Combining Finite Element methods and Neural Networks to solve elliptic problem on complex 2D geometries

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MIMESIS

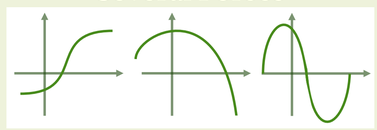
Scientific context - Create real-time digital twins of an organ (e.g. liver)

Current Objective : Develop hybrid **finite element** / **neural network** methods.
accurate quick + parameterized

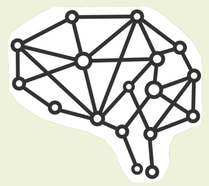
OFFLINE

Several Geometries

Several Forces

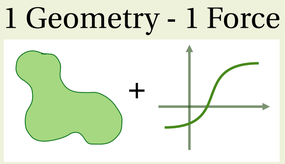


Train a PINNs CITE



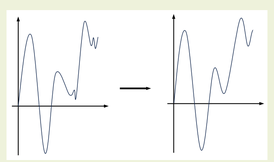
ONLINE

1 Geometry - 1 Force



Get PINNs prediction

Correct prediction with FEM



Poisson problem with Dirichlet boundary conditions

Find $u : \Omega \to \mathbb{R}^d (d = 1,2,3)$ such that

$$\begin{cases} -\Delta u(x) = f(x) & \text{in } \Omega, \\ u(x) = g(x) & \text{on } \Gamma \end{cases} \qquad (\mathscr{P})$$

with Δ the Laplace operator, Ω a smooth bounded open set and Γ its boundary.

PINNs - Physics-Informed Neural Networks

TODO ?

Finite Element methods

TODO ?

How to deal with complex geometry in PINNs ?

No mesh, so easy to go on complex geometry !

⚠ **In practice :** Not so easy ! We need to find **how to sample in the geometry**.

Solution : Approach by levelset. CITE

How can we improve PINNs prediction - Using FEM-type methods

TODO