Week 8: Introduction to Operations Research

Operations Research (O.R.), or operational research for the U.K./Europe, "is a discipline that deals with the application of advanced analytical methods to help make better decisions" [INFORMS]. The main focus is to determine maximum (e.g. profit, yield, performance) or minimum (e.g. cost, loss, risk). Various problem solving techniques such as mathematics optimization, simulation, queuing theory, Markov processes, statistics, neural networks, expert systems, and decision analysis are applied to enhance decision-making. Management science and decision science are often used as interchangeable terms. O.R. is the essential part of industrial engineering with similar objectives, techniques, and applications. In addition, O.R. has a strong tie with advanced analytics and computer science.

***A Brief History***

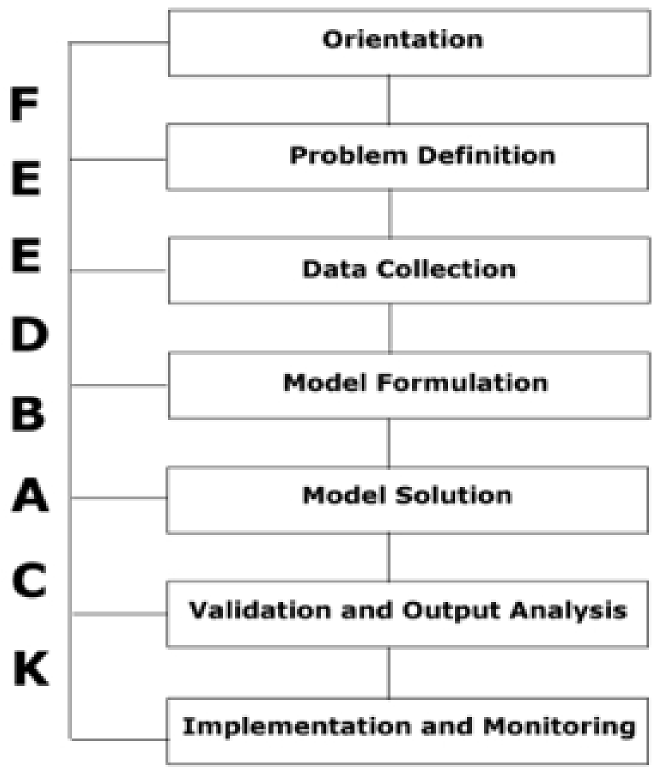
Operations research started in Britain during World War II, where teams of scientists studied the strategy and tactical problems related to military operations such as radar deployment, management of bombing, anti-submarine, and mining operations. The goal was to utilize limited military resources effectively and efficiently by means of quantitative techniques. After the war, numerous applications emerged in many industries. O.R. has been applied to many problems such as scheduling, inventory control, resource allocation, etc. A simplex method for solving linear programming, which was invented by George Dantzing in 1947, helped to ignite growth in O.R.
Following with the simplex method, other O.R. techniques have been developed and still utilized nowadays such as integer programming, dynamic programming, computer simulations, PERT/CPM, queuing theory, game theory, sequencing and scheduling algorithm, etc.

***The Value of Operations Research and Management Scienc***e

http://ocw.mit.edu/courses/sloan-school-of-management/15-053-optimization-methods-in-management-science-spring-2013/lecture-notes/MIT15_053S13_lec1.pdf

- Making sense of data (e.g. big data, social network, polls)
- Dealing with complexity and uncertainty (i.e. making intelligent choices under uncertainty)
- Applying mathematical methods to enhance thinking and develop insights

***The Methodology of Operations Research***




The Operations Research Approach
Source: [http://www.pitt.edu/~jrclass/or/or-intro.html#history](http://www.pitt.edu/~jrclass/or/or-intro.html#history)

1. Problem Orientation
   The objective of this step is to understand the problem and its relationship with respect to the system. This includes studying documents, literature, and adopting the method that successfully solved similar problems in the past before inventing new methods.
2. Problem Definition
   The purpose of this step is to revise the previous step to get precise and specific goals, including scope and the desired results.
3. Data Collection
   Data are collected from different sources such as observation (observe the operational system, surveys, questionnaires, etc.), and standard values that have been recorded in the company such as hourly rate, sell price.
4. Model Formulation
   The model refers to an abstract representation that captures selected characteristics of a system. Building a model requires experience so that all key elements are captured. At the same time, it should be simple enough to do the

analysis. You should be aware of the trade-off between accuracy and manageability of the system detail capturing process.

5. Model Solution
   It is important to know which techniques are applicable to the formulated model. General speaking, most O.R. techniques can be categorized as 1) simulation techniques for simulation models 2) mathematical analysis techniques (not always having clear objectives or constraints) such as regression analysis, statistical inference, ANOVA, queuing, Markov chains, etc. 3) optimum seeking techniques for finding the best values of the decision variables such as linear, nonlinear, integer, dynamic, and stochastic programming 4) heuristics techniques, which are the simple or rule of thumb that do not necessarily guarantee the best solutions but perhaps satisfactory good results. Techniques in this category include genetic algorithms, simulated annealing, evolutionary programming, etc.
6. Validation and Analysis
   This step covers ensuring that the model accurately represents the model (validation), verifying the solution whether it is acceptable, and the robustness of the solutions (e.g. sensitivity analysis).
7. Implement and evaluate recommendation
   The results are conveyed effectively for recommendation and implementation on the system.

***Basic Concepts***
O.R. represents the real world systems using mathematical models together with the quantitative methods for solving the models.
***Mathematical Programming***
The mathematical model consists of decision variables, objective function, constraints, parameters and data. Typically, the model is represented in the following standard format:

$$\text{Max or Min } f(x_1, x_2, \ldots, x_n)$$

$$\text{s.t.} \qquad g_i(x_1, x_2, \ldots, x_n) \begin{Bmatrix} \le \\ = \\ \ge \end{Bmatrix} b_i$$

$$x \in X$$

Terminology:
Decision variables are the quantities in which a decision maker can control and improve the objective, such as work schedules for each employee, level of portfolio investment, etc.

Objective functions (of the decision variable) are the value measure that can be used to rank different alternatives. Typically, the goal is to maximize or minimize the objective.

Constraints are limitations or restrictions on the values of the decision variables. Constraints are business rules (e.g. no more than 5 working days consecutively, prerequisite for a task) or physical laws (e.g. no negative values for time, production amount should be greater than zero)

A feasible solution is the values that are assigned to variables such that all constraints are satisfied. An optimal solution is the best feasible solution.

***Linear Programming (LP)*** The objective function is a linear function. Constraints are linear equalities or inequalities. Usually, a LP has non-negative constraints. But strict inequalities such as ($x > 0$) are not allowed. Many practical problems can be formulated as LP such as work scheduling, production process, oil refinery management, financial planning, inventory model, etc. The objectives for LP are to maximize (e.g. profit) or minimize (e.g. cost) the performance of the system. An algorithm called ***simplex*** method can be used to solve LP problems.

An example of a linear programming problem: Giapetto (from Winston 3.1, p.49)
Giapetto produces 2 wooden toy products: soldiers and trains.

Soldiers sell for $27 each and require $10 of raw materials, $14 of labor and overhead, 2 hours finishing labor, and 1 hour of carpentry labor

Trains sell for $21 each and require $9 of raw materials, $10 of labor and overhead, 1 hour finishing labor, and 1 hour of carpentry labor

Only 100 hours of finishing and 80 hours carpentry are available each week
At most 40 soldiers can be sold each week

How many of each toy should be made each week to maximize the profits?
The above problem is formulated as follows:

- Decision variables: the decision about numbers of soldiers and trains to be made each week.

  $x_1$ the number of soldiers produced each week
  $x_2$ the number of trains produced each week

- Objective function: the function of decision variables that we want to maximize or minimize. Here, the quantity that we want to maximize is the weekly profit.

$$profit = revenue - cost$$

  Revenue = $27x_1 + 21x_2$
  Cost = $(10 + 14)x_1 + (9 + 10)x_2$

  Profit = $(27 - 10 - 14)x_1 + (21 - 9 - 10)x_2$
  = $3x_1 + 2x_2$

- Constraints: the limitations on the decision variables values.

$$2x_1 + x_2 \leq 100 \quad \text{(finishing labor per week)}$$
$$x_1 + x_2 \leq 80 \quad \text{(carpentry labor per week)}$$
$$x_1 \leq 40 \quad \text{(weekend demand for soldiers)}$$
$$x_1, x_2 \geq 0 \text{ (non-negative values for soldiers and trains)}$$

Problem summary:

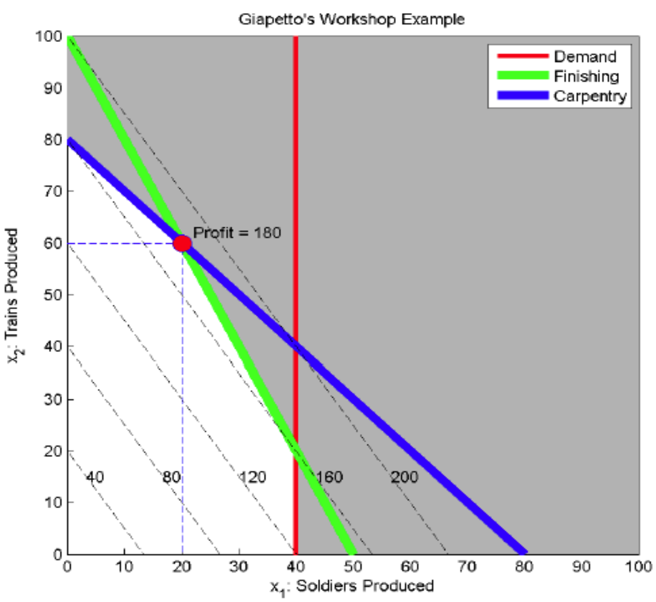
Maximize $3x_1 + 2x_2$

Subject to
$$2x_1 + x_2 \leq 100$$
$$x_1 + x_2 \leq 80$$
$$x_1 \leq 40$$
$$x_1, x_2 \geq 0$$

When solving an LP problem, one of these scenarios can happen:

- The LP has a unique optimal solution
- The LP has multiple optimal solutions
- The LP has no feasible solutions (the feasible region is empty)
- The LP is unbounded (e.g. there are points in the feasible region which is very large for a maximize problem)

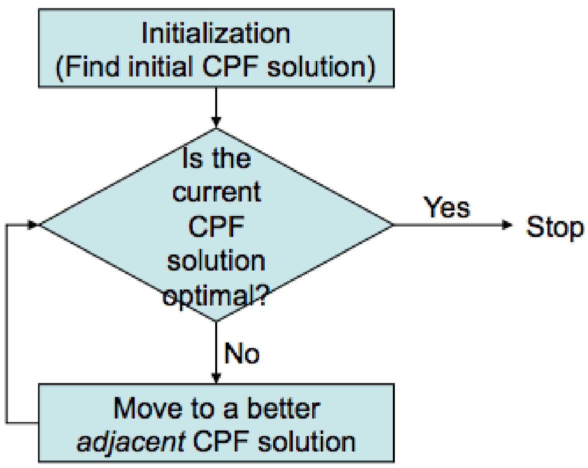
***Solving the LP problem by the graphical solution***:
From the Giapetto problem, there are only two variables so it can be solved graphically. A value of $(x_1, x_2)$ is considered to be in the feasible region if it satisfied all the constraints. An LP is called feasible if the feasible region is not empty. Otherwise, it is called infeasible.




Source: http://www3.nd.edu/~jeff/Teaching/ESTM60203/Lectures/Lecture_01.pdf
The solution for this problem is $(x_1, x_2) = (20{,}60)$. The maximum profit is \$180, which is the optimal solution.

***The Simplex Method***

The simplex method is a popular method for solving linear programming problems. It is an iterative algorithm, based on geometric concepts. Basically, the shape of the polytope (vertices corners of geometric objects) is defined by the constraints. The key idea of the method is moving along the adjacent vertex in the direction that has the better rate of improvement. Then, do the optimality test. If a corner-point feasible solution (CPF) has no adjacent solutions that are better, then it must be an optimal solution.




The Simplex Method in a Nutshell
Source: http://courses.washington.edu/inde310/Simplex%20method.ppt

Using Matlab to solve the Giapetto example can be found at
http://jckantor.github.io/CBE30338/pdf/Ch19_Giapetto_cvx.pdf

Using the above example and try to solve the problem on this website:
http://www.zweigmedia.com/RealWorld/simplex.html

***Solving the LP problems by Computational Solutions***

http://www.statistik.tuwien.ac.at/forschung/CS/CS-2012-1complete.pdf

- Open Source Solvers (examples):

  - GLPK (GNU Linear Programming Kit) is a free and open source software. It is written in ANSI C. A set of routines is organized in the form of a library. It can be used to solve linear programming (LP) and mixed integer programming (MIP) problems. In short, MIP requires that some of the unknown variables are integers, thus it is more general than LP.

  - LpSolve is an open source software to that can be used to solve LP (based on revised simplex method and Branch and bound methods) and MIP problems. It is implemented in C and compatible with Linux and Windows. Many external language interfaces for JAVA, MATLAB, Python, R, etc. are available.

- CBC (Coin-OR Branch and Cut) is a set of routines developed in C++. It can be used as a stand-alone or callable library. It can solve LP and MIP problems.

- SCIP: is a solver for a mixed integer programming based on branch and cut and branch and price framework. It is supported by many operating system platforms such as Linux, Mac, Sun and Windows.

- Commercial Solvers (examples):

- Cplex: is a commercial solver developed by IBM. It is designed to solve large scale (mixed integer) linear problems. It is a popular solver with APIs for several programming languages. It is free for academic usage.

- MATLAB: is matrix-oriented programming language for numerical computation. LP requires MATLAB Optimization Toolbox and routines such as LINPROG, BINTPROG.

- LINDO: is a large scale optimizer for linear programming, integer programming, nonlinear programming, stochastic programming, and global optimization. It has Excel add-in for linear, integer and nonlinear optimization.

- Xpress is a commercial software that available on most computer platforms. It offers interfaces for several programming languages. It also provides a standalone command line interface.

Others proprietary software include Excel Solver function, Mathcad, Mathematica, SAS/OR, Maple, MOSEK, Gurobi, etc.

***Other Terminology:***

***Optimization*** is also known as Mathematical Programming. It is one branch of O.R. that endeavors to optimize the performance of the system using mathematical techniques such as linear and nonlinear programming. Problems for optimization such as choosing a combination of investments to maximize expected return for a given level of risk, design a process to maximize the expected profit, maximize value of advertising, minimize the production cost, etc. Optimization problems are ubiquitous.

***Integer Programming*** is linear programming with some or all variables in the constraints being integer values. It is a NP-hard problem.

***Network Flow Programming*** is a special case of linear programming, where the focus is on utilizing computer time and space resources, for instance, the shortest path problem, the assignment problem, the transportation problem, the minimum cost flow problem.

***Nonlinear Programming*** The objective and/or some constraints are nonlinear. In fact, most real world applications are nonlinear models. However, nonlinear is very difficult to solve. Therefore, nonlinear models are approximated with linear functions.

***Dynamic Programming*** is a method for solving complex problems. In general, the problem is divided into simpler sub-problems. These sub-problems are solved, and the solutions are then combine to get the overall solution.

***Stochastic Processes*** The system attributes randomly change over time such as number of customers at the bank tellers, traffic congestion on a highway, a stock price at the given time, etc. The model is represented by the states of the system, where a state is a snapshot at a point in time of a system. The state is also described the attributes of the system. For instance, consider an automated teller machine system. Here, the state is presented by number of customers waiting for the machine. Events are arrivals and departures of the ATM customers.

***Markov Chains*** is random process, which represent a state transition from one state to another state. It is considered as memoryless, which means than the next state depends on the current state only. There are plethora applications of Markov chains such as physics (e.g. thermodynamics), information sciences (e.g. A mathematical theory of communications by the famous Claude Shannon), queuing theory (e.g. optimize the performance of networks with limited resources), Internet applications (e.g. pagerank by Google), economics and finance (e.g. asset prices model), games (e.g. snakes and ladders), etc.

***Time Series*** is a sequence of observations for a random variable periodically. Examples include course schedules and staff models to estimate the inflow of future students, inventory models to estimate the future demands of customers.

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