Chapter 6

CAPM

Reference: Elton, Gruber, Brown, and Goetzmann (2014) 13 and 15
Additional references: Danthine and Donaldson (2005) 7
More advanced material is denoted by a star (*). It is not required reading.

6.1 Beta Representation of Expected Returns

6.1.1 Beta Representation: Definition

The optimal portfolio weights (based on mean-variance preferences) are proportional to the tangency portfolio, $v = cw_T$ where c is a constant. We also know that the *first order conditions* for optimal portfolio choice are $\mathrm{E}\, R_i^e = k \operatorname{Cov}(R_i, R_v)$ where R_v is the optimal portfolio (and thus depends on v). Combining gives

$$\mathrm{E}\, R_i^e = kc \operatorname{Cov}(R_i, R_T), \text{ for } i = 1, \ldots, n, \tag{6.1}$$

where k is the risk aversion and $\operatorname{Cov}(R_i, R_T)$ is the covariance of R_i with the return of the tangency portfolio. The expression says that the portfolio weight on asset i should be increased until the marginal "cost" (driven by the increase in portfolio variance, $c\sigma_{iT}$) equals the marginal benefit (driven by $\mathrm{E}\, R_i^e$). A "beta representation" is a way to rewrite (6.1) in terms of a regression coefficient instead of a covariance.

The beta representation says that, for any asset, the expected excess return ($\mathrm{E}\, R_i^e$) is linearly related to the expected excess return on the tangency portfolio (μ_T^e) according to

$$\mathrm{E}\, R_i^e = \beta_i \mu_T^e \text{ or} \tag{6.2}$$

$$\mathrm{E}\, R_i = R_f + \beta_i \mu_T^e, \text{ where} \tag{6.3}$$

$$\beta_i = \sigma_{iT}/\sigma_{TT}. \tag{6.4}$$

(See below for a proof). It is important to acknowledge that this expression *does not say what causes what*, just how expected returns and betas (the covariance matrix) relate to each other.

The β_i is clearly the slope coefficient in a (time series) OLS regression

$$R_i^e = \alpha_i + \beta_i R_T^e + \varepsilon_i, \tag{6.5}$$

where time subscripts are suppressed. Plotting $\mathrm{E}\, R_i^e$ or $\mathrm{E}\, R_i$ against β_i gives the *security market line*, see Figure 6.1.

Remark 6.1 *(Covariance of portfolios) Let v_q and w_p be the vectors of portfolio weights for two different portfolios (p and q) and Σ the variance-covariance matrix of the investable assets. Then,* $\mathrm{Cov}(R_q, R_p) = v_q' \Sigma w_p$.

Remark 6.2 *(Calculating β_i from the covariance matrix*) The traditional way of estimating β_i is to run a regression. However, if we know the variance-covariance matrix Σ of the investable assets, then we can also use the fact that $\beta_i = \sigma_{iT}/\sigma_{TT}$ where $\sigma_{iT} = w_i' \Sigma w_T$. Using the asset price characteristics in Table (11.2), together with the weights of the tangency portfolio gives the β values in Figure 6.1.*

	μ, %	Σ, bp		
		A	B	C
A	11.5	166	34	58
B	9.5	34	64	4
C	6.0	58	4	100

Table 6.1: Characteristics of the assets in the MV examples. Notice that μ, % is the expected return in % (that is, $\times 100$) and Σ, bp is the covariance matrix in basis points (that is, $\times 100^2$).

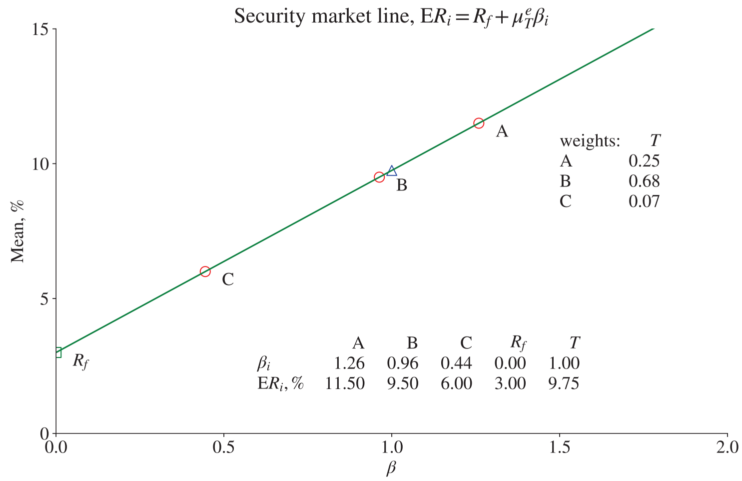


Figure 6.1: Security market line. The properties of the investable assets (A, B, and C) are shown in Table 11.2.

Example 6.3 *(Effect of β) With $R_f = 3\%$ and $\mu_T = 9.75\%$ (so $\mu^e_T = 6.75\%$) we get*

β_i	$\mathrm{E}\,R_i$	*Comment*
0.44	6.0%	*baseline case*
1.5	13.12%	*high β, high return*
1	9.75%	*same risk as market*
0	3%	*no risk*
−0.5	−0.38%	*the opposite of risk*

Proof. (of (6.2)) Recall that the optimal portfolio weights are proportional to the weights in the tangency portfolio: $v = cw_T$ where c is a constant. We can then write the first order condition for optimal portfolio choice as

$$\mu^e = kc\Sigma w_T.$$

Notice that if we premultiply the first order condition with w_T' then we get $w_T'\mu^e = kcw_T'\Sigma w_T$ which is the same as

$$\mu^e_T = kc\sigma_{TT} \text{ or } kc = \mu^e_T/\sigma_{TT}.$$

Notice that Σw_T is a vector of covariances of each asset with the tangency portfolio, which we denote S_T. For instance, with two assets (using w_i to denote the weights in the tangency portfolio)

$$\begin{bmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{12} & \sigma_{22} \end{bmatrix} \begin{bmatrix} w_1 \\ w_2 \end{bmatrix} = \begin{bmatrix} w_1\sigma_{11} + w_2\sigma_{12} \\ w_1\sigma_{12} + w_2\sigma_{22} \end{bmatrix} = \begin{bmatrix} \text{Cov}(R_1, w_1 R_1 + w_2 R_2) \\ \text{Cov}(R_2, w_1 R_1 + w_2 R_2) \end{bmatrix}.$$

The first order conditions can thus be written

$$\mu^e = kcS_T.$$

Combine with the expression for kc to get

$$\mu^e = S_T \mu^e_T / \sigma_{TT}.$$

which is (6.2) in vector form. ■

6.1.2 Betas of Portfolios

Recall that the beta of a portfolio is the portfolios of betas. That is, the portfolio with return

$$R_p = v'R + (1 - \mathbf{1}'v)R_f \text{ has the beta} \tag{6.6}$$

$$\beta_p = v'\beta. \tag{6.7}$$

(This follows directly from $\text{Cov}(\Sigma_{i=1}^n v_i R_i, R_T) = \Sigma_{i=1}^n v_i \text{Cov}(R_i, R_T)$ and that $\beta_p = \sigma_{pT}/\sigma_{TT}$.)

Example 6.4 *Let* $(\beta_1, \beta_2) = (1.2, 0.8)$. *The portfolio return* $R_p = 0.6R_1 + 0.4R_2$ *has the beta* $\beta_p = 0.6 \times 1.2 + 0.4 \times 0.8 = 1.04$.

In particular, consider the portfolios on the capital market line (CML): $R_{opt} = vR_T + (1 - v)R_f$. Using the result in (6.7) and noticing that the tangency portfolio has $\beta_T = 1$ gives that

$$\beta = v \text{ for any portfolio on the CML.} \tag{6.8}$$

This means that it is easy to create a portfolio with any desired β: just invest $v = \beta$ in the tangency portfolio and $1 - v$ in the riskfree.

Example 6.5 *(Creating a portfolio with* $\beta_p = 0.44$*) We can create a portfolio with* $\beta = 0.44$ *by investing 0.44 into the tangency portfolio and 0.56 in the riskfree.*

6.1.3 Beta Representation and the Capital Market Line

CAPM implies that an asset has the same average return as a MV efficient portfolio with the same systematic risk—although it may have a much higher volatility. See Figure 6.2 for an illustration. To formalise this, consider the CAPM regression (6.5) which has the usual property that the residual is uncorrelated with the regressor. We can therefore write the variance as

$$\sigma_{ii} = \beta_i^2 \sigma_{TT} + \sigma_{\varepsilon\varepsilon}. \tag{6.9}$$

This says that the variance of return i has two components: *systematic risk* (the comovement of R_i with R_T) and *idiosyncratic noise* (the movements of ε_i).

Now, consider a portfolio on the capital market line (an optimal portfolio) with the weight β_i on the tangency portfolio, where β_i is from (6.5): $R_{opt} = \beta R_T + (1-\beta) R_f$. Notice that this portfolio has the same systematic risk, $\beta_i^2 \sigma_T^2$, as asset i, but no idiosyncratic risk. CAPM means that the average excess return on asset i should be the same as on this optimal portfolio which is

$$\mu_{opt}^e = \beta_i \mu_T^e \text{ so } \mu_i^e = \beta_i \mu_T^e. \tag{6.10}$$

Example 6.6 *In Figure 6.2, we want to understand the mean return (vertical location) of asset C (taking its volatility and β as given). We notice that C has the same systematic risk as the efficient portfolio D. According to CAPM, C must then have the same average return as D.*

6.1.4 The Tangency Portfolio is the Market Portfolio

To determine the equilibrium asset prices (and therefore expected returns) we have to equate demand (the mean variance portfolios) with supply (exogenous). Since we assume a fixed and exogenous supply (say, 2000 shares of asset 1 and 407 shares of asset 2,...), prices (and therefore returns) are driven by demand.

Suppose all agents have the same beliefs about the asset returns (same expected returns and covariance matrix). They will then all mix the tangency portfolio with the riskfree—but possibly in different proportions due to different risk aversions.

In equilibrium, net supply of the riskfree assets is zero (lending = borrowing), so *the average investor must hold the tangency portfolio and no riskfree assets*. Therefore, *the tangency portfolio must be the market portfolio*, so we can replace R_T with R_m in all

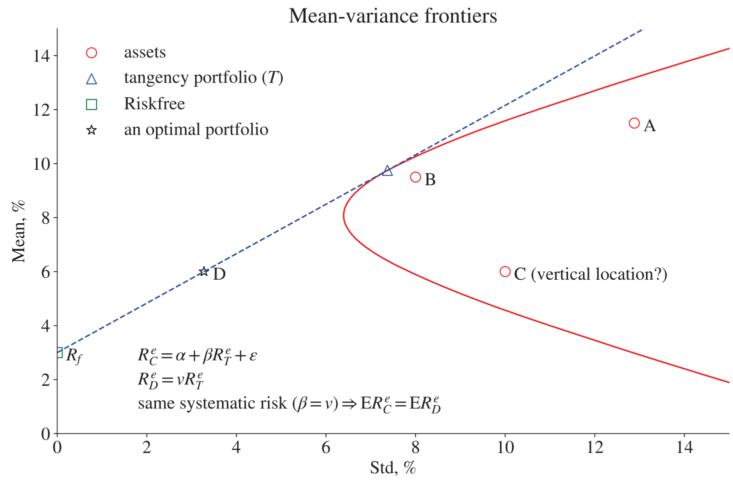


Figure 6.2: Mean-variance frontier and expected returns

expressions above. In particular, CAPM says

$$\mathrm{E}\, R^e_i = \mu^e_T \beta_i \text{ where} \tag{6.11}$$

$$\beta_i = \sigma_{im}/\sigma_{mm}. \tag{6.12}$$

As discussed before, this expression is just a characterisation of the equilibrium (the first order condition, really), but CAPM is silent on how that equilibrium is reached. One possible *story* is that β_i is driven by the firm characteristics (industry, size, leverage, etc.) and that equilibrium is reached as follows: high β assets are in low demand since they are too procyclical (pay off at the wrong time) which means that (in equilibrium) the share price will be low. For a given dividend, this means a higher dividend/price ratio, which contributes to a high average return.

6.1.5 Properties of the Market Portfolio

It is straightforward to show that the market risk premium (expected excess return) is proportional to the market volatility

$$\mathrm{E}\, R^e_m = k_m \sigma_{mm}, \tag{6.13}$$

where we used the subscript m to indicate that this is the market portfolio (which equals the tangency portfolio). This says that the market risk premium increases if the risk aver-

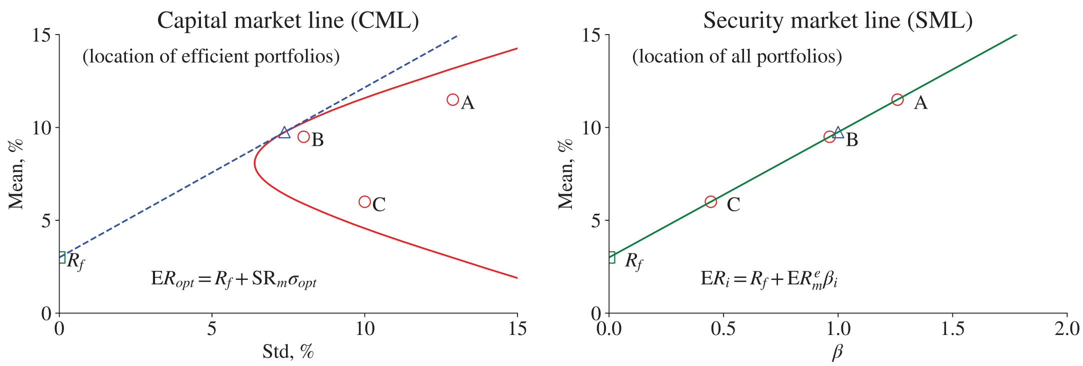


Figure 6.3: CML and SML

sion (k_m) or market variance does.

Proof. (of (6.13)) Recall the first order conditions for optimal portfolio choice for the investor with risk aversion k_T such that he/she holds the tangency portfolio

$$\begin{bmatrix} \mu_1^e \\ \mu_2^e \end{bmatrix} = k_T \begin{bmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{21} & \sigma_{22} \end{bmatrix} \begin{bmatrix} w_1 \\ w_2 \end{bmatrix}.$$

Premultiply both sides by $\begin{bmatrix} w_1 & w_2 \end{bmatrix}$ and set $m = T$ to get (6.13). ■

6.1.6 Summarizing MV and CAPM: CML and SML

According to MV analysis, the average return on all optimal (effective) portfolios (denoted opt) obey

$$\mathrm{E}\, R_{opt} = R_f + SR_m \sigma_{opt}. \tag{6.14}$$

The plot of $\mathrm{E}\, R_{opt}$ against σ_{opt} is called the *capital market line*. See Figure 6.3 for an example.

According to CAPM, the average return on all portfolios (optimal or not), obey the beta representation (6.3)

$$\mathrm{E}\, R_i = R_f + \mu_m^e \beta_i. \tag{6.15}$$

The plot of $\mathrm{E}\, R_i$ against β_i (for different assets, i) is called the *security market line*. See Figure 6.3 for an example.

6.1.7 Using CAPM to Measure Asset Performance

Consider a portfolio q that we want to evaluate—and write its average excess return as

$$\mathrm{E}\, R_q^e = \alpha + \beta_q \mu_m^e. \tag{6.16}$$

We can easily *replicate* the $\beta_q \mu_m^e$ part by constructing a portfolio

$$R_p = \beta_q R_m + (1 - \beta_q) R_f \text{, so} \tag{6.17}$$

$$\mathrm{E}\, R_p^e = \beta_q \mu_m^e. \tag{6.18}$$

Portfolio q will have both systematic and idiosyncratic volatility (see (6.9)), while portfolio p has the same systematic volatility but no idiosyncratic volatility. Hence, portfolio q must be at least as volatile as p.

However, portfolio q may still perform better (bring "value added") than p if $\alpha > 0$. This motivates why α can be used to *measure performance*. However, this assumes that *(i)* CAPM is (mostly) valid; *(b)* but some assets (like q) may deviate from CAPM.

Example 6.7 *(α as a performance measure) Suppose* $\mathrm{E}\, R_q^e = 10\%$*,* $\beta_p = 1.2$ *and* $\mu_m^e = 9\%$*. Construct a portfolio p so that* $R_p = 1.2 R_m - 0.2 R_f$ *(buy 1.2 of an ETF on the market index and borrow 0.2). Notice that* $\mathrm{E}\, R_p^e = 1.2 \times 9\% = 10.8\%$*. This means that asset q has* $\alpha = -0.8\%$*: the replicating portfolio has a higher return (and it can be shown that it will also have a lower volatility).*

6.1.8 Back to Prices (The Gordon Model)*

The gross return, $1 + R_{t+1}$, is defined as

$$1 + R_{t+1} = \frac{D_{t+1} + P_{t+1}}{P_t}, \tag{6.19}$$

where P_t is the asset price and D_{t+1} is the dividend it gives at the beginning of the next period. If we assume that expected returns are constant across time (denoted R, for instance 10%) and that dividends are expected to grow at the rate g (for instance, 2%), then it is straightforward to show that the asset price is

$$P_t = \mathrm{E}_t\, D_{t+1} \textstyle\sum_{s=1}^{\infty} \dfrac{(1+g)^{s-1}}{(1+R)^s} = \dfrac{\mathrm{E}_t\, D_{t+1}}{R - g}. \tag{6.20}$$

Clearly, higher (expected) dividends and/or a higher growth rate increases the asset price. In addition, a lower expected ("required") *future return* also increases *today's asset price*.

In CAPM, a lower expected return could be driven by a lower beta or by a lower riskfree rate. One way of interpreting this is as follows. If an asset (suddenly) gets a lower beta, that means that it has less systematic risk than before. It is therefore more useful in portfolio formation (more diversification benefits) and becomes more demanded—so the price level increases. With a higher price level, the dividend yield is lower, which contributes to a lower return (recall the return is the dividend yield plus the capital gains yield).

6.2 Testing CAPM

Reference: Elton, Gruber, Brown, and Goetzmann (2014) 15

Let $R^e_{it} = R_{it} - R_{ft}$ be the excess return on asset i in excess over the riskfree asset in period t, and let R^e_{mt} be the excess return on the market portfolio in the same period. (The time subscripts are written out to highlight that we use time series data to estimate and test the regression coefficients.) The basic implication of CAPM is that the expected excess return of an asset ($\mathrm{E}\, R^e_{it}$) is linearly related to the expected excess return on the market portfolio ($\mathrm{E}\, R^e_{mt}$) according to

$$\mathrm{E}\, R^e_{it} = \beta_i \,\mathrm{E}\, R^e_{mt}, \text{ where } \beta_i = \sigma_{im}/\sigma_{mm}. \tag{6.21}$$

Consider the regression

$$\begin{aligned} R^e_{it} &= \alpha_i + \beta_i R^e_{mt} + \varepsilon_{it}, \text{ where} \qquad (6.22)\\ \mathrm{E}\,\varepsilon_{it} &= 0 \text{ and } \mathrm{Cov}(R^e_{mt}, \varepsilon_{it}) = 0. \end{aligned}$$

The two last conditions are automatically imposed by LS. Take expectations of the regression (assuming we know the coefficients) to get

$$\mathrm{E}\, R^e_{it} = \alpha_i + \beta_i \,\mathrm{E}\, R^e_{mt}. \tag{6.23}$$

Notice that the LS estimate of β_i in (6.22) is the sample analogue to β_i in (6.21), since LS estimates a slope coefficient as the covariance of the dependent variable and the regressor, divided by the variance of the regressor. It is then clear that CAPM implies that the intercept (α_i) of the regression should be zero, which is also what empirical tests of CAPM focus on.

This test of CAPM can be given two interpretations. If we assume that R_{mt} is the correct benchmark (the tangency portfolio for which (6.21) is true by definition), then it is a test of whether asset R_{it} is correctly priced. This is typically the perspective in performance analysis of mutual funds. Alternatively, if we assume that R_{it} is correctly priced, then it is a test of the mean-variance efficiency of R_{mt}. That is, we test if the market portfolio is the correct "pricing factor" of all the test assets. This is the perspective of CAPM tests.

The test of the null hypothesis that $\alpha_i = 0$ uses the fact that, under fairly mild conditions, the t-statistic has an asymptotically normal distribution, that is

$$\frac{\hat{\alpha}_i}{\text{Std}(\hat{\alpha}_i)} \stackrel{d}{\to} N(0,1) \text{ under } H_0 : \alpha_i = 0. \tag{6.24}$$

In this expression, $\hat{\alpha}_i$ is the estimate of the intercept in (6.22) and $\text{Std}(\hat{\alpha}_i)$ its standard deviation (for instance, from the usual OLS results). Note that this is the distribution under the null hypothesis that the true value of the intercept is zero, that is, that CAPM is correct. We typically reject the null hypothesis ($\alpha_i = 0$) when the the t-statistic is very negative or very positive (for instance, lower than -1.96 or higher than 1.96).

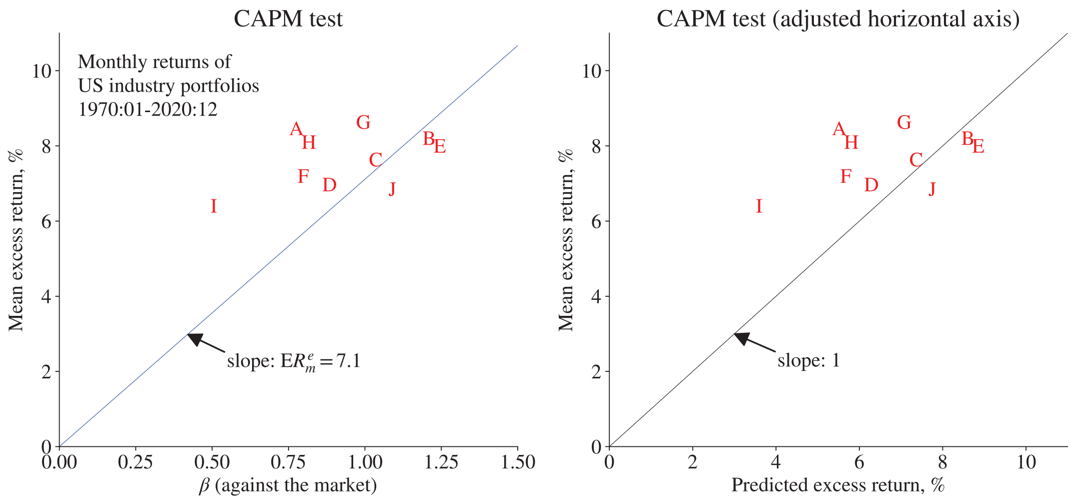
The test assets are typically portfolios of firms with similar characteristics, for instance, small size or having their main operations in the retail industry. There are two main reasons for testing the model on such portfolios: individual stocks are extremely volatile and firms can change substantially over time (so the beta changes). Moreover, it is of interest to see how the deviations from CAPM are related to firm characteristics (size, industry, etc), since that can possibly suggest how the model needs to be changed.

The empirical results from such tests vary with the test assets used. For US portfolios, CAPM seems to work reasonably well for some types of portfolios (for instance, portfolios based on firm size or industry), but much worse for other types of portfolios (for instance, portfolios based on firm dividend yield or book value/market value ratio). Figure 6.4 shows some results for US industry portfolios.

6.2.1 Several Assets

In most cases there are several (n) test assets, and we actually want to test if all the α_i (for $i = 1, 2, ..., n$) are zero. Ideally we then want to take into account the correlation of the different alphas.

While it is straightforward to construct such a test, it is also a bit messy. As a quick way out, the following will work fairly well. First, test each asset individually. Second,



	α	t-stat	σ
A (NoDur)	2.90	2.40	8.54
B (Durbl)	-0.46	-0.22	14.46
C (Manuf)	0.22	0.25	6.19
D (Enrgy)	0.63	0.28	15.88
E (HiTec)	-0.91	-0.57	11.36
F (Telcm)	1.47	0.98	10.61
G (Shops)	1.50	1.17	9.12
H (Hlth)	2.24	1.45	10.97
I (Utils)	2.76	1.68	11.63
J (Other)	-0.96	-1.01	6.75

CAPM: $R_i^e = \alpha_i + \beta_i R_m^e + e_i$
Predicted excess return: $\beta_i \mathrm{E}R_m^e$

α and σ (std of residual) are in annualized %

p-val for testing if all $\alpha_i = 0$: 0.379
10% crit. value (Bonferroni): 2.58

Figure 6.4: CAPM regressions on US industry indices

form a few different portfolios of the test assets (equally weighted, value weighted) and test these portfolios. Although this does not deliver one single test statistic, it provides plenty of information to base a judgment on. For a more formal approach, a SURE approach is useful. Alternatively, we can apply a *Bonferroni adjustment* of the individual t-stats: reject CAPM at the 5% significance level only if the largest t-stat (in absolute terms) exceeds the critical value at the $0.05/n$ significance level. For instance, with $n = 25$, the critical value from a standard normal distribution would be 3.09 instead of 1.96.

A quite different approach to study a cross-section of assets is to first perform a CAPM regression (6.22) and then the following cross-sectional regression

$$\bar{R}_i^e = \gamma + \lambda \hat{\beta}_i + u_i, \tag{6.25}$$

where $\bar{R}_i^e$ is the (sample) average excess return on asset i. Notice that the estimated betas

	1	2	3	4	5
1	−3.10	0.45	0.68	2.35	2.32
2	−2.00	0.97	1.83	2.27	1.71
3	−1.52	1.63	1.56	2.45	2.38
4	−0.23	0.55	1.27	2.02	1.39
5	0.09	1.08	1.45	−0.17	0.64

Table 6.2: t-stats for α in CAPM, 25 FF portfolios 1970:01-2020:12. NW uses 1 lag. The Bonferroni adjusted 10% and 5% critical values are 2.88 and 3.09.

are used as regressors and that there are as many data points as there are assets (n).

There are severe econometric problems with this regression equation since the regressor contains measurement errors (it is only an uncertain estimate), which typically tend to bias the slope coefficient towards zero. To get the intuition for this bias, consider an extremely noisy measurement of the regressor: it would be virtually uncorrelated with the dependent variable (noise isn't correlated with anything), so the estimated slope coefficient would be close to zero.

If we could overcome this bias (and we can by being careful), then the testable implications of CAPM is that $\gamma = 0$ and that λ equals the average market excess return. We also want (6.25) to have a high R^2—since it should be unity in a very large sample (if CAPM holds).

6.2.2 Representative Results of the CAPM Test

One of the more interesting studies is Fama and French (1993) (see also Fama and French (1996)). They construct 25 stock portfolios according to two characteristics of the firm: the size (by market capitalization) and the book-value-to-market-value ratio (BE/ME). In June each year, they sort the stocks according to size and BE/ME. They then form a 5×5 matrix of portfolios, where portfolio ij belongs to the ith size quintile *and* the jth BE/ME quintile:

$$\begin{bmatrix} \text{small size, low B/M} & \dots & \dots & \dots & \text{small size, high B/M} \\ \vdots & \ddots & & & \\ \vdots & & \ddots & & \\ \vdots & & & \ddots & \\ \text{large size, low B/M} & & & & \text{large size, high B/M} \end{bmatrix} \tag{6.26}$$

They run a traditional CAPM regression on each of the 25 portfolios (monthly data

1963–1991)—and then study if the expected excess returns are related to the betas as they should according to CAPM (recall that CAPM implies $\mathrm{E}\, R^e_{it} = \beta_i \lambda$ where λ is the risk premium (excess return) on the market portfolio).

However, it is found that there is almost no relation between $\mathrm{E}\, R^e_{it}$ and β_i (there is a cloud in the $\beta_i \times \mathrm{E}\, R^e_{it}$ space). This is due to the combination of two features of the data. First, *within a BE/ME quintile*, there is a positive relation (across size quantiles) between $\mathrm{E}\, R^e_{it}$ and β_i—as predicted by CAPM. Second, *within a size quintile* there is a negative relation (across BE/ME quantiles) between $\mathrm{E}\, R^e_{it}$ and β_i—in stark contrast to CAPM. Figure 6.4 shows some results for US industry portfolios and Figures 6.5–6.6 for US size/book-to-market portfolios.

In Figure 6.4, the results are presented in two different ways:

$$\begin{array}{lll} & \underline{\text{horizontal axis}} & \underline{\text{vertical axis}} \\ 1: & \beta_i & \sum_{t=1}^{T} R^e_i / T \\ 2: & \beta_i \sum_{t=1}^{T} R^e_m / T & \sum_{t=1}^{T} R^e_i / T \end{array} \tag{6.27}$$

In the first approach, CAPM 6.21 says that all data points (different assets, i) should cluster around a straight line with a slope equal to the average market excess return, $\sum_{t=1}^{T} R^e_m / T$. In the second approach, CAPM says that all data points should cluster around a 45-degree line. In either case, the vertical distance to the line is α_i (which should be zero according to CAPM).

6.2.3 Representative Results on Mutual Fund Performance

Mutual fund evaluations (estimated α_i) typically find *(i)* on average neutral performance (or less: trading costs&fees); *(ii)* large funds might be worse; *(iii)* perhaps better performance on less liquid (less efficient?) markets; and *(iv)* there is very little persistence in performance: α_i for one sample does not predict α_i for subsequent samples (except for bad funds).

Example 6.8 *(Steadman's funds*) "How can a fund be this bad?" (NYT, 1991) (the four Steadman funds rank among the six worst performers of the 244 stock funds tracked by Lipper Analytical Services for the 15 years that ended on Oct. 31. The Oceanographic Fund comes in at No. 243 and Steadman American Industry Fund, No. 244); "Steadman's creature just won't die" (Forbes, 1999); "Those awful Steadman's funds returning under a new name" (Baltimore Sun, 2002).*

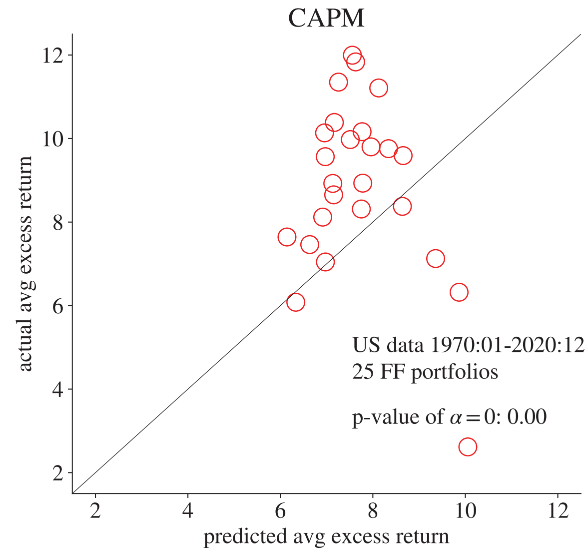


Figure 6.5: CAPM, FF portfolios

6.3 Appendix: Statistical Tables*

Tables 6.3 shows critical values for t-distributions (and the standard normal), while 6.4 shows critical values for chi-square distributions.

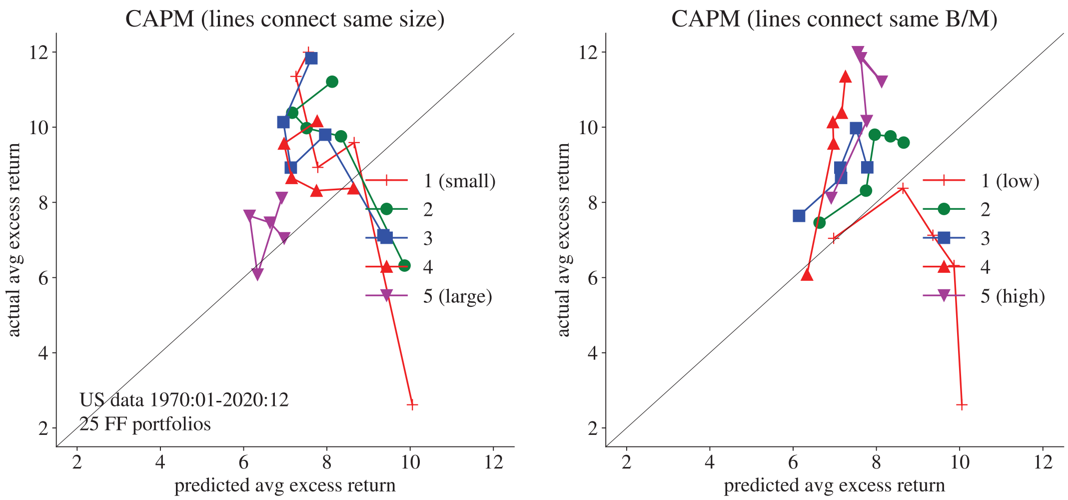


Figure 6.6: CAPM, FF portfolios

n	Significance level		
	10%	5%	1%
10	1.81	2.23	3.17
20	1.72	2.09	2.85
30	1.70	2.04	2.75
40	1.68	2.02	2.70
50	1.68	2.01	2.68
60	1.67	2.00	2.66
70	1.67	1.99	2.65
80	1.66	1.99	2.64
90	1.66	1.99	2.63
100	1.66	1.98	2.63
Normal	1.64	1.96	2.58

Table 6.3: Critical values (two-sided test) of t distribution (different degrees of freedom) and normal distribution.

n	Significance level		
	10%	5%	1%
1	2.71	3.84	6.63
2	4.61	5.99	9.21
3	6.25	7.81	11.34
4	7.78	9.49	13.28
5	9.24	11.07	15.09
6	10.64	12.59	16.81
7	12.02	14.07	18.48
8	13.36	15.51	20.09
9	14.68	16.92	21.67
10	15.99	18.31	23.21

Table 6.4: Critical values of chisquare distribution (different degrees of freedom, n).