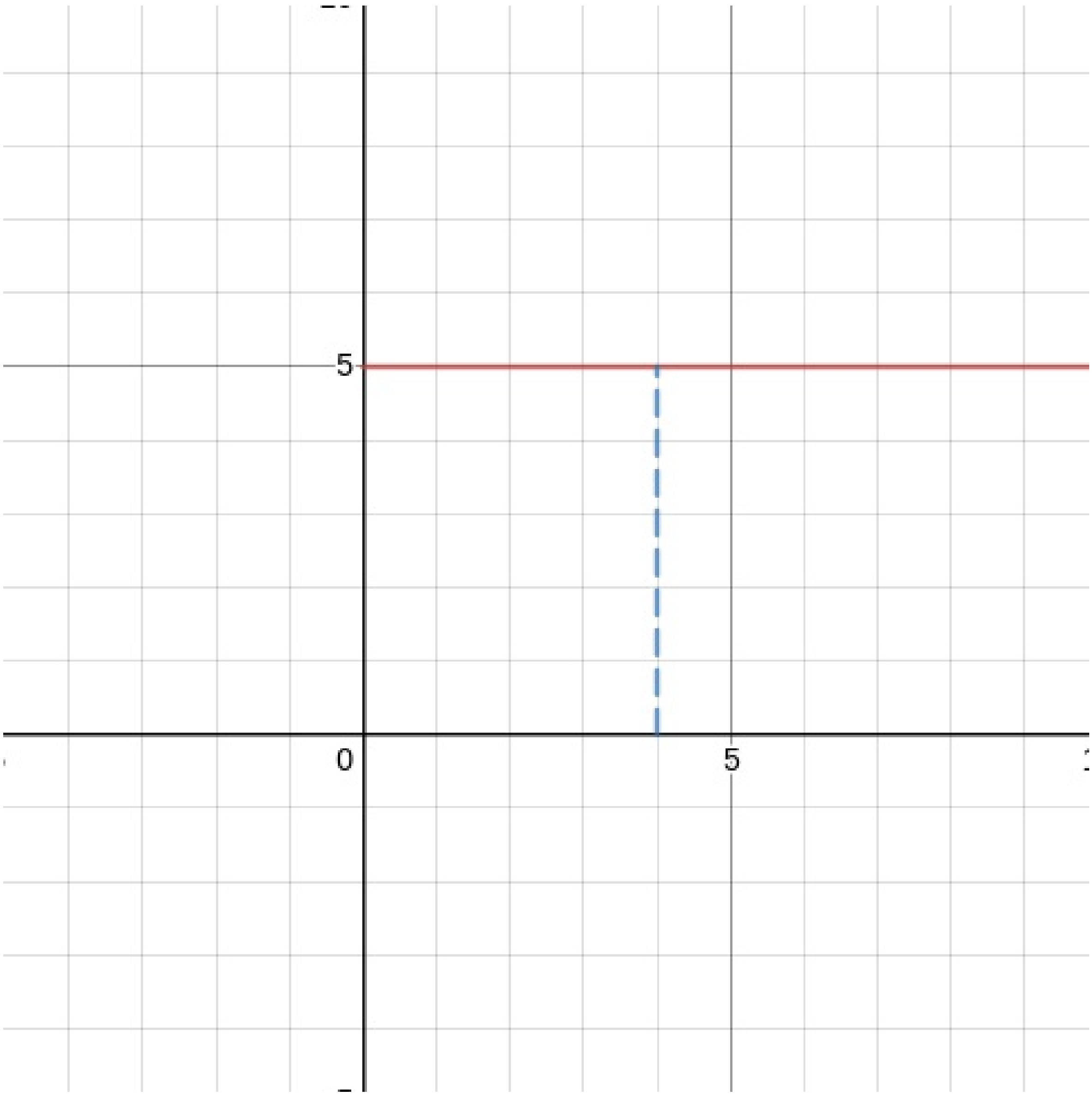
16.

LRAM: 87
RRAM: 87

18.

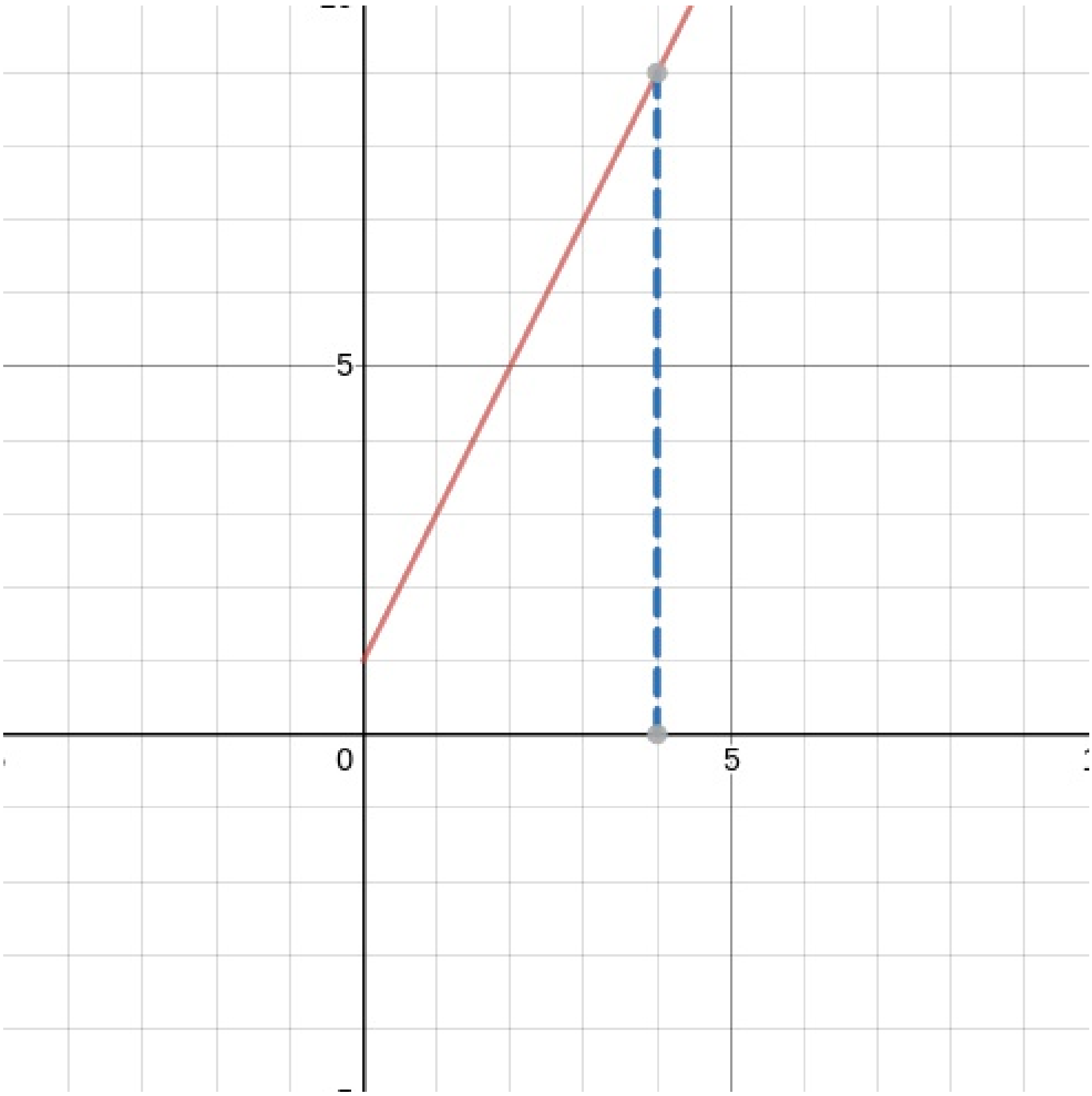
LRAM: 349
RRAM: 384
Ch 6.1 # 1 - 6

1. A particle starts at x = 0 and moves along the x-axis with velocity v(t) = 5 for time t >= 0. Where is the particle at t = 4?



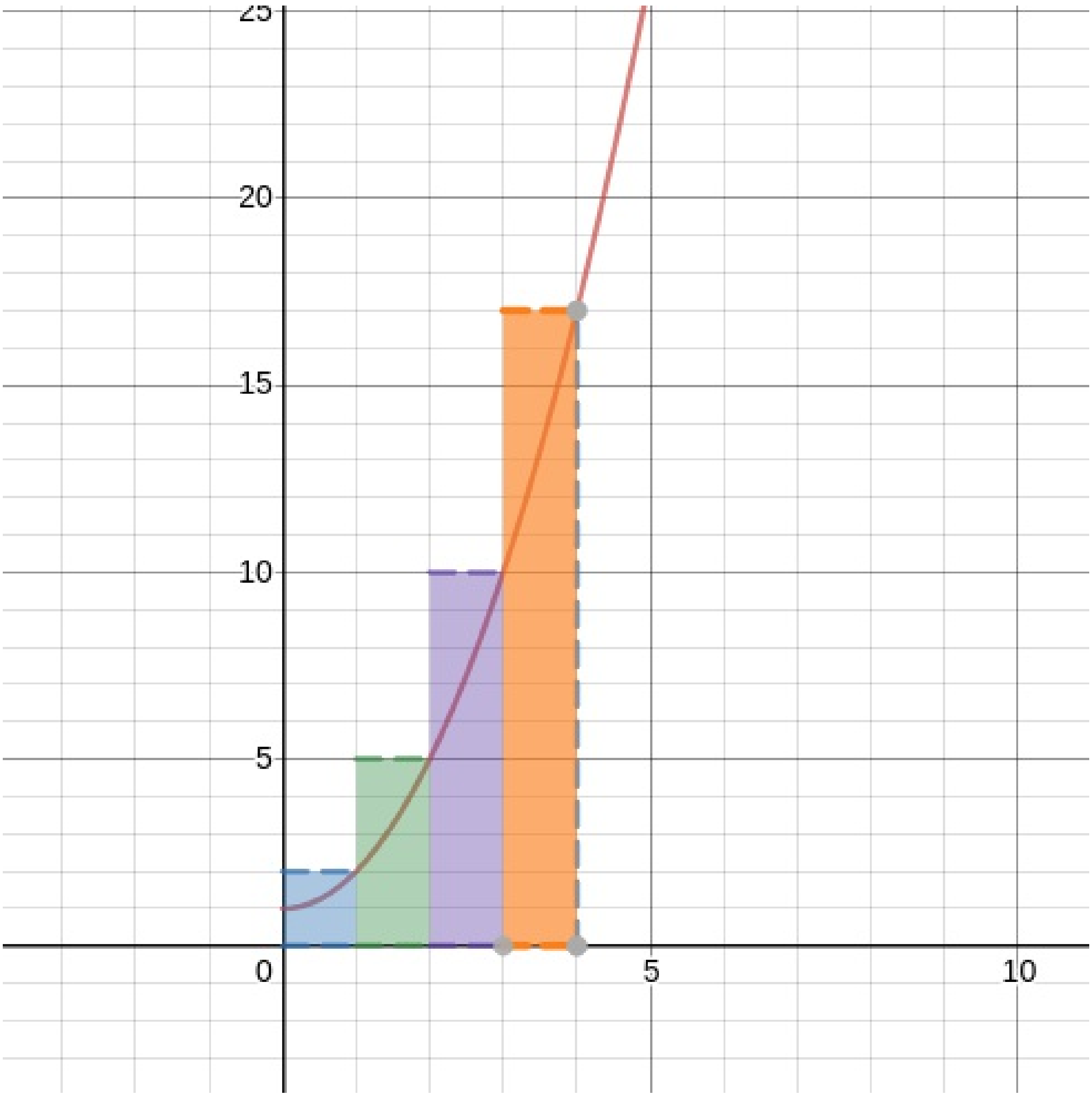
5 * 4 = 20
The particle is at 20 units when t = 4

2. A particle starts at x = 0 and moves along the x-axis with velocity v(t) = 2t + 1 for time t >= 0. Where is the particle at t = 4?



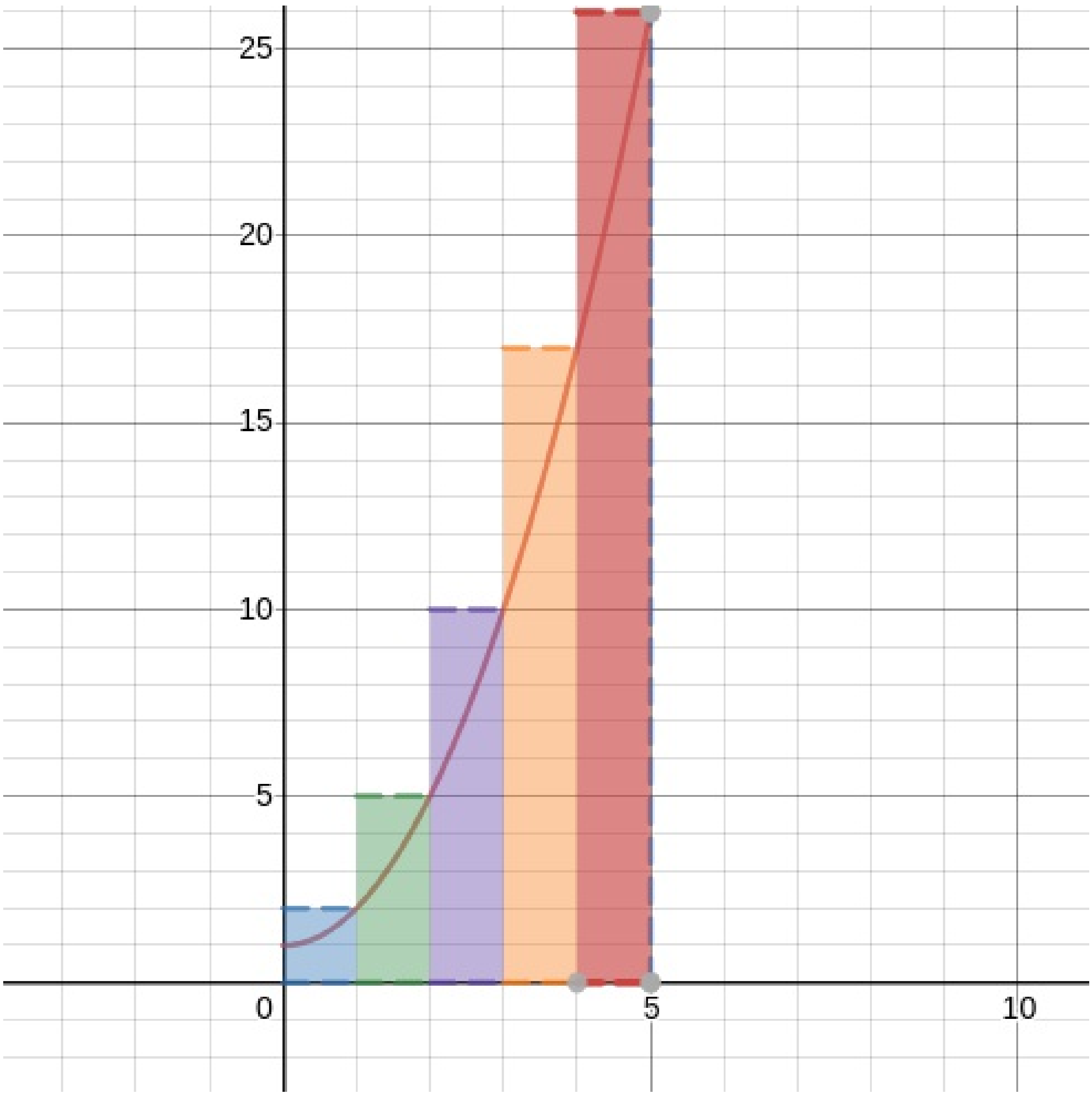
((1 + 9) / 2) * 4 = 20
The particle is at 20 units when t = 4

3. A particle starts at x = 0 and moves along the x-axis with velocity v(t) = t^2 + 1 for time t >= 0. Where is the particle at t = 4? Approximate the area under the curve by using four rectangles of equal width and heights determined by the right-endpoints of the intervals.



A = 2 + 5 + 10 + 17
A = 34

4. A particle starts at x = 0 and moves along the x-axis with velocity v(t) = t^2 + 1 for time t >= 0. Where is the particle at t = 5? Approximate the area under the curve by using five rectangles of equal width and heights determined by the right-endpoints of the intervals.

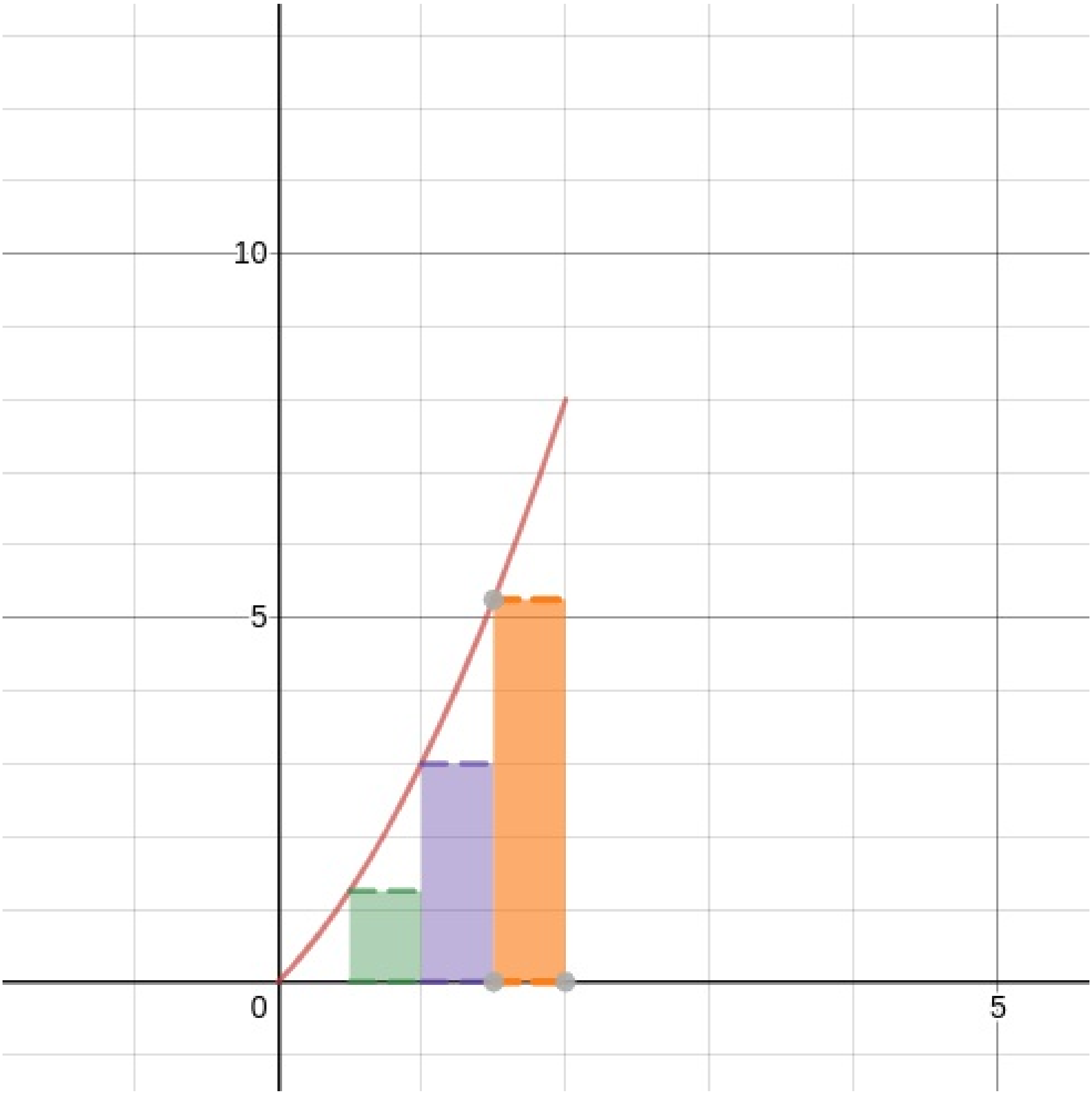


A = 2 + 5 + 10 + 17 + 26
A = 60

y = 2x - x^2 { x | 0 <= x <= 2 }

5.

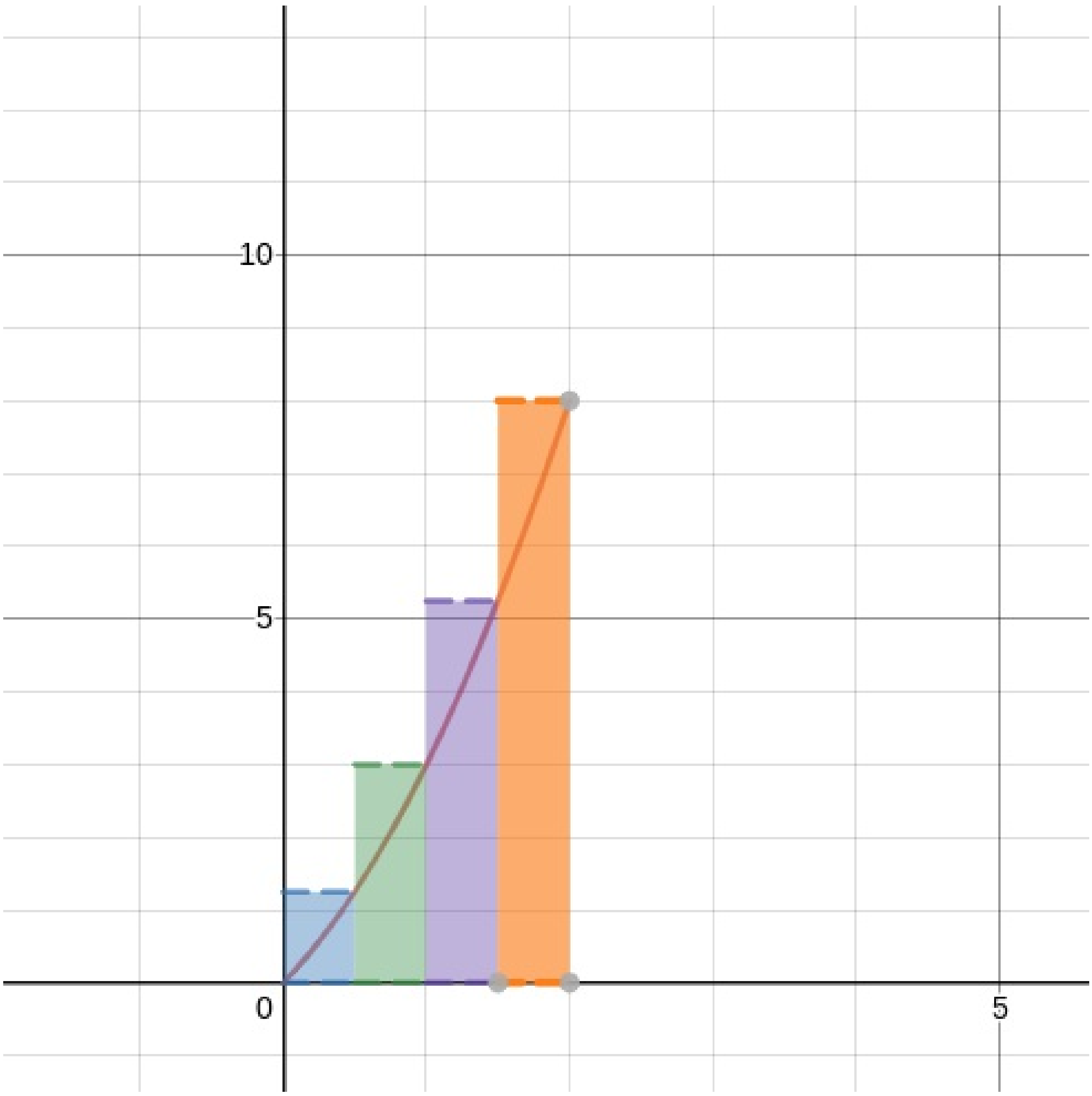
a. Sketch the region R



b. Partition [0, 2] into 4 subintervals and show the four rectangles that LRAM uses to approximate the area of R. Computer the LRAM sum without a calculator.

LRAM = 0 + 1.25 + 3 + 5.25
LRAM = 9.5

6. Repeat exercise 5b for RRAM
   (see fig 6.1.6)



RRAM = 1.25 + 3 + 5.25 + 8
RRAM = 17.8
Ch 6.2: Finding Integrals by Signed Areas

desmos: https://www.desmos.com/calculator/w4apywsgla

1. ∫(π, 2π) sin x dx
   -2; as this is simply the base equation shifted by π. In the sine wave this corresponds to a dip opposite of the first rise.

2. ∫(0, 2π) sin x dx
   0; over the period (0, 2π) the sine wave ends where it begins.

3. ∫(0, π/2) sin x dx
   1: 1/2 of the initial period results in 1/2 the area

4. ∫(0, 2π) (2 + sin x) dx
   12.57; not really sure how to explain this, but this is what I got from graphing it.

5. ∫(0, π) 2 sin x dx
   4; doubling the height of the original equation doubles the area.

6. ∫(2, π+2) sin(x - 2) dx
   2; this is just the base equation moved right by 2.

7. ∫(-π, π) sin u du
   0; the areas across the y-axis cancel each other out.

8. ∫(0, 2π) sin(x/2) dx
   4; this stretches the original wave's first rise out to 2π, doubling the area.

9. ∫(0, π) cos x dx
   0; (0, π) on a cosine wave goes from a peak to a trough, and the areas cancel out.

10. Suppose k is any positive number. Make a conjecture about ∫(-k, k) sin x dx
    ∫(-k, k) sin x dx will always be zero.
    Ch 6.3 # 1, 3, 4, 6, 8, 10, 11, 14, 15, 16

11.

a. 0
b. -6
c. -12
d. 10
e. -2
f. 16

3.

a. 5
b. 5sqrt(3)
c. -5
d. -5

4.

a. -sqrt(2)
b. sqrt(2)
c. -sqrt(2)
d. 1

6.

a. 6
b. 6

8.

9. If f is integrable, then int(a, b)f(x)dx <= f_max * (b-a), where f_max is 0, so int(a, b)f(x)dx <= 0

10. ∫0→√3: (x^2 - 1)dx = 0
    0 = x^2 - 1
    1 = x^2
    x = 1

11. $\int_0^3 ((x - 1)^2)dx = 3$
    $3 = (x - 1)^2$
    $\sqrt{3} = x - 1$
    $x = 1 + \sqrt{3}$

12.

0.5 * b * h
0.5 * 6 * 3
9

16.

$2 - \pi r^2$
$2 - \pi$

Ch 6.4.2 # 3, 6, 9, 12, 15, 18, 27 - 34, 42

Find dx/dy

3. $y = \int_{0}^{x} (\sin^2 t)\, dt$
   $dx/dy = \sin^2 x$

4. $y = \int_{4}^{x} (e^u \sec u)\, du$
   $dx/dy = e^x \sec x$

5. $y = \int_{0}^{x^2} (e^{t^2})\, dt$
   $dx/dy = e^{x^4} \cdot du/dx$
   $dx/dy = e^{x^4} \cdot 2x$
   $dx/dy = 2x\, e^{x^4}$

6. $y = \int_{\pi}^{\pi - x} ((1 + \sin^2 u) / (1 + \cos^2 u))\, du$
   $dx/dy = ((1 + \sin^2 (\pi - x)) / (1 + \cos^2 (\pi - x))) \cdot du/dx$
   $dx/dy = ((1 + \sin^2 (\pi - x)) / (1 + \cos^2 (\pi - x))) \cdot x$
   $dx/dy = x((1 + \sin^2 (\pi - x)) / (1 + \cos^2 (\pi - x)))$

7. $y = \int_{x^3}^{5} ((\cos t) / (t^2 + 2))\, dt$
   $dx/dy = - ((\cos x^3) / (x^5 + 2)) \cdot du/dx$
   $dx/dy = - ((\cos x^3) / (x^5 + 2)) \cdot 3x^2$
   $dx/dy = -3x^2 ((\cos x^3) / (x^5 + 2))$

8. $y = \int_{3x^2}^{5x} ((t^2 - 2t + 9) / (t^3 + 6))\, dt$
   $dx/dy = (((3x^2)^2 - 2(3x^2) + 9) / ((3x^2)^3 + 6)) \cdot du_1/dx - ((t^2 - 2t + 9) / (t^3 + 6)) \cdot du_2/dx$
   $dx/dy = (((3x^2)^2 - 2(3x^2) + 9) / ((3x^2)^3 + 6)) \cdot 6x - ((t^2 - 2t + 9) / (t^3 + 6)) \cdot 5$
   $dx/dy = (6x ((3x^2)^2 - 2(3x^2) + 9) / ((3x^2)^3 + 6)) - (5 (t^2 - 2t + 9) / (t^3 + 6))$

Evaluate each integral using FTC 2; support with NINT if unsure

27. $\int_{1/2}^{3} (2 - 1/x)\, dx$
    $F(x) = 2x - \ln |x| + C$
    $\int = F(3) - F(1/2)$
    $\int = (2(3) - \ln |3|) - (2(1/2) - \ln |1/2|)$
    $\int = 3.208$

28. $\int[2, -1] (3^x) dx$
$F(x) = 3^x / \ln 3$
$\int = F(-1) - F(2)$
$\int = (3^{(-1)} / \ln (-1)) - (3^{(2)} / \ln (2))$
$\int = -12.98$

29. $\int[0, 1] (x^2 + \sqrt{x}) dx$
$F(x) = x^3/3 + 2x^{(3/2)}/3$
$\int = F(1) - F(0)$
$\int = ((1)^3/3 + 2(1)^{(3/2)}/3) - ((0)^3/3 + 2(0)^{(3/2)}/3)$
$\int = 1$

30. $\int[0, 5] (x^{(3/2)}) dx$
$F(x) = 2x^{(5/2)}/5$
$\int = F(5) - F(0)$
$\int = (2(5)^{(5/2)}/5) - (2(0)^{(5/2)}/5)$
$\int = 22.36$

31. $\int[1, 32] (x^{(-6/5)}) dx$
$F(x) = -5/x^{(1/5)}$
$\int = F(32) - F(1)$
$\int = (-5/(32)^{(1/5)}) - (-5/(1)^{(1/5)})$
$\int = 5/2$

32. $\int[-2, -1] (2 / x^2) dx$
$F(x) = -2/x$
$\int = F(-1) - F(-2)$
$\int = (-2/-1) - (-2/-2)$
$\int = 1$

33. $\int[0, \pi] (\sin x) dx$
$F(x) = -\cos x$
$\int = F(\pi) - F(0)$
$\int = (-\cos \pi) - (-\cos 0)$
$\int = 2$

34. $\int[0, \pi] (1 + \cos x) dx$
$F(x) = x + \sin x$
$\int = F(\pi) - F(0)$
$\int = (\pi + \sin \pi) - (0 + \sin 0)$
$\int = \pi$

Find total area of region between curve and x-axis

42. $y = 3x^2 - 3, [-2, 2]$
$F(x) = x^3 - 3x$

$-\int[-1, 1] (3x^2 - 3) dx$
$-\int = F(1) - F(-1)$
$-\int = ((1)^3 - 3(1)) - ((-1)^3 - 3(-1))$
$\int = 4$

$2\int[1, 2] (3x^2 - 3)\, dx$
$\int = F(2) - F(1)$
$\int = ((2)^3 - 3(2)) - ((1)^3 - 3(1))$
$\int = 8$

total area = 12
Ch 6.4.3 # 19, 20, 35 - 40, 46, 47, 48

Find dy/dx

19. $\int[x^2, x^3] (\cos 2t)\, dt$
    $dt/dy = (\cos 2(x^3)) \cdot du_1/dx - (\cos 2(x^2)) \cdot du_2/dx$
    $dt/dy = (\cos 2(x^3)) \cdot 3x^2 - (\cos 2(x^2)) \cdot 2x$
    $dt/dy = 3x^2(\cos 2(x^3)) - 2x(\cos 2(x^2))$

20. $\int[\sin x, \cos x] (t^2)\, dt$
    $dt/dy = ((\cos x)^2) \cdot du_1/dx - ((\sin x)^2) \cdot du_2/dx$
    $dt/dy = ((\cos x)^2) \cdot -\sin x - ((\sin x)^2) \cdot \cos x$
    $dt/dy = -\sin x ((\cos x)^2) - \cos x ((\sin x)^2)$

Evaluate each integral using FTC 2; support with NINT if unsure

35. $\int[0, \pi/2] (2 \sec^2 \theta)\, d\theta$
    $F(x) = 2 \tan x$
    $\int = F(\pi/2) - F(0)$
    $\int = (2 \tan \pi/2) - (2 \tan 0)$
    $\int = \infty$

36. $\int[\pi/6, 5\pi/6] (\csc^2 \theta)\, d\theta$
    $F(x) = -\cot x$
    $\int = F(5\pi/6) - F(\pi/6)$
    $\int = (-\cot (5\pi/6)) - (-\cot (\pi/6))$
    $\int = 2\sqrt{3}$

37. $\int[\pi/4, 3\pi/4] (\csc x \cot x)\, dx$
    $F(x) = -\csc x$
    $\int = F(3\pi/4) - F(\pi/4)$
    $\int = (-\csc (3\pi/4)) - (-\csc (\pi/4))$
    $\int = 0$

38. $\int[0, \pi/3] (4 \sec x \tan x)\, dx$
    $F(x) = 4 \sec x$
    $\int = F(\pi/3) - F(0)$
    $\int = (4 \sec (\pi/3)) - (4 \sec (0))$
    $\int = 4$

39. $\int[-1, 1] (r + 1)^2\, dr$
    $F(x) = x^3/3 + x^2 + x$
    $\int = F(1) - F(-1)$
    $\int = ((1)^3/3 + (1)^2 + (1)) - ((-1)^3/3 + (-1)^2 + (-1))$
    $\int = 8/3$

40. $\int[0, 4] ((1 - \sqrt{u}) / \sqrt{u})\, dx$
    $F(x) = 2\sqrt{x} - x$
    $\int = F(4) - F(0)$

$\int = (2\sqrt{(4)} - (4)) - (2\sqrt{(0)} - (0))$
$\int = 0$

Find area

46. $[0, 1]: y = \sqrt{x}; [1, 2]: y = x^2$
    $F_a(x) = 2x^{(3/2)}/3$
    $F_b(x) = x^3/3$

$\int a = F_a(1) - F_a(0)$
$\int a = (2(1)^{(3/2)}/3) - (2(0)^{(3/2)}/3)$
$\int a = 2/3$

$\int b = F_b(2) - F_b(1)$
$\int b = ((2)^3/3) - ((1)^3/3)$
$\int b = 7/3$

area = $\int a + \int b$
area = 3

47. $[0, \pi]: y = 1 + \cos x$
    π

48. $[\pi/6, 5\pi/6]: f(x) = \sin x$
    $F(x) = -\cos x$
    $\int = F(5\pi/6) - F(\pi/6)$
    $\int = (-\cos (5\pi/6)) - (-\cos (\pi/6))$
    $\int = \sqrt{3}$
    area = $\sqrt{3} - (2\pi/3 * f(\pi/6))$
    area = $\sqrt{3} - (2\pi/3 * \sin \pi/6)$
    area = $\sqrt{3} - \pi/3$
    Chapter 6 Review

Part 1

2.

Time	Rate
0	525
1	700
3	800
4	1050
6	1350
7	1500
9	2000
12	2800

a = 1(525+700)/2 + 2(800+1050)/2 + 1(1350+1500)/2 + 3(2000+2800)/2
a = 11087.5

Part 2

1. $\int[0, 1]\ (3x)\ dx$
   $F(x) = 3x^2/2$
   $\int = F(1) - F(0)$
   $\int = 1.5$

2. $\int[-2, 3]\ (x - 5)\ dx$
   $F(x) = x^2/2 - 5x$
   $\int = F(3) - F(-2)$
   $\int = -22.5$

3. $\int[-1, 4]\ (x^2 + 2x - 1)\ dx$
   $F(x) = x^3/3 + x^2 - x$
   $\int = F(4) - F(-1)$
   $\int = 31\ \&\ 2/3$

4. $\int[0, 2]\ (2x - 5)^2\ dx$
   $F(x) = 4x^3/3 - 10x^2 + 25x$
   $\int = F(2) - F(0)$
   $\int = 4\ \&\ 2/3$

5. $\int[2, 3]\ (4/x^2 + 1)\ dx$
   $F(x) = x - 4/x$
   $\int = F(3) - F(2)$
   $\int = 1\ \&\ 2/3$

6. $\int[-2, -1]\ (x - 1/x^2)\ dx$
   $F(x) = x^2/2 + 1/x$
   $\int = F(-1) - F(-2)$
   $\int = -2$

7. $\int[1, 9]\ ((x - 2) / (\sqrt{x}))\ dx$
   $F(x) = 2/3 * (x - 6) \sqrt{x}$
   $\int = F(9) - F(1)$
   $\int = 9\ \&\ 1/3$

8. $\int[-2, 2]\ ({}^3\sqrt{x})\ dx$
   $F(x) = (3x\ {}^3\sqrt{x}) / 4$
   $\int = F(2) - F(-2)$
   $\int = 0$

9. $\int[0, 1]\ (t^{(2/3)} - t^{(1/3)})\ dt$
   $F(x) = (3t^{(5/3)} / 5) - (3t^{(4/3)} / 4)$
   $\int = F(1) - F(0)$
   $\int = -0.15$

10. $\int[0, 3]\ (|x - 2|)\ dx$
    $F(x) = \ldots?$

11. $\int[-\pi/2, \pi/2]\ (\cos(x))\ dx$
    $F(x) = \sin(x)$
    $\int = F(\pi/2) - F(-\pi/2)$
    $\int = 2$

12. $\int[0, \pi] (2x - \sin(x)) dx$
    $F(x) = x^2 + \cos(x)$
    $\int = F(\pi) - F(0)$
    $\int = 2.5\pi$

13. $\int[0, \pi/2] (3\sin(x) - 2\cos(x)) dx$
    $F(x) = -2\sin(x) - 3\cos(x)$
    $\int = F(\pi/2) - F(0)$
    $\int = 1$

14. $\int[0, \pi/4] (x - \sec^2(x)) dx$
    $F(x) = 1/2 * (x^2 - 2\tan(x))$
    $\int = F(\pi/4) - F(0)$
    $\int = -0.692$

15. $\int[0, \pi/3] (\sec(\theta) \tan(\theta)) dx$
    $F(x) = \sec(\theta)$
    $\int = F(\pi/3) - F(0)$
    $\int = 1$
    Ch 7.1.2

Complete the tables to find the value of dy/dx at the points (x, y). Then create a slope field and show the approximate solution passing through the given point.

1. $dy/dx = y$, (1, 1)

x, y	-2	-1	0	1	2
2	2	2	2	2	2
1	1	1	1	1	1
0	0	0	0	0	0
-1	-1	-1	-1	-1	-1
-2	-2	-2	-2	-2	-2

x, y	-2	-1	0	1	2
2					2
1				1	
0	0	0	0		
-1					
-2					

2. $dy/dx = x + y$ (1, 1)

x, y	-2	-1	0	1	2
2	0	1	2	3	4
1	-1	0	1	2	3
0	-2	-1	0	1	2
-1	-3	-2	-1	0	1
-2	-4	-3	-2	-1	0

x, y	-2	-1	0	1	2
2					
1	-1			2	
0		-1			
-1			-1		
-2					

3. dy/dx = -x/y, (0, 2)

x, y	-2	-1	0	1	2
2	1	0.5	0	-0.5	-1
1	2	1	0	-1	-2
0	∞	∞	∞	∞	∞
-1	-2	-1	0	1	2
-2	-1	-0.5	0	0.5	1

x, y	-2	-1	0	1	2
2		0.5	0	-0.5	
1					
0					
-1					
-2					

Solve the following differential equations: On the starred problem, take the derivative of the answer and show it gives the original DEQ.

1. dy/dx = 2x / y => y dy = 2x dx

2*. dy/dx = y^2 => ?

3. dy/dx = (x + sin(x)) / 3y^2 => 3y^2 dy = x + sin(x) dx

4*. dy/dx = 4y => ?

5. dy/dx = ky => dy/y = k dx

6*. dy/dx = xy => dy/y = x dx

(I'm really not sure about these)
Ch 7.2.2 # 25-40

25. ∫ dx / (1 - x)^2
    u = 1 - x
    du = -dx

- ∫ 1 / u^2 du
- -1/u
  1/u
  1 / (1 - x)

26. ∫ sec^2(x + 2) dx
    u = x + 2
    du = dx

$\int \sec^2(u)\, du$
$\tan(u)$
$\tan(x + 2)$

27. $\int \sqrt{\tan x} \sec^2 x\, dx$
    $u = \tan x$
    $du = -\log(\cos(x))\, dx$
    $\int \sqrt{u} \sec^2 x\, dx$
    ...

28. $\int \sec(\theta + \pi/2) \tan(\theta + \pi/2)\, d\theta$
    $u = \theta + \pi/2$
    $du = d\theta$
    $\int \sec(u) \tan(u)\, du$
    $\sec(u)$
    $\sec(\theta + \pi/2)$

29. $\int \tan(4x + 2)\, dx$
    $u = 4x + 2$
    $du = 4\, du$
    $1/4 \int 4\tan(u)\, dx$
    $1/4 \int \tan(u)\, du$
    $1/4\ -\log(\cos(u))$
    $-\log(\cos(4x + 2))/4$

30. $\int 3(\sin x)^{-2}\, dx$
    $u = \sin(x)$
    $du = -\cos(x)\, dx$
    ...

31. $\int \cos(3z + 4)\, dz$
    $u = 3z + 4$
    $du = 3\, dx$
    $1/3 \int \cos(u)\, du$
    $1/3 \sin(u)$
    $1/3 \sin(3z + 4)$

32. $\int \sqrt{\cot x} \csc^2 x\, dx$
    $u = \cot(x)$
    $du = \log(\sin(x))\, dx$
    ...

33. $\int (\ln^6 x) / x\, dx$
    $u = \ln^6(x)$
    ...

34. $\int \tan^7(x/2) \sec^2(x/2)\, dx$
    (I'm not sure how to do these...)

35. $\int s^{1/3} \cos(s^{4/3} - 8)\, ds$

36. $\int dx / (\sin^2 3x)$

37. $\int (\sin(2t + 1)) / (\cos^2(2t + 1))\, dt$

38. $\int (6 \cos t) / (2 + \sin t)^2 \, dt$

39. $\int dx / (x \ln x)$

40. $\int \tan^2 x \sec^2 x \, dx$
    Ch 7.2.3

Inner and outer functions

1. $\int (2x + 5) / \sqrt{x^2 + 5x + 1} \, dx$
   outside: $\sqrt{x}$
   inside: $x^2 + 5x + 1$

2. $\int dx / (\sqrt{x} (1 + \sqrt{x})^3)$
   outside: x^3
   inside: $1 + \sqrt{x}$

3. $\int (x^3 + x) (x^4 + 2x^2 + 7)^{(3/4)} \, dx$
   outside: $x^{(3/4)}$
   inside: $x^4 + 2x^2 + 7$

4. $\int (x \, dx) / \sqrt{x + 4}$
   outside: $\sqrt{x}$
   inside: $x + 4$

5. $\int x^3 (x^2 + 1)^9 \, dx$
   outside: x^9
   inside: $x^2 + 1$

Applying procedures for u-substitution

1. $\int (2x + 5) / \sqrt{x^2 + 5x + 1} \, dx$
   inside: $x^2 + 5x + 1$
   du: $2x + 5 \, dx$
   int: $\int 1 / \sqrt{u} \, du$

2. $\int dx / (\sqrt{x} (1 + \sqrt{x})^3)$
   inside: $1 + \sqrt{x}$
   du: $1/\sqrt{x} \, dx$
   int: $\int 1 / (u)^3 \, du$

3. $\int (x^3 + x) (x^4 + 2x^2 + 7)^{(3/4)} \, dx$
   inside: $x^4 + 2x^2 + 7$
   du: $4x^3 + 4x \, dx$
   int: $1/4 \int u^{(3/4)} \, du$

4. $\int (x \, dx) / \sqrt{x + 4}$
   inside: $x + 4$
   du: dx
   int: ?

5. $\int x^3 (x^2 + 1)^9 \, dx$
   inside: $x^2 + 1$
   du: $2x$
   int: $1/2 \int x^2 (u)^9 \, du$?

Finding antiderivatives and definite integrals

1. $\int \sqrt{x + 1}\, dx$
   $u = x + 1$
   $du = dx$
   $\int \sqrt{u}\, du$
   $(2 u^{(3/2)}) / 3$
   $(2 (x + 1)^{(3/2)}) / 3$

2. $\int 2x \sqrt{x^2 + 1}\, dx$
   $u = x^2 + 1$
   $du = 2x\, dx$
   $\int \sqrt{u}\, du$
   $(2 u^{(3/2)}) / 3$
   $(2 (x^2 + 1)^{(3/2)}) / 3$

3. $\int x^2 (x^3 - 1)^7\, dx$
   $u = x^3 - 1$
   $du = 3x^2\, dx$
   $1/3 \int (u)^7\, du$
   $1/3\ u^8/8$
   $1/3\ (x^3 - 1)^8/8$

4. $\int (x^2 + 2) / (x^3 + 6x + 1)^3\, dx$
   $u = x^3 + 6x + 1$
   $du = 3x^2 + 6\, dx$
   $1/3 \int 1 / u^3\, du$
   $-1 / 6u^2$
   $-1 / 6(x^3 + 6x + 1)^2$

5. $\int x^3 \sqrt[3]{x^2 + 4}\, dx$
   $u = x^2 + 4$
   $du = 2x\, dx$
   $1/2 \int x^2 \sqrt[3]{u}\, du$
   $3/8\ u^{(4/3)} x^2$
   $3/8\ (x^2 + 4)^{(4/3)} x^2$
   Ch 7.2

6. $\int (3x - 2)^4\, dx$
   $u = 3x - 2$
   $du = 3\, dx$
   $1/3 \int u^4\, du$
   $1/3\ u^5 / 5$
   $u^5 / 15$
   $1/15\ (3x - 2)^5$

7. $\int \sqrt{5x + 4}\, dx$
   $u = 5x + 4$
   $du = 5\, dx$
   $1/5 \int 5 \sqrt{u}\, dx$
   $1/5 \int \sqrt{u}\, du$
   $1/5\ 1 / (2 \sqrt{u})$
   $1 / (10 \sqrt{5x + 4})$

8. ∫ 4(6x - 1)^(2/3) dx
   u = 6x - 1
   du = 6 dx
   2/3 ∫ 6u^(2/3) dx
   2/3 ∫ u^(2/3) du
   2/3 2 / (3 ^3√(u))
   4 / (9 ^3√(u))
   4 / (9 ^3√(6x - 1))

9. ∫ x√(x^2 - 2) dx
   u = x^2 - 2
   du = 2x dx
   1/2 ∫ 2x√(u) dx
   1/2 ∫ √(u) du
   1/2 1 / (2 √(u))
   1 / (4 √(u))
   1 / (4 √(x^2 - 2))

10. ∫ x^2 √(1 - 4x^3) dx
    u = 1 - 4x^3
    du = 12x^2 dx
    1/12 ∫ 12x^2 √(u) dx
    1/12 ∫ √(u) du
    1/12 1 / (2 √(u))
    1 / (24 √(1 - 4x^3))

11. ∫ x / (^3√(2x^2 - 1)) dx
    u = 2x^2 - 1
    du = 4x dx
    1/4 ∫ 4x / (^3√(2x^2 - 1)) dx
    1/4 ∫ 1 / (^3√(u)) du
    1/4 3u / (2 ^3√(u))
    3u / (8 ^3√(u))
    3(2x^2 - 1) / (8 ^3√(2x^2 - 1))
    (6x^2 - 3) / (8 ^3√(2x^2 - 1))

12. ∫ x^(1/2) (x^(3/2) + 4)^9 dx
    u = x^(3/2) + 4
    du = 3√(x) / 2 dx
    ...

13. ∫ (x + 2) √(x^2 + 4x - 5) dx
    u = x^2 + 4x - 5
    du = 2x + 4 dx
    1/2 ∫ 2x + 4 √(u) dx
    1/2 ∫ √(u) du
    1/2 1 / 2√(u)
    1 / 4√(x^2 + 4x - 5)

14. ∫ x - √(3x) dx
    u = 3x
    du = 3 dx
    ...

15. $\int \sqrt{x^2 - 1}\, dx$
...
Ch 7 # 22, 24, 27, 28, 31, 32

22. An isotope of neptunium has a half life of 65 minutes. If the decay of Np-240 is modeled by the differential equation dy/dt = -ky, where t is measured in minutes, what is the decay constant k?

65 min = ln(2) / k
ln(2) / 65 min = k
k = 0.01066

24. A colony of bacteria is grown under ideal conditions in a laboratory so that the population increases exponentially with time. At the end of 3 hours there are 10,000 bacteria. At the end of 5 hours there are 40,000 bacteria. How many bacteria were present initially?

10k = y_0e^(k * 3)
40k = y_0e^(k * 5)

27. y = y_0e^(kt); (0, 2), (2, 5)
2 = y_0e^(k * 0)
2 = y_0

5 = y_0e^(k * 2)
5 = 2e^(k * 2)
5/2 = e^(k * 2)
ln(5/2) = k * 2
ln(5/2)/2 = k

y = 2e^(k * 0.458145)

28. y = y_0e^(kt); (-3, 3), (0, 1.1)
1.1 = y_0e^(k * 0)
1.1 = y_0

3 = y_0e^(k * -3)
3 = 1.1e^(k * -3)
3/1.1 = e^(k * -3)
ln(3/1.1) = k * -3
ln(3/1.1)/-3 = k

y = 1.1e^(k * -0.334434)

31. Suppose that a cup of soup cooled from 90°C to 60°C in 10 min in a room whose temperature was 20°C. Use Newton's law of cooling to answer the following questions.

T(t) = T_s + (T_0 – T_s) e^-kt
T(t) = 20°C + (90°C – 20°C) e^-kt
T(t) = 20°C + 70°C e^-kt

60°C = 20°C + 70°C e^(-k * 10)
40°C = 70°C e^(-k * 10)
4/7 = e^(-k * 10)

$\ln(4/7) = -10k$
$k = \ln(7/4) / 10$
$k = 0.055962$

a. How much longer would it take the soup to cool to 35°C?
27.527 min (solved with calculator)

b. Instead of being left to stand in the room, the cup of 90°C soup is put into a freezer whose temperature is -15°C. How long will it take the soup to cool from 90°C to 35°C?
13.258 min (solved with calculator)

32. The temperature of an ingot of silver is 60°C above room temperature right now. Twenty minutes ago, it was 70°C above room temperature. How far above room temperature will the silver be:

$60°C = 25°C + 45°C\, e^{(-k \cdot 20)}$
$35°C = 45°C\, e^{(-k \cdot 20)}$
$7/9 = e^{(-k \cdot 20)}$
$\ln(7/9) = -20k$
$k = \ln(9/7) / 20$
$k = 0.012566$

a. 15 minutes from now
53.987°C

b. 2 hours from now
32.748°C

c. When will the silver be 10°C above room temperature?
119.694 min
Ch 8.1.2 worksheet

$\int_{a}^{b} f'(x)\,dx = f(b) - f(a)$
$f(a) + \int_{a}^{b} f'(x)\,dx = f(b)$

Part 1: For each of the following situations, write the definite integral that will determine the requested quantity. Note: Do not evaluate the definite integrals yet.

1. There are 30 gallons of pollutant in a lake at t = 0. If the number of gallons, P(t) of the pollutant changes at the rate $P(t) = 1 - 3e^{(-0.3\sqrt{t})}$ gallons per day, where t is measured in days. Write a definite integral expression to determine the amount of pollutant in the lake after 12 days.
   $P(12) = 30 + \int_{0}^{12} (1 - 3e^{(-0.3\sqrt{t})})\,dt$

2. Water is leaking from a tank at a rate of $\sqrt{(t + 1)}$ gallons per minute. Ten minutes after the leak is discovered, there are 60 gallons of water in the tank. How much water was in the tank when the leak was initially discovered?
   $f(0) = 60 - \int_{0}^{10} \sqrt{(t + 1)}\,dt$

3. A pie is removed from an oven and, after sitting for 10 minutes, is at a temperature of 220 degrees Fahrenheit. The pie cools at a rate of $3\ln(x^2)$ degrees Fahrenheit per minute. What is the expected temperature of the pie 15 minutes after it was removed from the oven?
   $f(15) = 220 + \int_{10}^{15} (3\ln(x^2))\,dx$

4. Throughout the day, people have entered a craft fair at the rate of $E(t) = 30t^2 + 16t$ people per hour and have exited at a rate given by $L(t) = 12t - 3$ people per hour. If t is measured in hours since the fair opened, and there were 200 people at the fair at the end of the first hour, how many people should be at the fair 3 hours after it opened?
   $f(1) = 200 + \int_{1}^{3} (30t^2 + 4t + 3)\,dt$

Part 2: Some definite integrals can be evaluated by hand and others should be evaluated using the definite integral feature of your calculator. Two of the problems in Part I can be worked by hand, and two require the use of the calculator. Determine which methods will work for each of the four problems and then work each problem to determine the numerical answer.

Part 3: Read the problem situation and then match the integral expressions in the left column to their interpretations in the right column.

Situation 1
A pipeline company manufactures pipe that sells for $100 per meter. The cost of manufacturing a portion of the pipe varies with its distance from the beginning of the pipe. The company reports that the cost to produce a portion of the pipe that is x meters from the beginning of the pipe is C(x) dollars per meter. (Note: Profit is defined as the difference between the amount of money received by the company for selling the pipe and the amount it costs to manufacture the pipe.)

a. The difference in sales price, in dollars, between 100 meters of pipe and 50 meters of pipe
6

b. The cost, in dollars, of manufacturing 125 meters of pipe
1

c. The average profit, in dollars per meter, made on the sale of 125 meters of pipe
2

d. The difference in the cost, in dollars, of manufacturing 100 meters of pipe and 50 meters of pipe
4

e. The average rate of change, in dollars per meter per meter, in the cost per meter of pipe for 125 meters of pipe
3

f. The difference in profit, in dollars, between selling 100 meters of pipe and 50 meters of pipe
7

g. The average difference in the cost per meter, in dollars per meter, between manufacturing 100 meters of pipe and 50 meters of pipe
5

Situation 2
A cup of boiling water is removed from a microwave and set on the counter to cool. The initial temperature of the water is 100 degrees Celsius. The temperature of the water T, measured in degrees Celsius, as a function of time t, measured in minutes, is given by T(t).

a. An equation that can be solved to find k, the time in minutes, when the average temperature of the water is 85 degrees Celsius.
7

b. The rate of change in the temperature of the water, in degrees Celsius per minute, at t = 4 minutes
1

c. The average temperature of the water, in degrees Celsius, between t = 0 and t = 4 minutes.
3

d. The difference in the temperature of the water, in degrees Celsius, between t = 0 and t = 4 minutes
5

e. The temperature of the water, in degrees Celsius, at t = 4 minutes
6

f. The average rate of change in the temperature of the water, in degrees Celsius
2

g. The temperature of the water, in degrees Celsius, at t = k minutes.
Not sure, definitely missed a different one.

h. An equation that can be solved to determine the time k, in minutes, when the average temperature of the water, in degrees Celsius, was 85, over the time interval from t = 0 to t = k minutes8

Situation C
At 3 pm, when the box office opens, there are 75 people waiting in line to buy tickets for a new movie. The rate that the box office sells tickets and moves people out of the line is given by S(t), measured in people per hour. While tickets are being sold, more people are arriving and joining the line at a rate given by L(t), also measured in people per hour.

a. The number of people to whom tickets have been sold between 3 pm and 5 pm
2

b. The number of people standing in line at 5 pm
3

c. The total number of people who have stood in line beginning at 3 pm and ending at 5 pm
5

d. The change in the number of people in the line between 3 pm and 5 pm
1

e. The number of people who have arrived to join the line between 3 pm and 5 pm
4
Ch 8.2.2 # 1-15

Find the area of the region under the curves

1. $y = 1, y = \cos^2(x); [0, \pi]$
   $\int[0, \pi] (1 - \cos^2(x))\, dx$
   $(1 - 1/2(x + \sin(x)\cos(x)))\ [0, \pi]$
   $1 - 1/2\ \pi$

2. $y = 1/2 \sec^2(x), y = -4\sin^2(x); [-\pi/3, \pi/3]$
   $\int[-\pi/3, \pi/3] (1/2 \sec^2(x) - -4\sin^2(x))\, dx$
   $(\tan(x)/2 - (\sin(2x) - 2x))\ [-\pi/3, \pi/3]$

$(\tan(\pi/3)/2 - (\sin(2(\pi/3))) - 2(\pi/3)) - (\tan(-\pi/3)/2 - (\sin(2(-\pi/3)) - 2(-\pi/3)))$

3. $x = y^2, x = y^3; [0, 1]$
   $\int[0, 1] (y^2 - y^3)\, dy$

4. $x = 12y^2 - 12y^3, x = 2y^2 - 2y; [0, 1]$
   $\int[0, 1] (12y^2 - 12y^3 - (2y^2 - 2y))\, dy$

5. $y = 2x^2, y = x^4 - 2x^2; [-2, 2]$
   $\int[-2, 2] (2x^2 - (x^4 - 2x^2))\, dx$

6. $y = x^2, y = -2x^4; [-1, 1]$
   $\int[-1, 1] (x^2 - -2x^4)\, dx$

7. $y = \sin(x), y = 1 - x^2;$

8. $y = \cos(2x), y = x^2 - 2;$

9. $y = x, y = 1, y = x^2/4; [0, 2]$
   $\int[0, 2] (x - 1 - x^2/4)\, dx$

10. $y = x^2, x + y = 2; [0, 2]$
    $\int[0, 2] (x^2 - (2 - x))\, dx$

11. $y^2 = x + 1, y^2 = 3 - x;$

12. $y^2 = x + 3, y = 2x;$

13. $y = 2x^3 - x^2 - 5x, y = -x^2 + 3x; [-2, 0]$ R $[0, 2]$
    $\int[-2, 0] (2x^3 - x^2 - 5x - (-x^2 + 3x))\, dx + \int[0, 2] (-x^2 + 3x - (2x^3 - x^2 - 5x))\, dx$

14. $y = -x + 2, y = 4 - x^2; [-2, -1]$ R $[-1, 2]$ R $[2, 3]$
    $\int[-2, -1] (-x + 2 - (4 - x^2))\, dx + \int[-1, 2] (4 - x^2 - (-x + 2))\, dx + \int[0, 2] (-x + 2 - (4 - x^2))\, dx$

15. $y = x^2 - 2, y = 2;$

In-class FRQs

3. The continuous function f is defined on the closed interval $-6 <= x <= 5$. The figure above shows a portion of the graph of f, consisting of two line segments and a quarter of a circle centered at the point (5, 3). It is known that the point $(3, 3 - \sqrt{5}\,)$ is on the graph of f.

a. If $\int[-6, 5] f(x)\, dx = 7$, find the value of $\int[-6, -2] f(x)\, dx$. Show the work that leads to your answer.

slope of $-2 <= x <= 0$: -1
height at x = -2: 1
height at x = -6: 5
average height: 3
x-distance: 4
$\int[-6, -2] f(x)\, dx = 12$

b. Evaluate $\int[3, 5] (2f'(x) + 4)\, dx$.

$\int[3, 5]$ $(2f'(x) + 4)\,dx = [2f(x) + 4x][3, 5]$
$(2f(5) + 4(5)) - (2f(3) + 4(3))$
$0 - ((6 - 2\sqrt{5}) + 12)$
-13.53

c. The function g is given by $g(x) = \int[-2, x]\ f(t)\,dt$. Find the absolute maximum value of g on the interval $-2 <= x <= 5$. Justify your answer.

d. find $\lim[x\text{->}1]\ (10^x - 3f'(x)) / (f(x) - \arctan(x))$

$(10^{(1)} - 3f'(1)) / (f(1) - \arctan(1))$
$(10 - 6) / (1 - \pi/4)$
$4 / (4 - \pi)/4$
$16 / (4 - \pi)$