Bonus Material: Bond Trading

FINC 462/662

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Introduction

In this set of slides, we will talk about a few examples:

- Trading on beliefs
  - Steepener Trade
  - On-the-run/off-the-run
- Hedging liabilities that are sensitive to interest rates
  - Pension fund example
  - Perpetuity
  - Zero initial cash outlay

The Algorithm

Generally, trading on interest rates requires the following steps:

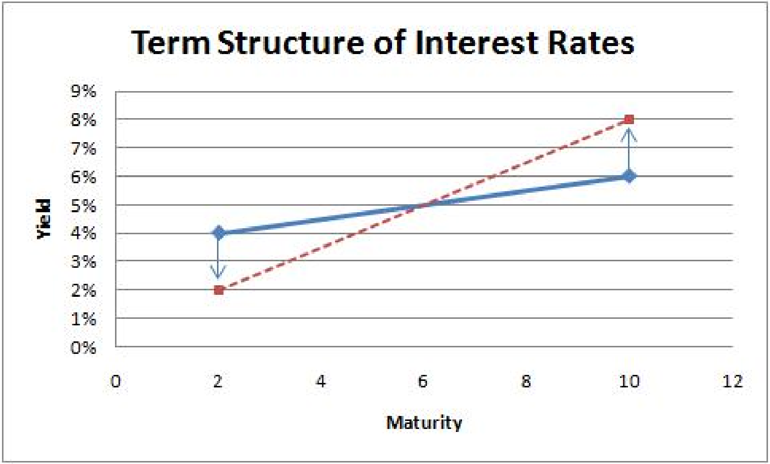
1. Identify your views.
2. Calculate measures of interest rate exposure (duration and/or convexity).
3. Determine the correct portfolio to hedge against certain types of interest rate exposures.
4. Verify that you did the trade correctly by doing some scenario analysis.

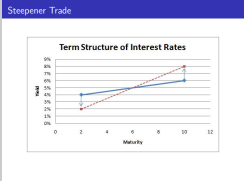
Steepener Trade

Sometimes our views might be about relative prices – equivalently, our views may be about relative interest rates. Let's consider an example.

- Suppose that the yield on a 2-year bond is 4% and the yield on a 10-year bond is 6%. (both zero coupon bonds; annually compounded yields)
- Suppose also that we believe that the yield on the 10-year bond will increase *relative* to the yield on the 2-year bond.
- We are not sure whether the overall level of yields will go up or down.

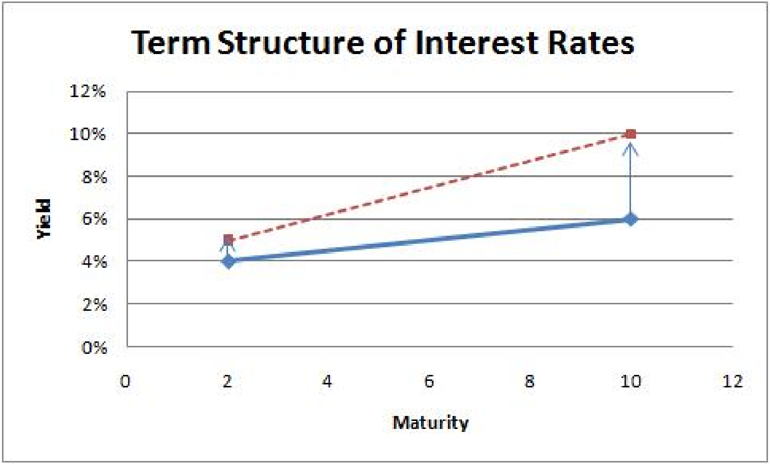
Steepener Trade





- 10-yr yield $\uparrow \Rightarrow$ Price $\downarrow$
- 2-yr yield $\downarrow \Rightarrow$ Price $\uparrow$
- Suggests that we want to long the 2-year and short the 10-yr.

Steepener Trade



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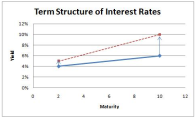
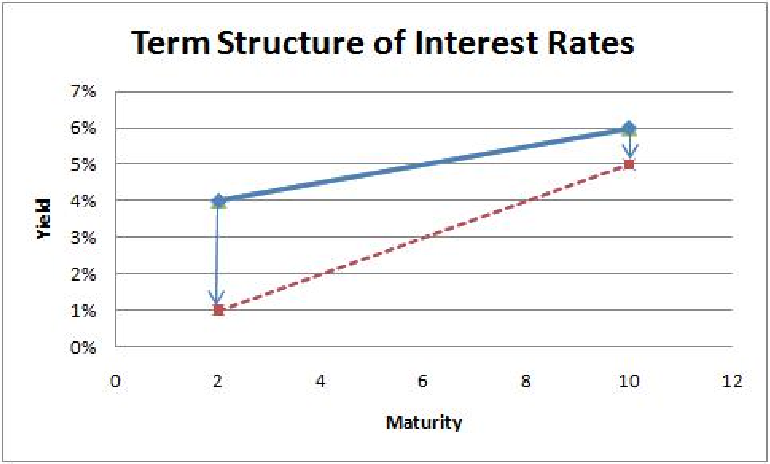


- Both yields go up, but 10yr yield goes up by more.
- Yield curve is steeper.

Steepener Trade



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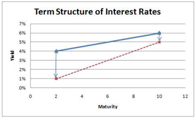
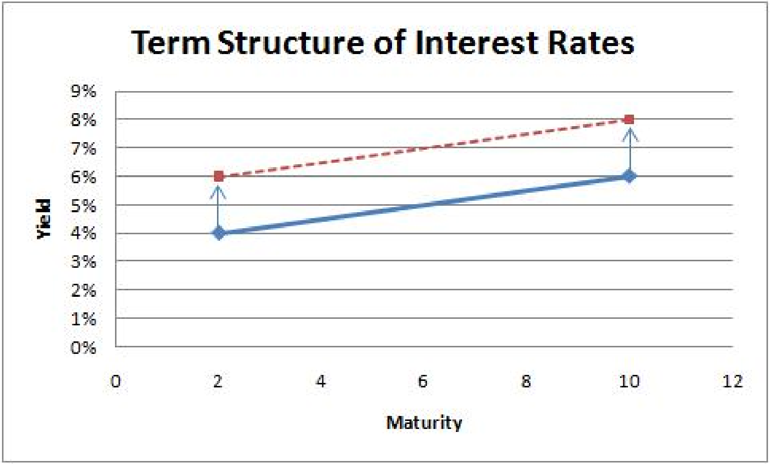


- Both yields go down, but 2yr goes down more.
- Yield curve is steeper.

Steepener Trade



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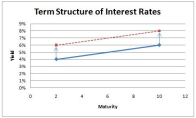
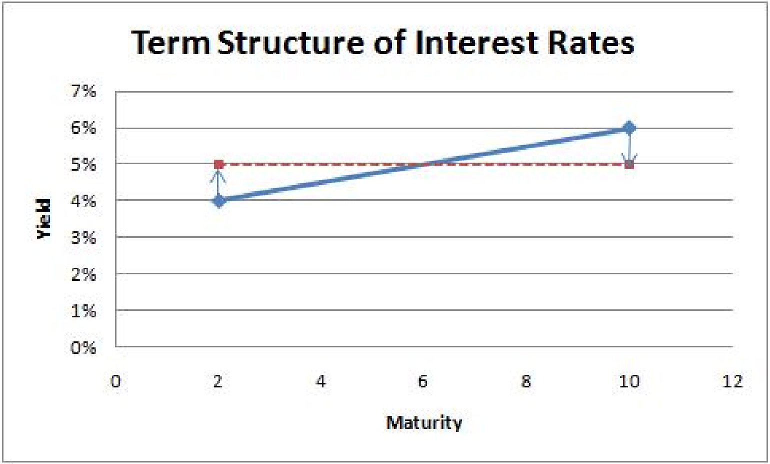
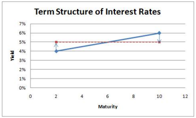


- Move in parallel.
- Same steepness.
- Break-even case: To be used to determine the proportion of each bond in the portfolio.

Steepener Trade



Steepener Trade



- Flatter
- Opposite of what we are betting on. Should expect to lose money.

Steepener Trade

- Step 1: We think the yield curve will get steeper. Thus, we want to buy the 2-year bond and short the 10-year bond.
  - But in what proportion? We want our exposure to the level of interest rates to roughly be zero. (If the 2-year rate and the 10-year rate change by the same amount, we want our portfolio value to remain close to constant.)
- Step 2: Some important quantities.
  - \$1000 face value of the 10-year bond is worth $\frac{\$1000}{(1.06)^{10}} = \$558.39$
  - $MD_{10} = \frac{10}{1.06} = 9.434$
  - $MD_2 = \frac{2}{1.04} = 1.923$

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  - $MD_2 = \frac{2}{1.04} = 1.923$

We could have also used the MD approximation formula to calculate MDs. For the 10-yr:

$$B(y+\Delta y) = \frac{1000}{1.061^{10}} = 553.1541$$

$$B(y-\Delta y) = \frac{1000}{1.059^{10}} = 563.6901$$

$$MD_{10} \approx -\frac{553.1541 - 563.6901}{2 \times .001} \times \frac{1}{558.39} = 9.434$$

Steepener Trade

<u>Step 3</u>: Let's consider a long position in the 2-year of \$x and a short position in the 10-year of \$558.39 (\$1000 face value):

Assets	Liabilities
\$x in 2-year	\$558.39 in 10-year
$MD_2 = 1.923$	$MD_{10} = 9.434$
If yield changes by Δy:	If yield changes by Δy:
$\frac{\Delta B}{B} \approx -1.923 \Delta y$	$\frac{\Delta B}{B} \approx -9.434 \Delta y$
x(-1.923)	(558.39)(-9.434)

Thus, x = \$2739.295 or \$2962.82 in face value of the 2-year bond.

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Thus, x = \$2739.295 or \$2962.82 in face value of the 2-year bond.

$$2739.295 = \frac{\text{Face value}}{1.04^2}$$

Steepener Trade

Step 4:

- Overall position value is: \$2180.90
- Let's see how well we have done in hedging level changes:
  - If $y_2 \to 6\%$ and $y_{10} \to 8\%$ (both yields increase by 2%), then the portfolio is worth \$2173.71.
  - If $y_2 \to 2\%$ and $y_{10} \to 4\%$ (both yields decrease by 2%), then the portfolio is worth \$2172.21.
- What if the yield curve steepens?
  - If $y_2 \to 4\%$ and $y_{10} \to 8\%$, then the portfolio is worth \$2276.10.

Steepener Trade

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- What if the yield curve steepens?
  - If $y_2 \to 4\%$ and $y_{10} \to 8\%$, then the portfolio is worth \$2276.10.

- Overall value $=$ 2739.29 - 558.39 $=$ 2180.90
- If $y_2 \to 6\%$ and $y_{10} \to 8\%$

$$\frac{2962.82}{1.06^2} - \frac{1000}{1.08^{10}} = 2173.71$$

- If $y_2 \to 2\%$ and $y_{10} \to 4\%$

$$\frac{2962.82}{1.02^2} - \frac{1000}{1.04^{10}} = 2172.21$$

- If $y_2 \to 4\%$ and $y_{10} \to 8\%$

$$\frac{2962.82}{1.04^2} - \frac{1000}{1.08^{10}} = 2276.10$$

Gain $\approx$ \$95

Steepener Trade Discussed

- We can use modified durations to make our portfolio (close to) insensitive to level changes.
- This allows us to bet on relative yields.
- In many ways, this is like a long-short equity strategy.

- Buy what we think is (relatively) too cheap.
- Short what we think is (relatively) too expensive.
- Buy in the right proportion to be “market-neutral.”

Steepener Trade Discussed

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Fixed Income Arbitrage

- One very popular trade is based on on-the-run versus off-the-run US Treasury bonds.
- On-the-run Treasuries are the most recently issued US Treasuries.
- Off-the-run Treasuries are all of the other Treasuries and tend to have low prices (high yields) relative to on-the-run Treasuries.
- Buy off-the-run Treasuries and short on-the-run Treasuries.

Caution: This trade is not an arbitrage in an academic (free money) sense.

2022-04-27

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Fixed Income Arbitrage Example

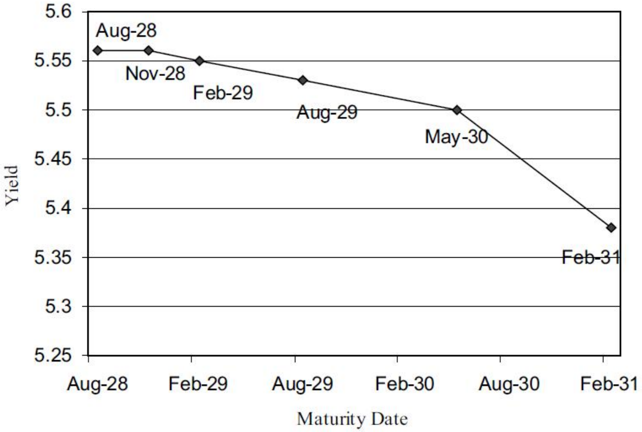

Fig. 1. The yield curve for the 30-year bond sector as of February 9, 2001.

Note the difference in yields between the May-30 (29.25yr) and Feb-31 (30yr) bonds.

Source: Krishnamurthy (2001)

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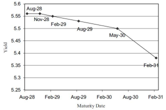
Bond Trading
└─Trading on Beliefs
└─Fixed Income Arbitrage
└─Fixed Income Arbitrage Example

Fixed Income Arbitrage Example



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- The idea is that these long maturity bonds are not very different from each other, so their yields should not be very different.
  - Notice the 13 bps gap between May-30 and Feb-31.
  - It just visually looks different.
- We might expect that the yield at the very right will go up a little.
- Undergrad: Clicker Q on next slide.

LTCM

"One of the fund's main strategies was to exploit tiny differences between the price of a newly issued ("on the run") 30-year American Treasury bond, and a similar one issued previously ("off the run"). There is little economic reason for these bonds to have different yields. Yet off-the-run Treasuries often trade slightly cheaper than on-the-run ones. LTCM bet that their yields would converge by buying off-the-run Treasuries and selling their on-the-run counterparts short."

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The Trade

Long (off-the-run)	Short (on-the-run)
\$x of 29.25yr bond	\$1000 of 30yr bond
$MD = \frac{29.25}{1.055} = 27.7251$	$MD = \frac{30}{1.0538} = 28.4684$
$\frac{\Delta B}{B} \approx -27.7251 \Delta y$	$\frac{\Delta B}{B} \approx -28.4684 \Delta y$

-27.7251x=-28.4684(1000)

$$x = 1026.81$$

Face values:
Off-the-run: $1026.81(1.055)^{29.25} = 4916.14$
On-the-run: $1000(1.0538)^{30} = 4816.66$

Portfolio value: 1026.81 - 1000 = 26.81

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- The bonds are not actually zero coupon in reality.
- Remember: On-the-run bond is the expensive one.

Balance Sheet

Long (Assets)	Short (Liabilities)
Off-the-run (29.25yr) bond	On-the-run (30yr) bond
\$4916.14 in face value	\$4816.66 in face value
\$1026.81 in market value	\$1000 in market value

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Bond Trading
└─Trading on Beliefs
└─Fixed Income Arbitrage
└─Balance Sheet

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\$1026.81 in market value	\$1000 in market value

Waiting for prices to converge

Suppose that we wait nine months and the yield of the on-the-run bond goes up to 5.47% while the yield of the off-the-run bond stays at 5.5%.

New portfolio value:
$\frac{4916.14}{1.055^{28.5}} - \frac{4816.66}{1.0547^{29.25}} = 54.45$

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- Nine months $=$ 0.75 years
- 5.47% makes the yield curve linear.

Hedging Interest Rate Risk

- Suppose that you are managing a pension fund.
- You have a liability of $100mm per year for the next 100 years.
- How do you create a portfolio of Treasury bonds to hedge your exposure to interest rate risk?

The US Government does not sell bonds with 100 year maturities, so we cannot just buy bonds with cash flows to match the liability.

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- Disney did issue a 100-yr bond known widely as the "Sleeping Beauty Bond."

Managing a Pension Fund

Let's first value the pension liability. It's an annuity. Let's assume that the discount rate is 5% regardless of maturity (term structure is flat).

$$\begin{aligned}\text{Value of Liability} &= 100 \times \frac{1}{0.05}\left[1 - \frac{1}{1.05^{100}}\right] \\ &= 1984.79102\end{aligned}$$

Let's also suppose that the pension fund currently has 1984.79102 in cash. That is, the pension fund is neither under- nor overfunded.

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- In reality, it's difficult to determine a default-free yield for $T > 30$.
  - Most models suggest that yields are mean-reverting, so we might just take a long-run average.
  - Vasicek: $dr_t = \kappa(\theta - r_t)dt + \sigma_r dZ_t$
- Underfunding has been a major issue for a lot of state pension plans.

Managing a Pension Fund

Next, let's calculate the modified duration of the pension fund. Recall, the formula for approximate modified duration:

$$MD \approx -\frac{B(y+\Delta y) - B(y-\Delta y)}{2\times \Delta y} \times \frac{1}{B(y)} \tag{1}$$

$$\begin{aligned}
&\text{Value of liability @ 5.0\%} = 1984.79102 \\
&\text{Value of liability @ 5.1\%} = 100 \times \frac{1}{0.051}\left[1 - \frac{1}{1.051^{100}}\right] = 1947.227482 \\
&\text{Value of liability @ 4.9\%} = 100 \times \frac{1}{0.049}\left[1 - \frac{1}{1.049^{100}}\right] = 2023.745478
\end{aligned}$$

$$MD \approx -\frac{1947.227482 - 2023.745478}{2\times 0.001} \times \frac{1}{1984.79102} = 19.2761$$

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$$MD \approx -\frac{1947.227482-2023.745478}{2\times 0.001}\times\frac{1}{1984.79102} = 19.2761$$

- Notice how we can use our MD approximation formula even though it's not a bond.

Managing a Pension Fund

We will use 1yr and 30yr zero-coupon bonds to form a portfolio that hedges this liability

Assets		Liabilities
1yr	30yr	Pension
\$x	\$z	\$1984.79102
$MD = \frac{1}{1.05} = 0.9524$	$MD = \frac{30}{1.05} = 28.5714$	$MD = 19.2761$

Recall that $\frac{\Delta B}{B} \approx -MD \times \Delta y$

Modified Duration Constraint:

$$-0.9524x - 28.5714z = -19.2761(1984.79102)$$

Assets = Liabilities Constraint:

$$x + z = 1984.79102$$

Solving: x = 667.9925, z = 1316.799
Face values: 1yr: 701.3921, 30yr: 5691.127

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Face values: 1yr: 701.3921 $[667.9925(1.05)]$, 30yr: 5691.127 $[1316.799(1.05)^{30}]$

How Did Our Hedge Do?

Suppose y changes to 6%:

$$\text{Value of assets} = \frac{701.3921}{1.06} + \frac{5691.127}{1.06^{30}} = 1652.574$$

$$\text{Value of liabilities} = 100 \times \frac{1}{0.06}\left[1 - \frac{1}{1.06^{100}}\right] = 1661.755$$

Suppose y changes to 4%:

$$\text{Value of assets} = \frac{701.3921}{1.04} + \frac{5691.127}{1.04^{30}} = 2429.096$$

$$\text{Value of liabilities} = 100 \times \frac{1}{0.04}\left[1 - \frac{1}{1.04^{100}}\right] = 2450.50$$

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Bond Trading
└─Hedging
└─Pension Fund Example
└─How Did Our Hedge Do?

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Hedging a Perpetuity

Suppose that you have a liability of \$100 per year in perpetuity and the current interest rate for discounting this perpetuity is 10%. To hedge the value of this perpetuity, you decide to buy a 10-year bond (which also has a discount rate of 10%). How much of a 10-year bond do you need to buy?

Note that the value of a perpetuity that pays \$1 each period is:

$$\text{Value of perpetuity} = \frac{1}{r}$$

Thus, the value of this perpetuity is:

$$\frac{100}{0.1} = 1000$$

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$$\frac{100}{0.1} = 1000$$

Modified Durations

Next, calculate the modified durations:

$$MD_{10} = \frac{10}{1.1} = 9.09$$

$$\text{Value of perpetuity @ 10.1\%} = \frac{100}{0.101} = 990.10$$

$$\text{Value of perpetuity @ 9.9\%} = \frac{100}{0.099} = 1010.10$$

$$MD_{perpetuity} \approx -\frac{990.10 - 1010.10}{2 \times 0.001} \times \frac{1}{1000} = 10$$

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$$\text{Value of perpetuity @ } 9.9\% = \frac{100}{0.099} = 1010.10$$
$$MD_{perpetuity} \approx -\frac{990.10 - 1010.10}{2 \times 0.001} \times \frac{1}{1000} = 10$$

Could have used MD approximation:

$$B(y + \Delta y) = \frac{1000}{1.101^{10}} = 382.0558$$
$$B(y - \Delta y) = \frac{1000}{1.099^{10}} = 389.0658$$
$$B(y) = \frac{1000}{1.1^{10}} = 385.5433$$
$$MD_{10} \approx -\frac{382.0558 - 389.0658}{2 \times .001} \times \frac{1}{385.5433} = 9.09$$

Immunizing

Assets	Liabilities
10-yr bond	Perpetuity
\$x worth of bonds	Market value $=$ \$1000
MD $= 9.09$	MD $= 10$
$\frac{\Delta B}{B} \approx -9.09\Delta y$	$\frac{\Delta B}{B} \approx -10\Delta y$
x(-9.09)Δy	(1000)(-10)Δy

x $=$ 1100

Assets	Liabilities
10-yr bond	Perpetuity
\$x worth of bonds	Market value = \$1000
MD = 9.09	MD = 10
$\frac{\Delta B}{B} \approx -9.09\Delta y$	$\frac{\Delta B}{B} \approx -10\Delta y$
x(-9.09)Δy	(1000)(-10)Δy

x = 1100

Zero Initial Cash Outlay

Sometimes, we will choose a hedge that is based on having a zero initial cash outplay. That is, the initial value of the short position is equal to the initial value of the long position. You will do something similar for your next assignment. Mathematically, this is the same as the earlier pension example.

Suppose we have the following yield curve for zero-coupon bonds:

T	yield
2	3%
5	5%
7	6%

Suppose that we are short $1000 in face value in the 5-year bond. Choose the correct proportion to buy in the 2-year and 7-year bonds such that the position is immunized against (small) level changes in yields and there is no initial cash outlay.

Zero Initial Cash Outlay

Sometimes, we will choose a hedge that is based on having a zero initial cash outplay. That is, the initial value of the short position is equal to the initial value of the long position. You will do something similar for your next assignment. Mathematically, this is the same as the earlier pension example.

Suppose we have the following yield curve for zero-coupon bonds:

T	yield
2	3%
5	5%
7	6%

Suppose that we are short \$1000 in face value in the 5-year bond. Choose the correct proportion to buy in the 2-year and 7-year bonds such that the position is immunized against (small) level changes in yields and there is no initial cash outlay.

- Basically, Assets = Liabilities

Immunizing

Assets		Liabilities
2yr	7yr	5yr
\$x	\$z	$\frac{\$1000}{(1.05)^5} = \$783.53$
$MD = \frac{2}{1.03} = 1.94$	$MD = \frac{7}{1.06} = 6.60$	$MD = \frac{5}{1.05} = 4.76$

Recall that $\frac{\Delta B}{B} \approx -MD \times \Delta y$.

2022-04-27

Bond Trading
└─Hedging
└─Zero Initial Cash Outlay
└─Immunizing

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Solving Equations

Modified Duration Constraint:

$$783.53(-4.76) = x(-1.94) + z(-6.60)$$

Zero Initial Cash Outlay Constraint:

$$783.53 = x + z$$

Solving for x and z:

$$x = 309.56$$
$$z = 473.97$$

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Constraints

As it turns out, we have been setting up problems using some combination of three constraints:

1. Modified Duration Constraint
2. Convexity Constraint
3. Assets = Liabilities Constraint

The key is to determine from the problem given, which constraints to use.

2022-04-27

Bond Trading
└─Hedging
  └─Zero Initial Cash Outlay
    └─Constraints

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Assignments

Recommended:

- Additional example in the slides that follow
- Practice Problems in the "Valuation Basics" section

Required:

- Assignment #1

2022-04-27

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- Additional example in the slides that follow
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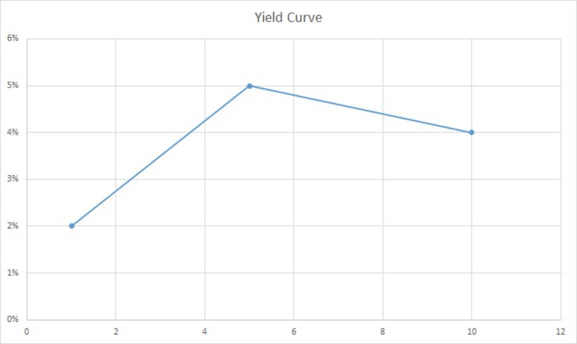
Required:

- Assignment #1

Betting on Yield Curve Shapes

Suppose that you observe the following yield curve:



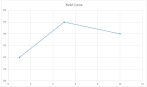
You are surprised by the hump shape, as yield curves are usually monotonically increasing. Construct a zero-cost trading strategy that bets on the yield curve becoming monotonically increasing, but does not bet on the overall level of the yield curve.

2022-04-27

Bond Trading
└─Additional Examples
└─A Hump-Shaped Yield Curve
└─Betting on Yield Curve Shapes

Betting on Yield Curve Shapes

Suppose that you observe the following yield curve:



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Betting on Yield Curve Shapes

<u>Identify your view</u>:

Which bond(s) do you buy? Which bond(s) do you short? Explain.

Betting on Yield Curve Shapes

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Which bond(s) do you buy? Which bond(s) do you short? Explain.

- Essentially, you view the 5-year bond yield as too high relative to the 1-year bond and 10-year bond.
- Remember that yields and prices are inversely related. So, you view the 5-year bond as too cheap (relative to the other bonds).

Betting on Yield Curve Shapes

Calculate the modified duration of the three bonds and write down your balance sheet. Assume that your position in the 5-year bond (either long or short) is \$1 in market value.

Betting on Yield Curve Shapes

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Assets	Liabilities	
5yr	1yr	10yr
\$1	\$x	\$z
$MD = \frac{5}{1.05} = 4.7619$	$MD = \frac{1}{1.02} = 0.9804$	$MD = \frac{10}{1.04} = 9.6154$

Betting on Yield Curve Shapes

Set-up two constraints: (1) Modified Duration Constraint and (2) Zero Initial Cash Outlay Constraint. Solve for the market values of the 1-year and 10-year bond positions.

Answer: \$0.5621 in 1yr, \$0.4379 in 10yr

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Answer: \$0.5621 in 1yr, \$0.4379 in 10yr

Modified Duration Constraint:

$$1(4.7619) = 0.9804x + 9.6154z$$

Zero Initial Cash Outlay Constraint:

$$1 = x + z$$

Solving for x and z:

$$x = 0.5621$$
$$z = 0.4379$$

Betting on Yield Curve Shapes

Keep in mind that all of the values solved for are market values. Convert them to face values. Write down a balance sheet of your positions.

Betting on Yield Curve Shapes

Keep in mind that all of the values solved for are market values. Convert them to face values. Write down a balance sheet of your positions.

$$5yr : 1 = \frac{FV_{5yr}}{1.05^5} \Rightarrow FV_{5yr} = 1.2763$$

$$1yr : 0.5621 = \frac{FV_{1yr}}{1.02} \Rightarrow FV_{1yr} = 0.5733$$

$$10yr : 0.4379 = \frac{FV_{10yr}}{1.04^{10}} \Rightarrow FV_{10yr} = 0.6482$$

Assets	Liabilities	
5yr	1yr	10yr
\$1 market value	\$0.5621 market value	\$0.4370 market value
\$1.2763 face value	\$0.5733 face value	\$0.6482 face value

Betting on Yield Curve Shapes

Suppose that the level of the yield curve increases (we had no view on this), but the 5yr yield drops relative to the other yields.

T	yield	Δ yield
1	5%	$+3$
5	6%	$+1$
10	7%	$+3$

Calculate the new value of your portfolio.

Answer: 0.0782

Betting on Yield Curve Shapes

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1	5%	+3
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10	7%	+3

Calculate the new value of your portfolio.

Answer: 0.0782

The value of the portfolio is now,

$$\frac{1.2763}{1.06^5} - \frac{0.5733}{1.05} - \frac{0.6482}{1.07^{10}} = 0.0782$$

Betting on Yield Curve Shapes

Suppose instead that the level of the yield curve increases (we had no view on this), but the 5yr is unchanged relative to the other yields.

T	yield	Δ yield
1	5%	$+3$
5	8%	$+3$
10	7%	$+3$

Calculate the new value of your portfolio.

Answer: -0.01

Betting on Yield Curve Shapes

Suppose instead that the level of the yield curve increases (we had no view on this), but the 5yr is unchanged relative to the other yields.

T	yield	Δ yield
1	5%	+3
5	8%	+3
10	7%	+3

Calculate the new value of your portfolio.

Answer: -0.01

$$\frac{1.2763}{1.08^5} - \frac{0.5733}{1.05} - \frac{0.6482}{1.07^{10}} = -0.013$$

Note that there is a slight change in portfolio value since modified duration hedging is not perfect.

Betting on Yield Curve Shapes

Suppose instead that the level of the yield curve increases (we had no view on this), but the 5yr yield increases relative to the other yields.

T	yield	Δ yield
1	3%	$+1$
5	8%	$+3$
10	5%	$+1$

Calculate the new value of your portfolio.

Answer: -0.0859

Betting on Yield Curve Shapes

Suppose instead that the level of the yield curve increases (we had no view on this), but the 5yr yield increases relative to the other yields.

T	yield	Δ yield
1	3%	+1
5	8%	+3
10	5%	+1

Calculate the new value of your portfolio.

Answer: -0.0859

$$\frac{1.2763}{1.08^5} - \frac{0.5733}{1.03} - \frac{0.6482}{1.05^{10}} = -0.0859$$

Yields moved in the opposite way of what we were betting on.