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Chapter 13

Efficient Markets

Reference (medium): Elton, Gruber, Brown, and Goetzmann (2014) 17 (efficient markets) and 19 (earnings estimation)

Additional references: Campbell, Lo, and MacKinlay (1997) 2 and 7; Cochrane (2001) 20.1

More advanced material is denoted by a star (*). It is not required reading.

13.1 The Efficient Market Hypothesis

The efficient market hypothesis (EMH) says that it is very *hard to predict future asset returns*. If this is true (evidence is discussed later), then active management (security analysis, market timing) is useless and costly (management fees, trading costs). Instead, it makes more sense to apply a passive approach that meets individual requirements (diversification, hedging background risk, appropriate risk level, etc). The practical implications are thus very significant.

13.1.1 Defining Expected Returns

Let P_t be the price of an asset at the end of period t, after any dividend in t has been paid (an ex-dividend price). The net return (R_{t+1}, like 0.05) of holding an asset with dividends (per current share), D_{t+1}, between t and $t+1$ is then defined as

$$R_{t+1} = \frac{P_{t+1} + D_{t+1}}{P_t} - 1. \tag{13.1}$$

The dividend can, of course, be zero in a particular period, so this formulation encompasses the case of daily stock prices with annual dividend payment.

Remark 13.1 *(Conditional expectations) The expected value of the random variable y_{t+1} conditional on the information set in t ($\mathrm{E}_t\, y_{t+1}$) is the best guess of y_{t+1} using the information in t. Example: suppose y_{t+1} equals $x_t + \varepsilon_{t+1}$, where x_t is known in t, but all we know about ε_{t+1} in t is that it is a random variable with a zero mean and some (finite) variance. In this case, the best guess of y_{t+1} based on what we know in t is equal to x_t.*

Take expectations of (13.1) based on the information set in t

$$\mathrm{E}_t\, R_{t+1} = \frac{\mathrm{E}_t\, P_{t+1} + \mathrm{E}_t\, D_{t+1}}{P_t} - 1. \tag{13.2}$$

This formulation is only a definition, but it will help us organize the discussion of how asset prices are determined.

13.1.2 Different Versions of the Efficient Market Hypothesis

The efficient market hypothesis (EMH) casts a long shadow on every attempt to forecast asset prices. In its simplest form it says that it is not possible to forecast asset prices, but there are several other forms with different implications. Before attempting to forecast financial markets, it is useful to take a look at the logic of the efficient market hypothesis.

The basic idea of the EMH is illustrated in Figure 13.1, which shows the asset price path around an event. If this is the typical (average) path, then both the overreaction and underreaction paths imply that returns are forecastable (after the event). For instance, after the event the overreaction path shows mean reversion of the price (so it would be profitable to short sell this asset just after the event), while the underreaction path shows the opposite. The EMH suggests that such (easy) opportunities do not exist (at least not systematically), so the (average) price path must be the solid line.

However, a precise formulation of the EMH needs to specify two things. First, what type of information is used in making those forecasts? Is it price and trading volume data (the weak form of the EMH), all public information (the semi-strong form), or perhaps all public and private information (the strong form)? Most modern analysis is focused on the weak or semi-strong forms (as private information is likely to have predictive power). Second, what is supposed to be unforecastable? Is it price changes, returns, or excess returns? This is discussion in some detail below.

If price changes are unforecastable, then $\mathrm{E}_t\, P_{t+1} - P_t$ equals a constant. Typically,

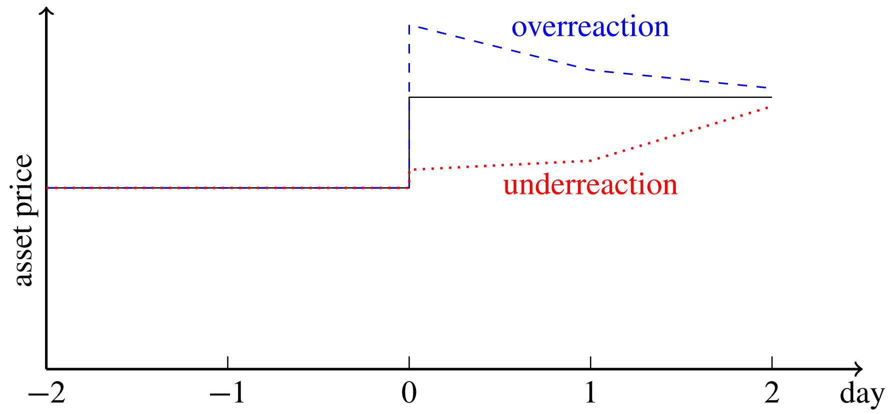

Figure 13.1: Asset price path around an event

this constant is taken to be zero. Use $\mathrm{E}_t\, P_{t+1} = P_t$ in (13.2) to get

$$\mathrm{E}_t\, R_{t+1} = \frac{\mathrm{E}_t\, D_{t+1}}{P_t}. \tag{13.3}$$

This says that the expected net return on the asset is the expected dividend divided by the current price. This is clearly implausible for daily data since it means that the expected return is zero for all days except those days when the asset pays a dividend (or rather, the day the asset goes ex dividend)—and then there is an enormous expected return for the one day. As a first step, we should probably refine the interpretation of the efficient market hypothesis to include the dividend so that $\mathrm{E}_t(P_{t+1} + D_{t+1}) = P_t$. Using that in (13.2) gives $\mathrm{E}_t\, R_{t+1} = 0$, which seems implausible for long investment horizons—although it is probably a reasonable approximation for short horizons (a week or less).

If returns are unforecastable, then $\mathrm{E}_t\, R_{t+1} = R$ (a constant). The main problem with this formulation is that it looks at every asset separately and that outside options are not taken into account. For instance, if the nominal interest rate changes from 5% to 10%, why should the expected (required) return on a stock be unchanged? In fact, most asset pricing models suggest that the expected return $\mathrm{E}_t\, R_{t+1}$ equals the riskfree rate plus compensation for risk.

If excess returns are unforecastable, then the compensation (over the riskfree rate) for risk is constant. This is a reasonable null hypothesis, which will be used in these notes.

Rejection of the EMH can have different sources: changes in risk or in risk aversion (both "rational" reasons) or in inefficiencies. It is typically very hard to disentangle these possible sources.

13.2 Autocorrelations and Autoregressions

Autocorrelations and autoregressions are tools for studying whether past and current returns can predict future returns (typically of the same asset).

13.2.1 Autocorrelation Coefficients

The autocovariances of the R_t process can be estimated as

$$\hat{\gamma}_s = \frac{1}{T}\sum_{t=1+s}^{T}\left(R_t - \bar{R}\right)\left(R_{t-s} - \bar{R}\right), \text{ with} \tag{13.4}$$

$$\bar{R} = \frac{1}{T}\sum_{t=1}^{T} R_t. \tag{13.5}$$

(We typically divide by T in (13.4) even if we have only $T-s$ full observations to estimate γ_s from.) Autocorrelations are then estimated as

$$\hat{\rho}_s = \hat{\gamma}_s / \hat{\gamma}_0. \tag{13.6}$$

The sampling properties of $\hat{\rho}_s$ are complicated, but there are several useful large sample results for Gaussian processes (these results typically carry over to processes which are similar to the Gaussian). When the true autocorrelations are all zero (not ρ_0, of course), then for any lag s different from zero

$$\sqrt{T}\hat{\rho}_s \to^d N(0,1), \tag{13.7}$$

so $\sqrt{T}\hat{\rho}_s$ can be used as a t-stat. See Figures 13.2–13.3.

Example 13.2 *(t-test) We want to test the hypothesis that* $\rho_1 = 0$*. Since the* $N(0,1)$ *distribution has 5% of the probability mass below -1.64 and another 5% above 1.64, we can reject the null hypothesis at the 10% level if* $\sqrt{T}|\hat{\rho}_1| > 1.64$.

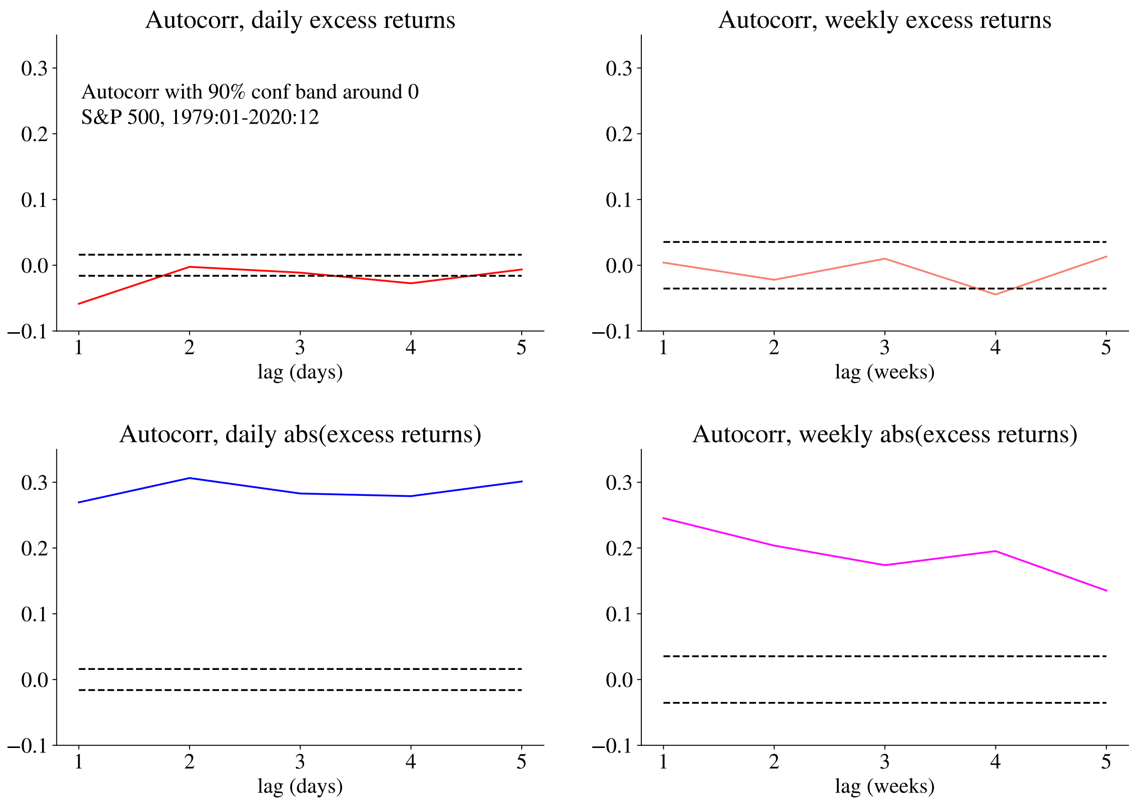

Figure 13.2: Predictability of US stock returns

13.2.2 Autoregressions

An alternative way of testing autocorrelations is to estimate an AR model

$$R_t = c + a_1 R_{t-1} + a_2 R_{t-2} + ... + a_p R_{t-p} + \varepsilon_t, \tag{13.8}$$

and then test if all slope coefficients $(a_1, a_2, ..., a_p)$ are zero with a χ^2 or F test. This approach is somewhat less general than testing if all autocorrelations are zero, but is easy to implement (and the difference is not large). See Table 13.1 for an illustration.

The autoregression can also allow for the coefficients to depend on the market situation. For instance, consider an AR(1), but where the autoregression coefficient may be

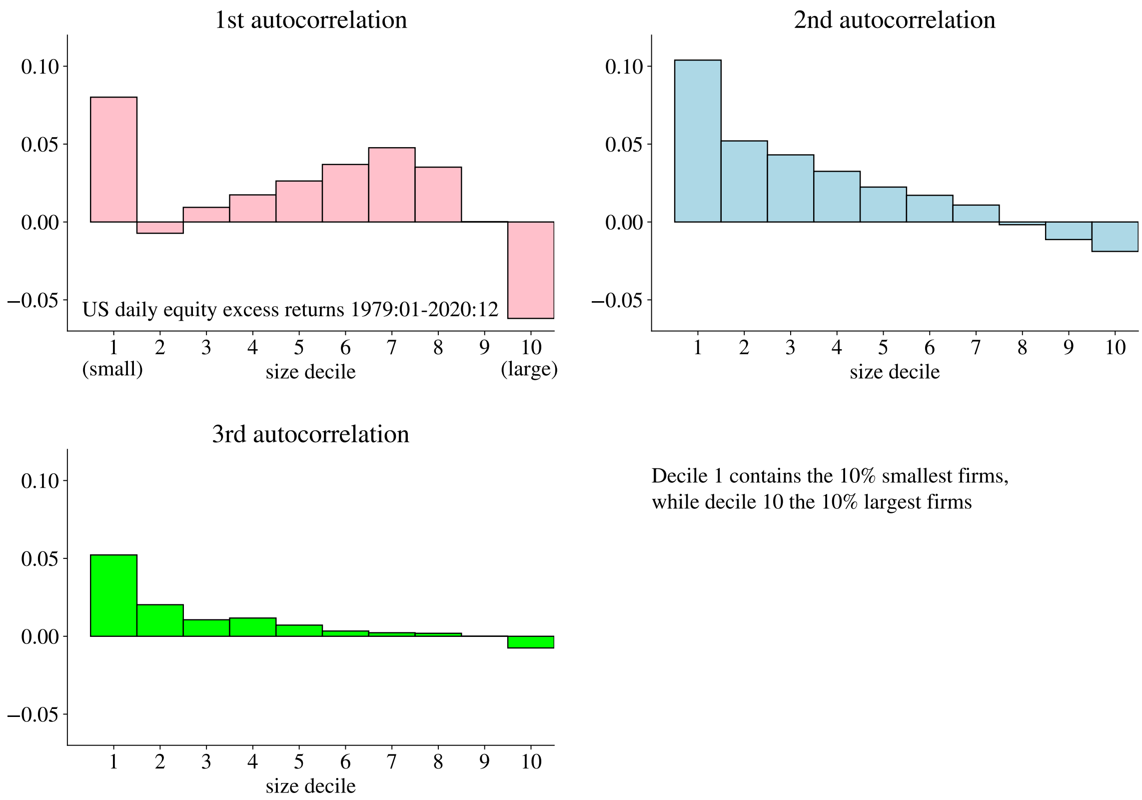

Figure 13.3: Predictability of US stock returns, size deciles

different depending on the sign of last period's return

$$R_t = \alpha + \beta Q_{t-1} R_{t-1} + \gamma (1 - Q_{t-1}) R_{t-1} + \varepsilon_t, \text{ where} \tag{13.9}$$

$$Q_{t-1} = \begin{cases} 1 \text{ if } R_{t-1} < 0 \\ 0 \text{ else.} \end{cases}$$

This is illustrated in Figure 13.4.

Autoregressions have also been used to study the predictability of long-run returns. See Figure 13.5 for an illustration.

13.3 Other Predictors and Methods

There are many other possible predictors of future stock returns. For instance, both the dividend-price ratio and nominal interest rates have been used to predict long-run returns, and lagged short-run returns on other assets have been used to predict short-run returns.

	(1)
lag 1	−0.07
	(−2.81)
lag 2	−0.02
	(−0.55)
lag 3	−0.02
	(−0.78)
lag 4	−0.04
	(−2.01)
lag 5	−0.01
	(−0.49)
c	0.03
	(2.98)
R^2	0.01
All slopes	0.00
obs	9945

Table 13.1: AR(5) of daily S&P returns 1979:01-2020:12. Numbers in parentheses are t-stats, based on Newey-West with 3 lags. All slopes is the p-value for all slope coefficients being zero.

13.3.1 Lead-Lags

Stock indices have more positive autocorrelation than (most) individual stocks: there should therefore be fairly strong cross-autocorrelations across individual stocks. Indeed, this is also what is found in US data where returns of large size stocks forecast returns of small size stocks. See Figure 13.6 for an illustration.

13.3.2 Earnings-Price Ratio as a Predictor

One of the most successful attempts to forecast long-run returns is a regression of future returns on the current earnings-price (or dividend-price) ratio (here in logs)

$$z^e_{s,t} = \alpha + \beta_q \ln(E_{t-s}/P_{t-s}) + \varepsilon_t, \tag{13.10}$$

where $z^e_{s,t}$ is the s-period excess log return over $t-s$ to t.

See Figure 13.7 for an illustration.

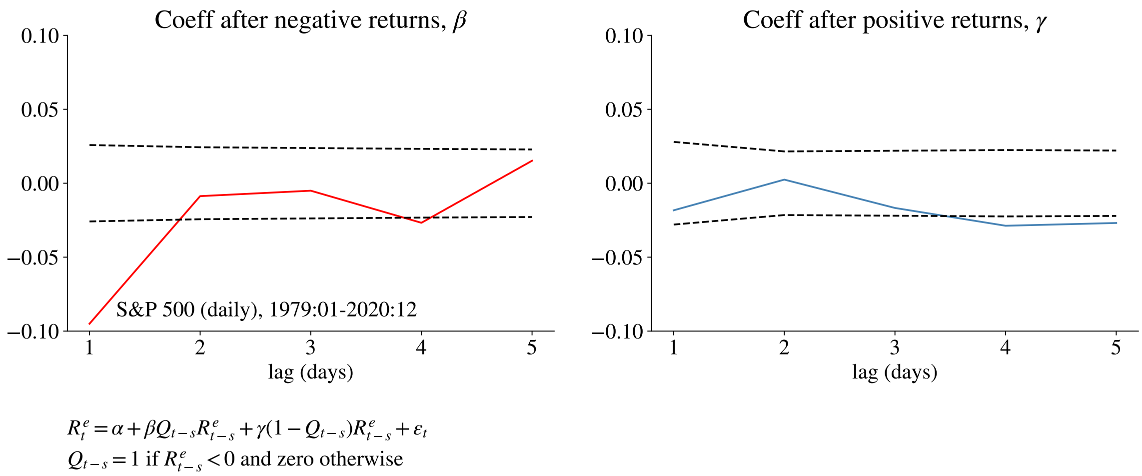

Figure 13.4: Predictability of US stock returns, results from a regression with interactive dummies

13.4 Out-of-Sample Forecasting Performance

13.4.1 In-Sample versus Out-of-Sample Forecasting

In-sample evidence on predictability can potentially be misleading because of *(a)* in-sample overfitting; and/or *(b)* structural breaks.

To gauge the out-of-sample predictability, estimate the prediction equation using data for a moving data window up to and including $t-1$ (for instance, $t-W$ to $t-1$), and then make a forecast for period t. The forecasting performance of the equation is then compared with a benchmark model (for instance, using the historical average as the predictor). Notice that this benchmark model is also estimated on data up to an including $t-1$, so it changes over time. See Figure 13.8.

To formalise the comparison, study the RMSE and the "out-of-sample R^2"

$$R^2_{OS} = 1 - \sum\nolimits_{t=s}^{T} (R_t - \hat{R}_t)^2 / \sum\nolimits_{t=s}^{T} (y_t - \tilde{R}_t)^2, \tag{13.11}$$

where s is the first period with an out-of-sample forecast, $\hat{R}_t$ is the forecast based on the prediction model (estimated on data up to and including $t-1$) and $\tilde{R}_t$ is the prediction from some benchmark model (also estimated on data up to and including $t-1$).

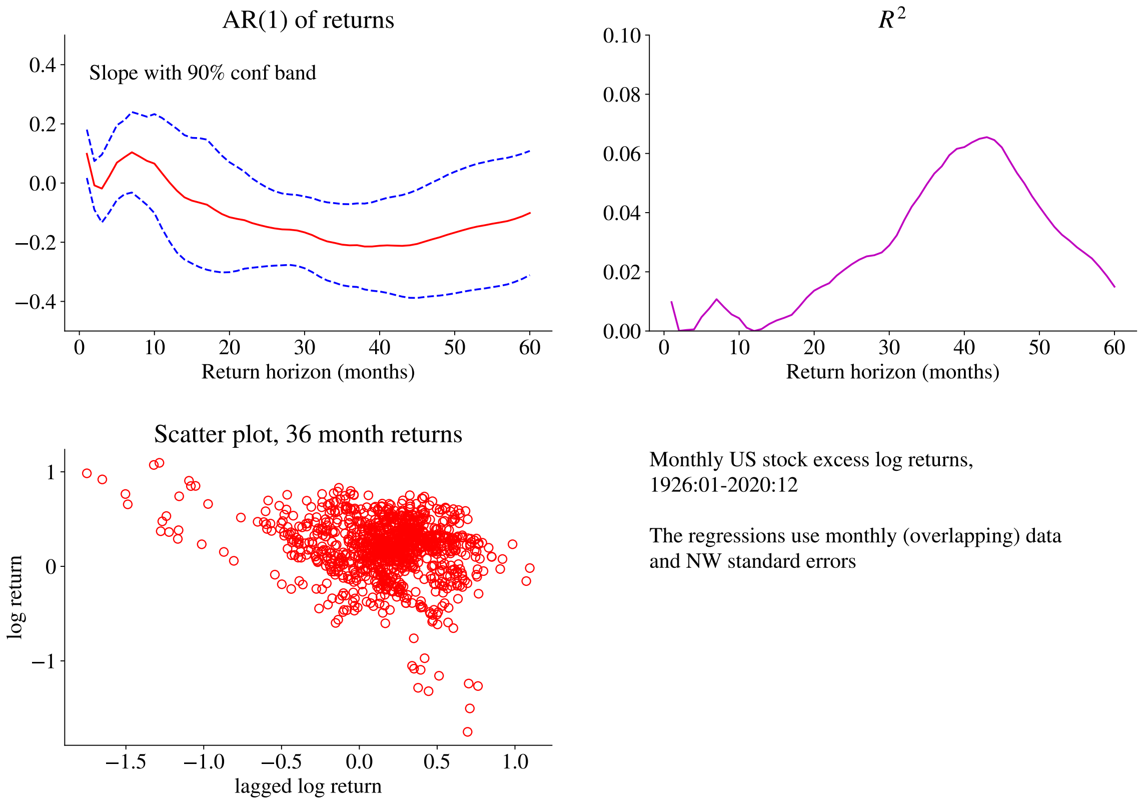

Figure 13.5: Predictability of long-run US stock returns

Example 13.3 *(R^2_{OS})*

$$R^2_{OS} = 1 - \frac{0.4}{0.5} = 0.2 \text{ (your model is better)}$$
$$R^2_{OS} = 1 - \frac{0.5}{0.4} = -0.25 \text{ (your model is worse)}$$

Goyal and Welch (2008) find that the evidence of predictability of equity returns disappears when out-of-sample forecasts are considered.

See Figures 13.9–13.10 for illustrations.

13.4.2 Trading Strategies

Another way to measure predictability and to illustrate its economic importance is to calculate the return of a *dynamic trading strategy*, and then measure the performance of this strategy in relation to some benchmark portfolios. The trading strategy should be based on the variables that are supposed to forecast returns.

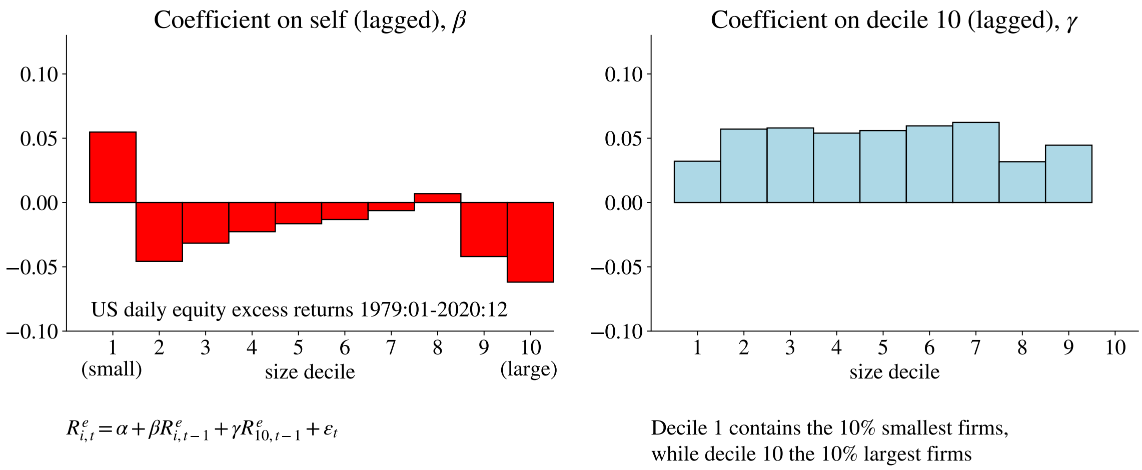

Figure 13.6: Coefficients from multiple prediction regressions

A common way to measure the performance of a portfolio is its *alpha* from a regression on the market excess return. Neutral performance requires $\alpha = 0$, which can be tested with a t test.

See Figure 13.11 for an empirical example. (In this example the alphas are almost the same as the excess return since a long-short equity portfolio has a beta close to zero.)

13.4.3 Technical Analysis

Main reference: Bodie, Kane, and Marcus (2002) 12.2; Neely (1997) (overview, foreign exchange market)
Further reading: Murphy (1999) (practical, a believer's view); The Economist (1993) (overview, the perspective of the early 1990's); Brock, Lakonishok, and LeBaron (1992) (empirical, stock market); Lo, Mamaysky, and Wang (2000) (academic article on return distributions for "technical portfolios")

Technical analysis is typically a data mining exercise which looks for local trends or systematic non-linear patterns. The basic idea is that markets are not instantaneously efficient: prices react somewhat slowly and predictably to news. In practice, technical analysis amounts to analysing different transformations (for instance, a moving average) of prices—and to spot patterns. This section summarizes some simple trading rules that are used.

Many trading rules rely on some kind of local trend which can be thought of as positive

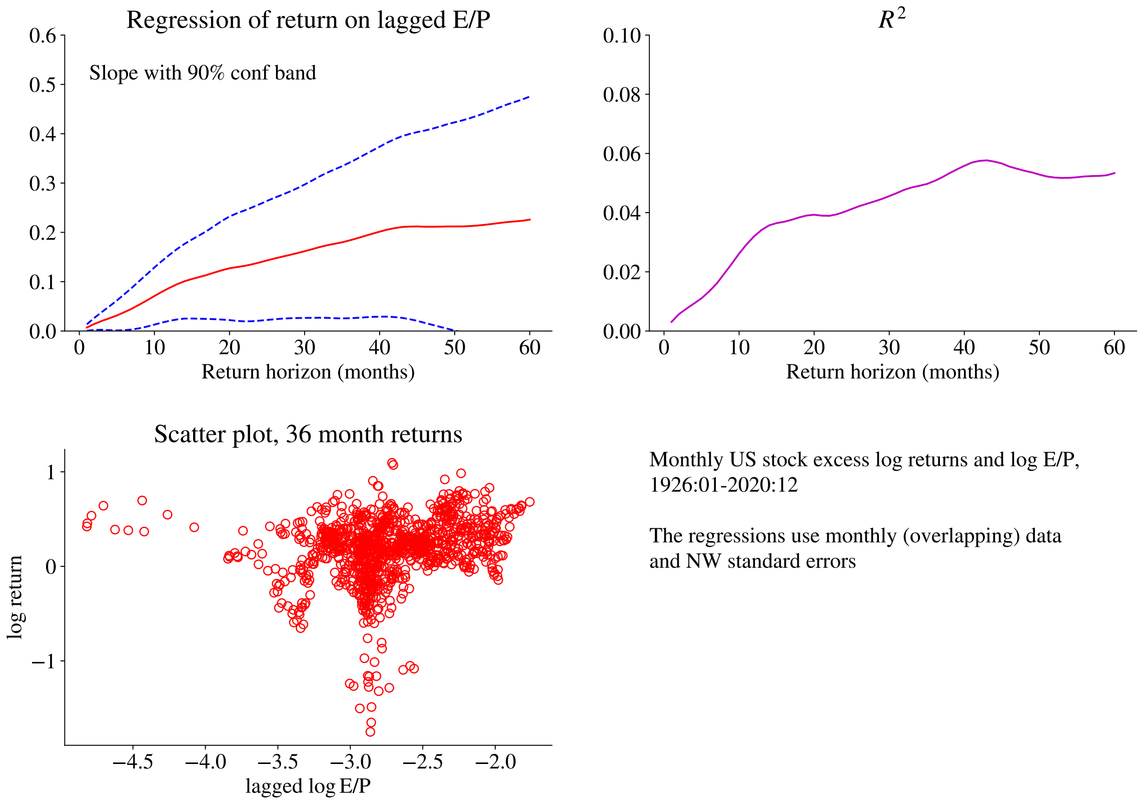

Figure 13.7: Predictability of long-run US stock returns

autocorrelation in price movements (also called momentum[1]).

A *moving average rule* is to buy if a short moving average (equally weighted or exponentially weighted) goes above a long moving average. The idea is that this signals a new upward trend. Let S (L) be the lag order of a short (long) moving average, with $S < L$ and let b be a bandwidth (perhaps 0.01). Then, a MA rule for period t could be

$$\begin{bmatrix} \text{buy in } t \text{ if} & MA_{t-1}(S) > MA_{t-1}(L)(1+b) \\ \text{sell in } t \text{ if} & MA_{t-1}(S) < MA_{t-1}(L)(1-b) \\ \text{no change} & \text{otherwise} \end{bmatrix}, \text{ where} \tag{13.12}$$

$$MA_{t-1}(S) = (p_{t-1} + \ldots + p_{t-S})/S.$$

The difference between the two moving averages is called an *oscillator*

$$\text{oscillator}_t = MA_t(S) - MA_t(L), \tag{13.13}$$

[1]In physics, momentum equals the mass times speed.

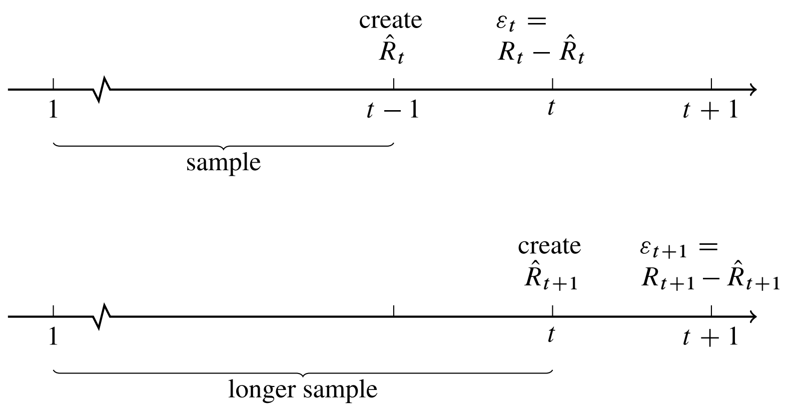

Figure 13.8: Out-of-sample forecasting

(or sometimes, moving average convergence divergence, MACD) and the sign is taken as a trading signal (this is the same as a moving average crossing, MAC). A version of the moving average oscillator is the *relative strength index*[2], which is the ratio of average price level (or returns) on "up" days to the average price (or returns) on "down" days—during the last z (14 perhaps) days. Yet another version is to compare the oscillator$_t$ to a moving average of the oscillator (also called a signal line).

The *trading range break-out rule* typically amounts to buying when the price rises above a previous peak (local maximum). The idea is that a previous peak is a *resistance level* in the sense that some investors are willing to sell when the price reaches that value (perhaps because they believe that prices cannot pass this level; clear risk of circular reasoning or self-fulfilling prophecies; round numbers often play the role as resistance levels). Once this artificial resistance level has been broken, the price can possibly rise substantially. On the downside, a *support level* plays the same role: some investors are willing to buy when the price reaches that value. To implement this, it is common to let the resistance/support levels be proxied by minimum and maximum values over a data

[2] Not to be confused with relative strength, which typically refers to the ratio of two different asset prices (for instance, an equity compared to the market).

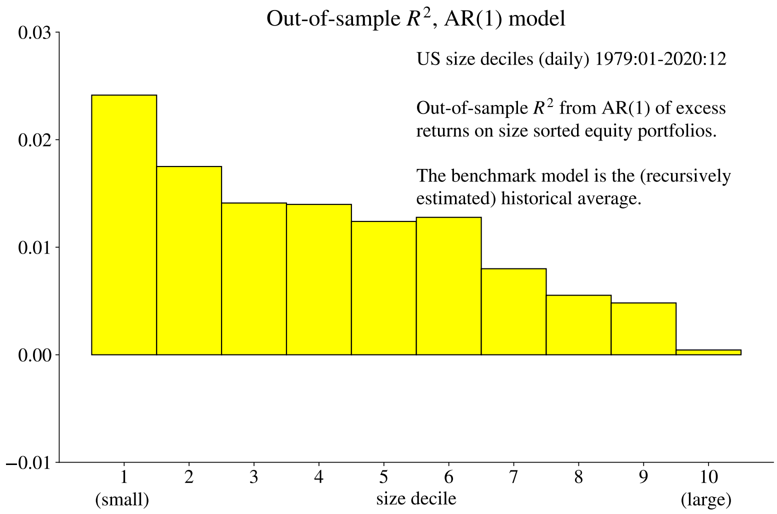

Figure 13.9: Short-run predictability of US stock returns, out-of-sample.

window of length L. With a bandwidth b (perhaps 0.01), the rule for period t could be

$$\begin{bmatrix} \text{buy in } t \text{ if} & P_t > M_{t-1}(1+b) \\ \text{sell in } t \text{ if} & P_t < m_{t-1}(1-b) \\ \text{no change} & \text{otherwise} \end{bmatrix}, \text{ where} \tag{13.14}$$

$$M_{t-1} = \max(p_{t-1}, \ldots, p_{t-S})$$
$$m_{t-1} = \min(p_{t-1}, \ldots, p_{t-S}).$$

When the price is already trending up, then the trading range break-out rule may be replaced by a *channel rule*, which works as follows. First, draw a *trend line* through previous lows and a *channel line* through previous peaks. Extend these lines. If the price moves above the channel (band) defined by these lines, then buy. A version of this is to define the channel by a *Bollinger band*, which is ± 2 standard deviations from a moving data window around a moving average.

If we instead believe in mean reversion of the prices, then we can essentially reverse the previous trading rules: we would typically sell when the price is high. See Figure 13.12 and Table 13.2.

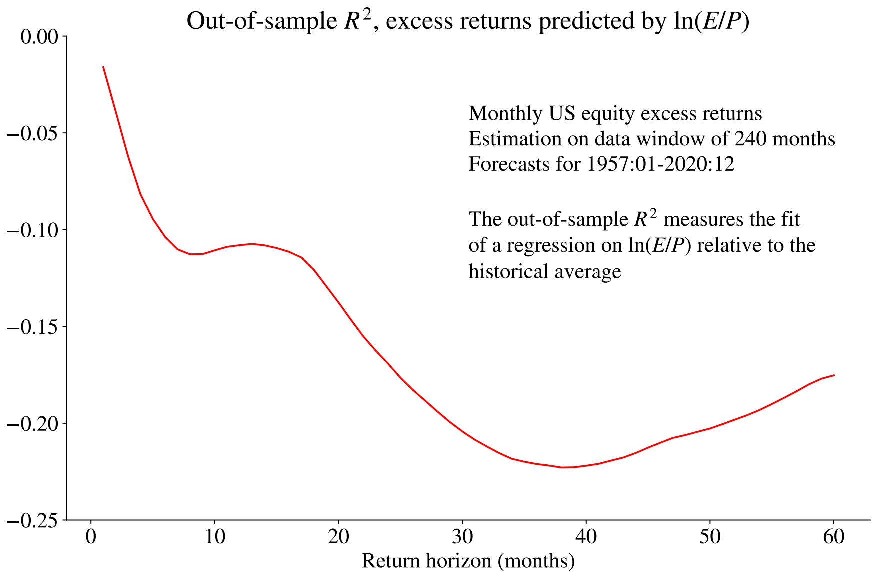

Figure 13.10: Predictability of long-run US stock returns, out-of-sample

13.5 Security Analysts

Reference: Makridakis, Wheelwright, and Hyndman (1998) 10 and Elton, Gruber, Brown, and Goetzmann (2014) 27

13.5.1 Evidence on Analysts' Performance

Makridakis, Wheelwright, and Hyndman (1998) show that there is little evidence that the average stock analyst beats (on average) the market (or a passive index portfolio). In fact, less than half of the analysts beat the market. However, there are analysts which seem to outperform the market for some time, but the autocorrelation in over-performance is weak. The evidence from mutual funds is similar.

It should be remembered that many analysts also are sales persons: either of a stock (for instance, since the bank is underwriting an offering) or of trading services. It could well be that their objective function is quite different from minimizing the squared forecast errors. (The number of litigations in the US after the technology boom/bust should serve as a strong reminder of this.)

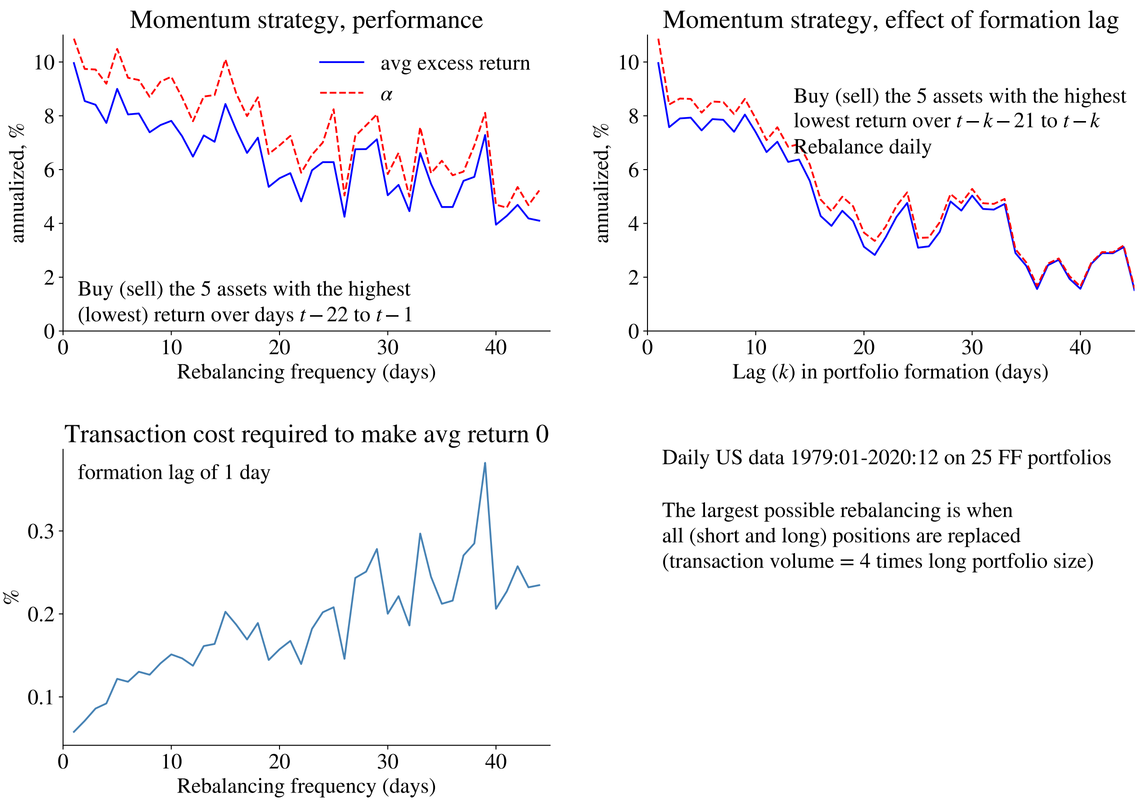

Figure 13.11: Predictability of US stock returns, momentum strategy

13.5.2 Do Security Analysts Overreact?

The paper by Bondt and Thaler (1990) compares the (semi-annual) forecasts (one- and two-year time horizons) with actual changes in earnings per share (1976-1984) for several hundred companies. The paper has regressions like

$$\text{Actual earnings change} = \alpha + \beta(\text{forecasted earnings change}) + \text{ residual},$$

and then studies the estimates of the α and β coefficients. With rational expectations (and a long enough sample), we should have $\alpha = 0$ (no constant bias in forecasts) and $\beta = 1$ (proportionality, for instance no exaggeration).

The main result is that $0 < \beta < 1$, so that the forecasted change tends to be too wild in a systematic way: a forecasted change of 1% is (on average) followed by a less than 1% actual change in the same direction. This means that analysts in this sample tended to be too extreme—to exaggerate both positive and negative news.

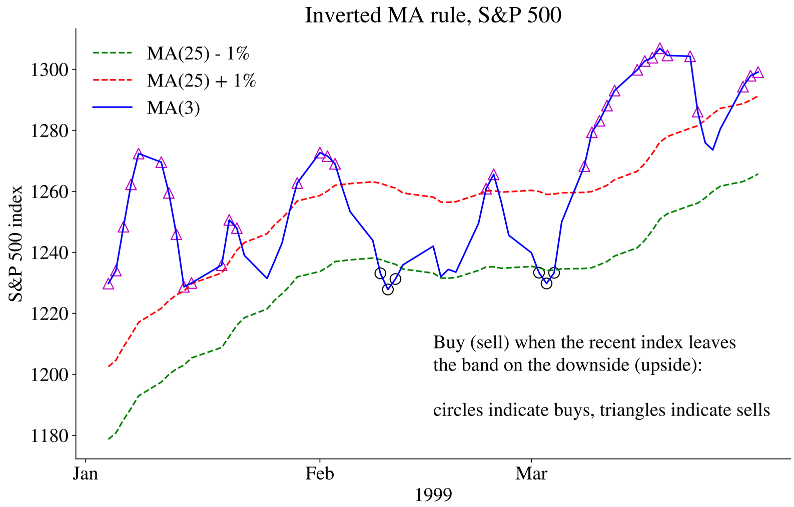

Figure 13.12: Example of a trading rule, illustration over short subsample

13.5.3 High-Frequency Trading Based on Recommendations from Stock Analysts

Barber, Lehavy, McNichols, and Trueman (2001) give a somewhat different picture. They focus on the profitability of a trading strategy based on analyst recommendations. They use a huge data set (some 360,000 recommendations, US stocks) for the period 1985–1996. They sort stocks in to five portfolios depending on the consensus (average) recommendation—and redo the sorting every day (if a new recommendation is published). They find that such a daily trading strategy gives an annual 4% abnormal return on the portfolio of the most highly recommended stocks, and an annual -5% abnormal return on the least favourably recommended stocks.

This strategy requires a lot of trading (a turnover of 400% annually), so trading costs would typically reduce the abnormal return on the best portfolio to almost zero. A less frequent rebalancing (weekly, monthly) gives a very small abnormal return for the best stocks, but still a negative abnormal return for the worst stocks. Chance and Hemler (2001) obtain similar results when studying the investment advise by 30 professional "market timers."

13.5.4 Economic Experts

Several papers, for instance, Bondt (1991) and Söderlind (2010), have studied whether economic experts can predict the broad stock markets. The results suggests that they cannot. For instance, Söderlind (2010) shows that the economic experts that participate in

	Mean	Std
All days	6.9	18.1
After buy signal	17.4	28.1
After neutral signal	5.3	14.5
After sell signal	2.7	13.7
Strategy	9.6	27.9
Transaction cost	0.1	

Table 13.2: Excess returns (annualized, in %) from technical trading rule (Inverted MA rule). Daily S&P 500 data 1990:01-2020:12. The trading strategy involves (a) on everay day: hold one unit of the index and short the riskfree; (b) on days after a buy signal: double the position in (a); (c) on days after a sell signal: short sell the position in (a), effectively having a zero investment. The transaction costs shows the cost (in %) of trade that the strategy can pay and still perform as well as the static holding of (a).

the semi-annual Livingston survey (mostly bank economists) *(ii)* forecast the S&P worse than the historical average (recursively estimated), and that their forecasts are strongly correlated with recent market data (which in itself, cannot predict future returns).

13.5.5 Analysts and Industries

Boni and Womack (2006) study data on some 170,000 recommendations for a very large number of U.S. companies for the period 1996–2002. Focusing on revisions of recommendations, the papers shows that analysts are better at ranking firms within an industry than ranking industries.

13.5.6 Insiders

Corporate insiders *used to* earn superior returns, mostly driven by selling off stocks before negative returns. (There is little/no systematic evidence of insiders gaining by buying before high returns.) Actually, investors who followed the insider's registered transactions (in the U.S., these are made public six weeks after the reporting period), also used to earn some superior returns. It seems as if these patterns have more or less disappeared.

13.5.7 Mutual Funds

The general evidence on mutual funds is that they, on average, have zero alphas (or worse, after fees), and that there is little persistence in overperformance, at least among good funds(possible exceptions: hedge funds and private equity funds), while bad funds can stay bad for a long while.

13.6 Event Studies*

Reference: Bodie, Kane, and Marcus (2005) 12.3 or Copeland, Weston, and Shastri (2005) 11
Reference (advanced): Campbell, Lo, and MacKinlay (1997) 4

13.6.1 Basic Structure

The idea of an event study is to study the effect of a special event by using a cross-section of such events. For instance, what is the average (across firms) effect of a negative earnings surprise on the return?

According to the efficient market hypothesis, only *news* should move the asset price, so it is often necessary to explicitly model the previous expectations to define the event. For earnings, the event is typically taken to be a dummy that indicates if the earnings announcement is smaller than (some average of) analysts' forecast.

To isolate the effect of the event, we typically study the *abnormal return* of asset i in period t

$$u_{it} = R_{it} - R_{it}^{normal}, \tag{13.15}$$

where R_{it} is the actual return and the last term is the normal return (which may differ across assets and time). The definition of the normal return is discussed in detail in Section 13.6.2.

Suppose we have a sample of n such events. To keep the notation simple, we "normalize" the time so period 0 is the time of the event (irrespective of its actual calendar time).

To study information leakage and slow price adjustment, the abnormal return is often calculated for some time before and after the event: the *event window* (often ± 20 days or so). For day s (that is, s days after the event time 0), the cross sectional average abnormal return is

$$\bar{u}_s = \textstyle\sum_{i=1}^{n} u_{is}/n. \tag{13.16}$$

For instance, $\bar{u}_2$ is the average abnormal return two days after the event, and $\bar{u}_{-1}$ is for one day before the event.

The *cumulative abnormal return* (CAR) of asset i is simply the sum of the abnormal return in (13.15) over some period around the event. It is often calculated from the beginning of the event window. For instance, if the event window starts at -20, then the 3-period (day?) car for firm i is

$$\text{car}_{i3} = u_{i,-20} + u_{i,-19} + u_{i,-18}. \tag{13.17}$$

More generally, if the event window starts at w (say, -20), then the q-period car for firm i is

$$\text{car}_{iq} = \textstyle\sum_{\tau=w}^{w+q-1} u_{i,\tau}. \tag{13.18}$$

The cross sectional average of the q-period car is

$$\overline{\text{car}}_q = \textstyle\sum_{i=1}^{n} \text{car}_{iq}/n. \tag{13.19}$$

See Figure 13.13 for an empirical example.

Example 13.4 *(Abnormal returns for ± day around event, two firms) Suppose there are two firms and the event window contains ±1 day around the event day, and that the abnormal returns (in percent) are*

Time	*Firm 1*	*Firm 2*	*Cross-sectional Average*
−1	0.2	−0.1	0.05
0	1.0	2.0	1.5
1	0.1	0.3	0.2

We have the following cumulative returns

Time	*Firm 1*	*Firm 2*	*Cross-sectional Average*
−1	0.2	−0.1	0.05
0	1.2	1.9	1.55
1	1.3	2.2	1.75

13.6.2 Models of Normal Returns

This section summarizes the most common ways of calculating the normal return in (13.15). The parameters in these models are typically estimated on a recent sample, the *estimation window*, which ends before the event window. See Figure 13.14 for an illustration. In this way, the estimated behaviour of the normal return should be unaffected by

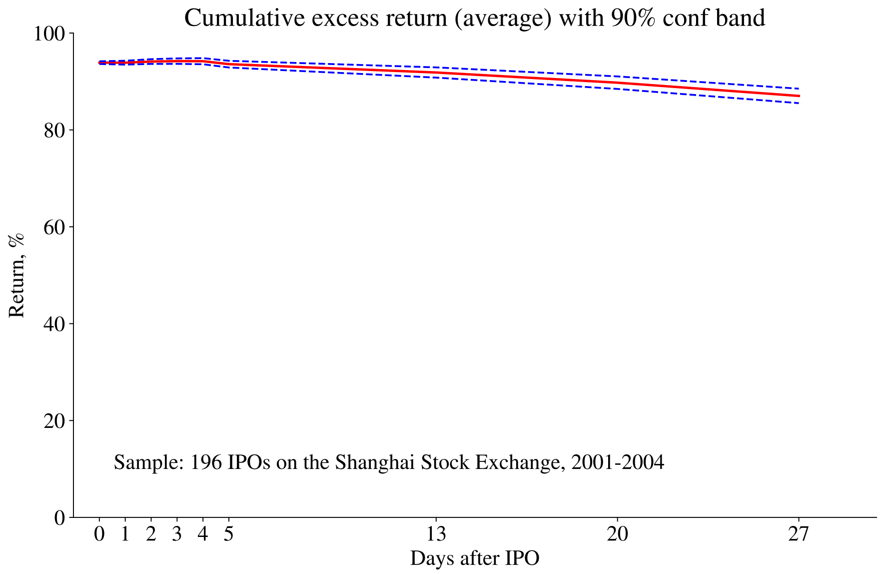

Figure 13.13: Event study of IPOs in Shanghai 2001–2004. (Data from Nou Lai.)

the event. It is almost always assumed that the event is exogenous in the sense that it is not due to the movements of the asset price during either the estimation window or the event window.

The *constant mean return model* assumes that the return of asset i fluctuates randomly around some mean μ_i

$$\begin{aligned} R_{it} &= \mu_i + \varepsilon_{it} \text{ with} \\ \mathrm{E}\,\varepsilon_{it} &= 0 \text{ and } \mathrm{Cov}(\varepsilon_{it}, \varepsilon_{i,t-s}) = 0. \end{aligned} \tag{13.20}$$

This mean is estimated by the sample average (during the estimation window). The normal return in (13.15) is then the estimated mean, $\hat{\mu}_i$, so the abnormal return (in the estimation window) is the fitted residual. During the event window, we calculate the abnormal return as

$$u_{it} = R_{it} - \hat{\mu}_i. \tag{13.21}$$

The standard error of this is estimated by the standard error of the fitted residual in the estimation window.

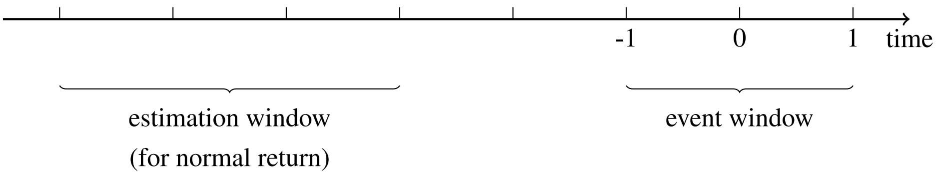

Figure 13.14: Event and estimation windows

The *market model* is a linear regression of the return of asset i on the market return

$$R_{it} = \alpha_i + \beta_i R_{mt} + \varepsilon_{it} \text{ with} \tag{13.22}$$
$$\mathrm{E}\,\varepsilon_{it} = 0 \text{ and } \mathrm{Cov}(\varepsilon_{it}, R_{mt}) = 0.$$

Notice that we typically do not impose the CAPM restrictions on the intercept in (13.22). The normal return in (13.15) is then calculated by combining the regression coefficients with the actual market return as $\hat{\alpha}_i + \hat{\beta}_i R_{mt}$, so that the abnormal return in the estimation window is the fitted residual. For the event window we calculate the abnormal return as

$$u_{it} = R_{it} - \hat{\alpha}_i - \hat{\beta}_i R_{mt}. \tag{13.23}$$

The standard error of this is estimated by the standard error of the fitted residual in the estimation window.

When we restrict $\alpha_i = 0$ and $\beta_i = 1$, then this approach is called the *market-adjusted-return model*. This is a particularly useful approach when there is no return data before the event, for instance, with an IPO. For the event window we calculate the abnormal return as

$$u_{it} = R_{it} - R_{mt} \tag{13.24}$$

and the standard error of it is estimated by $\mathrm{Std}(R_{it} - R_{mt})$ in the estimation window (if available).

Yet another approach is to construct a normal return as the actual return on assets which are very similar to the asset with an event. For instance, if asset i is a small manufacturing firm (with an event), then the normal return could be calculated as the actual return for other small manufacturing firms (without events). In this case, the abnormal return becomes the difference between the actual return and the return on the *matching* *portfolio*. For the event window we calculate the abnormal return as

$$u_{it} = R_{it} - R_{pt}, \tag{13.25}$$

where R_{pt} is the return of the matching portfolio. The standard error of it is estimated by the standard deviation of $R_{it} - R_{pt}$ in the estimation window.

High frequency data can be very helpful, provided the time of the event is known. High frequency data effectively allows us to decrease the volatility of the abnormal return since it filters out irrelevant (for the event study) shocks to the return while still capturing the effect of the event.

13.6.3 Testing the Abnormal Return

It is typically assumed that the abnormal returns are *uncorrelated across time and across assets*. The first assumption is motivated by the very low autocorrelation of returns. The second assumption makes sense if the events are not overlapping in time, so that the event of assets i and j happen at different (calendar) times. If the events are overlapping, then another approach is needed.

Let $\sigma_i^2 = \text{Var}(u_{it})$ be the variance of the abnormal return of asset i. The *variance of the cross-sectional* (across the n assets) *average*, $\bar{u}_s$ in (13.16), is then

$$\text{Var}(\bar{u}_s) = \textstyle\sum_{i=1}^{n} \sigma_i^2 / n^2, \tag{13.26}$$

since all covariances are assumed to be zero. In a large sample, we can therefore use a t-test since

$$\bar{u}_s / \text{Std}(\bar{u}_s) \to^d N(0,1). \tag{13.27}$$

The *cumulative abnormal return* over q period, $\text{car}_{i,q}$, can also be tested with a t-test. Since the returns are assumed to have no autocorrelation the variance of the $\text{car}_{i,q}$

$$\text{Var}(\text{car}_{iq}) = q\sigma_i^2. \tag{13.28}$$

This variance is increasing in q since we are considering cumulative returns (not the time average of returns).

The *cross-sectional average* $car_{i,q}$ is then (similarly to (13.26))

$$\text{Var}(\overline{\text{car}}_q) = q \textstyle\sum_{i=1}^{n} \sigma_i^2 / n^2, \tag{13.29}$$

if the abnormal returns are uncorrelated across time and assets.

Example 13.5 *(Variances of abnormal returns) If the standard deviations of the daily abnormal returns of the two firms in Example 13.4 are $\sigma_1 = 0.1$ and and $\sigma_2 = 0.2$, then we have the following variances for the abnormal returns at different days*

Time	*Firm 1*	*Firm 2*	*Cross-sectional Average*
-1	0.1^2	0.2^2	$\left(0.1^2 + 0.2^2\right)/4$
0	0.1^2	0.2^2	$\left(0.1^2 + 0.2^2\right)/4$
1	0.1^2	0.2^2	$\left(0.1^2 + 0.2^2\right)/4$

Similarly, the variances for the cumulative abnormal returns are

Time	*Firm 1*	*Firm 2*	*Cross-sectional Average*
-1	0.1^2	0.2^2	$\left(0.1^2 + 0.2^2\right)/4$
0	2×0.1^2	2×0.2^2	$2 \times \left(0.1^2 + 0.2^2\right)/4$
1	3×0.1^2	3×0.2^2	$3 \times \left(0.1^2 + 0.2^2\right)/4$

Example 13.6 *(Tests of abnormal returns) By dividing the numbers in Example 13.4 by the square root of the numbers in Example 13.5 (that is, the standard deviations) we get the test statistic for the abnormal returns*

Time	*Firm 1*	*Firm 2*	*Cross-sectional Average*
-1	2	-0.5	0.4
0	10	10	13.4
1	1	1.5	1.8

Similarly, the variances for the cumulative abnormal returns we have

Time	*Firm 1*	*Firm 2*	*Cross-sectional Average*
-1	2	-0.5	0.4
0	8.5	6.7	9.8
1	7.5	6.4	9.0