Monte Carlo Methods in Finance and Econometrics

1. **Foundations**
   (a) Elementary principles of Monte Carlo simulation methods
      i. Monte Carlo integration (computing expected values)
      ii. Central Limit Theorem and Law of Large Numbers
      iii. Efficiency of Monte Carlo estimates
   (b) Elementary principles of option pricing
      i. Black-Scholes (1973) framework of option pricing
      ii. Risk-neutral valuation
      iii. Connection to Monte Carlo methods (why these methods are useful in finance and economics.)

2. **Random number generation/Generating sample paths**
   (a) General sampling methods
      i. Inverse transform method
      ii. Alias method for discrete distributions
      iii. Acceptance-Rejection method
   (b) Generating normal random variables
      i. Some basic properties
      ii. Univariate normals
      iii. Multivariate normals
   (c) Heavy-tailed distributions
   (d) Copulas
      i. Correlation and co-movement/co-dependence
      ii. Examples of copulas: Gaussian, t-Copula, and Archimedean
   (e) Discretization schemes
   (f) Common models for asset price movement
      i. Brownian motion, Geometric Brownian motion, Ornstein-Uhlenbeck process (CTAR(1) model).
      ii. Square-root processes (e.g., used in interest rate modeling)
      iii. Additional examples that are non-standard

3. **Pricing standard options using Monte Carlo methods**
   (a) European option pricing
   (b) Examples of vanilla options: call and put options
   (c) Assessing the efficiency of Monte Carlo estimates
   (d) **Variance reduction methods**

i. Antithetic methods
ii. Stratification
iii. Control variates (also nonlinear controls and the delta method)
iv. Importance sampling

4. **Introduction to exotic options**
   (a) Path-dependent options: Asian options
   (b) Digital options
   (c) Options on multiple assets
   (d) The role of Monte Carlo in the valuation of exotic options.

5. **Discretization schemes for stochastic differential equations**
   (a) Focus will be on Ito diffusions
   (b) Euler discretization scheme
   (c) Exact schemes (for a few diffusions where it would be applicable)
   (d) Higher-order schemes: Milstein
   (e) Studies of the discretization error when doing simulation

6. **Sensitivity analysis in options pricing**
   (a) Sensitivity of option prices to parameter inputs
   (b) Evaluating the "Greeks" and relevance to options hedging
   (c) Applications of Monte Carlo methods in this setting

7. **Quasi Monte Carlo and low-discrepancy sequences**
   (a) Alternatives to random number generation in Monte Carlo methods
   (b) Implications for faster convergence rates
   (c) Examples: Halton numbers, Sobol' numbers

8. **Stochastic volatility models**
   (a) Extension of the constant volatility setting
   (b) Incomplete market environment
   (c) Discussion of some seminal papers in this area
   (d) Monte Carlo methods for stochastic volatility models

9. **American-style options**
   (a) Early-exercise features
   (b) Optimal stopping problem
   (c) The dynamic programming framework

(d) Monte Carlo methods for pricing American options (e.g., Longstaff and Schwartz (2001) least-squares Monte Carlo method)

10. **The role of particle filtering methods in econometrics**

    (a) Latent processes in diffusion models

    (b) Parameter estimation and statistical inference

    (c) Review the Bootstrap Filter Approach of Doucet et al (2001).

11. **Risk management**

    (a) Measuring risk in a portfolio of assets

    (b) Value at Risk (VaR) calculations

    (c) Applications of Monte Carlo and variance reduction techniques

12. **Current research areas**

    (a) Credit risk

    (b) Jump processes and other non-standard models: the case of electricity

    (c) Computational issues: advent of parallel processing

    (d) Statistical arbitrage

    (e) Nonparametric methods in finance and econometrics

Course Summary

This course will cover the use of Monte Carlo methodology in finance and econometrics. The emphasis will be on computational methods. The student must have familiarity with a package/language such as R, S-Plus, Matlab, or C/C++. We will cover standard applications of Monte Carlo methods to price derivative securities on assets. We will also explore several variance reduction techniques that improve the efficiency of Monte Carlo estimates. The valuation of exotic options and early-exercise options will be covered. Extensions to standard models used in financial time series will also be discussed with emphasis given to stochastic volatility models. The use of particle filtering methods will also be discussed in the stochastic volatility setting as a tool to aid statistical analysis and inference for non-linear and non-Gaussian time series. There will also be some discussion of risk management concepts and current research questions in this field.

The prerequisite for this course is STA 213 or a course that offers an introduction to advanced concepts in probability and statistical theory.