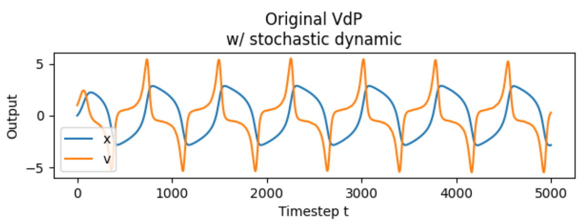
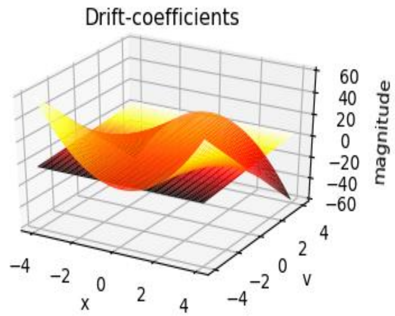
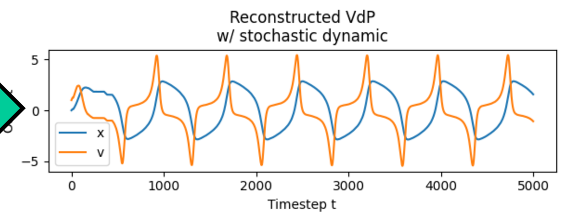
Model-Reconstruction

Retrieving system dynamics from it’s time series.

Overview







1. Gathering time-series
eg. the Van-der-Pol oscillator

2. Retrieve estimated
drift- & diffusion coefficients

3. Reconstruct the systems
time-series from the dynamics

Method used

The method used was proposed by J.Gradisek, S.Riegert, R.Freidrich, I.Grabec
in „Analysis of time series from stochastic processes", year 2000 [1]

Drift- and diffusion-coefficients can be directly derived from the time-series
by using their statistical definition: [2]

$$D_i^{(1)} = \lim_{\tau \to 0} \frac{1}{\tau} < X_i(t+\tau) - x_i >_{X(t)=\vec{x}}$$ ⬅ Drift-coefficient

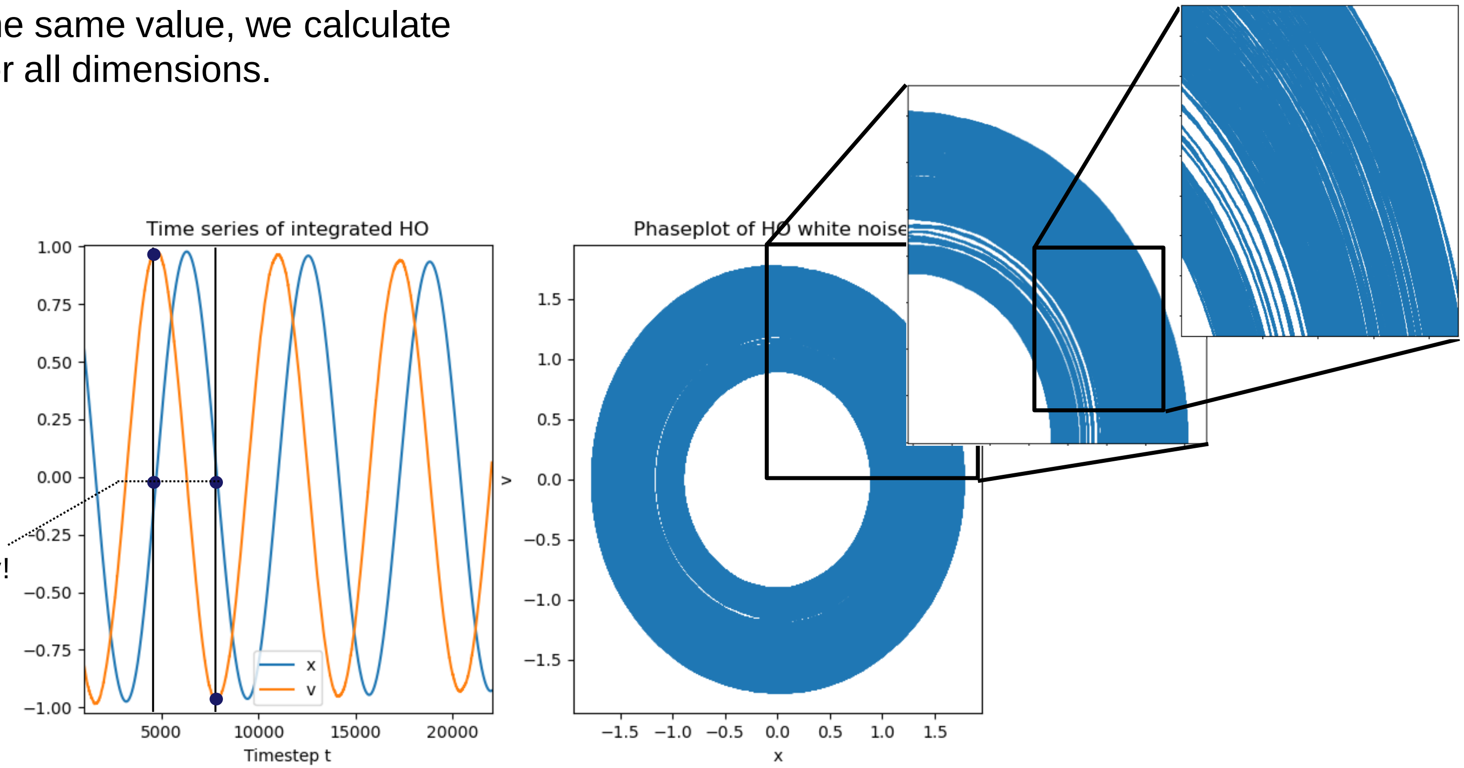
$$D_{ij}^{(2)} = \lim_{\tau \to 0} \frac{1}{\tau} < (X_i(t+\tau) - x_i)(X_j(t+\tau) - x_j) >_{X(t)=\vec{x}}$$ ⬅ Diffusion-coefficient

$<\cdot> \rightarrow$ arithmetic mean

For „good" results the process should be exposed to gaussian noise.

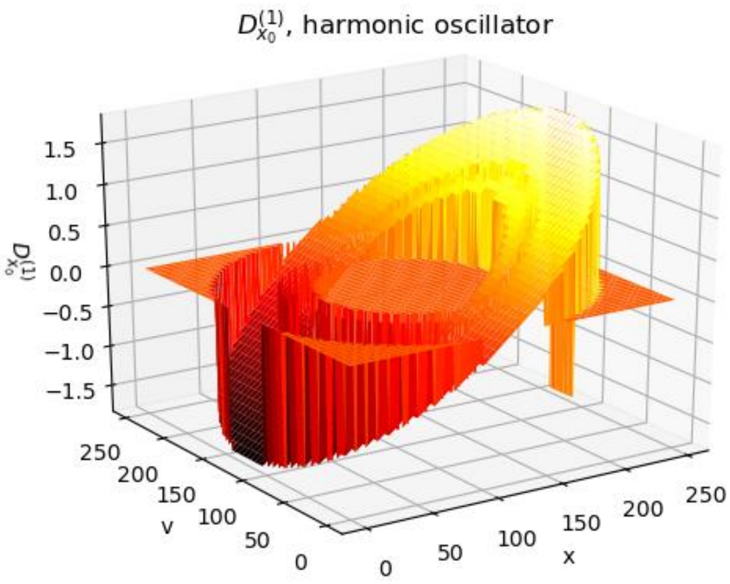
Method used

For any point in the series, where all dimensions have the same value, we calculate the time-derivate for all dimensions.



Same value in x, but not in v!

Method used



…receiving an array, where any state of the system $\begin{bmatrix} x \\ v \end{bmatrix}$ maps to the corresponding derivate

Drawbacks:

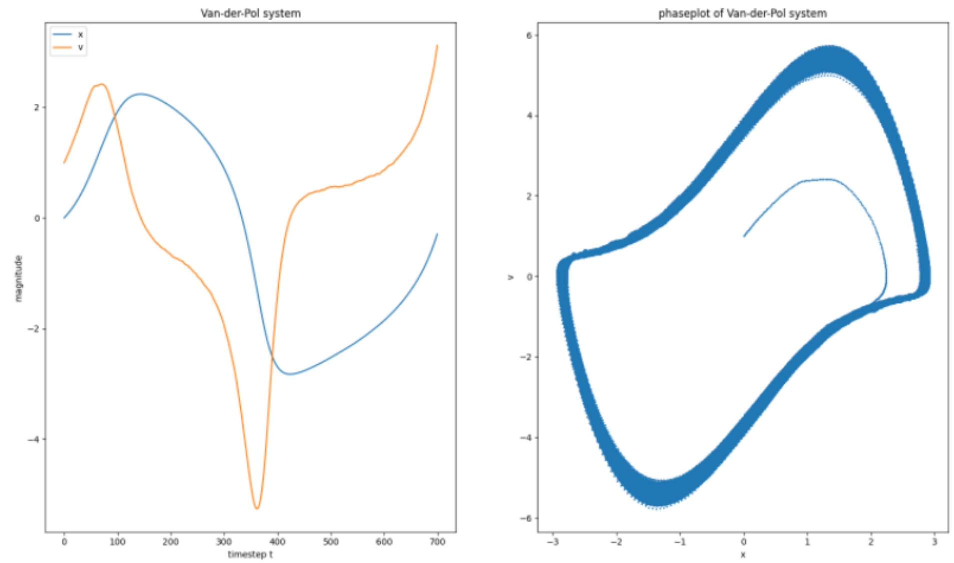
- Method is only applicable on periodic systems
- Method returns only „good" results if the system is exposed to noise $\Gamma(t)$ with $< \Gamma(t) > = 0$

Application

Practically applied to a simulated Van-der-Pol Oscillator.

$$\dot{x_1} = x_2$$
$$\dot{x_2} = \varepsilon(1 - x_1^2)x_2 - \omega^2 x_1 + g\Gamma(t)$$

$$\varepsilon = 2$$
$$\omega = 3$$
$$g = 10$$



Simulated over $100'000$ periods with $\Delta t = 0.01$
So $10'000'000$ timesteps.

Application

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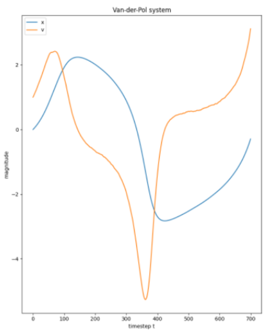
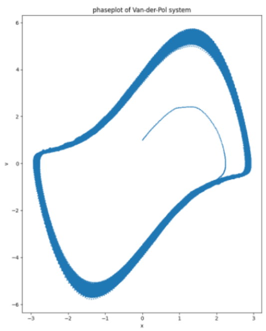
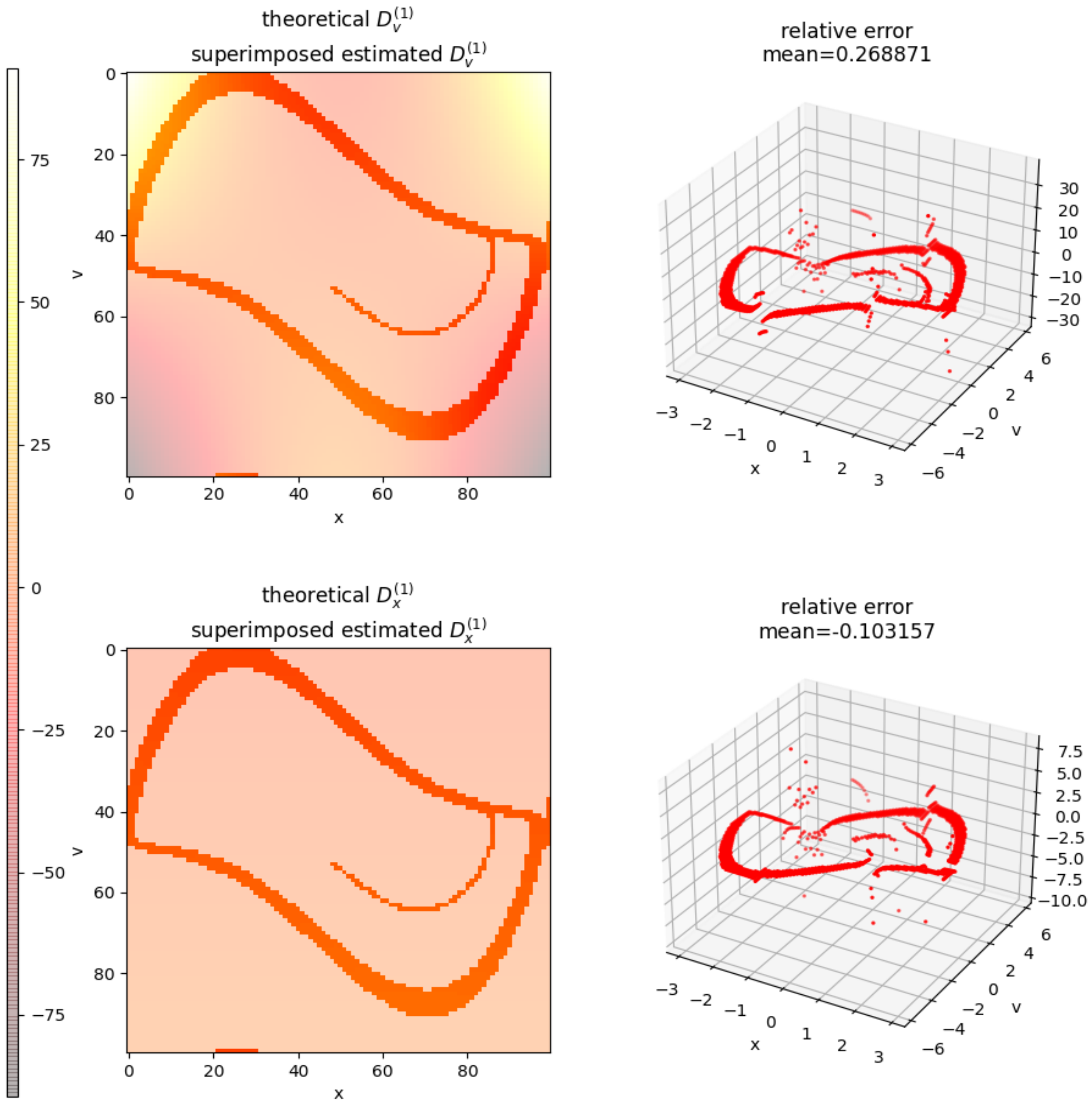






Reconstruction

With the estimated Drift-coefficient the reconstructioncan be easily done using: [1]

System‘s previous value

Stochastic proportion, noise

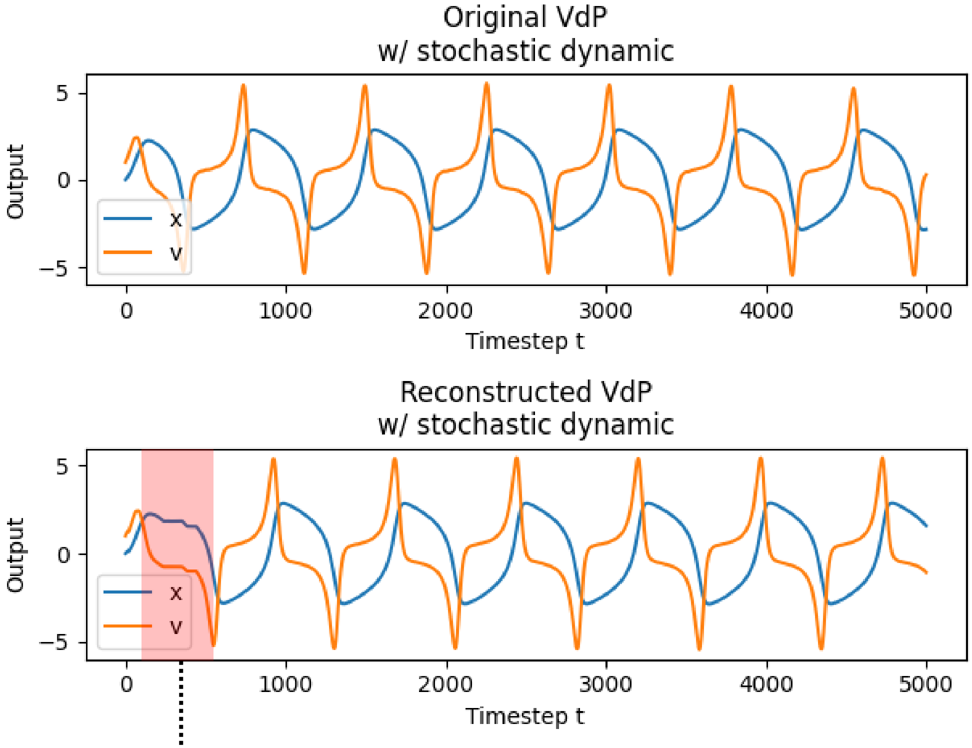
$$X_i(t+\Delta t) = \boxed{X_i(t)} + \boxed{D_i^{(1)}(\vec{X}(t))\Delta t} + \boxed{\sqrt{\Delta t}\sum_{j=1}^{i} g_{ij}(\vec{X}(t))\Gamma_j(t)}$$

Drift-coefficient at the previous state of the system

„Adding to the system‘s current values the estimated mean change at the system‘s state.“

Reconstruction

$$X_i(t+\Delta t) = X_i(t) + D_i^{(1)}(\vec{X}(t))\Delta t + \sqrt{\Delta t}\sum_{j=1}^{i} g_{ij}(\vec{X}(t))\Gamma_j(t)$$



„Settling process“

Fokker-Planck-Analysis

To proof the correctness of the estimated coefficients, we use the Fokker-Planck equation (FPE) inserting the estimated coefficients.

So far only done with analytical coefficients.

In case of the Van-der-Pol oscillator the FPE reads the following:

$$\frac{\partial W(x,\upsilon)}{\partial t} = -\frac{\partial}{\partial x} D_x^{(1)}(x,\upsilon)W(x,\upsilon) - \frac{\partial}{\partial \upsilon} D_\upsilon^{(1)}(x,\upsilon)W(x,\upsilon) + \frac{1}{2}\frac{\partial^2}{\partial \upsilon^2} D_{\upsilon\upsilon}^{(2)} W(x,\upsilon)$$

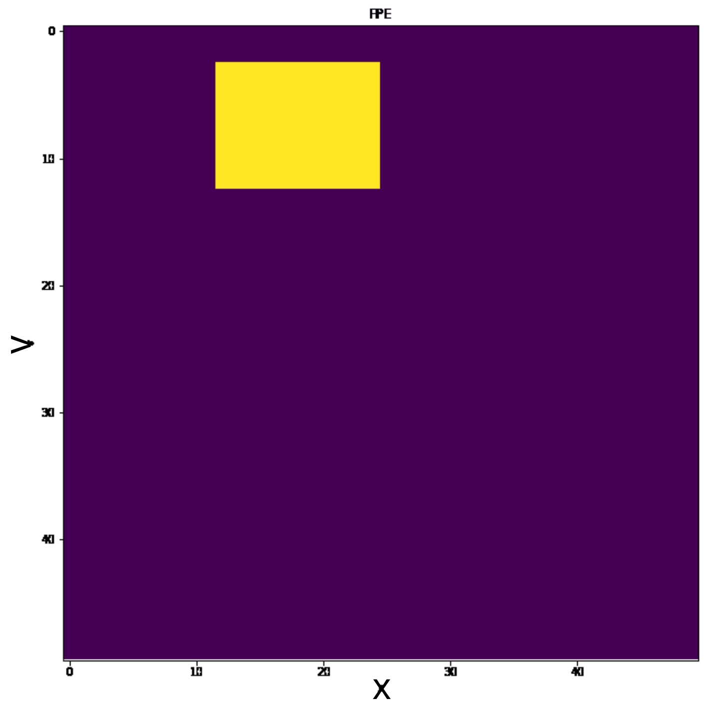
Probability density $W(x,\upsilon)$

Neumann boundary conditions

Problems occured with the „normal“ derivate (the system swinged open), so we used the second-order-upwind scheme to derivate.

Fokker-Planck-Analysis

Result with the analytic coefficients of the Van-der-Pol oscillator:



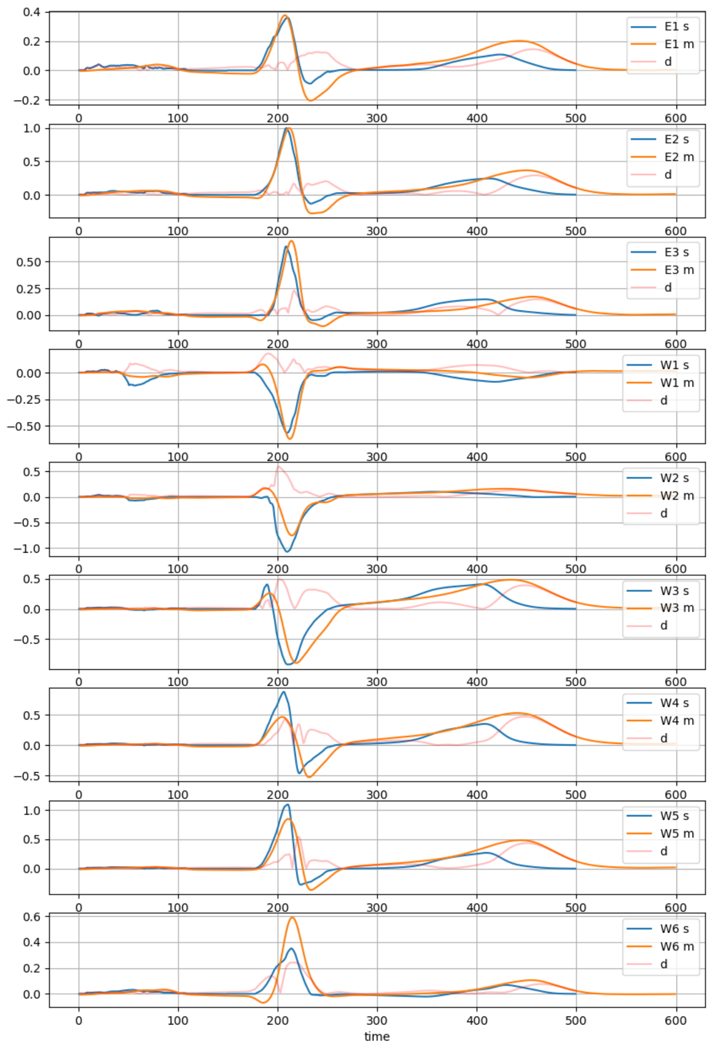
Not clearly visible here: But this results „runs“ over time to infinite values.

Next steps here:

- Evaluate FPE with estimated coefficients
- Comparing results with an analytical solution

AN ANALYTIC FPE SOLUTION WOULD BE ALSO GREAT HERE

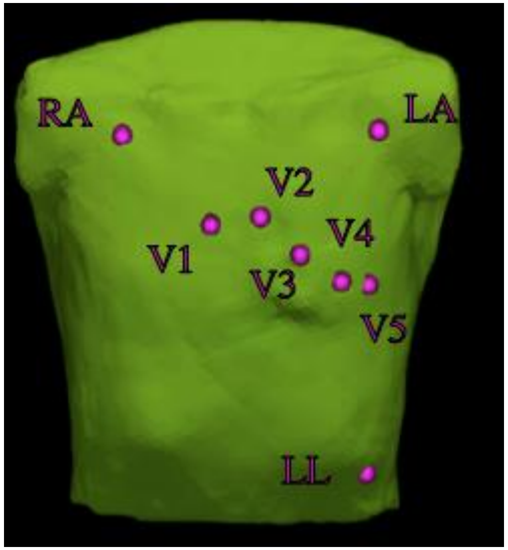
Outlook



Applying this method to simulated and experimental ECG data.

Data provided by Robin Moss.

Reconstruct the ECG System.



Electrodes positions

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