

Time Series (732A62) Lab1 Group 7

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02 October, 2019

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Assignment 1. Computations with simulated data

In table 1 a script for generation of data from simulation of the following state space model and implementation of the Kalman filter on the data is given.

$$Z_t = A_{t-1}Z_{t-1} + e_t$$

$$x_t = C_t z_t + \nu_t$$

$$\nu_t \sim N(0, R_t)$$

$$e_t \sim N(0, Q_t)$$

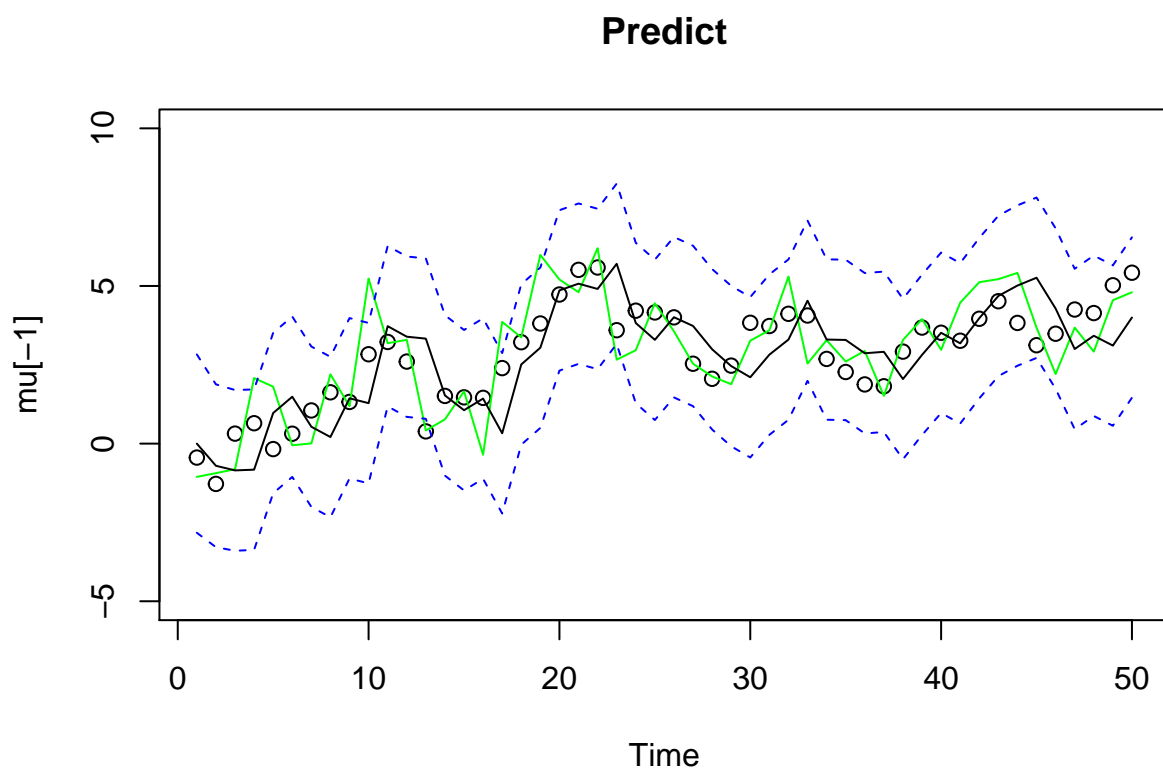
a) Write down the expression for the state space model that is being simulated.

$$z_k = z_{k-1} + N(0, Q_t)$$

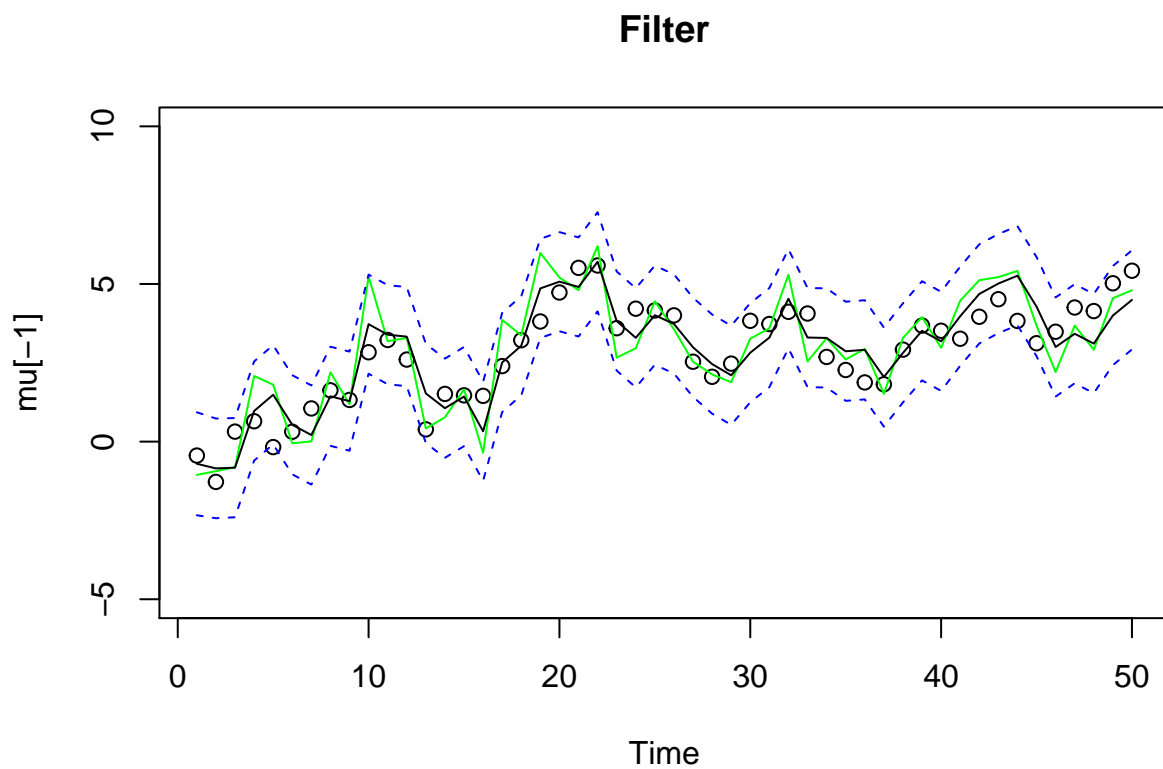
$$x_k = z_k + N(0, R_t)$$

b) Run this script and compare the filtering results with a moving average smoother of order 5.

```
# generate dataset
set.seed(1)
num = 50
w = rnorm(num+1,0,1)
v = rnorm(num ,0,1)
mu = cumsum(w) # state: mu[0], mu[1],..., mu[50]
y = mu[-1] + v # obs: y[1],..., y[50]
# filter and smooth (Ksmooth0 does both)
ks = Ksmooth0(num , y, A=1, mu0=0, Sigma0=1, Phi=1, cQ=1, cR=1)
# start figurepar(mfrow=c(3,1))
Time = 1:num
plot(Time , mu[-1], main="Predict", ylim=c(-5,10))
lines(Time ,y,col="green")
lines(ks$xp)
lines(ks$xp+2*sqrt(ks$Pp), lty=2, col=4)
lines(ks$xp -2*sqrt(ks$Pp), lty=2, col=4)
```

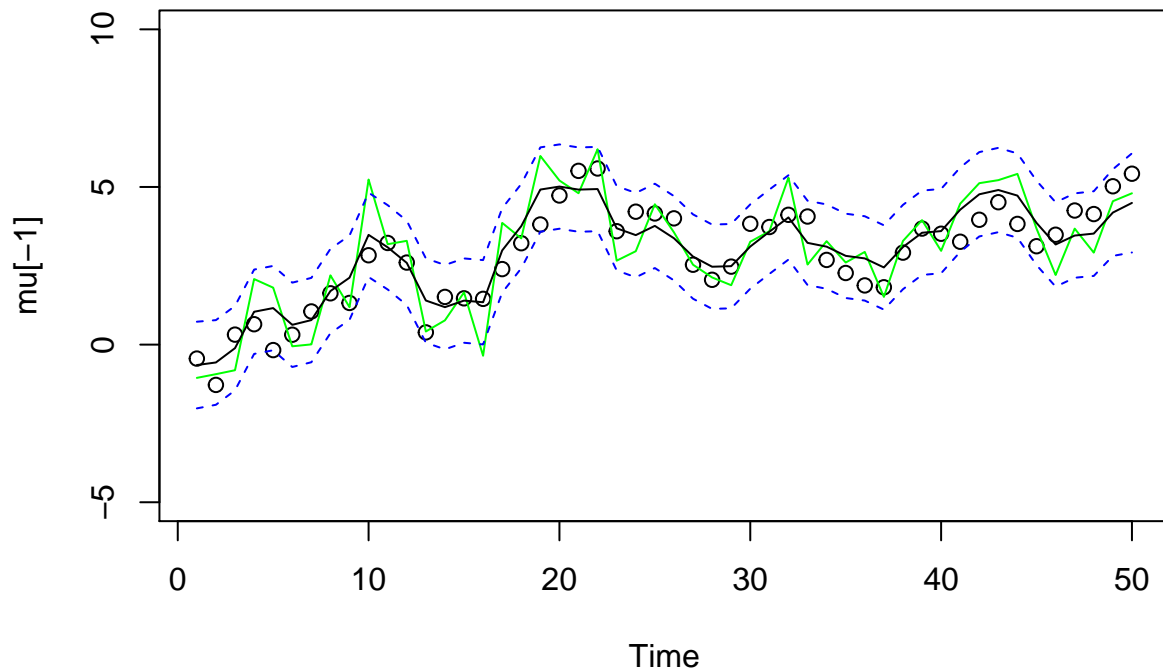


```
plot(Time , mu[-1], main="Filter", ylim=c(-5,10))
lines(Time ,y,col="green")
lines(ks$xf)
lines(ks$xf+2*sqrt(ks$Pf), lty=2, col=4)
lines(ks$xf -2*sqrt(ks$Pf), lty=2, col=4)
```



```
plot(Time , mu[-1], main="Smooth", ylim=c(-5,10))
lines(Time ,y,col="green")
lines(ks$xs)
lines(ks$xs+2*sqrt(ks$Ps), lty=2, col=4)
lines(ks$xs-2*sqrt(ks$Ps), lty=2, col=4)
```

Smooth



```
mu[-1]
```

```
## [1] -0.6264538
```

```
ks$x0n
```

```
## [1,]
```

```
## [1,] -0.3241541
```

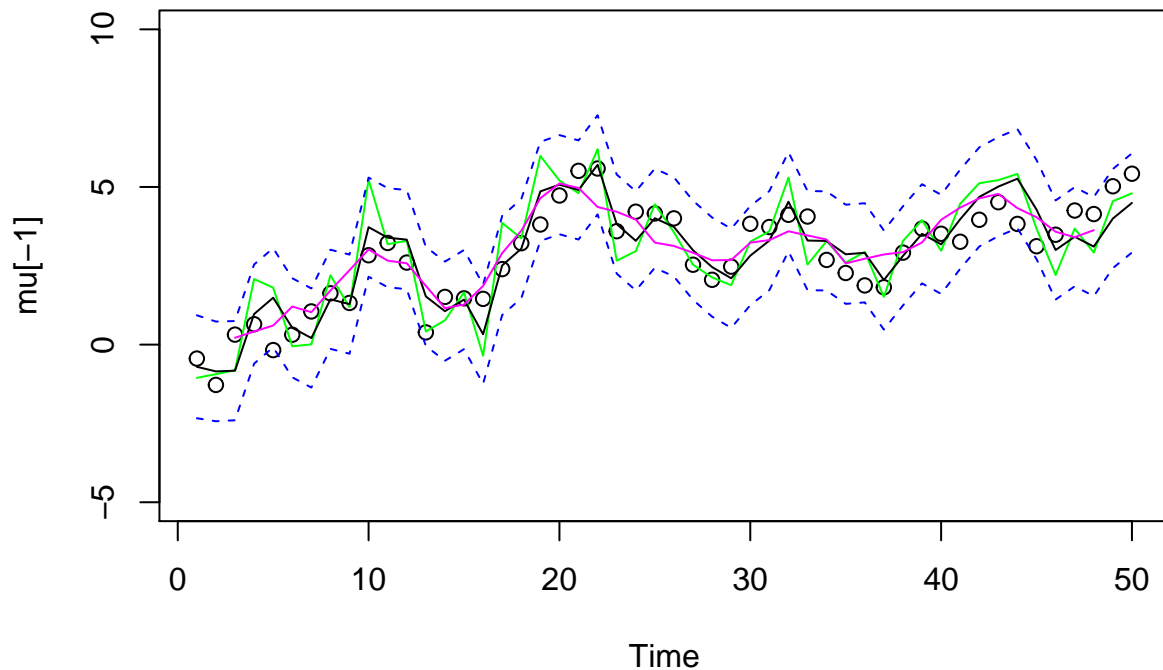
```
sqrt(ks$P0n) # initial value info
```

```
## [1,]
```

```
## [1,] 0.7861514
```

```
# filtering with moving average 5
plot(Time , mu[-1], ylim=c(-5,10), main="Moving average smoothing with order 5")
lines(Time ,y,col="green")
lines(ks$xf)
lines(ks$xf+2*sqrt(ks$Pf), lty=2, col=4)
lines(ks$xf -2*sqrt(ks$Pf), lty=2, col=4)
lines(ma(y, order=5, ), col=6)
```

Moving average smoothing with order 5

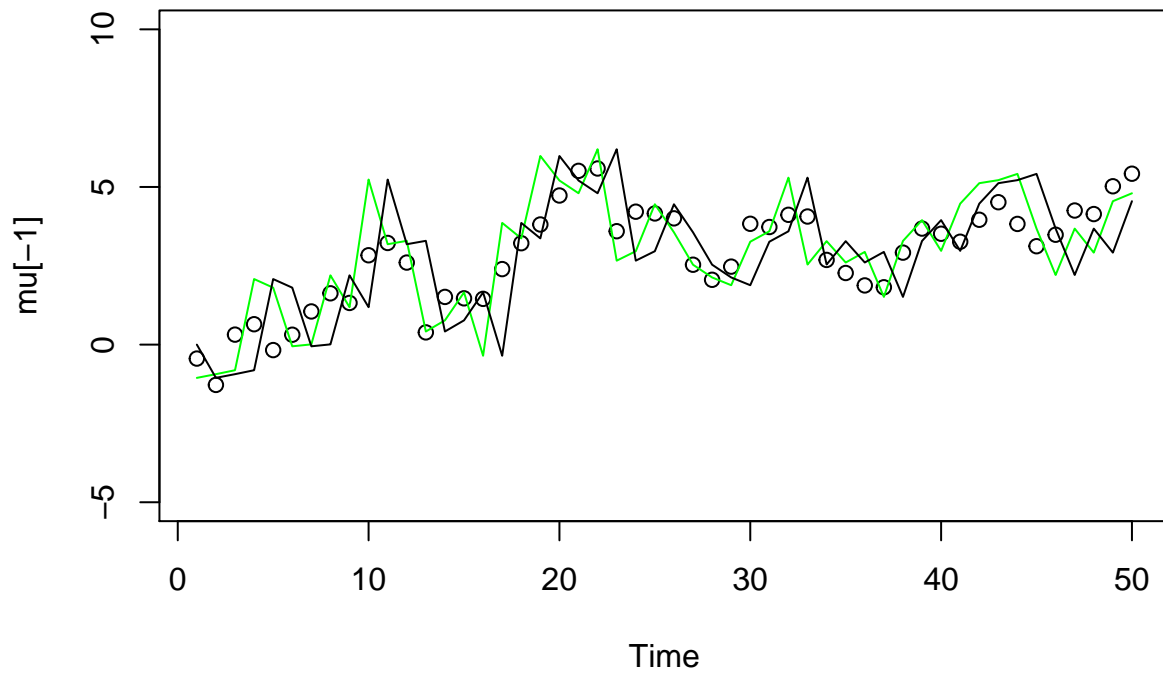


Analysis: We find that the moving average smoothing function with order 5 is the best fit but not overfitting.

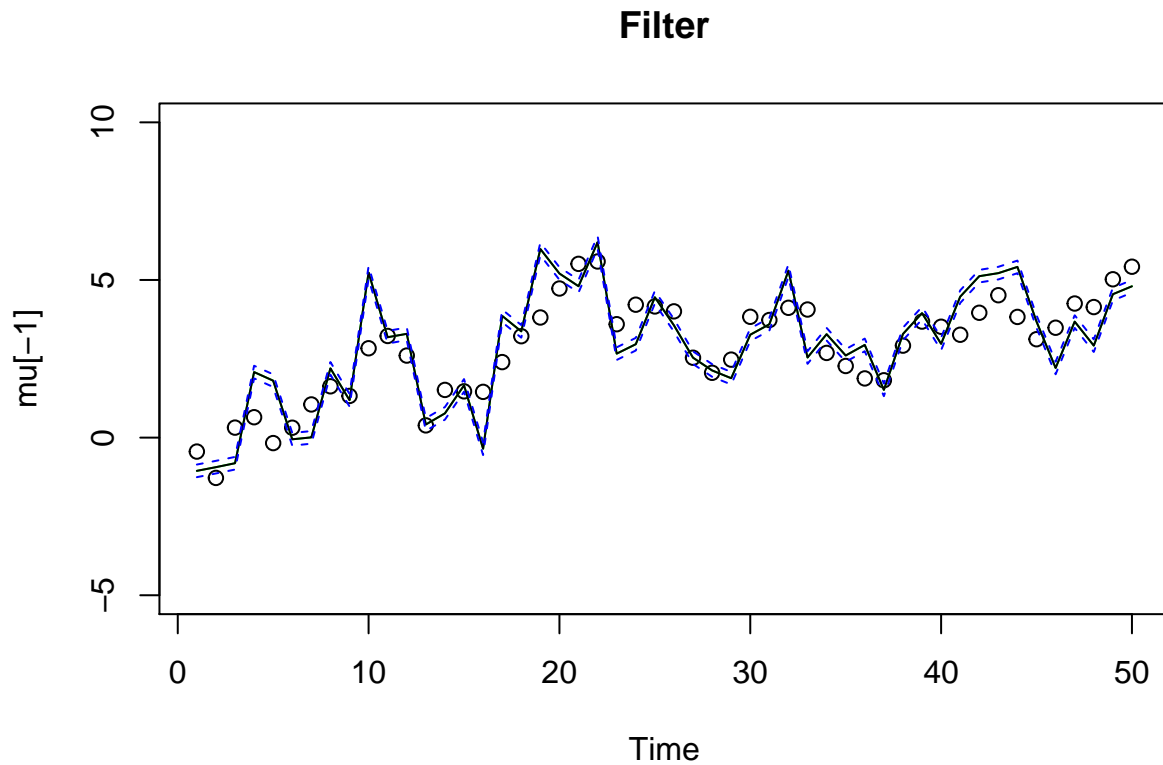
c) Also, compare the filtering outcome when R in the filter is 10 times smaller than its actual value, while Q in the filter is 10 times larger than its actual value. How does the filtering outcome varies?

```
# filter and smooth (Ksmooth0 does both)
ks = Ksmooth0(num , y, A=1, mu0=0, Sigma0=1, Phi=1, cQ=10, cR=0.1)
# start figurepar(mfrow=c(3,1))
Time = 1:num
plot(Time , mu[-1], main="Predict", ylim=c(-5,10))
lines(Time ,y,col="green")
lines(ks$xp)
lines(ks$xp+2*sqrt(ks$Pp), lty=2, col=4)
lines(ks$xp-2*sqrt(ks$Pp), lty=2, col=4)
```

Predict



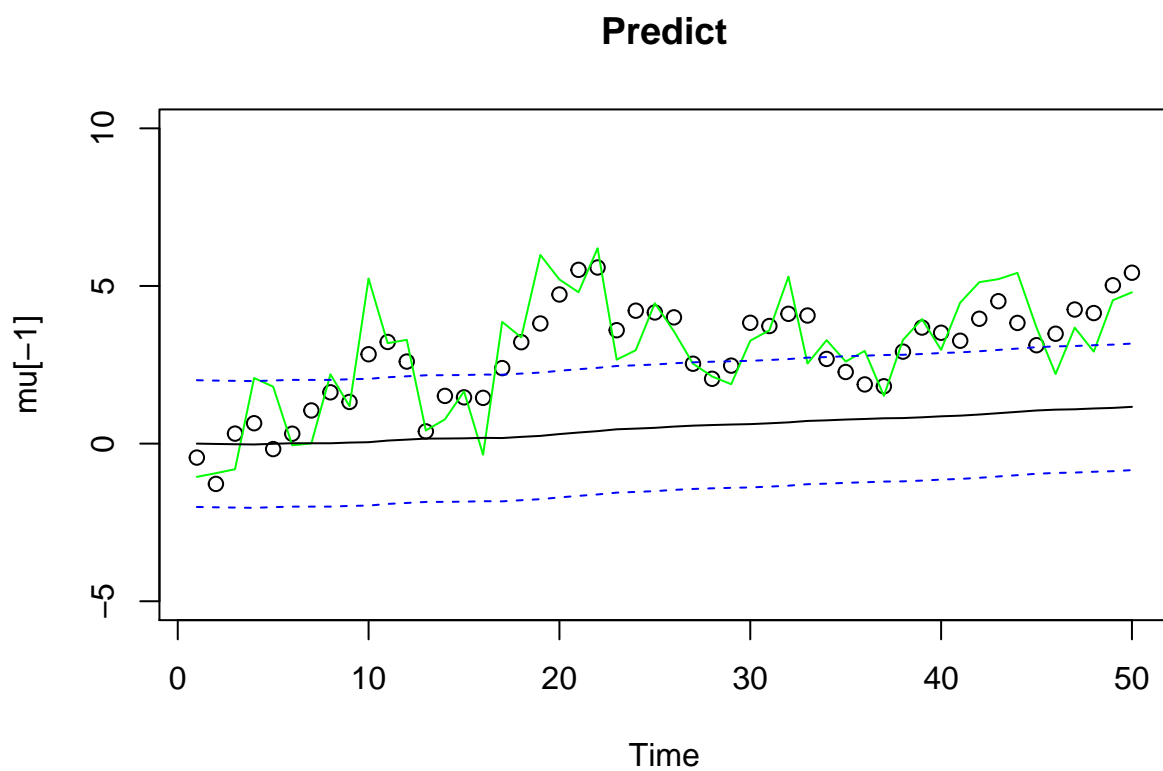
```
plot(Time , mu[-1], main="Filter", ylim=c(-5,10))
lines(Time ,y,col="green")
lines(ks$xf)
lines(ks$xf+2*sqrt(ks$Pf), lty=2, col=4)
lines(ks$xf -2*sqrt(ks$Pf), lty=2, col=4)
```



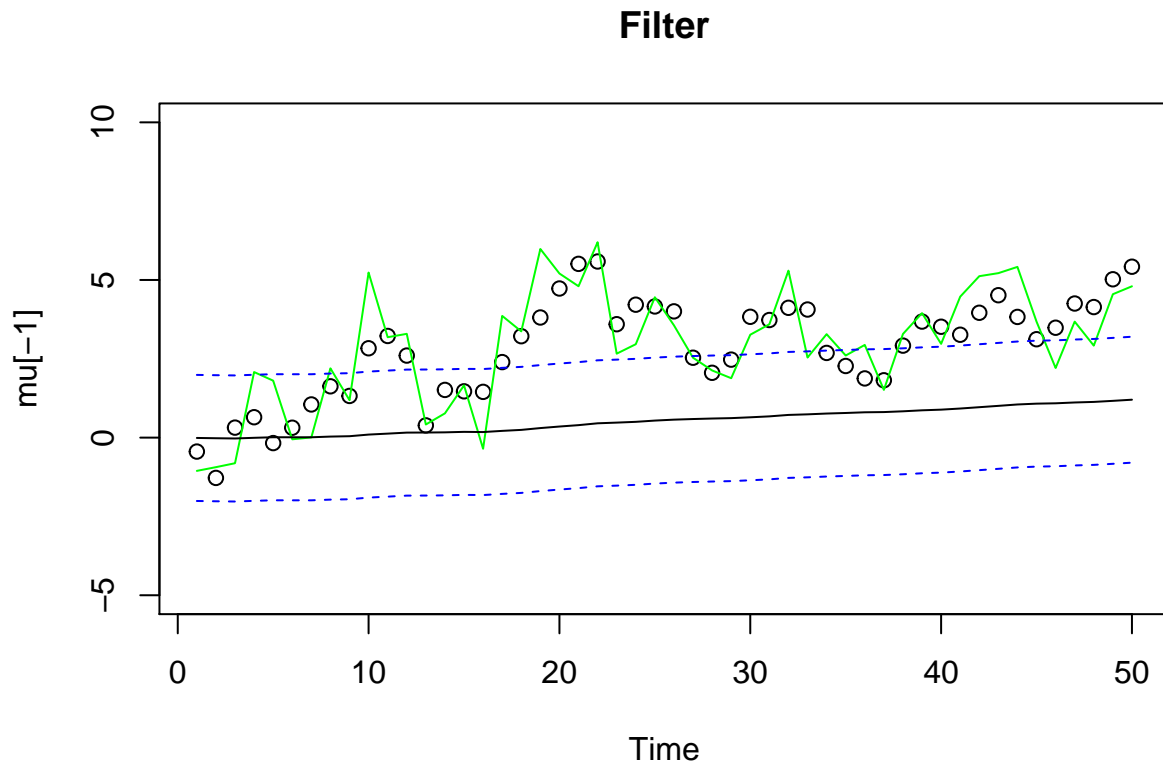
Analysis: We find that the filtering is resembles the true value much more than the previous values.

d) Now compare the filtering outcome when R in the filter is 10 times larger than its actual value, while Q in the filter is 10 times smaller than its actual value. How does the filtering outcome varies?

```
# filter and smooth (Ksmooth0 does both)
ks = Ksmooth0(num , y, A=1, mu0=0, Sigma0=1, Phi=1, cQ=0.1, cR=10)
# start figurepar(mfrow=c(3,1))
Time = 1:num
plot(Time , mu[-1], main="Predict", ylim=c(-5,10))
lines(Time ,y,col="green")
lines(ks$xp)
lines(ks$xp+2*sqrt(ks$Pp), lty=2, col=4)
lines(ks$xp-2*sqrt(ks$Pp), lty=2, col=4)
```

```
plot(Time , mu[-1], main="Filter", ylim=c(-5,10))
lines(Time ,y,col="green")
lines(ks$xf)
lines(ks$xf+2*sqrt(ks$Pf), lty=2, col=4)
lines(ks$xf -2*sqrt(ks$Pf), lty=2, col=4)
```



Analysis: We find that the filtering is far off from resembling the true value and hardly moving in terms of the predicted values.

e) Implement your own Kalman filter and replace ksmooth0 function with your script.

```
#Times: Number of time steps
#A: How the mean of z_t will be affected by z_(t-1) Used in transition model
#C: Scale the mean (z_t) in the emission model
#Q: Covariate matrix in the emission model
#R: Covariate matrix in the transition model
#y: All observations
#mu0: first mean
#sigma0: first variance
my_kalman <- function(Times, y, A, mu0, Sigma0, C, Q, R) {
  mu <- rep(1, Times)
  sigma <- rep(1, Times)
  my_unweighted <- rep(1, Times)
  sigma_unweighted <- rep(1, Times)
  kalman_gain <- rep(1, Times)

  # Our best guess is that my_1 is mu0 else it could be the first observation
  mu[1] <- mu0

  # We don't know what sigma_1 is, so we choose 1 else it would be provided
  sigma[1] <- Sigma0

  for (t in 2:Times) {
```

```

# Calculate the unweighted prediction of the mean
my_unweighted[t] <- A[t]%%mu[t - 1]

# Calculate the unweighted prediction of the covariate matrix
sigma_unweighted[t] <- A[t]%%sigma[t - 1]%%t(A[t]) + R[t]

# Kalman gain Used to weight between our unweighted prediction and the obs
kalman_gain[t] <- sigma_unweighted[t]%%t(C[t]) %% solve(C[t]%%sigma_unweighted[t]%%t(C[t] + Q[t]))

# Calculate the weighted mean, thus our prediction of the hidden state
mu[t] <- my_unweighted[t] + kalman_gain[t]%%(y[t] - C[t]%%my_unweighted[t])

# Calculate the weighted covariance matrix, thus our prediction of the prediction error
sigma[t] <- (1 - kalman_gain[t]%%C[t])%%sigma_unweighted[t]
}

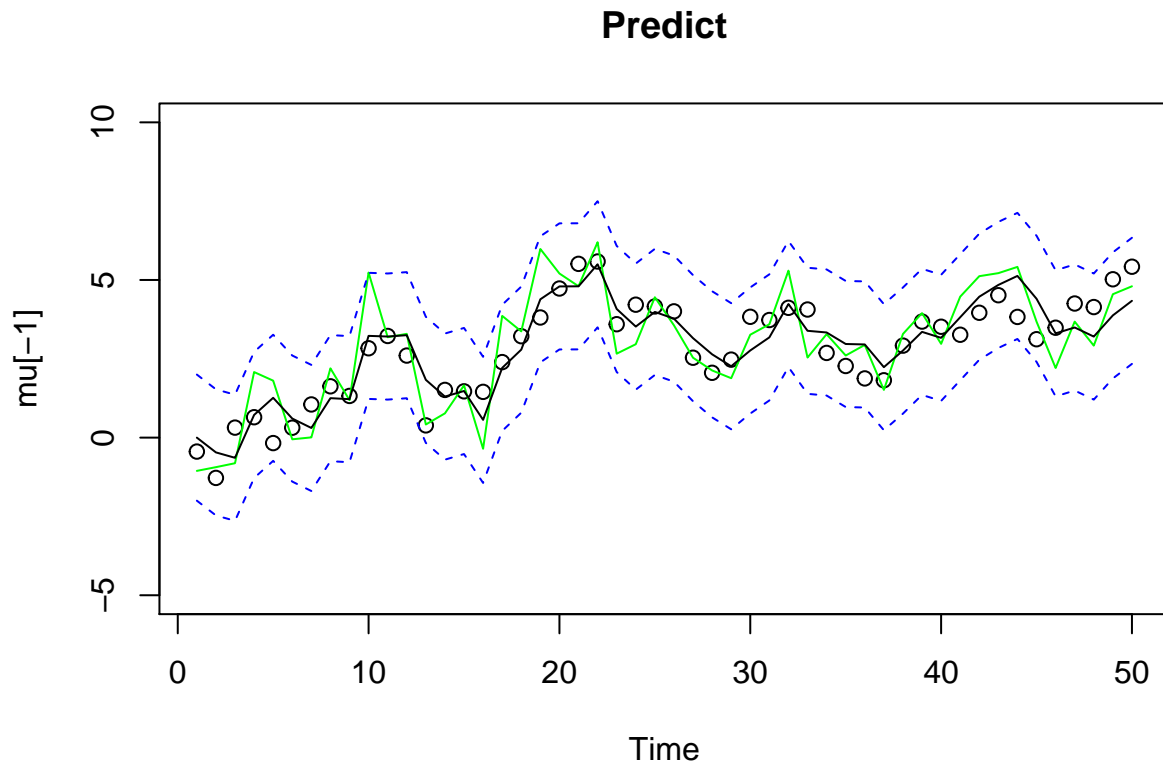
return (list(mu = mu, sigma = sigma))
}

# generate dataset
set.seed(1)
num = 50
w = rnorm(num+1,0,1)
v = rnorm(num ,0,1)
mu = cumsum(w) # state: mu[0], mu[1],..., mu[50]
y = mu[-1] + v # obs: y[1],..., y[50]

my_ks = my_kalman(Times=num , y=y, A=rep(1,num), mu0=0, Sigma0=1,
                  C=rep(1,num), Q=rep(1,num), R=rep(1,num))

plot(Time , mu[-1], main="Predict", ylim=c(-5,10))
lines(Time ,y,col="green")
lines(my_ks$mu)
lines(my_ks$mu+2*sqrt(my_ks$sigma), lty=2, col=4)
lines(my_ks$mu -2*sqrt(my_ks$sigma), lty=2, col=4)

```



f) How do you interpret the Kalman gain?

Analysis: Kalman gain is given by $K = \frac{P_k H_k^T}{P_k H_k^T + R_k}$ where you will realize that the relative magnitudes of matrices R_k and P_k control a relation between the filter's use of predicted state estimate z_t and measurement x_t .

When R_k tends to zero then $x_t = x_{t-1} + K(y_t - H_k)$ suggests that when the magnitude of R is small, meaning that the measurements are accurate, the state estimate depends mostly on the measurements.

When the state is known accurately, then numerator is small compared to R , and the filter mostly ignores the measurements relying instead on the prediction derived from the previous state.

Appendix

```
knitr::opts_chunk$set(echo = TRUE)
options(scipen=999)

library("tidyverse") #ggplot and dplyr
library("gridExtra") # combine plots
library("knitr") # for pdf
library("astsa") #dataset oil and gas is present here
library("matlib") # for inv and I
library("forecast")
```

```

# The palette with black:
cbbPalette <- c("#000000", "#E69F00", "#56B4E9", "#009E73",
               "#F0E442", "#0072B2", "#D55E00", "#CC79A7")
set.seed(12345)

# generate dataset
set.seed(1)
num = 50
w = rnorm(num+1,0,1)
v = rnorm(num ,0,1)
mu = cumsum(w) # state: mu[0], mu[1],..., mu[50]
y = mu[-1] + v # obs: y[1],..., y[50]
# filter and smooth (Ksmooth0 does both)
ks = Ksmooth0(num , y, A=1, mu0=0, Sigma0=1, Phi=1, cQ=1, cR=1)
# start figurepar(mfrow=c(3,1))
Time = 1:num
plot(Time , mu[-1], main="Predict", ylim=c(-5,10))
lines(Time ,y,col="green")
lines(ks$xp)
lines(ks$xp+2*sqrt(ks$Pp), lty=2, col=4)
lines(ks$xp -2*sqrt(ks$Pp), lty=2, col=4)
plot(Time , mu[-1], main="Filter", ylim=c(-5,10))
lines(Time ,y,col="green")
lines(ks$xf)
lines(ks$xf+2*sqrt(ks$Pf), lty=2, col=4)
lines(ks$xf -2*sqrt(ks$Pf), lty=2, col=4)
plot(Time , mu[-1], main="Smooth", ylim=c(-5,10))
lines(Time ,y,col="green")
lines(ks$xs)
lines(ks$xs+2*sqrt(ks$Ps), lty=2, col=4)
lines(ks$xs-2*sqrt(ks$Ps), lty=2, col=4)
mu[1]
ks$x0n
sqrt(ks$P0n) # initial value info

# filering with moving average 5
plot(Time , mu[-1], ylim=c(-5,10), main="Moving average smoothing with order 5")
lines(Time ,y,col="green")
lines(ks$xf)
lines(ks$xf+2*sqrt(ks$Pf), lty=2, col=4)
lines(ks$xf -2*sqrt(ks$Pf), lty=2, col=4)
lines(ma(y, order=5, ), col=6)

# filter and smooth (Ksmooth0 does both)
ks = Ksmooth0(num , y, A=1, mu0=0, Sigma0=1, Phi=1, cQ=10, cR=0.1)
# start figurepar(mfrow=c(3,1))
Time = 1:num
plot(Time , mu[-1], main="Predict", ylim=c(-5,10))
lines(Time ,y,col="green")
lines(ks$xp)
lines(ks$xp+2*sqrt(ks$Pp), lty=2, col=4)
lines(ks$xp -2*sqrt(ks$Pp), lty=2, col=4)

```

```

plot(Time , mu[-1], main="Filter", ylim=c(-5,10))
lines(Time ,y,col="green")
lines(ks$xf)
lines(ks$xf+2*sqrt(ks$Pf), lty=2, col=4)
lines(ks$xf -2*sqrt(ks$Pf), lty=2, col=4)
# filter and smooth (Ksmooth0 does both)
ks = Ksmooth0(num , y, A=1, mu0=0, Sigma0=1, Phi=1, cQ=0.1, cR=10)
# start figurepar(mfrow=c(3,1))
Time = 1:num
plot(Time , mu[-1], main="Predict", ylim=c(-5,10))
lines(Time ,y,col="green")
lines(ks$xp)
lines(ks$xp+2*sqrt(ks$Pp), lty=2, col=4)
lines(ks$xp -2*sqrt(ks$Pp), lty=2, col=4)
plot(Time , mu[-1], main="Filter", ylim=c(-5,10))
lines(Time ,y,col="green")
lines(ks$xf)
lines(ks$xf+2*sqrt(ks$Pf), lty=2, col=4)
lines(ks$xf -2*sqrt(ks$Pf), lty=2, col=4)
#Times: Number of time steps
#A: How the mean of z_t will be affected by z_(t-1) Used in transition model
#C: Scale the mean (z_t) in the emission model
#Q: Covariate matrix in the emission model
#R: Covariate matrix in the transition model
#y: All observations
#mu0: first mean
#sigma0: first variance
my_kalman <- function(Times, y, A, mu0, Sigma0, C, Q, R) {
  mu <- rep(1,Times)
  sigma <- rep(1,Times)
  my_unweighted <- rep(1,Times)
  sigma_unweighted <- rep(1,Times)
  kalman_gain <- rep(1,Times)

  # Our best guess is that my_1 is mu0 else it could be the first observation
  mu[1] <- mu0

  # We don't know what sigma_1 is, so we choose 1 else it would be provided
  sigma[1] <- Sigma0

  for (t in 2:Times) {
    # Calculate the unweighted prediction of the mean
    my_unweighted[t] <- A[t]*%*%mu[t - 1]

    # Calculate the unweighted prediction of the covariate matrix
    sigma_unweighted[t] <- A[t]*%*%sigma[t - 1]*%*%t(A[t]) + R[t]

    # Kalman gain Used to weight between our unweighted prediction and the obs
    kalman_gain[t] <- sigma_unweighted[t]*%*%t(C[t]) %*% solve(C[t]*%*%sigma_unweighted[t]*%*%t(C[t] + Q[t]))

    # Calculate the weighted mean, thus our prediction of the hidden state
    mu[t] <- my_unweighted[t] + kalman_gain[t]*%*%(y[t] - C[t]*%*%my_unweighted[t])
  }
}

```

```

    # Calculate the weighted covariance matrix, thus our prediction of the prediction error
    sigma[t] <- (1 - kalman_gain[t]%%C[t])%%sigma_unweighted[t]
  }

  return (list(mu = mu, sigma = sigma))
}

# generate dataset
set.seed(1)
num = 50
w = rnorm(num+1,0,1)
v = rnorm(num ,0,1)
mu = cumsum(w) # state: mu[0], mu[1],..., mu[50]
y = mu[-1] + v # obs: y[1],..., y[50]

my_ks = my_kalman(Times=num , y=y, A=rep(1,num), mu0=0, Sigma0=1,
                  C=rep(1,num), Q=rep(1,num), R=rep(1,num))

plot(Time , mu[-1], main="Predict", ylim=c(-5,10))
lines(Time ,y,col="green")
lines(my_ks$mu)
lines(my_ks$mu+2*sqrt(my_ks$sigma), lty=2, col=4)
lines(my_ks$mu -2*sqrt(my_ks$sigma), lty=2, col=4)

```