Empirical Tests of the CAPM

Teaching Demo

Zhengyu Cao

November 2022

Road Map

1. Review of CAPM
   - What Does CAPM Say?
   - Implications
2. Testing CAPM
   - Challenges
   - Preparing Data
   - Running Regressions
   - Empirical Results
3. Improving Empirical Performance
   - Using Portfolios
   - Fama-MacBeth Procedure
   - Conditional CAPM

CAPM

$$\mathbb{E}(r_i) = r_f + \beta_i \left[\mathbb{E}(r_M) - r_f\right]$$

where

- r_i is the return on asset i.
- r_f is the risk-free rate.
- r_M is the return on the market portfolio.
- β_i is the contribution of asset i to the variance of the market portfolio as a fraction of the total variance of the market portfolio.

$$\beta_i = \frac{cov(r_i, r_M)}{\sigma^2(r_M)}$$

CAPM Implications

Rearrange the equation

$$\underbrace{\mathbb{E}(r_i) - r_f}_{\mathbb{E}(R_i)} = \beta_i \underbrace{\left[\mathbb{E}(r_M) - r_f\right]}_{\mathbb{E}(R_M)}$$

$$\Rightarrow$$

$$\mathbb{E}(R_i) = \mathbb{E}(R_M)\beta_i$$

- The expected excess return on any asset is proportional to its β.
- The difference in expected excess return is explained by the difference in β only.

If we define α_i as

$$\alpha_i = \mathbb{E}(R_i) - \mathbb{E}(R_M)\beta_i$$

then CAPM implies that the α on every asset is zero.

To Test CAPM

Challenges

- The true market portfolio is not observable.
  - Use a market index, e.g. S&P 500, as a proxy
- The expected return is not observable.
  - Use the sample average of realized returns as an estimate
- β is not observable.
  - Estimate β

Are we really testing the CAPM?

- Not directly.
- We are testing the expected return-beta equation (the SML).

Getting Prepared

Setting up the sample data

- Determine a sample period.
  - Say 60 months, 2001.1-2005.12
- Pick a market portfolio proxy.
  - Say S&P 500
- Pick a risk-free rate proxy.
  - Say T-bills
- Choose testing assets.
  - Say 100 stocks. Randomly?
- Collect returns from a data vendor
  - Yahoo finance, CRSP, etc.

This constitutes a table of $102 \times 60 = 6{,}120$ rates of return.

S&P 500 (^GSPC)

SNP - SNP Real Time Price. Currency in USD

Follow

4,026.12 -1.14 (-0.03%)

At close: November 25 02:32PM EST

Summary | Chart | Conversations | **Historical Data** | Options | Components

Time Period: Jan 01, 2001 - Dec 31, 2005 | Show: Historical Prices | Frequency: Monthly | Apply

Currency in USD

Date	Open	High	Low	Close*	Adj Close**	Volume
Dec 01, 2005	1,249.48	1,275.80	1,246.59	1,248.29	1,248.29	41,756,130,000
Nov 01, 2005	1,207.01	1,270.64	1,201.07	1,249.48	1,249.48	45,102,870,000
Sep 30, 2005	1,228.81	1,233.34	1,168.20	1,207.01	1,207.01	49,793,790,000
Aug 31, 2005	1,220.33	1,243.13	1,205.35	1,228.81	1,228.81	44,777,510,000
Jul 31, 2005	1,234.18	1,245.86	1,201.07	1,220.33	1,220.33	42,030,090,000
Jun 30, 2005	1,191.33	1,245.15	1,183.55	1,234.18	1,234.18	37,464,670,000
May 31, 2005	1,191.50	1,219.59	1,188.30	1,191.33	1,191.33	40,334,040,000

The model predicts that

$$\mathbb{E}(R_i) = \mathbb{E}(R_M)\beta_i$$

The regression is

$$\bar{R}_i = \gamma_0 + \gamma_1\hat{\beta}_i + u_i$$

where $\bar{R}_i$ is the mean excess return over the sample period for asset i.

A further test

$$\bar{R}_i = \gamma_0 + \gamma_1\hat{\beta}_i + \gamma_2\hat{\beta}_i^2 + \gamma_3\sigma(\varepsilon_i) + u_i$$

According to the CAPM

- γ_0, γ_2, and γ_3 all should be zero.
- γ_1 should equal $\bar{R}_M$.

How to Get $\hat{\beta}_i$

Estimating the SCL for each asset i

$$R_{it} = a_i + \beta_i R_{Mt} + \varepsilon_{it}$$

- Obtain the coefficient $\hat{\beta}_i$
- Calculate the standard deviation of residuals $\sigma(\varepsilon_i)$

Popup quiz #1:
The CAPM applies to ________.

A individual stocks

B bonds

C well-diversified portfolios

D all assets

Popup quiz #2:
What is the β of a portfolio?

Two-Pass Regression

First-pass regression, for each i

$$R_{it} = a_i + \beta_i R_{Mt} + \varepsilon_{it} \qquad t = 1, \ldots, 60$$

Get the following statistics and use them in later analysis:

- $\hat{\beta}_i$, $\bar{R}_i$, $\bar{R}_M$, and $\sigma(\varepsilon_i)$

Second-pass regression

$$\bar{R}_i = \gamma_0 + \gamma_1 \hat{\beta}_i + u_i \qquad i = 1, \ldots, 100$$

or

$$\bar{R}_i = \gamma_0 + \gamma_1 \hat{\beta}_i + \gamma_2 \hat{\beta}_i^2 + \gamma_3 \sigma(\varepsilon_i) + u_i \qquad i = 1, \ldots, 100$$

Check if

- $\hat{\gamma}_0 = 0$; $\hat{\gamma}_2 = 0$; $\hat{\gamma}_3 = 0$;
- $\hat{\gamma}_1 = \bar{R}_M$

Empirical Performance of CAPM

Unfortunately, tests of the CAPM performed by various researchers all suggest that

- γ_0 is significantly different from zero.
- γ_1 is significantly different from (less than) $\bar{R}_M$.

Lintner (1965) and Miller and Scholes (1972) run a slightly different regression:

$$\bar{R}_i = \gamma_0 + \gamma_1 \hat{\beta}_i + \gamma_2 \sigma^2(\varepsilon_i) + u_i$$

Using annual data on 631 NYSE stocks for 10 years (1954-1963), they report the following testing results:

Coefficient:	$\gamma_0 = .127$	$\gamma_1 = .042$	$\gamma_2 = .310$
Standard error:	.006	.006	.026
Sample average:		$\overline{r_M - r_f} = .165$	

Do those results refute the CAPM?

Difficulties with this approach:

- Stock returns are extremely volatile.
- The proxy of the market portfolio is likely to be inefficient.
- Investors cannot borrow at the risk-free rate
- Beta is measured with error.
  - The slope coefficient will be biased downward while the intercept biased upward.

Can we save the CAPM, or rather, β?

Portfolio β

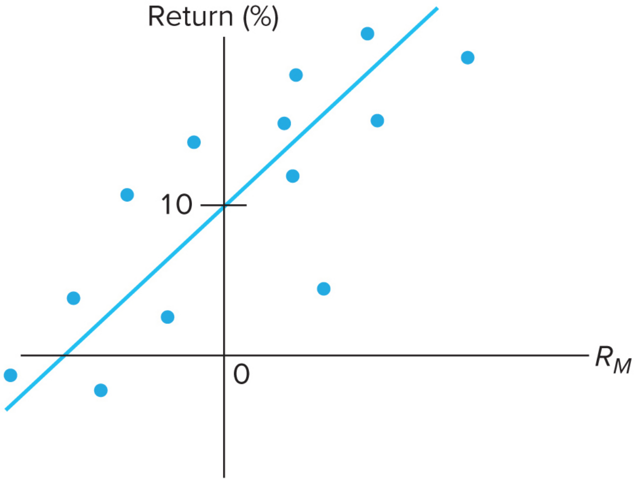
Combining securities into portfolios enhancs the precision of the β estimates.

- Black, Jensen, and Scholes (1972), Fama and MacBeth (1973)

Objective: group securities to get the largest dispersion of β without introducing selection bias.

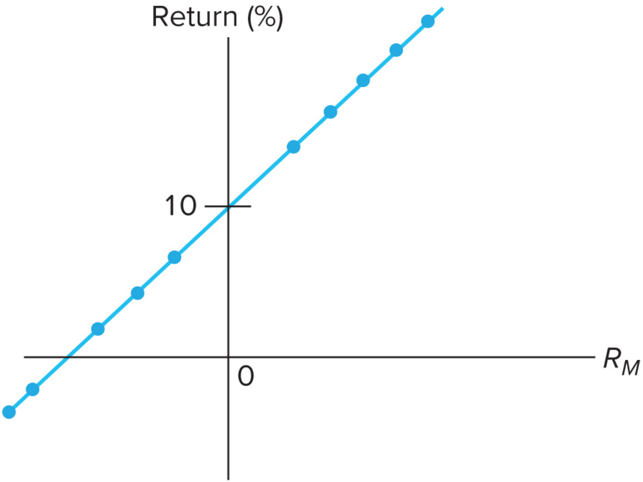
- Estimate $\hat{\beta}_i$ for each individual stock
- Rank stocks into portfolios based on $\hat{\beta}_i$
- Obtain $\hat{\beta}_p$ for each portfolio in the subsequent period
  - Regress portfolio return on market return
  - Average stock $\hat{\beta}_i$ within each portfolio
- Conduct tests using $\hat{\beta}_p$

Stock SCL



B: Single stock

Portfolio SCL



A: Well-diversified portfolio

Fama-MacBeth Procedure

Instead of estimating a single cross-sectional regression with sample averages in the second-pass,

$$\bar{R}_i = \gamma_0 + \gamma_1 \hat{\beta}_i + u_i$$

FM run a cross-sectional regression at each time period t

$$R_{it} = \gamma_{0t} + \gamma_{1t} \hat{\beta}_i + u_{it}$$

Then take the time-series average of the cross-sectional regression estimates

$$\hat{\gamma}_0 = \frac{1}{T} \sum_{t=1}^{T} \hat{\gamma}_{0t} \qquad \hat{\gamma}_1 = \frac{1}{T} \sum_{t=1}^{T} \hat{\gamma}_{1t}$$

Fama and MacBeth (1973) Results

Period	1935/6–1968	1935–1945	1946–1955	1956/6–1968
Average γ_0	8	10	8	5
t-statistic (testing $\gamma_0 = 0$)	0.20	0.11	0.20	0.10
Average $r_M - r_f$	130	195	103	95
Average γ_1	114	118	209	34
t-statistic (testing $\gamma_1 = r_M - r_f$)	1.85	0.94	2.39	0.34
Average γ_2	−26	−9	−76	0
t-statistic (testing $\gamma_2 = 0$)	−0.86	−0.14	−2.16	0
Average γ_3	516	817	−378	960
t-statistic (testing $\gamma_3 = 0$)	1.11	0.94	−0.67	1.11
Average *R*-square	0.31	0.31	0.32	0.29

Conditional CAPM

Allow β and the market risk premium to vary over time according to various conditions.

- Jagannathan and Wang (1996), Petkova and Zhang (2005)

In JW

$$\mathbb{E}(R_i) = c_0 + c_{\mathsf{size}} log(\mathsf{ME}) + c_{\mathsf{vw}}\beta^{\mathsf{vw}} + c_{\mathsf{credit}}\beta^{\mathsf{credit}} + c_{\mathsf{labor}}\beta^{\mathsf{labor}}$$

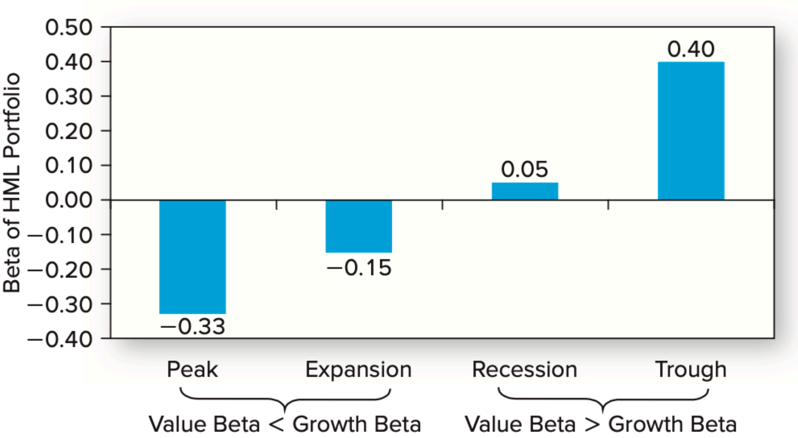
In PZ

$$\begin{aligned} R_{\mathsf{HML}} &= \alpha + \beta_t R_{Mt} + e_i \\ &= \alpha + \underbrace{\left[b_0 + b_1 \mathsf{DIV}_t + b_2 \mathsf{DEFLT}_t + b_3 \mathsf{TERM}_t + b_4 \mathsf{TB}_t\right]}_{\beta_t} R_{Mt} + e_i \end{aligned}$$

Jagannathan and Wang (1996) Results

Coefficient	c_0	c_{vw}	c_{credit}	c_{labor}	c_{size}	R^2
A. The Static CAPM without Human Capital						
Estimate	1.24	−0.10				1.35
t-statistic	5.16	−0.28				
Estimate	2.08	−0.32			−0.11	57.56
t-statistic	5.77	−0.94			−2.30	
B. The Conditional CAPM with Human Capital						
Estimate	1.24	−0.40	0.34	0.22		55.21
t-statistic	4.10	−0.88	1.73	2.31		
Estimate	1.70	−0.40	0.20	0.10	−0.07	64.73
t-statistic	4.14	−1.06	2.72	2.09	−1.30	

Petkova and Zhang (2005) Results



Takeaway

- We can test CAPM using a two-pass regression procedure.
- The empirical performance of CAPM is not satisfactory.
  - The estimated SML is too flat.
  - The pricing error is significantly different from zero.
- Beta measurement is usually noisy.
- The static CAPM can be extended to conditional CAPM.