present

FINC 462/662 - Fixed Income Securities

FINC-462/662: Fixed Income Securities

Term Structure of Interest Rates

Spring 2022

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Overview

Goals for today

- ☐ Understand what the term structure of interest rates means.
- ☐ Know how to use the term structure of interest rates to price bonds.
- ☐ Understand how to bootstrap the yield curve.
- ☐ Know how to use discount factors to price bonds.
- ☐ Understand how to price a bond by replication.

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Valuing Treasury Notes/Bonds

- Recall that to calculate the price P of a Treasury note/bond with T years to maturity, we need to calculate the present values (PV) of
  - all coupon cash flows
  - and the principal cash flow at maturity

$$P = \text{PV}(\text{Coupon cash flows}) + \text{PV}(\text{Par value})$$

- Specifically, given a discount rate r, and assuming that the bond pays semi-annual coupon cash flows (face value of 100), we calculate the bond price as follows:

$$P = \frac{C}{\left(1+\frac{r}{2}\right)^{2\times 0.5}} + \frac{C}{\left(1+\frac{r}{2}\right)^{2\times 1.0}} + \frac{C}{\left(1+\frac{r}{2}\right)^{2\times 1.5}} + \frac{C}{\left(1+\frac{r}{2}\right)^{2\times 2.0}} + \ldots + \frac{100+C}{\left(1+\frac{r}{2}\right)^{2\times T}}$$

- We assumed that there is just one interest rate r that we can use to discount all future cash flows.
- In reality, there are different interest rates r, for instance for different time horizons (e.g. 1-year vs. 5-year interest rates) and credit risk (e.g. Treasury securities vs. corporate bonds).

First, we are going to focus on the time dimension (and consider credit risk later).

- The idea is that because the coupon cash flows happen at different times in the future, it is not appropriate to use the same interest rate to discount all cash flows.
- Each cash flow should be discounted at a unique rate appropriate for the time period in which the cash flow will be received.
- This means that for each time-t cash flow in the future, there is a corresponding interest rate r.
- We will write the interest rate for time t as $r(t)$ or simply as r_t.
- When we plot the relation between time on the horizontal axis and the corresponding interest rate on the vertical axis, this is referred to as the **Term Structure of Interest Rates**.

- Thus, given interest rates $r(t)$, and assuming that the bond pays semi-annual coupon cash flows (face value of 100), we calculate the bond price as follows:

$$P = \frac{C}{\left(1+\frac{r_{0.5}}{2}\right)^{2\times 0.5}} + \frac{C}{\left(1+\frac{r_{1.0}}{2}\right)^{2\times 1.0}} + \frac{C}{\left(1+\frac{r_{1.5}}{2}\right)^{2\times 1.5}} + \frac{C}{\left(1+\frac{r_{2.0}}{2}\right)^{2\times 2.0}} + \ldots + \frac{100 + C}{\left(1+\frac{r_T}{2}\right)^{2}}$$

- We refer to the interest rate $r(t)$ as **zero-coupon yields** (sometimes also referred to as **spot rates**).
- Why?

- Recall that the price of a zero-coupon bond with T years to maturity and face value of C and semi-annually-compounded yield y_T is

$$P_T = \frac{C}{(1+\frac{y_T}{2})^{2T}}$$

- For example, the price of a 1-year zero-coupon bond is

$$P_1 = \frac{C}{(1+\frac{y_1}{2})^{2\times 1.0}}$$

- Let's compare this to the second term in the equation for P (on the previous slide).

$$\frac{C}{\left(1+\frac{r_{1.0}}{2}\right)^{2\times 1.0}} \qquad\qquad \frac{C}{(1+\frac{y_1}{2})^{2\times 1.0}}$$

- Thus, we see that the correct discount rate that we need to use to calculate the present value of the 1-year coupon cash flow r_1 is equal to the yield of a one-year zero-coupon bond y_1.

$$r_1 = y_1$$

- This is also true for all t

$$r_t = y_t$$

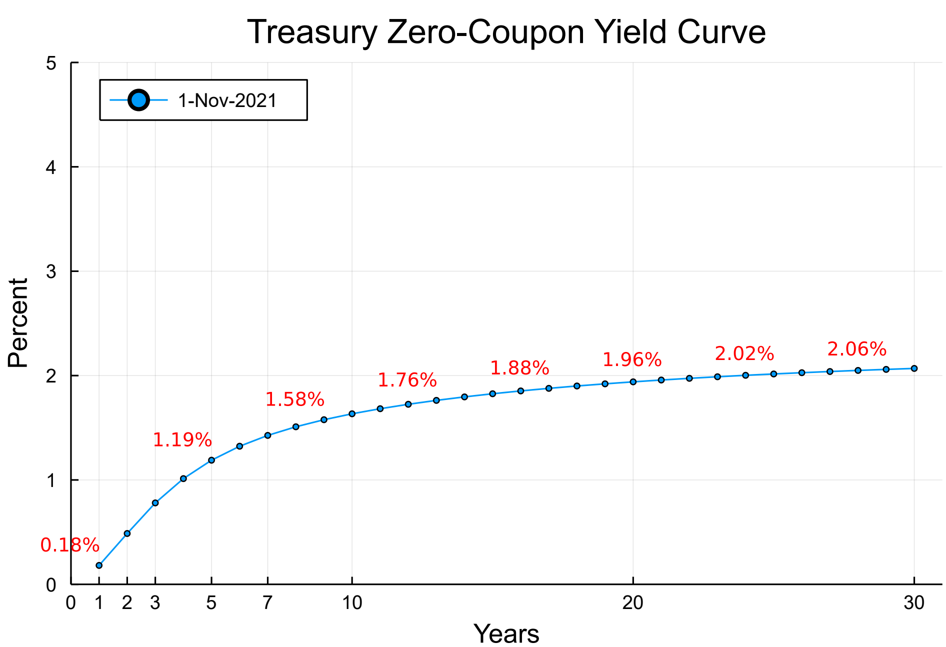
- This means that in order find values for $r(t)$, we need to use market data on zero-coupon yields.
- When we collect market data of Treasury zero-coupon yields y_t and make a graph with time t on the horizontal axis and the corresponding yield on the vertical axis, we call this the "Term Structure of Interest Rates" or "Zero-Coupon Yield Curve."

Term Structure of Interest Rates

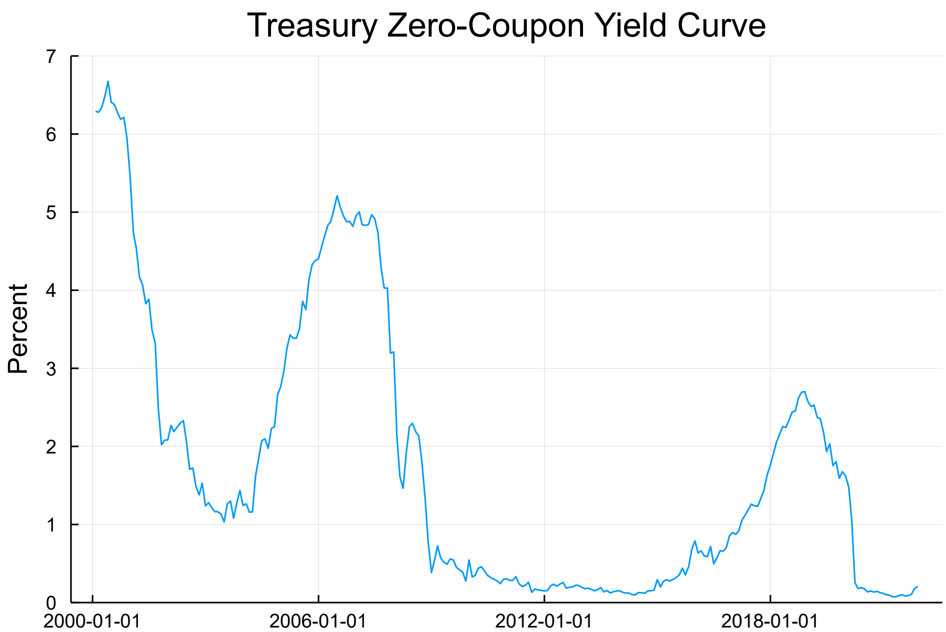
- Let's look at the term structure of zero-coupon interest rates of Treasury securities.
- The Federal Reserve provides zero-coupon yield curves on its **webpage**.

	Date	Year	Month	ZC_1	ZC_2	ZC_3	ZC_4	ZC_5
1	2000-01-31	2000	1	6.29348	6.54579	6.60405	6.6172	6.6264
2	2000-02-29	2000	2	6.27814	6.45442	6.50633	6.5328	6.55073
3	2000-03-31	2000	3	6.34299	6.40947	6.35167	6.28421	6.23502
4	2000-04-30	2000	4	6.48637	6.58408	6.53832	6.48099	6.43627
5	2000-05-31	2000	5	6.67495	6.65021	6.55047	6.49584	6.46749
6	2000-06-30	2000	6	6.40646	6.34298	6.23561	6.17505	6.15107
7	2000-07-31	2000	7	6.37748	6.2787	6.18816	6.13717	6.113
8	2000-08-31	2000	8	6.27304	6.114	5.99706	5.93223	5.90172
9	2000-09-30	2000	9	6.1877	5.94101	5.81337	5.77679	5.78955
10	2000-10-31	2000	10	6.21425	5.90783	5.77525	5.73961	5.7465
more								
263	2021-11-30	2021	11	0.2076	0.5661	0.872	1.0881	1.2387

- Date: 31-Oct-2021



- Date: 1 Year



Example

- Using the zero-coupon yield curve on October 31, 2021, let's now price a 2.5-year Treasury note. Suppose the Treasury note has a coupon rate of 2% (paid-semiannually) and face value of $100.
- On 31 October, 2021, the zero-coupon yield curve out to five years is

t [years]	r_t [%]
0.5	0.09
1.0	0.18
1.5	0.33
2.0	0.49
2.5	0.63
3.0	0.78
3.5	0.90
4.0	1.01
4.5	1.10
5.0	1.19

- The semi-annual coupon cash flows are $C = \frac{2\%}{2} \times 100 = 1$
- To price the Treasury note, we discount all coupon cash flows and the principal cash flow at maturity by the corresponding discount rate $r(t)$.

$$P = \frac{C}{\left(1+\frac{r_{0.5}}{2}\right)^{2\times 0.5}} + \frac{C}{\left(1+\frac{r_{1.0}}{2}\right)^{2\times 1.0}} + \frac{C}{\left(1+\frac{r_{1.5}}{2}\right)^{2\times 1.5}} + \frac{C}{\left(1+\frac{r_{2.0}}{2}\right)^{2\times 2.0}} + \frac{100+C}{\left(1+\frac{r_{2.5}}{2}\right)^{2\times 2.5}}$$

- Plugging in the values for r_t

$$P = \frac{1.0}{\left(1+\frac{0.09\%}{2}\right)^{2\times 0.5}} + \frac{1.0}{\left(1+\frac{0.18\%}{2}\right)^{2\times 1.0}} + \frac{1.0}{\left(1+\frac{0.33\%}{2}\right)^{2\times 1.5}} + \frac{1.0}{\left(1+\frac{0.49\%}{2}\right)^{2\times 2.0}} + \frac{}{\left(1+\right.}$$

- The result is

$$P = \$104.200513$$

Concept Question When we calculate the price of a bond, we calculate the

1. Full price.
2. Flat price.
3. Accrued interest.

Hint

Practice Problem Calculate the price of a coupon bond with the following terms: $1000 face value, 5% coupon rate (paid semi-annually), 3 years to maturity.

Time t	Yield [%]
0.5	0.06
1	0.120018
1.5	0.195093
2	0.270249
2.5	0.335465
3	0.400790

▶ [Click to open solution]

Discount Factors

- For long-term bonds (time to maturity T > 10 years), it becomes tedious to calculate the individual terms

$$\frac{C}{\left(1+\frac{r_T}{2}\right)^{2\times T}}$$

- We can use a short-cut using so-called **Discount Factors**.
- Then, we can write these terms simply as

$$C \times D(T)$$

- Specifically, the bond pricing equation becomes

$$P = C \times D(0.5) + C \times D(1.0) + C \times D(1.5) + \ldots + (C + 100) \times D(T)$$

- How do we get the discount factors $D(t)$?
- Compare the previous equation to the pricing equation we started with

$$P = \frac{C}{\left(1+\frac{r}{2}\right)^{2\times 0.5}} + \frac{C}{\left(1+\frac{r}{2}\right)^{2\times 1.0}} + \frac{C}{\left(1+\frac{r}{2}\right)^{2\times 1.5}} + \frac{C}{\left(1+\frac{r}{2}\right)^{2\times 2.0}} + \ldots + \frac{100+C}{\left(1+\frac{r}{2}\right)^{2\times T}}$$

- By comparing the terms, we get that

$$D(t) = \frac{1}{\left(1+\frac{r_t}{2}\right)^{2\times t}}$$

- Intuitively, what are discount factors?
- Let's look at the present value of $1 to be received in 1-year (assuming semi-annual compounding) when the discount rate is r_1.

$$PV = \frac{1}{(1+\frac{r_1}{2})^{2\times 1}}$$

- Let's compare this to the one-year discount factor $D(1)$.

$$D(1) = \frac{1}{(1+\frac{r_1}{2})^{2\times 1}}$$

- Since the right-hand sides are equal, we note that the discount factor $D(T)$ is the present value of $1 to be received at time T.
    - For instance, $D(3)$ is the value today of $1 paid in three years.

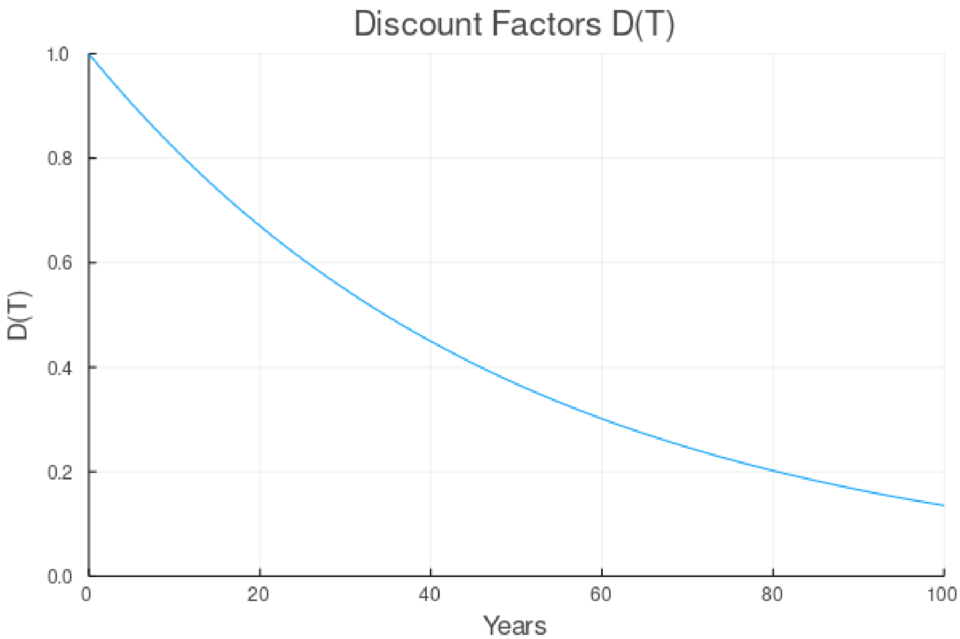
- What are the properties of discount factors $D(T)$?

1. D(T) will usually be less than 1, since the present value of $1 to be received in the future will in general be less than $1 today.
2. D(T) becomes smaller as T increases because the value today of $1 to received goes down the further in the future it is received.
3. Under continuous compounding, when the discount rate is r_T, the discount factor is

$$D(T) = \exp(-r_T \times T)$$

Example

- Discount rate r_T [% p.a.]: 2.0



Example

- To illustrate, let's consider the previous example, where we calculated the price of a 2.5-year Treasury note with coupon rate of 2% (paid-semiannually).
- Suppose now that instead of zero-coupon yields, we have discount factors.

t **[years]**	$D(t)$
0.5	0.9998
1.0	0.9991
1.5	0.9975
2.0	0.9951
2.5	0.9922

- Then, we calculate the bond price as follows.

$$P = C \times D(0.5) + C \times D(1.0) + C \times D(1.5) + C \times D(2.0) + (C + 100) \times D(2.5)$$

$$\downarrow$$

$$P = 1.0 \times 0.9998 + 1.0 \times 0.9991 + 1.0 \times 0.9975 + 1.0 \times 0.9951 + 101.0 \times 0.9922$$

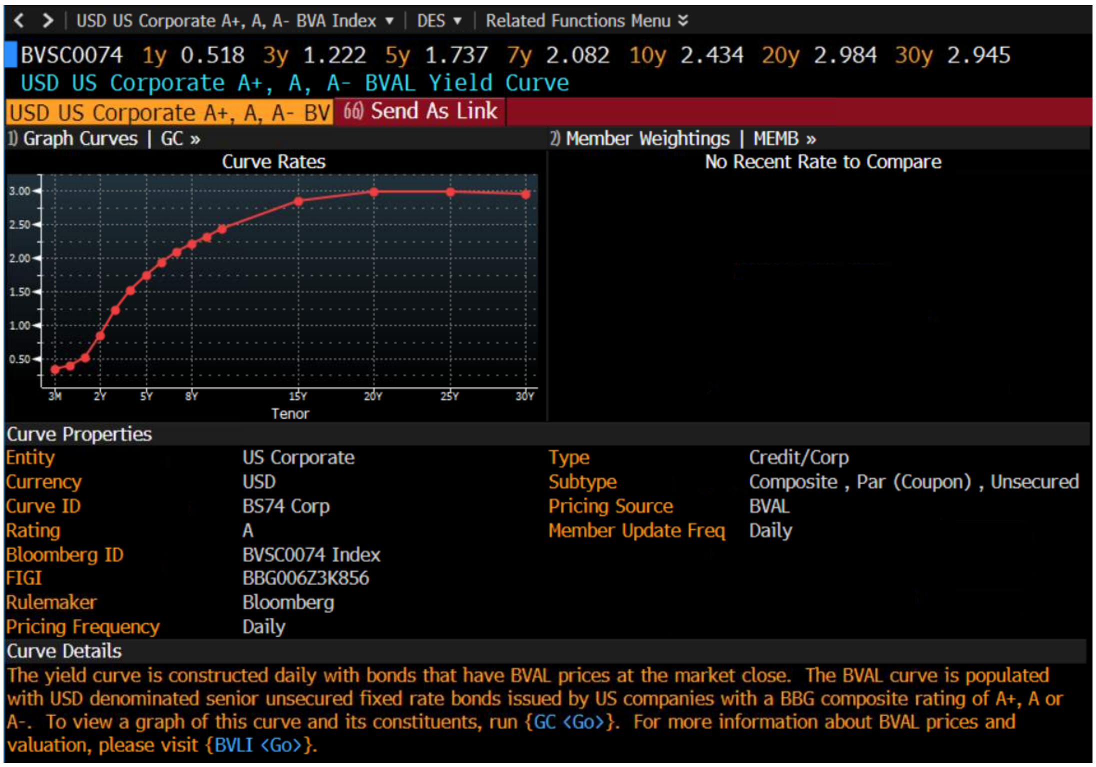
$$\downarrow$$

$$P = \$104.200513$$

Par Yields

- In the case of Treasury securities, we were given a zero-coupon yield curve.
- However, for corporate bonds, for instance, we typically do not observe zero-coupon yields directly.
- Instead, we can observe the **yields-to-maturity** of coupon bonds.
- In general, these are **not** zero-coupon yields that we can use to discount cash flows.
- Thus, we need a technique to get **zero-coupon yields** from yields of coupon bonds.
- This technique is referred to as **Boot-Strapping**.
- Before we discuss boot strapping, we need to talk about **par yields**, because the yield curves we observe in the market are often **par yield curves.**

Example: Corporate Bond Par Yield Curve



- What are **par yields?**
- The **Par Yield** is the yield to maturity of bonds that trade at **par** value.
  - To **trade** at par just means that the price is equal to par value, e.g.. the bond price is P=$100 if the bond has face value of $100.
- The **par yield curve** plots yield to maturity against term to maturity for current bonds trading at par.

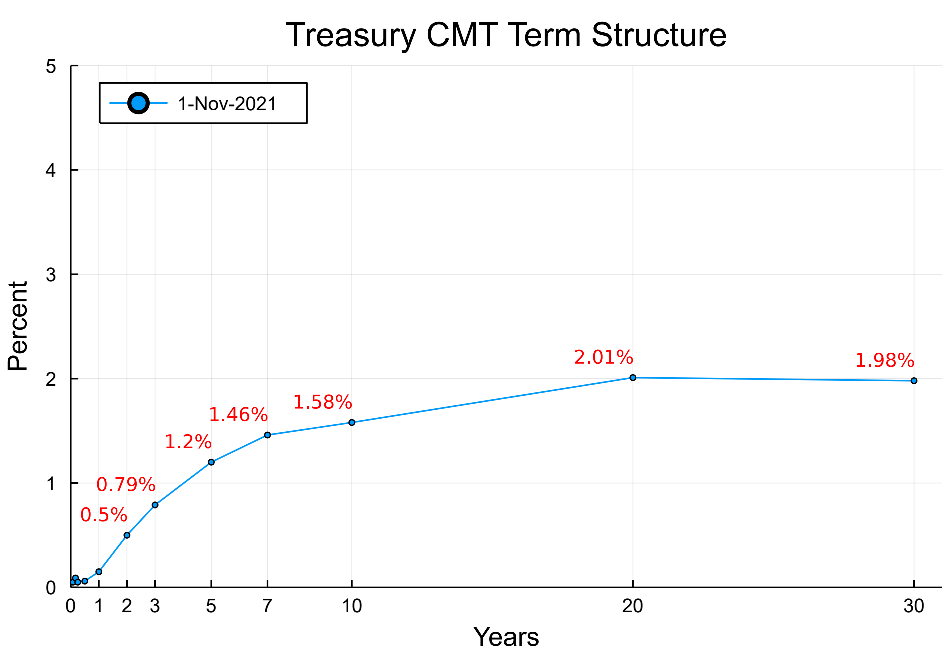
Treasury Constant Maturity Yield Curve

- Let's start by looking at one example of a **par yield curve** – the so-called **Treasury Constant Maturity (CMT)** yield curve.
- The Federal Reserve provides daily term structures on its **webpage**.
    - This specific term structure is referred as the "Constant Maturity Treasury" (CMT) yield curve.

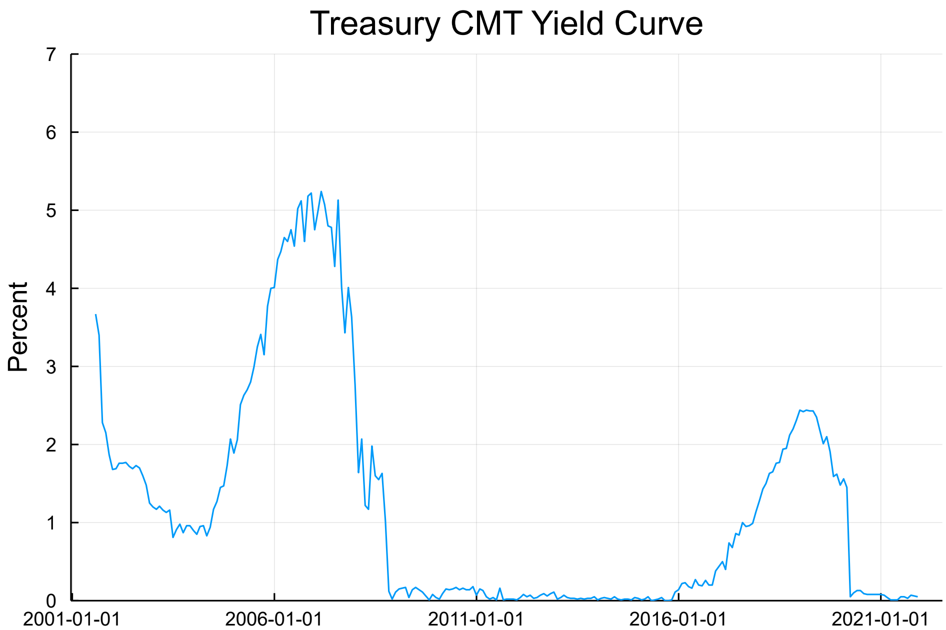
- Treasury Constant Maturity Curve on November 1, 2021.

Tenor	Time t	r [%]
1 Month	1/12=0.08	0.05
2 Months	2/12=0.17	0.09
3 Months	3/12=0.25	0.05
6 Months	6/12=0.50	0.06
1 Year	1.00	0.15
2 Years	2.00	0.50
3 Years	3.00	0.79
5 Years	5.00	1.20
7 Years	7.00	1.46
10 Years	10.00	1.58
20 Years	20.00	2.01
30 Years	30.00	1.98

- Date: 1-Nov-2021



- Date: 1 Month



- Treasury CMT rates at the St. Louis Fed: **CMT**

Bond prices and Yields

- Recall that the **par yield** is the *yield to maturity* of bonds that trade at **par** value, i.e. the bond price is equal to 100 (for bonds with 100 face value).
- To get a sense of how the bond price and the yield to maturity are related, let's consider a Treasury note with maturity in T years, coupon rate of c (paid semi-annually), principal value of $100 and yield to maturity y.

- Time to maturity T [years]: 5.0
- Coupon rate c [% p.a.]: 4.0
- Yield to maturity y [% p.a.]: 3.0

Reset

	Time	DiscountFactor	CashFlow	PresentValue	Calculation
1	0.5	0.985222	2.0	1.97044	"2.0 * 1/(1+3.0%/2)^(2*0.5)=1.9704"
2	1.0	0.970662	2.0	1.94132	"2.0 * 1/(1+3.0%/2)^(2*1.0)=1.9413"
3	1.5	0.956317	2.0	1.91263	"2.0 * 1/(1+3.0%/2)^(2*1.5)=1.9126"
4	2.0	0.942184	2.0	1.88437	"2.0 * 1/(1+3.0%/2)^(2*2.0)=1.8844"
5	2.5	0.92826	2.0	1.85652	"2.0 * 1/(1+3.0%/2)^(2*2.5)=1.8565"
6	3.0	0.914542	2.0	1.82908	"2.0 * 1/(1+3.0%/2)^(2*3.0)=1.8291"
7	3.5	0.901027	2.0	1.80205	"2.0 * 1/(1+3.0%/2)^(2*3.5)=1.8021"
8	4.0	0.887711	2.0	1.77542	"2.0 * 1/(1+3.0%/2)^(2*4.0)=1.7754"
9	4.5	0.874592	2.0	1.74918	"2.0 * 1/(1+3.0%/2)^(2*4.5)=1.7492"
10	5.0	0.861667	102.0	87.8901	"102.0 * 1/(1+3.0%/2)^(2*5.0)=87.89

- Bond Price: $ 104.6111

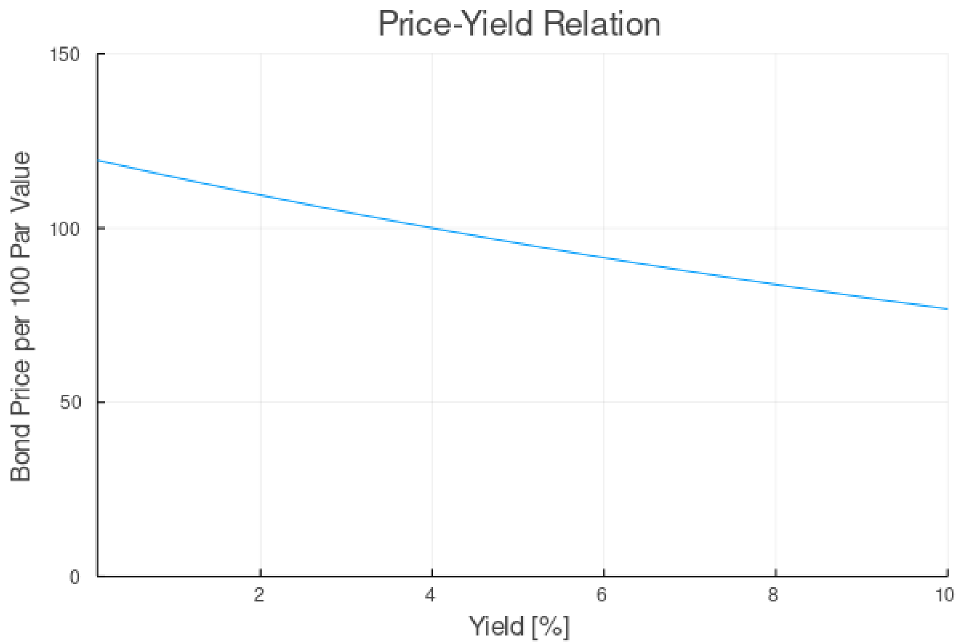
Bond Price-Yield Relation

- By varying the yield to maturity y and the coupon rate c, we notice the following:
- When the yield is **equal** to the coupon rate, the price of the bond is *equal* to its par value.
  - This is called a **par bond**, and the yield is called the **par yield**.
- If the yield is **greater** than the coupon rate, the price is *less* than par value.
  - This is called a **discount bond**, and the bond is *trading at a discount*.
- If the yield is **less** than the coupon rate, the price is *greater* than par, and
  - This is called a **premium bond**, and the bond is trading at a *premium*.

- Next, let's plot the yield to maturity on the horizontal axis and the bond price on the vertical axis.
- This is called the **price-yield relation**.

- Time to maturity T [years]: 5.0
- Coupon rate c [% p.a.]: 4.0

Reset



- We see that as the yield increases, the bond price decreases.
- This is referred to as **inverse** relation between prices and yields.
  - It just means that prices and yields move in opposite directions.
- We also see that the relation between prices and yields is not a straight line, but the relation has curvature. It is **convex**.

Example from the CFA Exam

Bond	Price	Coupon Rate	Time-to-Maturity
A	101.886	5%	2 years
B	100.000	6%	2 years
C	97.327	5%	3 years

Which bond offers the lowest yield to maturity?

Source: Petitt, Pinto, and Pirie (2015)

Hint

Bootstrapping the zero-coupon yield curve

- Now that we understand what **par yields** are and how prices and yields are related, let's now answer the question how we can calculate the **zero-coupon yield curve** from a **par yield curve**.
- To illustrate, let's consider the Treasury par yield curve on November 25, 2021 out to 5-years to maturity.

t [years]	c_t [%]
0.5	0.095
1.0	0.213
1.5	0.437
2.0	0.643
2.5	0.806
3.0	0.965
3.5	1.063
4.0	1.155
4.5	1.248
5.0	1.344

- **Note:** The column with the par yield curve rates is labeled c_t.
    - Recall that this is because the par yields are coupon rates of Treasury notes trading at par.
    - When a Treasury note is trading at par its yield is the same as its coupon rate.

- To **boostrap** the zero-coupon yield curve form the par yield curve above, we proceed in steps.
    - First, we calculate the discount factor $D(0.5)$ for the 6-month (t=0.5) maturity.
    - Next, we calculate the discount factor $D(1.0)$ for the 1-year (t=1) maturity.
    - Then, we calculate the discount factor $D(1.5)$ for the 1.5-year (t=1) maturity.
    - We continue this procedure until and including the 5-year maturity (in this example).
- In the last step, we convert all discount factors $D(t)$ to zero-coupon yields
    - We know

$$D(t) = \frac{1}{(1 + \frac{r_t}{2})^{2 \times t}}$$

thus

$$r_t = 2 \times \left(\left(\frac{1}{D(t)} \right)^{\frac{1}{2 \times t}} - 1 \right)$$

- **Step 1**: $t = 0.5$
    - The par yield is 0.095%. This means a Treasury coupon bond with a coupon rate of $c = 0.095\%$ has a price $P = \$100$.
    - Since coupon cash flows are semi-annual, the six-month bond has one remaining cash flow in $t = 0.5$ years of principal $F = 100$ plus coupon $C = \frac{0.095\%}{2} \times 100 = 0.0475$.
    - We are looking for the discount factor $D(0.5)$ that sets the present value of the Treasury note's final cash flow of 100.0475 equal to its price of 100.

$$100 \stackrel{!}{=} D(0.5) \times 100.0475 \rightarrow D(0.5) = 0.999525$$

- **Step 2**: $t = 1.0$
    - The par yield is 0.213%. This means a Treasury coupon bond with a coupon rate of $c = 0.213\%$ has a price $P = \$100$.
    - Since coupon cash flows are semi-annual, the one-year bond has two remaining cash flows in $t = 0.5$ years of $C = 0.1065$ and one in $t = 1.0$ year of principal plus coupon $F + C = 100.1065$.

$$100 \overset{!}{=} D(0.5) \times 0.1065 + D(1.0) \times 100.1065$$

- We use $D(0.5) = 0.999525$ from the first step.

$$100 \overset{!}{=} 0.999525 \times 0.1065 + D(1.0) \times 100.1065$$

$$\to D(1.0) = 0.997873$$

- **Step 3**: $t = 1.5$
    - The par yield is 0.437%. This means a Treasury coupon bond with a coupon rate of $c = 0.437\%$ has a price $P = \$100$.
    - Since coupon cash flows are semi-annual, the 1.5-year bond has three remaining cash flows. Two cash flows of $C = 0.2185$ in $t = 0.5$ and $t = 1.0$ years and one in $t = 1.5$ year of principal plus coupon $F + C = 100.2185$.

$$100 \overset{!}{=} D(0.5) \times 0.2185 + D(1.0) \times 0.2185 + D(1.5) \times 100.2185$$

- We use $D(0.5) = 0.999525$ from the first step.
- And we use $D(1.0) = 0.997873$ from the second step.

$$100 \overset{!}{=} 0.999525 \times 0.2185 + 0.997873 \times 0.2185 + D(1.5) \times 100.2185$$

$$\to D(1.5) = 0.993465$$

- We continue until $t = 5$ and in doing so, we get

	Time	DT
1	0.5	0.999525
2	1.0	0.997873
3	1.5	0.993465
4	2.0	0.98721
5	2.5	0.980019
6	3.0	0.97139
7	3.5	0.963365
8	4.0	0.954681
9	4.5	0.945134
10	5.0	0.934633

- Finally, we just need to get the zero coupon yields using the equation

$$r_t = 2 \times \left(\left(\frac{1}{D(t)} \right)^{\frac{1}{2 \times t}} - 1 \right)$$

- For instance, the $t = 0.5$ year zero-coupon yield is

$$r_{0.5} = 2 \times \left(\left(\frac{1}{D(0.5)} \right)^{\frac{1}{2 \times 0.5}} - 1 \right)$$

$$r_{0.5} = 2 \times \left(\left(\frac{1}{0.999525} \right)^{\frac{1}{2 \times 0.5}} - 1 \right) = 0.095\%$$

- Continuing with $t = 1.0 \ldots 5.0$, we get r_t for $t = 0.5, \ldots, 5$.
  - Note the value for r_t shown in the table below are in percent.

	Time	r_t
1	0.5	0.095
2	1.0	0.213063
3	1.5	0.437577
4	2.0	0.644636
5	2.5	0.808969
6	3.0	0.969911
7	3.5	1.06923
8	4.0	1.16283
9	4.5	1.25791
10	5.0	1.35662

Practice Problem

Bootstrap the zero-coupon yield curve out to five years (in 6-month intervals).

T	Maturity date	Coupon rate	Price	Yield
0.5	1/30/2020	0	98.81	0.024087
1	6/30/2020	0.025	100.53	0.019622
1.5	1/31/2021	0.02125	100.41	0.018466
2	6/30/2021	0.01125	98.69	0.017943
2.5	1/31/2022	0.01875	100.25	0.017723
3	6/30/2022	0.0175	99.97	0.017603
3.5	1/31/2023	0.02375	102.03	0.017742
4	6/30/2023	0.02625	103.22	0.017873
4.5	1/31/2024	0.025	103.03	0.017961
5	6/30/2024	0.02	100.84	0.018235

Solution

▶ [Click to open solution]

Pricing Notes/Bonds by Replication

- Replicate a 1-year bond, with a face value of 100 and a coupon rate of 5% using the following set of bonds.

T	Maturity date	Coupon rate	Price	Yield
0.5	1/30/2020	0	98.81	0.024087
1	6/30/2020	0.025	100.53	0.019622
1.5	1/31/2021	0.02125	100.41	0.018466
2	6/30/2021	0.01125	98.69	0.017943
2.5	1/31/2022	0.01875	100.25	0.017723
3	6/30/2022	0.0175	99.97	0.017603
3.5	1/31/2023	0.02375	102.03	0.017742
4	6/30/2023	0.02625	103.22	0.017873
4.5	1/31/2024	0.025	103.03	0.017961
5	6/30/2024	0.02	100.84	0.018235

- *Assume that we can buy fractions of one bond (e.g. we can buy a principal amount of $50 of the six-month bond above for a price of* 0.5 × *$98.81=$49.405.*

Let's start by writing down the cash flows that we are trying to replicate

Position	Units	t=0	t=0.5	t=1
1-yr bond, 5% coupon rate	1	?	2.5	102.5

Let's set up a replicating portfolio

Position	Units	t=0	t=0.5	t=1
1-yr bond, 5% coupon rate	1	P_1	2.5	102.5

Replicating Portfolio	Units	t=0	t=0.5	t=1
1-yr bond, 2.5% coupon rate				
0.5-yr bond, 0% coupon rate				

Let's try 1 unit of the 1-year bond in the replicating portfolio

Position	Units	t=0	t=0.5	t=1
1-yr bond, 5% coupon rate	1	P_1	2.5	102.5

Replicating Portfolio	Units	t=0	t=0.5	t=1
1-yr bond, 2.5% coupon rate	1	-100.53	1.25	101.25
0.5-yr bond, 0% coupon rate				
Total of Replicating Portfolio				

- Since, we are not matching the cash flows of 2.5 at t=0.5 and 102.5 at t=1, let's adjust the position in the 1-year 2.5% coupon bond.
- To match the 102.5, we more than one unit. Specifically, we need $\frac{102.5}{101.25} = 1.012345679$ units.
- The coupon cash flow at t=0.5 is then $1.012345679 \text{ units} \times 1.25 = 1.2654$.
- To match the cash flow of 2.5, we use the 0.5-year zero coupon bond.
- We need to make up the difference of $2.5 - 1.2654 = 1.2346$.
- Thus, we buy a 6-month zero coupon bond with face value of 1.2346. This costs, $1.2346 \times \frac{98.81}{100} = 1.2346 \times 0.9881 = 1.22$
- Thus, we have the replicating portfolio.

Position	Units	t=0	t=0.5	t=1
1-yr bond, 5% coupon rate	1	P_1	2.5	102.5

Replicating Portfolio	Units	t=0	t=0.5	t=1
1-yr bond, 2.5% coupon rate	1.01235	-100.53	1.2654	102.50
0.5-yr bond, 0% coupon rate	0.012346	-1.22	1.2346	0
Total of Replicating Portfolio		-102.99	2.5	102.50

- Thus, the price of the 1-year coupon with coupon rate of 5% must be equal to 102.99.

Wrap-Up

Our goals for today

- [x] Understand what the term structure of interest rates means.
- [x] Know how to use the term structure of interest rates to price bonds.
- [x] Understand how to bootstrap the yield curve.
- [x] Know how to use discount factors to price bonds.
- [x] Understand how to price a bond by replication.

Reading

Fabozzi, Fabozzi, 2021, Bond Markets, Analysis, and Strategies, 10th Edition
Chapter 6