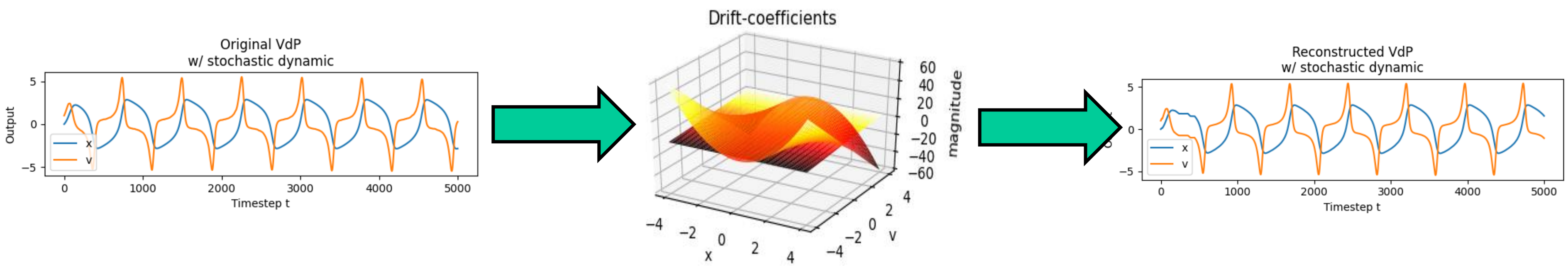
# Model-Reconstruction

**Retrieving system dynamics from it’s time series.**

# Overview




1. Gathering time-series
eg. the Van-der-Pol oscillator

2. Retrieve estimated
drift- & diffusion coefficients

3. Reconstruct the systems
time-series from the dynamics

# Method used

The method used was proposed by J.Gradisek, S.Riegert, R.Freidrich, I.Grabec
in „Analysis of time series from stochastic processes", year 2000 [1]

Drift- and diffusion-coefficients can be directly derived from the time-series
by using their statistical definition: [2]

$$D_i^{(1)} = \lim_{\tau \to 0} \frac{1}{\tau} < X_i(t+\tau) - x_i >_{X(t)=\vec{x}}$$ ← Drift-coefficient

$$D_{ij}^{(2)} = \lim_{\tau \to 0} \frac{1}{\tau} < (X_i(t+\tau) - x_i)(X_j(t+\tau) - x_j) >_{X(t)=\vec{x}}$$ ← Diffusion-coefficient

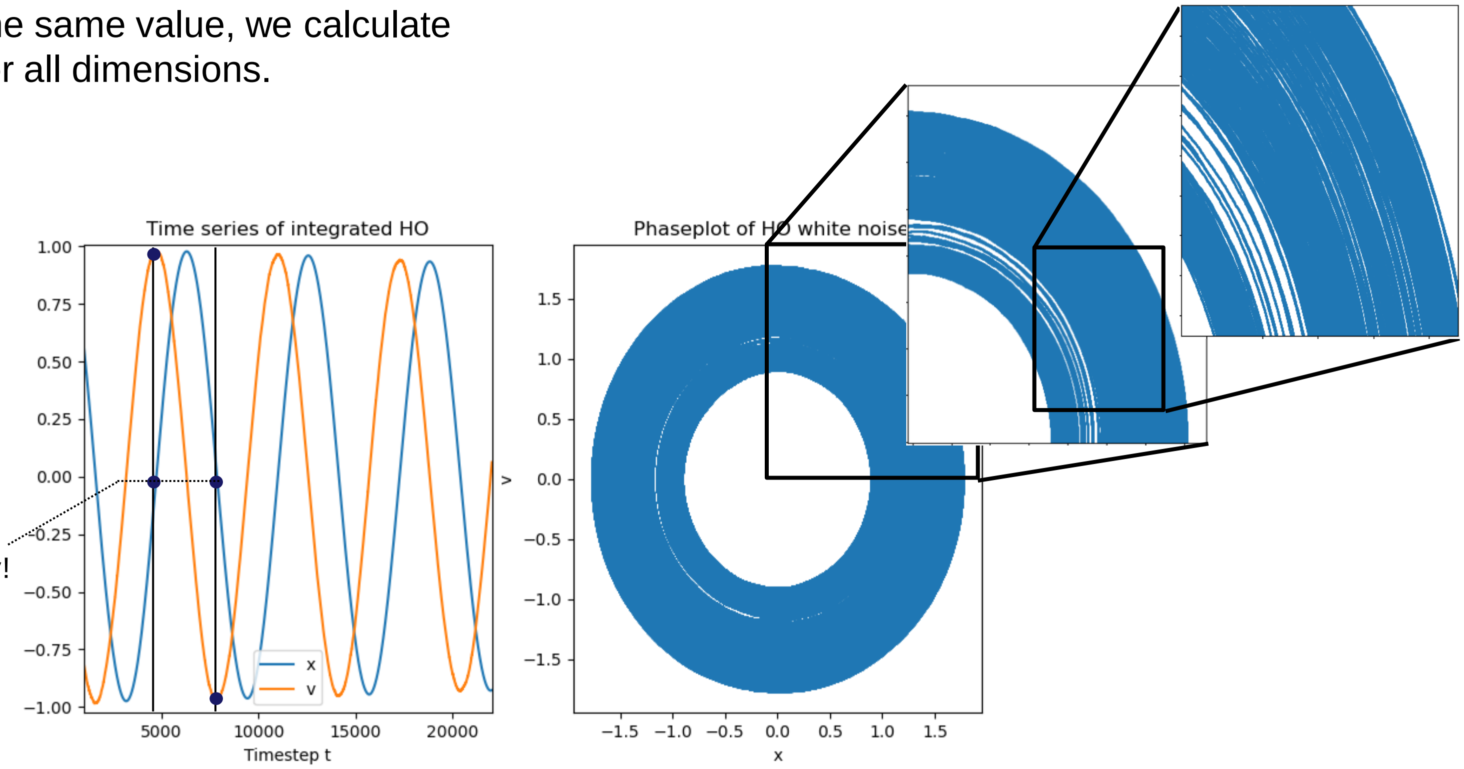
$<\cdot> \rightarrow$ arithmetic mean

For „good" results the process should be exposed to gaussian noise.

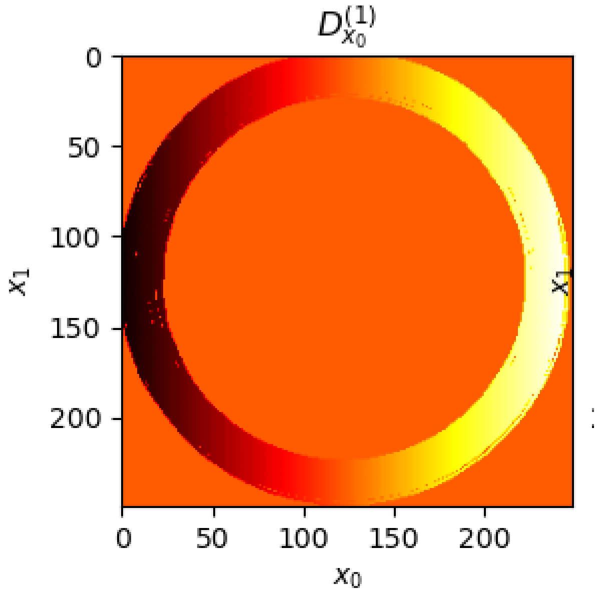
# Method used

For any point in the series, where all dimensions have the same value, we calculate the time-derivate for all dimensions.

Same value in x, but not in v!



# Method used




…receiving an array, where any state of the system $\begin{bmatrix} x \\ v \end{bmatrix}$ maps to the corresponding derivate

Drawbacks:

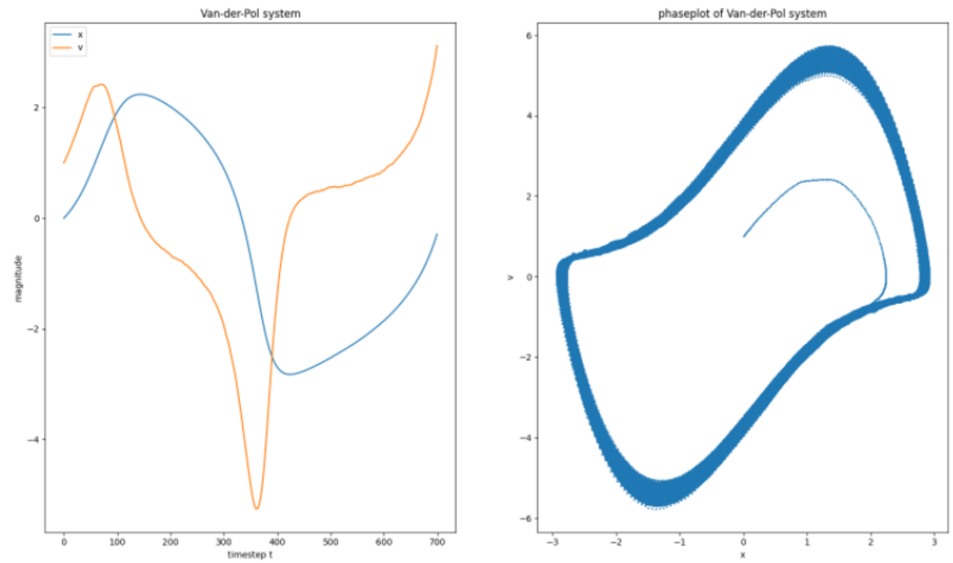
- Method is only applicable on periodic systems
- Method returns only „good“ results if the system is exposed to noise $\Gamma(t)$ with $< \Gamma(t) > = 0$

# Application

Practically applied to a simulated white-noised Van-der-Pol Oscillator.

$$\dot{x_1} = x_2$$
$$\dot{x_2} = \varepsilon(1 - x_1^2)x_2 - \omega^2 x_1 + g\Gamma(t)$$

$$\varepsilon = 2$$
$$\omega = 3$$
$$g = 10$$




Simulated over $100'000$ periods with $\Delta t = 0.01$
So $10'000'000$ timesteps.

# Application

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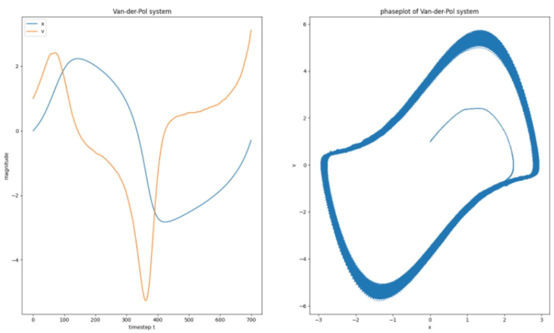
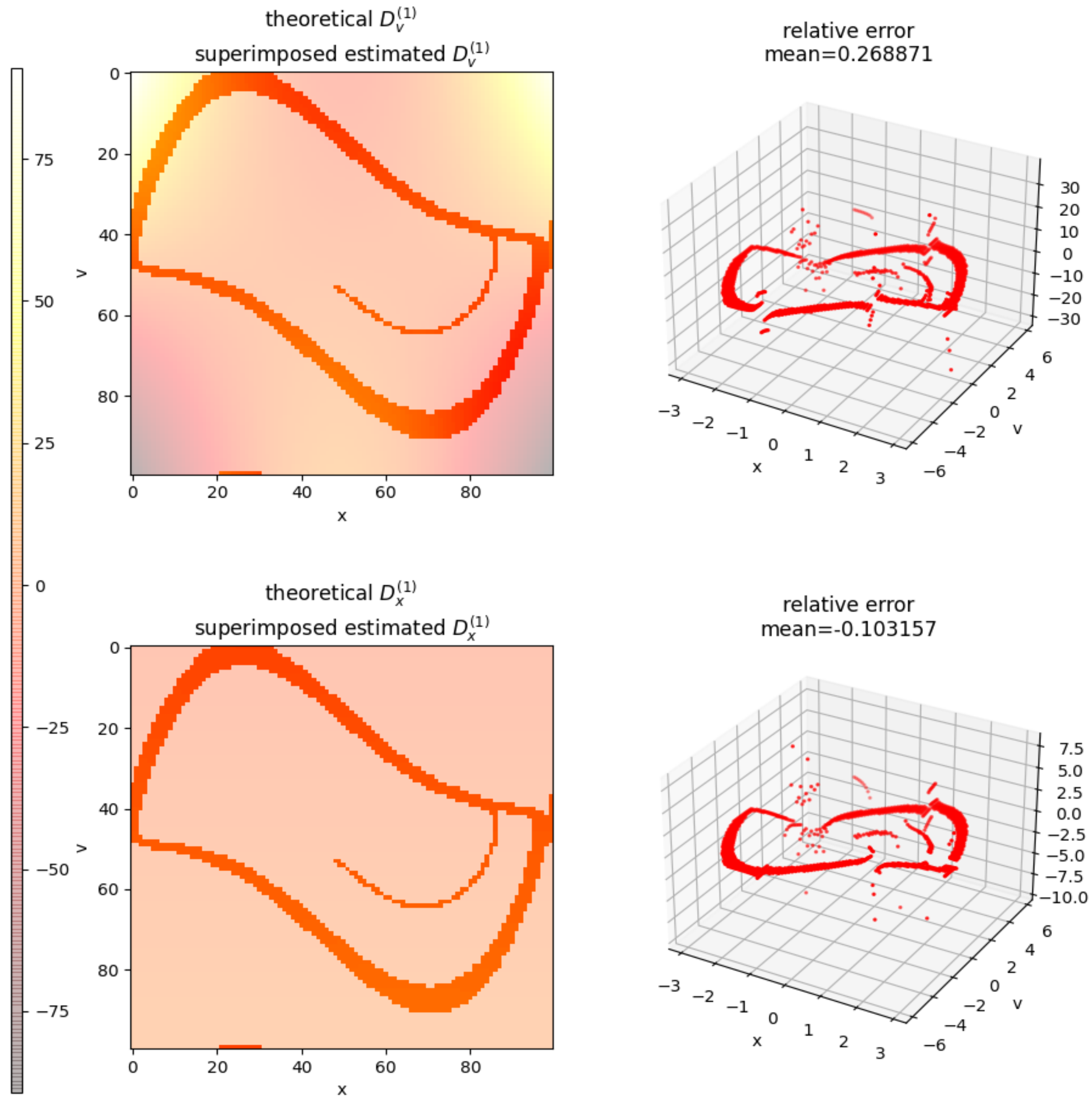


# Reconstruction

With the estimated Drift-coefficient the reconstructioncan be easily done using: [1]

System‘s previous value

Stochastic proportion, noise

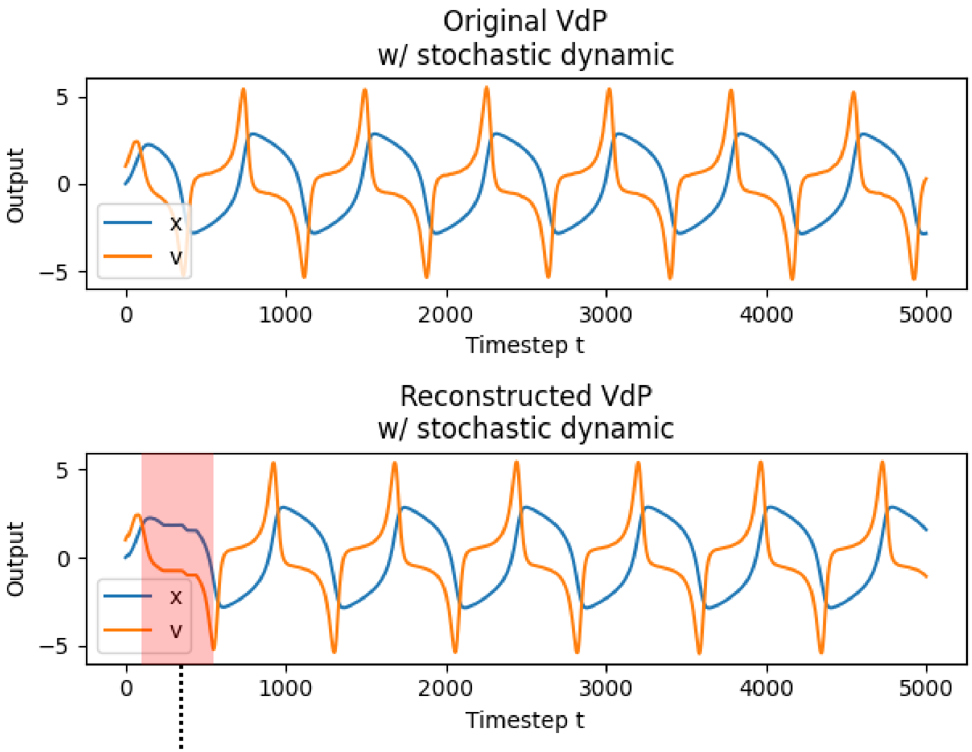
$$X_i(t+\Delta t) = \boxed{X_i(t)} + \boxed{D_i^{(1)}(\vec{X}(t))\Delta t} + \boxed{\sqrt{\Delta t}\sum_{j=1}^{i} g_{ij}(\vec{X}(t))\Gamma_j(t)}$$

Drift-coefficient at the previous state of the system

„Adding to the system‘s current values the estimated mean change at the system‘s state.“

# Reconstruction

$$X_i(t+\Delta t) = X_i(t) + D_i^{(1)}(\vec{X}(t))\Delta t + \sqrt{\Delta t}\sum_{j=1}^{i} g_{ij}(\vec{X}(t))\Gamma_j(t)$$




„Settling process“

# Fokker-Planck-Analysis

To proof the correctness of the estimated coefficients, we use the Fokker-Planck equation (FPE) inserting the estimated coefficients.

In case of the Van-der-Pol oscillator the FPE reads the following:

$$\frac{\partial W(x,\upsilon)}{\partial t} = -\frac{\partial}{\partial x} D_x^{(1)}(x,\upsilon) W(x,\upsilon) - \frac{\partial}{\partial \upsilon} D_\upsilon^{(1)}(x,\upsilon) W(x,\upsilon) + \frac{1}{2}\frac{\partial^2}{\partial \upsilon^2} D_{\upsilon\upsilon}^{(2)} W(x,\upsilon)$$

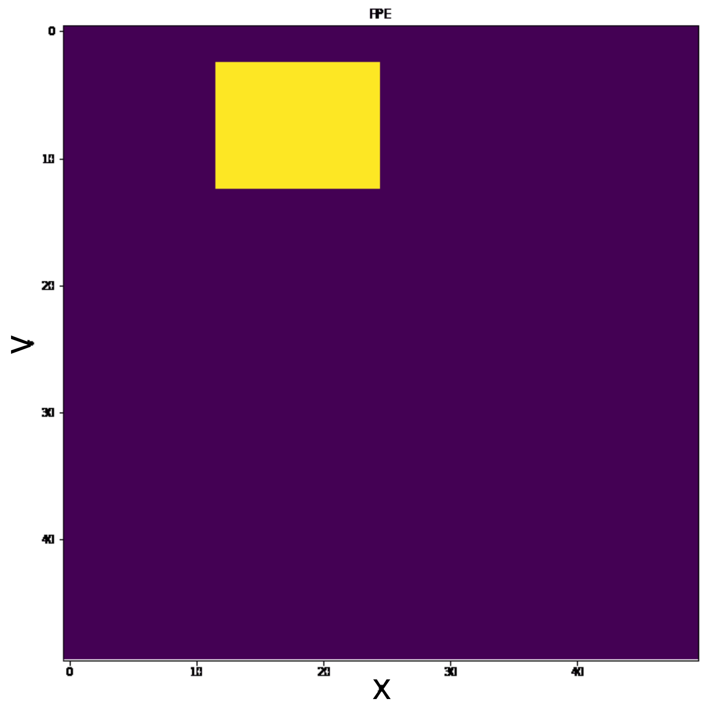
Probability density $W(x,\upsilon)$

Neumann boundary conditions

Problems occured with the „normal“ derivate (the system swinged open), so we used the second-order-upwind scheme to derivate.

# Fokker-Planck-Analysis

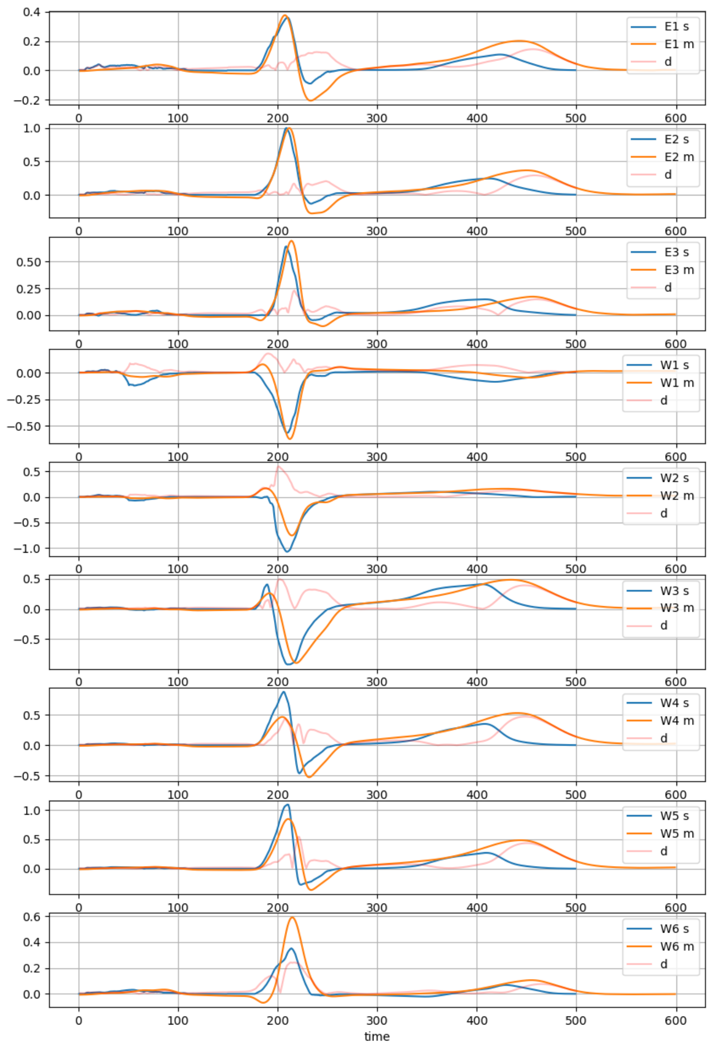
Result with the analytic coefficients of the Van-der-Pol oscillator:




Next steps here:

- Evaluate FPE with estimated coefficients
- Comparing results with an analytical solution

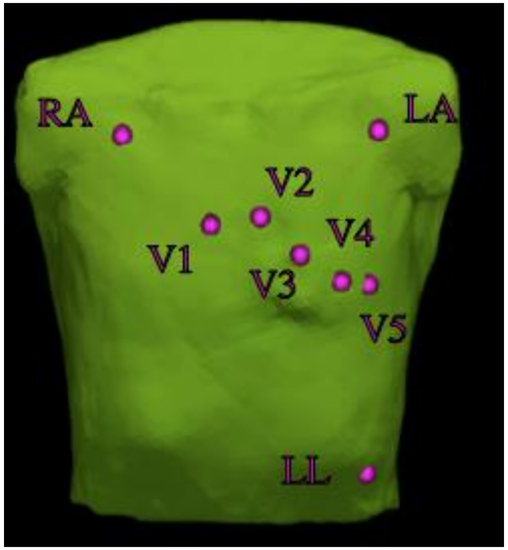
# Outlook




Applying this method to simulated and experimental ECG data.

Data provided by Robin Moss.

Reconstruct the ECG System.




Electrodes positions