# galois

# Exercise 1:

```
Salsa20.cry:
 quarterround: [4][32] -> [4][32]
 quarterround [y0, y1, y2, y3] = [z0, z1, z2, z3]
     z1 = y1 \wedge ((y0 + y3) <<< 0x7)
     z2 = y2 \wedge ((z1 + y0) <<< 0x9)
     z3 = y3 \wedge ((z2 + z1) <<< 0xd)
     z0 = v0 \wedge ((z3 + z2) <<< 0x12)
 rowround: [16][32] -> [16][32]
 rowround [y0, y1, y2, y3, y4, y5, y6, y7, y8, y9, y10, y11, y12, y13, y14, y15] =
           [z0, z1, z2, z3, z4, z5, z6, z7, z8, z9, z10, z11, z12, z13, z14, z15]
   where
                        = quarterround [y0, y1, y2, y3]
     [z0, z1, z2, z3]
     [z5, z6, z7, z4]
                        = quarterround [y5,y6,y7,y4]
     [z10,z11,z8,z9] = quarterround [y10,y11,y8,y9]
     [z15, z12, z13, z14] = quarterround [y15, y12, y13, y14]
 rowround_opt : [16][32] -> [16][32]
 rowround_opt ys = join [(quarterround (yi<<<i))>>>i
                          | yi <- split ys | i <- [0 .. 3]]
 columnround : [16][32] -> [16][32]
 columnround [x0, x1, x2, x3, x4, x5, x6, x7, x8, x9, x10, x11, x12, x13, x14, x15] =
              [y0, y1, y2, y3, y4, y5, y6, y7, y8, y9, y10, y11, y12, y13, y14, y15]
   where
     [y0, y4, y8, 12] = quarterround [x0, x4, x8, x12]
     [y5, y9, y13, y1] = quarterround [x5, x9, x13, x1]
     [y10, y14, y2, y6] = quarterround [x10, x14, x2, x6]
     [y15, y3, y7, y11] = quarterround [x15, x3, x7, x11]
 columnround_opt : [16][32] -> [16][32]
 columnround_opt xs = join (transpose [ (quarterround (xi<<<i))>>>i
                                         | xi <- transpose(split xs)
                                         | i <- [0 .. 3] ])
 doubleround : [16][32] -> [16][32]
 doubleround(xs) = rowround(columnround(xs))
 littleendian : [4][8] -> [32]
 littleendian b = join(reverse b)
 littleendian_inverse : [32] -> [4][8]
 littleendian_inverse b = reverse(split b)
 Salsa20 : [64][8] -> [64][8]
 Salsa20 xs = join ar
   where
     ar = [ littleendian_inverse words | words <- xw + zs@10 ]
     xw = [ littleendian xi | xi <- split xs ]</pre>
     zs = [xw] # [doubleround zi | zi <- zs]
 // Salsa 20 supports two key sizes, [16][8] and [32][8]
 Salsa20_expansion : \{a\} (a >= 1, 2 >= a) => ([16*a][8], [16][8]) -> [64][8]
 Salsa20_expansion(k, n) = z
```

```
where
      [s0, s1, s2, s3] = split "expand 32-byte k" : [4][4][8]
      [t0, t1, t2, t3] = split "expand 16-byte k" : [4][4][8]
      x = if(a == 2) then s0 # k0 # s1 # n # s2 # k1 # s3
                      else t0 # k0 # t1 # n # t2 # k0 # t3
      z = Salsa20(x)
      [k0, k1] = (split(k#zero)):[2][16][8]
  salsa20_encrypt: \{a, l\} (a>=1, 2>=a, l<=2^70) => ([16*a][8], [8][8], [1][8]) -> [1][8]
  Salsa20\_encrypt(k, v, m) = c
    where
      salsa = take (join [ Salsa20_expansion(k, v#(reverse (split i)))
                         | i <- [0, 1 ... ] ])
      c = m \wedge salsa
quarterround [0xd3917c5b, 0x55f1c407, 0x52a58a7a, 0x8f887a3b] =
              [0x3e2f308c, 0xd90a8f36, 0x6ab2a923, 0x2883524c]
rowround [0x08521bd6, 0x1fe88837, 0xbb2aa576, 0x3aa26365,
          0xc54c6a5b, 0x2fc74c2f, 0x6dd39cc3, 0xda0a64f6,
          0x90a2f23d, 0x067f95a6, 0x06b35f61, 0x41e4732e,
          0xe859c100, 0xea4d84b7, 0x0f619bff, 0xbc6e965a] =
         [0xa890d39d, 0x65d71596, 0xe9487daa, 0xc8ca6a86,
          0x949d2192, 0x764b7754, 0xe408d9b9, 0x7a41b4d1,
          0x3402e183, 0x3c3af432, 0x50669f96, 0xd89ef0a8,
          0x0040ede5, 0xb545fbce, 0xd257ed4f, 0x1818882d]
columnround [0x08521bd6, 0x1fe88837, 0xbb2aa576, 0x3aa26365,
              0xc54c6a5b, 0x2fc74c2f, 0x6dd39cc3, 0xda0a64f6,
              0x90a2f23d, 0x067f95a6, 0x06b35f61, 0x41e4732e,
              0xe859c100, 0xea4d84b7, 0x0f619bff, 0xbc6e965a] =
             [0x8c9d190a, 0xce8e4c90, 0x1ef8e9d3, 0x1326a71a,
              0x90a20123, 0xead3c4f3, 0x63a091a0, 0xf0708d69,
              0x789b010c, 0xd195a681, 0xeb7d5504, 0xa774135c,
              0x481c2027, 0x53a8e4b5, 0x4c1f89c5, 0x3f78c9c8]
littleendian [86, 75, 30, 9] = 0 \times 091 = 4 \times 160
Salsa20 [88,118,104,54,79,201,235,79,3,81,156,47,203,26,244,243,
         191, 187, 234, 136, 211, 159, 13, 115, 76, 55, 82, 183, 3, 117, 222, 37,
         86, 16, 179, 207, 49, 237, 179, 48, 1, 106, 178, 219, 175, 199, 166, 48,
         238, 55, 204, 36, 31, 240, 32, 63, 15, 83, 93, 161, 116, 147, 48, 113] =
        [0xb3, 0x13, 0x30, 0xca, 0xdb, 0xec, 0xe8, 0x87,
         0x6f, 0x9b, 0x6e, 0x12, 0x18, 0xe8, 0x5f, 0x9e,
         0x1a, 0x6e, 0xaa, 0x9a, 0x6d, 0x2a, 0xb2, 0xa8,
         0x9c, 0xf0, 0xf8, 0xee, 0xa8, 0xc4, 0xbe, 0xcb,
         0x45, 0x90, 0x33, 0x39, 0x1d, 0x1d, 0x96, 0x1a,
         0x96, 0x1e, 0xeb, 0xf9, 0xbe, 0xa3, 0xfb, 0x30,
         0x1b, 0x6f, 0x72, 0x72, 0x76, 0x28, 0x98, 0x9d,
         0xb4, 0x39, 0x1b, 0x5e, 0x6b, 0x2a, 0xec, 0x231
Salsa20_encrypt ([0x23,0x12,0x14,0x72,0xEE,0xEa,0x45,0x23,
                   0x4A, 0x2A, 0x6D, 0x55, 0xF2, 0xCC, 0xCA, 0xC2],
                  [0x11,0x78,0x8E,0x3B,0x77,0x63,0x3A,0x3C],
                  [0xDD, 0x34, 0x67, 0x33, 0x23, 0xC4, 0xD3, 0xEE]) =
                  [0x21,0xc1,0x66,0xcb,0x24,0x58,0x7e,0x34]
```

```
Salsa20_encrypt ([0x23,0x12,0x14,0x72,0xEE,0xEa,0x45,0x23,
                   0x4A, 0x2A, 0x6D, 0x55, 0xF2, 0xCC, 0xCA, 0xC2],
                  [0x11,0x78,0x8E,0x3B,0x77,0x63,0x3A,0x3C],
                  [0x21,0xC1,0x66,0xCB,0x24,0x58,0x7E,0x34]) =
                  [0xDD, 0x34, 0x67, 0x33, 0x23, 0xC4, 0xD3, 0xEE]
Exercise 2:
 Prove Salsa20 encrypts and decrypts correctly:
   Salsa20\_can\_encrypt\_and\_decrypt : [32][8] -> [8][8] -> [200][8] -> Bit
   property Salsa20_can_encrypt_and_decrypt k v m =
      Salsa20\_encrypt(k,v,Salsa20\_encrypt(k,v,m)) == m
  Main> :prove Salsa20_can_encrypt_and_decrypt
   0.E.D.
   (Total Elapsed Time: 1.024s, using "Z3")
 Prove if x1 \neq x2 then doubleround x1 \neq doubleround x2:
   property Salsa20 has no collisions x1 x2 =
      if(x1 != x2) then (doubleround x1) != (doubleround x2) else True
  Main> :prove Salsa20_has_no_collisions
   0.E.D.
   (Total Elapsed Time: 46.972s, using "Z3")
 Prove columnround is the transpose of rowround:
   property columnround_is_transpose_of_rowround ys =
      rowround ys == join(transpose(split`{4}(columnround xs)))
      where xs = join(transpose(split`{4} ys))
  Main> :prove columnround_is_transpose_of_rowround
   Q.E.D.
   (Total Elapsed Time: 0.014s, using "Z3")
 Prove rowround x == rowround_opt x:
   property rowround opt is rowround x = rowround x = rowround opt x
```

```
property rowround_opt_is_rowround x = rowround x == rowround_opt x
Main> :prove rowround_opt_is_rowround
Q.E.D.
(Total Elapsed Time: 0.017s, using "Z3")
```

### Prove littleendian is invertible:

```
property littleendian_is_invertable b =
    littleendian_inverse(littleendian b) == b
Main> :prove littleendian_is_invertable
Q.E.D.
(Total Elapsed Time: 0.010s, using "Z3")
```

# s1.saw: import "Salsa20.cry"; let alloc\_init ty v = do { p <- llvm\_alloc ty;</pre> llvm\_points\_to p (llvm\_term v); return p; **}**; let quarterround\_setup = do { y0 <- llvm\_fresh\_var "y0" (llvm\_int 32); y1 <- llvm\_fresh\_var "y1" (llvm\_int 32); y2 <- llvm\_fresh\_var "y2" (llvm\_int 32); y3 <- llvm\_fresh\_var "y3" (llvm\_int 32); p0 <- alloc\_init (llvm\_int 32) {{ y0 }};</pre> p1 <- alloc\_init (llvm\_int 32) {{ y1 }};</pre> p2 <- alloc\_init (llvm\_int 32) {{ y2 }};</pre> p3 <- alloc\_init (llvm\_int 32) {{ y3 }}; llvm\_execute\_func [p0, p1, p2, p3]; let $zs = \{\{ quarterround [y0,y1,y2,y3] \}\};$ llvm\_points\_to p0 (llvm\_term {{ zs@0 }}); llvm\_points\_to p1 (llvm\_term {{ zs@1 }}); llvm\_points\_to p2 (llvm\_term {{ zs@2 }}); llvm\_points\_to p3 (llvm\_term {{ zs@3 }}); }; let main = do { mm <- llvm\_load\_module "salsa20.bc";</pre> qr <- llvm\_verify mm "s20\_quarterround" [] false quarterround\_setup yices;</pre> print "Done!"; }; Run saw s1.saw: [12:00:43.266] Verifying s20\_quarterround ... [12:00:43.267] Simulating s20\_quarterround ... [12:00:43.273] Checking proof obligations s20\_quarterround ... [12:00:43.303] Proof succeeded! s20\_quarterround [12:00:43.303] Done! **Exercise 4:** s1.saw: import "S.cry"; let alloc\_init ty v = do { }; let quarterround\_setup = do { };

Exercise 3:

```
let ptr_to_fresh n ty = do {
    };
    let oneptr_update_func n ty f = do {
    };
    let rowround_setup =
       oneptr_update_func "y" (llvm_array 16 (llvm_int 32)) {{ rowround }};
    let columnround_setup =
       oneptr_update_func "x" (llvm_array 16 (llvm_int 32)) {{ columnround }};
    let doubleround_setup =
       oneptr_update_func "x" (llvm_array 16 (llvm_int 32)) {{ doubleround }};
    let salsa20_setup =
       oneptr_update_func "seq" (llvm_array 64 (llvm_int 8)) {{ Salsa20 }};
    let main = do {
       mm <- llvm_load_module "salsa20.bc";</pre>
       qr <- llvm_verify mm "s20_quarterround" [] false quarterround_setup yices;</pre>
       rr <- llvm_verify mm "s20_rowround" [] false rowround_setup yices;</pre>
       cr <- llvm_verify mm "s20_columnround" [] false columnround_setup yices;</pre>
       dr <- llvm_verify mm "s20_doubleround" [] false doubleround_setup yices;</pre>
       s20 <- llvm_verify mm "s20_hash" [] false salsa20_setup yices;</pre>
       print "Done!";
    };
 Run saw s1.saw:
   May not finish
Exercise 5:
 s1.saw (modify the following):
    s20 <- llvm_verify mm "s20_hash" [dr] false salsa20_setup yices;</pre>
 Run saw s1.saw:
   [13:18:50.749] Verifying s20_quarterround ...
   [13:18:50.750] Simulating s20_quarterround ...
   [13:18:50.756] Checking proof obligations s20_quarterround ...
   [13:18:50.782] Proof succeeded! s20_quarterround
   [13:18:50.823] Verifying s20_rowround ...
   [13:18:50.823] Simulating s20_rowround ...
   [13:18:50.833] Checking proof obligations s20_rowround ...
   [13:18:50.851] Proof succeeded! s20_rowround
   [13:18:50.889] Verifying s20_columnround ...
   [13:18:50.890] Simulating s20_columnround ...
   [13:18:50.899] Checking proof obligations s20_columnround ...
   [13:18:50.918] Proof succeeded! s20_columnround
   [13:18:50.955] Verifying s20_doubleround ...
   [13:18:50.956] Simulating s20_doubleround ...
   [13:18:50.974] Checking proof obligations s20_doubleround ...
   [13:18:54.593] Proof succeeded! s20_doubleround
   [13:18:54.632] Verifying s20_hash ...
   [13:18:54.633] Simulating s20_hash ...
```

```
[13:18:54.633] Registering overrides for `s20_doubleround`
                variant `Symbol "s20_doubleround"`
[13:18:54.633]
[13:18:54.647] Matching 1 overrides of s20_doubleround ...
[13:18:54.649] Branching on 1 override variants of s20_doubleround ...
[13:18:54.651] Applied override! s20_doubleround
[13:18:54.652] Matching 1 overrides of s20 doubleround ...
[13:18:54.652] Branching on 1 override variants of s20_doubleround ...
[13:18:54.653] Applied override! s20_doubleround
[13:18:54.654] Matching 1 overrides of s20_doubleround ...
[13:18:54.654] Branching on 1 override variants of s20_doubleround ...
[13:18:54.655] Applied override! s20_doubleround
[13:18:54.655] Matching 1 overrides of s20_doubleround ...
[13:18:54.655] Branching on 1 override variants of s20_doubleround ...
[13:18:54.656] Applied override! s20_doubleround
[13:18:54.657] Matching 1 overrides of s20_doubleround ...
[13:18:54.657] Branching on 1 override variants of s20_doubleround ...
[13:18:54.658] Applied override! s20_doubleround
[13:18:54.658] Matching 1 overrides of s20_doubleround ...
[13:18:54.658] Branching on 1 override variants of s20_doubleround ...
[13:18:54.659] Applied override! s20 doubleround
[13:18:54.659] Matching 1 overrides of s20_doubleround ...
[13:18:54.660] Branching on 1 override variants of s20_doubleround ...
[13:18:54.661] Applied override! s20_doubleround
[13:18:54.661] Matching 1 overrides of s20_doubleround ...
[13:18:54.661] Branching on 1 override variants of s20_doubleround ...
[13:18:54.662] Applied override! s20_doubleround
[13:18:54.662] Matching 1 overrides of s20_doubleround ...
[13:18:54.662] Branching on 1 override variants of s20_doubleround ...
[13:18:54.663] Applied override! s20_doubleround
[13:18:54.664] Matching 1 overrides of s20_doubleround ...
[13:18:54.664] Branching on 1 override variants of s20_doubleround ...
[13:18:54.665] Applied override! s20_doubleround
[13:18:54.688] Checking proof obligations s20_hash ...
[13:18:55.181] Proof succeeded! s20_hash
[13:18:55.182] Done!
```

## **Exercise 6:**

#### Run saw s1.saw:

```
[20:41:38.847] Proof succeeded! s20_expand32 [20:41:38.847] Done!
```

## Exercise 7:

#### Run saw s1.saw:

```
[20:43:40.749] Proof succeeded! s20_crypt32
[20:43:40.749] Done!
```