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Combining Finite Element Methods and Neural Networks to Solve Elliptic Problems on 2D Geometries

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Introduction

Scientific context

TODO

Parametric linear Elliptic PDE

TODO

Physics-Informed Neural Networks

TODO

Finite Element Method

TODO

Introduction   How improve PINN prediction with FEM ?   Numerical results - 2D Poisson problem   Numerical results - 2D anysotropic Elliptic problem   Conclusion   References

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How improve PINN prediction with FEM ?

Additive approach

TODO

Theorerical results

TODO

Numerical results - 2D Poisson problem

2D Poisson problem

TODO

Numerical results - 2D anysotropic Elliptic problem

2D anysotropic Elliptic problem

TODO

Conclusion

Conclusion

TODO

References

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Appendix

Appendix 1 : Standard FEM

Appendix 1 : General Idea

TODO