Power-Electronic Control Handbook

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Chapter 1

Preface

This is a collection of technical notes centered around control of power electronics and drives. test.

Chapter 2

Modulation

The modulation described here is based on space-vectors.

2.1 Two-Level Space Vectors

Two-level modulation has six active vectors and two zero-vectors. Each vector is calculated from the general voltage vector formula as here below. The voltage vector subscript denotes the switch level for phases a, b and c, where 1 means the voltage is clamped to high rail of the dc-link voltage and 0 means the voltage is clamped to the low rail.

$$
\begin{aligned}
\underline{v}_{100} &= \frac{2}{3}\left(\frac{V_d}{2} - \frac{V_d}{2}e^{j\frac{2\pi}{3}} - \frac{V_d}{2}e^{j\frac{4\pi}{3}}\right) \\
&= \frac{2}{3}\frac{V_d}{2}\left(1 - (-\frac{1}{2} + j\frac{\sqrt{3}}{2}) - (-\frac{1}{2} - j\frac{\sqrt{3}}{2})\right) \\
&= \frac{2}{3}\frac{V_d}{2}2 \\
&= \frac{2}{3}V_d
\end{aligned}
$$

Similarly, all the vectors are derived as:

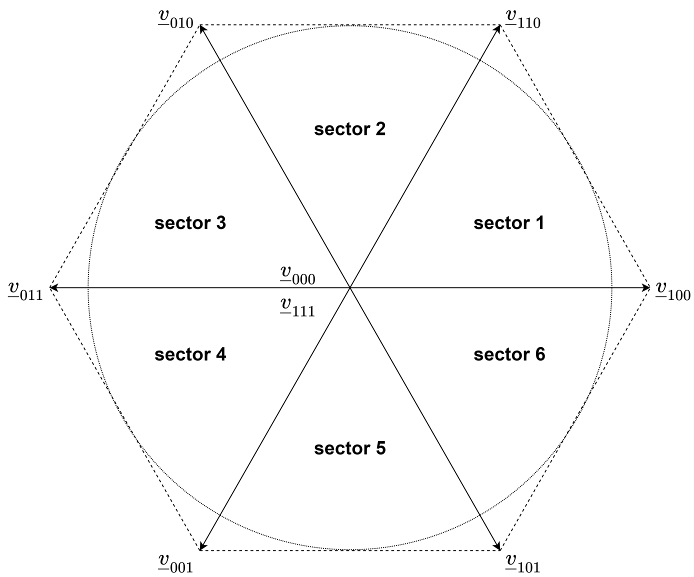

Figure 2.1: Hexagon with the six active vectors and two zero vector.

$$
\begin{aligned}
\underline{v}_{000} &= 0 \\
\underline{v}_{100} &= \frac{2}{3}V_d \\
\underline{v}_{110} &= \frac{2}{3}V_d(\frac{1}{2} + j\frac{\sqrt{3}}{2}) \\
\underline{v}_{010} &= \frac{2}{3}V_d(-\frac{1}{2} + j\frac{\sqrt{3}}{2}) \\
\underline{v}_{011} &= -\frac{2}{3}V_d \\
\underline{v}_{001} &= \frac{2}{3}V_d(-\frac{1}{2} - j\frac{\sqrt{3}}{2}) \\
\underline{v}_{101} &= \frac{2}{3}V_d(\frac{1}{2} - j\frac{\sqrt{3}}{2}) \\
\underline{v}_{111} &= 0
\end{aligned}
$$

2.2 Calculation of duty-cycles.

It is possible realize a voltage vector anywhere inside the hexagon on the figure above. Any reference voltage will be located in one of the six sectors, and as it generally desired to avoid unnecessary switching, the reference vector will be realized with the two neighboring active vectors and zero-vectors by geometrical considerations.

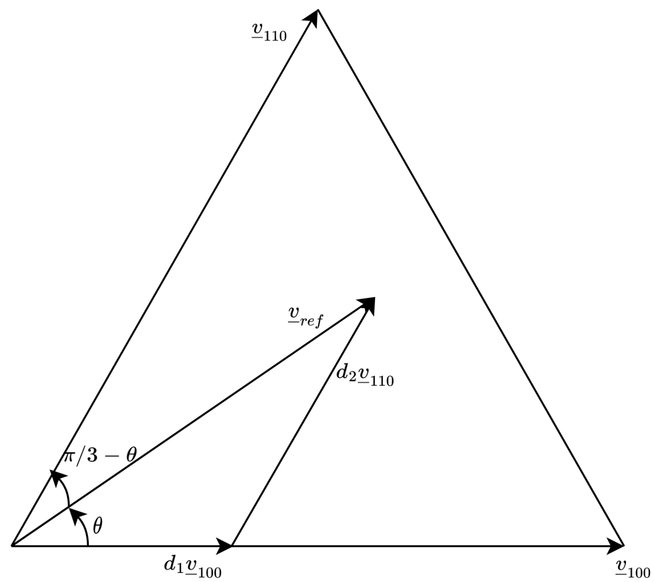

Figure 2.2: A sector of the modulation hexagon.

The duty-cycles are calculated here, from sector 1 on the figure above.

$$\begin{aligned}
\underline{v}_{ref} &= d_1\underline{v}_{100} + d_2\underline{v}_{110} \\
&= d_1\frac{2}{3}V_d + d_2\frac{2}{3}V_d(\frac{1}{2} + j\frac{\sqrt{3}}{2}) \\
&= \frac{2}{3}V_d(d_1 + d_2(\frac{1}{2} + j\frac{\sqrt{3}}{2})) \\
&= \frac{2}{3}V_d((d_1 + d_2\frac{1}{2}) + jd_2\frac{\sqrt{3}}{2})
\end{aligned}$$

Splitting into real and imaginary parts, we get from the imaginary part:

$$\begin{aligned}
|\underline{v}_{ref}|\sin\theta &= \frac{2}{3}V_d d_2\frac{\sqrt{3}}{2} \\
\Rightarrow |\underline{v}_{ref}|\sin\theta &= \frac{2}{3}V_d d_2\frac{\sqrt{3}}{2} \\
\Rightarrow d_2 &= \frac{|\underline{v}_{ref}|}{V_d/\sqrt{3}}\sin\theta
\end{aligned}$$

And from the real part:

$$
\begin{aligned}
|\underline{v}_{ref}|\cos\theta &= \frac{2}{3}V_d(d_1 + d_2\frac{1}{2}) \\
\Rightarrow d_1 &= \frac{3}{2}\frac{|\underline{v}_{ref}|}{V_d}\cos\theta - \frac{1}{2}d_2 \\
\Rightarrow d_1 &= \frac{\sqrt{3}}{2}\frac{|\underline{v}_{ref}|}{V_d/\sqrt{3}}\cos\theta - \frac{1}{2}d_2
\end{aligned}
$$

The modulation index is defined as $m_i = |\underline{v}_{ref}|/(V_d/\sqrt{3})$, so the duty-cycles can be formulated as:

$$
\begin{aligned}
d_2 &= m_i \sin\theta \\
d_1 &= \frac{\sqrt{3}}{2}m_i\cos\theta - \frac{1}{2}d_2
\end{aligned}
$$

Instead of projecting on the real and imaginary axes, we can make a projection onto the axis perpendicular to v_{110}:

$$
\begin{aligned}
d_1|\underline{v}_{110}|\frac{\sqrt{3}}{2} &= |\underline{v}_{ref}|\sin(\frac{\pi}{3} - \theta) \\
\Rightarrow d_1\frac{2}{3}V_d\frac{\sqrt{3}}{2} &= |\underline{v}_{ref}|\sin(\frac{\pi}{3} - \theta) \\
\Rightarrow d_1 &= \frac{|\underline{v}_{ref}|}{V_d/\sqrt{3}}\sin(\frac{\pi}{3} - \theta) \\
\Rightarrow d_1 &= m_i\sin(\frac{\pi}{3} - \theta)
\end{aligned}
$$

This way the duty-cycles can be calculated as:

$$
\begin{aligned}
d_1 &= m_i\sin(\frac{\pi}{3} - \theta) \\
d_2 &= m_i\sin\theta
\end{aligned}
$$

Furthermore, the zero-vector dutycycle is:

$$
d_0 = 1 - d_1 - d_2
$$

2.3 Space Vector Modulation (SVM).

In space vector modulation the active vectors are placed in the middle and the zero-vectors are evenly placed in the beginning and the end. An example is shown here:

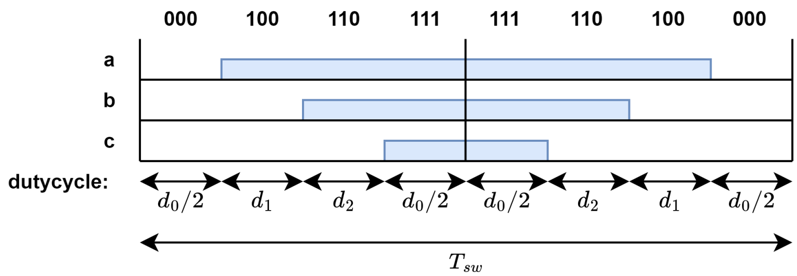

Figure 2.3: Pwm pulses over one switching period for sector 1 with spacevector modulation.

The duty-cycles for phases a, b and c are calculated as:

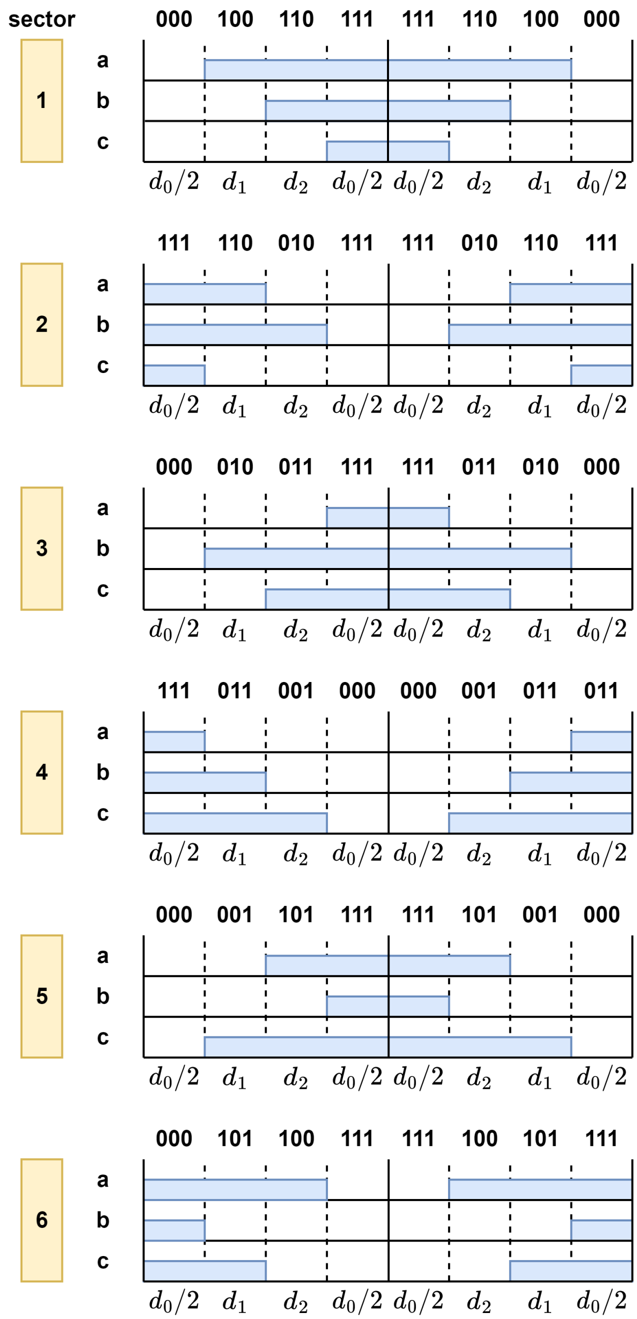

Figure 2.4: Spacevector modulation pulses for all six sectors.

sector 1

$$d_a = d_0/2 + d_2 + d_1$$
$$d_b = d_0/2 + d_2$$
$$d_c = d_0/2$$

sector 2

$$d_a = d_0/2 + d_1$$
$$d_b = d_0/2 + d_1 + d_2$$
$$d_c = d_0/2$$

sector 3

$$d_a = d_0/2$$
$$d_b = d_0/2 + d_2 + d_1$$
$$d_c = d_0/2 + d_2$$

sector 4

$$d_a = d_0/2$$
$$d_b = d_0/2 + d_1$$
$$d_c = d_0/2 + d_1 + d_2$$

sector 5

$$d_a = d_0/2 + d_2$$
$$d_b = d_0/2$$
$$d_c = d_0/2 + d_2 + d_1$$

sector 6

$$d_a = d_0/2 + d_1 + d_2$$
$$d_b = d_0/2$$
$$d_c = d_0/2 + d_1$$

2.4 Duty-cycle function expression.

The duty-cycles are reformulated for sector 1, by inserting $d_0 = 1 - d_1 - d_2$ in the expression above:

$$d_a = \frac{1}{2}(1 + d_1 + d_2)$$
$$d_b = \frac{1}{2}(1 - d_1 + d_2)$$
$$d_c = \frac{1}{2}(1 - d_1 - d_2)$$

For sector 1 we get:

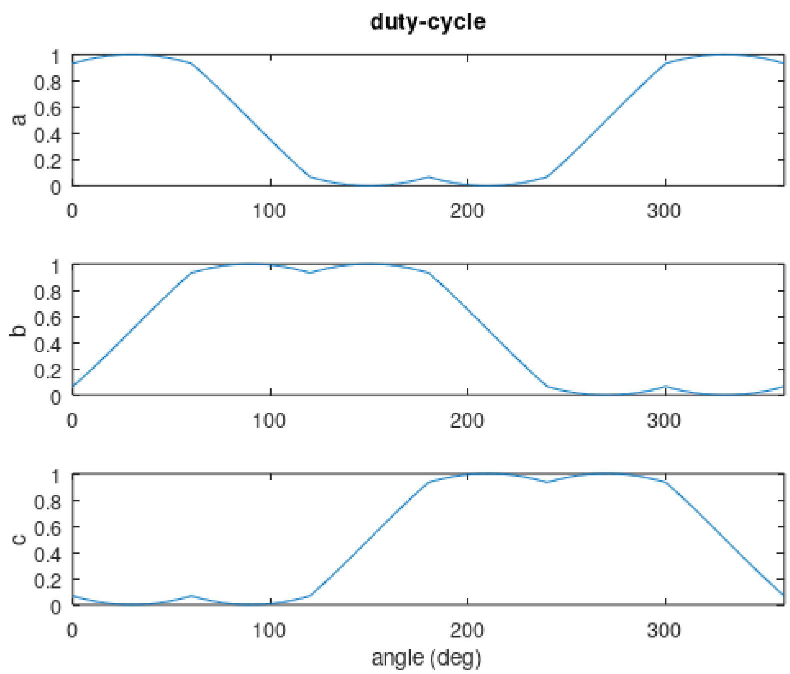

Figure 2.5: Spacevector modulation duty-cycle for phase a, b, and c, for modulation index = 1.

$$
\begin{aligned}
d_a &= \frac{1}{2}(1 + d_1 + d_2) \\
&= \frac{1}{2}(1 + (\frac{\sqrt{3}}{2} m_i \cos\theta - \frac{1}{2} d_2) + d_2) \\
&= \frac{1}{2}(1 + \frac{\sqrt{3}}{2} m_i \cos\theta + \frac{1}{2} m_i \sin\theta) \\
&= \frac{1}{2} + \frac{1}{2} m_i (\frac{\sqrt{3}}{2} \cos\theta + \frac{1}{2} \sin\theta)
\end{aligned}
$$

$$
\begin{aligned}
d_b &= \frac{1}{2}(1 - d_1 + d_2) \\
&= \frac{1}{2}(1 - (\frac{\sqrt{3}}{2} m_i \cos\theta - \frac{1}{2} d_2) + d_2) \\
&= \frac{1}{2}(1 - \frac{\sqrt{3}}{2} m_i \cos\theta + \frac{3}{2} m_i \sin\theta) \\
&= \frac{1}{2} - \frac{1}{2} m_i (\frac{\sqrt{3}}{2} \cos\theta - \frac{3}{2} \sin\theta)
\end{aligned}
$$

$$
\begin{aligned}
d_c &= \frac{1}{2}(1 - d_1 - d_2) \\
&= \frac{1}{2}(1 - (\frac{\sqrt{3}}{2} m_i \cos\theta - \frac{1}{2} d_2) - d_2) \\
&= \frac{1}{2}(1 - \frac{\sqrt{3}}{2} m_i \cos\theta - \frac{1}{2} m_i \sin\theta) \\
&= \frac{1}{2} - \frac{1}{2} m_i (\frac{\sqrt{3}}{2} \cos\theta + \frac{1}{2} \sin\theta)
\end{aligned}
$$

2.5 Calculation of duty-cycle from phase-voltage references.

As an alternative to calculate the duty-cycles from the space-vector, they can be calculated from the phase-voltage references directly.

$$
\begin{aligned}
d_a &= \frac{1}{2}\left(\frac{(v_{ref,a} - v_{cm})}{V_d/2} + 1\right) = \frac{1}{2} + \frac{1}{V_d}(v_{ref,a} - v_{cm}) \\
d_b &= \frac{1}{2}\left(\frac{(v_{ref,b} - v_{cm})}{V_d/2} + 1\right) = \frac{1}{2} + \frac{1}{V_d}(v_{ref,b} - v_{cm}) \\
d_c &= \frac{1}{2}\left(\frac{(v_{ref,c} - v_{cm})}{V_d/2} + 1\right) = \frac{1}{2} + \frac{1}{V_d}(v_{ref,c} - v_{cm})
\end{aligned}
$$

where: $v_{ref,a}$: reference voltage, phase a, phase-zero voltage.
$v_{ref,b}$: reference voltage, phase b, phase-zero voltage.

$v_{ref,c}$: reference voltage, phase c, phase-zero voltage.
v_{cm}: common-mode voltage.
V_d: Dc-link voltage.
d_a: duty-cycle, phase a, range 0-1.
d_b: duty-cycle, phase b, range 0-1.
d_c: duty-cycle, phase c, range 0-1.

The common-mode voltage can have any shape, and space-vector modulation is achieved with this common-mode signal, which is the mean value of the max and min phase voltage references:

$$v_{cm} = \frac{1}{2}(\min(v_a, v_b, v_c) + \max(v_a, v_b, v_c))$$

We can now show that this does give the same result as the previously derived expressions. As an example of this, we calculate the duty-cycle for phase a in sector 1. From the figure below we can see that phase a is the highest and phase c the lowest.

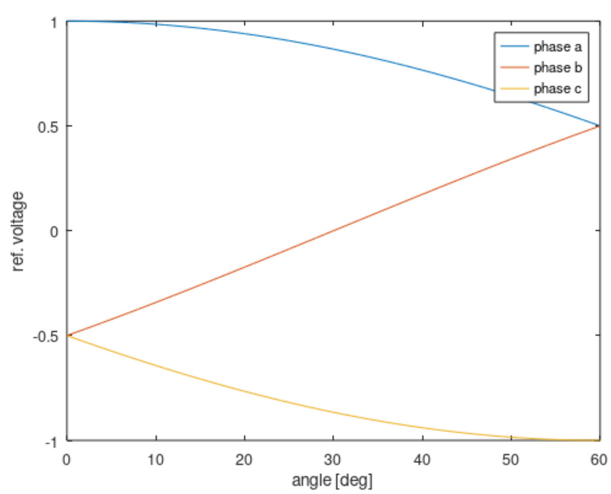

Figure 2.6: Voltage references for phase a, b, and c.

$$
\begin{aligned}
v_a - v_{cm} &= |\underline{v}_{ref}| \left(\cos\theta - \frac{1}{2}(\cos\theta + \cos{(\theta - 4\pi/3)}) \right) \\
&= |\underline{v}_{ref}| \left(\cos\theta - \frac{1}{2}(\cos\theta + \cos{(4\pi/3)}\cos\theta + \sin{(4\pi/3)}\sin\theta) \right) \\
&= |\underline{v}_{ref}| \left(\cos\theta - \frac{1}{2}(\cos\theta - \frac{1}{2}\cos\theta - \frac{\sqrt{3}}{2}\sin\theta) \right) \\
&= |\underline{v}_{ref}| (\frac{3}{4}\cos\theta + \frac{\sqrt{3}}{4}\sin\theta)
\end{aligned}
$$

Now the duty-cycle can be derived as:

$$
\begin{aligned}
d_a &= \frac{1}{2} + \frac{1}{V_d}(v_{ref,a} - v_{cm}) \\
&= \frac{1}{2} + \frac{1}{V_d}|\underline{v}_{ref}|(\frac{3}{4}\cos\theta + \frac{\sqrt{3}}{4}\sin\theta) \\
&= \frac{1}{2} + \frac{1}{2}\frac{|\underline{v}_{ref}|}{V_d/\sqrt{3}}(\frac{\sqrt{3}}{2}\cos\theta + \frac{1}{2}\sin\theta) \\
&= \frac{1}{2} + \frac{1}{2}m_i(\frac{\sqrt{3}}{2}\cos\theta + \frac{1}{2}\sin\theta)
\end{aligned}
$$

In the similar way, the expression can be derived for the other phases and other sectors.

Chapter 3

Induction Machine

3.1 Nomenclature

$\underline{v}_s$: Stator voltage vector.
$\underline{v}_r$: Rotor voltage vector.
R_s: Stator resistance.
R_r: Rotor resistance.
$\underline{\psi}_s$: Stator flux linkage.
$\underline{\psi}_r$: Rotor flux linkage.
ω_c: Angular frequency of reference frame.
ω_m: Angular frequency of shaft in electrical frame, i.e compensated for pole-pairs.
L_s: Stator inductance.
L_r: Rotor inductance.
L_m: Magnetizing inductance.
L_m: Magnetizing inductance.
$L_{s\sigma}$: Stator leakage inductance.
$L_{r\sigma}$: Rotor leakage inductance.
$1 - \frac{L_m^2}{L_s L_r}$: total leakage coefficient.

3.2 Machine Equations

3.2.1 Voltage Equations

$$\underline{v}_s = R_s \underline{i}_s + \frac{d\underline{\psi}_s}{dt} + j\omega_c \psi_s$$
$$\underline{v}_r = R_r \underline{i}_r + \frac{d\underline{\psi}_r}{dt} + j(\omega_c - \omega_m)\psi_s$$

3.2.2 Stator Equation

The stator equation in the stator reference frame is:

$$\underline{v}_s = R_s \underline{i}_s + \frac{d\underline{\psi}_s}{dt}$$

We want to express this in a general reference frame, where ρ_s is the angle from the stator fixed frame to the general reference frame. With the superscripts denoting the reference frames, s for stator, g for general, we can write:

$$\begin{aligned}
\underline{v}_s^s &= \underline{v}_s^g e^{j\rho_s} \\
\underline{i}_s^s &= \underline{i}_s^g e^{j\rho_s} \\
\underline{\psi}_s^s &= \underline{\psi}_s^g e^{j\rho_s} \\
\frac{d\underline{\psi}_s^s}{dt} &= \frac{d(\underline{\psi}_s^g e^{j\rho_s})}{dt} = \frac{d\underline{\psi}_s^g}{dt} e^{j\rho_s} + \underline{\psi}_s^g j\omega_g e^{j\rho_s}, \omega_g = \frac{d\rho_s}{dt}
\end{aligned}$$

By inserting these in the stator equation above, we get:

$$\begin{aligned}
\underline{v}_s^s &= R_s \underline{i}_s^s + \frac{d\underline{\psi}_s^s}{dt} \\
\underline{v}_s^g e^{j\rho_s} &= R_s \underline{i}_s^g e^{j\rho_s} + \frac{d\underline{\psi}_s^g}{dt} e^{j\rho_s} + \underline{\psi}_s^g j\omega_g e^{j\rho_s} \\
\underline{v}_s^g &= R_s \underline{i}_s^g + \frac{d\underline{\psi}_s^g}{dt} + j\underline{\psi}_s^g \omega_g
\end{aligned}$$

3.2.3 Rotor Equation

The rotor equation in the rotor reference frame is:

$$\underline{v}_r = R_r \underline{i}_r + \frac{d\underline{\psi}_r}{dt}$$

We want to express this in a general reference frame, where ρ_r is the angle from the rotor fixed frame to the general reference frame. With the superscripts denoting the reference frames, r for rotor, g for general, we can write:

$$\begin{aligned}
\underline{v}_r^r &= \underline{v}_r^g e^{j\rho_r} \\
\underline{i}_r^r &= \underline{i}_r^g e^{j\rho_r} \\
\underline{\psi}_r^r &= \underline{\psi}_r^g e^{j\rho_r} \\
\frac{d\underline{\psi}_r^r}{dt} &= \frac{d(\underline{\psi}_r^g e^{j\rho_r})}{dt} = \frac{d\underline{\psi}_r^g}{dt} e^{j\rho_r} + \underline{\psi}_r^g j(\omega_g - \omega_r) e^{j\rho_r}, \omega_g - \omega_r = \frac{d\rho_r}{dt}
\end{aligned}$$

By inserting these in the rotor equation above, we get:

$$\begin{aligned}
\underline{v}_r^r &= R_r \underline{i}_r^r + \frac{d\underline{\psi}_r^r}{dt} \\
\underline{v}_r^g e^{j\rho_r} &= R_r \underline{i}_r^g e^{j\rho_r} + \frac{d\underline{\psi}_r^g}{dt} e^{j\rho_r} + \underline{\psi}_r^g j(\omega_g - \omega_r) e^{j\rho_r} \\
\underline{v}_r^g &= R_r \underline{i}_r^g + \frac{d\underline{\psi}_r^g}{dt} + j\underline{\psi}_r^g (\omega_g - \omega_r)
\end{aligned}$$

3.3 Voltage equations in a rotating dq-reference frame

$$\begin{aligned}
\underline{v}_s &= R_s \underline{i}_s + \frac{d\underline{\psi}_s}{dt} + j\underline{\psi}_s \omega_{dq} \\
\underline{v}_r &= R_r \underline{i}_r + \frac{d\underline{\psi}_r}{dt} + j\underline{\psi}_r (\omega_{dq} - \omega_r)
\end{aligned}$$

3.3.1 Flux Equations

$$\begin{bmatrix} \psi_s \\ \psi_r \end{bmatrix} = \begin{bmatrix} L_s & L_m \\ L_m & L_r \end{bmatrix} \begin{bmatrix} i_s \\ i_r \end{bmatrix}$$

$$L_s = L_m + L_{s\sigma}$$

$$L_r = L_m + L_{r\sigma}$$

3.4 Torque Equations

The electrical torque can be expressed in terms of cross-product as:

$$T_e = -\frac{3}{2} N_{pp} L_m (\underline{i}_s \times \underline{i}_r)$$

3.5 Rotor-flux Oriented Model

For the rotor-flux Oriented Model, the d-axis is oriented with the rotor flux, so the rotor-flux has no q-axis component.

The rotor magnetizing current $\underline{i}_{mr}$ is defined as:

$$\underline{i}_{mr} = \frac{\underline{\psi}_r}{L_m} = \frac{L_m \underline{i}_s + L_r \underline{i}_r}{L_m} = \underline{i}_s + \frac{L_r}{L_m}\underline{i}_r$$

$$\underline{i}_r = \frac{L_m}{L_r}(\underline{i}_{mr} - \underline{i}_s)$$

The stator flux can be calculated as:

$$\begin{aligned}
\underline{\psi}_s &= L_s \underline{i}_s + L_m \underline{i}_r \\
&= L_s \underline{i}_s + L_m \frac{L_m}{L_r}(\underline{i}_{mr} - \underline{i}_s) \\
&= \frac{L_m^2}{L_r}\underline{i}_{mr} + L_s \underline{i}_s - \frac{L_m^2}{L_r}\underline{i}_s \\
&= \frac{L_m^2}{L_r}\underline{i}_{mr} + L_s(1 - \frac{L_m^2}{L_s L_r})\underline{i}_s \\
&= \frac{L_m^2}{L_r}\underline{i}_{mr} + \sigma L_s \underline{i}_s
\end{aligned}$$

the leakage constant being defined as: $\sigma = 1 - \frac{L_m^2}{L_s L_r}$

3.5.1 Rotor voltage equation (rotor-flux oriented)

$$\begin{aligned}
\underline{v}_r^{mr} &= R_r \underline{i}_r^{mr} + \frac{d\underline{\psi}_r^{mr}}{dt} + j(\omega_{mr} - \omega_r)\underline{\psi}_r^{mr} \\
&= R_r \frac{L_m}{L_r}(\underline{i}_{mr} - \underline{i}_s) + \frac{d(L_m \underline{i}_{mr})}{dt} + j(\omega_{mr} - \omega_r) L_m \underline{i}_{mr} \\
&= -\underline{i}_s R_r \frac{L_m}{L_r} + L_m \frac{d\underline{i}_{mr}}{dt} + R_r \frac{L_m}{L_r}\underline{i}_{mr} + j(\omega_{mr} - \omega_r) L_m \underline{i}_{mr}
\end{aligned}$$

$$\begin{aligned}
\frac{L_r}{R_r L_m}\underline{v}_r^{mr} &= -\underline{i}_s^{mr} + \frac{L_r}{R_r}\frac{d\underline{i}_{mr}}{dt} + \underline{i}_{mr} + j(\omega_{mr} - \omega_r)\frac{L_r}{R_r}\underline{i}_{mr} \\
\frac{L_r}{R_r L_m}\underline{v}_r^{mr} + \underline{i}_s^{mr} &= \frac{L_r}{R_r}\frac{d\underline{i}_{mr}}{dt} + \underline{i}_{mr} + j(\omega_{mr} - \omega_r)\frac{L_r}{R_r}\underline{i}_{mr}
\end{aligned}$$

$$\frac{L_r}{R_r L_m} v_{rd} + i_{sd} = \frac{L_r}{R_r} \frac{di_{mr}}{dt} + i_{mr}$$
$$\frac{L_r}{R_r L_m} v_{rq} + i_{sq} = (\omega_{mr} - \omega_r) \frac{L_r}{R_r} i_{mr}$$

3.5.2 stator voltage equation (rotor-flux oriented)

$$\begin{aligned}
\underline{v}_s^{mr} &= R_s \underline{i}_s^{mr} + \frac{d\underline{\psi}_s^{mr}}{dt} + j\omega_{mr} \underline{\psi}_s^{mr} \\
&= R_s \underline{i}_s^{mr} + \frac{d}{dt}(\frac{L_m^2}{L_r} \underline{i}_{mr}^{mr} + \sigma L_s \underline{i}_s) + j\omega_{mr}(\frac{L_m^2}{L_r} \underline{i}_{mr}^{mr} + \sigma L_s \underline{i}_s) \\
&= R_s \underline{i}_s^{mr} + \frac{d}{dt}((1-\sigma) L_s i_{mr} + \sigma L_s \underline{i}_s) + j\omega_{mr}((1-\sigma) L_s i_{mr} + \sigma L_s \underline{i}_s)
\end{aligned}$$

$$v_{sd} = R_s i_{sd} + \sigma L_s \frac{di_{sd}}{dt} + (1-\sigma) L_s \frac{di_{mr}}{dt} - \omega_{mr} \sigma L_s i_{sq}$$
$$v_{sq} = R_s i_{sq} + \sigma L_s \frac{di_{sq}}{dt} + \omega_{mr}((1-\sigma) L_s i_{mr} + \sigma L_s i_{sd})$$