present

FINC 462/662 -- Fixed Income Securities

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Interest Rate Swaps and Floating Rate Bonds

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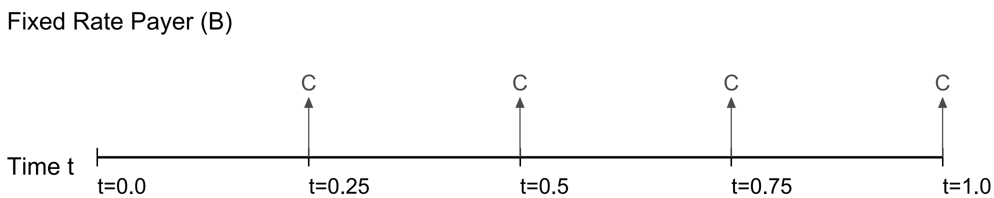
Overview

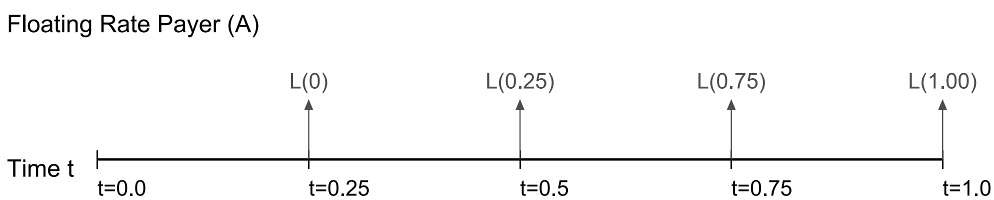
Goals for today

- [ ] Understanding what interest rate swaps and floating rate bonds are.
- [ ] Calculating the fair fixed rate in an interest rate swap contract.
- [ ] Calculating the price and modified duration of a floating rate bond.
- [ ] Relating interest rate swaps, floating rate bonds, and fixed coupon bonds.
- [ ] Calculating the value of an interest rate swap position after inception of the contract.
- [ ] Using interest rate swaps to hedge interest rate exposure.

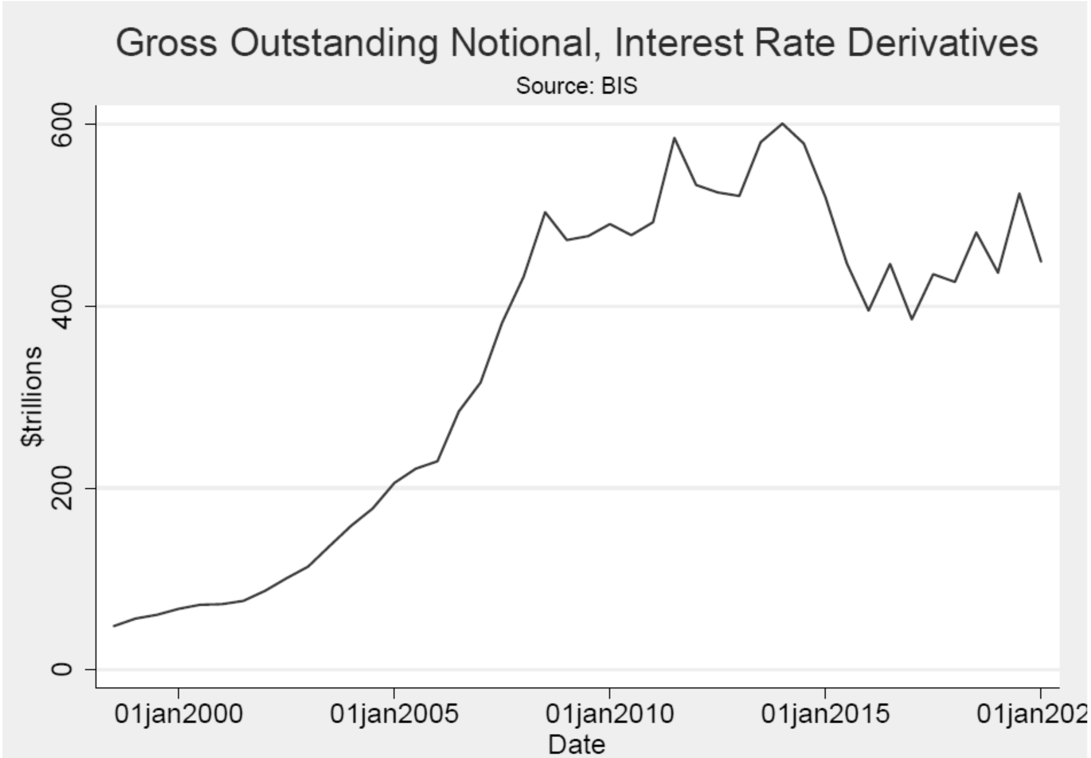
Interest Rate Swaps

- Each quarter (until maturity), B pays A a fixed payment C, which is agreed upon at the start of the interest rate swap contract.
- Each quarter, A pays B a payment that is based on the three-month interest rate at the start of the quarter (usually based on 3-month **LIBOR**).
  - This cash flow is written as $L(t)$
- Typically, there is no upfront payment.

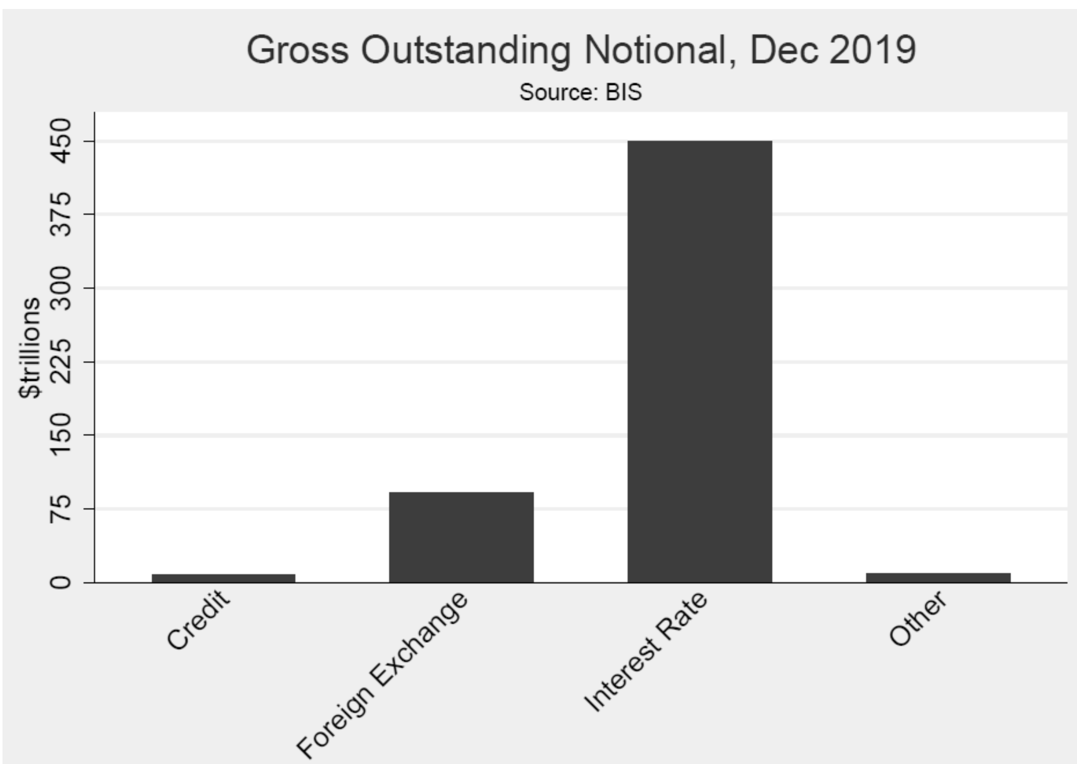




Size of the Interest Rate Swap Market



Size of OTC Markets



Source: BIS

GE’s Use of Interest Rate Swaps

DERIVATIVES AND HEDGING

Cash flow hedges – We use cash flow hedging primarily to reduce or eliminate the effects of foreign exchange rate changes on purchase and sale contracts in our industrial businesses and to convert foreign currency debt that we have issued in our financial services business back to our functional currency.

As part of our ongoing effort to reduce borrowings, we may repurchase debt that was in a cash flow hedge accounting relationship. At the time of determining that the debt cash flows are probable of not occurring any related OCI will be released to earnings.

Fair value hedges – These derivatives are used to hedge the effects of interest rate and currency exchange rate changes on debt that we have issued.

https://www.sec.gov/Archives/edgar/data/40545/000004054519000014/ge10-k2018.htm

NOTIONAL AMOUNT OF DERIVATIVES

The notional amount of a derivative is the number of units of the underlying (for example, the notional principal amount of the debt in an interest rate swap). The notional amount is used to compute interest or other payment streams to be made under the contract and is a measure of our level of activity. We generally disclose derivative notional amounts on a gross basis. The majority of the outstanding notional amount of $124 billion at December 31, 2018 is related to managing interest rate and currency risk between financial assets and liabilities in our financial services business. The remaining derivative notional amount primarily relates to hedges of anticipated sales and purchases in foreign currency, commodity purchases and contractual terms in contracts that are considered embedded derivatives.

NOTE 19. FAIR VALUE MEASUREMENTS

RECURRING FAIR VALUE MEASUREMENTS

Our assets and liabilities measured at fair value on a recurring basis include investment securities mainly supporting obligations to annuitants and policyholders in our run-off insurance operations and derivatives.

ASSETS AND LIABILITIES MEASURED AT FAIR VALUE ON A RECURRING BASIS *(In millions)*	Level 1	Level 2	Level 3(a)	Netting adjustment(d)	Net balance(b)
December 31, 2018					
Assets					
Investment securities	$ 126	$ 29,408	$ 4,301	$ —	$ 33,835
Derivatives	—	2,294	8	(2,001)	301
Total	$ 126	$ 31,701	$ 4,309	$ (2,001)	$ 34,136
Liabilities					
Derivatives	$ —	$ 1,913	$ 6	$ (1,234)	$ 686
Other(c)	—	722	—	—	722
Total	$ —	$ 2,635	$ 6	$ (1,234)	$ 1,408
December 31, 2017					
Assets					
Investment securities	$ 158	$ 34,126	$ 4,413	$ —	$ 38,696
Derivatives	—	3,343	21	(2,986)	378
Total	$ 158	$ 37,469	$ 4,433	$ (2,986)	$ 39,074
Liabilities					
Derivatives	$ —	$ 2,354	$ 7	$ (2,034)	$ 327
Other(c)	—	999	—	—	999
Total	$ —	$ 3,353	$ 7	$ (2,034)	$ 1,325

(a) Included debt securities classified within Level 3 of $3,498 million of U.S. corporate and $580 million of Government and agencies securities at December 31, 2018, and $3,629 million of U.S. corporate and $614 million of Government and agencies securities at December 31, 2017.

(b) See Notes 3 and 20 for further information on the composition of our investment securities and derivative portfolios.

(c) Primarily represents the liabilities associated with certain of our deferred incentive compensation plans.

(d) The netting of derivative receivables and payables is permitted when a legally enforceable master netting agreement exists. Amounts include fair value adjustments related to our own and counterparty non-performance risk.

FAIR VALUE OF DERIVATIVES	2018		2017	
December 31 (In millions)	Assets	Liabilities	Assets	Liabilities
Derivatives accounted for as hedges				
Interest rate contracts	$ 1,335	$ 23	$ 1,862	$ 148
Currency exchange contracts	175	121	160	70
Other contracts	—	—	—	—
	$ 1,511	$ 145	$ 2,021	$ 218
Derivatives not accounted for as hedges				
Interest rate contracts	28	2	93	8
Currency exchange contracts	747	1,562	1,111	2,043
Other contracts	16	211	139	91
	$ 791	$ 1,775	$ 1,343	$ 2,143
Gross derivatives recognized in Statement of Financial Position				
Gross derivatives	2,301	1,920	3,364	2,361
Gross accrued interest	209	6	469	(38)
	$ 2,511	$ 1,926	$ 3,833	$ 2,323
Amounts offset in Statement of Financial Position				
Netting adjustments(a)	(963)	(971)	(1,457)	(1,456)
Cash collateral(b)	(1,042)	(267)	(1,529)	(578)
	$ (2,005)	$ (1,238)	$ (2,986)	$ (2,034)

Example

- Suppose that we decide on 12/31/2017 to enter into a fixed-for-floating interest rate swap. The contract will have the following terms
  - Payment frequency: Quarterly
  - Maturity T : 1-year
  - Notional amount N: $ 1000000
  - Reference rate r: we assume that the reference rate is the 3-month Treasury rate.
  - Fixed rate f: 1.9873 %
- The zero-coupon yield curve (assume quarterly compounding) on 12/31/2017 was:

Time to maturity t	Spot rate r
0.25	1.39 %
0.5	1.53 %
0.75	1.76 %
1.0	1.99 %

- The fixed cash flow paid *each quarter* is

$$C = \frac{f}{4} \times N = \frac{1.9873\%}{4} \times \$1000000 = 4968.25$$

- Suppose we enter the swap as the fixed-rate payer

Cash flow date	3/31/2018	6/30/2018	9/30/2018	12/31/2018
Fixed Leg	-4968.25	-4968.25	-4968.25	-4968.25

- The end-of-quarter cash flows $L(t)$ on the floating leg of the contract (which we receive) are calculated using the 3-month Treasury rate at the beginning of the quarter.
  - This means that L(t) is based on $r(t-0.25)$.
- Specifically,

Cash flow date	3/31/2018	6/30/2018	9/30/2018	12/31/2018
Floating Leg	$L(3/31/2018)$	$L(6/30/2018)$	$L(9/30/2018)$	$L(12/31/2018)$
Based on	$r(0.25)$ on 12/31/2017	$r(0.25)$ on 3/31/2018	$r(0.25)$ on 6/30/2018	$r(0.25)$ on 9/30/2018

- The first cash flow on the floating leg is calculated using today's 3-month Treasury rate $r(0.25) = 1.39\%$.

$$L(3/31/2018) = \frac{1.39\%}{4} \times N = \frac{1.39\%}{4} \times 1000000 = \$3475.0$$

- Suppose now that we are on 3/31/2018.
- As the fixed rate payer, we pay a fixed cash flow of -4968.25.
- We receive a cash flow calculated using the 3-month Treasury rate which was set at the beginning of the quarter, i.e. this cash flow is based on the 3-month Treasury rate from three-months before the cash flow date.
  - Suppose that the three-month Treasury rate was 1.73%.
  - This means that the floating payment on 6/30/2018 will be

$$L(6/30/2018) = \frac{1.73\%}{4} \times N = \frac{1.73\%}{4} \times 1000000 = \$4325.0$$

- Let's suppose the 3-month Treasury rates on 6/30/2018, and on 9/30/2018 are 1.93% and 2.19% respectively. Thus, the floating rate cash flows are

$$L(9/30/2018) = \frac{1.93\%}{4} \times N = \frac{1.93\%}{4} \times 1000000 = \$4825.0$$

$$L(12/31/2018) = \frac{2.19\%}{4} \times N = \frac{2.19\%}{4} \times 1000000 = \$5475.0$$

- To summarize, the cash flows to this interest rate swap are

Cash flow date	**3/31/2018**	**6/30/2018**	**9/30/2018**	**12/31/2018**
Fixed Leg	-4968.25	-4968.25	-4968.25	-4968.25
Floating Leg	$L(3/31/2018) =$ 3475.0	$L(6/30/2018)$= 4325.0	$L(9/30/2018) =$ 4825.0	$L(12/31/2018) =$ 5475.0
Based on	$r(0.25)$ on 12/31/2017	$r(0.25)$ on 3/31/2018	$r(0.25)$ on 6/30/2018	$r(0.25)$ on 9/30/2018
	1.39 %	1.53 %	1.76 %	1.99 %

- It is important to keep in mind that each floating payment is only know three months in advance.
- When we first signed the contract, 3 of the 4 floating payments were unknown.

- In the example, we were given the interest rate on the fixed leg of the interest rate swap.
- To answer, the question how we can calculate this rate, we will discuss floating rate bonds. This is because knowing how to value floating rate bonds will simplify the calculation of the fixed swap rate.

Floating Rate Bonds

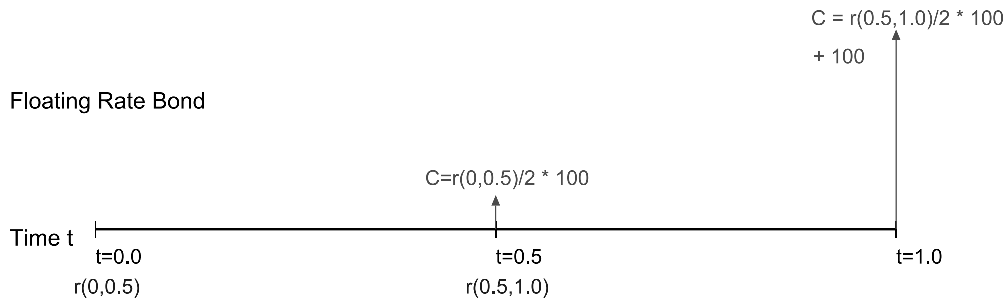
- The coupon cash flow of a floating rate bond/note (FRN) is based on prevailing interest rates in the market.
- Intuitively, in a plain-vanilla FRN, coupon cash flows payments go up if interest rates go up, and vice versa.
- Typically, the rate to be used to calculate the next coupon cash flow is set ("fixed") at the time of the previous coupon cash flow.
  - The dates on which the next coupon is determined are called *interest reset dates*.

- Let's consider an example of a one-year floating rate note (FRN).
- Suppose the FRN pays semi-annual coupon cash flows which are tied to the 6-month Treasury rate set at the beginning of each semi-annual coupon period.
- To show at which point in time spot rates $r(t)$ are known, we will follow the notation that we used for forward rates and write

$$r(T_1, T_2)$$

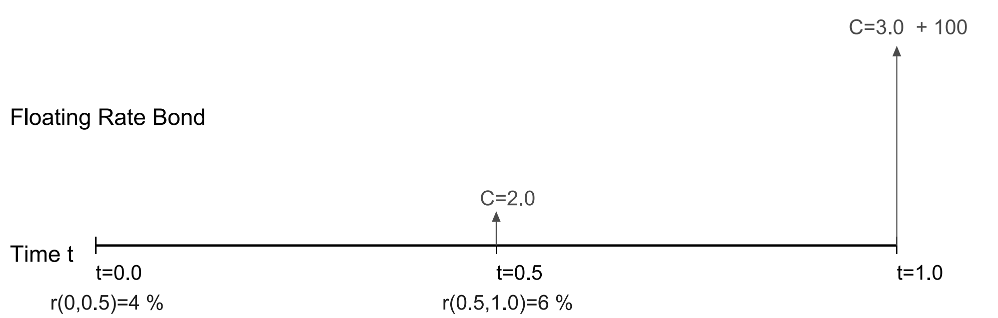
where T_1 is the time at which we know what the spot rate is and t is corresponding maturity from today.

- For example:
  - Today's 3-month spot rate which we have written as $r(t)$ is simply $r(0, 0.25)$.
  - The six-month spot rate in three months from now is $r(0.25, 0.75)$
  - The six-month spot rate in six months from now is $r(0.5, 1.00)$
  - The six-month spot rate in one year from now is $r(1.00, 1.50)$



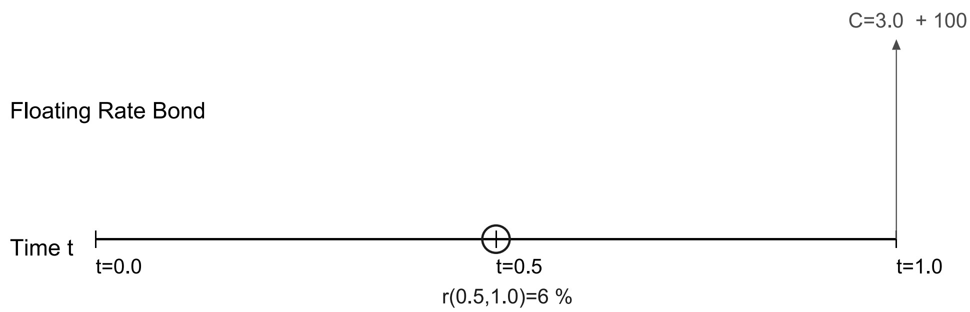
- The coupon cash flow paid at time $t = 0.5$ is $C = \frac{r(0,0.5)}{2} \times 100$ (known today).
- The coupon cash flow paid at time $t = 1.0$ is $C = \frac{r(0.5,1.0)}{2} \times 100$ (unknown today)

- Suppose that $r(0, 0.5) = 4\%$.
  - We get to observe this rate today.
- Suppose also that $r(0.5, 1) = 6\%$.
  - We do not get to observe this today, but find this out at time $t = 0.5$. At time 0, r (0.5, 1) is unknown.



Valuing a Floating Rate Note (FRN)

- Suppose we are in six months from now at $t = 0.5$.
- The first coupon has been paid, and there is one cash flow left which occurs at $t = 1.0$.



- The FRN has now six months to maturity.
- The final cash flow is

$$C + 100 = \frac{r(0.5,1)}{2} \times 100 + 100 = 100 \times \left(1 + \frac{r(0.5,1)}{2}\right)$$

- The key is that $r(0.5, 1)$ is known if we are at t=0.5.
- The value of the FRN at $t = 0.5$ is the present value of the final cash flow.
- We know what the final cash flow is and what the discount rate is since $r(0.5, 1)$ is known.
- Thus, the value of the FRN $P(t)$ at $t = 0.5$ is

$$P(0.5) = \frac{C + 100}{\left(1 + \frac{r(0.5,1)}{2}\right)^{2\times 0.5}} = \frac{100 \times \left(1 + \frac{r(0.5,1)}{2}\right)}{\left(1 + \frac{r(0.5,1)}{2}\right)^{1}} = 100$$

- Thus, the bond is trading at par.

- Next, suppose we are back at $t = 0$.
- In the previous step, we derived that the value of the FRN at $t = 0.5$ will be 100, right after the coupon cash flow is paid.
- Thus, the value today $P(0)$ of the FRN is the present value of 100 plus the coupon cash flow of $C = \frac{r(0.5,1)}{2} \times 100$.

$$P(0) = \frac{C + 100}{\left(1 + \frac{r(0,0.5)}{2}\right)^{2\times 0.5}} = \frac{\left(\frac{r(0.5,1)}{2} \times 100\right) + 100}{\left(1 + \frac{r(0,0.5)}{2}\right)^{1}} = \frac{\left(1 + \frac{r(0.5,1)}{2}\right) \times 100}{\left(1 + \frac{r(0,0.5)}{2}\right)^{1}} = 100$$

- The key takeaway is that just after interest reset dates, a floating rate note has a market value equal to its face value.
- This is true for floating rate Treasury bonds of arbitrary maturities as long as the coupon rate is based on Treasury rates.
- Another important property of floating rate bonds is their prices do not flucuate much in response to changes in interest rates.

Interest Rate Sensitivity of Floating Rate Notes

- To illustrate, suppose that we buy a 5-year FRN with semi-annual coupons and $r(0, 0.5) = 4\%$. The reset has just occurred.
- The first coupon will be $2, even if interest rates change tomorrow.
- The value of the bond will be $100 right after the next coupon reset (in 0.5 years).
- The current price of the bond is $100 since we are just after a coupon reset date and interest rates have not changed yet.
- Let's calculate the modified duration (MD) of the FRN.

- Recall that the modified duration (MD) can be calculated as
- Let's use $\Delta y = 0.001$

$$MD = -\frac{P(y+\Delta y) - P(y-\Delta y)}{2 \times \Delta y}$$

$$P(y) = \frac{100+2}{1+\frac{0.04}{2}} = 100$$

$$P(y+\Delta y) = \frac{100+2}{1+\frac{0.04+0.001}{2}} = 99.95100441$$

$$P(y-\Delta y) = \frac{100+2}{1+\frac{0.04-0.001}{2}} = 100.0490436$$

$$MD = -\frac{99.95100441 - 100.0490436}{2 \times 0.001} \times \frac{1}{100} = 0.4902$$

- As a rule of thumb, the modified duration of a FRN is the time to the next interest rate reset.

Interest Rate Swaps: Solving for the fair fixed rate

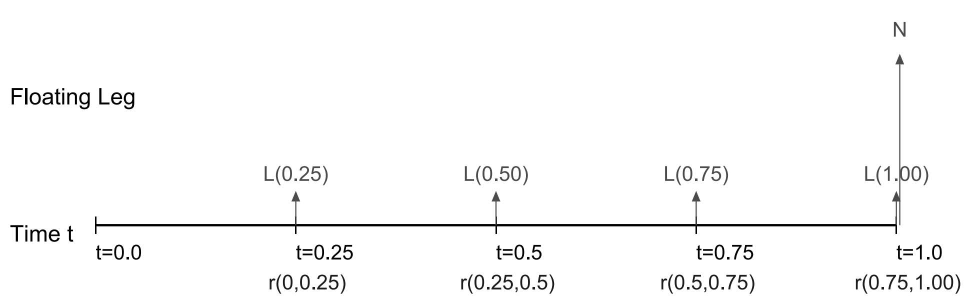
- Suppose that we decide on 12/31/2017 to enter into a fixed-for-floating interest rate swap. The contract will have the following terms
- Payment frequency: Quarterly
  - Maturity T : 1-year
  - Notional amount N: $ 1000000
  - Reference rate r: we assume that the reference rate is the 3-month Treasury rate.
  - Fixed rate f: 1.9873 %

- The zero-coupon yield curve on 12/31/2017 was as shown below. Assume quarterly compounding.

Time to maturity t	**Spot rate** r
0.25	1.39%
0.5	1.53%
0.75	1.76%
1.0	1.99%

- Without loss of generality, let's assume that the fixed and the floating leg exchange the notional amount N at maturity.

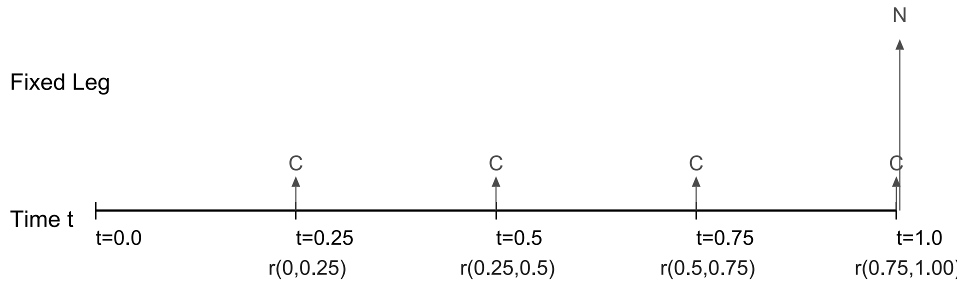
- Consider the floating leg of the swap first.



- The cash flows on the floating leg of the swap are similar to a floating rate note.
- Thus, we know that the value today at $t = 0$ of the floating leg is equal to the notional N.

$$P_{\text{Floating}} = N$$

- Let's next turn to the fixed leg of the swap.



- The cash flows on the fixed leg of the swap mirror those of a fixed-rate coupon bond.
- Recall that the term structure was given as

Time to maturity t	**Spot rate** r
0.25	1.39%
0.5	1.53%
0.75	1.76%
1	1.99%

- Thus, the value of the fixed leg is

$$P_{\text{Fixed}} = \frac{C}{\left(1+\frac{r_{0.25}}{4}\right)^{4\times 0.25}} + \frac{C}{\left(1+\frac{r_{0.50}}{4}\right)^{4\times 0.50}} + \frac{C}{\left(1+\frac{r_{0.75}}{4}\right)^{4\times 0.75}} + \frac{C+N}{\left(1+\frac{r_{1.00}}{4}\right)^{4\times 1.00}}$$

$$P_{\text{Fixed}} = \frac{C}{\left(1+\frac{1.39\%}{4}\right)^{4\times 0.25}} + \frac{C}{\left(1+\frac{1.53\%}{4}\right)^{4\times 0.50}} + \frac{C}{\left(1+\frac{1.76\%}{4}\right)^{4\times 0.75}} + \frac{C+1000000}{\left(1+\frac{1.99\%}{4}\right)^{4\times 1.00}}$$

- At the start of the interest rate swap at time $t=0$, the value of the fixed leg is equal to the value of the floating leg.
  - Recall that no cash flows occur at time $t=0$.
- Thus, setting both sides equal

$$P_{\text{Fixed}} \stackrel{!}{=} P_{\text{Floating}}$$

$$\frac{C}{\left(1+\frac{1.39\%}{4}\right)^{1}} + \frac{C}{\left(1+\frac{1.53\%}{4}\right)^{2}} + \frac{C}{\left(1+\frac{1.76\%}{4}\right)^{3}} + \frac{C+1000000}{\left(1+\frac{1.99\%}{4}\right)^{4}} \stackrel{!}{=} 1000000$$

- Solving for the cash flow C gives us

$$C = 496814.61$$

- and since $C = \frac{1}{4} \times f \times N$
- the fair swap rate is

$$f = 1.9873\%$$

- **Can we use discount factors to make the calculation less convoluted?**
- Consider again the fixed leg of the interest rate swap.

$$P_{\text{Fixed}} = \frac{C}{\left(1+\frac{r_{0.25}}{4}\right)^{4\times 0.25}} + \frac{C}{\left(1+\frac{r_{0.50}}{4}\right)^{4\times 0.50}} + \frac{C}{\left(1+\frac{r_{0.75}}{4}\right)^{4\times 0.75}} + \frac{C+N}{\left(1+\frac{r_{1.00}}{4}\right)^{4\times 1.00}}$$

- Recall that the discount factor $D(T)$ (quarterly compounded) os

$$D(T) = \frac{1}{\left(1+\frac{r_T}{4}\right)^{4\times T}}$$

- For instance, the 3-month discount factor is

$$D(0.25) = \frac{1}{\left(1+\frac{r_{0.25}}{4}\right)^{4\times 0.25}}$$

- Thus, let's rewrite the value of the fixed leg using discount factors.

$$P_{\text{Fixed}} = C \times D(0.25) + C \times D(0.5) + C \times D(0.75) + (C+N) \times D(1.0)$$

- Next, recall that that interest rate swap was fairly valued, such that $P_{\text{Fixed}} \stackrel{!}{=} P_{\text{Floating}}$
- We know that the value of the floating leg is par, $P_{\text{Floating}} = N$
- Thus,

$$N = C \times D(0.25) + C \times D(0.5) + C \times D(0.75) + (C+N) \times D(1.0)$$

- Let's solve for the cash flow on the fixed leg of the swap C

$$C = N \times \frac{1 - D(1.0)}{D(0.25) + D(0.50) + D(0.75) + D(1.0)}$$

- This equation gives us the *cash flow* on the fixed leg of the interest rate swap.
- To get the fair *swap rate*, let's use that the cash flow is

$$C = \frac{f}{4} \times N$$

- Plugging in the expression for C

$$\frac{f}{4} \times N = N \times \frac{1 - D(1.0)}{D(0.25) + D(0.50) + D(0.75) + D(1.0)}$$

- Thus, we have an equation for the fixed rate f on an interest rate swap.

$$f = 4 \times \frac{1 - D(1.0)}{D(0.25) + D(0.50) + D(0.75) + D(1.0)}$$

- Let's use this insight and calculate the fixed rate on the interest rate swap (which we already know is $f = 1.9873\%$.
- To begin, let's take the spot rates we are given and calculate the discount factors.

- Let's consider again the spot rates we are given and let's calculate the discount factors.

Time to maturity t	**Spot rate r**	**Discount Factor $D(t)$**	**Calculation**
0.25	1.39%	0.996537	$\frac{1}{(1+1.39\%/4)^{4\times0.25}}$
0.5	1.53%	0.992394	$\frac{1}{(1+1.53\%/4)^{4\times0.5}}$
0.75	1.76%	0.986915	$\frac{1}{(1+1.76\%/4)^{4\times0.75}}$
1.0	1.99%	0.980345	$\frac{1}{(1+1.99\%/4)^{4\times1.0}}$

- Thus, the fair rate on the fixed leg of the interest rate swap f is

$$f = 4 \times \frac{1 - D(1.0)}{D(0.25) + D(0.50) + D(0.75) + D(1.0)}$$

$$f = 4 \times \frac{1 - 0.9803}{0.9965 + 0.9924 + 0.9869 + 0.9803} = 1.9873\%$$

Example

- Suppose we need to determine the fair rate f on a three-year interest rate swap with $50 million notional.
- Suppose the yield curve today is

Time to maturity t	Spot rate $r(0, t)$
0.25	5.5%
0.5	5.62%
0.75	5.64%
1.0	5.65%
1.25	5.69%
1.5	5.74%
1.75	5.8%
2.0	5.86%
2.25	5.91%
2.5	5.96%
2.75	6.01%
3.0	6.06%

- Let's get the discount factors first.
- Let's consider again the spot rates we are given and let's calculate the discount factors.

Time to maturity t	Spot rate r	Discount Factor $D(t)$	Calculation
0.25	5.5%	0.986436	$\frac{1}{\left(1+5.5\%/4\right)^{4\times 0.25}}$
0.5	5.62%	0.972481	$\frac{1}{\left(1+5.62\%/4\right)^{4\times 0.5}}$
0.75	5.64%	0.958865	$\frac{1}{\left(1+5.64\%/4\right)^{4\times 0.75}}$
1.0	5.65%	0.94544	$\frac{1}{\left(1+5.65\%/4\right)^{4\times 1.0}}$
1.25	5.69%	0.931812	$\frac{1}{\left(1+5.69\%/4\right)^{4\times 1.25}}$
1.5	5.74%	0.918064	$\frac{1}{\left(1+5.74\%/4\right)^{4\times 1.5}}$
1.75	5.8%	0.90414	$\frac{1}{\left(1+5.8\%/4\right)^{4\times 1.75}}$
2.0	5.86%	0.890164	$\frac{1}{\left(1+5.86\%/4\right)^{4\times 2.0}}$
2.25	5.91%	0.876339	$\frac{1}{\left(1+5.91\%/4\right)^{4\times 2.25}}$
2.5	5.96%	0.862517	$\frac{1}{\left(1+5.96\%/4\right)^{4\times 2.5}}$
2.75	6.01%	0.848703	$\frac{1}{\left(1+6.01\%/4\right)^{4\times 2.75}}$
3.0	6.06%	0.834906	$\frac{1}{\left(1+6.06\%/4\right)^{4\times 3.0}}$

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- Let's get the discount factors first.
- Let's consider again the spot rates we are given and let's calculate the discount factors.
Time to maturity ``t``     | Spot rate ``r``   | Discount Factor ``D(t)`` | Calculation
------:|-------:|-------------------------------:|----------------------:
$(tVec_2[1])   | $(rVec_2[1])%  | $(roundmult(DTVec_2[1],1e-6)) | ``\$\\frac{1}{\\left(1+$(rVec_2[1])\\%/4 \\right)^{4 \\times $(tVec_2[1])}}\$``
$(tVec_2[2])     | $(rVec_2[2])% | $(roundmult(DTVec_2[2],1e-6)) | ``\$\\frac{1}{\\left(1+$(rVec_2[2])\\%/4 \\right)^{4 \\times $(tVec_2[2])}}\$``
$(tVec_2[3])   | $(rVec_2[3])%  | $(roundmult(DTVec_2[3],1e-6)) | ``\$\\frac{1}{\\left(1+$(rVec_2[3])\\%/4 \\right)^{4 \\times $(tVec_2[3])}}\$``
```

```
$(tVec_2[4])      | $(rVec_2[4])% | $(roundmult(DTVec_2[4],1e-6))| ``\$\\frac{1}{\\left(1+$(rVec_2[4])\\%/4 \\right)^{4 \\times $(tVec_2[4])}}\$``
$(tVec_2[5])      | $(rVec_2[5])% | $(roundmult(DTVec_2[5],1e-6))| ``\$\\frac{1}{\\left(1+$(rVec_2[5])\\%/4 \\right)^{4 \\times $(tVec_2[5])}}\$``
$(tVec_2[6])      | $(rVec_2[6])% | $(roundmult(DTVec_2[6],1e-6))| ``\$\\frac{1}{\\left(1+$(rVec_2[6])\\%/4 \\right)^{4 \\times $(tVec_2[6])}}\$``
$(tVec_2[7])      | $(rVec_2[7])% | $(roundmult(DTVec_2[7],1e-6))| ``\$\\frac{1}{\\left(1+$(rVec_2[7])\\%/4 \\right)^{4 \\times $(tVec_2[7])}}\$``
$(tVec_2[8])      | $(rVec_2[8])% | $(roundmult(DTVec_2[8],1e-6))| ``\$\\frac{1}{\\left(1+$(rVec_2[8])\\%/4 \\right)^{4 \\times $(tVec_2[8])}}\$``
$(tVec_2[9])      | $(rVec_2[9])% | $(roundmult(DTVec_2[9],1e-6))| ``\$\\frac{1}{\\left(1+$(rVec_2[9])\\%/4 \\right)^{4 \\times $(tVec_2[9])}}\$``
$(tVec_2[10])      | $(rVec_2[10])% | $(roundmult(DTVec_2[10],1e-6))| ``\$\\frac{1}{\\left(1+$(rVec_2[10])\\%/4 \\right)^{4 \\times $(tVec_2[10])}}\$``
$(tVec_2[11])      | $(rVec_2[11])% | $(roundmult(DTVec_2[11],1e-6))| ``\$\\frac{1}{\\left(1+$(rVec_2[11])\\%/4 \\right)^{4 \\times $(tVec_2[11])}}\$``
$(tVec_2[12])      | $(rVec_2[12])% | $(roundmult(DTVec_2[12],1e-6))| ``\$\\frac{1}{\\left(1+$(rVec_2[12])\\%/4 \\right)^{4 \\times $(tVec_2[12])}}\$``
")
```

- Next, we use that the fair swap rate f of a T-year interest rate swap (with quarterly cash flows) can be calculated using discount factors $D(t)$ as

$$f = 4 \times \frac{1-D(T)}{D(0.25)+D(0.50)+D(0.75)+\ldots+D(T)}$$

- Thus, the fair rate f on the fixed leg of the 3.0-year interest rate swap is

$$f = 4 \times \frac{1-D(3.0)}{D(0.25)+D(0.50)+\ldots+D(3.0)}$$

$$f = 4 \times \frac{1-0.8349}{0.9965+0.9924+0.9869+\ldots+0.9803} = 6.042\%$$

- The quarterly cash flows C on the fixed leg are

$$C = N \times \frac{f}{4} = 500000.0 \times \frac{6.042\%}{4} = 7552.4427$$

Valuing an Interest Rate Swap after Inception

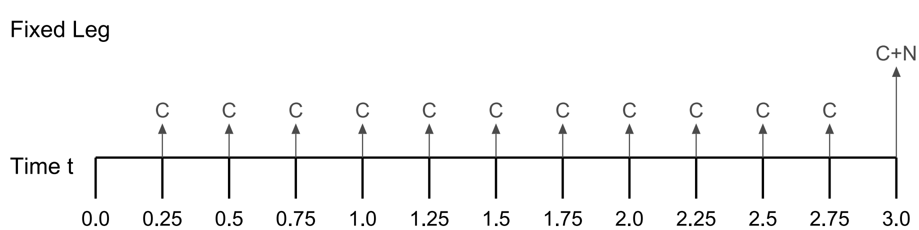
- We know that when the interest rate swap is executed (at inception of the contract), the value of the fixed leg is equal to the value of the floating leg.
- After some time passes, however, it is not necessarily the case that the values of fixed and floating remain equal.
- We will illustrate this next.

- Suppose that all spot rates increase by one percentage point in the instant right after the contract is agreed to.

Time to maturity t	**Spot rate** $r(0,t)$
0.25	6.5%
0.5	6.62%
0.75	6.64%
1.0	6.65%
1.25	6.69%
1.5	6.74%
1.75	6.8%
2.0	6.86%
2.25	6.91%
2.5	6.96%
2.75	7.01%
3.0	7.06%

- However, we have agreed to pay the fixed rate $f = 6.042\%$.
- Specifically, each quarter, we pay a fixed cash flow C equal to

$$C = N \times \frac{f}{4} = 500000.0 \times \frac{6.042\%}{4} = 7552.4427$$



- The cash flow stream C is analogous to fixed-rate coupon bond with quarterly coupon cash flows and 3.0 years to maturity.
- We calculate the present value of this cash flow stream by discounting the cash flows C using the new spot rates.
- Let's first calculate the discount factors.

- Let's get the discount factors first.
- Let's consider again the spot rates we are given and let's calculate the discount factors.

Time to maturity t	Spot rate r	Discount Factor $D(t)$	Calculation
0.25	6.5%	0.98401	$\frac{1}{\left(1+6.5\%/4\right)^{4\times0.25}}$
0.5	6.62%	0.967704	$\frac{1}{\left(1+6.62\%/4\right)^{4\times0.5}}$
0.75	6.64%	0.951809	$\frac{1}{\left(1+6.64\%/4\right)^{4\times0.75}}$
1.0	6.65%	0.936175	$\frac{1}{\left(1+6.65\%/4\right)^{4\times1.0}}$
1.25	6.69%	0.920412	$\frac{1}{\left(1+6.69\%/4\right)^{4\times1.25}}$
1.5	6.74%	0.904604	$\frac{1}{\left(1+6.74\%/4\right)^{4\times1.5}}$
1.75	6.8%	0.888696	$\frac{1}{\left(1+6.8\%/4\right)^{4\times1.75}}$
2.0	6.86%	0.872811	$\frac{1}{\left(1+6.86\%/4\right)^{4\times2.0}}$
2.25	6.91%	0.857146	$\frac{1}{\left(1+6.91\%/4\right)^{4\times2.25}}$
2.5	6.96%	0.841555	$\frac{1}{\left(1+6.96\%/4\right)^{4\times2.5}}$
2.75	7.01%	0.826046	$\frac{1}{\left(1+7.01\%/4\right)^{4\times2.75}}$
3.0	7.06%	0.810623	$\frac{1}{\left(1+7.06\%/4\right)^{4\times3.0}}$

```
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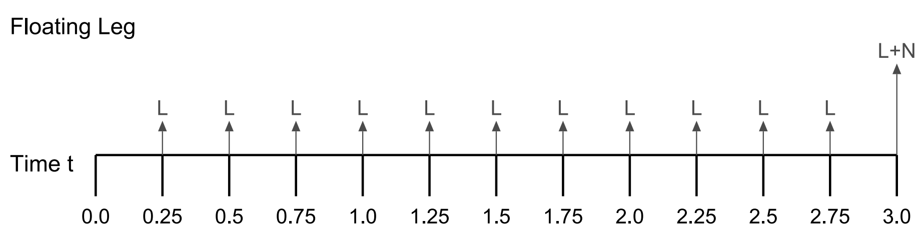
- Let's get the discount factors first.
- Let's consider again the spot rates we are given and let's calculate the discount factors.
Time to maturity ``t``      | Spot rate ``r``   | Discount Factor ``D(t)`` | Calculation
------:|-------:|------------------------------:|----------------------:
$(tVec_3[1])   | $(rVec_3[1])%  | $(roundmult(DTVec_3[1],1e-6)) | ``\$\\frac{1}{\\left(1+$(rVec_3[1])\\%/4 \\right)^{4 \\times $(tVec_3[1])}}\$``
$(tVec_3[2])     | $(rVec_3[2])% | $(roundmult(DTVec_3[2],1e-6)) | ``\$\\frac{1}{\\left(1+$(rVec_3[2])\\%/4 \\right)^{4 \\times $(tVec_3[2])}}\$``
$(tVec_3[3])    | $(rVec_3[3])%  | $(roundmult(DTVec_3[3],1e-6)) | ``\$\\frac{1}{\\left(1+$(rVec_3[3])\\%/4 \\right)^{4 \\times $(tVec_3[3])}}\$``
```

```
$`tVec_3[4]`      | $`rVec_3[4]`% | $`roundmult(DTVec_3[4],1e-6)`| ``\$\\frac{1}
{\\left(1+$`rVec_3[4]`\\%/4 \\right)^{4 \\times $`tVec_3[4]`}}\$``
$`tVec_3[5]`      | $`rVec_3[5]`% | $`roundmult(DTVec_3[5],1e-6)`| ``\$\\frac{1}
{\\left(1+$`rVec_3[5]`\\%/4 \\right)^{4 \\times $`tVec_3[5]`}}\$``
$`tVec_3[6]`      | $`rVec_3[6]`% | $`roundmult(DTVec_3[6],1e-6)`| ``\$\\frac{1}
{\\left(1+$`rVec_3[6]`\\%/4 \\right)^{4 \\times $`tVec_3[6]`}}\$``
$`tVec_3[7]`      | $`rVec_3[7]`% | $`roundmult(DTVec_3[7],1e-6)`| ``\$\\frac{1}
{\\left(1+$`rVec_3[7]`\\%/4 \\right)^{4 \\times $`tVec_3[7]`}}\$``
$`tVec_3[8]`      | $`rVec_3[8]`% | $`roundmult(DTVec_3[8],1e-6)`| ``\$\\frac{1}
{\\left(1+$`rVec_3[8]`\\%/4 \\right)^{4 \\times $`tVec_3[8]`}}\$``
$`tVec_3[9]`      | $`rVec_3[9]`% | $`roundmult(DTVec_3[9],1e-6)`| ``\$\\frac{1}
{\\left(1+$`rVec_3[9]`\\%/4 \\right)^{4 \\times $`tVec_3[9]`}}\$``
$`tVec_3[10]`      | $`rVec_3[10]`% | $`roundmult(DTVec_3[10],1e-6)`| ``\$\\frac{1}
{\\left(1+$`rVec_3[10]`\\%/4 \\right)^{4 \\times $`tVec_3[10]`}}\$``
$`tVec_3[11]`      | $`rVec_3[11]`% | $`roundmult(DTVec_3[11],1e-6)`| ``\$\\frac{1}
{\\left(1+$`rVec_3[11]`\\%/4 \\right)^{4 \\times $`tVec_3[11]`}}\$``
$`tVec_3[12]`      | $`rVec_3[12]`% | $`roundmult(DTVec_3[12],1e-6)`| ``\$\\frac{1}
{\\left(1+$`rVec_3[12]`\\%/4 \\right)^{4 \\times $`tVec_3[12]`}}\$``
")
```

$$P_{\text{Fixed}} = C \times D(0.25) + C \times D(0.5) + C \times D(0.75) + \ldots + (C + N) \times D(3.0)$$

$$P_{\text{Fixed}} = 7552.44 \times 0.984 + 7552.44 \times 0.9677 + 7552.44 \times 0.9518 + \ldots + (7552.44 + 500000.$$



- For the floating leg, we know that its value will be par at the next reset date at $t = 0.25$.
- We also know that the first cash flow on the floating leg at $t = 0.25$ is locked in at the 3-month interest rate (when we executed the contract, i.e. prior to the interest rate change)

$$L(0.25) = \frac{r_{0.25}}{4} \times N = \frac{5.5\%}{4} \times 500000.0 = 6875.0$$

- Thus, the value of the floating leg today is the present value of the first cash flow plus the present value of the par value.

$$P_{\text{Float}} = \frac{N + L(0.25)}{\left(1 + \frac{r_{0.25}}{4}\right)^{4 \times 0.25}} = \frac{500000.0 + 6875.0}{\left(1 + \frac{6.5\%}{4}\right)^{4 \times 0.25}} = 498769.9877$$

- Thus, the value of the floating leg is now $P_{\text{Float}} = 498769.99$ and the value of the fixed leg is $P_{\text{Fixed}} = 486587.6369$.
- Since we entered into the swap as the fixed rate payer, the interest rate swap now has a positive value to us.

$$P_{\text{Swap}} = 12182.35$$

- This is intuitive, since we are paying a fixed rate which was set when interest rates were lower than they are after the interest rate increase.

Another Practice Problem

- Solve for the fixed interest rate swap rate f of a three-year swap with a notional of $ 100 million. Assume that we enter the interst rate swap as the fixed rate payer.

Time to maturity t	Spot rate $r(0, t)$
0.25	4.05%
0.5	4.1%
0.75	4.25%
1.0	4.37%
1.25	4.47%
1.5	4.57%
1.75	4.65%
2.0	4.72%
2.25	4.8%
2.5	4.87%
2.75	4.94%
3.0	5.01%

- Let's get the discount factors first.

Time to maturity t	Spot rate r	Discount Factor $D(t)$	Calculation
0.25	4.05%	0.989976	$\frac{1}{(1+4.05\%/4)^{4\times0.25}}$
0.5	4.1%	0.979811	$\frac{1}{(1+4.1\%/4)^{4\times0.5}}$
0.75	4.25%	0.968791	$\frac{1}{(1+4.25\%/4)^{4\times0.75}}$
1.0	4.37%	0.957468	$\frac{1}{(1+4.37\%/4)^{4\times1.0}}$
1.25	4.47%	0.94595	$\frac{1}{(1+4.47\%/4)^{4\times1.25}}$
1.5	4.57%	0.93411	$\frac{1}{(1+4.57\%/4)^{4\times1.5}}$
1.75	4.65%	0.922281	$\frac{1}{(1+4.65\%/4)^{4\times1.75}}$
2.0	4.72%	0.910422	$\frac{1}{(1+4.72\%/4)^{4\times2.0}}$
2.25	4.8%	0.898205	$\frac{1}{(1+4.8\%/4)^{4\times2.25}}$
2.5	4.87%	0.886021	$\frac{1}{(1+4.87\%/4)^{4\times2.5}}$
2.75	4.94%	0.8737	$\frac{1}{(1+4.94\%/4)^{4\times2.75}}$
3.0	5.01%	0.861253	$\frac{1}{(1+5.01\%/4)^{4\times3.0}}$

- Next, we use that the fair swap rate f of a T-year interest rate swap (with quarterly cash flows) can be calculated using discount factors $D(t)$ as

$$f = 4 \times \frac{1 - D(T)}{D(0.25) + D(0.50) + D(0.75) + \ldots + D(T)}$$

- Thus, the fair rate f on the fixed leg of the 3.0-year interest rate swap is

$$f = 4 \times \frac{1 - D(3.0)}{D(0.25) + D(0.50) + \ldots + D(3.0)}$$

$$f = 4 \times \frac{1 - 0.8613}{0.99 + 0.9798 + 0.9688 + \ldots + 0.8613} = 4.9873\%$$

- The quarterly cash flows C on the fixed leg are

$$C = N \times \frac{f}{4} = 100.0 \times \frac{4.9873\%}{4} = 1.2468$$

- Suppose now that one year passes and that the spot rates are now

Time to maturity t	**Spot rate** $r(0,t)$
0.25	5.25%
0.5	5.49%
0.75	5.59%
1.0	5.69%
1.25	5.78%
1.5	5.86%
1.75	5.95%
2.0	6.05%
2.25	7.0%
2.5	7.05%
2.75	7.45%
3.0	7.55%

- Assume that the floating rate has just reset after the change in interest rates.
- What is the value of the interest rate swap now?

- First, note that we are locked into an interest rate swap at a fixed rate $f = 4.9873\%$. The swap has 2.0 years to maturity.
- The cash flows on the fixed leg are $C = 1.2468$ million.
- Thus, the value of the fixed leg is now

$$P_{\text{Fixed}} = C \times D(0.25) + C \times D(0.5) + C \times D(0.75) + \ldots + (C + N) \times D(2.0)$$

$$P_{\text{Fixed}} = 1.25 \times 0.987 + 1.25 \times 0.9731 + \ldots + (1.25 + 100.0) \times 0.8868 = 98.0353$$

- Since the floating rate has just reset after the change in interest rates, the value of the floating leg is par.

$$P_{\text{Float}} = N = 100$$

- Thus, the value of the interst rate swap to us as the fixed rate payer is now

$$P_{\text{Swap}} = P_{\text{Float}} - P_{\text{Fixed}} = 100 - 98.0353 = 1.9647$$

Hedging with Interest Rate Swaps

- Many firms use interest rate swaps to hedge exposures to interest rates (see e.g. GE).
- How can we use interest rate swaps to hedge interest rate risk?

- Let's think about a party that is paying fixed and receiving floating.
- This is equivalent to being long a floating rate bond and short a fixed rate bond.
- If interest rates increase:
  - The floating bond is almost unchanged in value. (It declines slightly in value.)
  - The fixed rate bond declines in value.
- If we pay fixed and receive floating, our overall change in value is positive.
  - Note that this was the case in the previous example.

- This means that if we own a portfolio of vanilla fixed coupon bonds, the value of our portfolio declines if interest rates increase.
- Fixed-for-floating interest rate swaps increase in value as interest rates increase.
- Thus, we can decrease our exposure to interest rates by entering into an interest rate swap, paying fixed and receiving floating.

Modified Duration of an Interest Rate Swap

- Our goal is to compute the duration of a three-year swap with a notional of $ 100 million.
- Assume that spot rates are as follow (quarterly compounded).

Time to maturity t	Spot rate $r(t)$
0.25	3.0%
0.5	3.2%
0.75	3.3%
1.0	3.5%
1.25	3.6%
1.5	3.6%
1.75	3.7%
2.0	3.8%
2.25	4.0%
2.5	4.1%
2.75	4.2%
3.0	4.3%

- To simplify the calculations, let's get the discount factors.

Time to maturity t	Spot rate r	Discount Factor $D(t)$	Calculation
0.25	3.0%	0.992556	$\frac{1}{(1+3.0\%/4)^{4\times0.25}}$
0.5	3.2%	0.98419	$\frac{1}{(1+3.2\%/4)^{4\times0.5}}$
0.75	3.3%	0.975653	$\frac{1}{(1+3.3\%/4)^{4\times0.75}}$
1.0	3.5%	0.965752	$\frac{1}{(1+3.5\%/4)^{4\times1.0}}$
1.25	3.6%	0.95619	$\frac{1}{(1+3.6\%/4)^{4\times1.25}}$
1.5	3.6%	0.947661	$\frac{1}{(1+3.6\%/4)^{4\times1.5}}$
1.75	3.7%	0.937581	$\frac{1}{(1+3.7\%/4)^{4\times1.75}}$
2.0	3.8%	0.927149	$\frac{1}{(1+3.8\%/4)^{4\times2.0}}$
2.25	4.0%	0.91434	$\frac{1}{(1+4.0\%/4)^{4\times2.25}}$
2.5	4.1%	0.903049	$\frac{1}{(1+4.1\%/4)^{4\times2.5}}$
2.75	4.2%	0.891457	$\frac{1}{(1+4.2\%/4)^{4\times2.75}}$
3.0	4.3%	0.879579	$\frac{1}{(1+4.3\%/4)^{4\times3.0}}$

- First, let's solve for the fixed interest rate swap rate f. Assume that we enter the interst rate swap as the fixed rate payer.
- We use that the fair swap rate f of a T-year interest rate swap (with quarterly cash flows) can be calculated using discount factors $D(t)$ as

$$f = 4 \times \frac{1 - D(T)}{D(0.25) + D(0.50) + D(0.75) + \ldots + D(T)}$$

- Thus, the fair rate f on the fixed leg of the 3.0-year interest rate swap is

$$f = 4 \times \frac{1 - D(3.0)}{D(0.25) + D(0.50) + \ldots + D(3.0)}$$

$$f = 4 \times \frac{1 - 0.8796}{0.9926 + 0.9842 + 0.9757 + \ldots + 0.8796} = 4.2721\%$$

- The quarterly cash flows C on the fixed leg are

$$C = N \times \frac{f}{4} = 100.0 \times \frac{4.2721\%}{4} = 1.068$$

- Note that as the fixed rate payer, we are essentially short a bond with quarterly coupon cash flows of C, and we are long one floating rate bond with face value equal to the notional N of the interest rate swap.
- Moreover, since the interest rate swap is faily valued at inception, the value of the floating leg is equal to the fixed leg. Thus, the market values are both equal to par.
- We can illustrate this using a balance sheet

Assets (Long position)	Liabilities (Shot position)
`Floating rate note: $ 100`	`3-year bond, coupon rate` f`: $ 100`

- Thus, an interest rate swap is essentially a portfolio of an FRN and fixed-rate bond.
- We know that the modified duration of a portfolio is the value-weighted average of the bonds in the portfolio.
- First, let's determine the modified duration of the FRN.

- Recall that the modified duration is given by

$$MD = -\frac{P(y + \Delta y) - P(y - \Delta y)}{2 \times \Delta y} \times \frac{1}{P(y)}$$

- For the floating leg of the swap, we again use that its value will be par on the next reset and that the next floating coupon cash flow is already set today at

$$L(0.25) = \frac{r_{0.25}}{4} \times N = \frac{3.0\%}{4} \times 100 = 0.75$$

- Hence,

$$P_{\text{Float}}(y) = N = 100$$

$$P_{\text{Float}}(y + \Delta y) = \frac{N + L(0.25)}{\left(1 + \frac{3.0\% + 0.1\%}{4}\right)} = \frac{100 + 0.75}{\left(1 + \frac{3.0\% + 0.1\%}{4}\right)} = 99.9752$$

$$P_{\text{Float}}(y - \Delta y) = \frac{N + L(0.25)}{\left(1 + \frac{3.0\% - 0.1\%}{4}\right)} = \frac{100 + 0.75}{\left(1 + \frac{3.0\% - 0.1\%}{4}\right)} = 100.0248$$

$$MD_{\text{Float}} = -\frac{99.9752 - 100.0248}{2 \times 0.001} \times \frac{1}{100} = 0.2481$$

- Next, let's determine the modified duration of the fixed leg of the swap.
- Recall that the fixed leg is similar to a fixed-rate coupon rate bond with quarterly coupon cash flows at an annual rate of $f = 4.2721\%$.
  - We have already calculated $C = N \times \frac{f}{4} = 100.0 \times \frac{4.2721\%}{4} = 1.068$

- The current value of the fixed leg at inception of the interest rate swap is par.

$$P_{\text{Fixed}}(y) = N = 100$$

$$P_{\text{Fixed}}(y + \Delta y) = \frac{C}{\left(1 + \frac{r_{0.25} + 0.1\%}{4}\right)^{4 \times 0.25}} + \ldots + \frac{C + N}{\left(1 + \frac{r_{3.0} + 0.1\%}{4}\right)^{4 \times 3.0}} = 99.7204$$

$$P_{\text{Fixed}}(y - \Delta y) = \frac{C}{\left(1 + \frac{r_{0.25} + 0.1\%}{4}\right)^{4 \times 0.25}} + \ldots + \frac{C + N}{\left(1 + \frac{r_{3.0} + 0.1\%}{4}\right)^{4 \times 3.0}} = 100.2805$$

$$MD_{\text{Fixed}} = -\frac{99.7204 - 100.2805}{2 \times 0.001} \times \frac{1}{100} = 2.8006$$

- Knowing the modified durations of the fixed leg and the floating leg of the swap, we can now calculate the modified duration MD_{Swap}.

$$MD_{\text{Swap}} = MD_{\text{Float}} - MD_{\text{Fixed}} = 0.2481 - 2.8006 = -2.5525$$

- Let's now use the interest rate swap to hedge a position in a three-year bond with coupon rate $c = 6\%$ (paid quarterly). The face value of the bond is $ 100.
- First, we need the modified duration MD_{Bond} of the bond.

$$P_{\text{Bond}}(y) = \frac{1.5}{\left(1+\frac{0.03}{4}\right)^{4\times 0.25}} + \frac{1.5}{\left(1+\frac{0.032}{4}\right)^{4\times 0.5}} + \ldots + \frac{1.5+100}{\left(1+\frac{0.043}{4}\right)^{4\times 3}} = 104.8707$$

$$P_{\text{Bond}}(y+\Delta y) = \frac{1.5}{\left(1+\frac{0.03+0.001}{4}\right)^{4\times 0.25}} + \frac{1.5}{\left(1+\frac{0.032+0.001}{4}\right)^{4\times 0.5}} + \ldots + \frac{1.5+100}{\left(1+\frac{0.043+0.001}{4}\right)^{4\times 3}}$$

$$P_{\text{Bond}}(y-\Delta y) = \frac{1.5}{\left(1+\frac{0.03-0.001}{4}\right)^{4\times 0.25}} + \frac{1.5}{\left(1+\frac{0.032-0.001}{4}\right)^{4\times 0.5}} + \ldots + \frac{1.5+100}{\left(1+\frac{0.043-0.001}{4}\right)^{4\times 3}}$$

$$MD_{\text{Bond}} = -\frac{104.5834-105.1589}{2\times 0.001} \times \frac{1}{104.8707} = 2.7438$$

- To hedge the bond using the interest rate swap, we require that change in the market value of the bond is offset by the change in value of the interest rate swap.
- Suppose the interst rate swap has notional $N = x$.
- Thus,

$$P_{\text{Bond}} \times MD_{\text{Bond}} \times \Delta y + x \times MD_{\text{Swap}} \times \Delta y \overset{!}{=} 0$$

- Hence, the notional N of the swap must be equal to

$$x = -\frac{P_{\text{Bond}} \times MD_{\text{Bond}}}{MD_{\text{Swap}}} = -\frac{104.8707 \times 2.7438}{-2.5525} = 112.7304$$

- Finally, let’s check how well the hedge works.
- Suppose now that all spot rates increase by 0.5 percent.

Time to maturity t	**Spot rate** $r(t)$
0.25	3.5%
0.5	3.7%
0.75	3.8%
1.0	4.0%
1.25	4.1%
1.5	4.1%
1.75	4.2%
2.0	4.3%
2.25	4.5%
2.5	4.6%
2.75	4.7%
3.0	4.8%

- The value of the bond is now

$$P = \frac{1.5}{\left(1 + \frac{0.035}{4}\right)^{4\times 0.25}} + \ldots + \frac{1.5 + 100}{\left(1 + \frac{0.048}{4}\right)^{4\times 3}} = 103.4432$$

- Before the change in interest rates, the bond price was 104.8707. Thus the change in value is

$$\Delta P_{\text{Bond}} = 103.4432 - 104.8707 = -1.4275$$

- The **floating leg** of the interest rate swap changes as follows.
  - Note that we entered into \$ 112.73 in notional of the interest rate swap, *not* 100.
  - To simplify the calculation, we will start with a \$ 100 notional. Then, we will make an adjustment. Basically, we use that a \$ 112.73 notional are 1.1273 units of \$ 100 notional.

$$\text{Price per 100 notional} = \frac{100 + 100 \times \frac{0.03}{4}}{1 + \frac{0.035}{4}} = 99.8761$$

$$\text{Price per 112.73 notional} = \frac{112.73}{100} \times \text{Price per 100 notional} = 112.73$$

- Recall that the previous market value of the floating leg was \$ 112.73.
- Thus, the change in value of the floating leg in the interest rate swap is

$$\Delta P_{\text{Float}} = 112.59 - 112.73 = -0.1397$$

- The **fixed leg** of the interest rate swap changes as follows.
  - Note that we entered into \$ 112.73 in notional of the interest rate swap, *not* 100.
  - To simplify the calculation, we will start with a \$ 100 notional. Then, we will make an adjustment. Basically, we use that a \$ 112.73 notional are 1.1273 units of \$ 100 notional.

$$\text{Price per 100 notional} = \frac{1.06802}{1 + \frac{0.035}{4}} + \ldots \frac{1.06802 + 100}{\left(1 + \frac{0.048}{4}\right)^{1}2} = 99.61068$$

$$\text{Price per 112.73 notional} = \frac{112.73}{100} \times \text{Price per 100 notional} = 111.1638$$

- The previous value of the fixed leg was \$ 112.73.
- Thus, the change in value of the fixed leg is

$$\Delta P_{\text{Fixed}} = 111.1638 - 112.73 = -1.566$$

- The total change in the swap and the bond is

$$(-1.4275 - 0.1397) - (-1.566) = -0.0012$$

- We can compare this to the change in the bond price without the interest rate swap hedge.

$$\Delta P_{\text{Bond}} = -1.4275$$

- Thus, by using an interest rate swap, we are able to reduce the change in the value of the bond when interest rates change.

Wrap-Up

Goals for today

- ☑ Understanding what interest rate swaps and floating rate bonds are.
- ☑ Calculating the fair fixed rate in an interest rate swap contract.
- ☑ Calculating the price and modified duration of a floating rate bond.
- ☑ Relating interest rate swaps, floating rate bonds, and fixed coupon bonds.
- ☑ Calculating the value of an interest rate swap position after inception of the contract.
- ☑ Using interest rate swaps to hedge interest rate exposure.

Reading

Fabozzi, Fabozzi, 2021, Bond Markets, Analysis, and Strategies, 10th Edition
Chapter 29