# How improve PINNs ?


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# Problem considered

**Poisson problem with homogeneous Dirichlet conditions :**
Find $u : \Omega \to \mathbb{R}^d (d = 1, 2, 3)$ such that

$$\begin{cases} -\Delta u(X) = f \quad \text{in } \Omega, \\ u(X) = g \quad \text{on } \partial\Omega \end{cases}$$

with $\Delta$ the Laplace operator, $\Omega$ a smooth bounded open set and $\Gamma$ its boundary.
**Standard PINNs :** We are looking for $\theta$ such that

$$\theta_u = \underset{\theta}{\operatorname{argmin}}\, w_r\, J_r(\theta) + w_{bc}\, J_{bc}(\theta)$$

where $w_r$ and $w_{bc}$ are the respective weights associated with

$$J_r = \int_\Omega (\Delta u + 1)^2 \;\text{ and }\; J_{bc} = \int_{\partial\Omega} u^2.$$

*Remark :* In practice, we use a Monte-Carlo method to discretize the cost function by random process.



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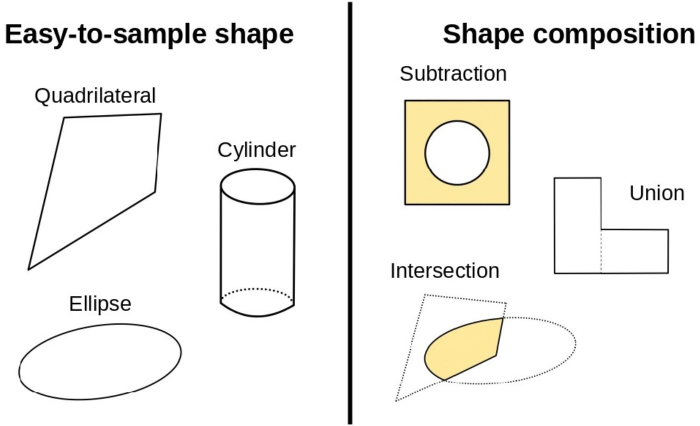
# Simple geometry

**Claim on PINNs :** No mesh, so easy to go on complex geometry !



# Complex geometry

**In practice :** Not so easy ! We need to find how to sample in the geometry.

## 1st approach : Mapping

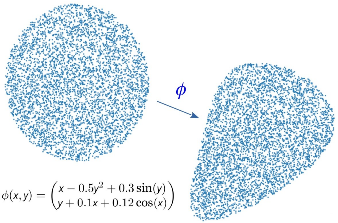
**Idea :**

➙ $\Omega_0$ a simple domain (as circle)

➙ $\Omega$ a target domain

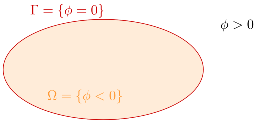
➙ A mapping from $\Omega_0$ to $\Omega$

$$\Omega = \phi(\Omega_0)$$




$$\phi(x,y) = \begin{pmatrix} x - 0.5y^2 + 0.3\sin(y) \\ y + 0.1x + 0.12\cos(x) \end{pmatrix}$$

## 2nd approach : LevelSet function




**Advantages :**

➙ Sample is easy in this case.

➙ Allow to impose in hard the BC :

$$u_\theta(X) = \phi(X) w_\theta(X) + g(X)$$

**Natural LevelSet :**

Signed Distance Function (SDF)

# LevelSet Approach

**Problem :**

SDF is a $\mathcal{C}^0$ function $\Rightarrow$ its derivatives explodes $\Rightarrow$ We need a regular levelset

**How construct smooth SDF ?**

- **1st solution :** Approximation theory ADD REFERENCE !!
  $\Delta u$ can be singular at the boundary. Sampling at $\epsilon$ to it solve the problem.
- **2nd solution :** Learn the levelset. How make that ? with a PINNs



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# Approximation theory

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# Learn LevelSet I

TO COMPLETE !

# Learn LevelSet I

TO COMPLETE !

# Learn LevelSet II

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# Learn LevelSet II

TO COMPLETE !



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# Conclusion

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Thank you !

# Bibliography

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# Appendix 1 : Polygonal domain

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# Appendix 2 : Curved domain

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