Project part 1 & 2: Image reconstruction using GMRFs & Image Segmentation using Mixture Models

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1 Part 1

1.1 Variograms

Using 10000 pixels, and approximately 50% observed pixels, we obtained the least squares estimate that follows in table 1.1.

Image \ Covariates	Intercept	Horizontal	Vertical
Rosetta	0.4391	0.0000	-0.0001
Titan	0.4099	-0.0002	0.0000

Estimating covariates with least squares, binning the residuals of our observed pixels, doing weighted least squares covariance estimation to get parameters and then fitting a matern variogram resulted in figure 1

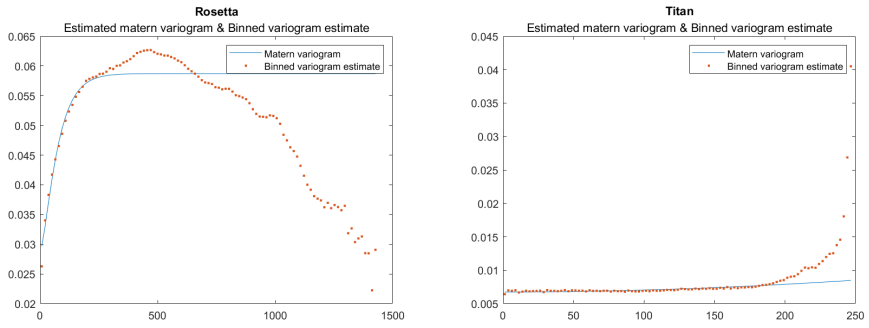

Figure 1: Variogram for both images.

1.2 Reconstructions

The LSE-estimate of κ was optimal for the Titan image, and very close to optimal for the Rosetta image. In the latter, approximately 0.03% lower error rating (sum of absolute difference in pixel values vs real image) was achieved by increasing the size of κ to 0.244 as shown in figure 2

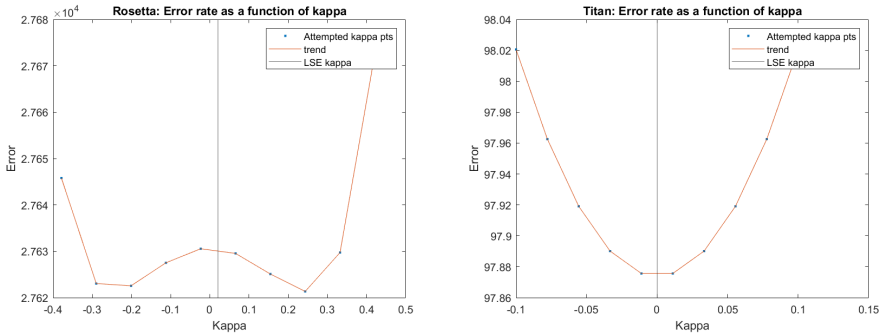

Figure 2: Error rate vs. kappa for both images.

Down below follows result of our reconstructed image for the κ estimated through least squares shown next to the original image and also differences in pixel values.

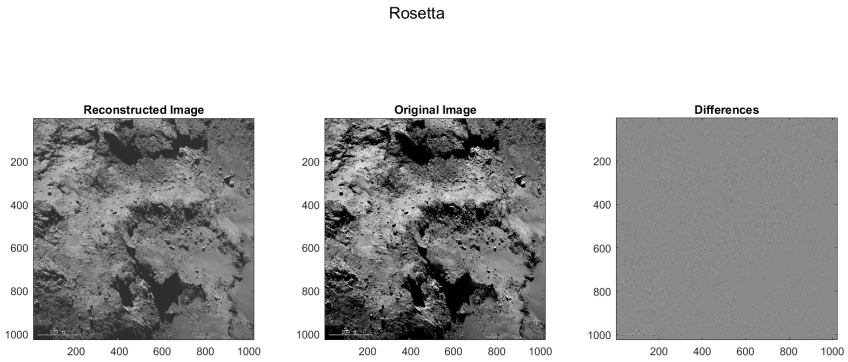

Figure 3: Reconstructed Rosetta image and differences for approximately 50% observed pixels, and subset size 10000 pixels.

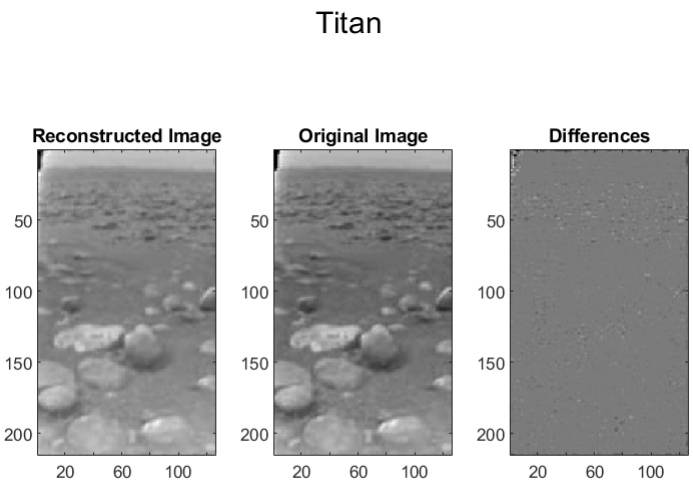

Figure 4: Reconstructed Titan image and differences for approximately 50% observed pixels, and subset size 10000 pixels.

1.3 Missing pixels

Below follows a series of reconstructions for different percentages of missing pixels for Rosetta and Titan respectively. Figure 5 with Rosetta images shows that for around 98% missing pixels you can still make out general contours, but all details have been distorted. In the image reconstructed from 95% missing pixels there is definitely more information to be extracted, but what counts as a "reasonable estimate of the true image" would probably depend on the situation and context. As for Titan, figure 6 shows that for around 70% missing pixels, details in the background start getting hard to make out. For 90%, only the most general features become distinguishable.

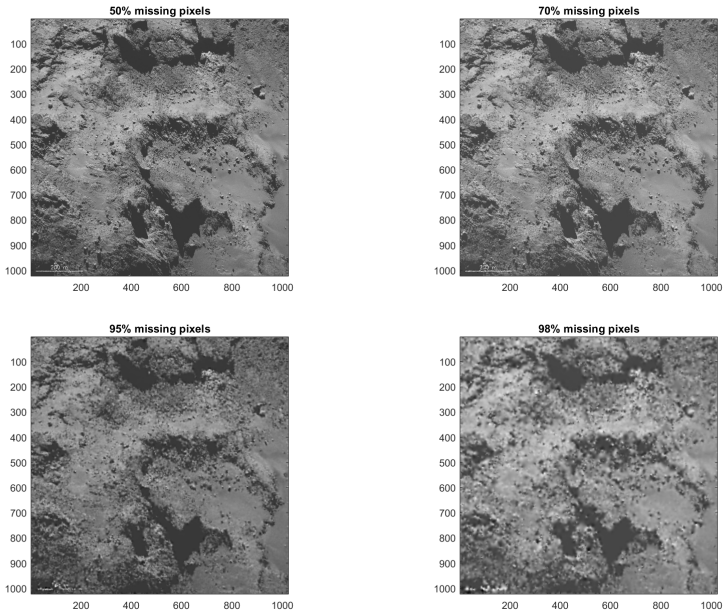

Figure 5: Image reconstruction of Rosetta image for different share of missing pixels.

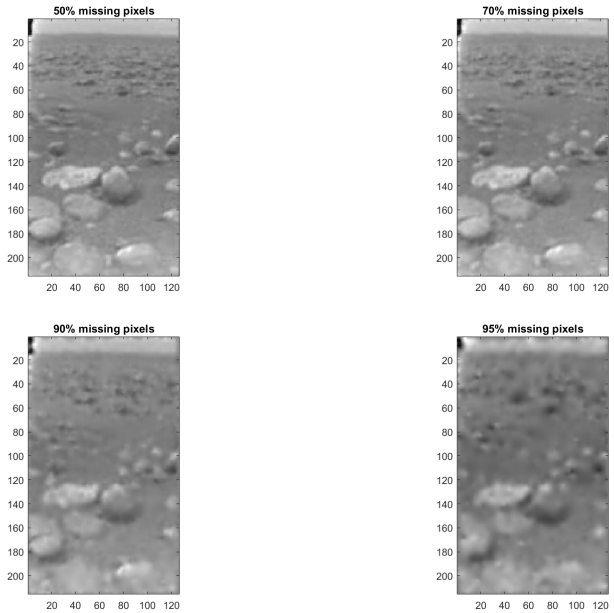

Figure 6: Image reconstruction of Titan image for different share of missing pixels.

2 Part 2

Below follows plots for segmentation of the image using K-means, Gaussian mixture and Markov random field. In the case of no noise, all three models seem to perform equally well. However, as noise increases, Markov random field model is superior. This is due to the fact that it sets label of a pixel depending on that pixels neighborhood.

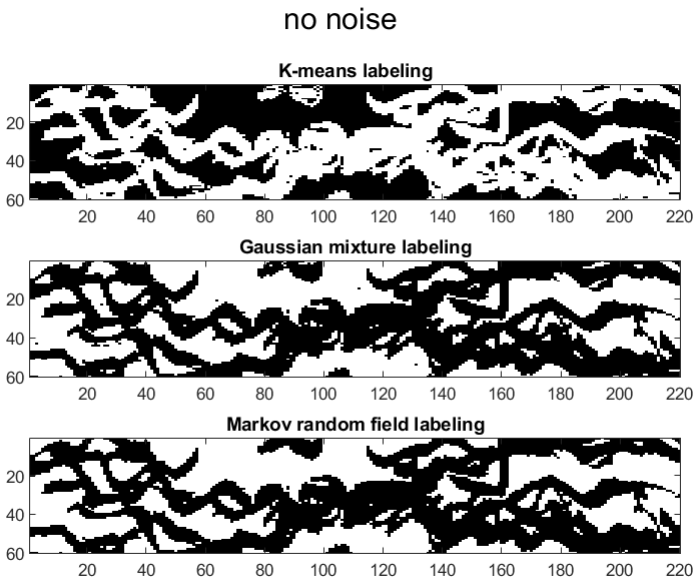

Figure 7: Image segmentation into two classes with no noise.

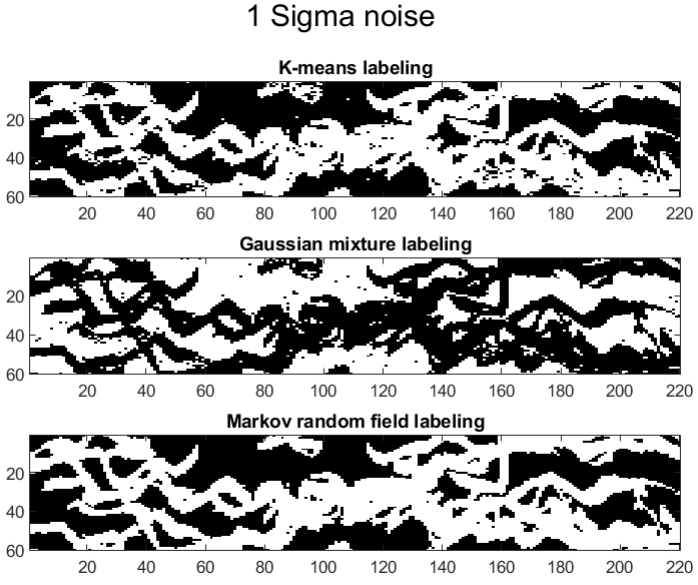

Figure 8: Image segmentation into two classes with σ noise.

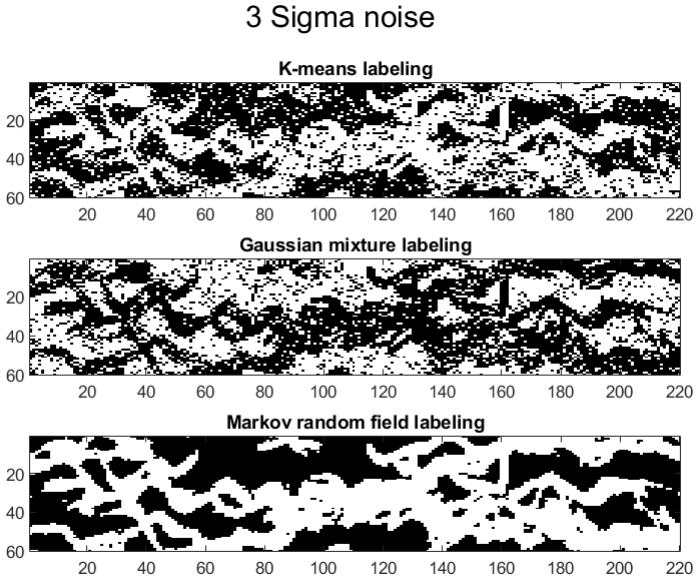

Figure 9: Image segmentation into two classes with 3σ noise.