

# Adaptively Secure, Universally Composable, Multi-Party Computation in Constant Rounds

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## Abstract

Cryptographic protocols with *adaptive security* ensure that security holds against an adversary who can dynamically determine which parties to corrupt as the protocol progresses—or even after the protocol is finished. In the setting where all parties may potentially be corrupted, and secure erasure is not assumed, it has been a long-standing open question to design secure-computation protocols with adaptive security running in *constant* rounds.

Here, we show a constant-round, universally composable protocol for computing any functionality, tolerating a malicious, adaptive adversary corrupting any number of parties. Interestingly, our protocol can compute *all* functionalities, not just adaptively well-formed ones.

## 1 Introduction

When designing and analyzing protocols for secure computation, there are several different adversarial models one can consider. The original definitions of security assume a *static* adversary who decides which parties to corrupt before execution of the protocol begins. Subsequently [3, 9], researchers began to consider the more challenging setting in which the adversary may *adaptively* decide which parties to corrupt as the protocol progresses—or even after the protocol ends. It is easy to come up with examples of protocols that are secure in a static-corruption model, but that are trivially insecure in the adaptive setting.

Even in a setting where adaptive corruptions are considered, there are different assumptions one can make. Initial work on adaptive security [3] made the assumption that honest parties can securely *erase* local data (e.g., randomness or other internal state) when no longer needed. Later work, led by Canetti et al. [9], sought to avoid this assumption, arguing that it is unwise to rely on other parties to erase data (since there is no way such erasure can be verified) and that it is generally difficult—even for an honest party who intends to erase data—to ensure that all traces of data are gone. Whether or not erasure is assumed has a significant impact on the complexity of adaptively secure protocols; for example, adaptively secure public-key encryption is fairly simple and efficient [3] if erasure is assumed, but much more complicated (and much less efficient) [9, 2, 15, 13] without this assumption. Similarly, adaptively secure two-party computation is much easier with the assumption of secure erasure [26] than without [11].

Designing protocols without the assumption of secure erasure is difficult, in part, due to the need to deal with *post-execution corruption* (PEC), whereby an adversary can corrupt parties

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(and hence obtain the randomness they used) even after execution of the protocol has concluded. Handling PEC is inherent to the setting of universal composability (UC) [7], and is important for ensuring sequential composition even in the stand-alone setting [6]. If secure erasure is assumed, the definition of adaptive security does not change whether or not PEC is allowed [8], but without erasure the requirement of dealing with PEC adds significant additional complications.

**Prior work.** We are interested in adaptive security, with PEC, in a model where secure erasure is not assumed. Some prior protocols for secure computation in this setting (e.g., [9, 2]) assume a majority of the parties remain uncorrupted. Other work [24, 23, 18, 21], including concurrent work of [16], allows *all but one* of the parties to be corrupted. While it may seem strange to worry about corruption of *all* parties, consideration of this case is important when a protocol  $\Pi_{\text{outer}}$  invokes some protocol  $\Pi_{\text{inner}}$  (not involving all parties running  $\Pi_{\text{outer}}$ ) as a subroutine. In this case, all parties running  $\Pi_{\text{inner}}$  may eventually be corrupted, and security of  $\Pi_{\text{outer}}$  should still be guaranteed.

To the best of our knowledge, all prior work giving adaptively secure protocols for general functionalities (without erasure), and tolerating an arbitrary number of corruptions, are based on the Goldreich-Micali-Wigderson [19] paradigm for semi-honest computation, and thus have round complexity linear in the depth of the circuit being computed. These include protocols in the common reference string model [11], the “sunspots” model [12], the key-registration model [1], and, more generally, based on adaptively secure UC puzzles [14]. In addition, all prior work in this setting handles only “adaptively well-formed functionalities” (see [11] for a definition).

## 1.1 Our Result

We show a constant-round, universally composable protocol for multi-party computation of arbitrary functionalities, with security against a malicious, adaptive adversary corrupting any number of parties. Once again, we stress that we do not assume secure erasure.

**Overview of our techniques.** The main difficulty in our setting is to construct a constant-round protocol with security against a *semi-honest*, adaptive adversary corrupting any number of parties. Given any such protocol, we can compile it as in [11] to obtain a universally composable protocol with security against a *malicious*, adaptive adversary, and still running in constant rounds. We may also assume secure channels, which can be implemented using adaptively secure encryption.

Our protocol in the semi-honest setting relies on a common reference string (CRS). While it would be more elegant to avoid this assumption, note that a CRS—or some other form of setup—is anyway needed [10] in order to obtain universally composable computation in the presence of *malicious* adversaries corrupting half or more of the parties, even in a static-corruption model. Thus, as far as our final result (i.e., our protocol with security in the malicious setting) is concerned, some form of setup is unavoidable. We remark further that results of Garg and Sahai [18] indicate that a CRS (or some other form of setup) is needed to obtain constant-round protocols with adaptive security even in the semi-honest case; see further discussion at the end of this section.

At its core, our protocol relies on the ability to make arbitrary algorithms *explainable*, an idea we explain in more detail now. Fix some randomized algorithm  $\text{Alg}$ . Informally, an *explainable* version of  $\text{Alg}$  is an algorithm  $\widetilde{\text{Alg}}$  along with an associated *explain* algorithm  $\text{Explain}$  such that, for any input, (1) the distributions over the outputs of  $\text{Alg}(\text{input})$  and  $\widetilde{\text{Alg}}(\text{input})$  are statistically close, and (2) choosing random coins  $r$ , computing  $\text{output} := \text{Alg}(\text{input}; r)$ , and outputting  $(\text{output}, r)$  is computationally indistinguishable from choosing random coins  $r$ , computing  $\text{output} := \widetilde{\text{Alg}}(\text{input}; r)$ ,

and then outputting  $(\text{output}, \widetilde{\text{Explain}}(\text{input}, \text{output}))$ . That is, the  $\widetilde{\text{Explain}}$  algorithm provides the ability to sample random coins for  $\widetilde{\text{Alg}}$  that “explain” any given input/output pair.

Sahai and Waters [27] introduced the notion of explainability for the specific case of public-key encryption schemes, in the context of constructing a deniable encryption scheme. We observe that their techniques can be suitably generalized to give an explainable version of *arbitrary* algorithms based on indistinguishability obfuscation for general circuits (and one-way functions). We refer the reader to Section 3 for a formal statement of this result.

Let  $C$  be a circuit taking  $n$ -bit inputs.<sup>1</sup> Consider the following functionality  $\text{NextMsg}$  that (essentially) computes the next-message function for a two-round secure-computation protocol for  $C$  based on garbled circuits:  $\text{NextMsg}$  takes as input a sequence of first-round messages  $\text{OT}_{1,1}, \dots, \text{OT}_{1,n}$  for a two-round, adaptively secure, oblivious-transfer (OT) protocol (e.g., the protocol of [11]); it then (1) computes a garbled circuit  $\text{GC}$  corresponding to  $C$ , along with input-wire labels  $\{(y_{i,0}, y_{i,1})\}_{i=1}^n$ , and (2) computes a sequence of OT responses  $\text{OT}_{2,1}, \dots, \text{OT}_{2,n}$ . (These responses allow the party that generated  $\text{OT}_{1,i}$  using input bit  $b$  to recover  $y_{i,b}$  while learning nothing about  $y_{i,1-b}$ .) The output of  $\text{NextMsg}$  is  $(\text{GC}, \text{OT}_{2,1}, \dots, \text{OT}_{2,n})$ . The CRS for our protocol will be  $\widetilde{\text{NextMsg}}$ , an explainable version of  $\text{NextMsg}$ .<sup>2</sup> We note that, in contrast to [27], in the real-world execution no parties have access to the  $\widetilde{\text{Explain}}$  algorithm corresponding to  $\widetilde{\text{NextMsg}}$ .

Our multi-party protocol computing  $C$  can now be described quite simply. The protocol proceeds in four rounds. Say we have  $n$  parties  $P_1, \dots, P_n$  holding inputs  $x_1, \dots, x_n$ , respectively. These parties generate first-round OT messages  $\text{OT}_{1,1}, \dots, \text{OT}_{1,n}$  (with the party who is supposed to provide the  $i$ th input generating  $\text{OT}_{1,i}$ ), and send these to  $P_n$ . Party  $P_n$  then runs  $\widetilde{\text{NextMsg}}(\text{OT}_{1,1}, \dots, \text{OT}_{1,n})$  to obtain  $\text{GC}, \text{OT}_{2,1}, \dots, \text{OT}_{2,n}$ , and sends  $\text{OT}_{2,i}$  to the corresponding party (which might be itself). Each party  $P_i$  then locally recovers  $y_i$ , the label for the  $i$ th input wire of the garbled circuit, and sends  $y_i$  to  $P_n$ . Finally,  $P_n$  evaluates the garbled circuit  $\text{GC}$  using the provided input-wire labels to obtain the output  $z$ , and sends  $z$  to all the other parties.<sup>3</sup> Only the third- and fourth-round messages need to be sent via a secure channel.

We now describe the simulator informally. Our simulator begins by generating  $\widetilde{\text{NextMsg}}$  along with its associated  $\widetilde{\text{Explain}}$  algorithm, and letting  $\widetilde{\text{NextMsg}}$  be the CRS. It simulates  $\text{OT}_{1,1}, \dots, \text{OT}_{1,n}$  and  $\text{OT}_{2,1}, \dots, \text{OT}_{2,n}$  using the simulator for the OT protocol (recall the OT protocol is adaptively secure), and uses these for the first two rounds of the protocol. Upon corruption of party  $P_i$ , the simulator corrupts that party in the ideal world and learns its input  $x_i$  and the output  $z$ . Then:

- If this is the first corruption, the simulator generates a simulated garbled circuit  $\text{GC}$  consistent with output  $z$ , along with  $n$  input-wire labels  $y_1, \dots, y_n$ . It also uses the  $\widetilde{\text{Explain}}$  algorithm to generate random coins  $r^*$  consistent with running  $\widetilde{\text{NextMsg}}$  on input  $\text{OT}_{1,1}, \dots, \text{OT}_{1,n}$  and obtaining output  $\text{GC}, \text{OT}_{2,1}, \dots, \text{OT}_{2,n}$ .
- The simulator uses the simulator for the OT protocol to generate internal state for  $P_i$  consistent with input  $x_i$  and output  $y_i$ , and returns this to the adversary. In addition, if  $P = P_n$  then it returns  $r^*$  to the adversary.

<sup>1</sup>We assume for simplicity here that  $C$  is deterministic. Randomized functionalities are handled in Section 4.

<sup>2</sup>As described, the CRS depends on the circuit  $C$ . However, by taking  $C$  to be a universal circuit, the CRS can be fixed independently of the “actual” function the parties wish to compute.

<sup>3</sup>As described, all parties learn the output of the computation. Standard techniques can be used to handle the general case in which each party learns a possibly different function of the inputs.

**Impossibility results?** We briefly mention two impossibility results regarding (constant-round) adaptively secure computation, and explain why they do not apply in our setting.

First, our protocol can compute *arbitrary* randomized functionalities, not just *adaptively well-formed* ones. (We refer to [11] for a definition of this term.) This may seem somewhat surprising in light of an impossibility result of Ishai et al. [22] showing that adaptively secure computation of *all* functionalities (and not just well-formed ones) is impossible. A closer examination of their result, however, reveals that it does *not* hold in the CRS model.<sup>4</sup>

Second, Garg and Sahai [18] show that no constant-round, adaptively secure, multi-party protocol can be proven secure using black-box techniques; although they only claim this result for protocols with security against malicious adversaries, their proof appears to extend to the case of semi-honest adversaries as well. Their impossibility result, though, explicitly only applies to the “plain” model where no setup is assumed, whereas in our work we assume a CRS.

## 1.2 Organization of the Paper

We review some standard cryptographic background and primitives in Section 2. In Section 3, we introduce the notion of an *explainable* algorithm, and show how the Sahai-Waters compiler [27] can be used to make any algorithm explainable. Finally, in Section 4 we present a constant-round multi-party computation protocol tolerating a semi-honest, adaptive adversary corrupting any number of parties. Applying the compiler of Canetti et al. [11] yields a constant-round protocol tolerating a *malicious*, adaptive adversary corrupting any number of parties.

## 2 Preliminaries

We let  $\lambda$  denote the security parameter. We refer to previous work [6, 8, 26] for definitions of secure computation in the adaptive-corruption setting (with PEC).

### 2.1 Garbled Circuits

We rely on the standard notion of garbled circuits [28]. However, we use slightly non-standard notation that we introduce here. Let  $C$  be a randomized circuit taking  $n$ -bit inputs and using  $\lambda$  bits of randomness. We abstract the construction/evaluation of a garbled circuit for  $C$  via algorithms  $\text{GenGC}$ ,  $\text{EvalGC}$  with the following properties.  $\text{GenGC}$  is a randomized algorithm that takes as input  $1^\lambda$  and  $C$ , and outputs a garbled circuit  $\text{GC}$  along with  $2n$  input-wire labels  $y_{1,0}, y_{1,1}, \dots, y_{n,0}, y_{n,1} \in \{0,1\}^\lambda$  and  $2\lambda$  random-wire labels  $w_{1,0}, w_{1,1}, \dots, w_{\lambda,0}, w_{\lambda,1} \in \{0,1\}^\lambda$ . Deterministic algorithm  $\text{EvalGC}$  takes as input  $\text{GC}$  and  $n + \lambda$  labels  $y_1, \dots, y_n, w_1, \dots, w_\lambda$ , and outputs a value  $z$ .

Correctness requires that for any  $\text{GC}$ ,  $(\{y_{i,0}, y_{i,1}\}_{i=1}^n, \{w_{i,0}, w_{i,1}\}_{i=1}^\lambda)$  output by  $\text{GenGC}(1^\lambda, C)$ , any  $x \in \{0,1\}^n$  and any  $r \in \{0,1\}^\lambda$ , we have

$$\text{EvalGC}\left(\text{GC}, \{y_{i,x_i}\}_{i=1}^n, \{w_{i,r_i}\}_{i=1}^\lambda\right) = C(x; r).$$

Security requires an efficient simulator  $\text{SimGC}$  such that for all  $x, r$ , the distribution

$$\left\{ \left( \text{GC}, \{(y_{i,0}, y_{i,1})\}_{i=1}^n, \{(w_{i,0}, w_{i,1})\}_{i=1}^\lambda \right) \leftarrow \text{GenGC}(1^\lambda, C) : \left( \text{GC}, \{y_{i,x_i}\}_{i=1}^n, \{w_{i,r_i}\}_{i=1}^\lambda \right) \right\}$$

is computationally indistinguishable from the output of  $\text{SimGC}(1^\lambda, C, C(x; r))$ .

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<sup>4</sup>Although Ishai et al. claim that their result holds in the CRS model, they only provide a proof in the plain model and their proof seems to break down in the CRS model.

## 2.2 Adaptively Secure Oblivious Transfer

Our protocol uses a two-round, semi-honest, adaptively secure OT protocol as a building block. A suitable construction can be found in [11].

A two-round OT protocol  $\Pi_{\text{OT}}$  comprises three algorithms: a receiver algorithm  $R_{\text{OT}}$ , a sender algorithm  $S_{\text{OT}}$ , and an evaluation algorithm  $E_{\text{OT}}$ . Algorithm  $R_{\text{OT}}$  takes as input a bit  $b$  and random coins  $r_R$ , and outputs initial message  $\text{OT}_1$ . Algorithm  $S_{\text{OT}}$  takes as input an initial message  $\text{OT}_1$ , a pair of  $\lambda$ -bit strings  $(y_0, y_1)$ , and randomness  $r_S$ , and outputs message  $\text{OT}_2$ . The evaluation algorithm  $E_{\text{OT}}$  takes as input  $b, r_R$ , and  $\text{OT}_2$  and outputs the  $\lambda$ -bit string  $y_b$ .

For our purposes we require the following property that is implied by semi-honest, adaptive security of  $\Pi_{\text{OT}}$ . There exists an efficient simulator  $\text{SimOT} = (\text{SimOT}_1, \text{SimOT}_2)$ , where  $\text{SimOT}_2$  is deterministic, such that (1)  $\text{SimOT}_1$  outputs a transcript  $(\text{OT}_1, \text{OT}_2)$  along with state  $\text{st}$  and (2)  $\text{SimOT}_2$ , given as input  $b, y$ , and  $\text{st}$ , outputs coins  $r_R$  for the receiver consistent with  $(\text{OT}_1, \text{OT}_2)$  and the receiver holding input  $b$  and obtaining output  $y$ ; for any  $b, y_0, y_1$ , the distribution

$$\{r_R, r_S \leftarrow \{0, 1\}^*; \text{OT}_1 := R_{\text{OT}}(b; r_R) : (r_R, \text{OT}_1, S_{\text{OT}}(\text{OT}_1, y_0, y_1; r_S))\}$$

is computationally indistinguishable from

$$\left\{ \begin{array}{l} (\text{OT}_1, \text{OT}_2, \text{st}) \leftarrow \text{SimOT}_1(1^\lambda); \\ r_R := \text{SimOT}_2(1^\lambda, b, y_b, \text{st}) \end{array} : (r_R, \text{OT}_1, \text{OT}_2) \right\}.$$

That is, we only require “one-sided security” [21] for adaptive corruption of the receiver.

If we define algorithm  $\text{SimOT}'_1(1^\lambda)$  to run  $\text{SimOT}_1(1^\lambda)$  and output only  $(\text{OT}_1, \text{st})$ , and define the algorithm  $\text{SimOT}'_2(1^\lambda, b, \text{st})$  to simply run  $\text{SimOT}_2(1^\lambda, b, 0^\lambda, \text{st})$ , then for any  $b$  the distribution  $\{r_R \leftarrow \{0, 1\}^* : (r_R, R_{\text{OT}}(b; r_R))\}$  is computationally indistinguishable from

$$\left\{ \begin{array}{l} (\text{OT}_1, \text{st}) \leftarrow \text{SimOT}'_1(1^\lambda); \\ r_R := \text{SimOT}'_2(1^\lambda, b, \text{st}) \end{array} : (r_R, \text{OT}_1) \right\}.$$

## 2.3 Indistinguishability Obfuscation

We use an indistinguishability obfuscator as a building block. A PPT machine  $\text{iO}$  is an *indistinguishability obfuscator* for a circuit class  $\{\mathcal{C}_\lambda\}$  if the following conditions are satisfied:

**Correctness.** For all  $\lambda$ , and all  $C \in \mathcal{C}_\lambda$ , it holds that  $C$  and  $\text{iO}(1^\lambda, C)$  compute the same function.

**Polynomial slowdown.** There is a polynomial  $p(\cdot)$  such that  $|\text{iO}(1^\lambda, C)| \leq p(\lambda) \cdot |C|$  for all  $C \in \mathcal{C}_\lambda$ .

**Indistinguishability.** For any sequence  $\{(C_{\lambda,0}, C_{\lambda,1}, \text{aux}_\lambda)\}_\lambda$  where  $C_{\lambda,0}, C_{\lambda,1} \in \mathcal{C}_\lambda$ ,  $C_{\lambda,0} \equiv C_{\lambda,1}$ , and  $|C_{\lambda,0}| = |C_{\lambda,1}|$ , and any PPT distinguisher  $D$ , there is a negligible function  $\text{negl}$  such that:

$$\left| \Pr[D(\text{iO}(1^\lambda, C_{\lambda,0}), \text{aux}_\lambda) = 1] - \Pr[D(\text{iO}(1^\lambda, C_{\lambda,1}), \text{aux}_\lambda) = 1] \right| \leq \text{negl}(\lambda).$$

When clear from the context, we will often omit the security parameter  $1^\lambda$  as an input to  $\text{iO}$  and as a subscript for  $C$ .

$\text{iO}$  is an *indistinguishability obfuscator* for  $\text{P/poly}$  if there is a polynomial  $p$  such that  $\text{iO}$  is an indistinguishability obfuscator for  $\{\mathcal{C}_\lambda\}$ , where  $\mathcal{C}_\lambda$  contains all circuits of size at most  $p(\lambda)$ . Garg et al. [17] have shown the first candidate construction of indistinguishability obfuscators for  $\text{P/poly}$ .

### 3 Explainability Compilers

Sahai and Waters [27] define a notion of *explainability* for public-key encryption, and show a compiler that transforms any public-key encryption scheme into an explainable version. Here, we generalize the notion of explainability for an *arbitrary* algorithm  $\text{Alg}$ , and show that the Sahai-Waters compiler can be used to transform any algorithm  $\text{Alg}$  into an explainable version  $\widetilde{\text{Alg}}$ .

At a high level, an explainability compiler takes as input (a description of) a randomized algorithm  $\text{Alg}$ , and outputs two algorithms  $\widetilde{\text{Alg}}, \text{Explain}$ . The first of these is a randomized algorithm computing the same functionality as  $\text{Alg}$ . The second algorithm, roughly speaking, takes an input/output pair  $\text{input}, \text{output}$  and produces random coins  $r$  consistent with running  $\text{Alg}(\text{input})$  and obtaining the result  $\text{output}$ . That is, the algorithm “explains” the input/output pair  $\text{input}, \text{output}$ . We now give a formal definition.

**Definition 1.** *A PPT algorithm  $\text{Comp}$  is an explainability compiler if for every efficient, randomized circuit  $\text{Alg}$ , the following hold:*

**Polynomial slowdown.** *There is a polynomial  $p(\cdot)$  such that, for any  $(\widetilde{\text{Alg}}, \text{Explain})$  output by  $\text{Comp}(1^\lambda, \text{Alg})$  it holds that  $|\widetilde{\text{Alg}}| \leq p(\lambda) \cdot |\text{Alg}|$ .*

**Statistical functional equivalence.** *With overwhelming probability over choice of  $(\widetilde{\text{Alg}}, \star)$  as output by  $\text{Comp}(1^\lambda, \text{Alg})$  the distribution of  $\widetilde{\text{Alg}}(\text{input})$  is statistically close to the distribution of  $\text{Alg}(\text{input})$  for all  $\text{input}$ .*

**Explainability.** *The success probability of every non-uniform, polynomial-time adversary  $\mathcal{A}$  in the following experiment is negligibly close to  $1/2$ :*

1.  $\mathcal{A}(1^\lambda)$  outputs  $\text{input}^*$  of its choice.
2.  $\text{Comp}(1^\lambda, \text{Alg})$  is run to obtain  $(\widetilde{\text{Alg}}, \text{Explain})$ .
3. Choose uniform coins  $r_0 \in \{0, 1\}^*$  and compute  $\text{output}^* := \widetilde{\text{Alg}}(\text{input}^*; r_0)$ .
4. Compute  $r_1 \leftarrow \text{Explain}(\text{input}^*, \text{output}^*)$ .
5. Choose a uniform bit  $b$  and give  $\widetilde{\text{Alg}}, \text{output}^*, r_b$  to  $\mathcal{A}$ .
6.  $\mathcal{A}$  outputs a bit  $b'$ , and succeeds if  $b' = b$ .

We highlight one key difference between our definition and the corresponding one from [27]: in our case  $\text{input}^*$  is an arbitrary length value (depending on the domain of  $\text{Alg}$ ) chosen by the adversary, whereas in [27] the input to the explainable algorithm is a *single bit* chosen uniformly (and given to the adversary). Because of this, and due to the way the explainability compiler is constructed, we require the adversary to choose  $\text{input}^*$  “non-adaptively,” i.e., before being given  $\widetilde{\text{Alg}}$ . This definition of explainability suffices for our eventual protocol.

#### 3.1 Constructing an Explainability Compiler

Following [27], we now show how to construct an explainability compiler. As in [27], we rely on an indistinguishability obfuscator,  $\text{iO}$ , for P/poly and three different pseudorandom function (PRF) variants (cf. Appendix A):

$\overline{\text{Alg}}$

**Hardwired constants:** Keys  $K_1$ ,  $K_2$ , and  $K_3$ .

**Input:** Input  $\text{input}$  and randomness  $u = (u[1], u[2])$ .

1. Let  $\text{input}', \text{output}', r' := F_3(K_3, u[1]) \oplus u[2]$ . If it is the case that  $\text{input} = \text{input}'$  and  $u[1] = F_2(K_2, (\text{input}', \text{output}', r'))$ , then output  $\text{output} := \text{output}'$  and end.
2. Else let  $x := F_1(K_1, (\text{input}, u))$  and output  $\text{output} := \text{Alg}(\text{input}; x)$  and end.

Figure 1: Program  $\overline{\text{Alg}}$

- A *puncturable, extracting* PRF  $F_1(K_1, \cdot)$  that accepts inputs of length  $\ell_1 + \ell_2 + \ell_{\text{in}}$ , and outputs strings of length  $\ell_r$ . It is extracting when the input min-entropy is greater than  $\ell_r + 2\lambda + 4$ , with statistical closeness less than  $2^{-(\lambda+1)}$ . Observe that  $\ell_{\text{in}} + \ell_1 + \ell_2 \geq \ell_r + 2\lambda + 4$ , and thus if one-way functions exist then such a PRF exists by Theorem 4.
- A *puncturable, statistically injective* PRF  $F_2(K_2, \cdot)$  that accepts inputs of length  $2\lambda + \ell_{\text{in}} + \ell_{\text{out}}$ , and outputs strings of length  $\ell_1$ . Observe that  $\ell_1 \geq 2 \cdot (2\lambda + \ell_{\text{in}} + \ell_{\text{out}}) + \lambda$ , and thus if one-way functions exist then such a PRF exists by Theorem 3.
- A *puncturable* PRF  $F_3(K_3, \cdot)$  that accepts inputs of length  $\ell_1$  and outputs strings of length  $\ell_2$ . If one-way functions exist, then such a PRF exists by Theorem 2.

We define  $\text{Comp}(1^\lambda, \text{Alg})$  as follows. Let  $\text{Alg} : \{0, 1\}^{\ell_{\text{in}}} \times \{0, 1\}^{\ell_r} \rightarrow \{0, 1\}^{\ell_{\text{out}}}$  be an algorithm with domain  $\{0, 1\}^{\ell_{\text{in}}}$ , range  $\{0, 1\}^{\ell_{\text{out}}}$ , and randomness length  $\ell_r$ . Our compiled program  $\overline{\text{Alg}}$  will take input  $\text{input} \in \{0, 1\}^{\ell_{\text{in}}}$  and randomness  $u = (u[1], u[2])$  of length  $\ell_1 + \ell_2$ , where  $|u[1]| = \ell_1 = 5\lambda + 2(\ell_{\text{in}} + \ell_{\text{out}}) + \ell_r$  and  $|u[2]| = \ell_2 = 2\lambda + \ell_{\text{in}} + \ell_{\text{out}}$ . Our compiler first samples keys  $K_1$ ,  $K_2$ , and  $K_3$  for PRFs  $F_1$ ,  $F_2$ , and  $F_3$ , respectively. It then defines algorithms  $\overline{\text{Alg}}$  and  $\overline{\text{Explain}}$  as in Figures 1 and 2, respectively. Finally, it computes  $\widetilde{\text{Alg}} \leftarrow \text{iO}(\overline{\text{Alg}})$  and  $\widetilde{\text{Explain}} \leftarrow \text{iO}(\overline{\text{Explain}})$ , and outputs  $(\widetilde{\text{Alg}}, \widetilde{\text{Explain}})$ .

The proofs of security for our compiler, given for completeness in Appendix B, follow closely along the lines of the analogous proofs in [27]. Specifically, the proof of statistical functional equivalence closely follows the proof used by Sahai and Waters to establish IND-CPA security of their deniable encryption scheme, and the proof of explainability follows the Sahai-Waters proof establishing explainability of their deniable encryption scheme. We highlight, however, that in our proof of explainability a difference arises because in our case the input  $\text{input}^*$  is an arbitrary length value (depending on the domain of  $\text{Alg}$ ), whereas in [27] the input is just a single bit. We are able to adapt the proof to this case because we do not allow  $\text{input}^*$  to depend on  $\overline{\text{Alg}}$ .

$\overline{\text{Explain}}$

**Hardwired constants:** Keys  $K_2$  and  $K_3$ .

**Input:** input, output, and randomness  $r \in \{0, 1\}^\lambda$ .

1. Set  $\alpha := F_2(K_2, (\text{input}, \text{output}, \text{PRG}(r)))$  and let  $\beta := F_3(K_3, \alpha) \oplus (\text{input}, \text{output}, \text{PRG}(r))$ . Output  $(\alpha, \beta)$ .

Figure 2: Program  $\overline{\text{Explain}}$

---

NextMsg

**Inputs:**  $\text{OT}_{1,1}, \dots, \text{OT}_{1,n}$ ; randomness  $r_1, \dots, r_\lambda \in \{0, 1\}$  and  $r_{\text{GC}}, r_{S,1}, \dots, r_{S,n} \in \{0, 1\}^*$ .

1. Run  $\text{GenGC}(1^\lambda, C; r_{\text{GC}})$  to produce the garbled circuit  $\text{GC}$  along with  $n$  pairs of input-wire labels  $\{(y_{i,0}, y_{i,1})\}_{i=1}^n$  and  $\lambda$  pairs of random-wire labels  $\{(w_{i,0}, w_{i,1})\}_{i=1}^\lambda$ .
2. For  $i \in [n]$ , run  $S$



### Semi-Honest, Adaptively Secure Multi-Party Computation

**Common input:**

- $CRS = \widetilde{\text{NextMsg}}$ .
- Description of a randomized circuit  $C$ .

**Private inputs:** Every party  $P_i$  has private input  $x_i \in \{0, 1\}$ .

[Each  $P_i$ :] *Compute first-round OT messages:*

- Sample random coins  $r_{R,i} \leftarrow \{0, 1\}^*$  of appropriate length.
- Compute  $\text{OT}_{1,i} := R_{\text{OT}}(x_i; r_{R,i})$  and, for  $i \in [n-1]$ , send  $\text{OT}_{1,i}$  to  $P_n$ .

[ $P_n$ :] *Compute garbled circuit and second-round OT messages:*

- Sample random coins  $r_n \leftarrow \{0, 1\}^*$  of appropriate length.
- Compute  $(\text{GC}, \text{OT}_{2,1}, \dots, \text{OT}_{2,n}, w_1, \dots, w_\lambda) := \widetilde{\text{NextMsg}}(\text{OT}_{1,1}, \dots, \text{OT}_{1,n}; r_n)$ .
- For  $i \in [n-1]$ , send  $\text{OT}_{2,i}$  to  $P_i$ .

[Each  $P_i$ :] *Recover OT output:*

- Compute  $y_i := E_{\text{OT}}(x_i, r_{R,i}, \text{OT}_{2,i})$  and, for  $i \in [n-1]$ , send  $y_i$  to  $P_n$  over a secure channel.

[ $P_n$ :] *Evaluate garbled circuit and broadcast output:*

- Compute  $z := \text{EvalGC}(\text{GC}, \{y_i\}_{i=1}^n, \{w_i\}_{i=1}^\lambda)$ .
- For  $i \in [n-1]$ , send  $z$  to  $P_i$  over a secure channel.

**Output:** Each party  $P_i$  outputs  $z$ .

Figure 4: Protocol  $\Pi$  for computing randomized circuit  $C$ .

- If this is the first party to be corrupted, compute  $(\text{GC}, \{y_i\}_{i=1}^n, \{w_i\}_{i=1}^\lambda) \leftarrow \text{SimGC}(1^\lambda, C, z)$  and  $r_n \leftarrow \text{Explain}((\text{OT}_{1,1}, \dots, \text{OT}_{1,n}), (\text{GC}, \text{OT}_{2,1}, \dots, \text{OT}_{2,n}, w_1, \dots, w_n))$ . Store these values to be used, as needed, in the rest of the simulation.
- In any case, compute  $r_{R,i} := \text{SimOT}_2(1^\lambda, x_i, y_i, \text{st}_i)$  and give  $x_i, z, y_i$ , and  $r_{R,i}$  to  $\mathcal{Z}$ . In addition, if  $i = n$  give  $\{y_i\}_{i=1}^{n-1}$  and  $r_n^*$  to  $\mathcal{Z}$ .

4. Output whatever  $\mathcal{Z}$  outputs.

We prove that the output of  $\mathcal{Z}$  when interacting with  $\mathcal{A}$  and parties in a real-world execution of protocol  $\Pi$  is indistinguishable from the output of  $\mathcal{Z}$  when interacting with  $\text{Sim}$  and the functionality  $C$  in an ideal-world execution of the protocol. We do so by considering a sequence of hybrid experiments, beginning with the real-world execution and ending with the ideal-world execution, and showing that each experiment is computationally indistinguishable from the preceding one.

**Hybrid 0.** This corresponds to the real-world execution of the protocol. We write the experiment in a format convenient for the proof. This experiment proceeds via the following steps:

1. Compute  $(\widetilde{\text{NextMsg}}, \text{Explain}) \leftarrow \text{Comp}(1^\lambda, \text{NextMsg})$ , and give  $\widetilde{\text{NextMsg}}$  to  $\mathcal{Z}$  as the CRS.  $\mathcal{Z}$  chooses inputs  $x_1, \dots, x_n$ .
2. For  $i \in [n]$ , sample coins  $r_{R,i}$  and compute  $\text{OT}_{1,i} := R_{\text{OT}}(x_i; r_{R,i})$ . Give the sequence of values  $\text{OT}_{1,1}, \dots, \text{OT}_{1,n-1}$  to  $\mathcal{Z}$  as the first-round message.
3. Sample coins  $r_n$  and compute

$$(\text{GC}, \text{OT}_{2,1}, \dots, \text{OT}_{2,n}, w_1, \dots, w_\lambda) := \widetilde{\text{NextMsg}}(\text{OT}_{1,1}, \dots, \text{OT}_{1,n}; r_n).$$

Give  $\text{OT}_{2,1}, \dots, \text{OT}_{2,n-1}$  to  $\mathcal{Z}$  as the second-round message.

4. When  $\mathcal{Z}$  requests to corrupt party  $P_i$ , compute  $y_i := E_{\text{OT}}(x_i, r_{R,i}, \text{OT}_{2,i})$  and give  $x_i, z, y_i$ , and  $r_{R,i}$  to  $\mathcal{Z}$ . In addition, if  $i = n$  then compute  $y_i := E_{\text{OT}}(x_i, r_{R,i}, \text{OT}_{2,i})$  for  $i \in [n-1]$  and give  $\{y_i\}_{i=1}^{n-1}$  and  $r_n$  to  $\mathcal{Z}$ .

**Hybrid 1.** This experiment is similar to the previous one, except that the  $\text{OT}_1$  messages and the random coins  $\{r_{R,i}\}$  are generated by the simulator for the OT protocol (cf. Section 2.2). That is, the experiment proceeds via the following steps:

1. Compute  $(\widetilde{\text{NextMsg}}, \text{Explain}) \leftarrow \text{Comp}(1^\lambda, \text{NextMsg})$ , and give  $\widetilde{\text{NextMsg}}$  to  $\mathcal{Z}$  as the CRS.  $\mathcal{Z}$  chooses inputs  $x_1, \dots, x_n$ .
2. Run  $\text{SimOT}'_1(1^\lambda)$  a total of  $n$  times to obtain  $\{(\text{OT}_{1,i}, \text{st}_i)\}_{i=1}^n$ . Give the sequence of values  $\text{OT}_{1,1}, \dots, \text{OT}_{1,n-1}$  to  $\mathcal{Z}$  as the first-round message.
3. Sample coins  $r_n$  and compute

$$(\text{GC}, \text{OT}_{2,1}, \dots, \text{OT}_{2,n}, w_1, \dots, w_\lambda) := \widetilde{\text{NextMsg}}(\text{OT}_{1,1}, \dots, \text{OT}_{1,n}; r_n).$$

Give  $\text{OT}_{2,1}, \dots, \text{OT}_{2,n-1}$  to  $\mathcal{Z}$  as the second-round message.

4. When  $\mathcal{Z}$  corrupts party  $P_i$ , compute  $r_{R,i} := \text{SimOT}'_2(1^\lambda, x_i, \text{st}_i)$  and  $y_i := E_{\text{OT}}(x_i, r_{R,i}, \text{OT}_{2,i})$ , and give  $x_i, z, y_i$ , and  $r_{R,i}$  to  $\mathcal{Z}$ . In addition, if  $i = n$  then for  $i \in [n-1]$  compute  $r_{R,i} := \text{SimOT}'_2(1^\lambda, x_i, \text{st}_i)$  and  $y_i := E_{\text{OT}}(x_i, r_{R,i}, \text{OT}_{2,i})$ , and give  $\{y_i\}_{i=1}^{n-1}$  and  $r_n$  to  $\mathcal{Z}$ .

It follows immediately by security of the OT protocol (and a straightforward hybrid argument) that this experiment is computationally indistinguishable from the previous one.

**Hybrid 2.** This experiment is similar to the previous one, except that we now use the **Explain** algorithm to generate the random coins  $r_n$ . That is, the experiment proceeds as follow:

1. Compute  $(\widetilde{\text{NextMsg}}, \text{Explain}) \leftarrow \text{Comp}(1^\lambda, \text{NextMsg})$ , and give  $\widetilde{\text{NextMsg}}$  to  $\mathcal{Z}$  as the CRS.  $\mathcal{Z}$  chooses inputs  $x_1, \dots, x_n$ .
2. Run  $\text{SimOT}'_1(1^\lambda)$  a total of  $n$  times to obtain  $\{(\text{OT}_{1,i}, \text{st}_i)\}_{i=1}^n$ . Give the sequence of values  $\text{OT}_{1,1}, \dots, \text{OT}_{1,n-1}$  to  $\mathcal{Z}$  as the first-round message.

3. Sample coins  $r_n$  and compute

$$(\text{GC}, \text{OT}_{2,1}, \dots, \text{OT}_{2,n}, w_1, \dots, w_\lambda) := \widetilde{\text{NextMsg}}(\text{OT}_{1,1}, \dots, \text{OT}_{1,n}; r_n).$$

In addition, let  $\text{input}^* = (\text{OT}_{1,1}, \dots, \text{OT}_{1,n})$  and  $\text{output}^* = (\text{GC}, \text{OT}_{2,1}, \dots, \text{OT}_{2,n}, w_1, \dots, w_\lambda)$ , and compute  $r^* \leftarrow \text{Explain}(\text{input}^*, \text{output}^*)$ .

Give  $\text{OT}_{2,1}, \dots, \text{OT}_{2,n-1}$  to  $\mathcal{Z}$  as the second-round message.

4. When  $\mathcal{Z}$  corrupts party  $P_i$ , compute  $r_{R,i} := \text{SimOT}'_2(1^\lambda, x_i, \text{st}_i)$  and  $y_i := E_{\text{OT}}(x_i, r_{R,i}, \text{OT}_{2,i})$ , and give  $x_i, z, y_i$ , and  $r_{R,i}$  to  $\mathcal{Z}$ . In addition, if  $i = n$  then for  $i \in [n-1]$  compute  $r_{R,i} := \text{SimOT}'_2(1^\lambda, x_i, \text{st}_i)$  and  $y_i := E_{\text{OT}}(x_i, r_{R,i}, \text{OT}_{2,i})$ , and give  $\{y_i\}_{i=1}^{n-1}$  and  $r_n^*$  to  $\mathcal{Z}$ .

Computational indistinguishability of this experiment from the previous one follows from the definition of explainability (cf. Definition 1), and the fact that **Comp** is an explainability compiler. To see this, say there is an efficient adversary  $\mathcal{Z}$  and a non-uniform, polynomial-time distinguisher  $D$  that distinguishes the outcome of Hybrid 1 from that of Hybrid 2. We show how to use this to construct an attacker  $\mathcal{A}'$  violating explainability.  $\mathcal{A}'$  works as follows: it runs  $\text{SimOT}'_1(1^\lambda)$  a total of  $n$  times to obtain  $\{(\text{OT}_{1,i}, \text{st}_i)\}_{i=1}^n$ , and outputs the value  $\text{input}^* = (\text{OT}_{1,1}, \dots, \text{OT}_{1,n})$ . Given  $\widetilde{\text{NextMsg}}, \text{output}^*, r$  in response, where  $\text{output}^* = (\text{GC}, \text{OT}_{2,1}, \dots, \text{OT}_{2,n}, w_1, \dots, w_\lambda)$ , it then does:

1. Give  $\widetilde{\text{NextMsg}}$  to  $\mathcal{Z}$  as the CRS.  $\mathcal{Z}$  chooses inputs  $x_1, \dots, x_n$ .
2. Give  $\text{OT}_{1,1}, \dots, \text{OT}_{1,n-1}$  to  $\mathcal{Z}$  as the first-round message, and  $\text{OT}_{2,1}, \dots, \text{OT}_{2,n-1}$  to  $\mathcal{Z}$  as the second-round message.
3. When  $\mathcal{Z}$  corrupts party  $P_i$ , compute  $r_{R,i} := \text{SimOT}'_2(1^\lambda, x_i, \text{st}_i)$  and  $y_i := E_{\text{OT}}(x_i, r_{R,i}, \text{OT}_{2,i})$ , and give  $x_i, z, y_i$ , and  $r_{R,i}$  to  $\mathcal{Z}$ . In addition, if  $i = n$  then for  $i \in [n-1]$  compute  $r_{R,i} := \text{SimOT}'_2(1^\lambda, x_i, \text{st}_i)$  and  $y_i := E_{\text{OT}}(x_i, r_{R,i}, \text{OT}_{2,i})$ , and give  $\{y_i\}_{i=1}^{n-1}$  and  $r$  to  $\mathcal{Z}$ .

Finally, run  $D$  on the output of  $\mathcal{Z}$  and output the result. It is easy to see that if the coins  $r$  are those used to run  $\widetilde{\text{NextMsg}}$ , then the view of  $\mathcal{Z}$  when run as a subroutine by  $\mathcal{A}'$  corresponds to Hybrid 1, whereas if the coins  $r$  are those output by **Explain**, then the view of  $\mathcal{Z}$  when run as a subroutine by  $\mathcal{A}'$  corresponds to Hybrid 2. Indistinguishability of the two experiments follows.

**Hybrid 3.** This is similar to the previous experiment, except that **NextMsg** and **Explain** are used in place of  $\widetilde{\text{NextMsg}}$ . That is, the experiment proceeds as follows:

1. Compute  $(\widetilde{\text{NextMsg}}, \text{Explain}) \leftarrow \text{Comp}(1^\lambda, \text{NextMsg})$ , and give  $\widetilde{\text{NextMsg}}$  to  $\mathcal{Z}$  as the CRS.  $\mathcal{Z}$  chooses inputs  $x_1, \dots, x_n$ .
2. Run  $\text{SimOT}'_1(1^\lambda)$  a total of  $n$  times to obtain  $\{(\text{OT}_{1,i}, \text{st}_i)\}_{i=1}^n$ . Give the sequence of values  $\text{OT}_{1,1}, \dots, \text{OT}_{1,n-1}$  to  $\mathcal{Z}$  as the first-round message.
3. Compute

$$(\text{GC}, \text{OT}_{2,1}, \dots, \text{OT}_{2,n}, w_1, \dots, w_\lambda) \leftarrow \text{NextMsg}(\text{OT}_{1,1}, \dots, \text{OT}_{1,n}).$$

In addition, let  $\text{input}^* = (\text{OT}_{1,1}, \dots, \text{OT}_{1,n})$  and  $\text{output}^* = (\text{GC}, \text{OT}_{2,1}, \dots, \text{OT}_{2,n}, w_1, \dots, w_\lambda)$ , and compute  $r^* \leftarrow \text{Explain}(\text{input}^*, \text{output}^*)$ .

Give  $\text{OT}_{2,1}, \dots, \text{OT}_{2,n-1}$  to  $\mathcal{Z}$  as the second-round message.

4. When  $\mathcal{Z}$  corrupts party  $P_i$ , compute  $r_{R,i} := \text{SimOT}'_2(1^\lambda, x_i, \text{st}_i)$  and  $y_i := E_{\text{OT}}(x_i, r_{R,i}, \text{OT}_{2,i})$ , and give  $x_i, z, y_i$ , and  $r_{R,i}$  to  $\mathcal{Z}$ . In addition, if  $i = n$  then for  $i \in [n-1]$  compute  $r_{R,i} := \text{SimOT}'_2(1^\lambda, x_i, \text{st}_i)$  and  $y_i := E_{\text{OT}}(x_i, r_{R,i}, \text{OT}_{2,i})$ , and give  $\{y_i\}_{i=1}^{n-1}$  and  $r_n^*$  to  $\mathcal{Z}$ .

Indistinguishability of this experiment from the previous one follows by statistical equivalence of  $\text{NextMsg}$  and  $\widetilde{\text{NextMsg}}$ .

**Hybrid 4.** In this experiment, we first make explicit the steps of  $\text{NextMsg}$ . (This is just a syntactic rewriting, and does not affect the experiment.) In addition, we now set  $y_i = y_{i,x_i}$  instead of computing  $y_i$  using the OT-evaluation algorithm  $E_{\text{OT}}$ . This experiment proceeds as follows:

1. Compute  $(\widetilde{\text{NextMsg}}, \text{Explain}) \leftarrow \text{Comp}(1^\lambda, \text{NextMsg})$ , and give  $\widetilde{\text{NextMsg}}$  to  $\mathcal{Z}$  as the CRS.  $\mathcal{Z}$  chooses inputs  $x_1, \dots, x_n$ .
2. Run  $\text{SimOT}'_1(1^\lambda)$  a total of  $n$  times to obtain  $\{(\text{OT}_{1,i}, \text{st}_i)\}_{i=1}^n$ . Give the sequence of values  $\text{OT}_{1,1}, \dots, \text{OT}_{1,n-1}$  to  $\mathcal{Z}$  as the first-round message.
3. Compute  $(\text{GC}, \{(y_{i,0}, y_{i,1})\}_{i=1}^n, \{(w_{i,0}, w_{i,1})\}_{i=1}^\lambda) \leftarrow \text{GenGC}(1^\lambda, C)$  and set  $y_i = y_{i,x_i}$  for all  $i$ . For  $i \in [n]$ , run  $\text{OT}_{2,i} \leftarrow S_{\text{OT}}(\text{OT}_{1,i}, y_{i,0}, y_{i,1})$ . Choose uniform  $r_1, \dots, r_\lambda \in \{0, 1\}$ , and let  $\text{input}^* = (\text{OT}_{1,1}, \dots, \text{OT}_{1,n})$  and  $\text{output}^* = (\text{GC}, \text{OT}_{2,1}, \dots, \text{OT}_{2,n}, w_{r_1}, \dots, w_{r_\lambda})$ . Compute  $r^* \leftarrow \text{Explain}(\text{input}^*, \text{output}^*)$ .  
Give  $\text{OT}_{2,1}, \dots, \text{OT}_{2,n-1}$  to  $\mathcal{Z}$  as the second-round message.
4. When  $\mathcal{Z}$  corrupts party  $P_i$ , compute  $r_{R,i} := \text{SimOT}'_2(1^\lambda, x_i, \text{st}_i)$ . Give  $x_i, z, y_i$ , and  $r_{R,i}$  to  $\mathcal{Z}$ . In addition, if  $i = n$  then give  $\{y_i\}_{i=1}^{n-1}$  and  $r_n^*$  to  $\mathcal{Z}$ .

Computational indistinguishability of this experiment from the previous one follows from security of the OT protocol.

**Hybrid 5.** In the previous experiment the  $\text{OT}_2$  messages were generated honestly as part of  $\text{NextMsg}$ . Here, we have the OT simulator output them instead. That is, we now do:

1. Compute  $(\widetilde{\text{NextMsg}}, \text{Explain}) \leftarrow \text{Comp}(1^\lambda, \text{NextMsg})$ , and give  $\widetilde{\text{NextMsg}}$  to  $\mathcal{Z}$  as the CRS.  $\mathcal{Z}$  chooses inputs  $x_1, \dots, x_n$ .
2. Run  $\text{SimOT}_1(1^\lambda)$  a total of  $n$  times to obtain  $\{(\text{OT}_{1,i}, \text{OT}_{2,i}, \text{st}_i)\}_{i=1}^n$ . Give the sequence of values  $\text{OT}_{1,1}, \dots, \text{OT}_{1,n-1}$  to  $\mathcal{Z}$  as the first-round message, and give  $\text{OT}_{2,1}, \dots, \text{OT}_{2,n-1}$  to  $\mathcal{Z}$  as the second-round message.
3. Compute  $(\text{GC}, \{(y_{i,0}, y_{i,1})\}_{i=1}^n, \{(w_{i,0}, w_{i,1})\}_{i=1}^\lambda) \leftarrow \text{GenGC}(1^\lambda, C)$  and set  $y_i = y_{i,x_i}$  for all  $i$ . Choose uniform values  $r_1, \dots, r_\lambda \in \{0, 1\}$ , and let  $\text{input}^* = (\text{OT}_{1,1}, \dots, \text{OT}_{1,n})$  and  $\text{output}^* = (\text{GC}, \text{OT}_{2,1}, \dots, \text{OT}_{2,n}, w_{r_1}, \dots, w_{r_\lambda})$ . Compute  $r^* \leftarrow \text{Explain}(\text{input}^*, \text{output}^*)$ .
4. When  $\mathcal{Z}$  corrupts party  $P_i$ , compute  $r_{R,i} := \text{SimOT}_2(1^\lambda, x_i, y_i, \text{st}_i)$ . Give  $x_i, z, y_i$ , and  $r_{R,i}$  to  $\mathcal{Z}$ . In addition, if  $i = n$  then give  $\{y_i\}_{i=1}^{n-1}$  and  $r_n^*$  to  $\mathcal{Z}$ .

Again, computational indistinguishability between this experiment and the previous one follows by security of the OT protocol.

**Hybrid 6.** Here we use the garbled-circuit simulator (cf. Section 2.1) instead of the garbled-circuit generation algorithm. Thus, the experiment now proceeds as follows:

1. Compute  $(\widetilde{\text{NextMsg}}, \text{Explain}) \leftarrow \text{Comp}(1^\lambda, \text{NextMsg})$ , and give  $\widetilde{\text{NextMsg}}$  to  $\mathcal{Z}$  as the CRS.  $\mathcal{Z}$  chooses inputs  $x_1, \dots, x_n$ .
2. Run  $\text{SimOT}_1(1^\lambda)$  a total of  $n$  times to obtain  $\{(\text{OT}_{1,i}, \text{OT}_{2,i}, \text{st}_i)\}_{i=1}^n$ . Give  $\text{OT}_{1,1}, \dots, \text{OT}_{1,n-1}$  to  $\mathcal{Z}$  as the first-round message, and  $\text{OT}_{2,1}, \dots, \text{OT}_{2,n-1}$  to  $\mathcal{Z}$  as the second-round message.
3. Compute  $(\text{GC}, \{y_i\}_{i=1}^n, \{w_i\}_{i=1}^\lambda) \leftarrow \text{SimGC}(1^\lambda, C, z)$ . Let  $\text{input}^* = (\text{OT}_{1,1}, \dots, \text{OT}_{1,n})$  and  $\text{output}^* = (\text{GC}, \text{OT}_{2,1}, \dots, \text{OT}_{2,n}, w_{r_1}, \dots, w_{r_\lambda})$ . Compute  $r^* \leftarrow \text{Explain}(\text{input}^*, \text{output}^*)$ .
4. When  $\mathcal{Z}$  corrupts party  $P_i$ , compute  $r_{R,i} := \text{SimOT}_2(1^\lambda, x_i, y_i, \text{st}_i)$ . Give  $x_i, z, y_i$ , and  $r_{R,i}$  to  $\mathcal{Z}$ . In addition, if  $i = n$  then for  $i \in [n-1]$  give  $\{y_i\}_{i=1}^{n-1}$  and  $r_n^*$  to  $\mathcal{Z}$ .

Computational indistinguishability between this experiment and the previous one follows from security of garbled circuits.

We conclude the proof by noting that Hybrid 6 is simply a syntactic rewriting of the ideal-world execution involving the simulator originally defined.  $\blacksquare$

## 5 Conclusions and Open Questions

In this work we have shown the first constant-round, universally composable protocol tolerating a malicious, adaptive adversary that can corrupt any number of parties, in a setting where secure erasure is not assumed. In addition, we have shown the first adaptively secure protocol, regardless of round complexity, that can compute arbitrary functionalities (and not only adaptively well-formed ones) in the presence of any number of corruptions and without erasures.

Several interesting open questions remain. Although a CRS (or some other form of setup) is necessary if we wish to obtain a universally composable protocol with security against malicious adversaries corrupting an arbitrary number of parties, it is still possible that the CRS can be avoided in the semi-honest case, or in the stand-alone setting. Moreover, our protocol assumes that the CRS depends on the circuit  $C$  being computed or, if we let  $C$  be a universal circuit (cf. footnote 2), an a priori bound on the size of the circuit being computed. It would be interesting to see if this can be avoided.

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## A Puncturable PRFs

Puncturable PRFs are a type of constrained PRF [4, 5, 25] whereby it is possible to generate a key that defines the function everywhere except on some polynomial-size set of inputs:

**Definition 2.** A puncturable family of PRFs is defined by polynomials  $n(\cdot)$  and  $m(\cdot)$  and a triple of Turing machines  $\text{Key}_F$ ,  $\text{Puncture}_F$ , and  $\text{Eval}_F$ , and satisfying the following conditions:

**Functionality preserved under puncturing.** For all polynomial-size sets  $S \subseteq \{0,1\}^{n(\lambda)}$  and all  $x \in \{0,1\}^{n(\lambda)} \setminus S$ , we have:

$$\Pr [K \leftarrow \text{Key}_F(1^\lambda), K_S = \text{Puncture}_F(K, S) : \text{Eval}_F(K, x) = \text{Eval}_F(K_S, x)] = 1.$$

**Pseudorandom at punctured points.** For every PPT adversary  $(A_1, A_2)$  such that  $A_1(1^\lambda)$  outputs a set  $S \subseteq \{0,1\}^{n(\lambda)}$  and state  $\sigma$ , consider an experiment where  $K \leftarrow \text{Key}_F(1^\lambda)$  and  $K_S = \text{Puncture}_F(K, S)$ . Then we have

$$\left| \Pr [A_2(\sigma, K_S, S, \text{Eval}_F(K, S)) = 1] - \Pr [A_2(\sigma, K_S, S, U_{m(\lambda) \cdot |S|}) = 1] \right| \leq \text{negl}(\lambda)$$

where  $\text{Eval}_F(K, S)$  denotes the concatenation of  $\text{Eval}_F(K, x_1), \dots, \text{Eval}_F(K, x_k)$ , and  $S = \{x_1, \dots, x_k\}$  is an enumeration of the elements of  $S$  in lexicographic order.

For ease of notation, we write  $F(K, x)$  to represent  $\text{Eval}_F(K, x)$ . We also represent the punctured key  $\text{Puncture}_F(K, S)$  by  $K(S)$ .

As observed by [4, 5, 25], the GGM construction [20] of PRFs from one-way functions yields puncturable PRFs. Thus:

**Theorem 2.** [4, 5, 25] If one-way functions exist, then for all polynomials  $n(\lambda)$  and  $m(\lambda)$  there exists a puncturable PRF family that maps  $n(\lambda)$  bits to  $m(\lambda)$  bits.

We follow [27] for the following definitions of puncturable PRFs with enhanced properties:

**Definition 3.** A statistically injective (puncturable) PRF family with failure probability  $\epsilon(\cdot)$  is a family of (puncturable) PRFs  $F$  such that with probability  $1 - \epsilon(\lambda)$  over the random choice of key  $K \leftarrow \text{Key}_F(1^\lambda)$ , we have that  $F(K, \cdot)$  is injective.

**Definition 4.** An extracting (puncturable) PRF family with error  $\epsilon(\cdot)$  for min-entropy  $k(\cdot)$  is a family of (puncturable) PRFs  $F$  mapping  $n(\lambda)$  bits to  $m(\lambda)$  bits such that for all  $\lambda$ , if  $X$  is any distribution over  $n(\lambda)$  bits with min-entropy greater than  $k(\lambda)$ , then the statistical distance between  $(K \leftarrow \text{Key}_F(1^\lambda), F(K, X))$  and  $(K \leftarrow \text{Key}_F(1^\lambda), U_{m(\lambda)})$  is at most  $\epsilon(\lambda)$ .

The following results were proved in [27]:

**Theorem 3** ([27]). If one-way functions exist, then for all efficiently computable functions  $n(\lambda)$ ,  $m(\lambda)$ , and  $e(\lambda)$  such that  $m(\lambda) \geq 2n(\lambda) + e(\lambda)$ , there exists a puncturable statistically injective PRF family with failure probability  $2^{-e(\lambda)}$  that maps  $n(\lambda)$  bits to  $m(\lambda)$  bits.

**Theorem 4.** If one-way functions exist, then for all efficiently computable functions  $n(\lambda)$ ,  $m(\lambda)$ ,  $k(\lambda)$ , and  $e(\lambda)$  such that  $n(\lambda) \geq k(\lambda) \geq m(\lambda) + 2e(\lambda) + 2$ , there exists an extracting puncturable PRF family that maps  $n(\lambda)$  bits to  $m(\lambda)$  bits with error  $2^{-e(\lambda)}$  for min-entropy  $k(\lambda)$ .

## B Proof of Security for Our Explainability Compiler

In this section we prove security of our explainability compiler. We must show two properties: statistical functional equivalence and explainability. (Polynomial slowdown is obvious.) The proof of statistical functional equivalence is largely identical to the analogous proof in [27], so is omitted. Instead, we focus on explainability.

We first state the following lemma, whose proof is the same as in [27].



**Lemma 1.** *Except with negligible probability over the choice of key  $K_2$ , the following hold:*

1. *For any fixed  $u[1] = \alpha$ , there exists at most one pair  $(\text{input}, \beta)$  such that the input  $\text{input}$  with randomness  $u = (\alpha, \beta)$  will cause the Step 1 check of  $\widetilde{\text{Alg}}$  to be satisfied.*
2. *There are at most  $2^{2\lambda + \ell_{\text{in}} + \ell_{\text{out}}}$  values for the randomness  $u$  that can cause the Step 1 check of  $\widetilde{\text{Alg}}$  to be satisfied.*

Given the above, we prove:

**Theorem 5.** *If  $F_1, F_2, F_3$  are PRFs that satisfy the properties specified in Section 3.1, and  $\text{iO}$  is an indistinguishability obfuscator for  $\text{P/poly}$ , then our construction  $\text{Comp}(\cdot, \cdot)$  satisfies explainability.*

**Proof:** Recall the explainability game from Definition 1:

1.  $\mathcal{A}(1^\lambda)$  outputs  $\text{input}^*$  of its choice.
2.  $\text{Comp}(1^\lambda, \text{Alg})$  is run to obtain  $(\widetilde{\text{Alg}}, \text{Explain})$ .
3. Choose random coins  $r_0 \leftarrow \{0, 1\}^*$ , and compute  $\text{output}^* \leftarrow \widetilde{\text{Alg}}(\text{input}^*; r_0)$ .
4. Compute  $r_1 \leftarrow \text{Explain}(\text{input}^*, \text{output}^*)$ .
5. Choose a uniform bit  $b$  and give  $\widetilde{\text{Alg}}, \text{output}^*, r_b$  to  $\mathcal{A}$ .
6.  $\mathcal{A}$  outputs a bit  $b'$ , and succeeds if  $b' = b$ .

Let  $\text{Expl}_{\text{Alg}, \mathcal{A}}$  be a random variable set to 1 if  $\mathcal{A}$  succeeds in outputting  $b' = b$  in the above game. Security of  $\text{Comp}(1^\lambda, \text{Alg})$  requires that for every PPT  $\mathcal{A}$  and for every efficient algorithm  $\text{Alg}$ , we have  $\Pr[\text{Expl}_{\text{Alg}, \mathcal{A}} = 1] \leq 1/2 + \text{negl}(\lambda)$ .

Assume towards a contradiction that there is some PPT adversary  $\mathcal{A}$  and some efficient algorithm  $\text{Alg}$  such that  $\Pr[\text{Expl}_{\text{Alg}, \mathcal{A}} = 1] \geq 1/2 + \varepsilon(\lambda)$ , for non-negligible  $\varepsilon(\cdot)$ . Then, we shall arrive at a contradiction through several hybrids. To maintain ease of verification for the reader, we present a full description of each hybrid experiment, each one given on a separate page. The change between each hybrid and the previous hybrid will be denoted in underlined font. The hybrids are chosen so that the indistinguishability of each successive hybrid experiment follows in a relatively straightforward manner.

We unwrap the explainability game specifically with respect to our construction  $\text{Comp}$ . Recall that we consider an adversary whose objective is to output  $b' = b$  in the following game.

**Original Game.** We consider the probability that  $b' = b$  in the following game:

1.  $b \leftarrow \{0, 1\}$ .
2.  $\text{input}^* \leftarrow \mathcal{A}(1^\lambda)$ .
3. Choose  $K_1, K_2, K_3$  at random.
4. Select  $u^*$  at random. Select  $r^*$  at random.

5. - If  $F_3(K_3, u[1]) \oplus u[2] = (\text{input}', \text{output}', r')$  for (proper length) strings  $\text{output}', r', \text{input}'$ , and  $\text{input}' = \text{input}^*$ , and  $u[1] = F_2(K_2, (\text{input}', \text{output}', r'))$ , then let  $\text{output}^* = \text{output}'$  and jump to Step 5. Otherwise, perform the following Step.
  - Let  $x^* = F_1(K_1, (\text{input}^*, u^*))$  and let  $\text{output}^* = \text{Alg}(\text{input}^*; x^*)$ .
6. Do the following. Set  $\alpha^* = F_2(K_2, (\text{input}^*, \text{output}^*, \text{PRG}(r^*)))$ . Let  $\beta^* = F_3(K_3, \alpha^*) \oplus (\text{input}^*, \text{output}^*, \text{PRG}(r^*))$ . Set  $e^* = (\alpha^*, \beta^*)$ .
7. Let  $\widetilde{\text{Alg}} \leftarrow \text{iO}(\overline{\text{Alg}})$  for  $\overline{\text{Alg}}$  as in Figure 1. Let  $\text{Explain} \leftarrow \text{iO}(\overline{\text{Explain}})$  for  $\overline{\text{Explain}}$  as in Figure 2.
8. If  $b = 0$ , set  $b' \leftarrow \mathcal{A}(\widetilde{\text{Alg}}, \text{output}^*, u^*)$ .  
If  $b = 1$ , set  $b' \leftarrow \mathcal{A}(\widetilde{\text{Alg}}, \text{output}^*, e^*)$ .

Next, we jump to Hybrid 0, where we eliminate Step 1 check from the  $\overline{\text{Alg}}$  program when preparing the outputs of the fixed challenge input  $\text{input}^*$ . Hybrid 0 is statistically close to the original Explainability game by Lemma 1.

**Hybrid 0.** We consider the probability that  $b' = b$  in the following game:

1.  $b \leftarrow \{0, 1\}$ .
2.  $\text{input}^* \leftarrow \mathcal{A}(1^\lambda)$ .
3. Choose  $K_1, K_2, K_3$  at random.
4. Select  $u^*$  at random. Select  $r^*$  at random.
5. - If  $F_3(K_3, u[1]) \oplus u[2] = (\text{input}', \text{output}', r')$  for (proper length) strings  $\text{output}', r', \text{input}'$ , and  $\text{input}' = \text{input}^*$ , and  $u[1] = F_2(K_2, (\text{input}', \text{output}', r'))$ , then let  $\text{output}^* = \text{output}'$  and jump to Step . Otherwise, perform the following Step.
  - Let  $x^* = F_1(K_1, (\text{input}^*, u^*))$  and let  $\text{output}^* = \text{Alg}(\text{input}^*; x^*)$ .
6. Do the following. Set  $\alpha^* = F_2(K_2, (\text{input}^*, \text{output}^*, \text{PRG}(r^*)))$ . Let  $\beta^* = F_3(K_3, \alpha^*) \oplus (\text{input}^*, \text{output}^*, \text{PRG}(r^*))$ . Set  $e^* = (\alpha^*, \beta^*)$ .
7. Let  $\widetilde{\text{Alg}} \leftarrow \text{iO}(\overline{\text{Alg}})$  for  $\overline{\text{Alg}}$  as in Figure 5. Let  $\text{Explain} \leftarrow \text{iO}(\overline{\text{Explain}})$  for  $\overline{\text{Explain}}$  as in Figure 6.
8. If  $b = 0$ , set  $b' \leftarrow \mathcal{A}(\widetilde{\text{Alg}}, \text{output}^*, u^*)$ .  
If  $b = 1$ , set  $b' \leftarrow \mathcal{A}(\widetilde{\text{Alg}}, \text{output}^*, e^*)$ .

**Hybrid 1.** In this hybrid, we modify the  $\overline{\text{Alg}}$  program as follows: First, we add constants  $\text{input}^*, \text{output}^*, u^*, e^*$  to the program. Then, we add an “if” statement at the start that outputs  $\text{output}^*$  if the input is either  $(\text{input}^*, u^*)$  or  $(\text{input}^*, e^*)$ , as this is exactly what the original  $\overline{\text{Alg}}$  program would do by our choice of  $u^*, e^*$ . Because this “if” statement is in place, we know that  $F_1$  cannot be evaluated at either  $(\text{input}^*, u^*)$  or  $(\text{input}^*, e^*)$ , within the program, and therefore we can safely puncture key  $K_1$  at these two positions.

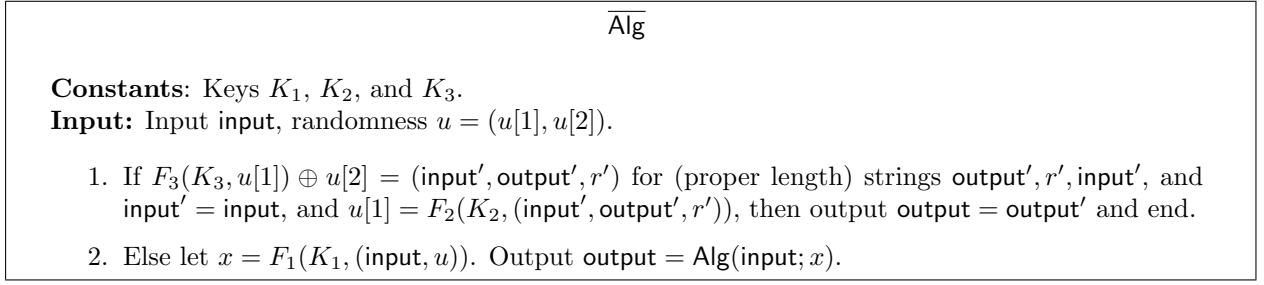


Figure 5: Program  $\overline{\text{Alg}}$

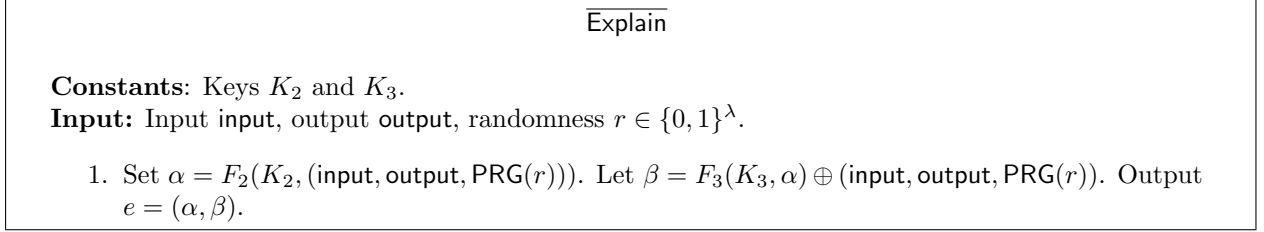


Figure 6: Program  $\overline{\text{Explain}}$

By construction, the new  $\overline{\text{Alg}}$  program is functionally equivalent to the original  $\overline{\text{Alg}}$  program. Therefore the indistinguishability of Hybrid 0 and Hybrid 1 follows by the security of  $\text{iO}$ . Thus, the probabilities that  $\mathcal{A}$  outputs  $b' = b$  in the two hybrids differ by a negligible amount.

Note: Implicitly, all “if” statements that are added to programs with multiple checks are written in lexicographic order; that is, if  $u^* < e^*$  in lexicographic order, we write it as “If  $(\text{input}, u) = (\text{input}^*, u^*)$  or  $(\text{input}, u) = (\text{input}^*, e^*)$ ,” otherwise we write it as “If  $(\text{input}, u) = (\text{input}^*, e^*)$  or  $(\text{input}, u) = (\text{input}^*, u^*)$ .”

1.  $b \leftarrow \{0, 1\}$ .
2.  $\text{input}^* \leftarrow \mathcal{A}(1^\lambda)$ .
3. Choose  $K_1, K_2, K_3$  at random.
4. Select  $u^*$  at random. Select  $r^*$  at random.
5. Let  $x^* = F_1(K_1, (\text{input}^*, u^*))$  and let  $\text{output}^* = \text{Alg}(\text{input}^*; x^*)$ .
6. Do the following. Set  $\alpha^* = F_2(K_2, (\text{input}^*, \text{output}^*, \text{PRG}(r^*)))$ . Let  $\beta^* = F_3(K_3, \alpha^*) \oplus (\text{input}^*, \text{output}^*, \text{PRG}(r^*))$ . Set  $e^* = (\alpha^*, \beta^*)$ .
7. Let  $\widetilde{\text{Alg}} \leftarrow \text{iO}(\overline{\text{Alg}})$  for  $\overline{\text{Alg}}$  as in Figure 7. Let  $\widetilde{\text{Explain}} \leftarrow \text{iO}(\overline{\text{Explain}})$  for  $\overline{\text{Explain}}$  as in Figure 8.
8. If  $b = 0$ , set  $b' \leftarrow \mathcal{A}(\widetilde{\text{Alg}}, \text{output}^*, u^*)$ .  
 If  $b = 1$ , set  $b' \leftarrow \mathcal{A}(\widetilde{\text{Alg}}, \text{output}^*, e^*)$ .

**Hybrid 2.** Here, the value  $x^*$  is chosen uniformly instead of as the output of  $F_1(K_1, (\text{input}^*, u^*))$ . The indistinguishability of Hybrid 2 from Hybrid 1 follows immediately from the pseudorandomness property of the punctured PRF  $F_1$  (Definition 2). Thus, the difference in the probability  $\mathcal{A}$  outputs  $b' = b$  in the two hybrids is by a negligible amount.

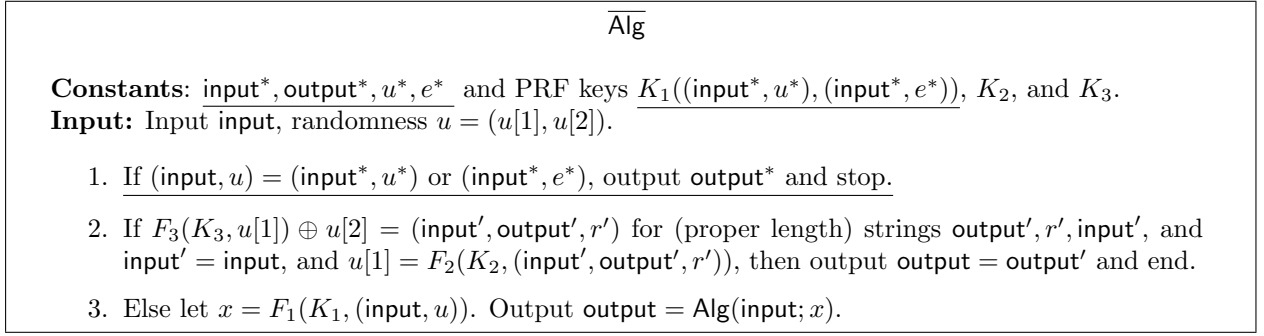


Figure 7: Program  $\overline{\text{Alg}}$

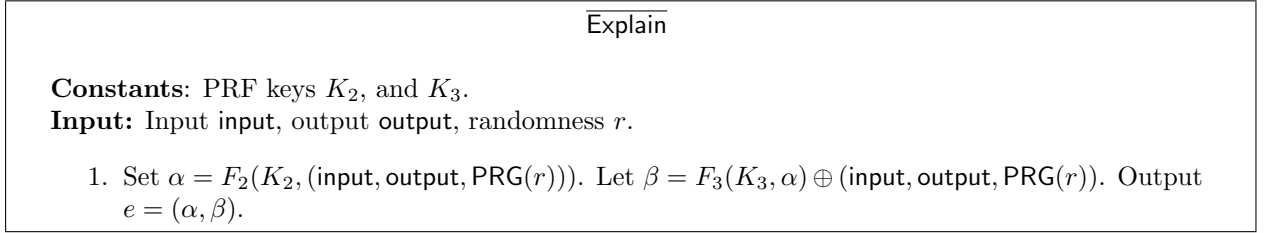


Figure 8: Program  $\overline{\text{Explain}}$

1.  $b \leftarrow \{0, 1\}$ .
2.  $\text{input}^* \leftarrow \mathcal{A}(1^\lambda)$ .
3. Choose  $K_1, K_2, K_3$  at random.
4. Select  $u^*$  at random. Select  $r^*$  at random.
5. Let  $x^*$  be chosen randomly and let  $\text{output}^* = \text{Alg}(\text{input}^*; x^*)$ .
6. Do the following. Set  $\alpha^* = F_2(K_2, (\text{input}^*, \text{output}^*, \text{PRG}(r^*)))$ . Let  $\beta^* = F_3(K_3, \alpha^*) \oplus (\text{input}^*, \text{output}^*, \text{PRG}(r^*))$ . Set  $e^* = (\alpha^*, \beta^*)$ .
7. Let  $\widetilde{\text{Alg}} \leftarrow \text{iO}(\overline{\text{Alg}})$  for  $\overline{\text{Alg}}$  as in Figure 9. Let  $\widetilde{\text{Explain}} \leftarrow \text{iO}(\overline{\text{Explain}})$  for  $\overline{\text{Explain}}$  as in Figure 10.
8. If  $b = 0$ , set  $b' \leftarrow \mathcal{A}(\widetilde{\text{Alg}}, \text{output}^*, u^*)$ .  
If  $b = 1$ , set  $b' \leftarrow \mathcal{A}(\widetilde{\text{Alg}}, \text{output}^*, e^*)$ .

**Hybrid 3.** In this hybrid, instead of picking  $r^*$  at random and applying a PRG to it, a value  $\tilde{r}$  is chosen at random from the co-domain of the PRG. The indistinguishability of Hybrid 2 and Hybrid 3 follows immediately from the security of the PRG. Thus, the difference in the probability  $\mathcal{A}$  outputs  $b' = b$  in the two hybrids differs by a negligible amount.

1.  $b \leftarrow \{0, 1\}$ .
2.  $\text{input}^* \leftarrow \mathcal{A}(1^\lambda)$ .
3. Choose  $K_1, K_2, K_3$  at random.

$\overline{\text{Alg}}$

**Constants:**  $\text{input}^*, \text{output}^*, u^*, e^*$  and PRF keys  $K_1((\text{input}^*, u^*), (\text{input}^*, e^*))$ ,  $K_2$ , and  $K_3$ .

**Input:** Input  $\text{input}$ , randomness  $u = (u[1], u[2])$ .

1. If  $(\text{input}, u) = (\text{input}^*, u^*)$  or  $(\text{input}^*, e^*)$ , output  $\text{output}^*$  and stop.
2. If  $F_3(K_3, u[1]) \oplus u[2] = (\text{input}', \text{output}', r')$  for (proper length) strings  $\text{output}', r', \text{input}'$ , and  $\text{input}' = \text{input}$ , and  $u[1] = F_2(K_2, (\text{input}', \text{output}', r'))$ , then output  $\text{output} = \text{output}'$  and end.
3. Else let  $x = F_1(K_1, (\text{input}, u))$ . Output  $\text{output} = \text{Alg}(\text{input}; x)$ .

Figure 9: Program  $\overline{\text{Alg}}$

$\overline{\text{Explain}}$

**Constants:** PRF keys  $K_2$ , and  $K_3$ .

**Input:** Input  $\text{input}$ , output  $\text{output}$ , randomness  $r$ .

1. Set  $\alpha = F_2(K_2, (\text{input}, \text{output}, \text{PRG}(r)))$ . Let  $\beta = F_3(K_3, \alpha) \oplus (\text{input}, \text{output}, \text{PRG}(r))$ . Output  $e = (\alpha, \beta)$ .

Figure 10: Program  $\overline{\text{Explain}}$

4. Select  $u^*$  at random. Select  $\tilde{r}$  at random.
5. Let  $x^*$  be chosen randomly and let  $\text{output}^* = \text{Alg}(\text{input}^*; x^*)$ .
6. Do the following. Set  $\alpha^* = F_2(K_2, (\text{input}^*, \text{output}^*, \tilde{r}))$ . Let  $\beta^* = F_3(K_3, \alpha^*) \oplus (\text{input}^*, \text{output}^*, \tilde{r})$ . Set  $e^* = (\alpha^*, \beta^*)$ .
7. Let  $\widetilde{\text{Alg}} \leftarrow \text{iO}(\overline{\text{Alg}})$  for  $\overline{\text{Alg}}$  as in Figure 11. Let  $\widetilde{\text{Explain}} \leftarrow \text{iO}(\overline{\text{Explain}})$  for  $\overline{\text{Explain}}$  as in Figure 12.
8. If  $b = 0$ , set  $b' \leftarrow \mathcal{A}(\widetilde{\text{Alg}}, \text{output}^*, u^*)$ .  
If  $b = 1$ , set  $b' \leftarrow \mathcal{A}(\widetilde{\text{Alg}}, \text{output}^*, e^*)$ .

$\overline{\text{Alg}}$

**Constants:**  $\text{input}^*, \text{output}^*, u^*, e^*$  and PRF keys  $K_1((\text{input}^*, u^*), (\text{input}^*, e^*))$ ,  $K_2$ , and  $K_3$ .

**Input:** Input  $\text{input}$ , randomness  $u = (u[1], u[2])$ .

1. If  $(\text{input}, u) = (\text{input}^*, u^*)$  or  $(\text{input}^*, e^*)$ , output  $\text{output}^*$  and stop.
2. If  $F_3(K_3, u[1]) \oplus u[2] = (\text{input}', \text{output}', r')$  for (proper length) strings  $\text{output}', r', \text{input}'$ , and  $\text{input}' = \text{input}$ , and  $u[1] = F_2(K_2, (\text{input}', \text{output}', r'))$ , then output  $\text{output} = \text{output}'$  and end.
3. Else let  $x = F_1(K_1, (\text{input}, u))$ . Output  $\text{output} = \text{Alg}(\text{input}; x)$ .

Figure 11: Program  $\overline{\text{Alg}}$

| $\overline{\text{Explain}}$  |
|--|
| <p><b>Constants:</b> PRF keys <math>K_2</math>, and <math>K_3</math>.</p> <p><b>Input:</b> Input input, output output, randomness <math>r</math>.</p> <ol style="list-style-type: none"> <li>1. Set <math>\alpha = F_2(K_2, (\text{input}, \text{output}, \text{PRG}(r)))</math>. Let <math>\beta = F_3(K_3, \alpha) \oplus (\text{input}, \text{output}, \text{PRG}(r))</math>. Output <math>e = (\alpha, \beta)</math>.</li> </ol> |

Figure 12: Program  $\overline{\text{Explain}}$

**Hybrid 4.** In this hybrid, the  $\overline{\text{Alg}}$  and  $\overline{\text{Explain}}$  programs are modified as shown below. In Lemma 2, (proven below after all hybrids are given), we argue that except with negligible probability over choice of constants, these modifications do not alter the functionality of either program.

Thus, the indistinguishability of Hybrid 3 and Hybrid 4 follows from the  $\text{iO}$  security property. and so the difference in the probability  $\mathcal{A}$  outputs  $b' = b$  in the two hybrids differs by a negligible amount.

1.  $b \leftarrow \{0, 1\}$ .
2.  $\text{input}^* \leftarrow \mathcal{A}(1^\lambda)$ .
3. Choose  $K_1, K_2, K_3$  at random.
4. Select  $u^*$  at random. Select  $\tilde{r}$  at random.
5. Let  $x^*$  be chosen randomly and let  $\text{output}^* = \text{Alg}(\text{input}^*; x^*)$ .
6. Do the following. Set  $\alpha^* = F_2(K_2, (\text{input}^*, \text{output}^*, \tilde{r}))$ . Let  $\beta^* = F_3(K_3, \alpha^*) \oplus (\text{input}^*, \text{output}^*, \tilde{r})$ . Set  $e^* = (\alpha^*, \beta^*)$ .
7. Let  $\widetilde{\text{Alg}} \leftarrow \text{iO}(\overline{\text{Alg}})$  for  $\overline{\text{Alg}}$  as in Figure 13. Let  $\widetilde{\text{Explain}} \leftarrow \text{iO}(\overline{\text{Explain}})$  for  $\overline{\text{Explain}}$  as in Figure 14.
8. If  $b = 0$ , set  $b' \leftarrow \mathcal{A}(\widetilde{\text{Alg}}, \text{output}^*, u^*)$ .  
If  $b = 1$ , set  $b' \leftarrow \mathcal{A}(\widetilde{\text{Alg}}, \text{output}^*, e^*)$ .

**Hybrid 5.** In this hybrid, the value  $e^*[2]$ , denoted  $\beta^*$ , is chosen at random instead of being chosen as  $\beta^* = F_3(K_3, \alpha^*) \oplus (\text{input}^*, \text{output}^*, \tilde{r})$ . The indistinguishability of Hybrid 4 and Hybrid 5 follows immediately from the pseudorandomness property of the puncturable PRF  $F_3$ . Thus, the difference in the probability  $\mathcal{A}$  outputs  $b' = b$  in the two hybrids differs by a negligible amount.

1.  $b \leftarrow \{0, 1\}$ .
2.  $\text{input}^* \leftarrow \mathcal{A}(1^\lambda)$ .
3. Choose  $K_1, K_2, K_3$  at random.
4. Select  $u^*$  at random. Select  $\tilde{r}$  at random.
5. Let  $x^*$  be chosen randomly and let  $\text{output}^* = \text{Alg}(\text{input}^*; x^*)$ .

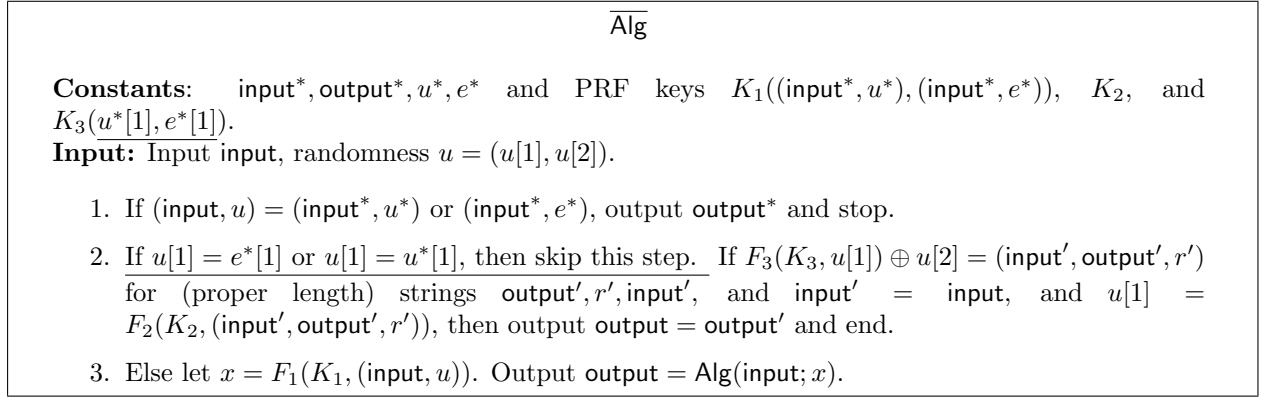


Figure 13: Program  $\overline{\text{Alg}}$

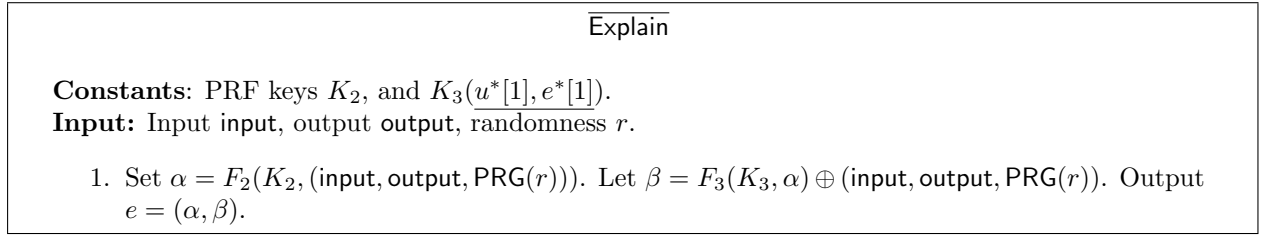


Figure 14: Program  $\overline{\text{Explain}}$

6. Do the following. Set  $\alpha^* = F_2(K_2, (\text{input}^*, \text{output}^*, \tilde{r}))$ . Let  $\beta^*$  be random. Set  $e^* = (\alpha^*, \beta^*)$ .
7. Let  $\widetilde{\text{Alg}} \leftarrow \text{iO}(\overline{\text{Alg}})$  for  $\overline{\text{Alg}}$  as in Figure 15. Let  $\widetilde{\text{Explain}} \leftarrow \text{iO}(\overline{\text{Explain}})$  for  $\overline{\text{Explain}}$  as in Figure 16.
8. If  $b = 0$ , set  $b' \leftarrow \mathcal{A}(\widetilde{\text{Alg}}, \text{output}^*, u^*)$ .  
If  $b = 1$ , set  $b' \leftarrow \mathcal{A}(\widetilde{\text{Alg}}, \text{output}^*, e^*)$ .

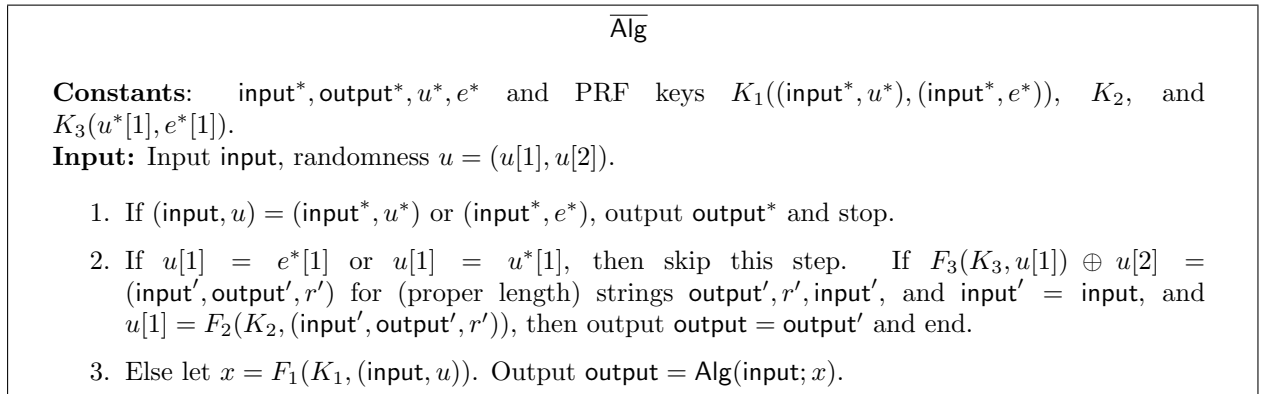


Figure 15: Program  $\overline{\text{Alg}}$

**Hybrid 6.** In this hybrid, first we modify the  $\overline{\text{Alg}}$  program to add a condition to Step 2 check to determine if the decrypted  $(\text{input}', \text{output}', r') = (\text{input}^*, \text{output}^*, \tilde{r})$ , and if so, to skip this check. This

| $\overline{\text{Explain}}$  |
|--|
| <p><b>Constants:</b> PRF keys <math>K_2</math>, and <math>K_3(u^*[1], e^*[1])</math>.</p> <p><b>Input:</b> Input <math>\text{input}</math>, output <math>\text{output}</math>, randomness <math>r</math>.</p> <ol style="list-style-type: none"> <li>1. Set <math>\alpha = F_2(K_2, (\text{input}, \text{output}, \text{PRG}(r)))</math>. Let <math>\beta = F_3(K_3, \alpha) \oplus (\text{input}, \text{output}, \text{PRG}(r))</math>. Output <math>e = (\alpha, \beta)</math>.</li> </ol> |

Figure 16: Program  $\overline{\text{Explain}}$

new check does not change the functionality of the program, because  $e^*[1] = F_2(K_2, (\text{input}^*, \text{output}^*, \tilde{r}))$ , and therefore the check could not pass if  $(\text{input}', \text{output}', r') = (\text{input}^*, \text{output}^*, \tilde{r})$ , since Step 2 is skipped entirely if  $u[1] = e^*[1]$ . Then, both the  $\overline{\text{Alg}}$  and  $\overline{\text{Explain}}$  programs are modified to have keys  $K_2$  punctured at the points  $(\text{input}^*, \text{output}^*, \tilde{r})$ . This puncturing does not change the functionality of the  $\overline{\text{Alg}}$  program because of the new “if” condition just implemented. With high probability over the choice of  $\tilde{r}$ , it is true that  $\tilde{r}$  is not in the image of the PRG, and therefore this puncturing also does not change the functionality of the  $\overline{\text{Explain}}$  program.

Thus, the indistinguishability of Hybrid 5 and Hybrid 6 follows from the  $i\mathcal{O}$  security property. Thus, the difference in the probability  $\mathcal{A}$  outputs  $b' = b$  in the two hybrids differs by a negligible amount.

1.  $b \leftarrow \{0, 1\}$ .
2.  $\text{input}^* \leftarrow \mathcal{A}(1^\lambda)$ .
3. Choose  $K_1, K_2, K_3$  at random.
4. Select  $u^*$  at random. Select  $\tilde{r}$  at random.
5. Let  $x^*$  be chosen randomly and let  $\text{output}^* = \text{Alg}(\text{input}^*; x^*)$ .
6. Do the following. Set  $\alpha^* = F_2(K_2, (\text{input}^*, \text{output}^*, \tilde{r}))$ . Let  $\beta^*$  be random. Set  $e^* = (\alpha^*, \beta^*)$ .
7. Let  $\widetilde{\text{Alg}} \leftarrow i\mathcal{O}(\overline{\text{Alg}})$  for  $\overline{\text{Alg}}$  as in Figure 17. Let  $\widetilde{\text{Explain}} \leftarrow i\mathcal{O}(\overline{\text{Explain}})$  for  $\overline{\text{Explain}}$  as in Figure 18.
8. If  $b = 0$ , set  $b' \leftarrow \mathcal{A}(\widetilde{\text{Alg}}, \text{output}^*, u^*)$ .  
If  $b = 1$ , set  $b' \leftarrow \mathcal{A}(\widetilde{\text{Alg}}, \text{output}^*, e^*)$ .

**Hybrid 7.** In our final hybrid, we modify  $e^*[1]$ , denoted  $\alpha^*$ , to be randomly chosen, instead of being set as  $\alpha^* = F_2(K_2, (\text{input}^*, \text{output}^*, \tilde{r}))$ . The indistinguishability of Hybrid 6 and Hybrid 7 follows immediately from the pseudorandomness property of the puncturable PRF  $F_2$ . Thus, the difference in the probability  $\mathcal{A}$  outputs  $b' = b$  in the two hybrids differs by a negligible amount.

1.  $b \leftarrow \{0, 1\}$ .
2.  $\text{input}^* \leftarrow \mathcal{A}(1^\lambda)$ .
3. Choose  $K_1, K_2, K_3$  at random.



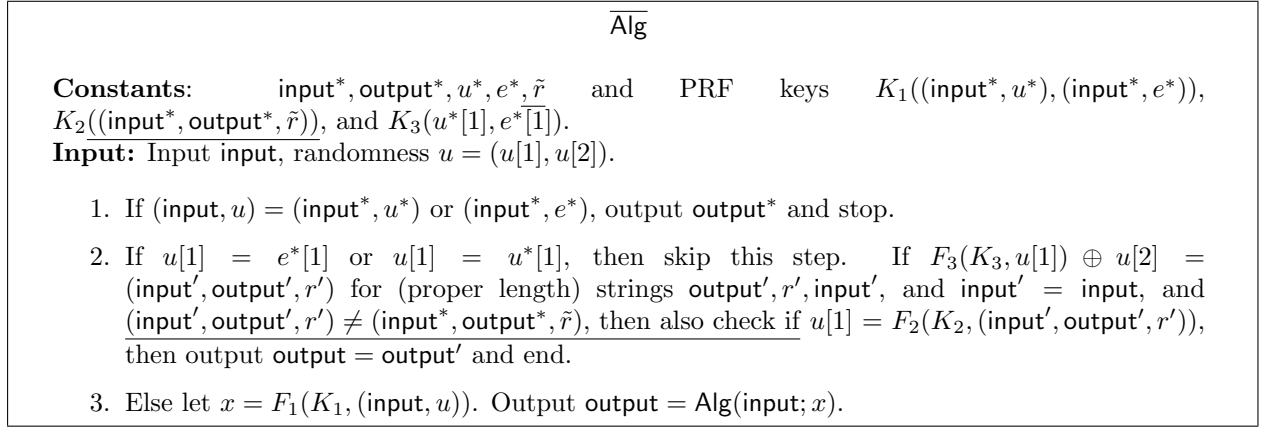


Figure 17: Program  $\overline{\text{Alg}}$

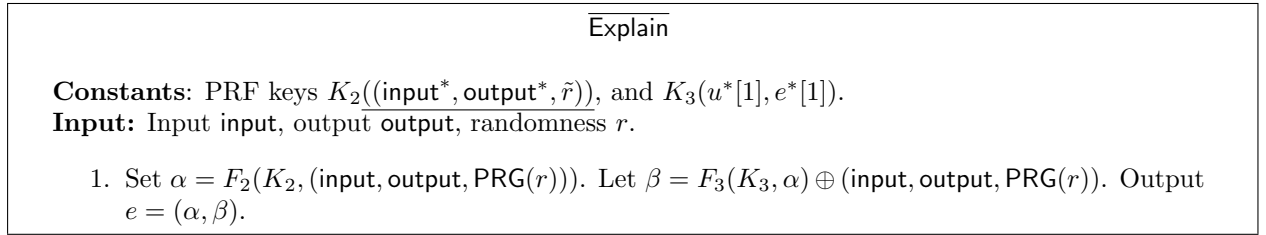


Figure 18: Program  $\overline{\text{Explain}}$

4. Select  $u^*$  at random. Select  $\tilde{r}$  at random.
5. Let  $x^*$  be chosen randomly and let  $\text{output}^* = \text{Alg}(\text{input}^*; x^*)$ .
6. Do the following. Let  $\alpha^*$  be random. Let  $\beta^*$  be random. Set  $e^* = (\alpha^*, \beta^*)$ .
7. Let  $\widetilde{\text{Alg}} \leftarrow \text{iO}(\overline{\text{Alg}})$  for  $\overline{\text{Alg}}$  as in Figure 19. Let  $\widetilde{\text{Explain}} \leftarrow \text{iO}(\overline{\text{Explain}})$  for  $\overline{\text{Explain}}$  as in Figure 20.
8. If  $b = 0$ , set  $b' \leftarrow \mathcal{A}(\widetilde{\text{Alg}}, \text{output}^*, u^*)$ .  
If  $b = 1$ , set  $b' \leftarrow \mathcal{A}(\widetilde{\text{Alg}}, \text{output}^*, e^*)$ .

In Hybrid 7 we observe that the variables  $e^*, u^*$  are now both independently uniformly random strings, and they are treated entirely symmetrically. (Recall that the “if” statements above have the conditions written in lexicographic order, so they do not reveal any asymmetry between  $e^*$  and  $u^*$ .) Thus, the distributions output by this Hybrid for  $b = 0$  and  $b = 1$  are identical, and therefore even an unbounded adversary outputs  $b = b'$  with probability exactly  $1/2$ . ■

The proof above made use of the following lemma for arguing that the programs obfuscated by the indistinguishability obfuscator in Hybrid 3 are equivalent to the corresponding programs in Hybrid 4.

**Lemma 2.** *Except with negligible probability over the choice of  $u^*[1]$  and  $e^*[1]$ , the  $\overline{\text{Alg}}$  and  $\overline{\text{Explain}}$  programs in Hybrid 4 are equivalent to the  $\overline{\text{Alg}}$  and  $\overline{\text{Explain}}$  programs in Hybrid 3.*

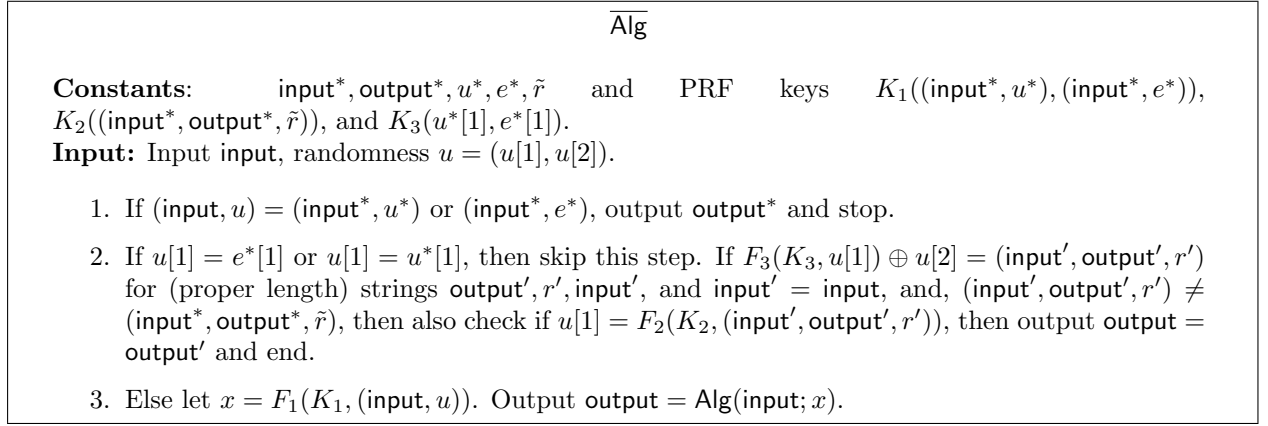


Figure 19: Program  $\overline{\text{Alg}}$

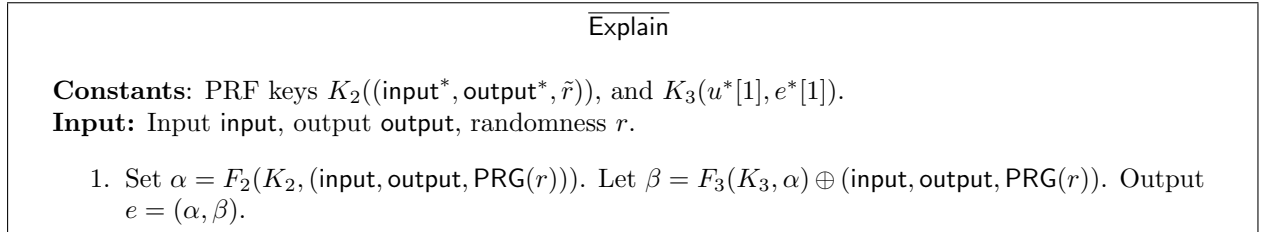


Figure 20: Program  $\overline{\text{Explain}}$

**Proof:** We consider below each change to the programs.

First, an “if” statement is added to Step 2 of the  $\overline{\text{Alg}}$  program, to skip Step 2 check if, either  $u[1] = e^*[1]$  or  $u[1] = u^*[1]$ . To see why this change does not affect the functionality of the program, let us consider each case in turn. Observe that by Lemma 1, if  $u[1] = e^*[1]$ , then the only way the Step 2 check can be satisfied is if  $\text{input} = \text{input}^*$  and  $u[2] = e^*[2]$ . But this case is already handled in Step 1, therefore skipping Step 2 if  $u[1] = e^*[1]$  does not affect functionality. On the other hand, recall that every  $u^*[1]$  is chosen at random, and therefore the probability that  $u^*[1]$  would be in the image of  $F_2(K_2, \cdot)$  is negligible, therefore with high probability over the choice of constants  $u^*[1]$ , Step 2 check cannot be satisfied if  $u[1] = u^*[1]$ . Therefore, the addition of this “if” statement does not alter the functionality of the  $\overline{\text{Alg}}$  program.

Also, the key  $K_3$  is punctured at  $u^*[1], e^*[1]$  in both the  $\overline{\text{Alg}}$  and  $\overline{\text{Explain}}$  programs. The new “if” statement above ensures that  $F_3(K_3, \cdot)$  is never called at these values in the  $\overline{\text{Alg}}$  program. Recall that the  $\overline{\text{Explain}}$  program only calls  $F_3(K_3, \cdot)$  on values computed as  $F_2(K_2, (\text{input}, \text{output}, \text{PRG}(r)))$  for some bit  $\text{input}$  and strings  $\text{output}$  and  $r$ . Furthermore,  $F_2$  is statistically injective with a very sparse image set, by our choice of parameters. Since every  $u^*[1]$  is randomly chosen, it is very unlikely to be in the image of  $F_2(K_2, \cdot)$ . Since every  $e^*[1]$  is chosen based on a random  $\tilde{r}$  value instead of a PRG output, it is very unlikely to correspond to  $F_2(K_2, (\text{input}, \text{output}, \text{PRG}(r)))$  for any  $(\text{input}, \text{output}, r)$ . Thus, these values are not called by the  $\overline{\text{Explain}}$  program, except with negligible probability over the choice of these constants  $u^*[1]$  and  $e^*[1]$ . ■