# Time Series (732A62) Lab1

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# Contents

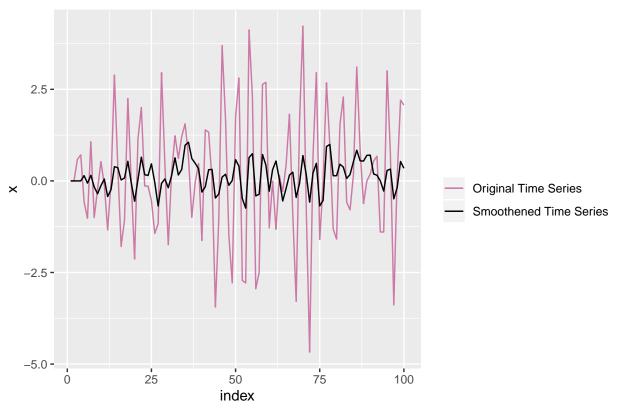
Assignment 1. Computations with simulated data	2
Assignment 2. Visualization, detrending and residual analysis of Rhine data.	6
Assignment 3. Analysis of oil and gas time series.	22
Appendix	30

### Assignment 1. Computations with simulated data

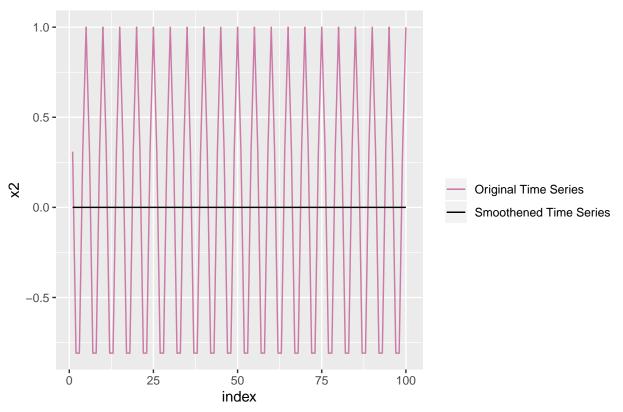
a) Generate two time series  $x_t = -0.8x_{t-2} + w_t$ , where  $x_0 = x_1 = 0$  and  $x_t = \cos(\frac{2\pi t}{5})$  with 100 observations each. Apply a smoothing filter  $v_t = 0.2(x_t + x_{t-1} + x_{t-2} + x_{t-3} + x_{t-4})$  to these two series and compare how the filter has affected them.

```
set.seed(12345)
n = 100
x <- vector(length = n)
x2 <- vector(length = n)</pre>
x[1] <- 0
x[2] < 0
#first series generation
for(i in 3:n){
 x[i] \leftarrow -0.8 * x[i-2] + rnorm(1,0,1)
#second series generation
for(i in 1:n){
 x2[i] <- cos(0.4*pi*i)
# smoothing filter function
smoothing_filter <- function(x){</pre>
v <- vector(length = length(x))</pre>
for(i in 5:length(x)){
 v[i] = 0.2 * (x[i] + x[i-1] + x[i-2] + x[i-3] + x[i-4])
}
return(v)
}
#generate smoothed series
smooth_x <- smoothing_filter(x)</pre>
smooth_x2 <- smoothing_filter(x2)</pre>
#adding everything to a dataframe
df <- cbind(x,x2,smooth x,smooth x2) %>% as.data.frame() %>% mutate(index=1:100)
ggplot(df, aes(x=index)) +
  geom_line(aes(y=x, color="Original Time Series")) +
  geom_line(aes(y=smooth_x, color="Smoothened Time Series")) +
  ggtitle("Plot of 1st time series and its smoothened version") +
    scale_colour_manual("", breaks = c("Original Time Series", "Smoothened Time Series"),
                         values = c("#CC79A7", "#000000"))
```

### Plot of 1st time series and its smoothened version



#### Plot of 2nd time series and its smoothened version



b) Consider time series  $x_t - 4x_{t-1} + 2x_{t-2} + x_{t-5} = w_t + 3w_{t-2} + w_{t-4} - 4w_{t-6}$ . Write an appropriate R code to investigate whether this time series is casual and invertible.

Causality: ARMA(p,q) is causal iff roots  $\phi(z') = 0$  are outside unit circle. eg:  $x_t = 0.4x_{t-1} + 0.3x_{t-2} + w_t$ , roots are  $-> 1 - 0.4B + 0.3B^2$ 

equation is:  $\phi(Z) = 1 - 4B + 2B^2 + 0B^3 + 0B^4 + B^5$ 

```
z = c(1,-4,2,0,0,1)
polyroot(z)
```

- ## [1] 0.2936658+0.000000i -1.6793817+0.000000i 1.0000000-0.0000000i
- ## [4] 0.1928579-1.410842i 0.1928579+1.410842i

#### any(Mod(polyroot(z))<=1)</pre>

## [1] TRUE

Invertible: ARMA(p,q) is causal iff roots  $\theta(z') = 0$  are outside unit circle.

equation is:  $\theta(Z) = 1 + 3B^2 + B^4 - 4B^6$ 

$$z = c(1,0,3,0,1,0,-4)$$
polyroot(z)

- ## [1] 0.1375513+0.6735351i -0.1375513+0.6735351i -0.1375513-0.6735351i
- ## [4] 0.1375513-0.6735351i 1.0580446+0.0000000i -1.0580446+0.0000000i

```
any(Mod(polyroot(z))<=1)</pre>
```

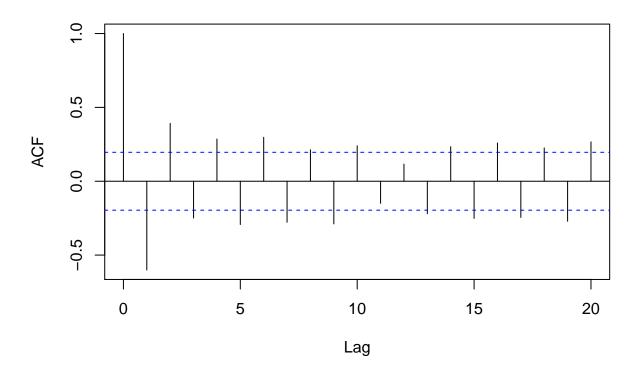
#### ## [1] TRUE

Analysis: Baring one of the roots all are inside the unit circle. Thus the time series is not invertiable.

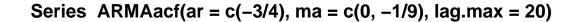
c) Use built-in R functions to simulate 100 observations from the process  $x_t + \frac{3}{4}x_{t-1} = w_t - \frac{1}{9}w_{t-2}$  compute sample ACF and theoretical ACF, use seed 54321. Compare the ACF plots.

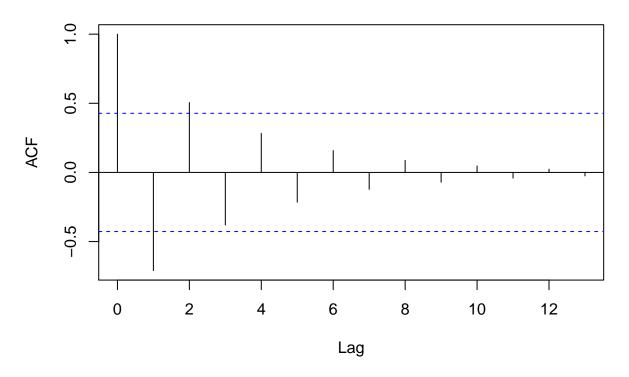
```
set.seed(54321)
series <- arima.sim(n = 100, list(ar = c(-3/4), ma = c(0,-1/9)))
acf(series)</pre>
```

## Series series



```
acf(ARMAacf(ar = c(-3/4), ma = c(0,-1/9), lag.max = 20))
```



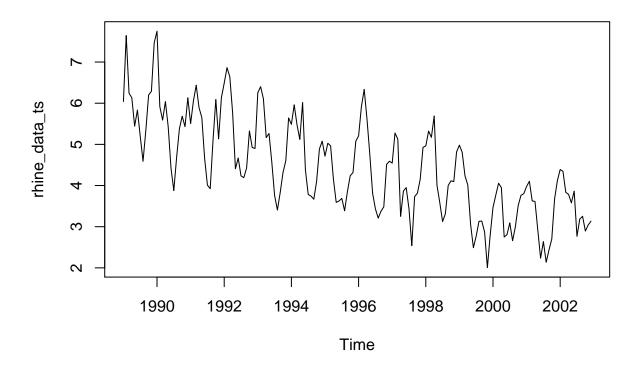


Analysis: In the theortical ACF, only the 1 and 2nd lag components were significant, while using the sample ACF function we get many more lag components as significant.

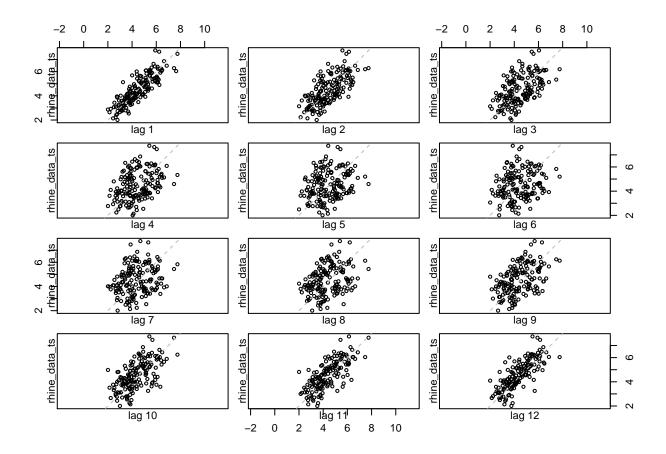
# Assignment 2. Visualization, detrending and residual analysis of Rhine data.

The data set Rhine.csv contains monthly concentrations of total nitrogen in the Rhine River in the period 1989-2002.

a) Import the data to R, convert it appropriately to ts object (use function ts()) and explore it by plotting the time series, creating scatter plots of  $x_t$  against  $x_{t-1},...x_{t-12}$ . Analyze the time series plot and the scatter plots: Are there any trends, linear or seasonal, in the time series? When during the year is the concentration highest? Are there any special patterns in the data or scatterplots? Does the variance seem to change over time? Which variables in the scatterplots seem to have a significant relation to each other?

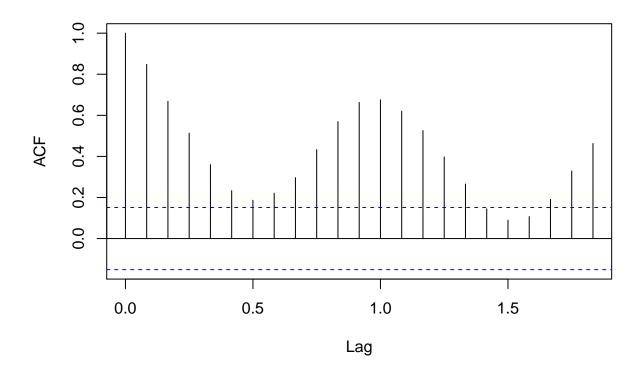


lag.plot(rhine\_data\_ts,lags = 12)



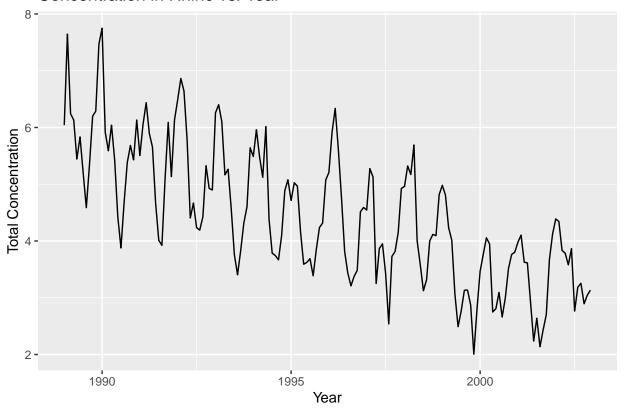
acf(rhine\_data\_ts)

# Series rhine\_data\_ts

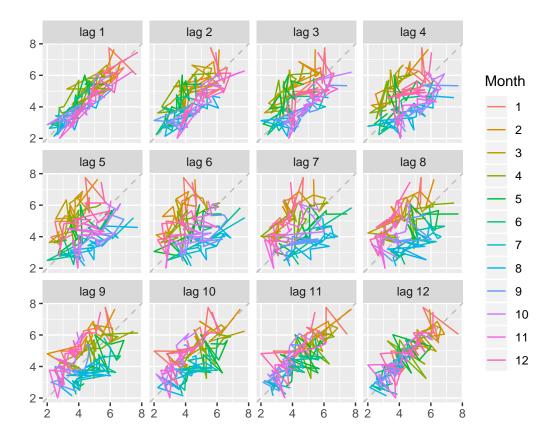


```
#alternative
autoplot(rhine_data_ts) + ylab("Total Concentration") +xlab("Year") +
ggtitle("Concentration in Rhine vs. Year")
```

# Concentration in Rhine vs. Year

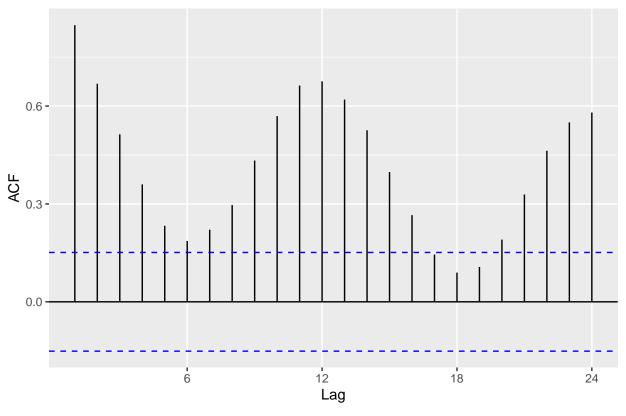


gglagplot(rhine\_data\_ts, lags = 1, set.lags = 1:12, color=FALSE)



ggAcf(rhine\_data\_ts)

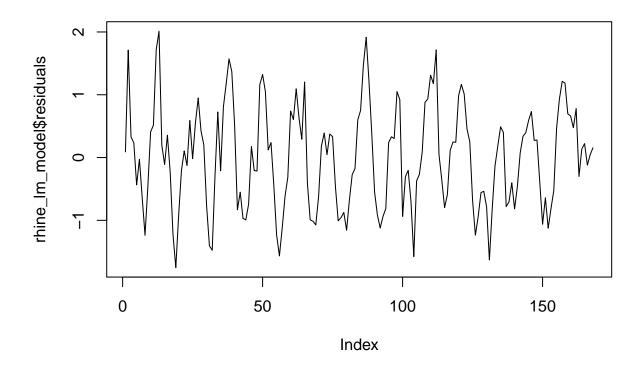
## Series: rhine\_data\_ts



#### Analysis:

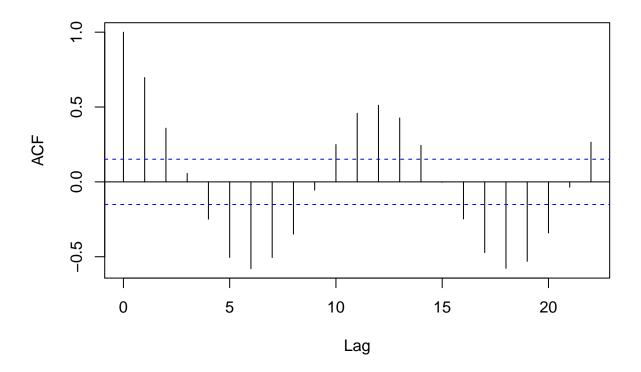
b) Eliminate the trend by fitting a linear model with respect to t to the time series. Is there a significant time trend? Look at the residual pattern and the sample ACF of the residuals and comment how this pattern might be related to seasonality of the series.

```
rhine_lm_model <- lm(TotN_conc~Time, data=rhine_data)
plot(rhine_lm_model$residuals, type = 'l')</pre>
```



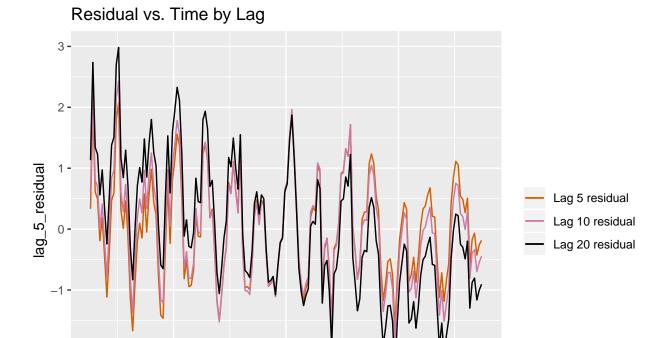
acf(rhine\_lm\_model\$residuals)

### Series rhine\_Im\_model\$residuals



c) Eliminate the trend by fitting a kernel smoother with respect to t to the time series (choose a reasonable bandwidth yourself so the fit looks reasonable). Analyze the residual pattern and the sample ACF of the residuals and compare it to the ACF from step b). Conclusions? Do residuals seem to represent a stationary series?

```
set.seed(12345)
model_smooth_lag_5 <- ksmooth(x = rhine_data$Time, y = rhine_data$TotN_conc, bandwidth=5)</pre>
model_smooth_lag_10 <- ksmooth(x = rhine_data$Time, y = rhine_data$TotN_conc, bandwidth=10)
model_smooth_lag_20 <- ksmooth(x = rhine_data$Time, y = rhine_data$TotN_conc, bandwidth=20)
model smooth lag 5 residual <- rhine data$TotN conc - model smooth lag 5$y
model_smooth_lag_10_residual <- rhine_data$TotN_conc - model_smooth_lag_10$y
model_smooth_lag_20_residual <- rhine_data$TotN_conc - model_smooth_lag_20$y</pre>
residual_df <- cbind(model_smooth_lag_5_residual, model_smooth_lag_10_residual,
                     model_smooth_lag_20_residual, rhine_data$Time) %>% as.data.frame()
colnames(residual_df) <- c("lag_5_residual", "lag_10_residual", "lag_20_residual", "Time")</pre>
ggplot(residual_df, aes(x=Time)) +
  geom_line(aes(y=lag_5_residual, color="Lag 5 residual")) +
  geom_line(aes(y=lag_10_residual, color="Lag 10 residual")) +
  geom_line(aes(y=lag_20_residual, color="Lag 20 residual")) +
  ggtitle("Residual vs. Time by Lag") +
    scale_colour_manual("", breaks = c("Lag 5 residual", "Lag 10 residual", "Lag 20 residual"),
```



2000

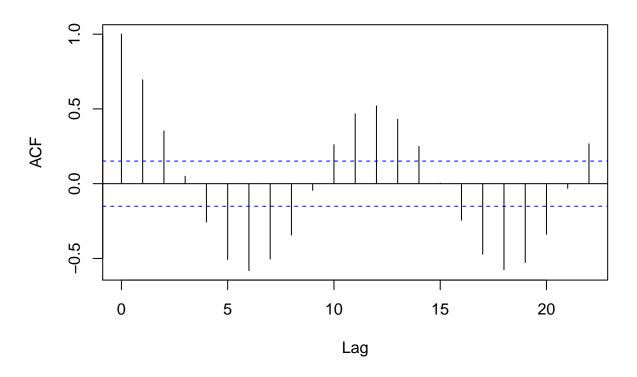
1996 Time

acf(model\_smooth\_lag\_5\_residual)

1992

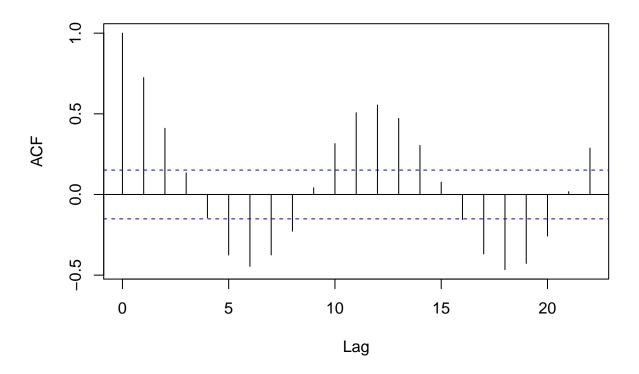
-2 **-**

# Series model\_smooth\_lag\_5\_residual



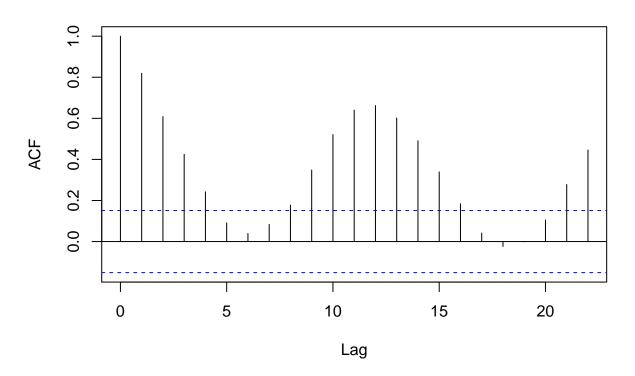
acf(model\_smooth\_lag\_10\_residual)

# Series model\_smooth\_lag\_10\_residual

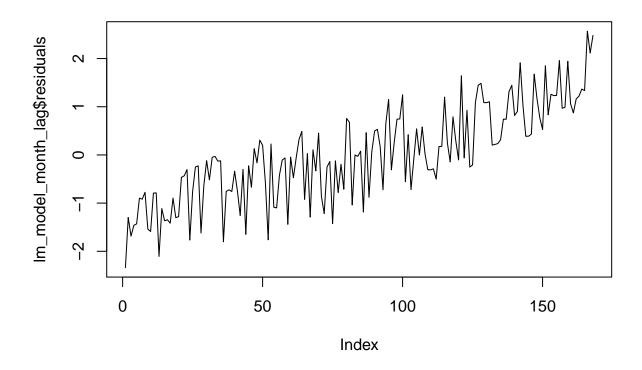


acf(model\_smooth\_lag\_20\_residual)

# Series model\_smooth\_lag\_20\_residual

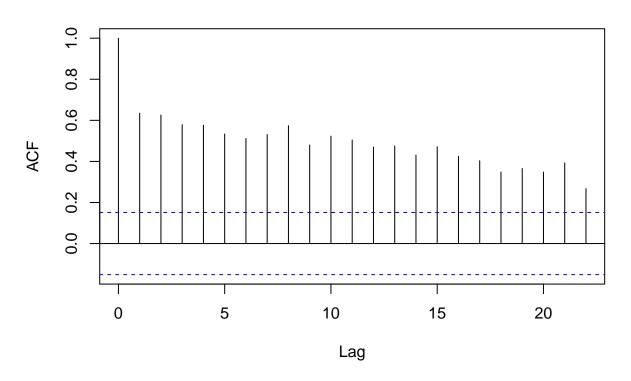


d) Eliminate the trend by fitting the following so-called seasonal means model:  $x_t = \alpha_0 + \alpha_1 t + \beta_1 I(month = 2) + \dots + \beta_{12} I(month = 12) + w_t$ , where I(x)=1 is an identity function. Fitting of this model will require you to augment data with a categorical variable showing the current month, and then fitting a usual linear regression. Analyze the residual pattern and the ACF of residuals.



acf(lm\_model\_month\_lag\$residuals)

# Series Im\_model\_month\_lag\$residuals



#### Analysis:

e) Perform stepwise variable selection in model from step d). Which model gives you the lowest AIC value? Which variables are left in the model?

```
## Start: AIC=17.5
## TotN_conc ~ Month_1 + Month_2 + Month_3 + Month_4 + Month_5 +
##
       Month_6 + Month_7 + Month_8 + Month_9 + Month_10 + Month_11 +
##
       Month_12
##
##
## Step: AIC=17.5
  TotN_conc ~ Month_1 + Month_2 + Month_3 + Month_4 + Month_5 +
##
       Month_6 + Month_7 + Month_8 + Month_9 + Month_10 + Month_11
##
##
              Df Sum of Sq
                              RSS
## - Month 4
                    0.0064 161.63 15.506
               1
## - Month_1
               1
                    0.9497 162.57 16.484
```

```
## - Month 3
                    0.9775 162.60 16.512
                           161.62 17.499
## <none>
## - Month 11
                    2.1676 163.79 17.738
## - Month_5
                    2.2167 163.84 17.788
               1
## - Month_2
               1
                    2.7569 164.38 18.341
## - Month 10 1
                    3.0705 164.69 18.661
## - Month 6
                    6.7169 168.34 22.340
               1
## - Month 9
               1
                    7.1009 168.72 22.723
## - Month_7
               1
                   12.6417 174.26 28.151
## - Month_8
               1
                   15.0349 176.66 30.443
##
## Step: AIC=15.51
## TotN_conc ~ Month_1 + Month_2 + Month_3 + Month_5 + Month_6 +
##
       Month_7 + Month_8 + Month_9 + Month_10 + Month_11
##
##
              Df Sum of Sq
                              RSS
## - Month_1
                    1.3720 163.00 14.926
               1
## - Month 3
                    1.4106 163.04 14.966
## <none>
                           161.63 15.506
## - Month 11 1
                    2.7357 164.37 16.326
## - Month_5
               1
                    2.7993 164.43 16.391
## - Month 2
                    3.8546 165.49 17.465
               1
## - Month_10 1
                    3.9098 165.54 17.521
## - Month 6
                    8.6824 170.31 22.297
               1
## - Month 9
               1
                    9.1865 170.82 22.793
## - Month 7
               1
                   16.4796 178.11 29.817
## - Month_8
                   19.6363 181.27 32.768
               1
##
## Step: AIC=14.93
## TotN_conc ~ Month_2 + Month_3 + Month_5 + Month_6 + Month_7 +
##
       Month_8 + Month_9 + Month_10 + Month_11
##
##
              Df Sum of Sq
                              RSS
## - Month_3
                    0.7151 163.72 13.661
               1
## <none>
                           163.00 14.926
## - Month 2
                    2.7831 165.78 15.770
               1
## - Month 11 1
                    4.7022 167.70 17.704
## - Month_5
                    4.7906 167.79 17.792
               1
## - Month 10
                    6.3071 169.31 19.304
              1
## - Month_6
                   12.5278 175.53 25.366
               1
## - Month_9
               1
                 13.1690 176.17 25.979
## - Month 7
                   22.2773 185.28 34.447
               1
## - Month 8
               1
                   26.1552 189.16 37.927
##
## Step: AIC=13.66
## TotN_conc ~ Month_2 + Month_5 + Month_6 + Month_7 + Month_8 +
##
       Month_9 + Month_10 + Month_11
##
##
              Df Sum of Sq
                              RSS
                                     AIC
## <none>
                           163.72 13.661
                    2.2640 165.98 13.969
## - Month_2
               1
## - Month_11 1
                    6.0413 169.76 17.749
## - Month 5
                    6.1447 169.86 17.852
               1
## - Month 10 1
                    7.9079 171.62 19.586
```

```
15.0069 178.72 26.396
## - Month_6
               1
## - Month_9
                   15.7313 179.45 27.075
               1
## - Month_7
                   25.9388 189.66 36.370
## - Month_8
                   30.2531 193.97 40.148
colnames(lm_model_month_lag_step$model)
                                                                    "Month_8"
## [1] "TotN_conc" "Month_2"
                                "Month_5"
                                            "Month_6"
                                                        "Month_7"
## [7] "Month_9"
                   "Month_10"
                               "Month_11"
```

Analysis: The final terms in the model are given above, this model had the least AIC.

# Assignment 3. Analysis of oil and gas time series.

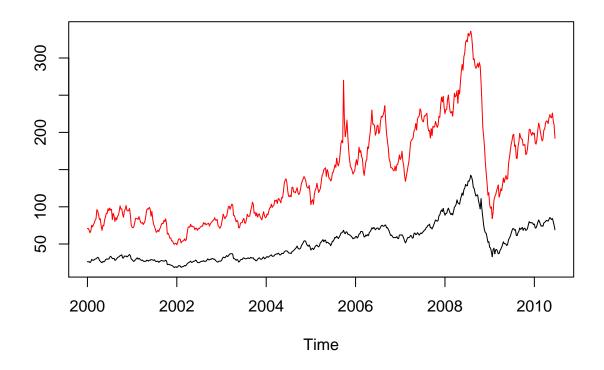
Weekly time series oil and gas present in the package astsa show the oil prices in dollars per barrel and gas prices in cents per dollar.

a) Plot the given time series in the same graph. Do they look like stationary series? Do the processes seem to be related to each other? Motivate your answer.

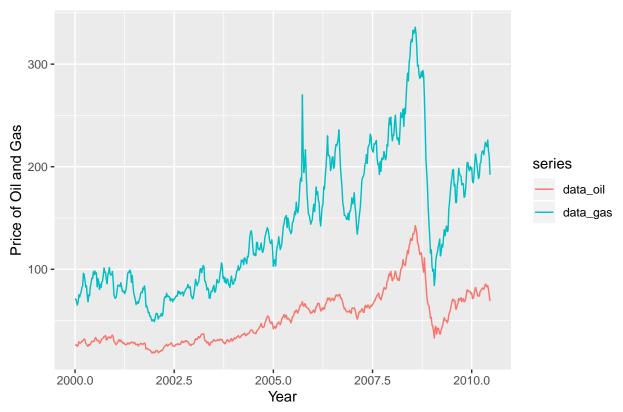
```
set.seed(12345)

data_oil <- astsa::oil
data_gas <- astsa::gas

ts.plot(data_oil, data_gas, gpars = list(col = c("black", "red")))</pre>
```

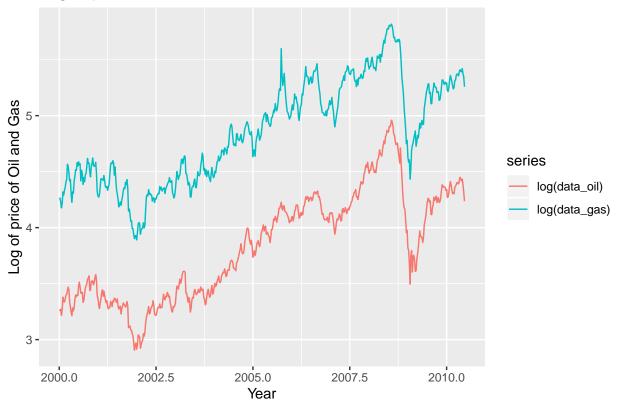


### Price of Oil and Gas vs. Years



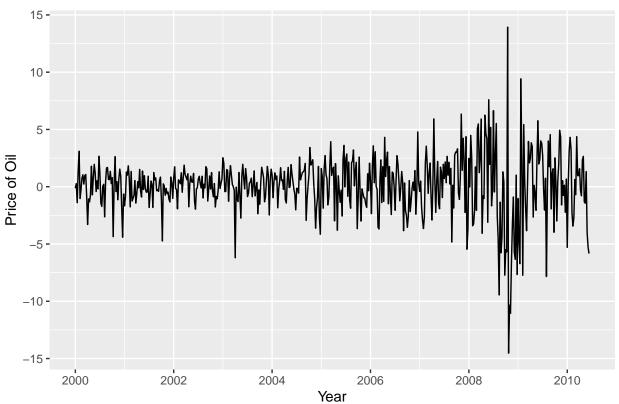
b) Apply log-transform to the time series and plot the transformed data. In what respect did this transformation made the data easier for the analysis?

# Log of price of Oil and Gas vs. Years



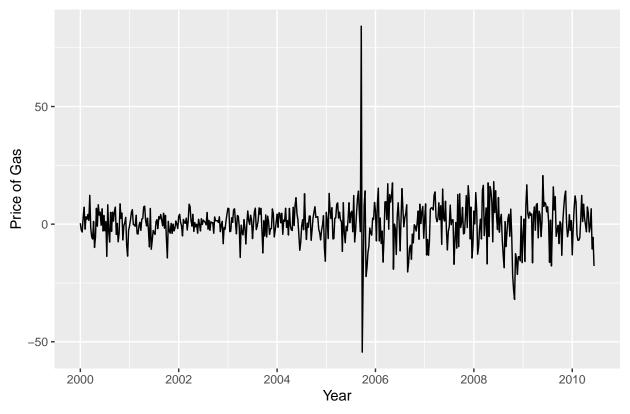
c) To eliminate trend, compute the first difference of the transformed data, plot the detrended series, check their ACFs and analyze the obtained plots. Denote the data obtained here as  $x_t(\text{oil})$  and  $y_t(\text{gas})$ .

## Price of Oil with Diff 1 vs. Years



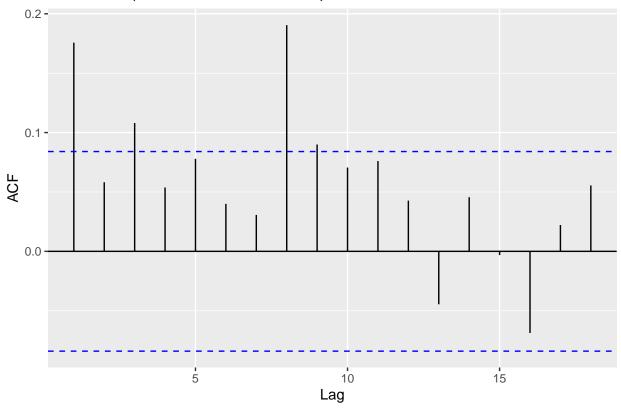
```
autoplot(ts(diff(data_gas, differences = 1), start = 2000, frequency = 52)) +
    ylab("Price of Gas") +xlab("Year") +
    ggtitle("Price of Gas with Diff 1 vs. Years")
```

Price of Gas with Diff 1 vs. Years



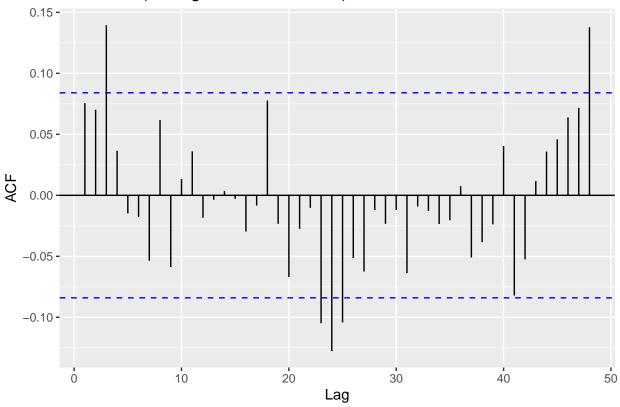
ggAcf(diff(data\_oil, differences = 1), data\_oil)

# Series: diff(data\_oil, differences = 1)



ggAcf(diff(data\_gas, differences = 1), data\_gas)

### Series: diff(data\_gas, differences = 1)



d) Exhibit scatterplots of  $x_t$  and  $y_t$  for up to three weeks of lead time of  $x_t$  include a nonparametric smoother in each plot and comment the results: are there outliers? Are the relationships linear? Are there changes in the trend?

e) Fit the following model:  $y_t = \alpha_0 + \alpha_1 I(x_t > 0) + \beta_1 x_1 + \beta_2 x_{t-1} + w_t$  and check which coefficients seem to be significant. How can this be interpreted? Analyze the residual pattern and the ACF of the residuals.

# Appendix

```
knitr::opts_chunk$set(echo = TRUE)
options(scipen=999)
library("tidyverse") #ggplot and dplyr
library("gridExtra") # combine plots
library("knitr") # for pdf
library("fpp2") #timeseries with autoplot and stuff
library("reshape2") #reshape the data
library("MASS") #StepAIC
library("astsa") #dataset oil and gas is present here
# The palette with black:
cbbPalette <- c("#000000", "#E69F00", "#56B4E9", "#009E73",
                 "#F0E442", "#0072B2", "#D55E00", "#CC79A7")
set.seed(12345)
set.seed(12345)
n = 100
x <- vector(length = n)</pre>
x2 <- vector(length = n)</pre>
x[1] <- 0
x[2] < 0
#first series generation
for(i in 3:n){
 x[i] \leftarrow -0.8 * x[i-2] + rnorm(1,0,1)
#second series generation
for(i in 1:n){
  x2[i] <- cos(0.4*pi*i)
}
# smoothing filter function
smoothing_filter <- function(x){</pre>
v <- vector(length = length(x))</pre>
for(i in 5:length(x)){
  v[i] = 0.2 * (x[i] + x[i-1] + x[i-2] + x[i-3] + x[i-4])
return(v)
}
#generate smoothed series
smooth_x <- smoothing_filter(x)</pre>
smooth_x2 <- smoothing_filter(x2)</pre>
```

```
#adding everything to a dataframe
df <- cbind(x,x2,smooth_x,smooth_x2) %>% as.data.frame() %>% mutate(index=1:100)
ggplot(df, aes(x=index)) +
  geom_line(aes(y=x, color="Original Time Series")) +
  geom_line(aes(y=smooth_x, color="Smoothened Time Series")) +
  ggtitle("Plot of 1st time series and its smoothened version") +
    scale_colour_manual("", breaks = c("Original Time Series", "Smoothened Time Series"),
                         values = c("\#CC79A7", "\#000000"))
ggplot(df, aes(x=index)) +
  geom_line(aes(y=x2, color="Original Time Series")) +
  geom_line(aes(y=smooth_x2, color="Smoothened Time Series")) +
  ggtitle("Plot of 2nd time series and its smoothened version") +
    scale_colour_manual("", breaks = c("Original Time Series", "Smoothened Time Series"),
                        values = c("#CC79A7", "#000000"))
z = c(1,-4,2,0,0,1)
polyroot(z)
any(Mod(polyroot(z))<=1)</pre>
z = c(1,0,3,0,1,0,-4)
polyroot(z)
any(Mod(polyroot(z))<=1)</pre>
set.seed(54321)
series \leftarrow arima.sim(n = 100, list(ar = c(-3/4), ma = c(0,-1/9)))
acf(series)
acf(ARMAacf(ar = c(-3/4), ma = c(0,-1/9), lag.max = 20))
set.seed(12345)
rhine_data <- read.csv2("Rhine.csv")</pre>
rhine_data_ts <- ts(data = rhine_data$TotN_conc,</pre>
                    start = c(1989,1),
                    frequency = 12)
plot.ts(rhine_data_ts)
lag.plot(rhine_data_ts,lags = 12)
acf(rhine data ts)
#alternative
autoplot(rhine_data_ts) + ylab("Total Concentration") +xlab("Year") +
  ggtitle("Concentration in Rhine vs. Year")
gglagplot(rhine_data_ts, lags = 1, set.lags = 1:12, color=FALSE)
ggAcf(rhine_data_ts)
set.seed(12345)
rhine_lm_model <- lm(TotN_conc~Time, data=rhine_data)</pre>
plot(rhine_lm_model$residuals, type = 'l')
acf(rhine_lm_model$residuals)
set.seed(12345)
```

```
model_smooth_lag_5 <- ksmooth(x = rhine_data$Time, y = rhine_data$TotN_conc, bandwidth=5)</pre>
model_smooth_lag_10 <- ksmooth(x = rhine_data$Time, y = rhine_data$TotN_conc, bandwidth=10)
model_smooth_lag_20 <- ksmooth(x = rhine_data$Time, y = rhine_data$TotN_conc, bandwidth=20)
model_smooth_lag_5_residual <- rhine_data$TotN_conc - model_smooth_lag_5$y
model_smooth_lag_10_residual <- rhine_data$TotN_conc - model_smooth_lag_10$y</pre>
model_smooth_lag_20_residual <- rhine_data$TotN_conc - model_smooth_lag_20$y
residual_df <- cbind(model_smooth_lag_5_residual, model_smooth_lag_10_residual,
                     model_smooth_lag_20_residual, rhine_data$Time) %>% as.data.frame()
colnames(residual_df) <- c("lag_5_residual", "lag_10_residual", "lag_20_residual", "Time")</pre>
ggplot(residual_df, aes(x=Time)) +
  geom_line(aes(y=lag_5_residual, color="Lag 5 residual")) +
  geom_line(aes(y=lag_10_residual, color="Lag 10 residual")) +
  geom_line(aes(y=lag_20_residual, color="Lag 20 residual")) +
  ggtitle("Residual vs. Time by Lag") +
    scale_colour_manual("", breaks = c("Lag 5 residual", "Lag 10 residual", "Lag 20 residual"),
                        values = c("#CC79A7", "#000000", "#D55E00"))
acf(model_smooth_lag_5_residual)
acf(model_smooth_lag_10_residual)
acf(model_smooth_lag_20_residual)
set.seed(12345)
rhine_data_wide <- rhine_data</pre>
rhine_data_wide$dummy <- "1"</pre>
rhine_data_wide$Month <- paste0("Month_",rhine_data_wide$Month)</pre>
rhine_data_wide <- dcast(rhine_data_wide, formula = TotN_conc+Year+Time~Month, value.var = "dummy", fil
lm_model_month_lag <- lm(data=rhine_data_wide,</pre>
                    TotN_conc~Month_1+Month_2+Month_3+Month_4+Month_5+Month_6+Month_7+
                      Month_8+Month_9+Month_10+Month_11+Month_12)
plot(lm model month lag$residuals, type = 'l')
acf(lm_model_month_lag$residuals)
set.seed(12345)
lm_model_month_lag_step <- stepAIC(lm_model_month_lag, scope = list(upper = ~Month_1+Month_2+</pre>
                                                                    Month_3+Month_4+Month_5+Month_6+Month
                                                                      Month_8+Month_9+Month_10+Month_11+M
                                                                    lower = ~1), trace = TRUE,
                                    direction="backward")
colnames(lm_model_month_lag_step$model)
set.seed(12345)
data_oil <- astsa::oil
data_gas <- astsa::gas
```

```
ts.plot(data_oil, data_gas, gpars = list(col = c("black", "red")))
#alternative
autoplot(ts(cbind(data_oil, data_gas), start = 2000, frequency = 52)) +
           ylab("Price of Oil and Gas") +xlab("Year") +
           ggtitle("Price of Oil and Gas vs. Years")
set.seed(12345)
autoplot(ts(cbind(log(data_oil), log(data_gas)), start = 2000, frequency = 52)) +
           ylab("Log of price of Oil and Gas") +xlab("Year") +
           ggtitle("Log of price of Oil and Gas vs. Years")
set.seed(12345)
autoplot(ts(diff(data_oil, differences = 1), start = 2000, frequency = 52)) +
           ylab("Price of Oil") +xlab("Year") +
           ggtitle("Price of Oil with Diff 1 vs. Years")
autoplot(ts(diff(data_gas, differences = 1), start = 2000, frequency = 52)) +
           ylab("Price of Gas") +xlab("Year") +
           ggtitle("Price of Gas with Diff 1 vs. Years")
ggAcf(diff(data_oil, differences = 1), data_oil)
ggAcf(diff(data_gas, differences = 1), data_gas)
set.seed(12345)
oil_price_one_diff <- diff(data_oil, differences = 1)</pre>
gas_price_one_diff <- diff(data_gas, differences = 1)</pre>
df <- data.frame(oil_price_one_diff=as.matrix(oil_price_one_diff),</pre>
           gas_price_one_diff=as.matrix(gas_price_one_diff),
           time=time(oil_price_one_diff))
one_week_lag <- ksmooth(x = df\soil_price_one_diff, y = lag(df\square\gammagas_price_one_diff), bandwidth = 0.04, ke
two_week_lag <- ksmooth(x = df$oil_price_one_diff, y = lag(df$gas_price_one_diff,2), bandwidth = 0.04,
three_week_lag <- ksmooth(x = df$oil_price_one_diff, y = lag(df$gas_price_one_diff,3), bandwidth = 0.04
set.seed(12345)
```