HW# 5

Aaron Segura
Frederick Lee

November 16, 2018

1 Task 1: The Effects of Flock Size and Ordinary Differential Equation (ODE) System Parameters on Angry Bird Flock Diameter

1.1 What

In this assignment, we will be numerically modeling a system of Ordinary Differential Equations (ODEs) which simulates the flocking behavior of birds in two-dimensional space. This simulation of ”Angry Birds” may appear to be simple and whimsical but this problem has actual applications in regards to modeling the dynamics of diseases, magnets, populations, the N-body problem, etc. Here, we will describe and simulate the dynamics involved in bird flocking such the natural attraction to food, the leader of the flock, and the repulsively of other birds in the flock to avoid bird-on-bird collisions.

First, we include the set of ODEs which describes the flocking behavior of our flock size of N birds letting

$$B_k(t) = (x_k(t), y_k(t)) \tag{1}$$

be the location of the kth bird where $k = 1, 2, .., N$ and $B_1(t)$ is the location of our bird leader in the flock. In other words, the bird leader $B_1(t)$ is the first bird in the set. Additionally, this first bird is unique in that it is not influenced by the position of the rest of the birds in the set and all other birds are compelled to follow this leader bird by definition.

Interestingly, the force of attraction to the leader bird by subsequent birds in the set can be explained by the notion of evolutionary selection pressure. This evolutionary advantage the following birds, B_k for $k = 2, 3, ..., N$, receive when they follow the leader bird $B_1(t)$ can be thought of as a positive survival factor. This is where a bird who follows will be more likely to survive and reproduce if they follow the leader bird than if they did not follow. Stated more clearly, the positive survival factor received when following is the increased probability of being exposed to resources such as the food the leader bird is following, a potentially increased opportunity to mate with neighboring birds within the flock, and any protection from predators that the leader bird or the flock dynamics may provide. Therefore, we can say that it is advantageous for a bird in the set to follow our leader bird since it will increase the chances that the following bird's genes will remain in the gene pool, i.e. it confers a survival benefit by following our leader bird. This is the reasoning as to why we constructed our ODE system such that all the following birds will only be attracted to our leader bird rather than the food itself, objects or entities outside the scope of simulation, or to one another.

In this simulation, we will focus on a special case where the leader bird will behave in such a way that it is always "hungry" or continually is attracted to a food source. He/she will not tire, speed up, or stop for any reason during the duration of our simulation of time t. As stated previously, this also means that the birds that follow him/her do so because they will have an increased chance of encountering the desired food. Here, we will define the location of the food source to be at $C(t)$, where $C(t)$ is a function of $x_c(t)$ and $y_c(t)$ i.e. $C(t)=(x_c(t),y_c(t))$. So, in our simulation, being always "hungry", at any instance of time t translates to $B_1(t)$ constantly approaching $C(t)$ thereby reducing its distance to $C(t)$ as described by the following:

$$F_1^{food}(t) = \gamma_1(C(t) - B_1(t)) \tag{2}$$

Here, the parameter γ_1 is a positive constant whose magnitude depends on how desirable the food is to our leader bird. In other words, this term encourages the leader to fly towards the food and so increasing the magnitude of γ_1 will simulate a leader bird that is proportionally more attracted to the food location $C(t)$ than he/she would be at a lower γ_1 and will behave according. Also, note that this force is added to the right hand side of the ODE system and only applies to our leader bird.

Additionally, we can talk about the positions of these two bodies, the food $C(t)$ and the leader bird $B_1(t)$, relative to one another in the coordinate system of our simulation. We do this by first denoting x_c to be the x-component of the location of the food at time t and x_1 to be the x-component of the location of the leader bird at the same time t. In other words, x_c is the $x_c(t)$ component of $C(t) = (x_c(t), y_c(t))$ and x_1 is the $x_1(t)$ of $B_1(t) = (x_1(t), y_1(t))$. One way of thinking about these two positions relative to one another would be to take the case where $x_c > x_1$. In this case, the leader bird is to the left of the food at some time t. Also note that this case only considers the two body's location relative to the x-axis and so it may be the case that the leader bird is to the left of the food but it is not specified as whether it is above or below or at the same level relative to the food at time t. In other words, the condition $x_c > x_1$ can be be true regardless of the y-components of the food and the leader bird.

Returning back to our description of the ODE system, by definition the subsequent birds in the set, i.e. after the leader bird, will follow our leader bird due to some sort of intrinsic trait of the leader bird. Hence, we need an equation that describes the movement of the birds following our leader bird. In this case, we will use the following equation to represent how the follow birds' behavior will be governed over time t:

$$F_k^{follow}(t) = \gamma_2(B_1(t) - B_k(t)), k = 2, 3, ..., N \tag{3}$$

where γ_2 is a positive value and simulates the attractive force factor that encourages the birds to move towards the leader be it evolutionary selection pressure, attractiveness for mating purposes, protection effects against predators, charisma etc. In this regard, a γ_2 of greater magnitude will proportionally increase the attractive force the leader bird has on the birds that follow it causing the following birds to aggregate more closely to their leader bird than if we had a γ_2 of smaller magnitude.

To further describe the properties of our flock, we will denote the middle of the flock as it's center of mass by the following equation:

$$\overrightarrow{B}(t) = \sum_{k=1}^{N} B_k(t)/N \tag{4}$$

where $\overrightarrow{B}$ represents the center of the "flock" of birds on our grid. Here, we include a flocking force where being closer to the center of mass will afford safety to the birds. This "safety" can be thought of as the protective effect the flock provides in the case that a predator attacks such that the likelihood of a bird near the center of the flock being attacked decreases. Therefore, it is advantageous for a bird to approach the center of the flock since the flock confers safety to the bird. We can represent a flocking force which encourages the follow birds to aggregate towards the center by the following equation:

$$F_k^{fl}(t) = \kappa(\overrightarrow{B}_k(t) - B_k(t)), k = 2, 3, ..., N \tag{5}$$

Note that this flocking force does not influence our leader bird, however, the leader bird does affect the flocking force applied to the following birds. Additionally, this flocking force will be added to the right hand side of our ODE system.

Another factor that must be taken into account is the notion of physicality i.e. these birds take up space and cannot occupy the same space at any given time in the real world. However, instead of giving the birds size such as an area in the 2-dimensional sense or an individual volume in the 3-dimensional, we will include in our simulation the notion that any bird in the system can should not collide. This force will help prevent the birds from occupying the same space with any other bird in the system but unfortunately in our simulation we do not explicitly control for this. In other words, we may observe in our simulation the case where a bird may be able to occupy the same space as another bird in the set. Additionally, this repulsive force is important since it would not be an interesting system for modeling purposes if we only had attractive forces. If we were to model only attractive forces, all follow birds would then converge to the location of the leader bird and we would observe the superposition of the bird set on top of the leader bird.

So here we will include in our simulation the prevention of collisions among birds by including a specific repelling force which will effectively increase the distance among birds in the following way. Say the k-th bird B_k is in a group with its 5 nearest neighbors. If an additional bird gets close to the group of 6 birds then each of the 6 birds will be pushed away from the center of the group of 6 to allow room for the new bird. This effect of repulsion will the reduce the amount of aggregation among groups of birds. Note that this same principle can be applied to a bird that is undesirable to the other birds or "smelly" as discussed further in the extra credit portion of this assignment.

This relationship will affect the behavior of all birds B_k from $k = 2, 3...,$ to K excluding the leader bird. To implement this, as stated previously assume that the kth bird B_k is repelled by its 5 closest neighbors and let the five closest neighbors of the kth bird be the set $\{l_1^k, l_2^k, l_3^k, l_4^k, l_5^k\}$. Then, we will add to the right hand side of the ODE,

$$F_k^{rep}(t) = \sum_{i=1}^{5} \frac{\rho(B_k(t) - B_{l_i^k}(t))}{(B_k(t) - B_{l_i^k}(t))^2 + \delta}, k = 2, 3, ..., N \tag{6}$$

where ρ is a constant that determines the magnitude of the repelling force. In other words, if we increase the magnitude of ρ there we will see a greater repulsive force acting on the 5 nearest neighbors relative to the kth bird. Interestingly, this force of repulsion for each bird will be different with respect to time. There may also be additional forces at play, such as predator avoidance, obstacle avoidance forces, grouping forces, etc. that can also govern the flocking behavior and movement of individual birds but we will not include them here in our simulation.

In this assignment we will also experiment with a few variants of the mathematical model. We will compute the derived system of ODEs, with some randomly chosen initial locations for the birds $B_k(0) = (x_k(0), y_k(0))$,for $k = 1, 2, ...N$ using the classic fourth order Runge-Kutta method (RK4). The general ODE system is

$$\overrightarrow{y}(t) = \overrightarrow{f}(t, \overrightarrow{y}(t)), \tag{7}$$

$$\overrightarrow{y}(t_0) = \overrightarrow{y_0}, \tag{8}$$

where $\overrightarrow{f}(t, \overrightarrow{y}(t))$ is the right hand side of the ODE. The previously mentioned terms form the right hand side of the equation as discussed above. Here the term $\overrightarrow{y}$ is a vector of individual values of $B_k(t)$ which can be represented in the following vector:

$$\overrightarrow{y} = \begin{bmatrix} B_1(t) \\ B_2(t) \\ \vdots \\ B_N(t) \end{bmatrix} \tag{9}$$

In this task, the focus of our attention will be on the effects of changing the ODE system parameters α, γ_1, γ_2, κ, and δ has on flock diameter D. Here, the flock diameter is defined by the following:

$$D = 2r$$

$$r = max(B_N(t) - \overrightarrow{B}(t)) \tag{10}$$

Note that we use *2 times* the maximum value of the difference of $B_N(t)$ from $\overrightarrow{B}(t)$ to represent our flock diameter. This is because the maximum value in the set of differences from the center of the flock $\overrightarrow{B}(t)$ and the location of each of the birds at $B_N(t)$ is the greatest difference between these two locations which represents the distance of the bird farthest away from the center of the flock. In other words, the distance of the bird farthest from the center of the flock will define the flock radius r and two times our flock radius r will be our flock diameter D.

Before we begin our comparison of the effects that different ODE system parameters has on flock diameter, we provide a table of definitions for the ease of interpretation for each of our parameters α, γ_1, γ_2, κ, and δ (as well as σ and Π):

Parameter	Description/Interpretation
α	Movement Speed of Food ”Fast food”
γ_1	Attractive Force of Leader Bird to Food ”How Hungry”
γ_2	Attractive Force of Follow Birds to Leader Bird ”Charisma”
κ	Attractive Force Among Follow Birds to Flock Center ”Groupies”
ρ	Repulsive Force from 5 Nearest Neighbors ”Personal space/Angry Birds”
δ	- Contributes to ρ ”More Angry Birds”-
σ*	Repulsive Force of Smelly Bird ”Strength of Smelly Bird BO”
Π*	Repulsive Force of Predator ”How Intimidating our Predator is”

*Parameter used in the Extra Credit section of this assignment

Table 1: List Description with Interpretations of ODE system parameters α, γ_1, γ_2, κ, and δ (as well as σ and Π)

This table captures the parameters used in our ODE system simulation as well as the parameters used in our extra credit section with an interpretation/comment on what each parameter might mean in the real sense.

$Center\ of\ Flock$

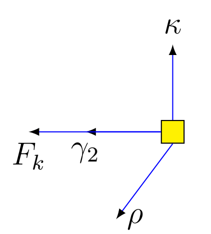


Figure 1: Free body diagram of the forces on bird $B_N(t)$ for $N = 2, 3, ...N$

Above, we have also provided a free body diagram describing the forces acting on one of our follow birds, B_k for $k = 2, 3, ..., N$. Consistent with our parameters above, we have represented the attractive force towards the Center of the Flock by κ, the attractive force of the follow birds to the leader bird γ_2, and the repulsive force from the 5 nearest neighbors as ρ. Here, F_k is the sum of the multiple forces acting on the follow bird.

As you can see, at this time step our follow bird is inclined to follow the leader bird despite the attractive force towards the Center of the flock and the repulsive force from the 5 nearest neighbors. This is because the sum of the forces on the follow bird F_k points in the general direction of our leader bird. Thus, in our simulation the follow birds will follow the leader bird as expected given the fixed set of ODE parameters. Note that as the positions of each of our birds and ODE parameters change we will observe variations in the follow birds' behavior and this will affect our overall flock diameter.

In the section below, we describe the effects that changing these ODE parameters has on the flock diameter. In other words, we will experiment with different values of α, γ_1, γ_2, κ, and δ and note the changes in flock diameter D.

1.2 How

To accomplish this task, we used the instructor provided homework template code and implemented the required functionality for testing the first six default flock parameters as defined in Table 1. For this study, we aim to gain an understanding of each tunable parameter and how the value of each effects the behavior of the flock diameter over time.

The following figures and tables offer an illustration of how changing each tunable parameter in our ODE system simulation affects the general behavior of the system through time as represented by the flock diameter D. For each figure-table pair, we explore the effects that differing parameter values might have on this flock of birds. In particular, we present data that tracks the diameter of our flocks through time, subject to our $flock\_diameter$ function which is defined as computing the diameter of a circle whose origin is located at the center of the flock and whose radius is the greatest distance between the flock center and any bird within the flock.

In the next part of this section, we present a total of seven parameters that were explored by taking 5 different parameter values. When then compute the average flock diameter over the time period of our data. In other words, we will be plotting the flock diameter D over time t and then determine the average time $\overline{D}$.

For each parameter, except for the *food_flag* and bird count parameter, N, we experiment with 5 different parameters, one of which being the default parameter value. Note that the boldface font for a parameter in each given table of parameters and flock distance indicates the **default** parameters provided by the instructor.

For simplicity, we do not include the time stepping bounds or step resolution while testing the parameters in our study. Additionally, for all runs conducted in this test:

Initial time: $t_0 = 0.0$ seconds
Final time: $T = 10.0$ seconds
Step size: $\Delta t = 0.2$ seconds

Lastly, the *NumPy* Python library random number generator, given an arbitrary seed value of 2018, was used to initialize the system for each run.

Table of Default Parameters

Parameter	N	α	γ_1	γ_2	κ	ρ	δ	*Food Flag*	σ*	Π*
Default Value	30	0.4	2.0	8.0	2.0	4.0	0.5	1	8.0	4.0

* extra credit parameter

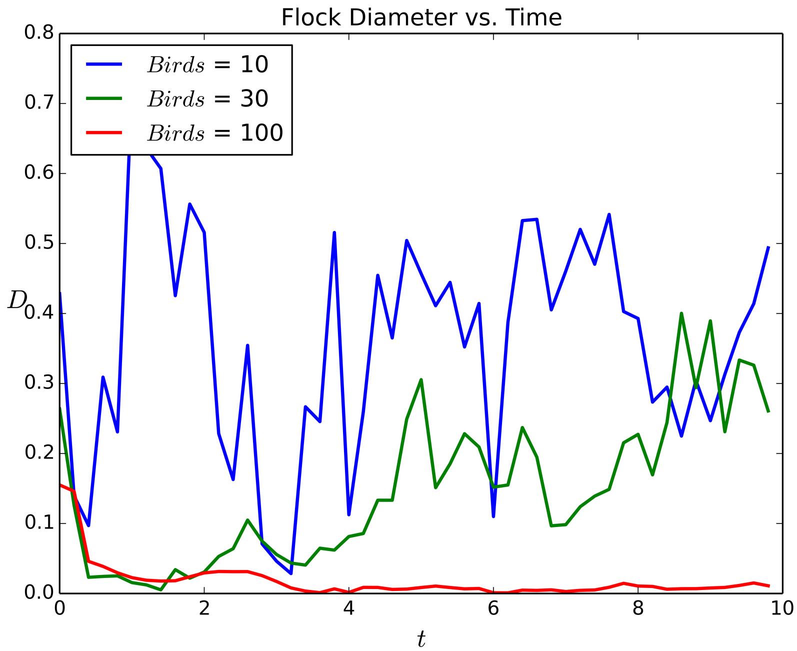


Figure 2: Flock Diameter (D) Vs Time (t) for the Number of Birds (N)

N	$\overline{D}$
10	3.61208e-01
30	**1.46906e-01**
100	1.84314e-02

Table 2: Average Flock Diameter ($\overline{D}$) Over Time (t) for the Number of Birds (N)

As you can see here, the greatest diameter was achieved on average by the flock size of $N = 10$ and the smallest diameter was achieved by a flock size of $N = 100$. This suggests that flock size is negatively related to the number of birds in the flock, although the we do not statistically test for correlation between these two parameters in our study.

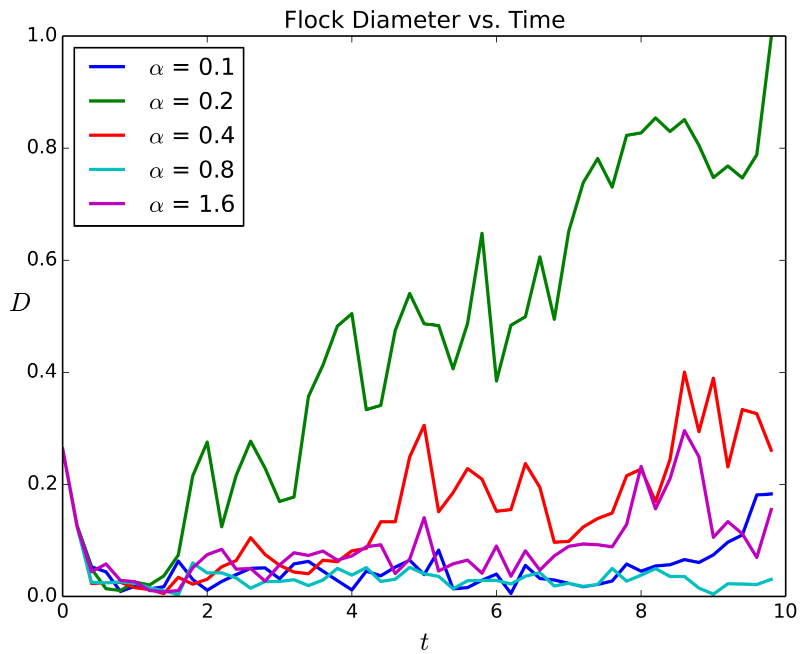


Figure 3: Flock Diameter (D) Vs Time (t) For the Movement Speed of Food (α)

α	$\overline{D}$
0.1	5.33763e-02
0.2	4.53381e-01
0.4	**1.46906e-01**
0.8	3.59452e-02
1.6	9.02547e-02

Table 3: Average Flock Diameter ($\overline{D}$) Over Time (t) For the Movement Speed of Food (α)

Here, we can see that the flock diameter is not clearly related to speed of the food which is represented by α. This can be stated since the diameter $\overline{D}$ inconsistently changes with α values. In other words, we do not see a consistent pattern or relationship between these two parameters.

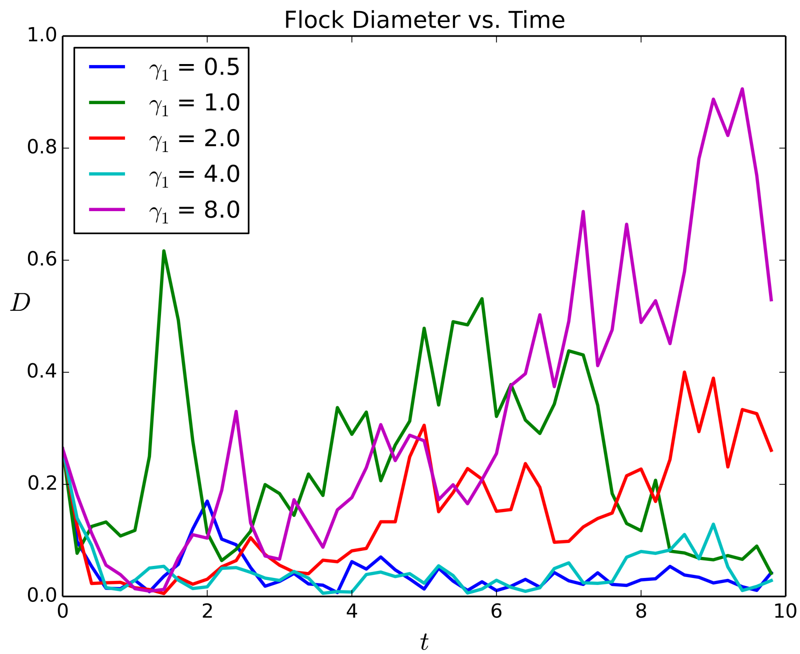


Figure 4: Flock Diameter (D) Vs Time (t) For the Attractive Force of the Leader Bird to Food (γ_1)

γ_1	$\overline{D}$
0.5	4.39458e-02
1.0	2.37929e-01
2.0	**1.46906e-01**
4.0	4.59095e-02
8.0	3.18652e-01

Table 4: Average Flock Diameter ($\overline{D}$) over Time (t) For the Attractive Force of the Leader Bird to Food (γ_1)

In this case, how "hungry" our leader bird is, or the magnitude of the γ_1 parameter, does not affect the flock diameter size. In other words, as γ_1 changes we do not see a consistent change in the average flock diameter $\overline{D}$ which may suggest that the two are unrelated.

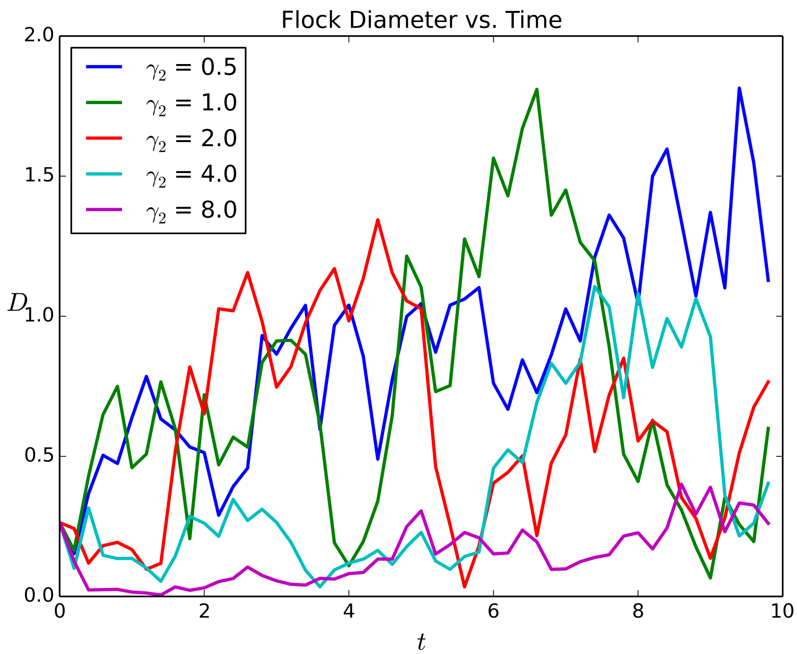


Figure 5: Flock Diameter (D) Vs Time (t) For the Attractive Force of the Follow Birds to the Leader Bird (γ_2)

γ_2	$\overline{D}$
0.5	8.87978e-01
1.0	7.09394e-01
2.0	6.06284e-01
4.0	3.93497e-01
8.0	**1.46906e-01**

Table 5: Average Flock Diameter ($\overline{D}$) over Time (t) For the Attractive Force of the Follow Birds to the Leader Bird (γ_2)

Here, we see that the "charisma" of the leader bird is related to the average diameter of the flock. This is what we would expect to see since this is a major attractive force to a particular point in the grid space applied to all follow birds, $B_k(t)$ for $k = 2, 3, ..., N$. Thus, a large γ_2 decreases the distance of our birds to an optimal point decreasing the flock diameter.

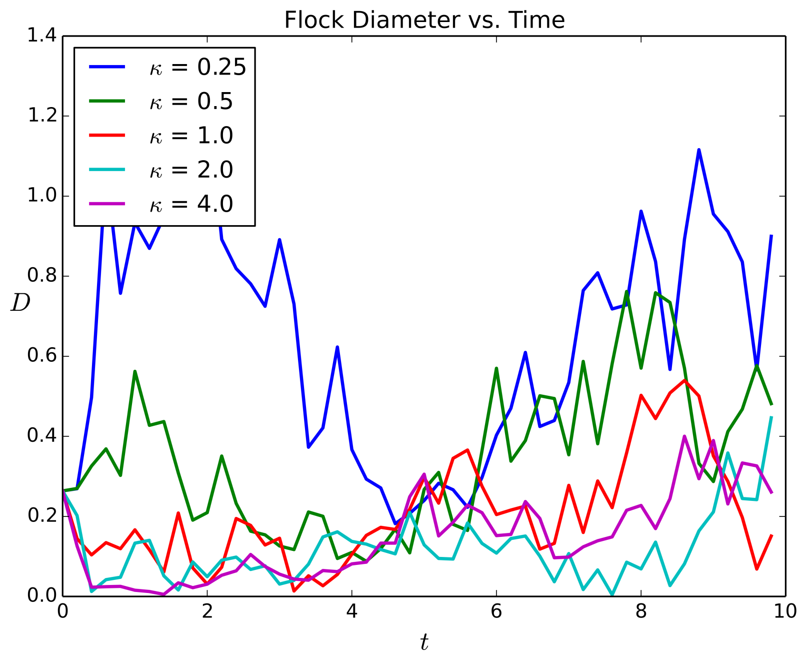


Figure 6: Average Flock Diameter ($\overline{D}$) Over Time (t) For the Attractive Force Among the Follow Birds (κ)

κ	$\overline{D}$
0.25	6.43995e-01
0.50	3.46939e-01
1.00	2.07474e-01
2.00	1.19518e-01
4.00	**1.46906e-01**

Table 6: Average Flock Diameter ($\overline{D}$) over Time (t) For the Attractive Force Among the Follow Birds (κ)

From the table above we see that on average, smaller κ values gives rise to a larger flock diameter in our simulation. This is what one would expect since decreasing an attractive force that keeps the flock together will allow for the flock to be more widely distributed in the domain space. Interestingly, the plot also shows that the diameter of the flock at $t = 5$ seconds is relatively the same for all κ values tested although it is not clear as to why we see this phenomenon in our data.

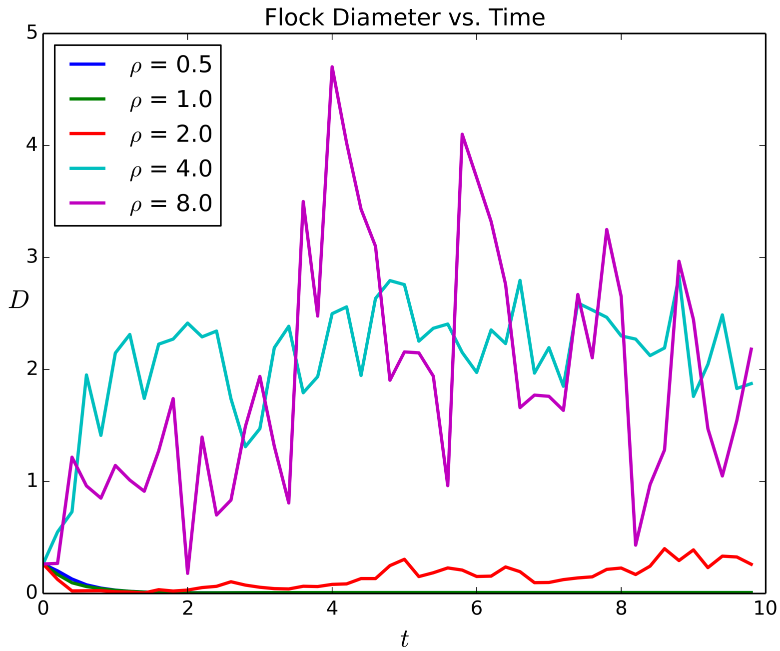


Figure 7: Flock Diameter (D) Vs Time (t) For the Repulsive Force from 5 Nearest Neighbors (ρ)

ρ	$\overline{D}$
0.5	2.25769e-02
1.0	2.06781e-02
2.0	**1.46906e-01**
4.0	2.09061e+00
8.0	1.88795e+00

Table 7: Average Flock Diameter ($\overline{D}$) over Time (t) For the Repulsive Force from 5 Nearest Neighbors (ρ)

Here, we see that as ρ values increase we observe an increase in the average flock diameter. Therefore, there may be an association between ρ the 5-nearest neighbors repulsion force scaling factor and the average flock diameter. Notably, at $\rho = 4.0$, we observe a relatively greater average flock diameter than the other ρ values tested.

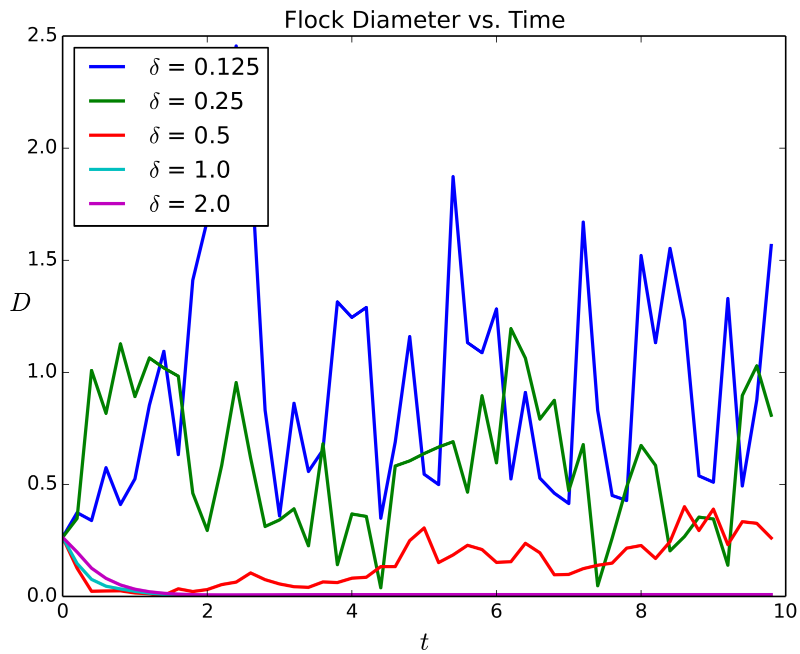


Figure 8: Flock Diameter (D) Vs Time (t) For δ

δ	$\overline{D}$
0.125	9.47923e-01
0.250	5.91929e-01
0.500	**1.46906e-01**
1.000	1.91281e-02
2.000	2.26925e-02

Table 8: Average Flock Diameter ($\overline{D}$) over Time (t) For δ

Here we provide data on the δ value used in our calculation of ρ. We use this δ value to prevent the division by zero error for the case when $B_k = B_{l_i^k}(t)$ in the equation $F_k^{rep}(t) = \sum_{i=1}^{5} \frac{\rho(B_k(t) - B_{l_i^k}(t))}{(B_k(t) - B_{l_i^k}(t))^2 + \delta}, k = 2, 3, ..., N$. If this were the case, then we would have an almost infinitely large repulsive force applied to the 5 nearest neighbors of a bird $B_k(t)$ causing flock behavior such that we would observe an average diameter that would be unrealistic to measure since there may be divergence of the positions of our birds. In other words, the birds would be pushed beyond the limits of what the computer can represent numerically in our simulation.

Also, from the figure above we see that δ has an effect on how the flock diameter behaves over time. So it seems that there may be a relationship between increasing values of δ and decreasing values of flock diameter average over time as an indirect result of the effect that δ has on ρ. Additionally, a relationship can be seen in the figure since the lower δ values appear to be associated with an increased variation in flock diameter. This may suggest that decreasing δ will cause both an increase in average flock diameter as well as an increase in the change of position each flock member may take over each time step. This is interesting in that the observed chaotic behavior of the $\delta = 0.125$ in plot might suggest at a potential loss of stability in our simulation in the case where our time stepping resolution is not high enough. However, the focus of our assignment does not concern the analysis of the time stepping resolution. Hence, we do not investigate the stability of our time-stepping method (fourth order Runge Kutta method or RK4).

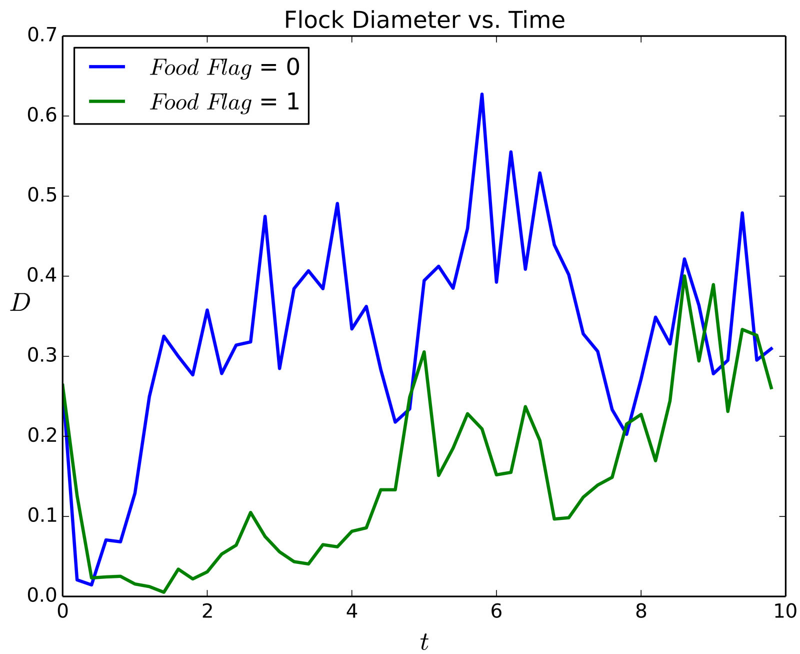


Figure 9: Flock Diameter (D) Vs Time (t) For the Food Flag

Food Flag	$\overline{D}$
0	3.25951e-01
1	**1.46906e-01**

Table 9: Average Flock Diameter ($\overline{D}$) over Time (t) For *Food Flag*

For this parameter study, we have a parameter space size of two because the *food_flag* parameter is essentially a Boolean value, where 0 indicates non-moving food, and 1 indicates a food source that has circular orbit around the origin of the plot in a clockwise manner. From the figure and the average diameter data, we see that the non-moving food source resulted in a generally larger flock diameter. In other words, in the case where the food flag is stationary, we observe a greater average flock diameter. Also, since we used a relatively small flock size of $N = 30$ birds, we may not be able to state how the system behaves for the cases where flock size is much greater than 30. In other words, we cannot state how flock size will affect the average diameter of the flock given different food flag parameters.

1.2.1 Task 2: Extra Credit - A Smelly Bird and A Predator

In this portion of the assignment, we will simulate two special cases of our system of ODEs. This first case is where we have a bird that is less desirable than the other birds and will repel the others from its location. Since being smelly may confer an underlying disease process whether it be due to the malodorous bacterial, fungi, or weakened immune system allowing for opportunistic pathogens to invade the host, or simply unappealing, we will classify this attribute as being smelly or a smelly bird. Here, it is not advantages for the other birds to be near this bird because they may catch its stink or illness since it may be a disease vector. So being close to the smelly bird will make our birds even more angry than they already were and proceed to flock away from our smelly bird. The repulsive force of the smelly bird on the other members of the flock is described by the following:

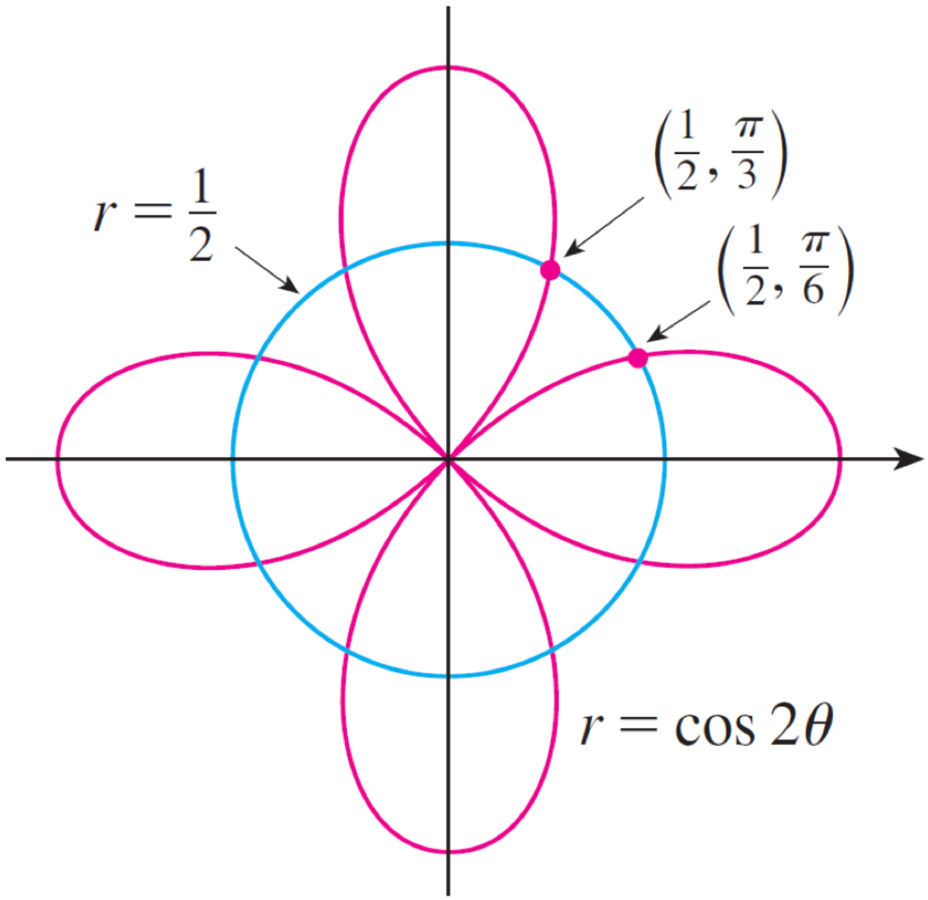
$$F_k^{smelly} = \frac{\sigma(B_k(t) - B_2(t))}{(B_k(t) - B_2(t))^2 + \delta}, k = 1, 3, ...N \tag{11}$$

Here, σ is the parameter that influences the force of repulsion that the smelly bird has on the rest of the birds in the flock. Also, we are using the δ factor in this equation to prevent any division by zero error in cases where $B_k(t) = B_2(t)$ and the denominator would become zero. So, as we increase the smelliness of the bird, by increasing σ, we would expect the other birds to be repelled away from the smelly birds location at each time step t.

In our second case, we have a predator that moves along the grid and repels all the birds in the set $B_N(t)$. In other words, the predator effectively repels every member of the flock without receiving any of the dynamic forces previously mentioned in section 1.1. This force will be simulated by adding an avoidance force to our system of ODEs:

$$F_k^{pr} = \frac{\Pi(B_k(t) - P(t))}{(B_k(t) - P(t))^2 + \delta}, k = 1, 2, ...N \tag{12}$$

where $P(t) = P(x_p(t), y_p(t))$ and $x_p(t)$ denotes the x-coordinate of the predator at time t and $y_p(t)$ denotes the y-coordinate of the predator at time t. Here, the parameter Π is a ODE system parameter that denotes how intimidating our predator is perceived by the birds in the flock. This could also be interpreted as how deadly or dangerous the predator is in terms of how effective it would be at catching and destroying its prey (Angry Birds). We will include plots and movies of these two additional cases below.



Path of Our Predator and Our Food Path

Above is a graph of the path that our predator takes (in pink) and the path that the bird's food takes (in blue). We chose the former path because we wanted a path that would transverse the path of the food more than once. Thus, adding to our simulation the notion that the predator is chasing the birds near and around the path of the bird's food.

1.2.2 How

We implemented the flock smelly bird and predator avoidance force in the same manner as the flock 5 nearest neighbor repulsion force was implemented. We kept the δ term in the denominator to be the default $delta$, changing only the scaling factor σ for the smelly bird force and π for the predator avoidance scaling factor. In other words, we continue our parameter changing study for these two parameters, keeping all other system parameters at their default values as well as the same time conditions such as the time span and time step resolution.

Similar to what we have shown above, we plot flock diameter and tabulate the average flock diameter for both the smelly bird and the predator. This results of which are shown below.

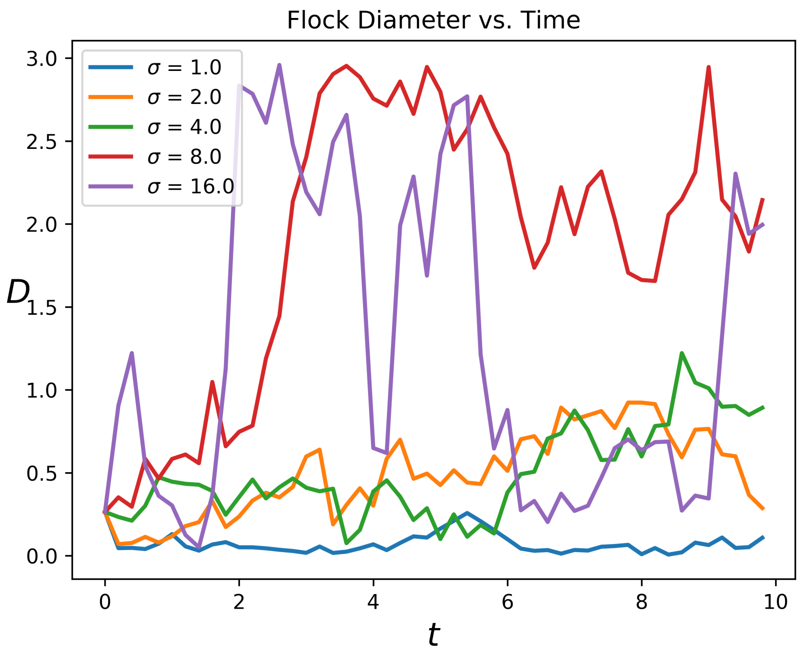


Figure 10: Flock Diameter (D) Vs Time (t) For the Repulsive Force of Smelly Bird (σ)

Π	$\overline{D}$
1.0	7.20187e-02
2.0	4.92705e-01
4.0	4.94733e-01
8.0	**9.75234e-02**
16.0	1.24773e+00

Table 10: Average Flock Diameter ($\overline{D}$) Over Time (t) for the Repulsive Force of Smelly Bird (σ)

Above we see flock behaviors similar to what we observed we observed in the flock 5 nearest neighbors repulsive force time plot. Here, we see that when $\sigma = 1.0$ there is a relatively small average flock diameter as well as a more stable flock diameter value throughout the entirety of the run. At $\sigma = 16.0$, we see an increase in average flock diameter and more variation in the flock diameter.

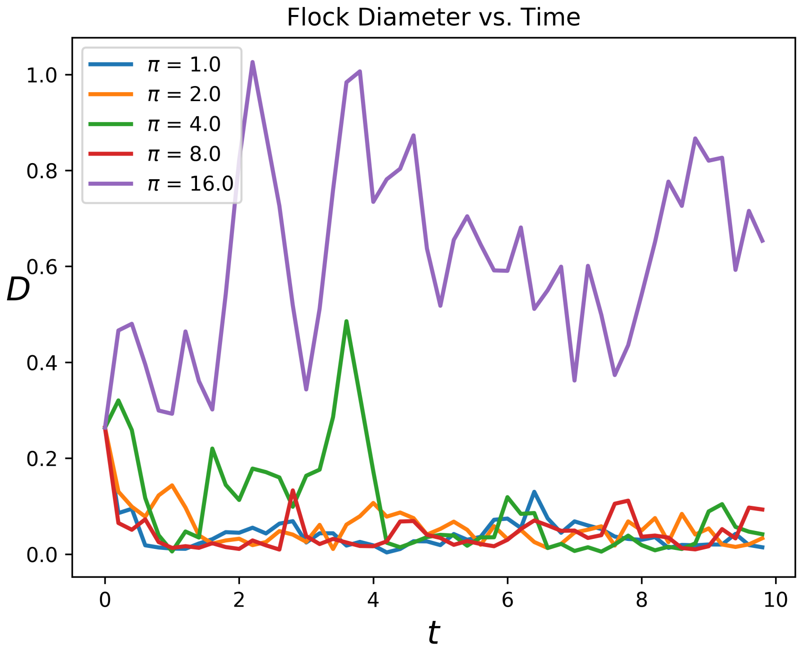


Figure 11: Flock Diameter (D) Vs Time (t) For the Repulsive Force of Predator (Π)

Π	$\overline{D}$
1.0	4.35020e-02
2.0	5.73096e-02
4.0	**9.75234e-02**
8.0	4.51218e-02
16.0	6.15143e-01

Table 11: Average Flock Diameter ($\overline{D}$) Over Time (t) for the Repulsive Force of Predator (Π)

Here, we see that at the largest intimidating factor $\Pi = 16.0$, we have the largest flock diameter which suggests that our flock becomes more widely spread if the predator is effective in scaring off the birds. We also note that at $\Pi = 16.0$, the diameter has greater variation relative to the other parameter values tested. A greater variation in diameter can be interpreted as the bird in the flock furthest from the center approaching the center at a subinterval within the time interval while also fleeing from our predator at other time subintervals.

1.3 Why

The purpose of this assignment was to experiment with an ODE system to determine the effects that attractive and repulsive forces has on the bodies of the system. This is important because we could then generalize these methods to simulate ODE systems that behave in a similar manner. In our system, we limit the number of influences so that we can learn the fundamentals of how an ODE system is modeled in practice. ODE system simulations in general are often dynamic and useful in that they can approximate the phenomenon of what one would find in the natural world such as modeling N-bodies in space, the dynamics of disease propagation, changes in population, or in this case how the Angry Birds of a flock influence one another when following their leader bird.

From the graphs above we have shown that there is a wealth of complexity to explore in studying the behavior of our ODE mathematical model of how bird flocks might behave. Also, we found that interconnected and dynamic nature of our bird flock model leads itself to further analysis that is outside the scope of this course. In particular, further analytical studies could help answer questions such as how does the system behave when we increase the resolution in our singly changing parameter studies? Is there an optimal set of parameters that will provide the largest stable average flock diameter? Is there a better way to measure the average flock diameter itself? (such as a linear regression model that could take the least squares difference of each of our birds position and provide us with a line through the flock of birds that may better represent the appearant flock diameter). Should time series analysis be used to better describe the data that we obtain in this system for certain time steps? Additional directions to pursue may be to include having the nearest neighbors-based flock repulsion be tunable i.e. changing the number of neighbors to account for such as the number of birds repelled. Thus, there may be numerous statistical and analysis methods that could help provide us with a better understanding and description of how this system behaves relative to changing parameters of the system.

Also, in future studies, one could consider examining the effects of random sampling since our initial conditions are a random distribution of birds. In other words, how does the initial conditions affect our ODE model.

Also, we could include other more natural phenomena such as altitude, wind speeds, time of day, the effect that trees may provide protection/additional food sources, species specific characteristics, other natural selection pressures such as mating behavior or social dynamics of the flock, multiple competing leader birds, multiple smelly birds or competing predators, poisonous food, and the health or disease states of the population of birds and predators etc. Since we did not include these numerous natural parameters that affect the movement behavior of birds, our simulation is limited in this regard. In other words, these factors are unaccounted for here but do indeed play a role in the real world when describing bird flock dynamics.

In terms of limitations of our analysis, we were given the liberty of determining how the flock diameter would be defined and there may be more appropriate ways to arrive at definitions of flock diameter. One way may be to consider defining the flock diameter as the sum of the length of each dimension of an axis aligned bounding box around our flock. Another possible definition, as mentioned previously, involves using linear regression to define a diameter line and then take length along the boundaries of the line to represent the diameter. Other limitations may be that this system of ODEs describes the behavior of objects moving in space but are allow to occupy the same time and space giving rise to an unrealistic simulation.

Below is our final task which was to create movies of our Angry Bird simulations. The directions of how to find the movie.mp4 files are described below.

2 Task 3: Movies of Angry Birds

To view the movies of our simulations the ODE system above, first navigate to the subfolder in our hw5 folder located within our class LoboGit repository named */movies*. In our */movies* folder, there will be three MPEG4 format files named $default.mp4$, $smelly.mp4$, and $predator.mp4$.

Each movie captures, in 150 frames, time-iterated behavior of the bird flock given default simulation parameters through 30 simulation seconds. $default.mp4$ is the movie that includes only the non-extra credit actors in our simulation, which includes a single leader bird and 29 flock birds. $smelly.mp4$ includes a smelly bird within the flock of 30. $predator.mp4$ includes a predator that follows the fixed path specified earlier in this report and also includes the flock of 30 birds.

3 Summary

All in all, we found this to be a very interesting assignment where in which we could experiment with parameters of the model and develop our own method to measure the flock diameter as well as create two new simulations in the extra credit portion. We have shown how both attractive and repulsive forces affect objects in space over time using an Ordinary Differential Equation model. Here, we also have found interesting complexity in our analysis of the movement of the Angry Birds and speculated on limitations of our study. We also included a brief discussion on how the ODE system could include more parameters to simulate a more realistic system.

Ultimately, our ODE system is extremely useful in that we can create a model that simulates real world systems without having to conduct the actual experiment, saving time and money at the expense of computation. This can be applied to numerous scenarios in science e.g. radioactive materials research, nuclear weapons testing, flock dynamics simulation, aircraft aerodynamics simulations etc. Thus, in scientific computing, being able to conduct a simulation of a system on a computer can help us to efficiently describe how a system works and identify relationships between parameters within the system.