present

FINC 462/662 -- Fixed Income Securities

FINC-462/662: Fixed Income Securities

Bond Pricing

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Overview

Our goals for today

- ☐ Calculate the price of a Treasury note/bond.
- ☐ Know what the yield to maturity of a bond is.
- ☐ Price securities using the observed prices of other securities and the Law of One Price.
- ☐ Construct an arbitrage trade if the Law of One Price is violated.

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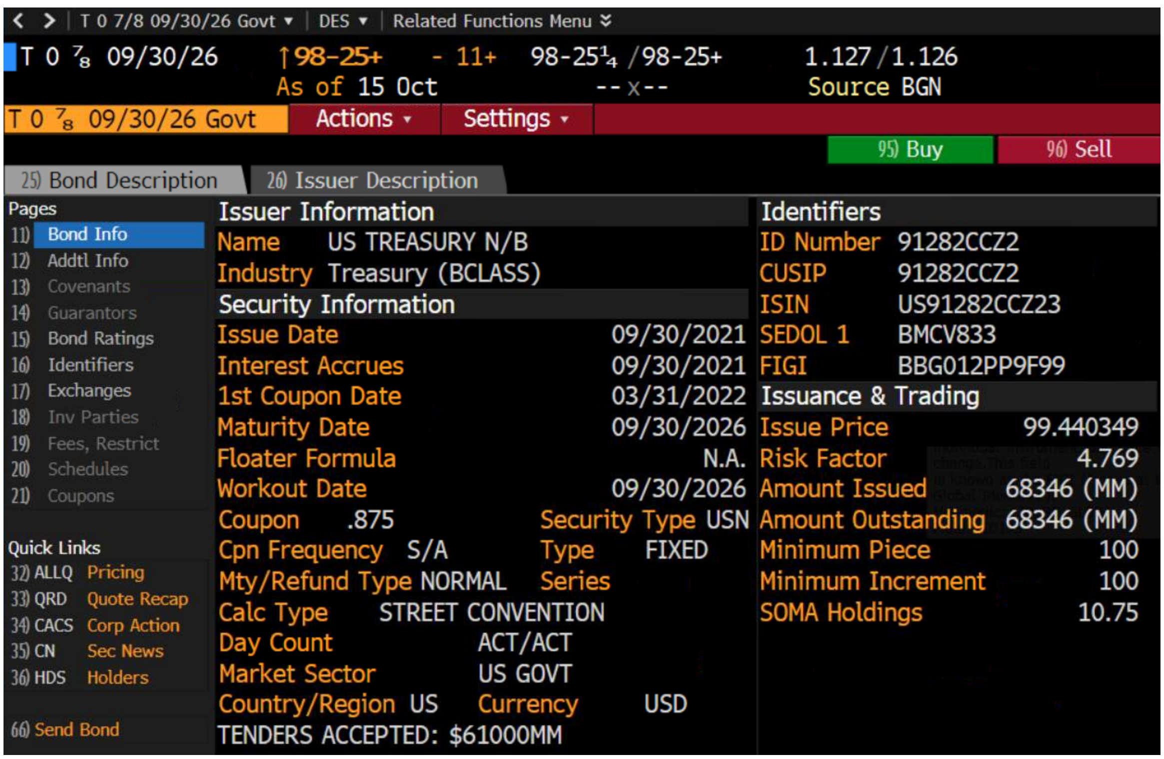
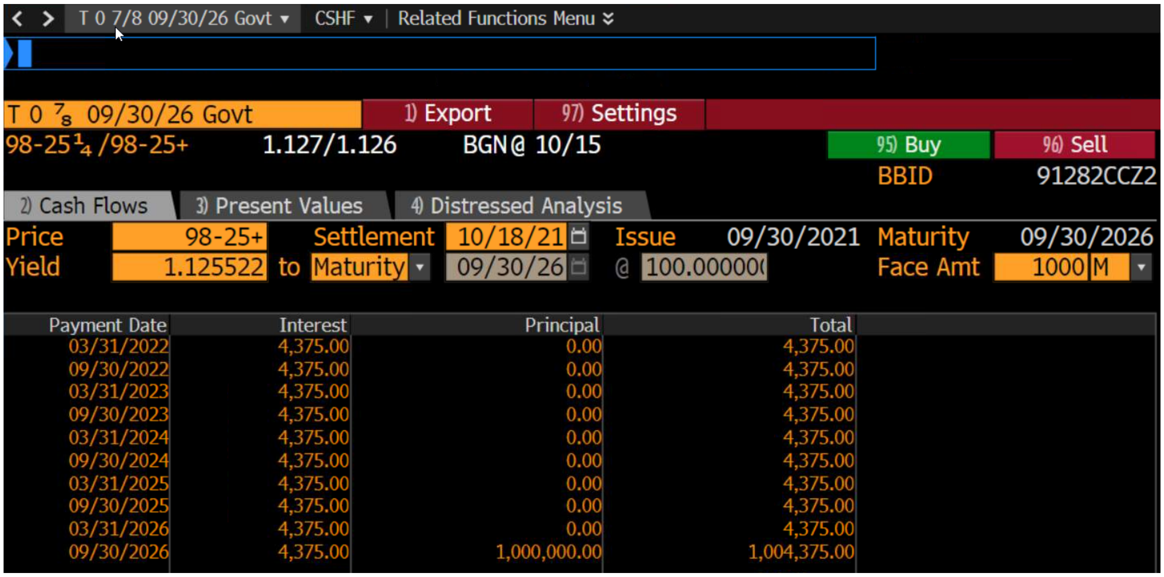
Pricing Treasury Notes/Bonds

- In the last lecture, we covered the "building blocks" (annuities, perpetuities, compounding) which we will now use to price bonds
- Our focus is on fixed coupon **Treasury notes/bonds** without option-like features.
- Unless otherwise noted, coupon interest is paid semi-annually
- We are going to assume that Treasury bonds have no credit risk.





Example

Set Coupon Rate

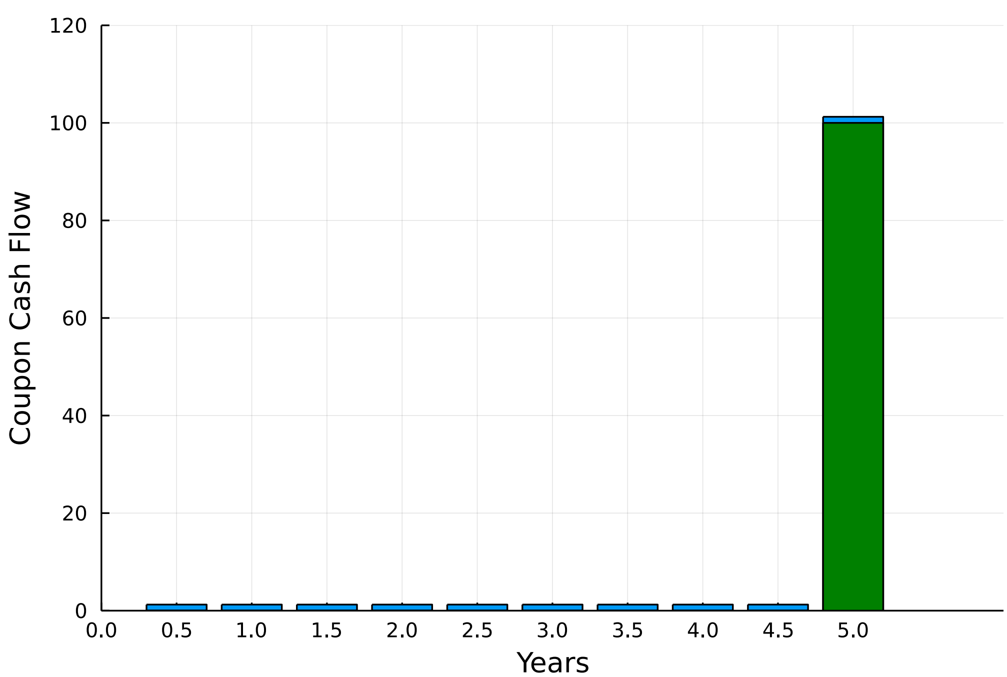
2.5

Coupon Rate: 2.5%

Set Time to Maturity

5.0

Time to Maturity: 5.0 years



Valuing Treasury notes/bonds

- To calculate the price P of a Treasury note/bond, we need to calculate the present values (PV) of
  - all coupon cash flows
  - and the principal cash flow at maturity

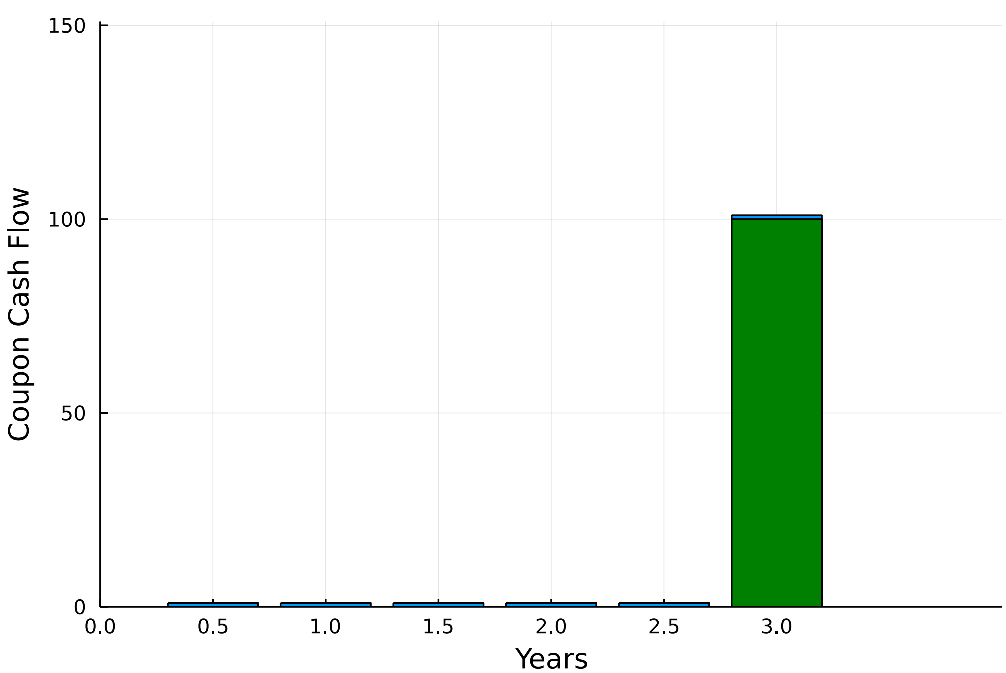
$$P = \text{PV}(\text{Coupon cash flows}) + \text{PV}(\text{Par value})$$

Example

- Face Value F [$]: 100
- Coupon Rate c [% p.a.]: 2.0
- Discount rate r [% p.a.]: 4.0
- Time to maturity T [years]: 3

Reset

- To illustrate, suppose we want to calculate the price of a 3-year Treasury note (T =3) with coupon rate $c = 2.0\%$ (paid semiannually) and principal amount (face value) of $F =$ $ 100.
- Assume that the discount rate ("interest rate") is $r = 6\%$ per annum (semi-annually compounded).



- The cash flows are

	Time	CashFlow	PresentValue	Calculation
1	0.5	1.0	0.980392	"1.0 * 1/(1+4.0%/2)^(2*0.5)=0.9804"
2	1.0	1.0	0.961169	"1.0 * 1/(1+4.0%/2)^(2*1.0)=0.9612"
3	1.5	1.0	0.942322	"1.0 * 1/(1+4.0%/2)^(2*1.5)=0.9423"
4	2.0	1.0	0.923845	"1.0 * 1/(1+4.0%/2)^(2*2.0)=0.9238"
5	2.5	1.0	0.905731	"1.0 * 1/(1+4.0%/2)^(2*2.5)=0.9057"
6	3.0	101.0	89.6851	"101.0 * 1/(1+4.0%/2)^(2*3.0)=89.6851"

Present Value = 0.9804 + 0.9612 + 0.9423 + 0.9238 + 0.9057 + 89.6851 = 94.398569

Shortcut using the annuity formula

- As the time to maturity T increases, manually calculating all present values becomes tedious.
- Instead, we can use the annuity formula.

Present Value of Annuity

Recall

The present value today (time $t = 0$) of an annuity paying a dollar cash flow of C for T years is

$$\text{PV} = \left(\frac{C}{r}\right)\left(1 - \frac{1}{(1+r)^T}\right)$$

Time t	0	1	2	3	4	...	T	T+1	...
Cash Flow	0	C	C	C	C	...	C	0	0

- We can use the annuity formula to calculate the present values of all coupon cash flows.
- To calculate the bond price, we need to add the present value of the principal cash flow.
- First, the terms in the annuity formula are
  - C=1.0
  - T=3,
  - r=4.0
- Second, the present value of the principal amount is $F/(1+\frac{r}{2})^{2T}$
- The sum of these two is the bond price P.

$$P = \frac{C}{r/2}\left(1-\frac{1}{\left(1+\frac{r}{2}\right)^{2T}}\right) + \frac{F}{(1+\frac{r}{2})^{2T}}$$

Price of a semi-annual coupon bond

Important

The price P of a T-year bond with principal value F paying semi-annual coupon interest at an annual rate of c (semi-annual cash flows of $C = c/2 \times F$) and semi-annually-compounded discount rate r is

$$P = \frac{C}{r/2}\left(1-\frac{1}{\left(1+\frac{r}{2}\right)^{2T}}\right) + \frac{F}{(1+\frac{r}{2})^{2T}}$$

Time t	0	0.5	1	1.5	2	...	T
Cash Flow	0	C	C	C	C	$\ldots$	$C+F$

$$P = \frac{C}{\frac{r}{2}}\left(1-\frac{1}{\left(1+\frac{r}{2}\right)^{2T}}\right) + \frac{F}{(1+\frac{r}{2})^{2T}} = \frac{1.0}{0.02}\left(1-\frac{1}{(1+0.02)^{2\times 3}}\right) + \frac{100}{(1+0.02)^{2\times 3}} = 9$$

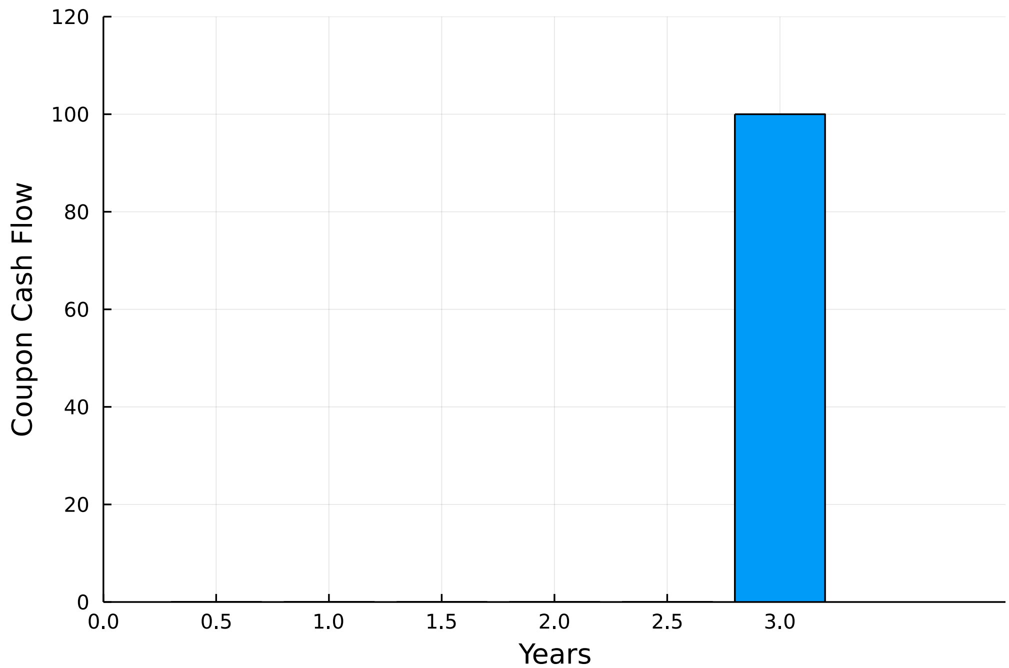
Valuing Zero-coupon bonds

- A zero coupon bond has one single cash flow at maturity T.

Example

- Face Value F [$]: 100
- Discount rate r [% p.a.]: 4.0
- Time to maturity T [years]: 3

Reset



- To value a zero-coupon bond, we need to compute the present value of the single principal cash flow at maturity T.

Price of a zero-coupon bond

Important

The price P of a T-year zero-coupon bond with principal value F and annually-compounded discount rate r is

$$P = \frac{F}{(1+r)^T}$$

With semi-annually-compounded discount rate r, the price of the zero-coupon bond is

$$P = \frac{F}{(1+\frac{r}{2})^{2\times T}}$$

Example: Price of Zero-Coupon bond

- Continuing with the example from above and assuming that the discount rate r is semi-annually compounded, the price P of a T=3 year zero-coupon bond with face value of 100 is

$$P = \frac{F}{(1+\frac{r}{2})^{2\times T}} = \frac{100}{(1+\frac{0.04}{2})^{2\times 3}} = 79.031453$$

Yield to Maturity

- Thus far, the discount rate r was given.
- Suppose now that we observe the bond price P, but are not given the discount rate.
- We can use the bond price P to calculate what discount rate–call it y–the bond price implies.
- In other words, we ask what discount rate investors are using to arrive at the bond price.
- This discount rate y is referred to as the **yield to maturity** of the bond.

- Let's first consider a zero-coupon bond (face value 100) with maturity in T=3 years.
- Suppose the market price of this zero-coupon bond P=90
- What is the market-implied discount rate y (annually-compounded)?
- This is the **yield to maturity**.

- We first write down the equation for the price of the zero-coupon bond.
- Then, we set the price equal to the market price and solve for the yield y.

$$P = \frac{F}{(1+y)^T}$$

$$90 = \frac{100}{(1+y)^3} \rightarrow (1+y)^3 = \frac{100}{90} \rightarrow (1+y) = \left(\frac{100}{90}\right)^{(1/3)} \rightarrow y = 0.035744 = 3.574417\%$$

Thus, the yield on the zero coupon bond is 3.574417 percent (annually compounded).

- Next, let's consider a Treasury note/bond paying semi-annual coupon interest.
- To start, we can apply the same approach as in the case of the zero-coupon bond.
- To illustrate, we use the Treasury note (T =3) with coupon rate $c = 2.0\%$ (paid semiannually) and principal amount (face value) of $F = \$\,100$ from the previous example.
- Assume that the Treasury note has a price of $ 94.398569.

$$P = \frac{C}{\frac{y}{2}}\left(1 - \frac{1}{(1+\frac{y}{2})^{2T}}\right) + \frac{F}{(1+\frac{y}{2})^{2T}}$$

$$94.398569 = \frac{1.0}{y/2}\left(1 - \frac{1}{(1+\frac{y}{2})^{2\times 3}}\right) + \frac{100}{(1+\frac{y}{2})^{2\times 3}}$$

- We need to find the value for y such that the right hand side of the equation is equal to 94.398569.
- It turns out that we cannot easily solve for y, and that we need to use a numerical method.
- Excel and financial calculators have functions that can calculate the yield to maturity.
- Let's illustrate how these function work.
- Essentially, it is by trial-and-error. We pick a value for y and change it until the right hand side is equal to the price of 94.398569.

- Discount rate y [% p.a.]: 4.0

$$94.398569 = \frac{1.0}{\frac{0.04}{2}}\left(1 - \frac{1}{(1+\frac{0.04}{2})^{2\times 3}}\right) + \frac{100}{(1+\frac{0.04}{2})^{2\times 3}} = 94.398569$$

Reset

- By varying the discount rate y, we find that the right-hand side is equal to the price of 94.398569 when the discount rate is y = 4.0%.
- This is the bond's yield to maturity.

- To summarize, the yield to maturity y is the discount rate that will make the present value of the bond's cash flows equal to its price.
- We can think of the yield to maturity as the "return" an investor earns by buying the bond today at its market price and holding it until maturity of the bond.
    - We can also think of the yield to maturity as the internal rate of return (IRR) of the bond.
- The yield to maturity is specific to each bond (i.e. we take a bond and calculate the yield to maturity of this specific bond). Do not simply use the yield to maturity you calculated for one bond and use it to get the price of a different bond
- Note that the yield-to-maturity has limitations as a measure of "return".
- Consider the following example.

The “Sleeping Beauty” Bond

- On July 21, 1993, Disney issued a 100-year bond
- They sold 300,000,000 worth of debt at an annual yield of 7.55%.
- For reference, the 30-year Treasury bond yield was approximately 6.6% at that time.
- Their bond was graded AA.
- Disney has an option to call or redeem the bonds beginning July 15, 2023 at 103.02% of their face value.
- A lot of interest from pension funds, insurance companies and other institutional investors

PRICING SUPPLEMENT
(To Prospectus Supplement and
Prospectus each dated August 18, 1992)

$300,000,000

The Walt Disney Company

7.55% SENIOR DEBENTURES DUE 2093

Interest payable January 15 and July 15

The Senior Debentures (the "Debentures") will mature on July 15, 2093, and will be redeemable on not less than 30 nor more than 60 days' notice, at the option of the The Walt Disney Company ("Disney"), in whole or in part, at any time on or after July 15, 2023 at the redemption prices set forth herein, plus accrued and unpaid interest to the redemption date. See "Description of the Debentures" herein, "Description of the Notes" in the Prospectus Supplement and "Description of the Debt Securities" in the Prospectus.

The Debentures will be represented by global securities (the "Global Securities") registered in the name of a nominee of The Depository Trust Company, as Depositary. Beneficial interests in the Debentures will be shown on, and transfers thereof will be effected only through, records maintained by the Depositary or its participants. Except as described under "Description of the Notes" in the Prospectus Supplement, owners of beneficial interests in the Global Securities will not be entitled to receive physical delivery of the Debentures in definitive form. See "Description of the Notes" in the Prospectus Supplement.

THESE SECURITIES HAVE NOT BEEN APPROVED OR DISAPPROVED BY THE SECURITIES AND EXCHANGE COMMISSION OR ANY STATE SECURITIES COMMISSION NOR HAS THE SECURITIES AND EXCHANGE COMMISSION OR ANY STATE SECURITIES COMMISSION PASSED UPON THE ACCURACY OR ADEQUACY OF THIS PRICING SUPPLEMENT OR THE PROSPECTUS SUPPLEMENT OR THE PROSPECTUS TO WHICH IT RELATES. ANY REPRESENTATION TO THE CONTRARY IS A CRIMINAL OFFENSE.

PRICE 100% AND ACCRUED INTEREST

	Price to Public(1)	Underwriting Discounts and Commissions(2)	Proceeds to Disney(1)(3)
Per Debenture	100.000%	1.125%	98.875%
Total	$300,000,000	$3,375,000	$296,625,000

(1) Plus accrued interest from July 15, 1993.
(2) Disney has agreed to indemnify the Underwriters against certain liabilities, including liabilities under the Securities Act of 1933, as amended. See "Underwriters."
(3) Before deducting expenses payable by Disney.

The Debentures are offered, subject to prior sale, when, as and if accepted by the Underwriters and subject to approval of certain legal matters by counsel for the Underwriters. It is expected that delivery of the Debentures will be made on or about July 28, 1993 through the book-entry facilities of The Depository Trust Company, against payment therefor in New York funds.

MORGAN STANLEY & CO. *Incorporated*

MERRILL LYNCH & CO.

July 21, 1993

Yield to maturity in Excel

- We are given a 3-year Treasury note with a coupon rate of $c = 4$ (paid semi-annually) with a face value F=$1,000.
- The market price of the Treasury note is P=$1,029.17,
- Let's calculate the yield to maturity.
- In Excel, we can do this with the function `YIELD`.

- In order to use this function, we need to provide the following:
  - The current date (can be arbitrary). Let's pick January 1, 2020.
  - The maturity date (in the example, the maturity date has to be 3 years after the current date). Let's pick January 1, 2023.
  - The coupon rate, expressed as an annual rate in decimals.
  - The bond price per 100 par value (in the example, it is 102.917).
  - The principal value 100.0
  - The number of times interest is compounding during the year (for semi-annual bonds, this number is 2).
- Thus, writing in an Excel cell: `=YIELD(DATE(2020,1,1),DATE(2023,1,1),0.04,102.917,100,2)`
- The result is: `2.9765%`
- For more information on this Excel function, see **Yield Function**

- Let's verify whether Excel calculated the yield to maturity correctly.
- We know that the yield to maturity is the discount rate r that sets the present value of the bond's coupon cash flows equal to its market price.

$$P = \frac{C}{\left(1+\frac{r}{2}\right)^{2\times 0.5}} + \frac{C}{\left(1+\frac{r}{2}\right)^{2\times 1.0}} + \ldots + \frac{C+100}{\left(1+\frac{r}{2}\right)^{2\times T}}$$

- Plugging in the numbers using $C = 0.04/2 \times 1000 = 40$ and $T = 3$, the right hand side is equal to

$$\frac{20}{\left(1+\frac{2.9765\%}{2}\right)^{2\times 0.5}} + \frac{20}{\left(1+\frac{2.9765\%}{2}\right)^{2\times 1.0}} + \ldots + \frac{20+1000}{\left(1+\frac{2.9765\%}{2}\right)^{2\times T}} = 1029.17$$

- Indeed, this matches the market price $P = \$\,1029.17$.

Law of One Price and Pricing by Replication

The law of one price says that all portfolios with the same payoff have the same price. (Principles of Financial Economics, by Stephen F. LeRoy)

- We will use this fundamental idea to price bonds and other securities.
- To illustrate the concept, suppose there are three bonds A, B, and C.
- Suppose we do not know the price of bond C, but that we know the price of A and B.
- Assume that when we buy both bond A and bond B, the resulting coupon/principal cash flows are the same as those of bond C.
- Then, it must be true that the price of bond C is the same as the sum of the price of bond A and bond B.

Example

- Consider two portfolios

Portfolio 1

Security	Payoff t=1	Payoff t=2
C	25	50

Portfolio 2

Security	Payoff t=1	Payoff t=2
A	25	0
B	0	50

Suppose that the price of A is \$24 and the price of B is \$44. What is the price of C?

Hint

- Next, suppose that the price of C is 70.
- This is a violation of the law of one price. We can take advantage of this and earn an **arbitrage** profit.
- We do this by buying the security that costs less and short-sell the security that costs more.
- We earn a riskfree profit by doing so.

Short-selling

- To illustrate this concept, let's consider again the securities A, B, and C from the previous example.
- Let's also suppose that we observe prices on all three securities.

Portfolio 1

Security	Price	Payoff t=1	Payoff t=2
C	71	25	50

Portfolio 2

Security	Price	Payoff t=1	Payoff t=2
A	24	25	0
B	44	0	50

- Portfolio 1 (Security C) is too expensive, so we **short-sell** it.
- Short-selling involves borrowing a security and selling it at the market price.

1. Borrow Security C

Assets	Liabilities
Security C, $71	Security C, $71

2. Sell Security C on the market and get 71 in cash

Assets	Liabilities
Security C, $0	Security C, $71
Cash C, $71	

3. Use part of the cash to buy securities A and B (which cost 24 + 44 = 68)

Assets	Liabilities
Security C, $0	Security C, $71
Cash C, $3	
Security A, $24	
Security B, $44	

- What are the resulting cash flows?

Position	$t = 0$ (today)	$t = 1$	$t = 2$
Buy 1 unit of A	-24	25	0
Buy 1 unit of B	-44	0	50
Short 1 unit of C	71	-25	-50
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Total	3	0	0

- The difference 3 is a riskfree arbitrage profit.

Practice Problem

1. Is the Law of One Price satisfied here?
2. Construct a long-short strategy to take advantage of mispricing.

Portfolio 1

Security	Price	Payoff t=1	Payoff t=2
C	71	25	50

Portfolio 2

Security	Price	Payoff t=1	Payoff t=2
A	24	25	0
B	44	0	50

Hint

CFA Practice Problem

Consider the following two bonds that pay interest annually and suppose the discount rate is $r = 4\%$

Bond	Coupon Rate	Time-to-Maturity
A	5%	2 years
B	3%	2 years

- The price difference between Bond A and Bond B per 100 of par value is closest to:

  a. 2.00

  b. 3.77

  c. 4.00

Hint

Hint

Wrap-Up

Our goals for today

- [x] Calculate the price of a Treasury note/bond.
- [x] Know what the yield to maturity of a bond is.
- [x] Price securities using the observed prices of other securities and the Law of One Price.
- [x] Construct an arbitrage trade if the Law of One Price is violated.

Reading

Fabozzi, Fabozzi, 2021, Bond Markets, Analysis, and Strategies, 10th Edition
Chapter 2