Chapter 1
The Foundations: Logic and Proofs

MAD101

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1 Propositional Logic

Proposition (mệnh đề) is a declarative sentence that is either **true** or **false** but **not both**.

1. Hà Nội is the capital of Việt Nam.
2. Biden is not president of USA.
3. It is going to rain!
4. What times is it?

Definition

Proposition (mệnh đề) is a declarative sentence that is either **true** or **false** but **not both**.

1. Hà Nội is the capital of Việt Nam.
2. Biden is not president of USA.
3. It is going to rain!
4. What times is it?

1\. 2. are propositions and 3. 4. are not.

Truth table

A proposition can be true (**True/T/1**) or false (**False/F/0**).

p
True/ T / 1
False / F / 0

a. Hà Nội is the capital of Việt Nam.
b. Biden is not president of USA.

a. is true and b. is false

Negation (phủ định)

Negation of proposition p is the statement " It is not case that p". Notation: $\neg p$ (or $\overline{p}$)

p	$\overline{p}$
1	0
0	1

Conjunction (hội)

Conjunction of propositions p and q is the proposition " p and q" and denoted by $p \wedge q$

p	q	p∧q
0	0	0
0	1	0
1	0	0
1	1	1

(Chỉ đúng khi cả 2 cùng đúng)

Disjunction (tuyển)

Disjunction of propositions p and q is the proposition " p or q" and denoted by $p \vee q$

p	q	p∨q
0	0	0
0	1	1
1	0	1
1	1	1

(Chỉ sai khi cả 2 cùng sai)

Exclusive-or(xor)(tuyển loại trừ)

Exclusive-or (xor) of propositions p and q, denoted by $p \oplus q$

p	q	p ⊕ q
0	0	0
0	1	1
1	0	1
1	1	0

(Chỉ đúng khi cả hai khác chân trị)

Implication (kéo theo)

Implication: $p \to q$ (p implies q)

p	q	$p \to q$
0	0	1
0	1	1
1	0	0
1	1	1

(Chỉ sai khi p đúng q sai)

Example:"If 1+3=7, then cats can fly"

Implication (kéo theo)

$p \to q$ can be expressed as

1. If p, then q (or q if p) (nếu p thì q)
2. p only if q (p kéo theo q)
3. p is sufficient for q (p là điều kiện đủ của q)
4. q is neccessary condition for p (q là điều kiện cần của p)

other expresses,

"if p, then q"	"p implies q"
"if p, q"	"p only if q"
"p is sufficient for q"	"a sufficient condition for q is p"
"q if p"	"q whenever p"
"q when p"	"q is necessary for p"
"a necessary condition for p is q"	"q follows from p"
"q unless $\neg p$"	"q provided that p"

Implication (kéo theo)

Let P is the statement “Mr. Bean has an invention”

Let Q is the statement “Mr. Bean gets a prize”

Which of the following English statements can be used for "$P \to Q$"?

(i) He will get a price whenever he has an invention

(ii) A prize is necessary for having an invention

Select one:

- a. (i)
- b. (ii)
- c. Both
- d. None

Implication (kéo theo)

Let P is the statement “Mr. Bean has an invention”

Let Q is the statement “Mr. Bean gets a prize”

Which of the following English statements can be used for "P → Q"?

(i) He will get a price whenever he has an invention

(ii) A prize is necessary for having an invention

Select one:

- a. (i)
- b. (ii)
- c. Both
- d. None

Ans c

Biconditional statement(tương đương)

Biconditional statement $p \leftrightarrow q$ is the proposition " p if and only if q". Other notation $p \equiv q$ (if $p \leftrightarrow q$ is True)

p	q	p→q	q→p	(p→q) ^ (q→p)	p ↔ q
0	0	1	1	1	1
0	1	1	0	0	0
1	0	0	1	0	0
1	1	1	1	1	1

(Chỉ đúng khi cả 2 cùng chân trị)

Examle:"1+1=1 if and only if 2+2=2" is True

Precedence of Logical Operators (thứ tự ưu tiên)

1. Parentheses from inner to outer (dấu "()" từ trong ra ngoài)
2. $\neg$
3. $\wedge$ and $\oplus$
4. $\vee$
5. $\rightarrow$
6. $\leftrightarrow$

Example: $\neg p \vee q \wedge r$ mean $(\neg p) \vee (q \wedge r)$

Quizz

Which the following propositions is FALSE:

Select one:

- a. 1+1 = 1 if and only if 2 + 2 = 2
- b. If 1<0, then 1 = 0
- c. If 1 + 1 = 3 or 1 + 1 = 2, then 2 + 2 = 4 and 2 + 2 = 1
- d. If 2 + 1 = 3, then 2 = 3 - 1

Quizz

Which the following propositions is FALSE:

Select one:

- a. 1+1 = 1 if and only if 2 + 2 = 2
- b. If 1<0, then 1 = 0
- c. If 1 + 1 = 3 or 1 + 1 = 2, then 2 + 2 = 4 and 2 + 2 = 1
- d. If 2 + 1 = 3, then 2 = 3 - 1

Ans c

- We can use a **bit** to represent a truth value: **bit 1** for **true** and **bit 0** for **false.**
- A **Boolean variable** has value either true or false, and can be represented by a bit.
- By replacing true by 1 and false by 0 in the truth tables of logical operators, we obtain the corresponding tables for bit operations.
- The operators $\neg, \wedge, \vee$ and $\oplus$ are also denoted by NOT, AND, OR and XOR.

Bit Operations (phép toán bit)

- A **bit string** (xâu bit) is a sequence of zero or more bits. The **length** of a bit string is the number of bits in the string.
- The bitwise AND, OR and XOR of two strings of the same length is the string whose bits are the AND, OR and XOR of the corresponding bits of the two strings

Example. The bitwise AND, OR and XOR of
01 1001 0110 and 11 0001 1101

Bit Operations (phép toán bit)

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Example. The bitwise AND, OR and XOR of
01 1001 0110 and 11 0001 1101

- AND 0100010100
- OR 1110011111
- XOR 1010001011

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Example

Let p: **Today is Sunday**. Find truth table of $p \vee \neg p$.

Note:

- $\neg p$: Today is not Sunday
- $p \vee \neg p$: Today is Sunday or not

p	¬p	p ∨ ¬p
T	F	T
F	T	T

$p \vee \neg p$ is called **tautology** (hằng đúng)

Example

Show that each of the following propositions is tautology

a. $(p \vee q) \wedge \neg p \to q$

b. $(p \to q) \wedge \neg q \to \neg p$

c. $(p \to q) \wedge (\neg p \to q) \to q$

Tautology and contradiction

- **Tautology** (hằng đúng) is a proposition that is **always true**
- **Contradiction** (mâu thuẫn) is a proposition that is **always false**

Thus, p and q are **logical equivalent** if and only if $p \leftrightarrow q$ is a **tautology**

Notes. The propositions p and q are logically equivalent if they have the same truth tables. We also write $p \equiv q$.

Example

2 Propositional Equivalences

Find truth table of proposition $p \wedge T$ (T is true-proposition)

p	T	$p \wedge$ T
T	T	T
F	T	F

Thus, $p \wedge T \equiv p$

The Laws of Logic

2 Propositional Equivalences

Equivalence	Name
$p \wedge T \equiv p$ $\quad p \vee F \equiv p$	Identity laws (đồng nhất)
$p \vee T \equiv T$ $\quad p \wedge F \equiv F$	Domination Laws (nuốt)
$p \vee p \equiv p$ $\quad p \wedge p \equiv p$	Idempotent Laws (lũy đẳng)
$\neg(\neg p) \equiv p$	Double Negation Laws (phủ định kép)
$p \vee q \equiv q \vee p$ $\quad p \wedge q \equiv q \wedge p$	Commutative Laws (giao hoán)
$(p \vee q) \vee r \equiv p \vee (q \vee r)$ $(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$	Associative Laws (kết hợp)
$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$ $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$	Distributive Laws (phân phối)

Equivalence		Name
$\neg (p \wedge q) \equiv \neg p \vee \neg q$ $\neg (p \vee q) \equiv \neg p \wedge \neg q$		De Morgan Laws
$p \vee (p \wedge q) \equiv p$	$p \wedge (p \vee q) \equiv p$	Absorption Laws (hấp thụ)
$p \vee \neg p \equiv T$	$p \wedge \neg p \equiv F$	Negation Laws (phần tử bù)

Example. Show that $p \wedge (p \vee q) \equiv p$ (Absortion Laws)

$$\begin{aligned} p \wedge (p \vee q) &\equiv (p \vee 0) \wedge (p \vee q) \\ &\equiv p \vee (0 \wedge q) \\ &\equiv p \vee (q \wedge 0) \\ &\equiv p \vee 0 \\ &\equiv p \end{aligned}$$

Homeworks

Equivalences	Equivalences
$p \to q \equiv \neg p \vee q$	$p \leftrightarrow q \equiv (p \to q) \wedge (q \to p)$
$p \to q \equiv \neg q \to \neg p$	$p \leftrightarrow q \equiv \neg p \leftrightarrow \neg q$
$p \vee q \equiv \neg p \to q$	$p \leftrightarrow q \equiv (p \wedge q) \vee (\neg p \wedge \neg q)$
$p \wedge q \equiv \neg (p \to \neg q)$	$\neg p \leftrightarrow q \equiv p \leftrightarrow \neg q$
$\neg(p \to q) \equiv p \wedge \neg q$	
$(p \to q) \wedge (p \to r) \equiv p \to (q \wedge r)$	
$(p \to r) \wedge (q \to r) \equiv (p \vee q) \to r$	
$(p \to q) \vee (p \to r) \equiv p \to (q \vee r)$	
$(p \to r) \vee (q \to r) \equiv (p \wedge q) \to r$	

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3 Predicates and Quantifiers

Example

Which statements are propositions:

1. Minh loves ice cream.
2. X loves ice cream.
3. Everyone loves ice cream.
4. Someone loves ice cream.

Example

Which statements are propositions:

1. Minh loves ice cream.
2. X loves ice cream.
3. Everyone loves ice cream.
4. Someone loves ice cream.

1. Yes; 2. No; 3. Yes; 4.Yes

Example

Which statements are propositions:

1. $3 + 2 = 5$
2. $X + 2 = 5$
3. $X + 2 = 5$ for any choice of X in $\{1, 2, 3\}$
4. $X + 2 = 5$ for some X in $\{1, 2, 3\}$

Example

Which statements are propositions:

1. $3 + 2 = 5$
2. $\mathrm{X} + 2 = 5$
3. $\mathrm{X} + 2 = 5$ for any choice of X in $\{1, 2, 3\}$
4. $\mathrm{X} + 2 = 5$ for some X in $\{1, 2, 3\}$

1. Yes; 2. No; 3. Yes; 4. Yes

Example

3 Predicates and Quantifiers

Which statements are propositions:

1. $12 > 4$.
2. X $>$ 4.
3. X $>$ 4 for any choice of X in $\{3, 4, 5\}$.
4. X $>$ 4 for some X in $\{1, 2, 3\}$.

Example

Which statements are propositions:

1. $12 > 4$.
2. $X > 4$.
3. $X > 4$ for any choice of X in $\{3, 4, 5\}$.
4. $X > 4$ for some X in $\{1, 2, 3\}$.

1. Yes; 2. No;3. Yes; 4. Yes

The efficiency of Predicates.

- An eats pizza at least once a week.
- Bình eats pizza at least once a week.
- Hà eats pizza at least once a week.
- Minh eats pizza at least once a week.
- Thư eats pizza at least once a week.
- Huy eats pizza at least once a week.
- Việt eats pizza at least once a week.

$\rightarrow$ **Define.** P(x) = "x eats pizza at least once a week." **Universe of Discourse -** x is a student in Discrete Math class.
Note that P(x) is not a proposition, P(Binh) is.

Predicate (vị từ)

A predicate, or propositional function, is a function defined on a set U and returns a proposition as its value.
The set U is called the **universe of discourse.**

- We often denote a predicate by P(x)
- Note that P(x) is not a proposition, but P(a) where a is some fixed element of U is a proposition with well determined truth value

Example

Let Q(x,y) = "x > y". Which statements are propositions:

1. Q(x,y)
2. Q(3,4)
3. Q(x,9)

1. No; 2. Yes; 3. No
Q(x,y) is a predicates in two free variables x and y in $\mathbb{R}$

- Predicates are pre-conditions and post-conditions of a program.
- If x>0 then x:=x+1
  - Predicate: “x>0” ➔ P(x)
  - Pre-condition: P(x)
  - Post-condition: P(x)
- T:=X;
  X:=Y;
  Y:=T;
  - Pre-condition: “x=a and y=b” ➔ P(x, y)
  - Post-condition: “x=b and y=a” ➔ Q(x, y)

> Pre-condition (P(...)) : condition describes valid input.
> Post-condition (Q(...)) : condition describes valid output of the codes.
> **Show the verification that a program always produces the desired output:**
> P(...) is true
> Executing Step 1.
> Executing Step 2.
> .....
> Q(...) is true

Quizz

3 Predicates and Quantifiers

Study the following computer code segment:

```
x:= 5
y:= 6
If (1+1=0) OR (2+2=1) then x:=x+1
If (1+1=2) XOR (1+2=3) then y=y+1
```

What are values of x and y after the codes execute?

Select one:

- a. 5; 7
- b. 6; 6
- c. 6; 8
- d. 5; 6

Ans: d

The universal quantifier (lượng từ phổ dụng hay với mọi: $\forall$)

Let P(x) be a predicate on some universe of discourse U

- One way to obtain a proposition from P(x) is to substitute x by a fixed element of U.
- Another way to obtain a proposition from P(x) is to use the universal quantifier.

Consider the statement: "P(x) is true for all x in the universe of discourse."

- We write it $\forall x P(x)$ and say "for all x, P(x) "
- The symbol $\forall$ is the **universal quantifier**.

The universal quantifier (lượng từ phổ dụng hay với mọi: $\forall$)

3 Predicates and Quantifiers

Defintion. Let P(x) be a predicate on some universe of discourse U. Consider the statement "P(x) is true for all x in the universe of discourse."
We write it $\forall x P(x)$, and say "for all x, P(x) "
The proposition $\forall x P(x)$ is called the **universal quantification of the predicate P(x)** (lượng từ phổ dụng hóa của vị từ $P(x)$). It is

- TRUE if P(a) is true when we subsitute x by any element a in U
- FALSE if there is an element a in U for which P(a) is false.

Example

3 Predicates and Quantifiers

The following propositions True or False

a. Let P(x) be the predicate x + 1 > x, where the universe of discourse are the real numbers.

b. Let Q(x) be the predicate x < 1, where the universe of discourse are the real numbers.

a. $\forall x P(x)$ True; b. $\forall x Q(x)$ False

The existential quantifier (lượng từ tồn tại): $\exists$

Let P(x) is a predicate on some universe of discourse.
The **existential quantification of P(x)** (lượng từ tồn tại hóa của vị từ P(x)) is the proposition:
"There exists an element x in the universe of discourse such that P(x) is true."
We write it $\exists x P(x)$, and say "for some x, P(x)". $\exists$ is called the **existential quantifier**. It is

- $\exists x P(x)$ is FALSE if P(x) is false for every single x.
- $\exists x P(x)$ is TRUE if there is an x for which P(x) is true.

Example

The following propositions True or False

a. Let P(x) be the predicate x $>$ 3, where the universe of discourse are the real numbers.

b. Let Q(x) be the predicate x $=$ x $+$ 1, where the universe of discourse are the real numbers.

a. $\exists x P(x)$ True; b. $\exists x Q(x)$ False

U is finite

In the special case that the universe of discourse, U, is finite, say $U = \{x_1, x_2, x_3, \ldots, x_n\}$. Then

$$\exists x\, P(x)$$

corresponds to the proposition:

$$P(x_1) \vee P(x_2) \vee \ldots \vee P(x_n)$$

We can write a program to loop through the elements in the universe and check each for truthfulness.

The program will stop when we find an element x_i such that $P(x_i)$ is true. In this case the Proposition is true.

Example

Suppose the universe of discourse is all creatures, and define the following:

- L(x) = “x is a lion.”
- F(x) = “x is fierce.”
- C(x) = “x drinks coffee.”

1. All lions are fierce → $\forall x(L(x) \rightarrow F(x))$
2. Some lions don't drink coffee → $\exists x(L(x) \wedge \neg C(x))$
3. Some fierce creatures don't drink coffee → $\exists x(F(x) \wedge \neg C(x))$

Example

3 Predicates and Quantifiers

- B(x) = “x is a hummingbird.”
- L(x) = “x is a large bird.”
- H(x) = “x lives on honey.”
- R(x) = “x is richly colored.”

1. All hummingbirds are richly colored $\rightarrow \forall x(B(x) \rightarrow R(x))$
2. No large birds live on honey $\rightarrow \neg\exists x(L(x) \wedge H(x))$
3. Birds that do not live on honey are dully colored $\rightarrow \forall x(\neg H(x) \rightarrow \neg R(x))$

Negations

3 Predicates and Quantifiers

No large birds live on honey $\rightarrow \neg\exists x(L(x) \wedge H(x))$

- $\exists x P(x)$ means "P(x) is true for some x."
- What about $\neg\exists x P(x)$?
- Not["P(x) is true for some x."]
- "P(x) is not true for all x."
- $\forall x \neg P(x)$

So, $\neg\exists x P(x)$ is the same as $\forall x \neg P(x)$

Negations

3 Predicates and Quantifiers

- $\forall x P(x)$ means "P(x) is true for every x."
- What about $\neg\forall x P(x)$?
- Not["P(x) is true for every x."]
- "There is an x for which P(x) is not true."
- $\exists x \neg P(x)$

So, $\neg\forall x P(x)$ is the same as $\exists x \neg P(x)$

Quantifier negation

No large birds live on honey $\rightarrow \neg\exists x(L(x) \wedge H(x))$
So, $\neg\exists x P(x)$ is the same as $\forall x \neg P(x)$
and, $\neg\forall x P(x)$ is the same as $\exists x \neg P(x)$
General rule: to negate a quantification,

- Move negation ($\neg$) to the right,
- Change the quantifier from $\exists$ to $\forall$, and from $\forall$ to $\exists$.

Example

$\neg\forall x\exists y(xy = 1)$?

$$\neg \forall x \exists y (xy=1) \equiv \exists x \neg \exists y (xy=1)$$
$$\equiv (\exists x)(\forall y) \neg (xy=1)$$
$$\equiv (\exists x)(\forall y)(xy \neq 1)$$

Example

No large birds live on honey

$$\neg \exists x (L(x) \wedge H(x))$$

- $\equiv \forall x \neg (L(x) \wedge H(x))$ (Negation rule)
- $\equiv \forall x (\neg L(x) \vee \neg H(x))$ (DeMorgan's)
- $\equiv \forall x (L(x) \rightarrow \neg H(x))$ $(p \vee q \equiv \neg p \rightarrow q)$

$\rightarrow$ **Large birds do not live on honey.**

Biding Variables

A variable is bound (ràng buộc) if it is known or quantified. Otherwise, it is free (tự do).

Example

- P(x) x is free
- P(5) x is bound to 5
- $\forall x P(x)$ x is bound by quantifier

Note. In a proposition, all variables must be bound.

Biding Variable

To bind(ràng buộc) many variables, use many quantifiers.

Example. P(x,y) = "x > y"

- $\forall x P(x, y)$ NOT a proposition
- $\forall x \forall y P(x, y)$ FALSE proposition
- $\forall x \exists y P(x, y)$ TRUE proposition
- $\forall x P(x, 3)$ FALSE proposition

The meaning of multiple quantifiers

- "$\forall x \forall y P(x, y)$" means P(x,y) is true for every possible combination of x and y.
- "$\exists x \exists y P(x, y)$" means P(x,y) is true for some choice of x and y (together).
- "$\forall x \exists y P(x, y)$" means for every x we can find a (possibly different) y so that P(x,y) is true.
- "$\exists x \forall y P(x, y)$" means there is (at least one) particular x for which P(x,y) is always true.

Note. Quantifier order is not interchangeable! (không hoán đổi thứ tự lượng từ)

Example

3 Predicates and Quantifiers

P(x,y) = "x is sitting next to y"

- $\forall x \forall y P(x, y)$ - everyone is sitting next to everyone else. FALSE
- $\exists x \exists y P(x, y)$ - there are two people sitting next to each other TRUE
- $\forall x \exists y P(x, y)$ - every person is sitting next to somebody TRUE
- $\exists x \forall y P(x, y)$ - a particular person is sitting next to everyone else FALSE

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The following statements are true:

I am Duong

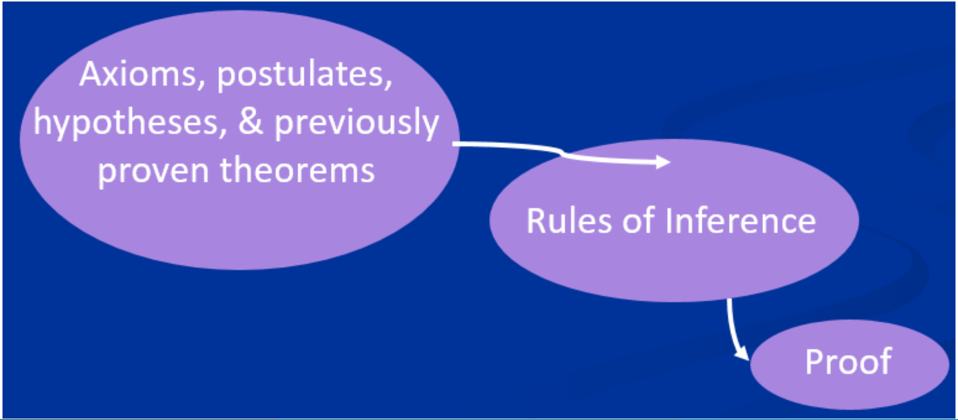
If I am Duong, then I am a Mathematics Lecture.

What do we know?

I am a Mathematics Lecture.

How do we know?

A **theorem** is a statement that can be shown to be true. A **proof** is the means of doing so.



Introduction

The following statements are true:

I am Duong

If I am Duong, then I am a Mathematics Lecture.

What do we know?

I am a Mathematics Lecture.

How do we know?

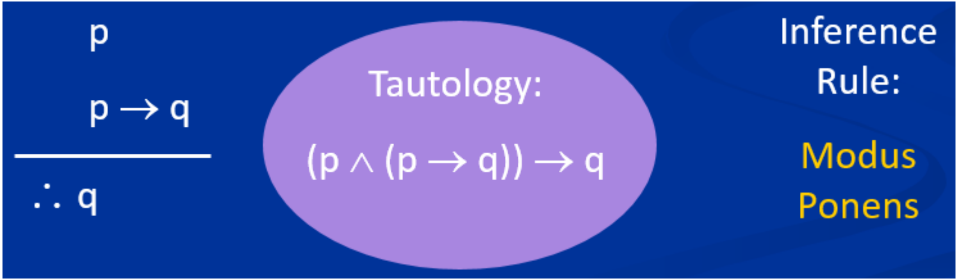
What rule of inference can we use to argue?

Modus Ponens (khẳng định)

I am Duong.

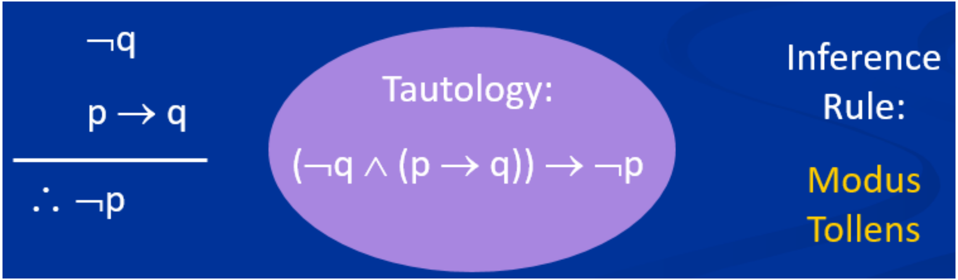
If I am Duong, then I am a Mathematics Lecture.

$\therefore$ I am a Mathematics Lecture.



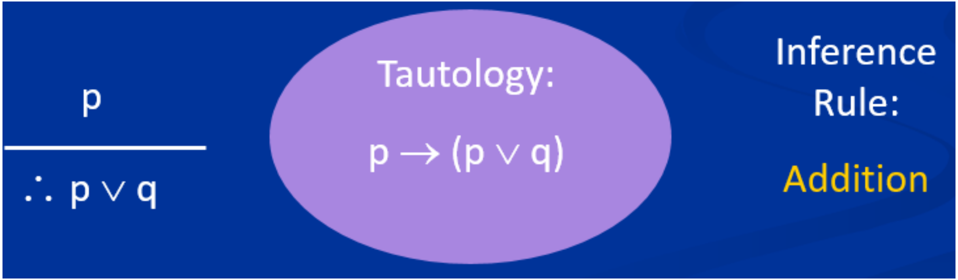
Modus Tollens (phủ định)

I am not a great football striker.
If I am Henry, then I am a great football striker.
$\therefore$ I am not Henry!



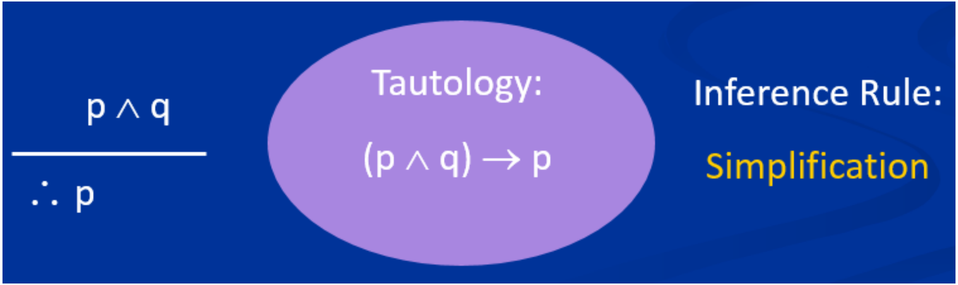
Addition (thêm)

I am not a great football striker.
$\therefore$ I am not a great football striker or I am tall.



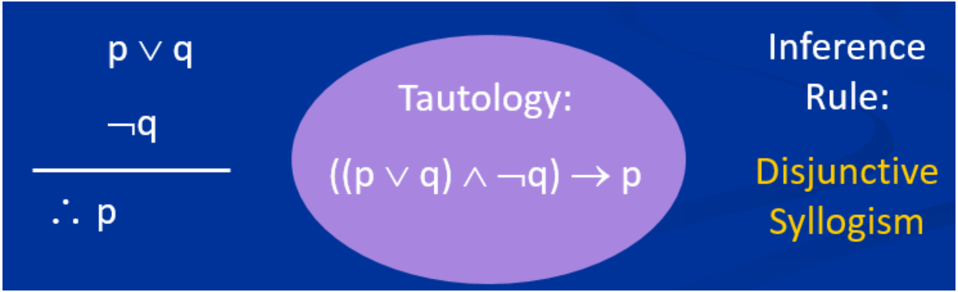
Simplification

I am not a great football striker and you are sleepy.
$\therefore$ you are sleepy.



Disjunctive Syllogism (loại trừ)

I am teacher or doctor.
I am not teacher.
$\therefore$ I am doctor.



Hypothetical Syllogism (tam đoạn luận)

If you are teacher, then you must teach MAD101.
If you teach MAD101, then you are good in logic.
$\therefore$ If you are teacher, then you are good in logic.

$$\begin{array}{l} p \to q \\ q \to r \\ \hline \therefore\ p \to r \end{array}$$

Tautology:

$$((p \to q) \wedge (q \to r)) \to (p \to r)$$

Inference Rule:

Hypothetical Syllogism

Fallacies

4 Rules of Inference

$14 + 6 - 20 = 21 + 9 - 30$

$\rightarrow 2(7 + 3 - 10) = 3(7 + 3 - 10)$

$\rightarrow 2 = 3$

- Rules of inference, appropriately applied give **valid** arguments.
- Mistakes in applying rules of inference are called **fallacies.**(ngụy biện)

Fallacies

If I am Descartes, then I am a mathematician

I am a mathematician!

$\therefore$ I am Descartes

$(p \to q) \land q \to p$ not at tautology $\to$ **fallacies**

If you don't give me \$10, I bite your ear.

I bite your ear!

∴ You didn't give me \$10.

Fallacies

If it rains then it is cloudy.

It does not rain.

$\therefore$ It is not cloudy

Fallacies

4 Rules of Inference

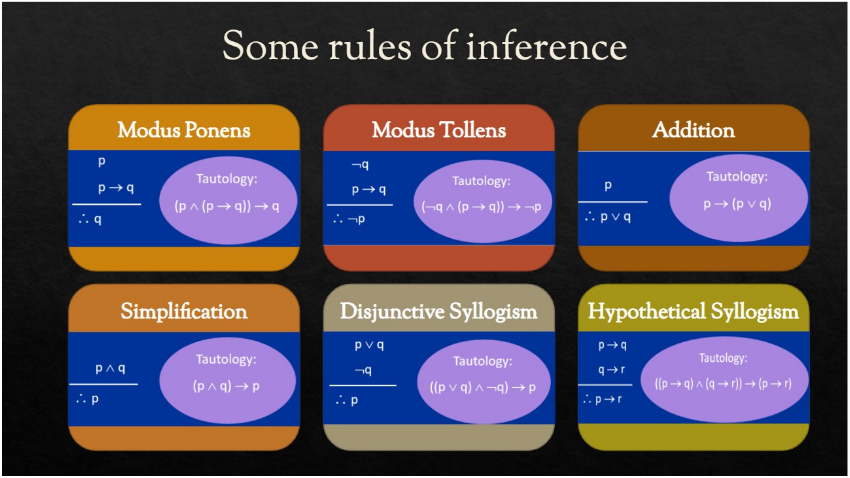
If it is a bicycle, then it has 2 wheels.

It is not a bicycle.

$\therefore$ It doesn't have 2 wheels.

Review

4 Rules of Inference



Quantified Statements

4 Rules of Inference

Rule	Name
$\dfrac{\forall x P(x)}{\therefore P(c)}$	Universal Instantiation
$\dfrac{P(c) \text{ for arbitrary } c}{\therefore \forall x P(x)}$	Universal generalization
$\dfrac{\exists x P(x)}{\therefore P(c) \text{ for some element } c}$	Existential instantiation
$\dfrac{P(c) \text{ for some element } c}{\therefore \exists x P(x)}$	Existential generalization

Universal Instantiation. If $\forall x P(x)$ is true, then P(c) is true for any choice of c in the universe of discourse.

Universal generalization. If P(c) is true for any choice of c in the universe

Example

From the premises:

Every student who is in this MAD class has passed the MAE course

Quân is a student in this MAD

Show that : " Quân has passed the MAE course"

Solution.

$\forall x(MAD(x) \rightarrow MAE(x))$

MAD(Quân)

$\therefore$ MAE(Quân)

Example.

$\forall x(MAD(x) \rightarrow MAE(x)) \rightarrow$ **Premise**

MAD(Quân) $\rightarrow$ MAE(Quân) $\rightarrow$ **Universal Instantiation**

MAD (Quân) $\rightarrow$ **Premise**

MAE(Quân) $\rightarrow$ **Modus Ponens**

Therefore, Quân has passed the MAE course.

All men are mortal

Socrates is a man

Therefore, Socrates is mortal.

Premise: Everyone, who is over 30, can read newspaper.

Premise: Mr. Bean can read newspaper.

Premise: Mrs. Bean is 36 years old.

Conclusion 1: he is over 30.

Conclusion 2: she can read newspaper.

Select one:

- a. The conclusion 1 is logical and the conclusion 2 is not.
- b. Both conclusions are logical.
- c. The conclusion 2 is logical and the conclusion 1 is not.
- d. Both conclusions are illogical.

Ans: c

Quizz

4 Rules of Inference

Study the following arguments:

(i) If Mr. Bean can speak English, then he is good. he can't speak English. Therefore, he is not good.

(ii) All Cr7 fans love FC MU. Mr. Bean doesn't love FC MU. Therefore, he is not a CR7 fan.

(i) is ... and (ii) is ...

Select one:

- a. illogical, illogical
- b. illogical, logical
- c. logical, illogical
- d. logical, logical

Ans: b (use modus ponens and tollens)

Table of Contents

5 Problems

1. Which of these sentences are propositions? What are the truth values of those that are propositions?

a) Boston is the capital of Massachusetts.

b) Miami is the capital of Florida.

c) $2 + 3 = 5$.

d) $5 + 7 = 10$.

e) $x + 2 = 11$.

f) Answer this question.

2. What is the negation of each of these propositions?

a) Mei has an MP3 player.

b) There is no pollution in New Jersey.

c) $2 + 1 = 3$.

d) The summer in Maine is hot and sunny.

3. Let p and q be the propositions

p : I bought a lottery ticket this week.

q : I won the million dollar jackpot.

Express each of these propositions as an English sentence.

4. Let p and q be the propositions

p : It is below freezing.

q : It is snowing.

Write these propositions using p and q and logical connectives (including negations).

a) It is below freezing and snowing.

b) It is below freezing but not snowing.

c) It is not below freezing and it is not snowing.

d) It is either snowing or below freezing (or both).

Propositional Logic

5 Problems

5. Let p and q be the propositions

p : You drive over 65 miles per hour.

q : You get a speeding ticket.

Write these propositions using p and q and logical connectives (including negations).

a) You do not drive over 65 miles per hour.

b) You drive over 65 miles per hour, but you do not get a speeding ticket.

c) You will get a speeding ticket if you drive over 65 miles per hour.

d) If you do not drive over 65 miles per hour, then you will not get a speeding ticket.

e) Driving over 65 miles per hour is sufficient for getting a speeding ticket.

f) You get a speeding ticket, but you do not drive over 65 miles per hour.

g) Whenever you get a speeding ticket, you are driving over 65 miles per hour.

7. Determine whether each of these conditional statements is true or false.

a) If $1 + 1 = 2$, then $2 + 2 = 5$.

b) If $1 + 1 = 3$, then $2 + 2 = 4$.

c) If $1 + 1 = 3$, then $2 + 2 = 5$.

d) If monkeys can fly, then $1 + 1 = 3$.

8. Determine whether each of these conditional statements is true or false.

a) If $1 + 1 = 3$, then unicorns exist.

b) If $1 + 1 = 3$, then dogs can fly.

c) If $1 + 1 = 2$, then dogs can fly.

d) If $2 + 2 = 4$, then $1 + 2 = 3$.

9. Write each of these statements in the form "if p, then q"

a) It is necessary to wash the boss's car to get promoted.

b) Winds from the south imply a spring thaw.

c) A sufficient condition for the warranty to be good is that you bought the computer less than a year ago.

d) Willy gets caught whenever he cheats.

e) You can access the website only if you pay a subscription fee.

f) Getting elected follows from knowing the right people.

g) Carol gets seasick whenever she is on a boat.

10. How many rows appear in a truth table for each of these compound propositions?

a) $p \to \neg p$ b) $(p \vee \neg r) \wedge (q \vee \neg s)$ c) $q \vee p \vee \neg s \vee \neg r \vee \neg t \vee u$

d) $(p \wedge r \wedge t) \leftrightarrow (q \wedge t)$ e) $(q \to \neg p) \vee (\neg p \to \neg q)$ f) $(p \vee \neg t) \wedge (p \vee \neg s)$

g) $(p \to r) \vee (\neg s \to \neg t) \vee (\neg u \to v)$ h) $(p \wedge r \wedge s) \vee (q \wedge t) \vee (r \wedge \neg t)$

11. Construct a truth table for each of these compound propositions.

a) $p \land \neg p$ b) $p \lor \neg p$ c) $(p \lor \neg q) \to q$ d) $(p \lor q) \to (p \land q)$

e) $(p \to q) \leftrightarrow (\neg q \to \neg p)$ f) $(p \to q) \to (q \to p)$

12. Construct a truth table for each of these compound propositions.

a) $p \to \neg p$ b) $p \leftrightarrow \neg p$ c) $p \oplus (p \lor q)$ d) $(p \land q) \to (p \lor q)$

e) $(q \to \neg p) \leftrightarrow (p \leftrightarrow q)$ f) $(p \leftrightarrow q) \oplus (p \leftrightarrow \neg q)$

13. What is the value of x after each of these statements is encountered in a computer program, if x = 1 before the statement is reached?

a) if x + 2 = 3 then x := x + 1

b) if (x + 1 = 3) OR (2x + 2 = 3) then x := x + 1

c) if (2x + 3 = 5) AND (3x + 4 = 7) then x := x + 1

d) if (x + 1 = 2) XOR (x + 2 = 3) then x := x + 1

14. Find the bitwise OR, bitwise AND, and bitwise XOR of each of these pairs of bit strings.

a) 101 1110, 010 0001

b) 1111 0000, 1010 1010

c) 00 0111 0001, 10 0100 1000

d) 11 1111 1111, 00 0000 0000

15. Evaluate each of these expressions.

a) $1\ 1000 \wedge (0\ 1011 \vee 1\ 1011)$

b) $(0\ 1111 \wedge 1\ 0101) \vee 0\ 1000$

c) $(0\ 1010 \oplus 1\ 1011) \oplus 0\ 1000$

d) $(1\ 1011 \vee 0\ 1010) \wedge (1\ 0001 \vee 1\ 1011)$

Propositional Equivalences

5 Problems

1. Show that each of these conditional statements is a tautology by using truth tables.

a) $(p \wedge q) \rightarrow p$ b) $p \rightarrow (p \vee q)$ c) $\neg p \rightarrow (p \rightarrow q)$

d) $(p \wedge q) \rightarrow (p \rightarrow q)$ e) $\neg(p \rightarrow q) \rightarrow p$ f) $\neg(p \rightarrow q) \rightarrow \neg q$

2. Show that each of these conditional statements is a tautology by using truth tables.

a) $[\neg p \wedge (p \vee q)] \rightarrow q$ b) $[(p \rightarrow q) \wedge (q \rightarrow r)] \rightarrow (p \rightarrow r)$

c) $[p \wedge (p \rightarrow q)] \rightarrow q$ d) $[(p \vee q) \wedge (p \rightarrow r) \wedge (q \rightarrow r)] \rightarrow r$

3. Determine whether $(\neg p \wedge (p \rightarrow q)) \rightarrow \neg q$ is a tautology.

4. Determine whether $(\neg q \wedge (p \rightarrow q)) \rightarrow \neg p$ is a tautology.

5. Show that $(p \vee q) \wedge (\neg p \vee r) \rightarrow (q \vee r)$ is a tautology.

6. Show that $(p \rightarrow q) \rightarrow r$ and $p \rightarrow (q \rightarrow r)$ are not logically equivalent.

7. Show that $(p \wedge q) \rightarrow r$ and $(p \rightarrow r) \wedge (q \rightarrow r)$ are not logically equivalent.

Predicates and Quantifiers

5 Problems

1. Let $P(x)$ denote the statement "$x \leq 4$". What are these truth values?

a) $P(0)$ b) $P(4)$ c) $P(6)$

2. Let $P(x)$ be the statement "the word x contains the letter a." What are these truth values?

a) P (orange) b) P (lemon) c) P (true) d) P (false)

3. Let $Q(x, y)$ denote the statement "x is the capital of y". What are these truth values?

a) Q(Denver, Colorado) b) Q(Detroit, Michigan)

c) Q(Massachusetts, Boston) d) Q(New York, New York)

4. State the value of x after the statement if $P(x)$ then $x := 1$is executed, where $P(x)$ is the statement "$x > 1$", if the value of x when this statement is reached is

a) $x = 0$. b) $x = 1$. c) $x = 2$.

5. Let $P(x)$ be the statement "x spends more than five hours every weekday in class," where the domain for x consists of all students. Express each of these

a) $\exists x P(x)$ b) $\forall x P(x)$ c) $\exists x \neg P(x)$ d) $\forall x \neg P(x)$

7. Let $P(x)$ be the statement "$x = x^2$". If the domain consists of the integers, what are these truth values?

a) $P(0)$ b) $P(1)$ c) $P(2)$

d) $P(-1)$ e) $\exists x P(x)$ f) $\forall x P(x)$

8. Let $Q(x)$ be the statement "$x + 1 > 2x$." If the domain consists of all integers, what are these truth values?

a) $Q(0)$ b) $Q(-1)$ c) $Q(1)$

d) $\exists x Q(x)$ e) $\forall x Q(x)$ f) $\exists x \neg Q(x)$

Predicates and Quantifiers

5 Problems

9. Determine the truth value of each of these statements if the domain consists of all integers.

a) $\forall n(n + 1 > n)$ b) $\exists n(2n = 3n)$ c) $\exists n(n = -n)$ d) $\forall n(3n \leq 4n)$

10. Determine the truth value of each of these statements if the domain consists of all real numbers.

a) $\exists x(x^3 = -1)$ b) $\exists x(x^4 < x^2)$

c) $\forall x((-x)^2 = x^2)$ d) $\forall x(2x > x)$

11. Determine the truth value of each of these statements if the domain for all variables consists of all integers.

a) $\forall n(n^2 \geq 0)$ b) $\exists n(n^2 = 2)$ c) $\forall n(n^2 \geq n)$ d) $\exists n(n^2 < 0)$

12. Suppose that the domain of the propositional function P(x) consists of the integers 0, 1, 2, 3, and 4. Write out each of these propositions using disjunctions, conjunctions, and negations.

a) $\exists x P(x)$ b) $\forall x P(x)$ c) $\exists x \neg P(x)$

d) $\forall x \neg P(x)$ e) $\neg\exists x P(x)$ f) $\neg\forall x P(x)$

13. Suppose that the domain of the propositional function P(x) consists of the integers 1, 2, 3

Rules of Inference

5 Problems

1. Find the argument form for the following argument and determine whether it is valid. Can we conclude that the conclusion is true if the premises are true?

If Socrates is human, then Socrates is mortal.

Socrates is human.

∴ Socrates is mortal.

2. Use rules of inference to show that the hypotheses "Randy works hard", "If Randy works hard, then he is a dull boy", and "If Randy is a dull boy, then he will not get the job" imply the conclusion "Randy will not get the job".

3. For each of these collections of premises, what relevant conclusion or conclusions can be drawn? Explain the rules of inference used to obtain each conclusion from the premises.

a) “If I take the day off, it either rains or snows”. “I took Tuesday off or I took Thursday off”. “It was sunny on Tuesday”. “It did not snow on Thursday”.

b) “If I eat spicy foods, then I have strange dreams”. “I have strange dreams if there is thunder while I sleep”. “I did not have strange dreams”.

c) “I am either clever or lucky”. “I am not lucky”. “If I am lucky, then I will win the lottery”

d) “Every computer science major has a personal computer”. “Ralph does not have a personal computer”. “Ann has a personal computer”.

e) “What is good for corporations is good for the United States”. “What is good for the United States is good for you”. “What is good for corporations is for you to buy lots of stuff”.

f) “All rodents gnaw their food”. “Mice are rodents”. “Rabbits do not gnaw their food”. “Bats are not rodents.

Rules of Inference

5 Problems

4. Determine whether each of the following arguments is valid or not valid.

a) All parrots like fruit. My pet bird is not a parrot. Therefore, my pet bird does not like fruit.

b) Everyone who eats granola every day is healthy. Linda is not healthy. Therefore, Linda does not eat granola every day.

c) No man is an island. Manhattan is an island. Therefore, Manhattan is not a man.

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d) A convertible car is fun to drive. Isaac's car is not a convertible. Therefore, Isaac's car is not fun to drive.

e) If Mai knows French, Mai is smart. But Mai doesn't know French. So, she is not smart.

Q&A

Thank you for listening!