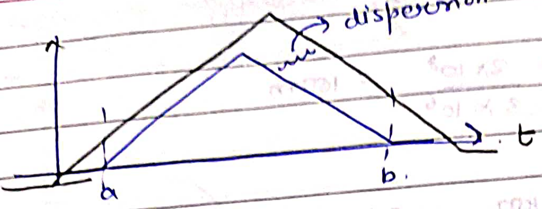
Date: / /
Page No.

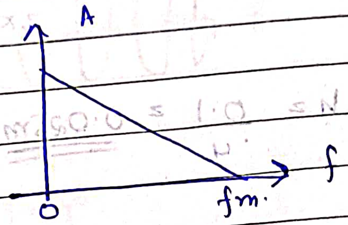
DTM.



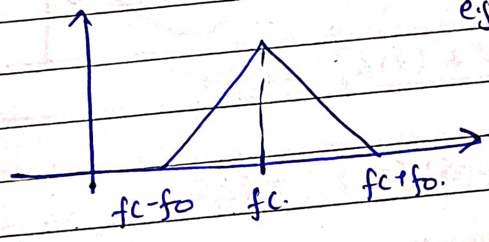
(Linear distortion).

→ Mainly occurs when we try to send the two pulses.

* Base band signal :-



* Bandpass signal :-



e.g. visible light. or radio spectrum

→ Height of Antenna

$$h = \frac{\lambda}{4}$$ → wavelength of the signal.

voice = 30Hz → 3300 Hz.

$$\lambda = \frac{c}{v} = \frac{3\times10^8}{3300} = \frac{3\times10^8}{\approx 3\times10^3} = 10^5 m.$$

$= 100 km.$

$$h = \frac{100}{4} = 25 km$$ → practically not possible.

Date: / /
Page No.

e.g. if $\nu = 3MHz$.

$$\lambda = \frac{3\times10^8}{3\times10^6} = 100m.$$

$$h = \frac{100}{4} = 25m = \underline{\underline{0.25\,km}}.$$ practically possible.

if $\nu = 3GHz \Rightarrow \lambda = \frac{3\times10^8}{3\times10^9} = \underline{\underline{0.1m}}.$

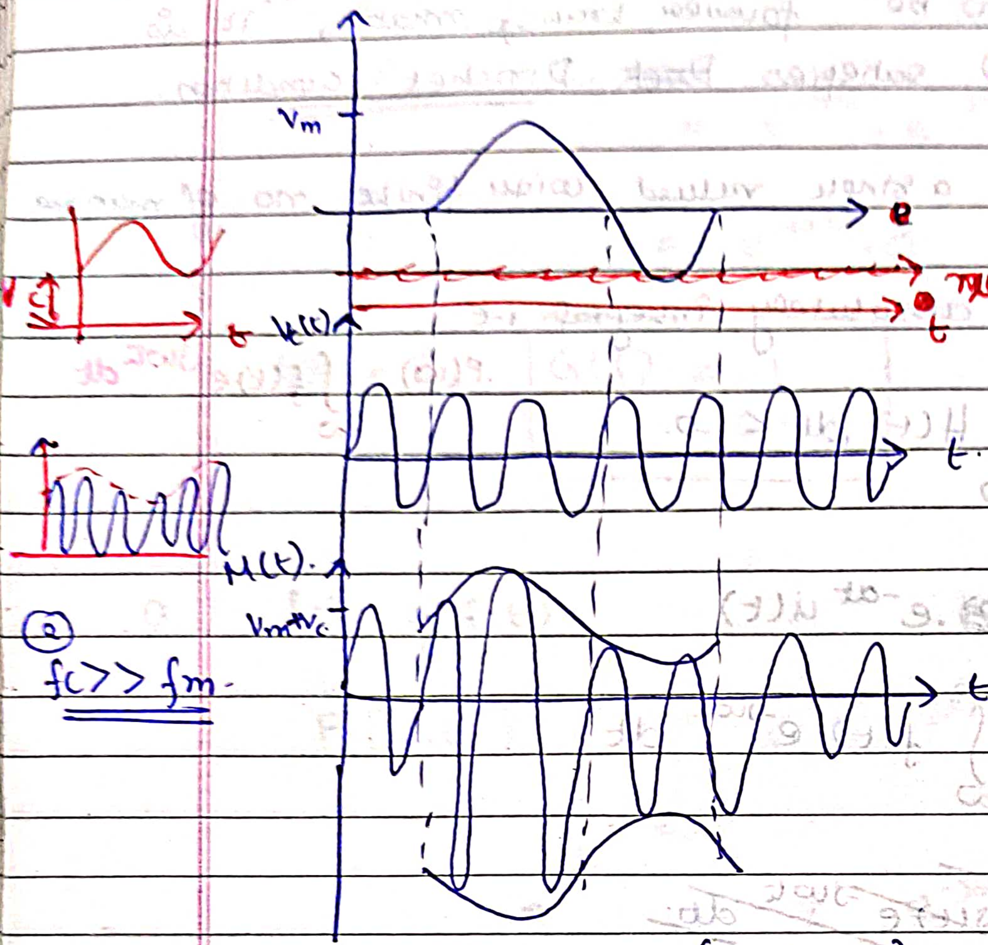
$$h = \frac{0.1}{4} = \underline{\underline{0.025m}}.$$ practically possible.

→ low freq. signal is thats why super impossed on high freq. signal so, that the height of antenna is practically possible.

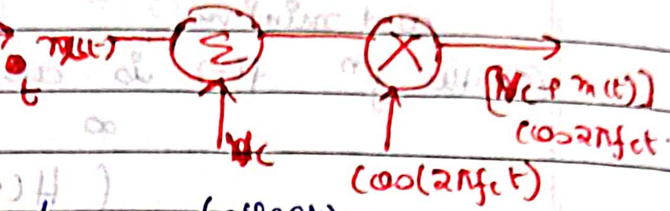
Amplitude Modulation.

OR DOUBLE SIDE BAND FULL CARRIER.

$M(t) = x(t)\cos\omega_c t + V_c \cos\omega_c t$



② $f_c >> f_m$



(offset) ↳ not zero crossing

→ If offset is 0, then it is DSB-SC.

① $(V_c + x(t) > 0)$.
↳ avoid zero crossing.

$$M(t) = (x(t) + V_c)\cos\omega_c t$$

$$M(t) = x(t)\cos\omega_c(t) + V_c\cos\omega_c(t)$$

$$\boxed{V_c \cos\omega_c(t) = V_c\,\pi\{\delta(\omega-\omega_c) + \delta(\omega+\omega_c)\}} \rightarrow ①$$

$$x(t)\cos\omega_c(t) = x(t)\left[\frac{e^{j\omega_c(t)} + e^{-j\omega_c(t)}}{2}\right]$$

$$= \frac{1}{2}x(t)\left[e^{j\omega_c(t)} + e^{-j\omega_c(t)}\right]$$

$$= \frac{1}{2}\left[X(\omega-\omega_c) + X(\omega+\omega_c)\right].$$

Date: / /
Page No.

FOURIER TRANSFORM. → used for aperiodic signal.

→ for a f^n $f(t)$ to be fourier transformable, it is sufficient that $f(t)$ satisfies ~~Both~~ Dirichet's condition.

(1) The f^n $f(t)$ is a single valued with finite no. of maxima and minima.

(2) The f^n $f(t)$ is absolutely integral i.e.

$$\int_{-\infty}^{\infty} |f(t)|\,dt < \infty.$$

$$F(\omega) = \int_{-\infty}^{\infty} f(t) e^{-j\omega t}\,dt$$

e.g. find f.T. of $e^{-at}u(t)$

$$F(\omega) = \int_{-\infty}^{\infty} f(t)\, e^{-j\omega t}\, dt$$

$$= \cancel{\int_{-\infty}^{\infty} f(t)\, e^{-j\omega t}\, dt}$$

$$= \int_{-\infty}^{\infty} e^{-at} u(t)\, e^{-j2\pi f t}\, dt$$

$$= \int_{0}^{\infty} e^{-at} e^{-j2\pi f t}\, dt$$

$$= \int_{0}^{\infty} e^{-(at+j2\pi f t)}\, dt = \frac{-1}{a+j2\pi f} \left. e^{-(a+j2\pi f)t} \right]_0^{\infty}$$

But ~~$|e^{-j2\pi ft}| = 1$~~ $= \frac{1}{a+j2\pi f}[0-1] = \frac{1}{a+j2\pi f}$

$\therefore$ as $t \to \infty$

$$e^{-(a+j2\pi f)t} = e^{-at} \cdot e^{-j2\pi ft}$$

$$= e^{-at} \cdot 0 = \underline{\underline{0}}$$

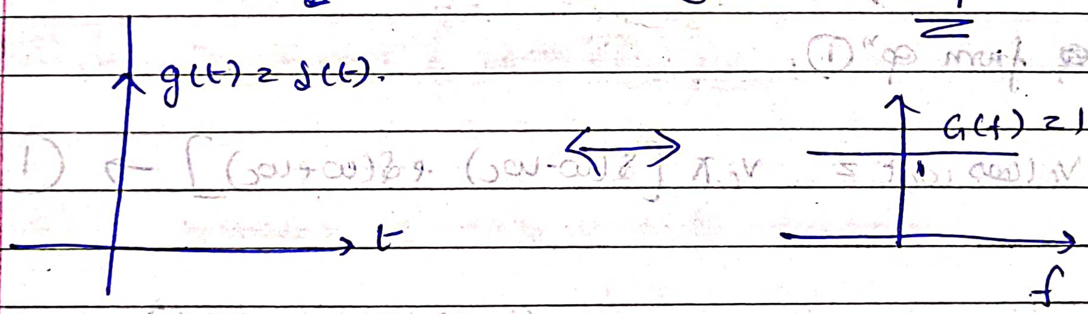
$\therefore$ $\boxed{G(f) = \frac{1}{a+j\omega}}$

Q. F.T. of $\delta(t)$.

$$F[\delta(t)] = \int_{-\infty}^{\infty} \delta(t)\, e^{-j2\pi ft}\, dt$$

↳ at $t = 0$.

$$= e^{-j2\pi f(0)} = \underline{\underline{1}}.$$



Q. F.T. of $\cos 2\pi f_0$.

$$\cos 2\pi f_0 t = \frac{1}{2}\left[e^{j2\pi f_0 t} + e^{-j2\pi f_0 t}\right]$$

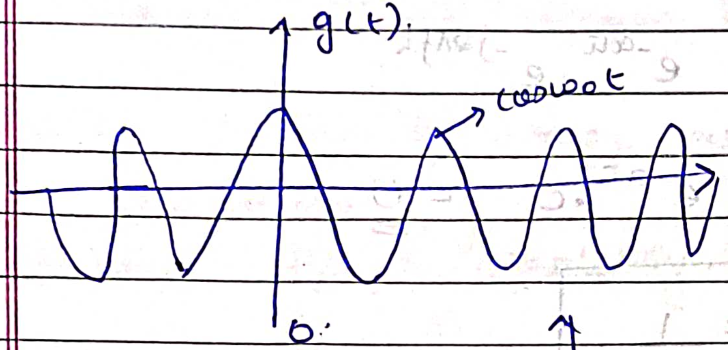
↳ Euler formula.

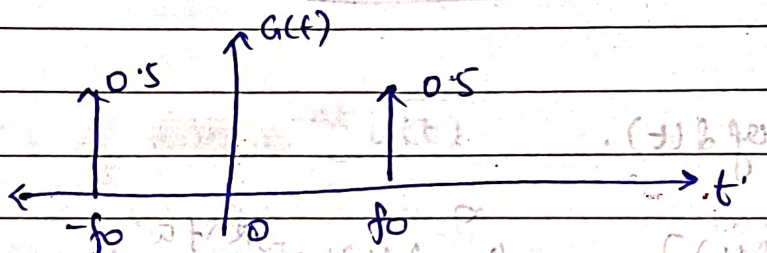
$\delta(f-f_0) \leftrightarrow e^{j2\pi f_0 t}$.

$\delta(f+f_0) \leftrightarrow e^{-j2\pi f_0 t}$.

Date : / /
Page No.

$$\boxed{\cos 2\pi f_0 t \leftrightarrow \frac{1}{2}\left[\delta(f+f_0) + \delta(f-f_0)\right]}$$





A.M.

→ from eqn (1)

$$V_c \cos\omega_c t = V_c\pi\left[\delta(\omega-\omega_c) + \delta(\omega+\omega_c)\right] \to (1)$$

$$x(t)\cos\omega_c(t) = x(t)\left[\frac{e^{j\omega_c t} + e^{-j\omega_c t}}{2}\right]$$

$$= \frac{1}{2}\left[X(\omega-\omega_c) + X(\omega+\omega_c)\right] \to (2)$$

adding (1) + (2)

$$M(t) = V_c\pi\left[\delta(\omega-\omega_c) + \delta(\omega+\omega_c)\right] + \frac{1}{2}\left[X(\omega-\omega_c) + X(\omega+\omega_c)\right]$$

Date : / /
Page No.

$\boxed{\text{Single tone Modulation}}$ → Single freq. of message signal.

$x(t) = V_m \cos \omega_m t$

$\omega_m = 2\pi f_m$

$M(t) = [V_m \cos \omega_m t + V_c] \cos \omega_c t$

$M(t) = V_c \cos \omega_c t \left[\frac{V_m}{V_c} \cos \omega_m t + 1 \right]$

$\boxed{\frac{V_m}{V_c} = m}$ → modulation index.

[It defines to what extent carrier signal is modulated by msg signal]

$m \leq 1$ → avoid zero crossing

$\Rightarrow M(t) = V_c \cos \omega_c t \left[1 + m \cos \omega_m t \right]$ → ③

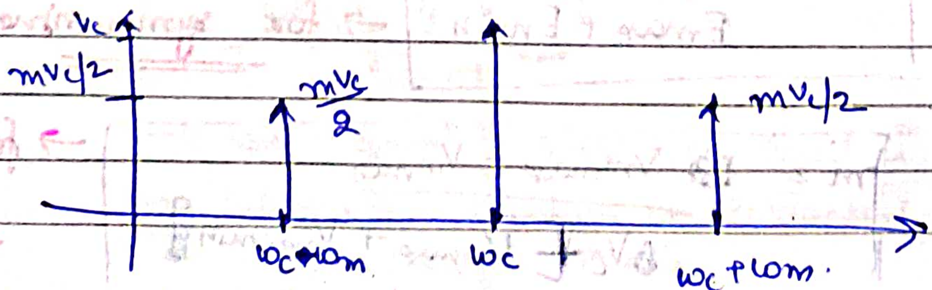
~~$M(t) = V_c \cos \omega_c t \left[1 + m\, x(t) \right]$~~

$M(t) = V_c \cos \omega_c t + m V_c \cos \omega_c t \cos \omega_m t$

$= V_c \cos \omega_c t + \frac{m V_c}{2} \cdot 2 \cos \omega_c t \cdot \cos \omega_m t$

carrier signal.

$M(t) = V_c \cos \omega_c t + \frac{m V_c}{2} \left[\cos(\omega_c + \omega_m)t + \cos(\omega_c - \omega_m)t \right]$



Date: / /
Page No.

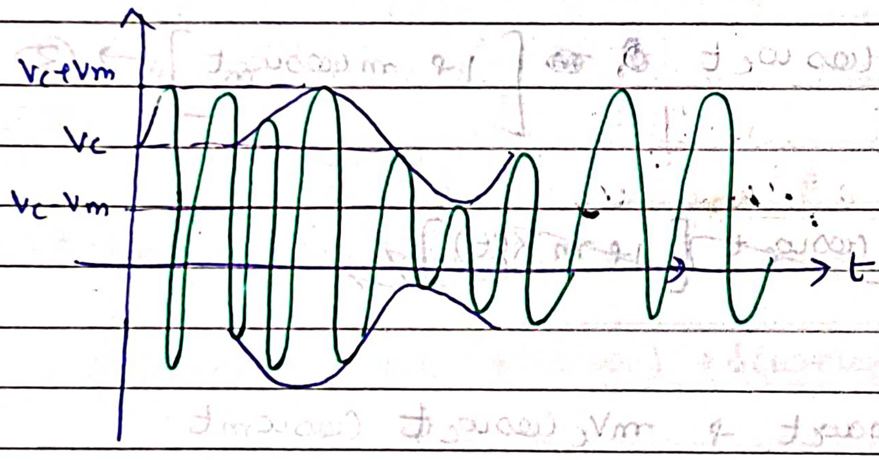
$M(t) = V_c \cos\omega_c t + \frac{mV_c}{2}\left[\cos(\omega_c - \omega_m) + \cos(\omega_c + \omega_m)\right]$

B.W of the modulated signal :-
↓
total range of freq.

B.W $= \omega_c + \omega_m - (\omega_c - \omega_m) = \underline{2\omega_m}$.

$\boxed{B.W = 2\omega_m}$ → B.W depends on the message signal.

Modulation index :- $\boxed{m = \frac{V_m}{V_c}}$



$E_{max} = V_c + V_m$.
$E_{min} = V_c - V_m$.

$V_c = \frac{E_{max} + E_{min}}{2}$

$V_m = \frac{E_{max} - E_{min}}{2}$

$\boxed{m = \frac{E_{max} - E_{min}}{E_{max} + E_{min}}}$ → for symm. signal

$\boxed{m = \frac{V_{m,max} - V_{m,min}}{2V_c + V_{m,max} + V_{m,min}}}$ → for Asymm. signal.

Date : / /
Page No.

⇒ (i) If ~~m~~ $m < 1$

$\frac{V_m}{V_c} < 1$

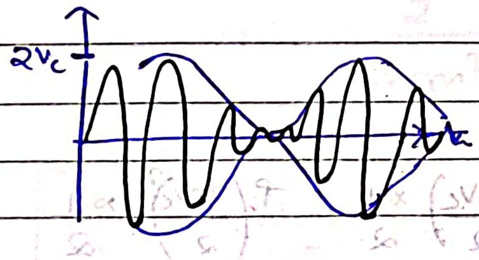
$V_m < V_c$

So, $V_c - V_m \rightarrow +ve$ ⇒ above diagram represents undermodulated.

(ii) $m = 1$

$\frac{V_m}{V_c} = 1$ ⇒ $V_m = V_c$.

h.h. ⇒ $V_c - V_m = V_c - V_c = 0$.

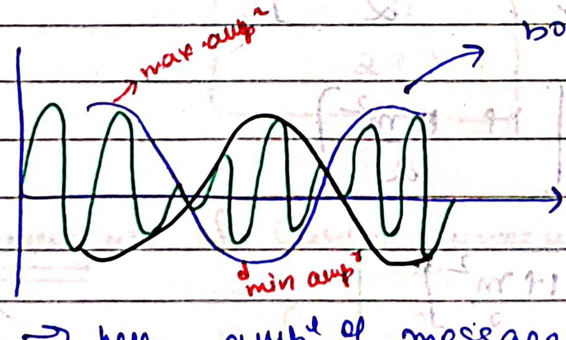


→ critical modulated (all info. is there in +ve wave).

(iii) $m > 1$

$\frac{V_m}{V_c} > 1$ ⇒ $V_m > V_c$.

⇒ $V_c - V_m < 0$



both the waves are overlapped.

⇒ over modulated.

→ here amp^d of message signal is in -ve side.

→ information cannot be extracted completely from seeing +ve side.

Date: / /
Page No.

Under modulated signal is better than over modulated signal.

POWER :-

$$P_c \to V_c \cos\omega_c t = \frac{V_c^2}{2} \quad \text{—(1)}$$

In the modulated signal :-

$$M(t) = V_c \cos\omega_c t + \frac{mV_c}{2}\left[\cos(\omega_c - \omega_m) + \cos(\omega_c + \omega_m)\right]$$

So, Total power :-

$$P_T = \frac{V_c^2}{2} + \left(\frac{mV_c}{2}\right)^2 \times \frac{1}{2} + \left(\frac{mV_c}{2}\right)^2 \times \frac{1}{2}$$

$$P_T = \frac{1}{2}\left[V_c^2 + \frac{m^2V_c^2}{4} + \frac{m^2V_c^2}{4}\right]$$

$$= \frac{1}{2}\left[V_c^2 + \frac{m^2V_c^2}{2}\right]$$

$$P_T = \frac{V_c^2}{2}\left[1 + \frac{m^2}{2}\right]$$

from (1)

$$\boxed{P_T = P_c\left[1 + \frac{m^2}{2}\right]}$$

Date : / /
Page No.

efficieny → Power tx by upper side band divided by ~~Total pow~~ ~~lower side band~~ total power.

$$\eta = \frac{P_{SB}}{P_T} = \frac{\left(\frac{m^2 V_c^2}{4} + \frac{m^2 V_c^2}{4}\right)\frac{1}{2}}{\frac{1}{2} V_c^2 \left[1 + \frac{m^2}{2}\right]}$$

$$= \frac{\frac{\cancel{V_c^2}}{\cancel{2}\times 4}\left[m^2 + m^2\right]}{\frac{\cancel{V_c^2}}{\cancel{2}}\left[1 + \frac{m^2}{2}\right]}$$

$$= \frac{\frac{m^2}{2}}{1 + \frac{m^2}{2}}$$

$$\boxed{\eta = \frac{m^2}{2 + m^2}}$$

Reduncy →

$$R = 1 - \eta$$

$$= 1 - \frac{m^2}{2 + m^2}$$

$$\boxed{R = \frac{2}{2 + m^2}}$$

Total current & Carrier current :-

$$\text{as } P_T = P_c \left[1 + \frac{m^2}{2}\right]$$

Radiation resistance of antenna.

$$I_T^2 \cancel{R} = I_c^2 \cancel{R} \left(1 + \frac{m^2}{2}\right)$$

$$I_T^2 = I_c^2 \left[1 + \frac{m^2}{2}\right]$$

$$\boxed{I_T = I_c \sqrt{1 + \frac{m^2}{2}}}$$

MULTI-TONE A.M. → here modulated msg. signal has multiple freq. components.

$$x(t) = V_1 \cos \omega_1 t + V_2 \cos \omega_2 t + V_3 \cos \omega_3 t$$

→ In single tone modulation :-

$$M(t) = \left\{1 + \frac{V_m}{V_c} \cos \omega_m t\right\} V_c \cos \omega_c t.$$

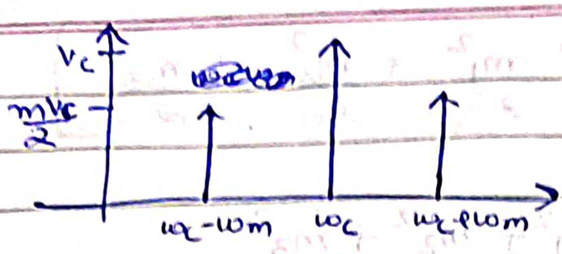
where $\frac{V_m}{V_c} = m$.

$$c(t) = V_c \cos \omega_c t.$$

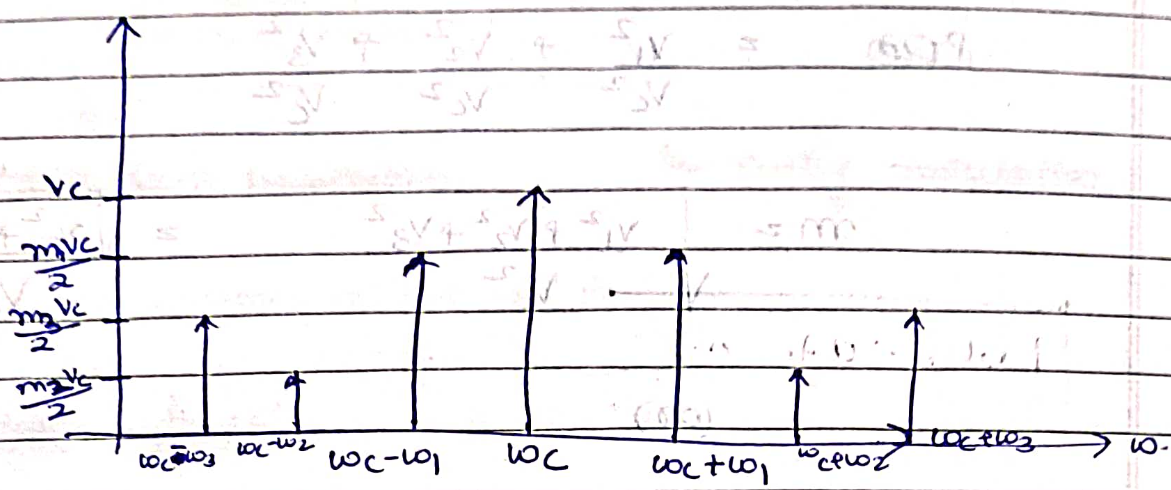
→ for multi tone :-

$$M'(t) = \left[1 + \frac{V_{m1}}{V_c} \cos \omega_1 t + \frac{V_2}{V_c} \cos \omega_2 t + \frac{V_3}{V_c} \cos \omega_3 t\right] V_c \cos \omega_c t.$$

Date : / /
Page No.



→ Single.



→ sidebands depends on the freq. components presents.

$$M'(t) = V_c \cos \omega_c t + \frac{m_1 V_c}{2}\left[\cos(\omega_c - \omega_1)t + \cos(\omega_c + \omega_1)t\right]$$

$$+ \frac{m_2 V_c}{2}\left[\cos(\omega_c - \omega_2)t + \cos(\omega_c + \omega_2)t\right]$$

$$+ \frac{m_3 V_c}{2}\left[\cos(\omega_c - \omega_3)t + \cos(\omega_c + \omega_3)t\right]$$

OR

$$\boxed{m_T = \sqrt{m_1^2 + m_2^2 + m_3^2 \dots m_n^2}}$$

Modulation index :- $m_1 = \frac{V_1}{V_c}$, $m_2 = \frac{V_2}{V_c}$, $m_3 = \frac{V_3}{V_c}$. $\boxed{m_T = \sqrt{m_1^2 + m_2^2 + m_3^2}}$

as two side bands.

Power :- $$P_T = \frac{V_c^2}{2} + \left(\frac{m_1 V_c}{2}\right)^2 \frac{1}{2} \times 2 + \left(\frac{m_2 V_c}{2}\right)^2 \frac{1}{2} \times 2 + \left(\frac{m_3 V_c}{2}\right)^2 \frac{1}{2} \times 2$$

$$P_T = \frac{V_c^2}{2} + \frac{m_1^2 V_c^2}{4} + \frac{m_2^2 V_c^2}{4} + \frac{m_3^2 V_c^2}{4}$$

Date : / /
Page No.

$$P_T = \frac{V_c^2}{2}\left[1 + \frac{m_1^2}{2} + \frac{m_2^2}{2} + \frac{m_3^2}{2}\right]$$

assume :- $m^2 = m_1^2 + m_2^2 + m_3^2$

$$= \frac{V_1^2}{V_c^2} + \frac{V_2^2}{V_c^2} + \frac{V_3^2}{V_c^2}$$

$$m = \sqrt{\frac{V_1^2 + V_2^2 + V_3^2}{V_c^2}} = \frac{\sqrt{V_1^2 + V_2^2 + V_3^2}}{V_c}.$$

(or)

$$m = \sqrt{m_1^2 + m_2^2 + m_3^2}$$

$$\boxed{P_T = P_C\left[1 + \frac{m^2}{2}\right]} \rightarrow \text{Power.}$$

$\eta = \frac{P_S}{P_T}$ $\rightarrow$ Power of side band
$\rightarrow$ Total power.

$$= \frac{\frac{m_1^2 V_c^2}{4} + \frac{m_2^2 V_c^2}{4} + \frac{m_3^2 V_c^2}{4}}{\frac{V_c^2}{2} + \frac{m_1^2 V_c^2}{4} + \frac{m_2^2 V_c^2}{4} + \frac{m_3^2 V_c^2}{4}}$$

$$= \frac{\frac{m_1^2}{2} + \frac{m_2^2}{2} + \frac{m_3^2}{2}}{1 + \frac{m_1^2 + m_2^2 + m_3^2}{2}} \Rightarrow \boxed{\eta = \frac{m_1^2 + m_2^2 + m_3^2}{2 + m_1^2 + m_2^2 + m_3^2}}$$

$$\boxed{R = \frac{2}{2+m_1^2+m_2^2+m_3^2}}$$

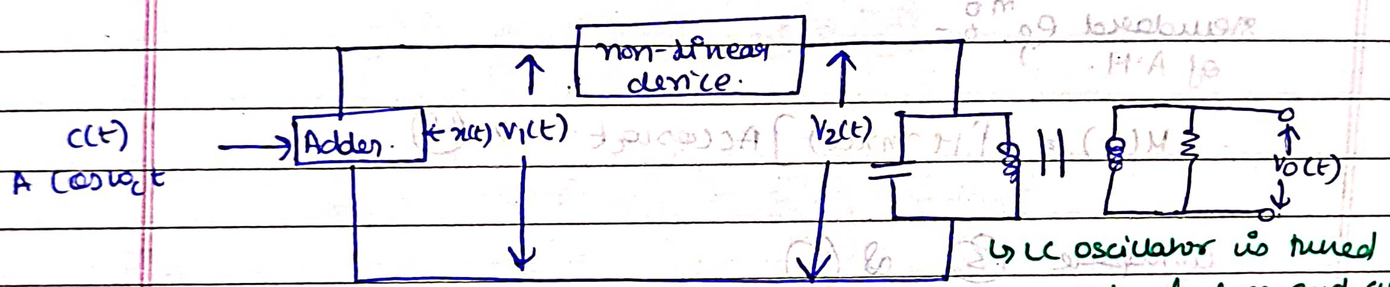
AM GENERATION. (PRACTICALLY).

Square law modulation. — Switching modulation.

↓

for non-linear device e.g. R, C, L

Circuit:



↳ LC oscillator is tuned at fc freq. and act as bandpass filter.

$V_2(t) = a V_1(t) + b V_1^2(t)$ [according to non linear device]

o/p voltage depends on $V_2(t)$.

$V_1(t) = x(t) + A_c \cos \omega_c t$

$V_2(t) = a[x(t) + A_c \cos \omega_c t] + b[(x(t) + A_c \cos \omega_c t)^2]$

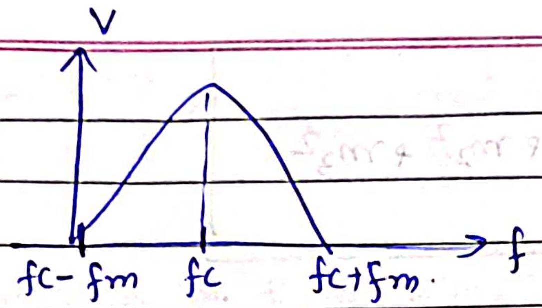
$= a x(t) + a A_c \cos \omega_c t + b[x^2(t) + A_c^2 \cos^2 \omega_c t + 2x(t) A_c \cos \omega_c t]$

$= a x(t) + a A_c \cos \omega_c t + b x^2(t) + b A_c^2 \cos^2 \omega_c t + 2b x(t) A_c \cos \omega_c t$

X ✓ X X

$1 + \cos 2\omega_c t$ → win f

Date : / /
Page No.



→ 'graph of BPf.

As BPf will only allow the freq. band and msg signal is low freq. signal so, wont be transmitted.

$$V_0(t) = aA_c \cos\omega_c t + 2bx(t)\cos\omega_c t$$

$$\boxed{V_0(t) = aA_c \cos\omega_c t\left[1 + \frac{2b}{a}x(t)\right]} \rightarrow (3)$$

standard eqn of A.M. :-

$$M(t) = [1 + m x(t)]A_c \cos\omega_c t \rightarrow (4)$$

compare (3) & (4).

$$\boxed{m = \frac{2b}{a}}$$

NUMERICALS

Q. Sketch $M(t)$ for modulation indexes of $\mu = 0.5$ and $\mu = 1$, when $m(t) = b\cos\omega_m t$. This case is referred to as tone modulation because the modulating signal is a pure sinusoidal signal.

As.

$E_{max} = b$.

$E_{min} = -b$.

$$m = \frac{E_{max} - E_{min}}{2A + E_{max} + E_{min}}$$

$$= \frac{b - (-b)}{2A + b + (-b)} = \frac{b + b}{2A + b - b} = \frac{\cancel{2}b}{\cancel{2}A}$$

$$\boxed{m = \frac{b}{A}} \Rightarrow b = mA.$$

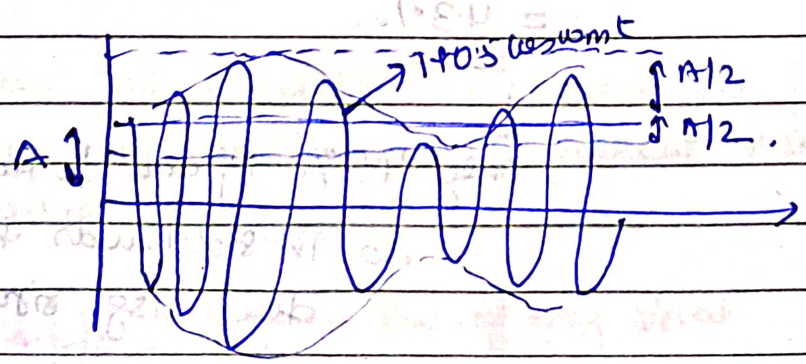
$$m(t) = b\cos\omega_m t = mA\cos\omega_m t$$

$$\therefore x(t) = [A + m(t)]\cos\omega_c t = \cos\omega_c t\,[mA\cos\omega_m t + A]$$

$$x(t) = A[1 + m\cos\omega_m t]\cos\omega_c t$$

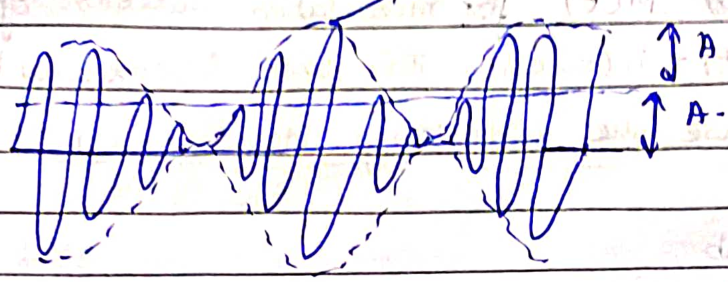
→ for $m = 0.5$, $m < 1$ → Undermodulated.

$$x(t) = A[1 + 0.5\cos\omega_m t]\cos\omega_c t$$



Date : / /
Page No. 21

→ for $m=1$. → criticaly modulated.



Q. Determine η and the % of the total power carried by the sidebands of the AM wave for tone modulation when $m=0.5$ and when $m=0.3$.

As. $\eta = \frac{m^2}{2+m^2} = \frac{(0.5)^2}{2+(0.5)^2}$

% Total power $= \eta \times 100\%$

$= 11\%$.

↓

only 11% of the total power in in the sidebands.

$m=0.3$ -

$\eta = \frac{(0.3)^2}{2+(0.3)^2}$

% of total power $= \eta \times 100\%$

$= 4.3\%$.

↓

~~only 4.3% of the total power~~

is in sidebands that contain the msg signal.

Date : / /
Page No.

AMPLITUDE MODULATION
(DSB-SC)

→ Carrier comm. → where carrier is used to transfer the msg signal.

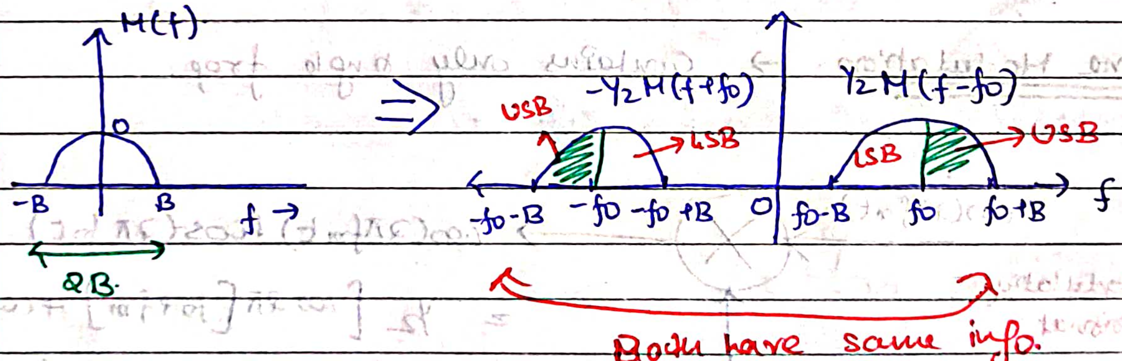
Depending on the freq. spectra, four types:-
1.) DSB-SC
2.) DSC-FC or AM
3.) SSB
4.) VSB

$m(t)\cos(2\pi f_0 t + \phi) \Rightarrow$ MODULATED SIGNAL.

$m(t)$ → msg signal
$\cos(2\pi f_0 t+\phi)$ → carrier signal.

$m(t) \leftrightarrow M(f)$.

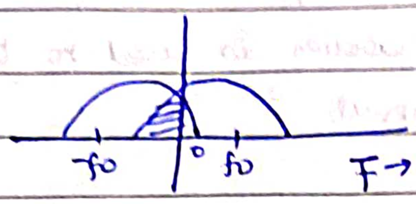
$m(t)\cos 2\pi f_0 t \longleftrightarrow \frac{1}{2}\left[M(f-f_0) + M(f-f_0)\right]$



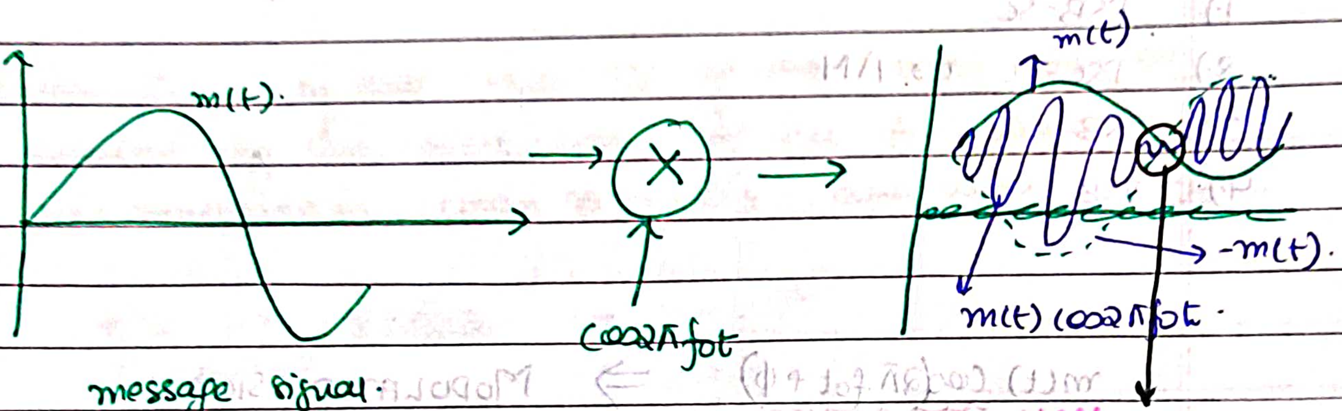
→ In this no discrete component is present at freq. f_0. If such freq. component is present then represent by δ f^n. So, called as DSB-fc.

carrier freq. ← $\boxed{f_0 \geq B}$ → B.W. of msg signal.

If $f_0 < B$.



→ overlapping will happen and info. will be lost



phase reversed at the zero crossing.

$$m(t) \longrightarrow \otimes \longrightarrow m(t)\cos 2\pi f_0 t$$

modulating signal: $m(t)$

carrier signal: $\cos 2\pi f_0 t$

Tone Modulation. → contains only single freq.

$$\cos(2\pi f_m t) \longrightarrow \otimes \longrightarrow \cos(2\pi f_m t)\cos(2\pi f_0 t)$$

$$= \tfrac{1}{2}\left[\cos 2\pi[f_0+f_m] + \cos[f_0-f_m]\right]$$

Modulating signal: $\cos(2\pi f_m t)$

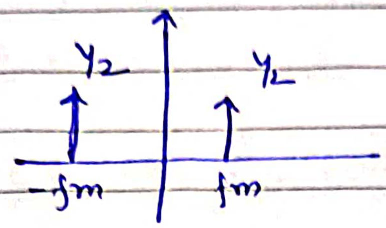
carrier signal: $\cos(2\pi f_0 t)$

Modulated signal ↓ ①

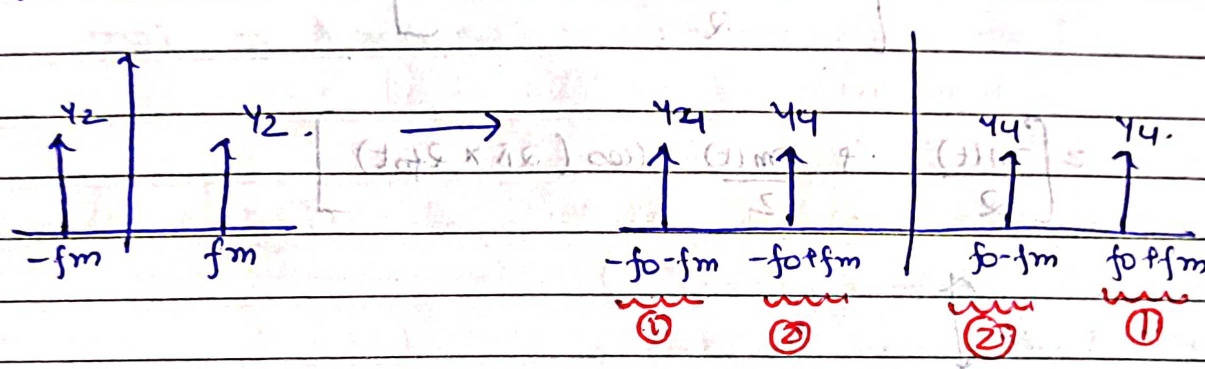
Date: / /
Page No.

by euler form :-

$$\cos(2\pi f_m t) = \frac{1}{2}\left[e^{j2\pi f_m t} + e^{-j2\pi f_m t}\right] \longleftrightarrow \frac{1}{2}\left[\delta(f-f_m) + \delta(f+f_m)\right]$$



for ①



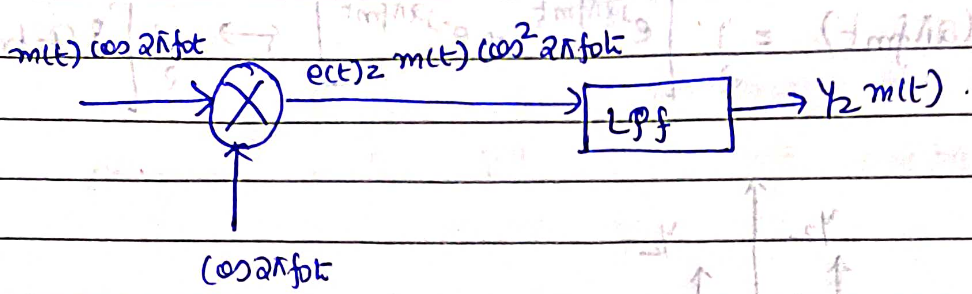
$$\cos(2\pi f_m t)\cos(2\pi f_o t) = \frac{1}{2}\Big[\underbrace{\cos(2\pi(f_o+f_m)t)}_{①} + \underbrace{\cos(2\pi(f_o-f_m)t)}_{②}\Big].$$

$$\left[\frac{1}{2}\left\{\delta(f-(f_o+f_m)) + \delta(f+(f_o+f_m))\right\}\right]$$

$$\left[\frac{1}{2}\left[\delta(f-(f_o-f_m)) + \delta(f+(f_o-f_m))\right.\right.$$

Date : / /
Page No.

Demodulation at Rx.

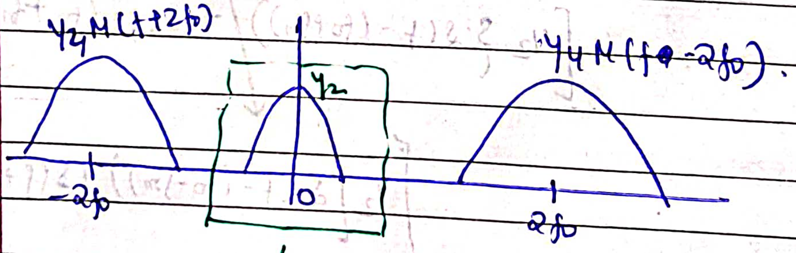


$e(t) = m(t) \cos^2 2\pi f_0 t$

$= m(t) \left[\frac{1 + \cos(2\pi \times 2f_0 t)}{2} \right]$

$= \left[\frac{m(t)}{2} + \frac{m(t)}{2} \cos(2\pi \times 2f_0 t) \right]$

$= \frac{M(f)}{2} + \frac{1}{2} \left[\frac{1}{2} M(f - 2f_0) + \frac{1}{2} M(f + 2f_0) \right]$



↳ after passing through LPF we will receive msg signal

This type of Modulation is called as Synchronous Demodulation (or) Coherent Demodulation

(bcoz carrier freq. of same freq. & phase is same for demodulation)

→ Also locally generated carrier freq. should be ~~same~~ in synchronism as that of msg signal freq. So, increases the complexity. As for long distance freq. or phase can be shifted.

So, original signal after transmission at demodulator looks like :-

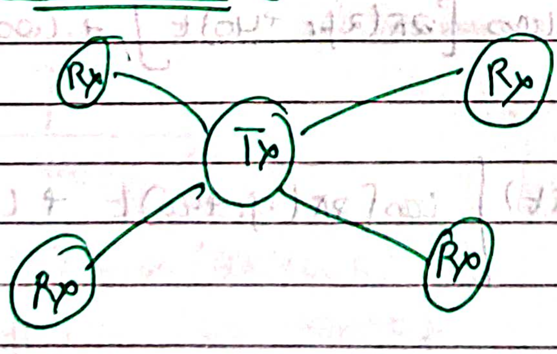
$$m(t)\cos 2\pi f_0 t \rightarrow \underbrace{A}_{\text{attenuation.}}\, m(t-\underbrace{t_d}_{\text{time shift}})\cos\left[2\pi(f_0+\underbrace{\Delta f}_{\text{freq. shift.}})(t-t_d)\right]$$

$$(\text{or}) \quad = A\, m(t-t_d)\cos\left[2\pi(f_0+\Delta f)t - \phi_d\right]$$

$\boxed{\cos(2\pi(f_0+\Delta f)t - \phi_d)}$ ⇒ So, in this local oscillator should generated this freq. from the Rxved signal which increases cost & complexity of Rxver.

This type of complexity is ok for pt. to pt. comm. when cost of Tx & Rx are same.

→ But for Broadcasting :-

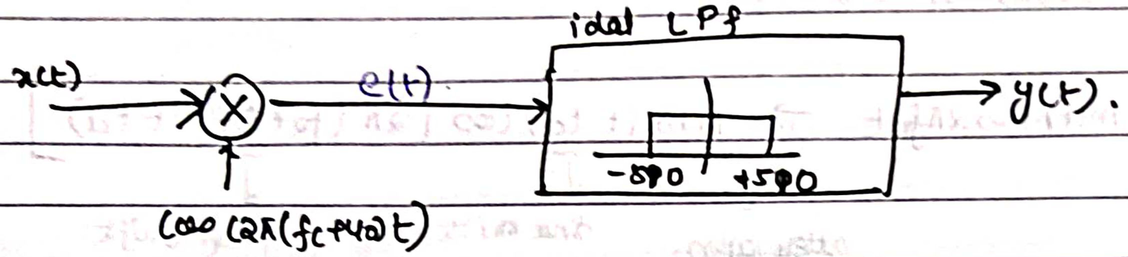


So, it is not economical.

⇒ So, carrier signal should be send with the Modulated signal, so that no need to generate at Rx end. It is called as DSB-RC (or) AM.

Date : / /
Page No.

Q.2. The modulated signal $x(t) = m(t)\cos 2\pi f_c t$, w/ an carrier freq 1MHz and $m(t) = 4\cos(1000\pi t)$ is transmitted by the transmitter. At the Rx, the signal $x(t)$ is passed through the demodulator as shown in fig. The O/P of the demodulator $y(t)$ is



$$e(t) = x(t)\cos(2\pi(f_c+40)t)$$

$$x(t) = m(t)\cos 2\pi f_c t.$$
$$m(t) = 4\cos(1000\pi t).$$

$$e(t) = m(t)\cos 2\pi f_c t \cos(2\pi(f_c+40)t).$$
$$= \underset{m(t)}{\cancel{4\cos(1000\pi t)}}\cos 2\pi f_c t\cos(2\pi(f_c+40)t)$$
$$= \frac{m(t)}{2}\left[\cos[2\pi(2f_c+40)t] + \cos[2\pi(f_c+40-f_c)t]\right]$$

$$= \frac{m(t)}{2}\left[\cos[2\pi(2f_c+40)t] + \cos[2\pi 40t]\right]$$

$$= 2\cos(1000\pi t)\left[\cos[2\pi(2f_c+40)t + \cos(2\pi 40t)\right]$$

$$= 2\cos(1000\pi t)\cos(80\pi t) + 2\cos(1000\pi t)\cdot\cos[2\pi(2f_c+40)t]$$

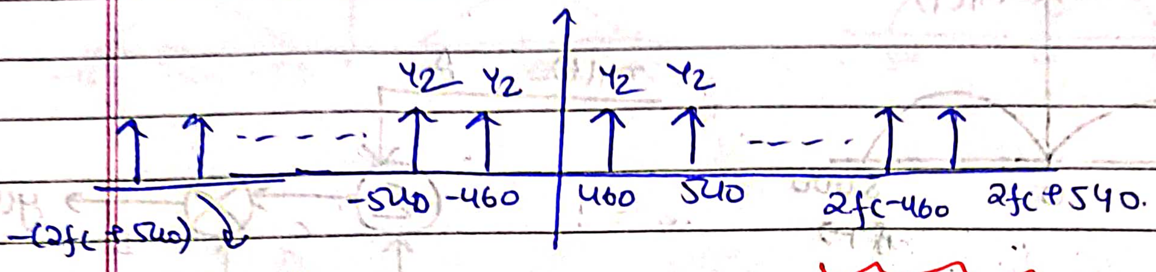
$$= \cos(1080\pi t) + \cos(920\pi t).$$
$$+ \cos(2\pi(2f_c+40+500)t] + \cos(2\pi(2f_c+40-500)t]$$

Date: / /
Page No.

$= \cos(1080\pi t) + \cos(920\pi t) + \cos(2\pi(2f_c+540)t) + \cos(2\pi(2f_c-460t)$

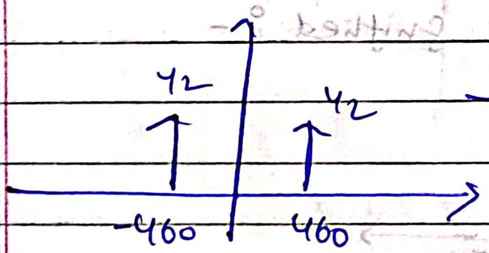
$2f_c$ $2f_c$

$f = 540$ $f = 460$



As carrier freq. is 1MHz, so these two terms are quiet far away.

$\boxed{LPF}$ → cut off freq. → 510 Hz.

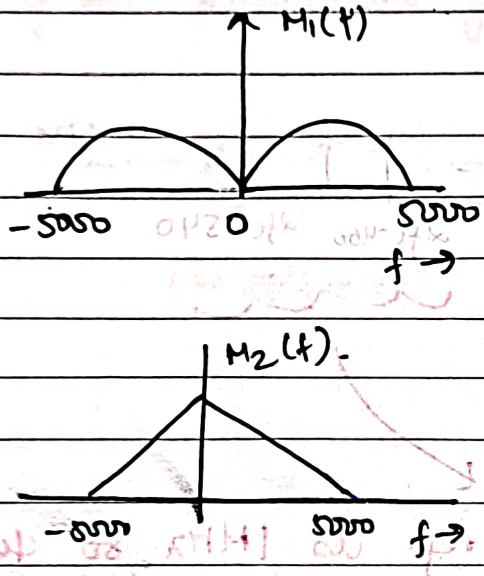


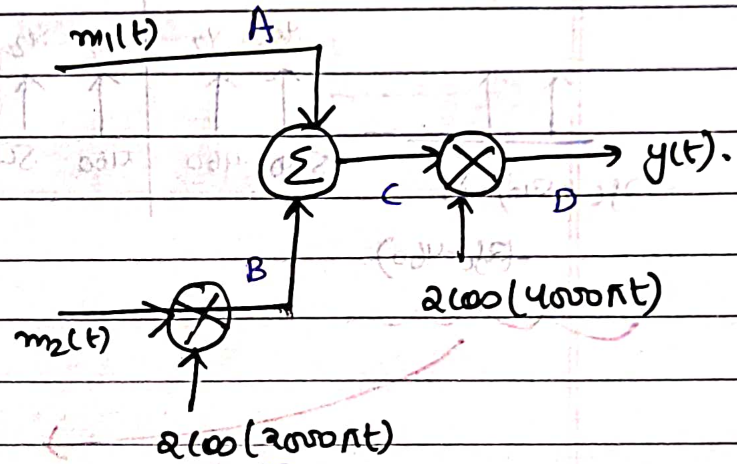
→ demodulator o/p.

$y(t) = \cos(2\pi 460 t)$.

$y(t) = \cos(920\pi t)$.

Q.2 Two signals $m_1(t)$ & $m_2(t)$, both band limited to 5000 Hz are transmitted simultaneously over the channel as shown in fig. The min. required B.W of the channel (in Hz) to transmit the modulated signal is ____.



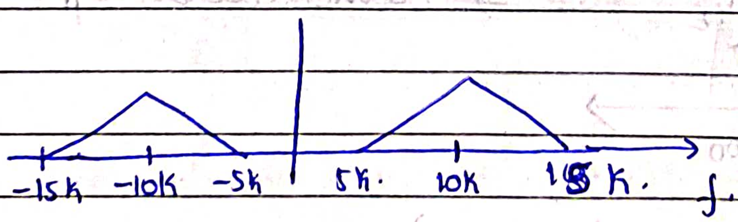


we will check freq. spectrum at A, B, C, D

<u>At B:</u>

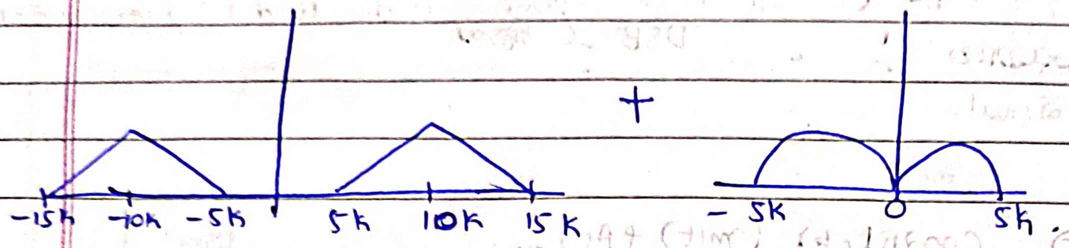
$f_c = 10KHz.$

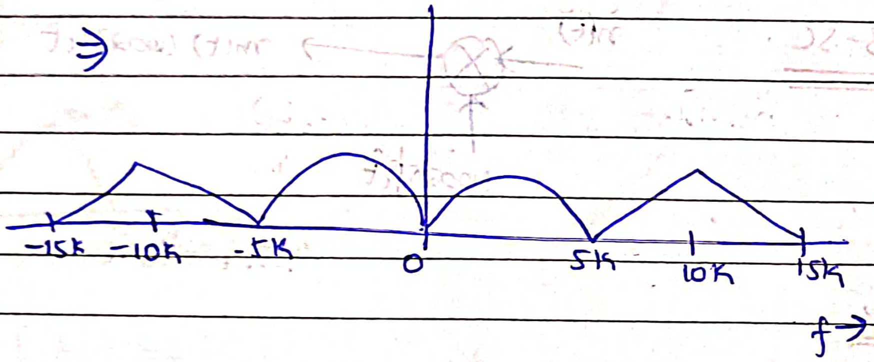
after multiplication $M_2(f)$ is shifted :-



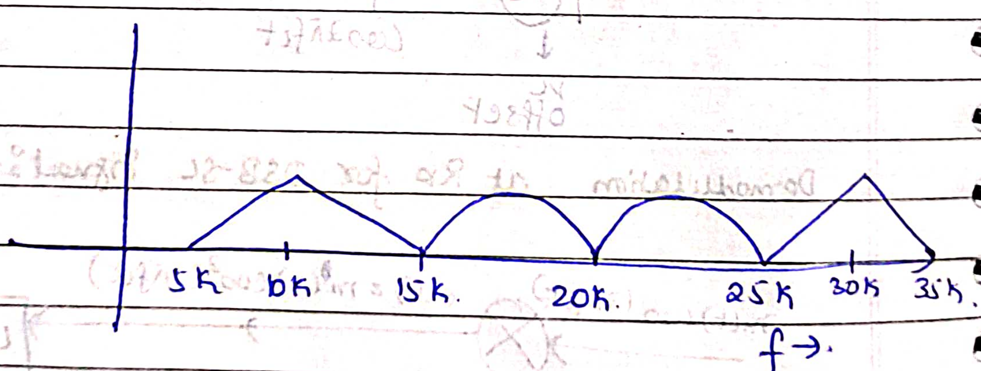
Date : / /
Page No.

At pt. C.





So, after multiplication with 2nd carrier whose freq. is $f_c = 20KHz$



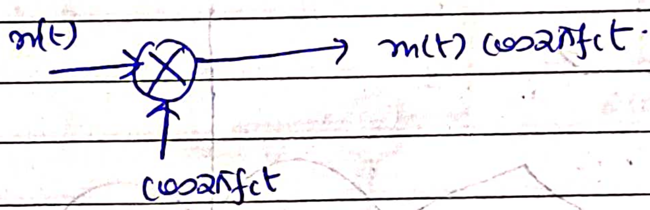
$B.W = 35K - 5KHz$
$= \underline{30KHz}$

AMPLITUDE MODULATION.

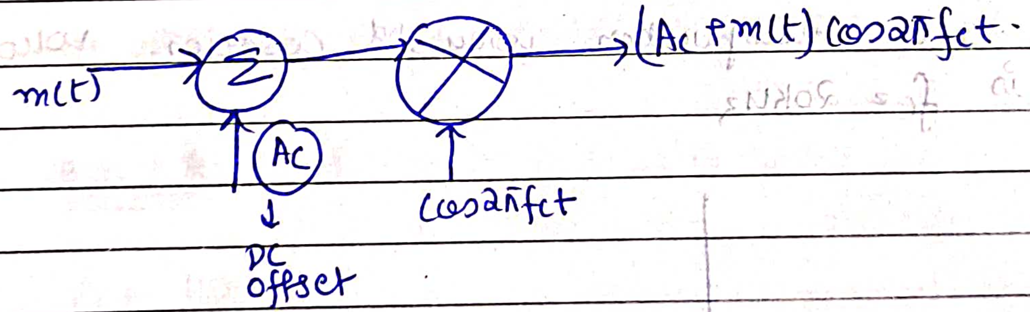
$\underbrace{m(t)\cos(2\pi f_c t)}_{\text{carrier signal}} + \underbrace{A_c\cos(2\pi f_c t)}_{\text{DSB-SC signal}} \Rightarrow$ AM / DSB-FC

$\Rightarrow \cos 2\pi f_c t \,(m(t) + A_c)$

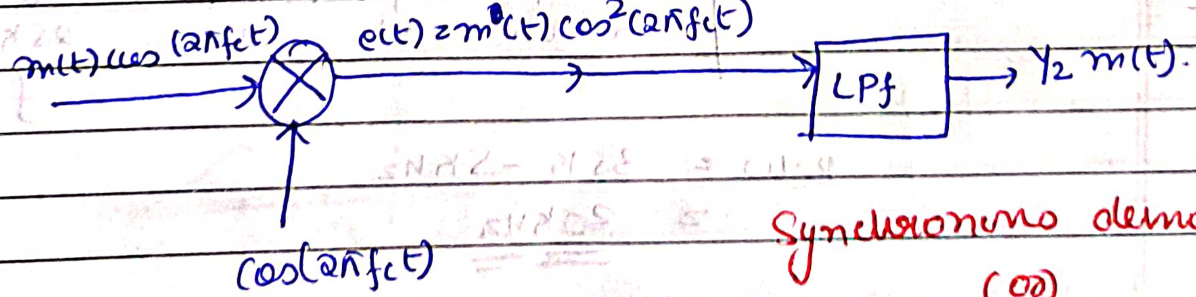
DSB-SC.



AM.



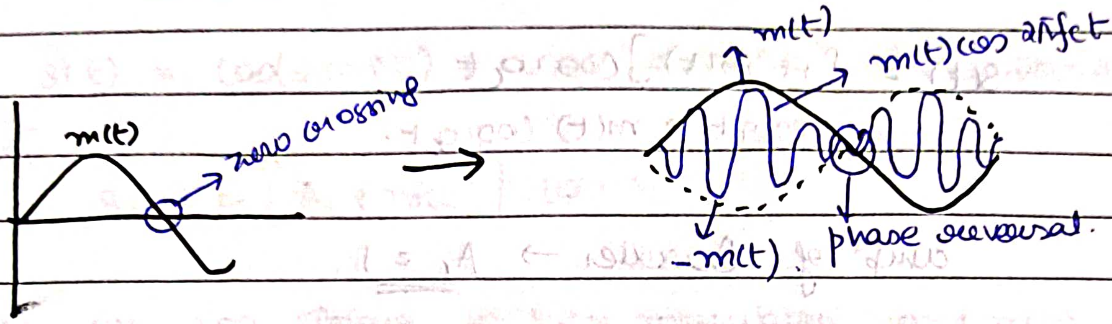
Demodulation at Rx for DSB-SC signal:-

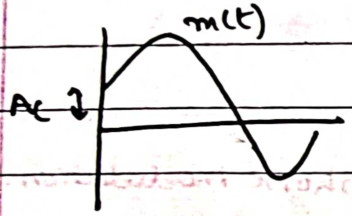


Synchronous demodulation
(or)
coherent demodulation.

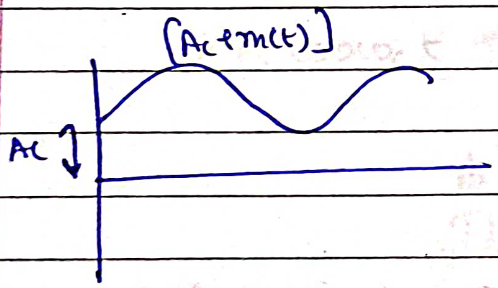
Broadcast Communication → we have one tx and several Rxs

DSB-SC.





$[A_c + m(t)] > 0$ in order to cancel out phase reversal



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Questions.

Date: / /
Page No.

Q1. A msg signal $m(t) = 0.5\cos(\omega_1 t) - 0.5\sin(\omega_2 t)$ is amplitude modulated with the carrier of freq. ω_c to generate $s(t) = [1+m(t)]\cos\omega_c t$. The power η achieved by this modulation scheme is?

$$s(t) = [1+m(t)]\cos\omega_c t$$
$$= \cos\omega_c t + m(t)\cos\omega_c t.$$

ampl of carrier $\rightarrow$ $A_c = 1$.

$$m(t) = 0.5\cos(\omega_1 t) - 0.5\sin(\omega_2 t)$$

Two freq. so multitone modulation.

$$|A_{m1}| = 0.5$$
$$|A_{m2}| = 0.5$$

$$\mu = \frac{|A_m|}{|A_c|}$$

$$\mu_1 = \frac{A_{m1}}{A_c} = \frac{0.5}{1} = 0.5, \quad \mu_2 = \frac{A_{m2}}{A_c} = \frac{0.5}{1} = 0.5$$

$$\mu_T = \sqrt{\mu_1^2 + \mu_2^2} = \sqrt{\left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2} = \frac{1}{\sqrt{2}}$$

$$\eta\% = \frac{\mu^2}{2+\mu^2} \times 100 = \frac{(1/\sqrt{2})^2}{2+(1/\sqrt{2})^2} \times 100 = 20\%.$$

Q2. Consider the following amplitude modulated signal $s(t) = \cos(2000\pi t) + 4\cos(2400\pi t) + \cos(2800\pi t)$. The ratio of the power of the msg signal to the power of carrier signal is ____?

Ans: $s(t) = \cos(2000\pi t) + 4\cos(2400\pi t) + \cos(2800\pi t)$ → (2)

$s(t) = [A_c + m(t)] \cos\omega_c t$

All are cos terms so tone modulation and m(t) will be of the form $A_m\cos\omega_m t$

$s(t) = A_c\cos\omega_c t + m(t)\cos\omega_c t$ (Let $m(t) = A_m\cos\omega_m t$)

$= A_c\cos\omega_c t + A_m\cos\omega_m t\cos\omega_c t$

$= A_c\cos\omega_c t + \underbrace{\frac{1}{2}A_m\cos(\omega_c-\omega_m)t}_{\text{Low freq.}} + \underbrace{\frac{1}{2}A_m\cos(\omega_c+\omega_m)t}_{\text{high freq}}$

(1)

from (1) & (2)

$A_c\cos\omega_c t = 4\cos(2400\pi t)$

$A_c = 4$, $\omega_c = 2400\pi$

$\frac{A_m}{2}\cos(\omega_c-\omega_m) = \cos(2000\pi t)$

$\frac{A_m}{2} = 1$

$A_m = 2$

$\omega_c - \omega_m = 2000\pi$

$2400\pi - \omega_m = 2000\pi$

$\omega_m = 400\pi$

Date : / /
Page No.

Sol: $c(t) = 4\cos(2400\pi t)$.

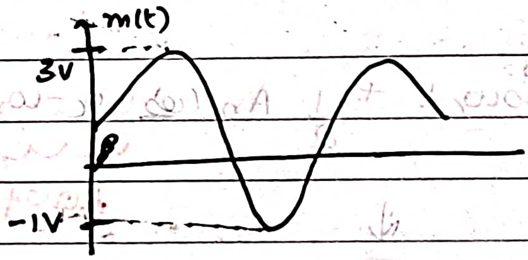
$m(t) = 2\cos(400\pi t)$

$$P_c = \frac{A_c^2}{2R} = \frac{(4)^2}{2R} = \frac{8}{R}$$

$$P_m = \frac{A_m^2}{2R} = \frac{(2)^2}{2R} = \frac{2}{R}$$

$$\frac{P_m}{P_c} = \frac{2/R}{8/R} = \frac{1}{4} = \underline{0.25}$$

Q3. The ampl modulated signal can be given as $[2+m(t)]\cos(2000\pi$
The msg signal $m(t)$ is shown below. The modulation index of the modulation signal is ____



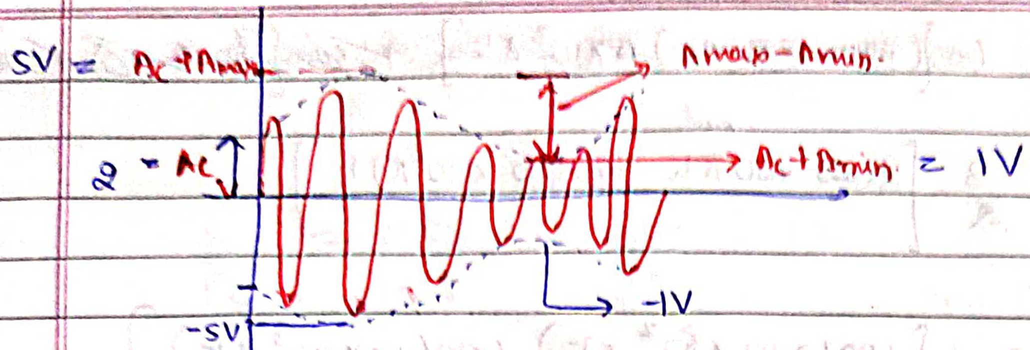
$$[2+m(t)]\cos(2000\pi t) = [A_c + m(t)]\cos\omega_c t$$

as we know $\boxed{\mu = \frac{A_m}{A_c}}$ → this formula is valid when msg signal is symm
But in above there is some offset present.

X not valid.

$A_c = 2$

Date: / /
Page No.



$$\mu = \frac{E_{max} - E_{min}}{E_{max} + E_{min}}$$

$$\mu = \frac{A_{max} - A_{min}}{2A_c + A_{max} + A_{min}} = \frac{3-(-1)}{2(2)+3-1} = \frac{4}{6} = \frac{2}{3}$$

Q.2. A signal $x(t)$ is described below,

$$x(t) = \frac{3}{2}\cos(190\times10^3\pi t) + 5\cos(200\times10^3\pi t) + \frac{3}{2}\cos(210\times10^3\pi t)$$

a) Show that $x(t)$ is an AM signal.

b) Determine ~~power~~ the ratio P_s/P_c where P_s is the power in sidebands and P_c is the power in the carrier.

c) What is the power η in the AM signal?

Signal $x(t)$ is :-

$$x(t) = 5\cos(200\times10^3\pi t) + \frac{3}{2}\cos(190\times10^3\pi t) + \frac{3}{2}\cos(210\times10^3\pi t)$$

$\cos(a+b)$
$\cos(a-b) = 2\cos a\cos b$

$= \cancel{5\cos(200\times10^3\pi t) +}$

2/3

Date : / /
Page No.

$\cancel{\cos[(200-10)\times 10^3\pi t]} + \cancel{\cos[(200+10)\times 10^3\pi t]}$

$\frac{3}{2}\left[2\cos 200\times 10^2\pi t \cos(10\times 10^3\pi)t\right]$

$= 3\left[\cos(200\times 10^3\pi t)\cos(10\times 10^3\pi t)\right]$

$x(t) = \underbrace{5\cos(200\times 10^3\pi t)}_{\text{Carrier signal}} + \underbrace{3[\cos(200\times 10^3\pi t)}_{\text{msg signal}}\cos(10\times 10^3\pi t)]$

$= 5\cos(200\times 10^3\pi t)\left[1+\frac{3}{5}\cos(10\times 10^3\pi t)\right]$

$= 5\left[1+0.6\cos(10\times 10^3\pi t)\right]\cos(200\times 10^3\pi t)$

$[A_c + m(t)]\cos\omega_c t$

$\mu = \frac{A_m}{A_c} = \frac{3}{5} = 0.6$

$f_c = 100\,kHz$

$f_m = 5\,kHz$

$A = 5V$

b) $P_s = \frac{\overline{m^2(t)}}{2}$ $\quad m(t) = 3\cos(10\times 10^3\pi t)$

$\overline{m(t)} = \frac{3}{\sqrt{2}}$

$\overline{m^2(t)} = \frac{9}{2}$

$\boxed{P_s = \frac{9}{4}}$

Date : / /
Page No.

$$P_c = \frac{A_c^2}{2} = \frac{(5)^2}{2} = 12.5W$$

$$\frac{P_s}{P_c} = \frac{2.25}{12.5} = 0.18.$$

(c) $$\eta = \frac{P_s}{P_s + P_c} = 0.1525$$

(or) $\mu = 0.6.$

$$\eta = \frac{\mu^2}{2+\mu^2} = \frac{0.36}{2.36} = 0.1525.$$

IMPORTANT FORMULAS

1.) Modulation index :-

$$\mu = \frac{A_m}{A_c} = \frac{E_{max} - E_{min}}{E_{max} + E_{min}} = \frac{A_{max} - A_{min}}{2A_c + A_{max} + A_{min}}$$

for multitone :-

$$\mu_n = \sqrt{\mu_1^2 + \mu_2^2 + \cdots \mu_n^2}$$

2.) Power.

$$Power = \frac{V_{rms}^2}{R}.$$

$$\boxed{P_c = \left(\frac{A_c}{\sqrt{2}}\right)^2 \times \frac{1}{R} = \frac{A_c^2}{2R}}$$

$$V_{rms} = \frac{A_c}{\sqrt{2}}$$

$$P_s = \left(\frac{\mu A_c}{2\sqrt{2}}\right)^2 \times \frac{1}{R} = \frac{\mu^2 A_c^2}{8R}$$

$$P_s = 2 \times \frac{\mu^2 A_c^2}{8R} = \frac{\mu^2 A_c^2}{4R}$$

Date : / /
Page No.

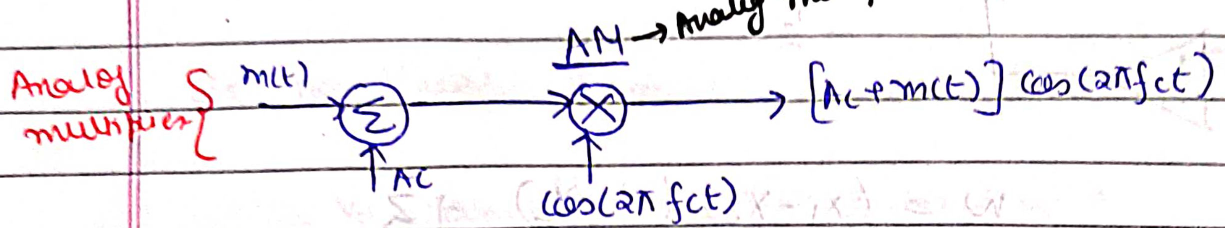
$$\boxed{P_T = P_C + P_{SB} = P_C + P_C \times \frac{\mu^2}{2} = P_C\left(1 + \frac{\mu^2}{2}\right)}$$

3.) $$\eta = \frac{P_{SB}}{P_{SB} + P_C} = \frac{P_C \times \frac{\mu^2}{2}}{P_C\left[1 + \frac{\mu^2}{2}\right]}$$

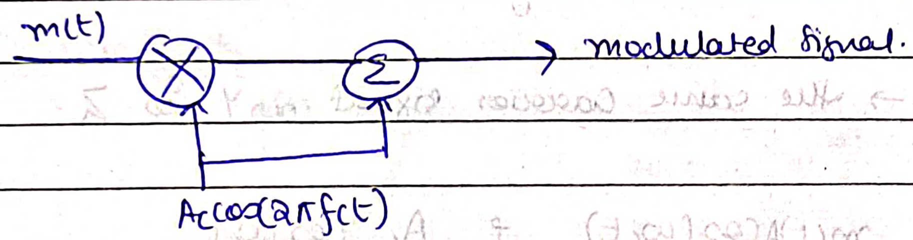
$$\boxed{\eta\% = \frac{\mu^2}{2 + \mu^2} \times 100}$$

AM GENERATION.

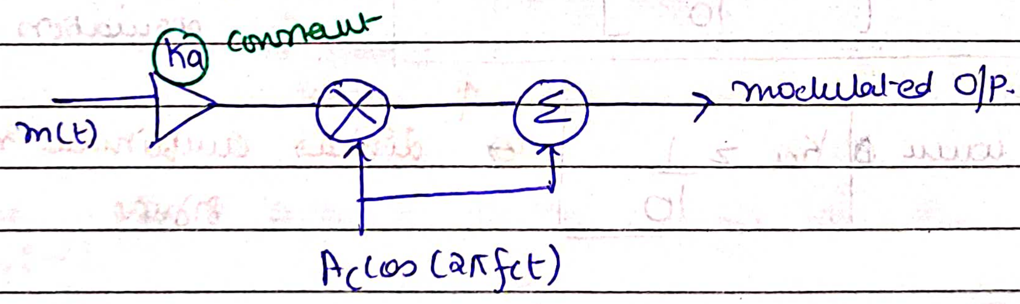
1) Using Analog multiplier.
2) Square law modulator → non linear modulation.
3) Switching modulator.



(OR)



during AM generation msg signal is multiplied by some constant factor.



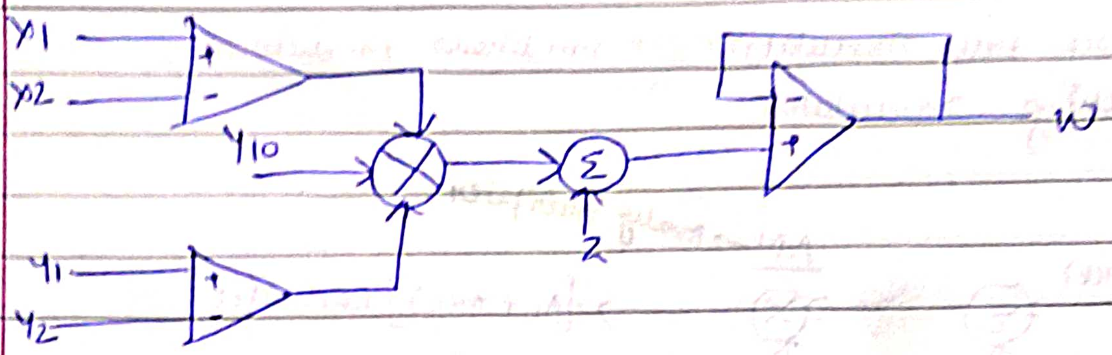
$$= A_c \underbrace{k_a}_{\text{amplitude sensitivity.}} m(t) \cos(2\pi f_c t) + A_c \cos(2\pi f_c t).$$

$$= A_c\left[1 + k_a m(t)\right] \cos(2\pi f_c t)]$$ another way

$\boxed{k_a = 1/A_c}$ (from previous general expression).

Date : / /
Page No.

AM generation using AD633.



$$w = \frac{(x_1 - x_2)(y_1 - y_2)}{10} + z$$

let x is msg signal & y is the carrier signal.

→ the same carrier signal as y is z.

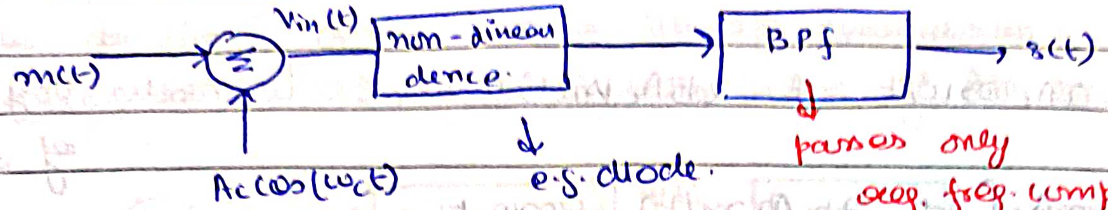
$$s(t) = \frac{m(t) A_c \cos(\omega_c t)}{10} + A_c \cos \omega_c t$$

$$\boxed{s(t) = A_c \left[1 + \frac{m(t)}{10} \right] \cos \omega_c t}$$ → modulated signal generation.

where $\boxed{K_a = \frac{1}{10}}$ → linear amplitude modulated signal.

Date: / /
Page No.

SQUARE LAW MODULATOR.

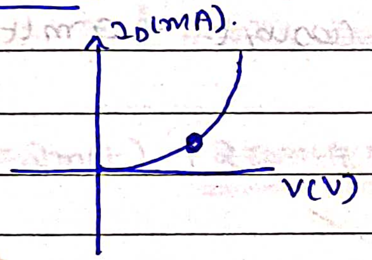


for non-linear devices :-

$$V_o = aV_{in}(t) + bV_{in}^2(t) + cV_{in}^3(t) + \ldots$$

$$V_o = aV_{in}(t) + bV_{in}^2(t).$$ [when i/p Vin is small then higher terms can be neglected

e.g. Diode.



$$I_D = I_S\left[e^{V_D/V_T} - 1\right]$$

$V_D = V_{DO} + v_{in}(t).$

V_{DO} → DC biasing voltage.

$v_{in}(t)$ → i/p signal

by Taylor series :- $I_D = a_1 V_{in} + a_2 V_{in}^2 + a_3 V_{in}^3 + \ldots$

so, $V_o = aV_{in}(t) + bV_{in}^2(t).$ → (2)

$V_{in}(t) = m(t) + A_c \cos\omega_c t$ → (1)

put (1) in (2)

$$V_o = a\left[m(t) + A_c\cos\omega_c t\right] + b\left[m(t) + A_c\cos\omega_c t\right]^2$$

Date: / /
Page No.

by BPf

$= a[m(t) + A_c \cos \omega_c t] + b[m^2(t)] + 2bA_c m(t)\cos\omega_c t + bA_c^2 \cos^2\omega_c t$

$= aA_c \cos\omega_c t + 2bA_c m(t)\cos\omega_c t$ (consider only terms of requirement).

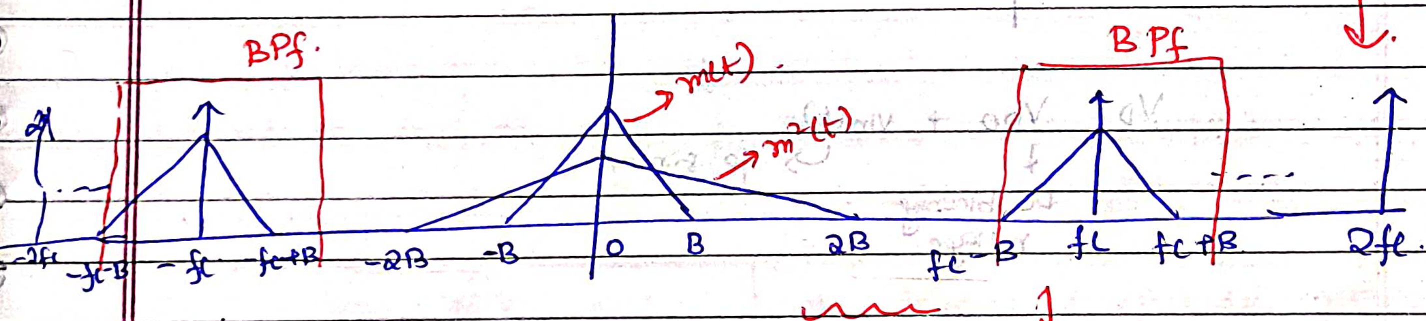
$= aA_c\left[1 + \frac{2b}{a} m(t)\right]\cos\omega_c t$

comparing with the general AM eqn.

$$K_a = \frac{2b}{a}$$

freq. spectrum :-

$V_0 = aA_c\left[1 + \frac{2b}{a} m(t)\right]\cos\omega_c t + a\,m(t) + b\,m^2(t) + bA_c^2\cos(2\pi f_c t)$



BPf should be designed in such a way that center freq. is at fc and B.W of 2B.

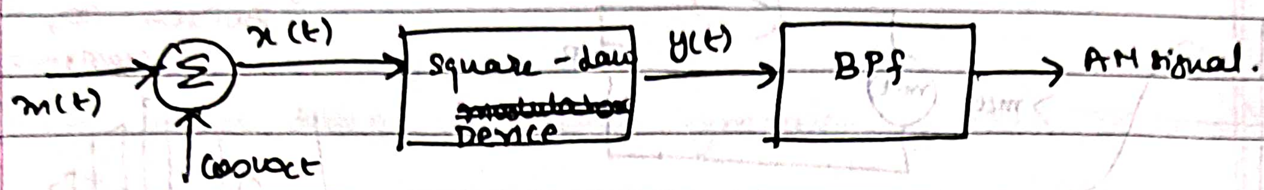
during designing of BPf $\Leftarrow$ $f_c - B > 2B$

so, that overlapping can be avoided.

$\underline{f_c > 3B}$

Date : / /
Page No.

Q.2 Consider the s/m shown in the fig. below. The modulating signal is a sinusoidal signal with max. amplitude of 2V. The non-linear device has a i/p-o/p characteristics $y(t) = 2x(t) + 0.2x^2(t)$. The modulation index of AM signal is ____



$$y(t) = 2x(t) + 0.2x^2(t).$$

$$x(t) = m(t) + \cos\omega_c t$$
$$m(t) = 2\sin\omega_m t$$

$$y(t) = 2[m(t) + \cos\omega_c t] + 0.2[m(t) + \cos\omega_c t]^2$$

$$= 2m(t) + \underline{2\cos\omega_c t} + 0.2m^2(t) + 0.2\cos^2\omega_c t + \underline{0.2 \times 2m(t)\cos\omega_c t}.$$

AM signal.

$$= 2\cos\omega_c t + 0.2 \times 2m(t)\cos\omega_c t.$$
$$= 2\left[1 + \frac{0.4}{2}m(t)\right]\cos\omega_c t.$$

$$= 2[1 + 0.2m(t)]\cos\omega_c t.$$

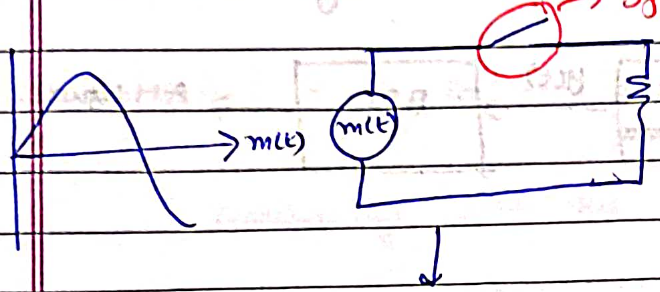
$$\boxed{\begin{aligned} \mu &= k_a \cdot |m(t)|_{max}. \\ &= 0.2 \times 2 = \underline{\underline{0.4}} \end{aligned}}$$

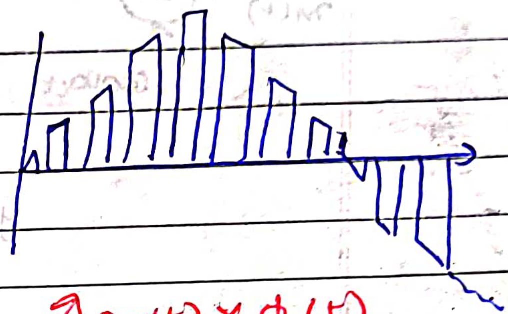
Date: / /
Page No.

SWITCHING MODULATION.

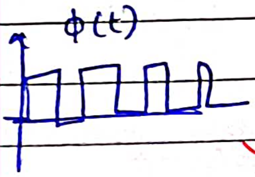
→ Using switch we can multiply msg signal & carrier.

→ If we periodically on/off the switch so, o/p across resistor is :-





when switch is closed then only msg signal we will get :

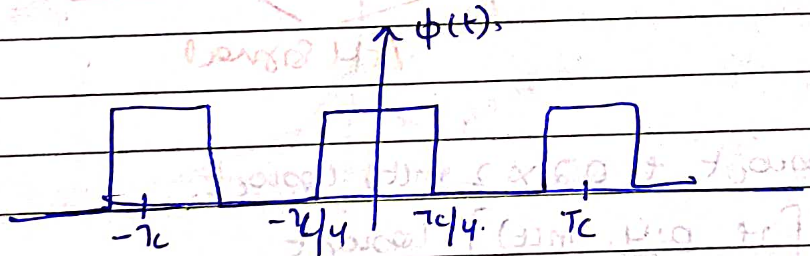


$m(t) \times \phi(t)$

$= m(t) \times \cos \omega_c t$:-

→ If we multiple msg signal with pulse train

Pulse train using fourier series :-



$$\boxed{\phi(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos(n\omega_c t) + \sum_{n=1}^{\infty} b_n \sin(n\omega_c t)}$$

$$\omega_c = \frac{2\pi}{T_c}$$

Date : / /
Page No.

$$a_0 = \frac{1}{T_c}\int_{T_c} \phi(t)\,dt \quad , \quad a_n = \frac{2}{T_c}\int_{T_c} \phi(t)\cos(n\omega_c t)\,dt$$

$$b_n = \frac{2}{T_c}\int_{T_c} \phi(t)\sin(n\omega_c t)\,dt.$$

Since pulse train is even symmetric, this means

$$\underline{\underline{b_n = 0}}.$$

$$a_0 = \frac{1}{T_c}\int_{-T_c/2}^{+T_c/2} 1\cdot dt$$

$$= \frac{1}{T_c}\int_{-T_c/4}^{T_c/4} 1\cdot dt = \frac{1}{2}$$

$$a_n = \frac{2}{T_c}\int_{-T_c/4}^{T_c/4} 1\cdot\cos(n\omega_c t)\,dt = \frac{2}{T_c}\left[\frac{\sin(n\omega_c t)}{n\omega_c}\right]_{-T_c/4}^{T_c/4}$$

$$= \frac{2}{T_c}\times\frac{1}{n\times\frac{2\pi}{T_c}}\left\{\left[\sin n\times\frac{2\pi}{T_c}\times\frac{T_c}{4}\right]-\left[-\sin n\times\frac{2\pi}{T_c}\times\frac{T_c}{4}\right]\right. = \frac{1}{n\pi}\left[\sin\left(\frac{n\pi}{2}\right)-\left(-\sin\left(\frac{n\pi}{2}\right)\right)\right]$$

$$= \left[\frac{2}{n\pi}\left(\sin\left(\frac{n\pi}{2}\right)\right)\right]$$

n is varying from $1 \to \infty$.

→ for even values this term $= 0$.

for $n = 1 \Rightarrow a_1 = \frac{2}{\pi}\sin\frac{\pi}{2} = \frac{2}{\pi}$

$$a_3 = \frac{2}{3\pi}\sin\left(\frac{3\pi}{2}\right) = -\frac{2}{3\pi}$$

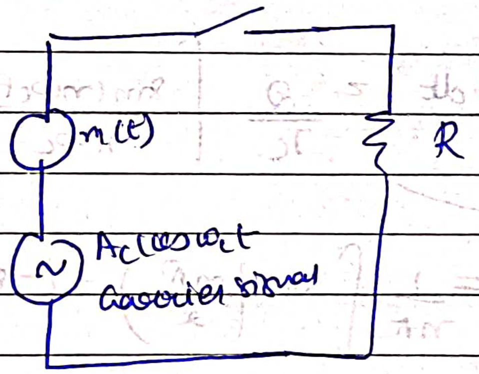
Date : / /
Page No.

$$\phi(t) = \frac{1}{2} + \frac{2}{\pi}\cos(\omega_c t) - \frac{2}{3\pi}\cos(3\omega_c t) + \ldots$$

$$m(t)\times\phi(t) = m(t)\left[\frac{1}{2} + \frac{2}{\pi}\cos 2\pi f_c t - \frac{2}{3\pi}\cos(2\pi\times 3 f_c t) + \ldots\right]$$

$$= \frac{m(t)}{2} + \underbrace{\frac{2}{\pi} m(t)\cos 2\pi f_c t}_{\text{our original modulated signal filtered out using BPF}} - \frac{2m(t)}{3\pi}\times\cos(3\omega_c t)$$

$\therefore$ We can see this is DSB-SC modulated signal so, carrier needs to be added in switching modulated signal.



$$\left[m(t) + A_c\cos(2\pi f_c t)\right]\times\left[\frac{1}{2} + \frac{2}{\pi}\cos(2\pi f_c t) - \frac{2}{3\pi}\cos(2\pi\times 3 f_c t) + \ldots\right]$$

$$= \frac{m(t)}{2} + \underbrace{\frac{2}{\pi}\times m(t)\cos(2\pi f_c t) + \frac{A_c}{2}\cos(2\pi f_c t)}_{\text{AM signal}} + \frac{2A_c}{\pi}\times\cos^2(2\pi f_c t)$$

$$= \frac{2}{\pi} m(t) \cos 2\pi f_c t + \frac{A_c}{2} \cos(2\pi f_c t)$$

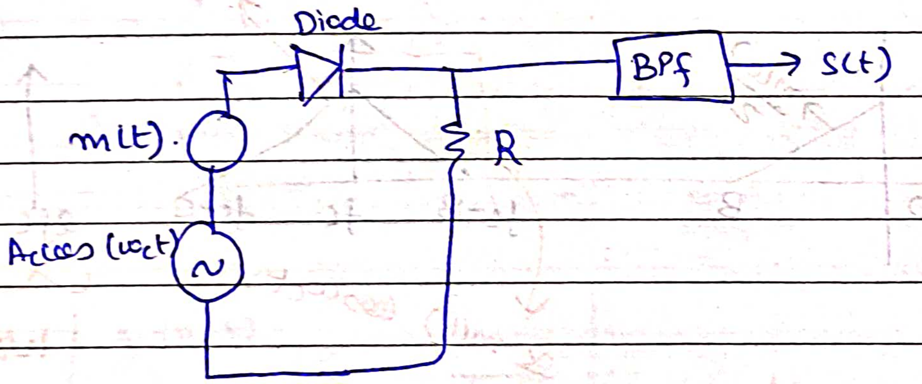
$$= \frac{A_c}{2}\left[1 + \left(\frac{4}{\pi A_c}\right) m(t)\right] \cos(2\pi f_c t)$$

Compare with standard AM.

$$\boxed{K_a = \frac{4}{\pi A_c}}$$

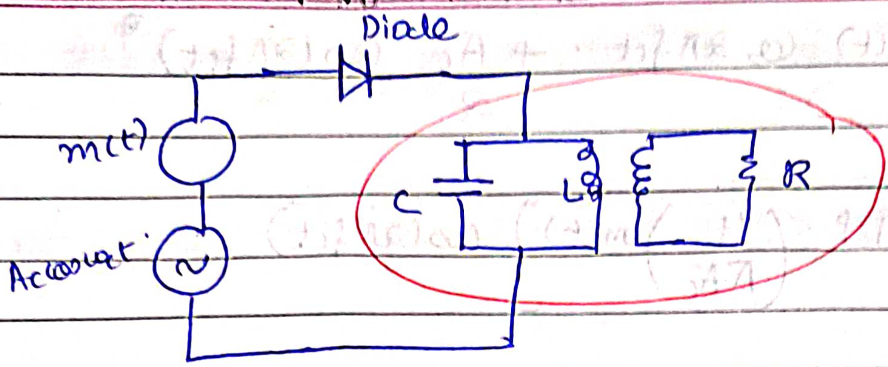
Carrier suppressed by 2.

⇒ Simple Diode can be used as SWITCH.



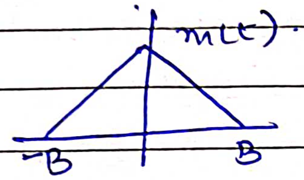
If $c(t) \gg m(t)$.
↓
amp of carrier signal

So, ON-off f^n of diode is trigger by the carrier.

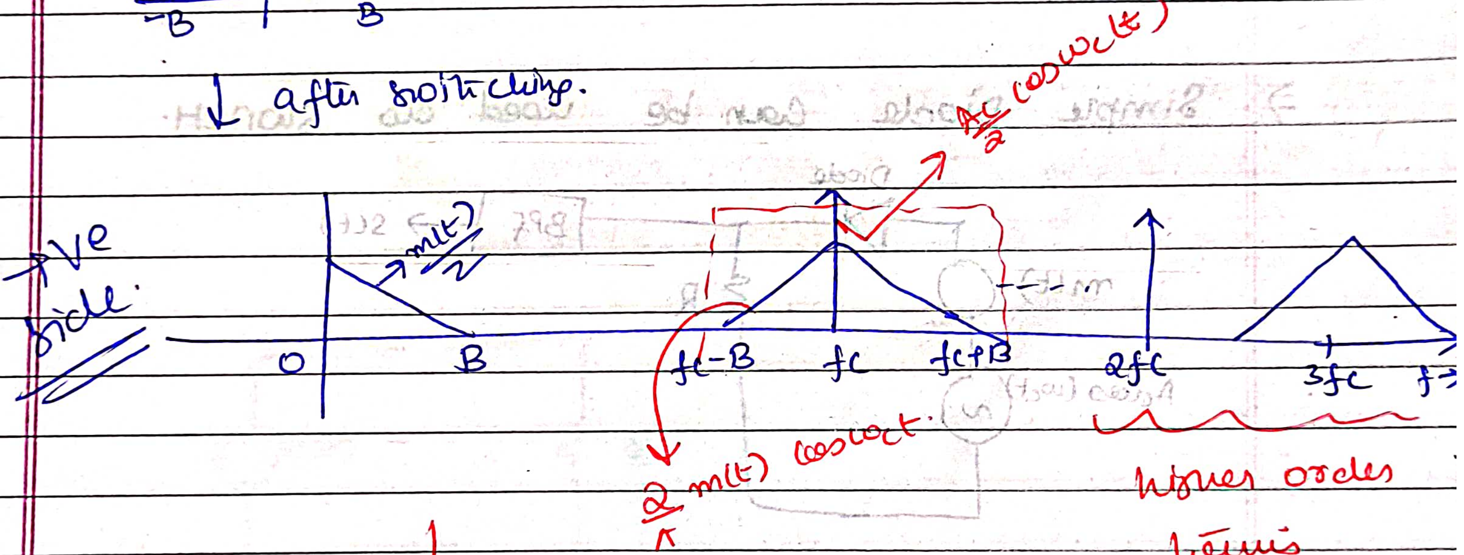


BPF.

In FREQ. DOMAIN



↓ after switching.



then pass through BPF.

Date : / /
Page No.

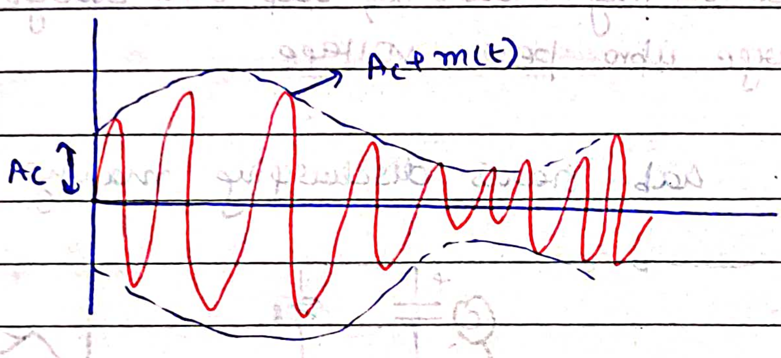
DEMODULATION of AM signal

Envelope Detector → non-coherent bcoz carrier is not req. in the demodulation.

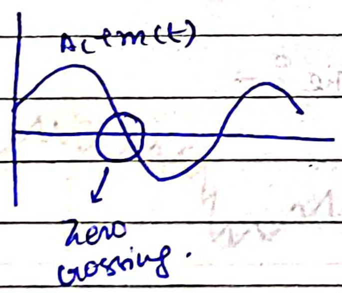
Gen. Expression :- $A_c [1 + K_a (m(t))] \cos \omega_c t$

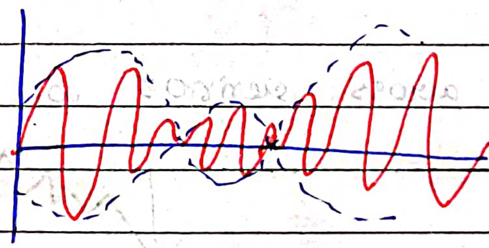
if $K_a = \frac{1}{A_c}$

$\Rightarrow [A_c + m(t)] \cos \omega_c t$

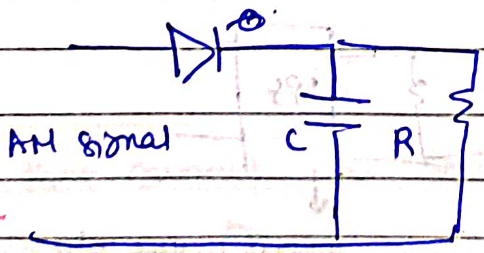


⇒ Envelope ~~detector~~ detector works only when
$A_c + m(t) > 0$ (or). $\mu < 1$.





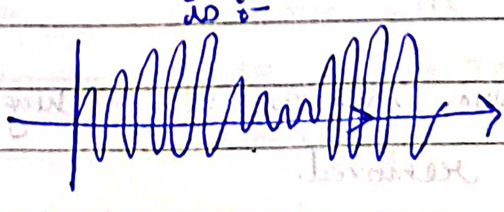
$\mu < 1$



TONE MODULATION.

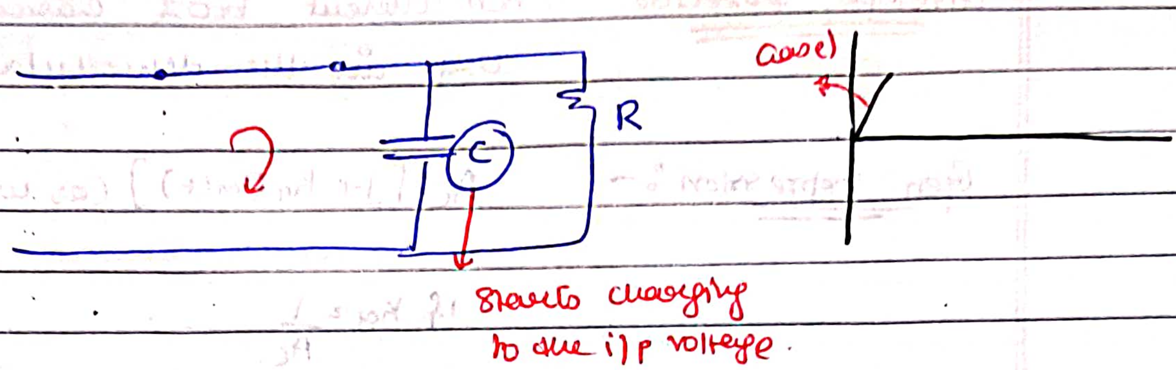
Assume diode is ideal.

* If AM signal → +ve → diode ~~is~~.
o/p across the capacitor is :-



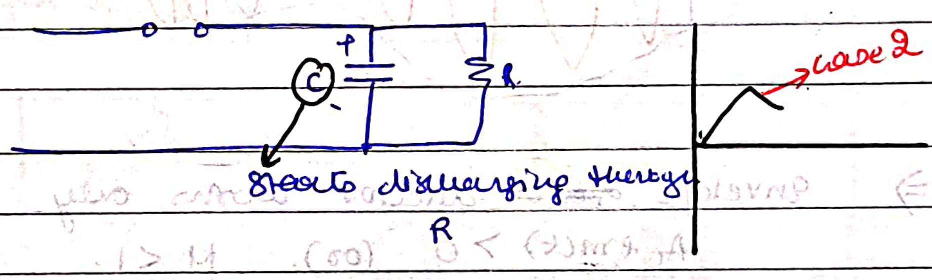
Case 1 :→ if ~~diode~~ AM signal ⇒ +ve .

Diode is S.C.



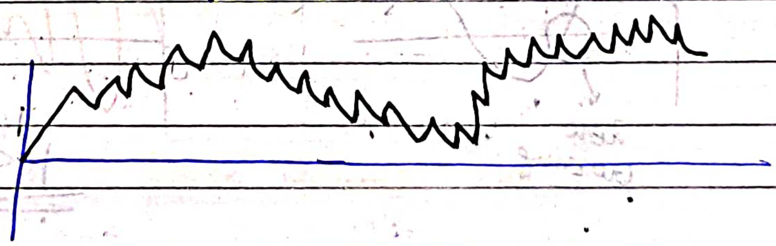
→ but in practical diode → we have some f/w resistance
rd which is very small. So, cap. will charge very quickly.
and charge upto peak voltage.

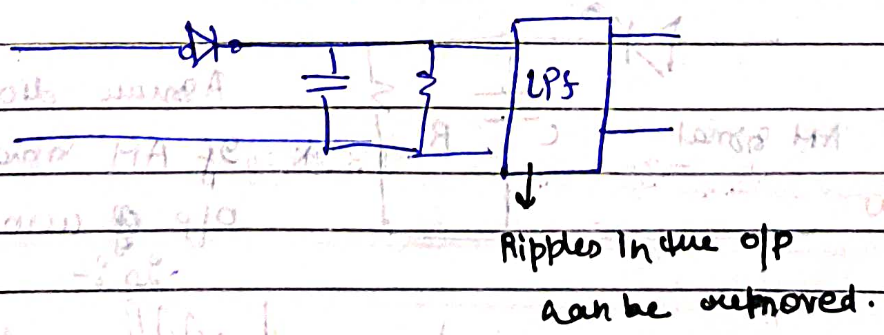
→ After this cap. starts discharging making diode as O.C.



The above step repeats.

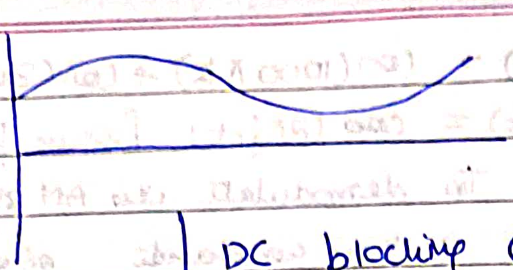
→ o/p across resistor will look like :-



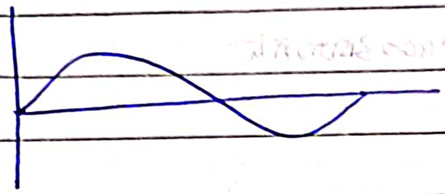


→ further using the DC blocking capacitor → DC component can be removed.

Date : / /
Page No.



↓ DC blocking capacitor



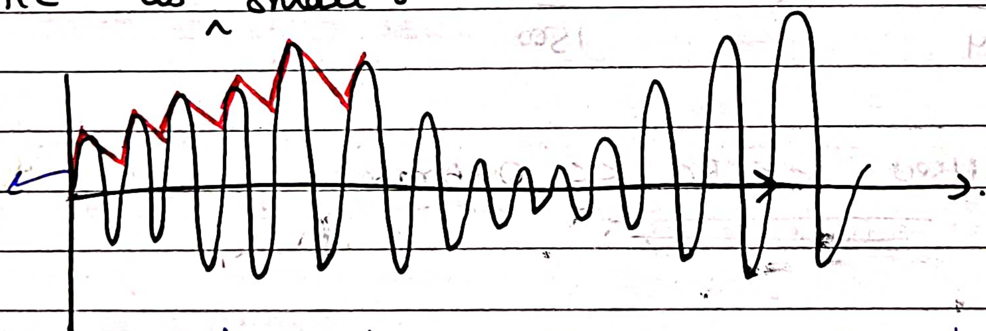
→ $\underline{\underline{f_c > f_m}}$ → for envelope detector

$$\boxed{\frac{1}{f_c} \ll \underbrace{RC}_{} \ll \frac{1}{f_m}}$$

DETECTION OF AM SIGNAL.

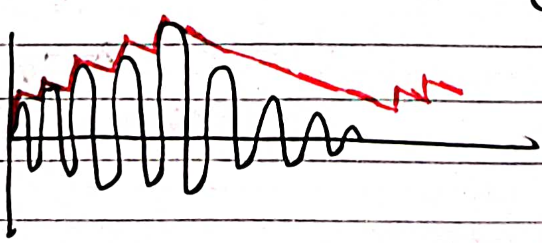
RC time constant.

→ If RC is too small :-



Capacitor will discharge rapidly and we will see more ripples at the o/p.

→ If RC time constant is too large :-



Capacitor will take longer time to discharge.

Q2. A msg signal $m(t) = \cos(1000\pi t) + \cos(3000\pi t)$ modulates the carrier signal $c(t) = \cos(2\pi f_c t)$ [where $f_c = 2MHz$], to generate the AM signal. To demodulate the AM signal using the envelope detector, the RC time constant of the ckt should satisfy :-

$$m(t) = \cos 1000\pi t + \cos 3000\pi t$$

$$2\pi f = 1000\pi$$

$$f_{m_1} = 500Hz$$

$$f_{m_2} = 1500Hz$$

$$f_{m\,max} = 1500Hz$$

$$f_c = 2MHz$$

$$\frac{1}{f_c} << RC << \frac{1}{f_m}$$

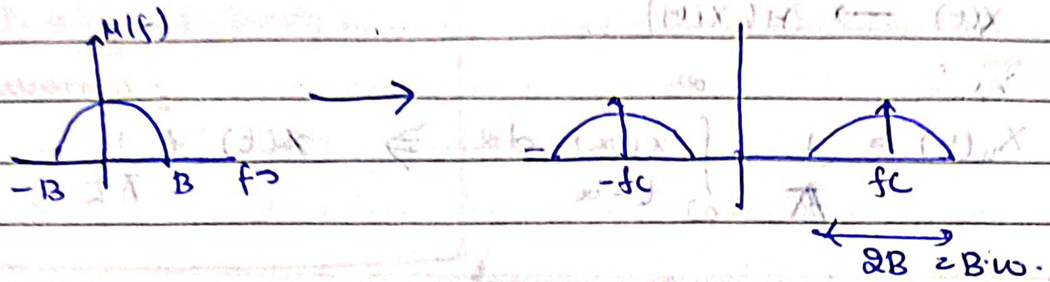
$$\frac{1}{2M} << RC << \frac{1}{1500}$$

$$0.5\mu sec << RC << 0.66\,msec$$

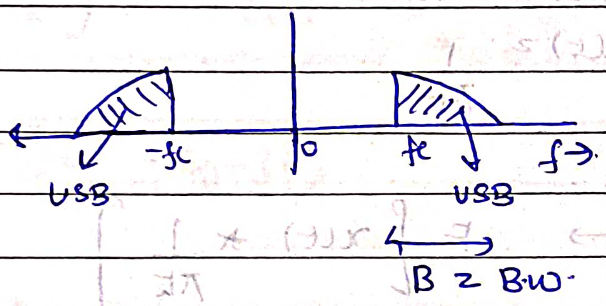
Date : / /
Page No.

SINGLE SIDE BAND MODULATION (SSB)

→ B.W efficient scheme

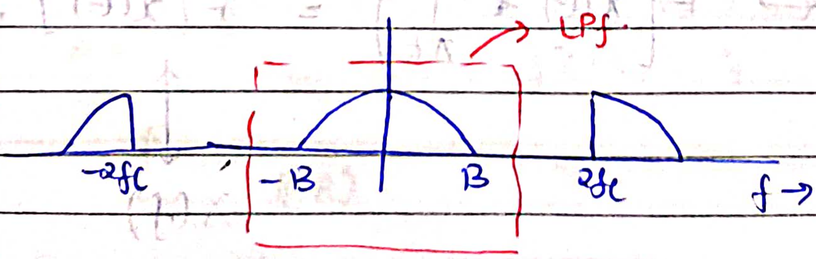


In SSB → either LSB / USB is transmitted. as both have same info.



→ SSB is B.W efficient.
→ Used for voice signal.
→ Total Power is also less.

→ SSB ⇒ Carrier signal is not transmitted. So, in demodulation it can be recovered using Synchronous Detector
↓
$\phi(t) \cdot \cos \omega_c t$



Date : / /
Page No.

Mathematical Representation of SSB :-

HILBERT TRANSFORM

$x(t) \longrightarrow H(x(t))$

$$x_h(t) = \frac{1}{\pi}\int_{-\infty}^{\infty} \frac{x(\alpha)}{t-\alpha}\, d\alpha \Rightarrow x(t) * \frac{1}{\pi t} \rightarrow (1)$$

convolution.

$$\boxed{x_1(t) * x_2(t) = \int_{-\infty}^{\infty} x_1(\tau)\, x_2(t-\tau)\, d\tau}$$ → convolution formula

In (1) $x_1(t) = x(t)$

$x_2(t) = \frac{1}{\pi t}$

$$F[x_h(t)] \longleftrightarrow F\left[x(t) * \frac{1}{\pi t}\right]$$

Convolution property :-

$g_1(t) \longleftrightarrow G_1(f)$ $\quad$ $g_2(t) \longleftrightarrow G_2(f)$

$F[g_1(t) * g_2(t)] \longleftrightarrow G_1(f) \cdot G_2(f)$

$$F[x_h(t)] \leftrightarrow F\left[x(t) * \frac{1}{\pi t}\right] = F[x(t)] \cdot F\left[\frac{1}{\pi t}\right]$$

$X(f)$

Date : / /
Page No.

$F[\mathrm{sgn}(t)] = \frac{1}{j\pi f}$

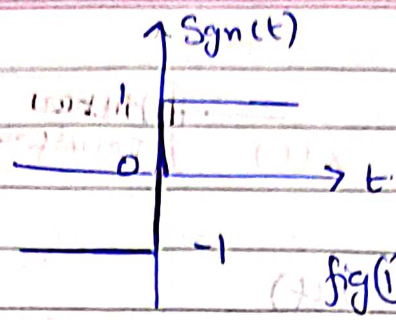


fig (1)

Duality property :-

$g(t) \leftrightarrow G(f)$.
$G(t) \leftrightarrow g(-f)$.

using duality property.

$F\left[\frac{1}{j\pi t}\right] = \underbrace{\mathrm{Sgn}}(-f)$ → odd f^n.

$\frac{1}{j\pi t} = -\mathrm{Sgn}(f)$.

$F\left[\frac{1}{\pi t}\right] \leftrightarrow -j\,\mathrm{Sgn}\,(f)$.

$F[x_h(t)] \leftrightarrow F\left[x(t) * \frac{1}{\pi t}\right] = X(f)\,[-j\,\mathrm{Sgn}(f)]$

from fig (1)

$$\boxed{\begin{aligned} F[x_h(t)] &= -j\,X(f)\,; \quad f>0 \\ &= j\,X(f)\,; \quad f<0 \end{aligned}}$$

from the above graph.

(OR)

$$\begin{aligned} &= X(f)\,e^{-j\pi/2} \quad ; f>0 \\ &= X(f)\,e^{j\pi/2} \quad ; f<0 \end{aligned}$$

Date : / /
Page No.

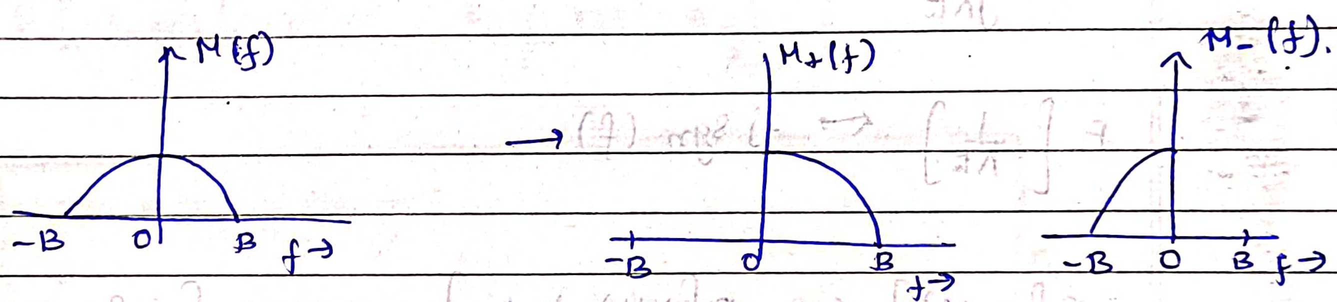
$x(t) \rightarrow$ [Hilbert Transform] $\rightarrow x_h(t)$.

$X(f)$ $\qquad\qquad$ $\underbrace{X(f)}_{X(f)} \underbrace{(-j\, Sgn(f))}_{H(f)}$

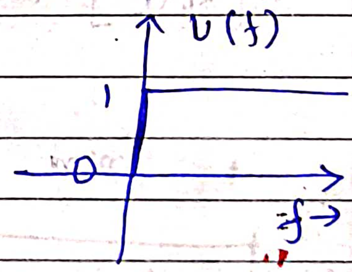
$$\boxed{\begin{aligned} H(f) &= -j\, Sgn(f). \\ &= e^{-j\pi/2} \quad f>0 \\ &\ e^{j\pi/2} \quad f<0 \end{aligned}}$$

→ Ideal phase shifter where every component shifts by $\pi/2$

SSB

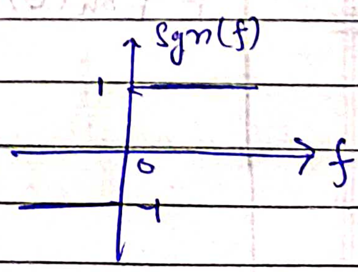


$M_+(f) = M(f) \cdot u(f)$.
$M_-(f) = M(f) \cdot u(-f)$.



→ Unit step f^n in form of Signum f^n :-

$$U(f) = \frac{1}{2}\left[1 + Sgn(f)\right]$$



Date: / /
Page No.

$$\underline{M_+(f)} = M(f)\cdot U(f).$$

$$= \frac{M(f)}{2}\left[1 + \operatorname{sgn}(f)\right]$$

$$= \frac{1}{2}M(f) + \frac{1}{2}M(f)\operatorname{sgn}(f).$$

$$= \frac{1}{2}M(f) + \frac{M(f)}{2} \times \left[\frac{-j\operatorname{sgn}(f)}{-j}\right] \Rightarrow M_h(f)$$

$$= \frac{1}{2}M(f) + \frac{M_h(f)}{-2j}$$

$$= \frac{M(f)}{2} + j\frac{M_h(f)}{2}.$$

$$M_-(f) = \frac{M(f)}{2}\left[1 - \operatorname{sgn}(f)\right]$$

$$= \frac{M(f)}{2} - j\frac{M_h(f)}{2}$$

$$\phi_{USB}(f) = M^+(f-f_c) + M_-(f-f_c)$$

$$= \frac{1}{2}\left[M(f-f_c) + j\,M_h(f-f_c)\right] + \frac{1}{2}\left[M(f+f_c) - j\,M_h(f+f_c)\right]$$

$$= \frac{1}{2}\left[M(f-f_c) + M(f+f_c)\right] - \frac{1}{2j}\left[M_h(f-f_c) - M_h(f+f_c)\right]$$

By freq. shifting property :-

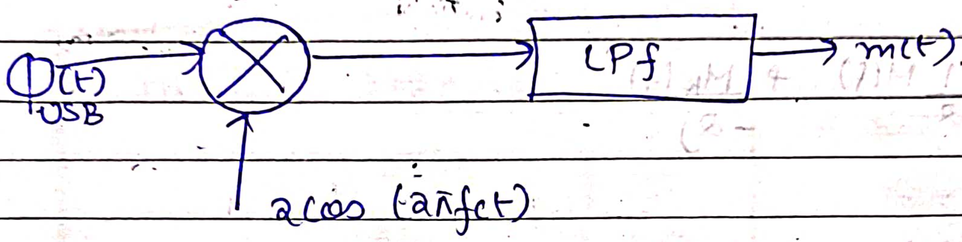
$$g(t) \leftrightarrow G(f).$$
$$g(t)e^{j2\pi f_0 t} \leftrightarrow G(f-f_0).$$

$$= m(t)\left[\frac{e^{j2\pi f_c t} + e^{-j2\pi f_c t}}{2}\right] - \frac{m_h(t)}{2}\left[e^{j2\pi f_c t} - e^{j2\pi f_c t}\right]$$

$$\boxed{\begin{aligned} \phi_{USB}(t) &= m(t)\cos\omega_c t - m_h(t)\sin\omega_c t \\ \phi_{LSB}(t) &= m(t)\cos\omega_c t + m_h(t)\sin\omega_c t \end{aligned}}$$

Mathematical representation of SSB signal

SSB DEModulation



$$\phi_{USB}(t) \times 2\cos(2\pi f_c t) = m(t)\cos\omega_c t \times 2\cos(\omega_c t) - m_h(t)\sin\omega_c t \times 2\cos\omega_c t$$

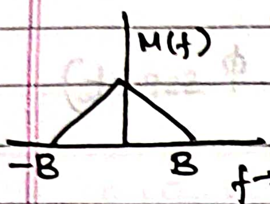
$$= m(t)\left[1+\cos(2\omega_c t)\right] - m_h(t)\left[\sin 2\omega_c t\right]$$

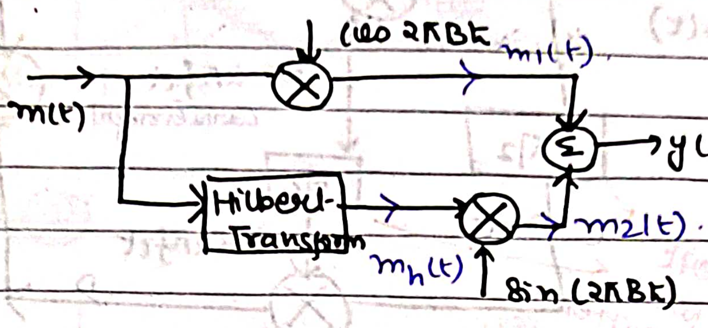
$$= m(t) + \left[m(t)\cos 2\omega_c t - m_h(t)\sin 2\omega_c t\right]$$

$\downarrow$ LPf.

$\underline{\underline{m(t)}}$.

Q.2 As shown below, if M(f) is the freq. spectrum of m(t) then Y(f), the freq. spectrum of y(t) is :-



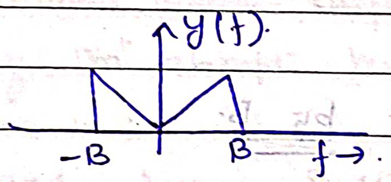


$y(t) = m_1(t) + m_2(t)$.

$y(t) = m(t)\cos(2\pi Bt) + m_h(t)\sin(2\pi Bt)$

~~LSB suppress carrier.~~

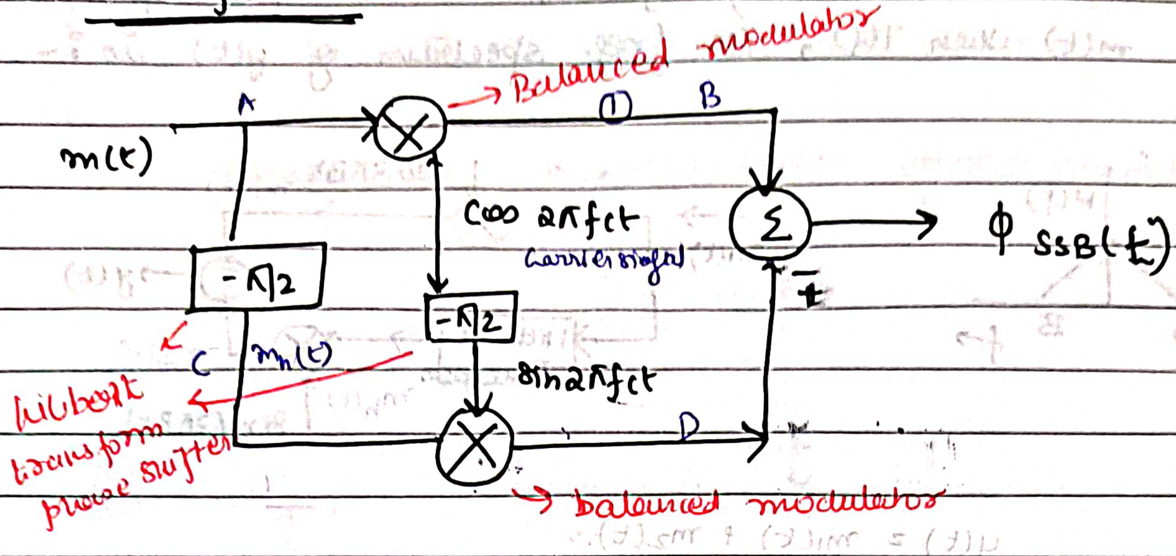
Carrier freq. = B.



SSB GENERATION.

1.) Phase Shift method
2.) Selective filtering method.
3.) Third method (Weaver's Method).

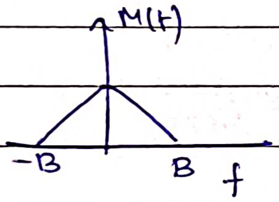
1) Phase-Shift Method.



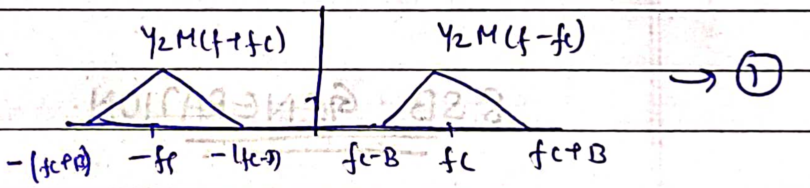
$$\phi_{SSB}(t) = \underbrace{m(t)\cos(2\pi f_c t)}_{①} \mp \underbrace{m_h(t)\sin(2\pi f_c t)}_{②}$$

freq. spectrum

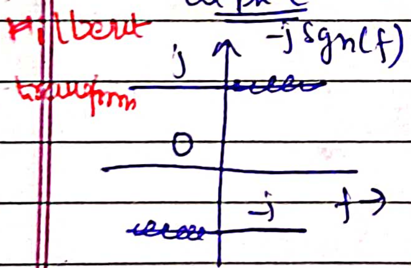
at pt. A.



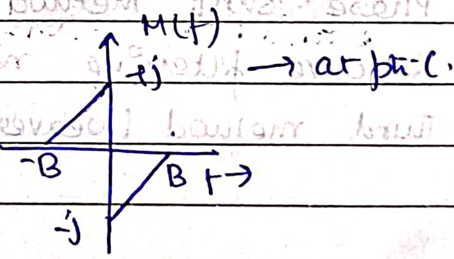
at pt. B.



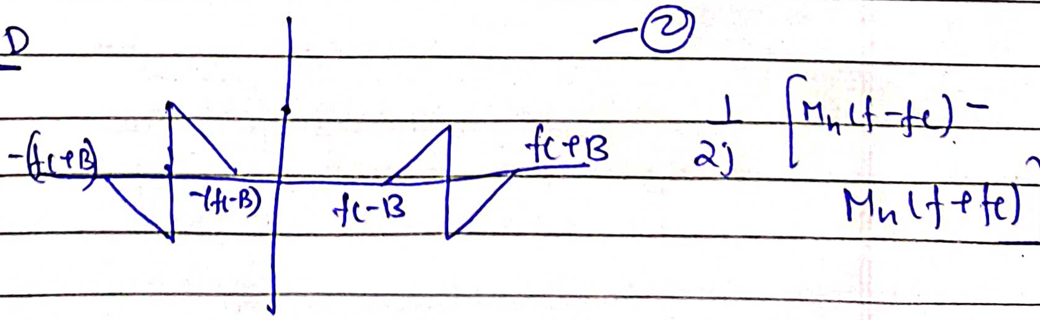
at pt. C.



A×C →

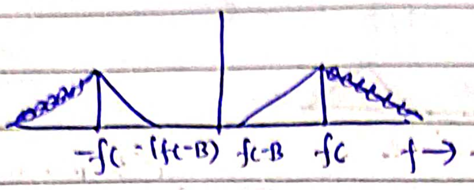


at pt. D



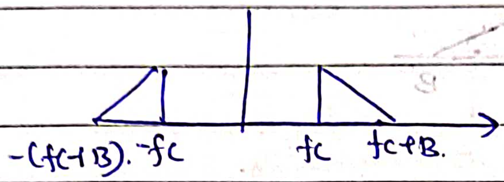
$$\frac{1}{2j}\left[M_h(f-f_c) - M_h(f+f_c)\right]$$

Adding (1) & (2)



LSB

Subtracting (1) & (2)



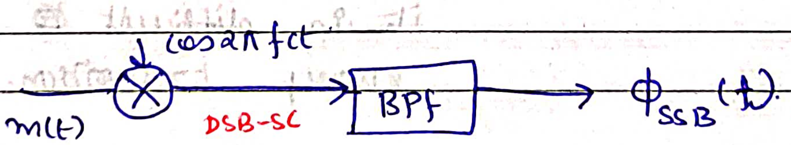
USB.

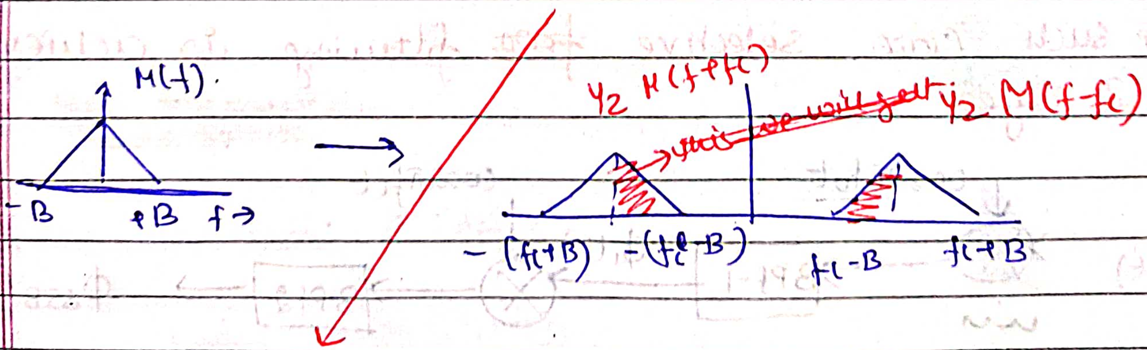
Practically generation of SSB with phase shift method there limitations are:-

(1) ideal hilbert phase shifter is not possible. practically 180° abrupt phase shift is not possible.

(2) wide band phase shifter is very difficult as it needs complexity.

(2) SELECTIVE FILTERING METHOD.





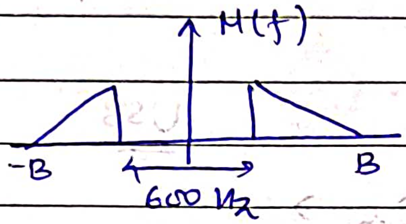
LSB is suppressed and we get USB.

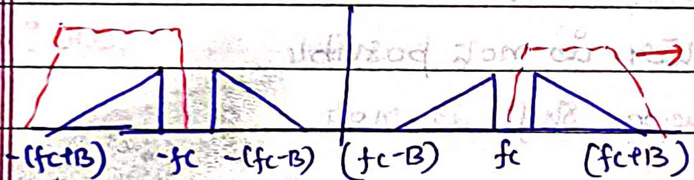
LPF. → USB is suppressed and we get LSB.

Date : / /
Page No.

Actual BPf. → doesnot have sharp suppressing

e.g. Voice signal → 300 Hz to 3500 Hz

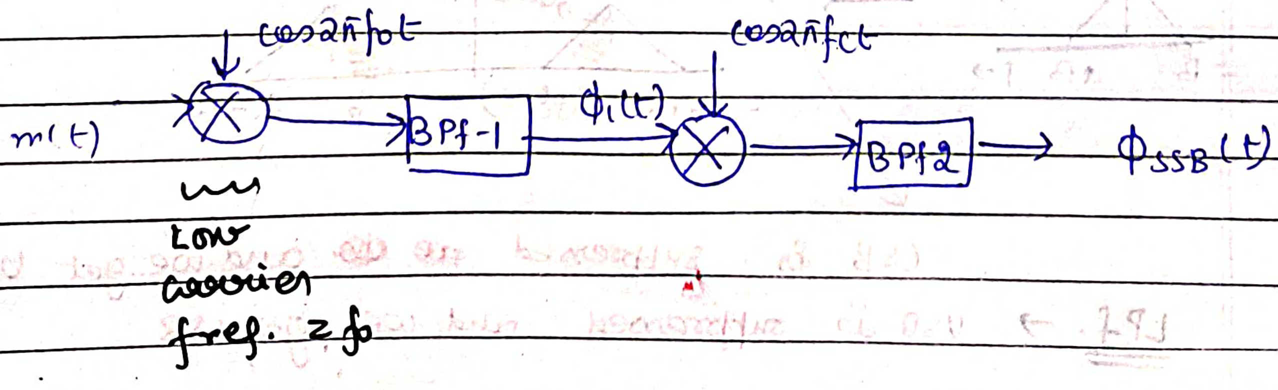


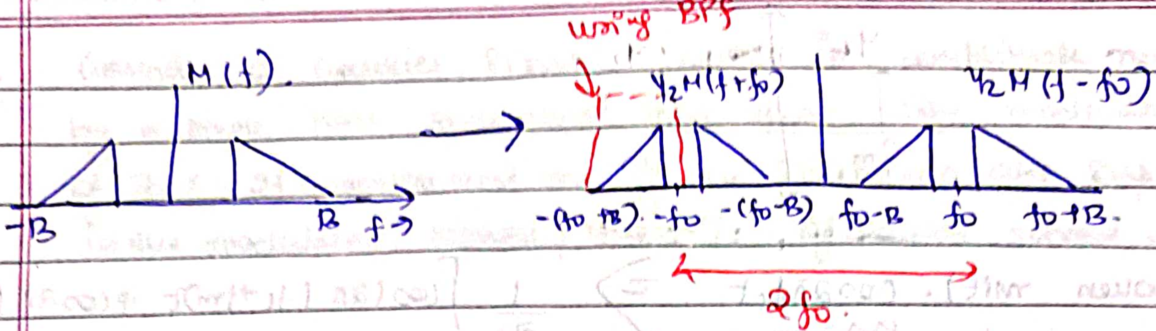


→ to avoid any interferance of unwanted band, the filter should suppress the unwanted band by 40dB

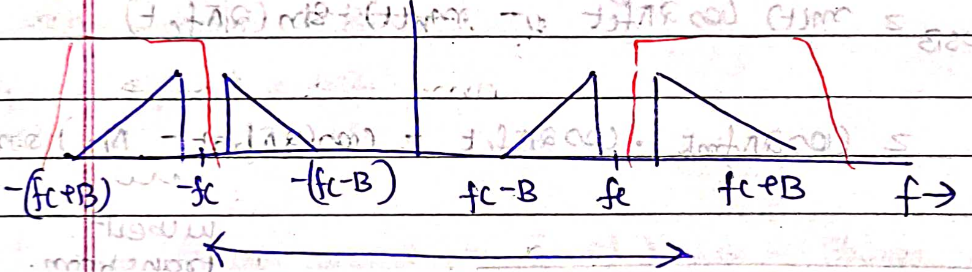
- At low carrier freq. it is possible.
- But at high carrier freq. e.g. 1MHz it is difficult to achieve sharp transition.

In such case selective ~~freq~~ filtering is achieved in 2 steps.





O/P of 2nd Balanced Modulator.



2fc > 600 Hz (voice signal example).

↓

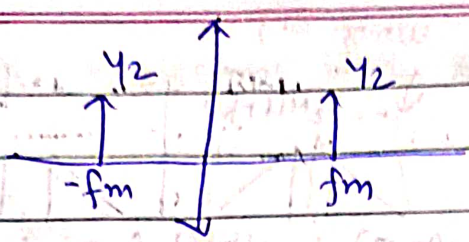
so, possible to generate the modulated signal.

Tone Modulation.

let, $m(t) = \cos(2\pi f_m t)$

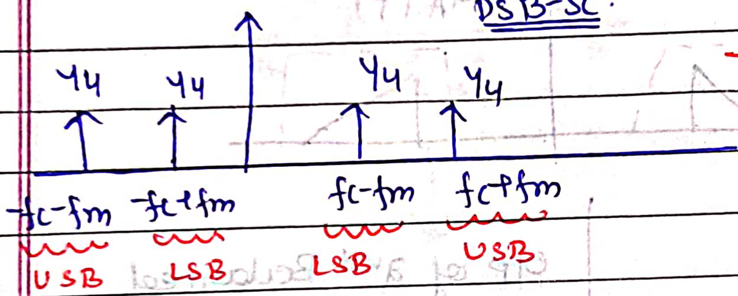
using euler identity :-

$$\cos(2\pi f_m t) = \frac{1}{2}\left[e^{j2\pi f_m t} + e^{-j2\pi f_m t}\right] \leftrightarrow \frac{1}{2}\left[\delta(f-f_m) + \delta(f+f_m)\right]$$



when $m(t) \cdot \underbrace{\cos 2\pi f_c t}_{\text{carrier signal}} \Rightarrow \frac{1}{2}\left[\cos(2\pi(f_c+f_m)t + \cos 2\pi(f_c-f_m)t\right]$

This is DSB-SC. → an SSB-SC



$$\phi_{USB} = m(t)\cos 2\pi f_c t - m_h(t)\sin(2\pi f_c t)$$

$$= \cos 2\pi f_m t \cdot \cos 2\pi f_c t - \underbrace{\cos(2\pi f_m t - \pi/2)}_{\text{hilbert transform.}}\sin 2\pi f_c t$$

$$\boxed{\phi_{USB} = \cos 2\pi(f_c + f_m)t}$$

$$\boxed{\phi_{LSB} = \cos 2\pi(f_c - f_m)t}$$

Date : / /
Page No.

Q. Consider a carrier signal which is amplitude modulated by a single tone sinusoidal msg signal with modulation index of 70%. If carrier and one of the sidebands are suppressed in the modulated signal, then % of power saved is :-

Ans. Total transmitted Power :- Sideband + Carrier power

$$P_{SB} = \frac{P_c \mu^2}{2}$$

$$P_T = P_{SB} + P_c = P_c\left[1+\frac{\mu^2}{2}\right]$$

SSB:-

$$P_T = \frac{P_{SB}}{2} \text{ (power of one side band)}$$

$$= \frac{P_c \mu^2}{4}$$

Power saved = Total power − Power after suppression.

$$\%\text{ of power saved} = \frac{\text{Total power} - \text{Power after suppression}}{\text{Total power}} \times 100$$

$$P_{AM} = P_c\left[1+\frac{\mu^2}{2}\right]$$

$$P_{SSB} = \cancel{B}\ P_c\frac{\mu^2}{4}$$

$$\%\text{ power saved} = \frac{P_c\left[1+\frac{\mu^2}{2}\right] - \frac{P_c\mu^2}{4}}{P_c\left[1+\frac{\mu^2}{2}\right]} \times 100$$

$$= \frac{P_c\left[1+\mu^2/4\right]}{P_c\left[1+\mu^2/2\right]} \times 100$$

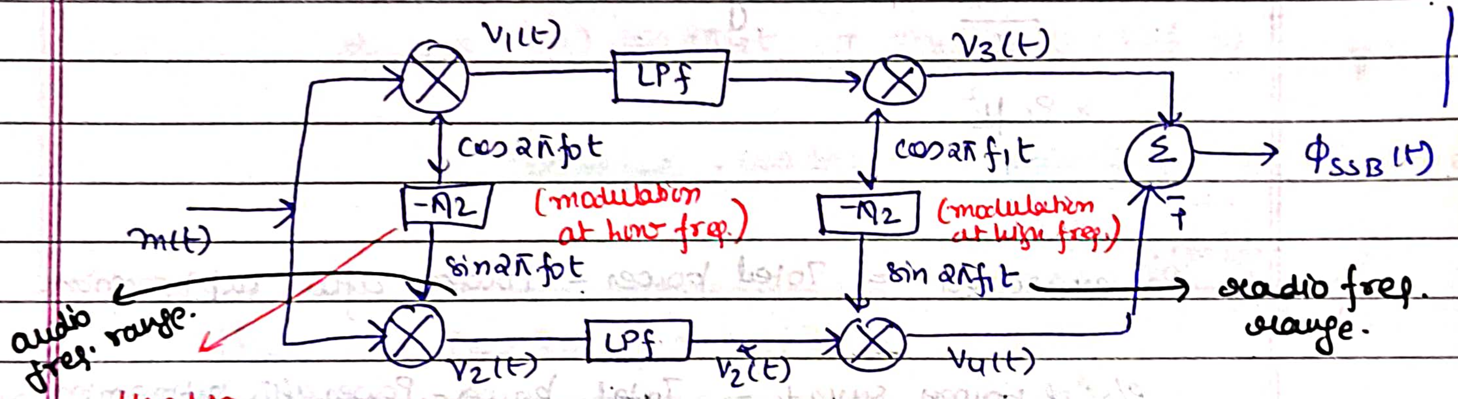
$$= \frac{1+\mu^2/4}{1+\mu^2/2} \times 100$$

$$\Rightarrow \mu = 70\% \text{ or } \underline{0.7}$$

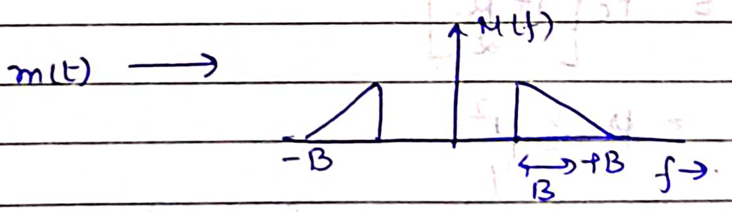
$$\% \text{ of power saved} = \frac{1+(0.7)^2/4}{1+(0.7)^2/2} \times 100 = \underline{\underline{90.16\%}}$$

(3) WEAVER METHOD.

→ In this method we don't need to provide $90°$ phase shift to entire msg signal as in the case of phase shift method.



Used to provide $90°$ phase shift to single freq. so, complexity of the ckt. will reduce.

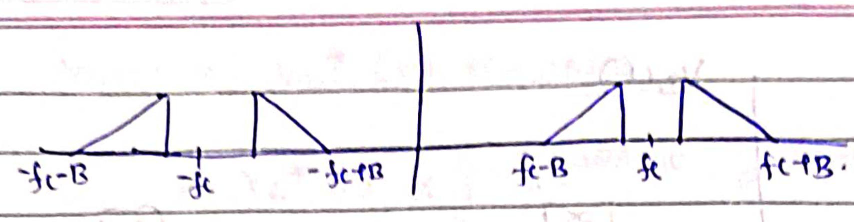


max. freq = B

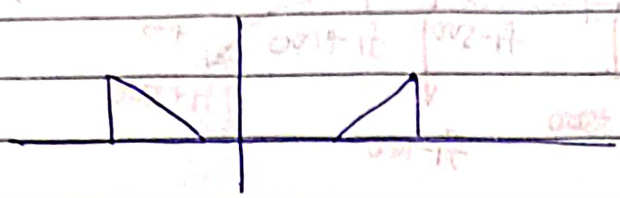
cut off freq. of LPf = B/2

Carrier freq. f_0 = B/2

Date : / /
Page No.



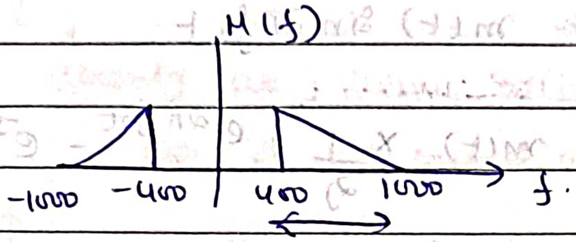
$\rightarrow$ DSB-SC



$\rightarrow$ SSB-SC

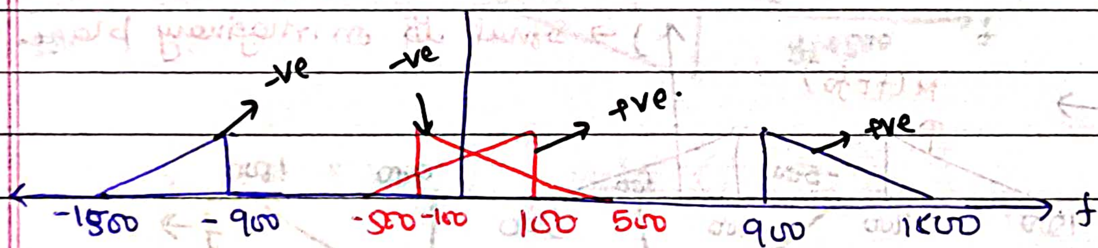
* e.g.

$m(t) \rightarrow$

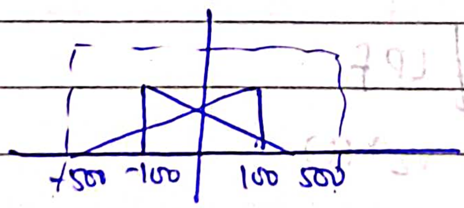


B = 1000.
f_0 = 500 Hz.
cut off freq. of LPF.

$V_1(t) \rightarrow$ It will shift on both +ve side & -ve side.

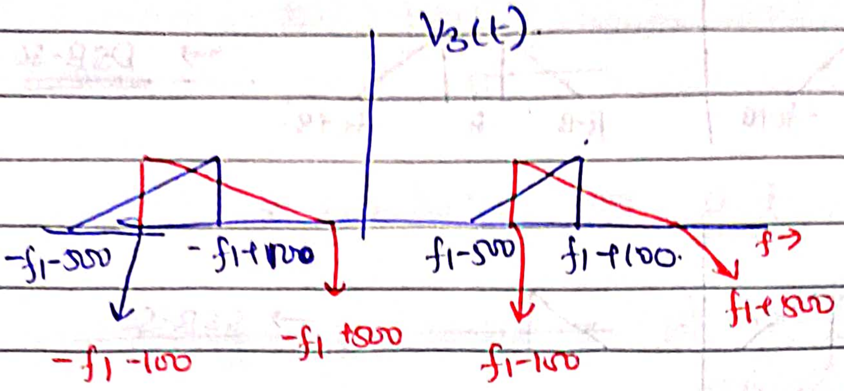


LPF.



$\downarrow$ 2nd Balanced modulator

Date: / /
Page No.



→ 1

<u>2nd half</u>..

$V_2(t) = m(t)\sin 2\pi f_0 t$.

$= m(t) \times \frac{1}{2j}\left[e^{j2\pi f_0 t} - e^{-j2\pi f_0 t}\right]$

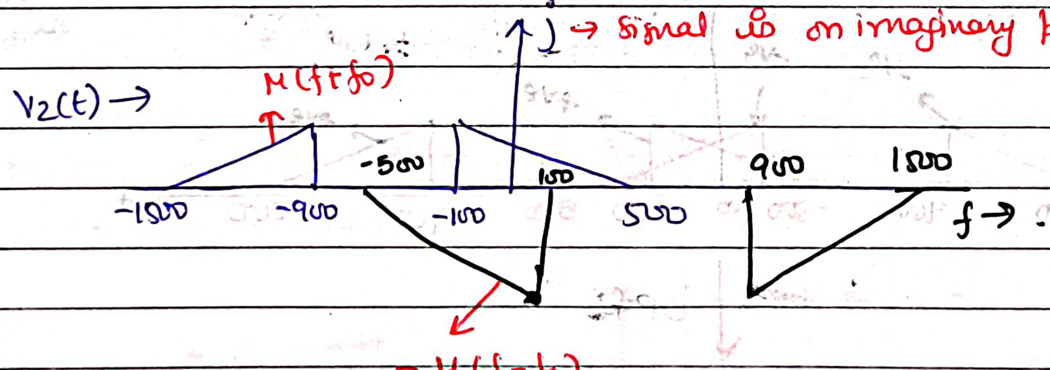
$\updownarrow$

$\frac{-j}{2}\left[M(f-f_0) - M(f+f_0)\right] \longleftrightarrow \frac{j}{2}\left[M(f+f_0) - M(f-f_0)\right]$

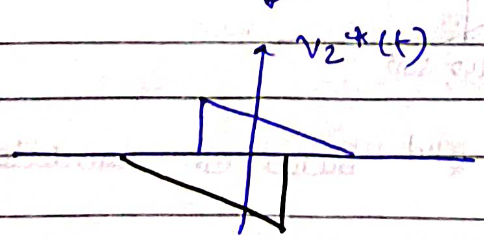
90° phase apart.

as, $j = e^{j\pi/2}$

j → signal is on imaginary plane



LPF



$$V_4(t) = V_2^*(t)\sin 2\pi f_1 t$$

$$= V_2^*(t) \times \frac{1}{2j}\left[e^{j2\pi f_1 t} - e^{-j2\pi f_1 t}\right]$$

$$\updownarrow$$

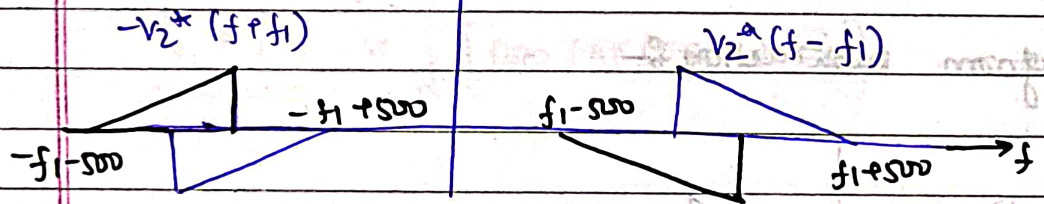
$$= \frac{1}{2j}\left[V_2^*(f-f_1) - V_2^*(f+f_1)\right]$$

already as j term. so if divided by 2j it will cancel out.

$$\updownarrow$$

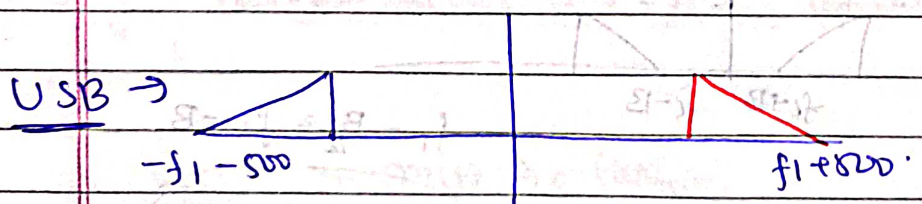
$$\frac{1}{2}\left[V_2^*(f-f_1) - V_2^*(f+f_1)\right]$$

~~real plane as j is cancelled before.~~

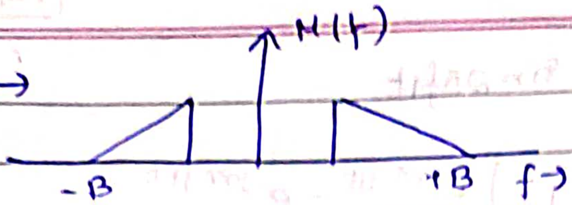


— ②

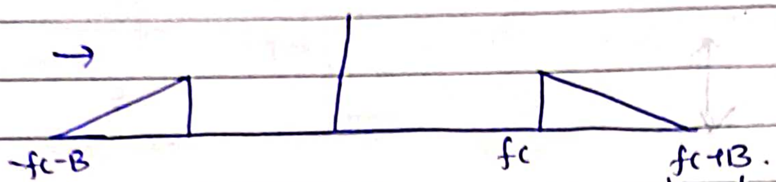
$V_3(t) + V_4(t)$ [add ① + ②]



$m(t) \rightarrow$

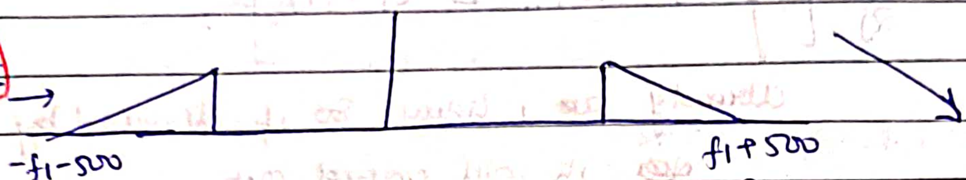


SSB-SC $\rightarrow$



$\underbrace{f_c+B}$ maximum freq. component.

USB $\rightarrow$



$B = 1000$

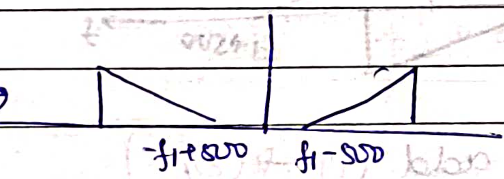
$f_1 + \frac{B}{2} = f_c + B$

$$\boxed{f_1 = f_c + \frac{B}{2}}$$

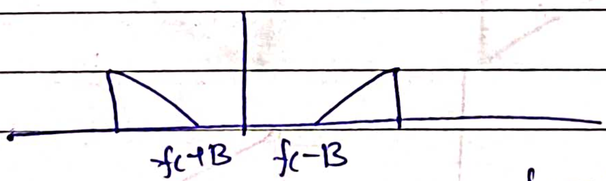
when f_1 is equal to $f_c + B/2$ then we get $f_1 + 500$.

If we perform subtraction :-

LSB $\rightarrow$



SSB $\rightarrow$



$f_1 - B/2 = f_c - B$

$$f_1 = f_c - B/2$$

Date : / /
Page No.

Tone Modulation.

(Mathematical Steps).

Let $m(t) = \cos 2\pi f_m t$.

$$V_1(t) = m(t) \cos 2\pi f_0 t.$$
$$= \cos 2\pi f_m t \cdot \cos 2\pi f_0 t.$$
$$= \frac{1}{2}\left[\cos(2\pi(f_0 + f_m)t + \cos(2\pi(f_0 - f_m)t)\right]$$

$$f_0 = \frac{B}{2} = \frac{f_m}{2}$$

$$\underline{\underline{V_1(t)}} = \frac{1}{2}\left[\cos(2\pi(1.5 f_m)t) + \cos(2\pi(0.5 f_m)t)\right]$$

$\downarrow$ after LPF

$$\underline{\underline{V_1^*(t)}} = \frac{1}{2}\left[\cos(2\pi(0.5 f_m)t)\right]$$

$$V_3(t) = V_1^*(t) \cos 2\pi f_1 t$$
$$= \frac{1}{2}\left[\cos(2\pi(0.5 f_m)t\right] \cdot \cos 2\pi f_1 t$$

$$\underline{\underline{V_3(t)}} = \frac{1}{4}\left[\cos(2\pi(f_1 + 0.5 f_m)t) + \cos(2\pi(f_1 \cancel{-} 0.5 f_m)t\right]$$

$$\underline{\underline{V_2(t)}} = m(t) \sin(2\pi f_0 t)$$
$$= \cos(2\pi f_m t) \sin(2\pi f_0 t).$$
$$= \frac{1}{2}\left[\sin(2\pi(f_0 + f_m)t + \sin(2\pi(f_0 - f_m)t)\right]$$

$$f_0 = f_m/2$$

$$= \frac{1}{2}\left[\sin(2\pi(1.5 f_m)t - \sin(2\pi(0.5 f_m)t\right]$$

$$\Big\downarrow \text{LPF}$$

$$v_2^*(t) = -\tfrac{1}{2}\sin(2\pi(0.5 f_m)t)$$

$$v_4(t) = v_2^*(t)\ \sin(2\pi f_1 t)$$

$$= -\tfrac{1}{2}\sin(2\pi(0.5 f_m)t)\cdot\sin(2\pi f_1 t)$$

$$= -\tfrac{1}{4}\left[\cos(2\pi(f_1 - 0.5 f_m)t) - \cos(2\pi(f_1 + 0.5 f_m)t\right]$$

$$v_4(t) = \tfrac{1}{4}\left[\cos(2\pi(f_1 + 0.5 f_m)t - \cos(2\pi(\cancel{f_1} - 0.5 f_m)t\right]$$

$$v_3(t) + v_4(t)$$

$$\boxed{O/P = \tfrac{1}{2}\cos\left(2\pi(f_1 + 0.5 f_m)t\right]} \rightarrow \text{USB}$$

as, $f_1 = f_c + \frac{B}{2}$

$$= f_c + \frac{f_m}{2}$$

$$O/P = \frac{1}{2}\cos\left[2\pi\left(f_c + \frac{f_m}{2} + \frac{f_m}{2}\right)t\right)$$

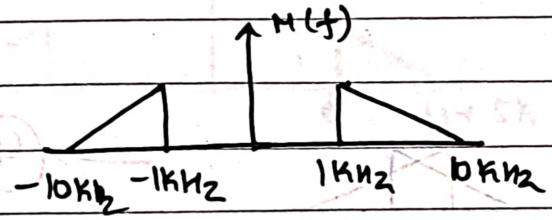
$$\boxed{O/P = \frac{1}{2}\cos\left[2\pi(f_c + f_m)t\right]}$$

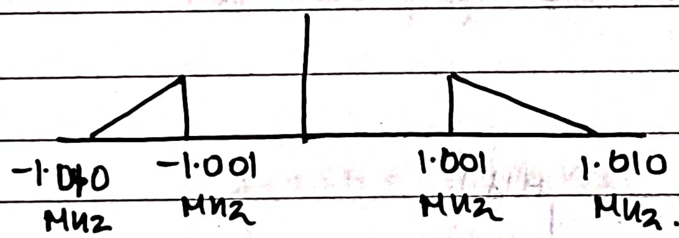
Date : / /
Page No.

of $v_3(t) - v_4(t)$.

$$\boxed{o/p = \frac{1}{2}\cos\left[2\pi(f_c - f_m)t\right]}$$

Q.2 for the given msg signal m(t), using SSB it is required to generate the upper sideband as shown below. The upper sideband is generated using the Weaver's method and the block diagram of the same is also shown below.
To generate upper sideband, the req. freq. of f_0 and f_1.





Block diagram of weaver.

As.2 $f_c = 1MHz$.

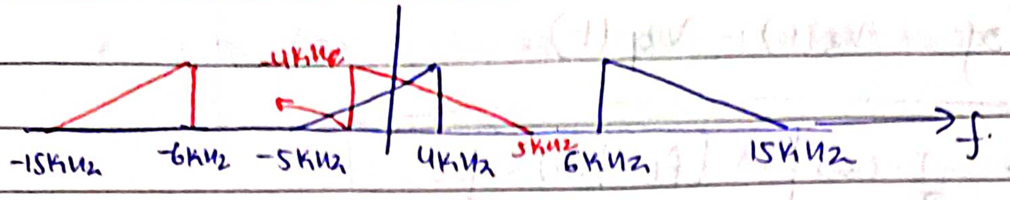
B = max. freq. of msg signal.

Carrier freq. $= \frac{B}{2}$

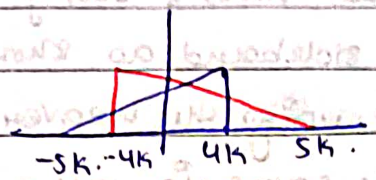
$B = 10KHz$.

$f_c = 5KHz$.

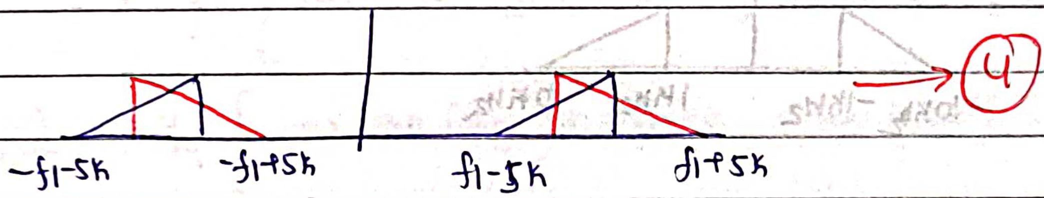
Date : / /
Page No.



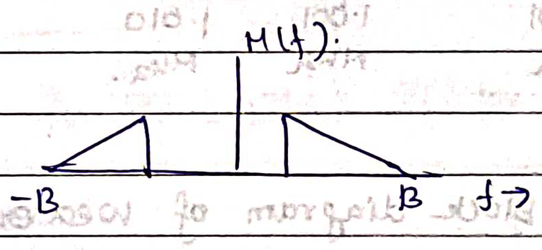
LPF (fc = 5K) .

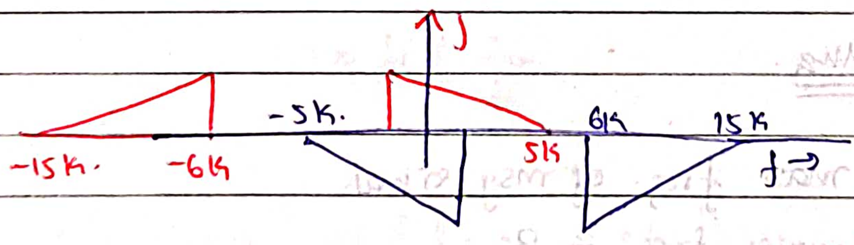


2nd Modulator

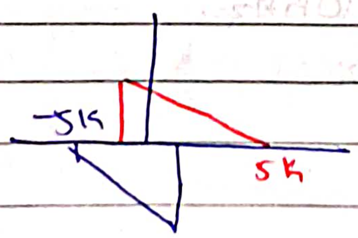


2nd part

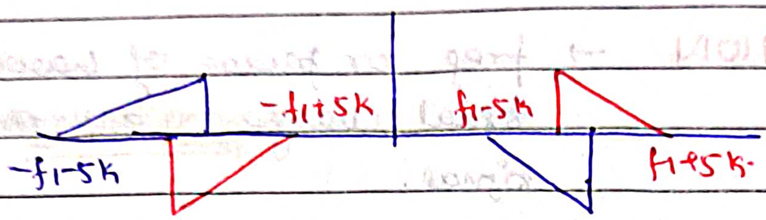




LPF (fc = 5K)

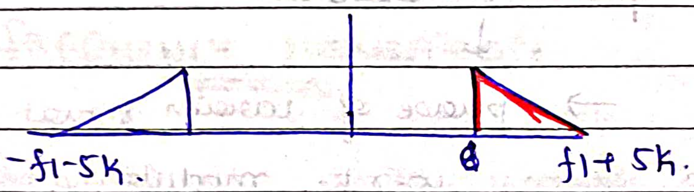


after other modulator



— (3)

(3) + (4)



maximum freq. in USB = 1.01MHz
= 1010kHz

$f_1 + 5k = 1010kHz$

$f_1 = \underline{\underline{1005kHz}}$

$f_0 = B/2 = \underline{\underline{5kHz}}.$

Date : / /
Page No.

FREQUENCY MODULATION

→ FM has ∞ B.W.

ANGLE MODULATION

→ freq or phase of carrier signal changes w.r.t. modulating signal.

freq. Modulation

↓

→ freq. ~~changes~~ of ~~xxxx~~ Carrier signal changes w.r.t. modulating signal.

of $y(t) = A_c \cos(\omega_c t + \phi)$

~~carrier~~

$\omega_c(t) = F[m(t)]$

freq. is function of msg signal.

phase modulation.

↓

→ phase of carrier signal changes w.r.t. modulating signal

$y(t) = A_c \cos(\omega_c t + \phi(t))$

$\phi(t) = F[m(t)]$

Advantages of Angle modulation.

1. Noise reduction → In AM amplitude of carrier is changing w.r.t. to modulating signal. Max. defect in amplitude only.
2. Improved system fidelity → in terms of security.

Date : / /
Page No.

(3) Efficient use of power. → we don't send separate carrier signal. we only send the modulating signal.

Applications :-

1.) Radio broadcasting.
2.) TV sound transmission.
3.) Cellular audio.

FREQUENCY MODULATION

→ In FM, freq. of the carrier signal $c(t)$ changes w.r.t. modulating signal $m(t)$

→ we have carrier signal.

$$c(t) = E_c \cos(\omega_c t + \phi).$$
$$= E_c \cos(2\pi \boxed{f_c} t + \phi).$$
$$= E_c \cos \theta(t).$$

→ angle

$$\theta(t) = 2\pi f_c t + \underbrace{2\pi k_f \int_0^t x(t)\, dt}_{\text{deviation}}.$$

modulated signal :-

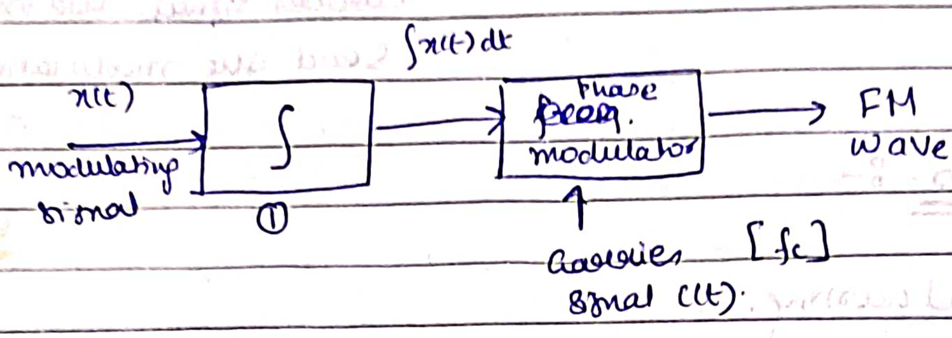
$$y_{FM}(t) = E_c \cos\left[2\pi f_c t + 2\pi k_f \int_0^t x(t)\, dt\right]$$

k_f ← freq. sensitivity. (Hz/V).

→ If k_f is high deviation of freq. is high

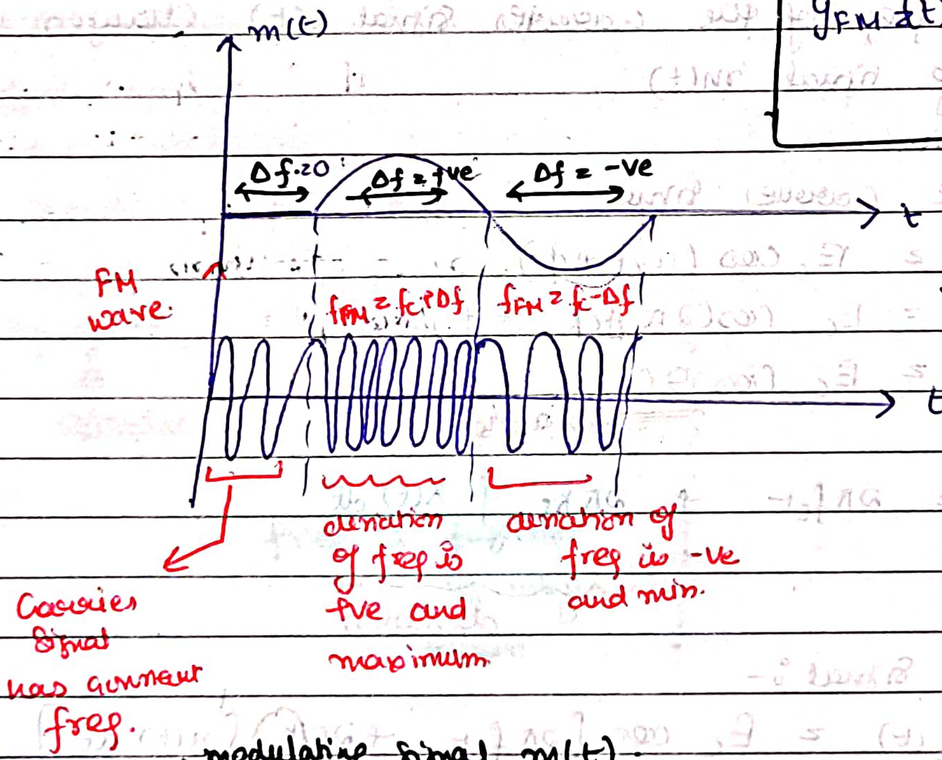
→ If k_f is low deviation of freq. is low.

freq. Modulator



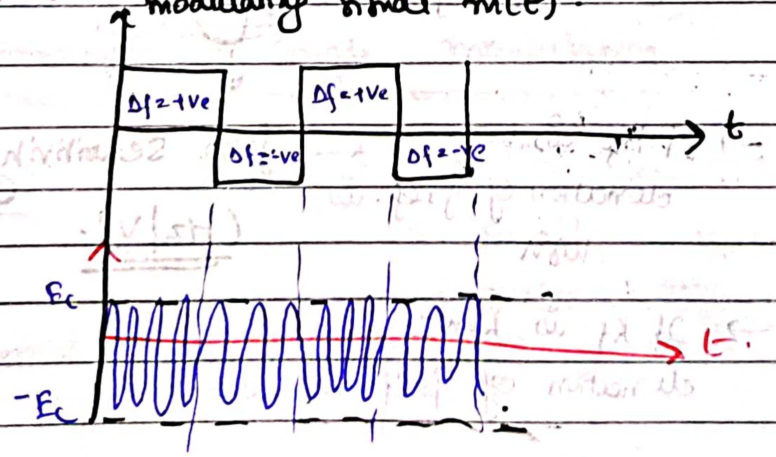
$$\cos\Big[\underbrace{2\pi f_c}_{\text{phase of carrier.}} + \underbrace{2\pi k_f \int x(t)\,dt}_{\text{msg signal phase}}\Big]$$

$$\boxed{y_{FM}(t) = E_c \cos\Big(2\pi f_c t + \underbrace{2\pi k_f \int_0^t x(t)\,dt}_{\text{deviation of freq. } \Delta f}\Big]}$$



$$\boxed{\omega_i(t) = \omega_c + k_f\, m(t)}$$

instaneous angular freq.



PHASE MODULATION

→ In PM, the phase of carrier signal changes w.r.t. modulating signal $m(t)$.

→ If we have carrier signal :-

$$c(t) = E_c \cos(\omega_c t + \phi).$$
$$= E_c \cos(2\pi f_c t + \phi(t))$$
$$= E_c \cos(\theta(t))$$

→ for phase modulation :-

$$\theta(t) = 2\pi f_c t + K_p x(t).$$

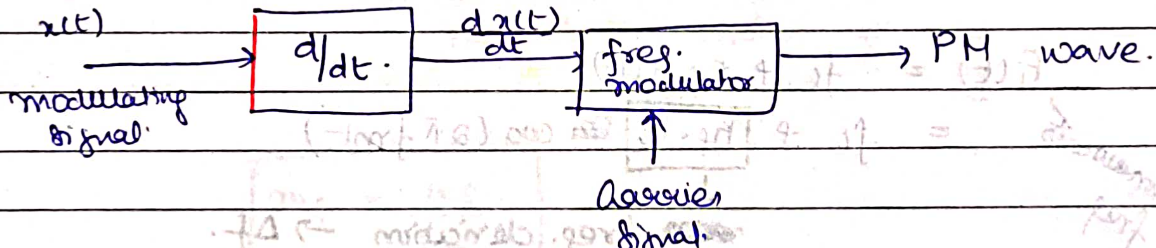
→ PM wave :-

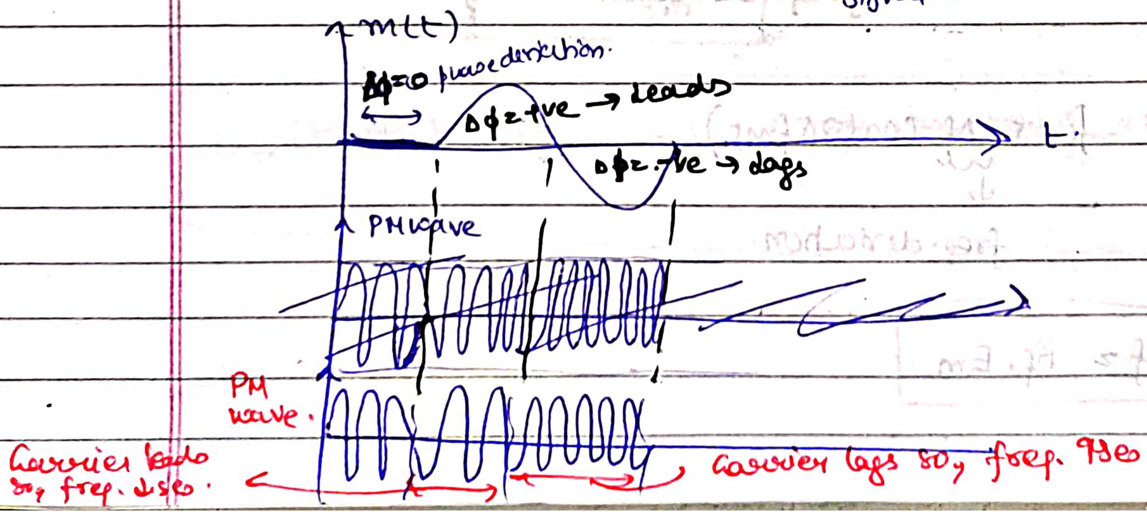
$$\boxed{y_{PM}(t) = E_c \cos(2\pi f_c t + \underbrace{K_p}_{\text{phase sensitivity (rad/V)}} x(t))}$$

if $x(t) = +ve$.
⇒ Carrier will lag w.r.t. $m(t)$.

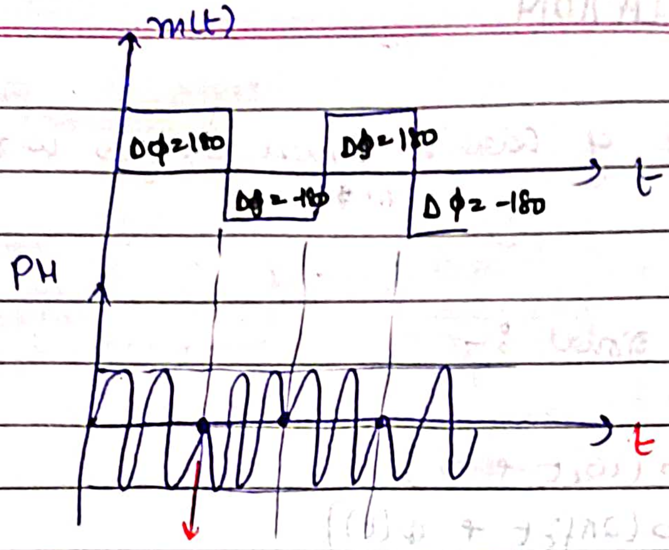
if $x(t) = -ve$.
⇒ Carrier will lead w.r.t. $m(t)$.

instantaneous angular freq.: $\boxed{\omega_i(t) = \omega_c + K_p m(t)}$





Date : / /
Page No.



phase reversal in PM wave.

FREQUENCY DEVIATION & MODULATING INDEX of FM

let $x(t)$:-

$$x(t) = E_m \cos(2\pi f_m t) \quad -(1)$$

Carrier signal :-

$$c(t) = E_c \sin(2\pi f_c t)$$

freq. of FM signal :-

$$F_i(t) = f_c + k_f x(t)$$

instantaneous freq.

$$= f_c + \boxed{k_f \cdot E_m} \cos(2\pi f_m t)$$

freq. deviation $\rightarrow \Delta f$.

$$= f_c + \Delta f \cos(2\pi f_m t)$$

freq. deviation.

$$\boxed{\Delta f = K_f \cdot E_m}$$

Date : / /
Page No.

→ max. freq. diaviation $= f_c + \Delta f$
→ min freq. deviation $= f_c - \Delta f$.

⇒ as $\boxed{y_{FM}(t) = E_c \sin(\omega_c t + K_f . 2\pi \int_0^t x(t)\, dt)}$ → (2)

put (1) in (2)

$$y_{FM}(t) = E_c \sin(\omega_c t + K_f\, 2\pi \int_0^t E_m \cos(2\pi f_m t)\, dt)$$

$$= E_c \sin(\omega_c t + \frac{K_f . \cancel{2\pi} E_m}{\cancel{2\pi} f_m} [\sin(2\pi f_m t)]$$

$$= E_c \sin\left[\omega_c t + \frac{K_f E_m}{f_m} [\sin(2\pi f_m t)]\right]$$

$$y_{FM}(t) = E_c \sin\left[\omega_c t + \boxed{\frac{\Delta f}{f_m}} \sin(2\pi f_m t)\right]$$

↓

modulation index of FM.

m_f.

$$\boxed{m_f = \frac{\Delta f}{f_m}}$$

$$\boxed{y_{FM}(t) = E_c \sin\left[\omega_c t + m_f \sin(2\pi f_m t)\right]}$$

FM signal.

Date : / /
Page No.

$$\boxed{\text{Deviation ratio} = \frac{\text{max. deviation}}{\text{max. modulating freq.}}}$$

$$\boxed{\%\ \text{Modulation of FM} = \frac{\text{Actual freq. deviation}}{\text{max. allowed deviation}}}$$

→ Spectral components of Angle modulated signal :-

→ In Angle modulation, there is freq. & phase modulation.
→ Spectral components are identical for freq. & phase modulation

for Phase modulation :-

$$\boxed{e(t) = E_c \sin(\omega_c t + \underbrace{m}_{\text{modulation index} = \frac{\Delta f}{f_m}} \cos \omega_m t)}$$

B.W Requirements as per Carson's rule.

→ If m_f is high so, B.W is required high.

$m = \frac{\Delta f}{f_m}$ → deviation in freq.
f_m → max. freq. of message signal.

→ modulation index has three categories :-
(1) Low (less than 1) $m_f < 1$
(2) Medium $1 \leq m_f < 10$
(3) High $m_f > 10$

Date : / /
Page No.

→ minimum B.W. :-

$\boxed{B.W = 2 f_m}$ $(m<1)$

→ for high modulation index :-

$\boxed{B.W = 2\Delta f}$ $(m>10)$.

As per Carson's Rule:- minimum B.W. that will:-

$\boxed{B.W = 2[\Delta f + f_m]}$ → This rule accomodates almost 98% of transmitted power.

In AM, max. B.W = $2 f_m$
for SSB → B.W = f_m.

⇒ Angle modulation has higher B.W than A.M but it has greater noise immunity. Better η w.r.t. transmitted power.

AVERAGE POWER REQUIREMENT FOR ANGLE MODULATION.

→ As per phase Modulation :-

$$\boxed{e(t) = E_c \sin(\omega_c t + m \cos \omega_m t)} \rightarrow (1)$$

Spectral components as per Bessel f^n.

$$e(t) = E_c [J_0 \sin \omega_c t + J_1 (\sin(\omega_c + \omega_m)t + \sin(\omega_c - \omega_m)t)$$
$$+ J_2 (\sin(\omega_c + 2\omega_m)t + \sin(\omega_c - 2\omega_m)t)$$
$$+ J_3 (\sin(\omega_c + 3\omega_m)t + \sin(\omega_c - 3\omega_m)t).$$
$$+ \ldots$$

→ Carrier signal has amplitude = $E_c J_0$

→ amp. with sidebands :- $E_c J_1$, $E_c J_2$, $E_c J_3$, ——

Date : / /
Page No.

as per eqⁿ ①

Power with carrier :-

$$\boxed{P_c = \frac{E_c^2}{2R}}$$ where R is load resistance.

→ for modulated spectrum :- $E_0 = E_c J_0$
$E_1 = E_c J_1$
$E_2 = E_c J_2$
⋮

Total power of modulated signal $= P_C + P_{SBs}$

$$= P_0 + \underbrace{P_1 + P_1}_{} + \underbrace{P_2 + P_2}_{} + \underbrace{P_3 + P_3}_{} + \ldots$$

$$= P_0 + 2P_1 + 2P_2 + 2P_3 + \ldots$$

$$= \frac{E_0^2}{2R} + 2\left[\frac{E_1^2}{2R}\right] + 2\left[\frac{E_2^2}{2R}\right] + 2\left[\frac{E_3^2}{2R}\right] + \ldots$$

$$\boxed{P_T = \frac{1}{R}\left[\frac{E_0^2}{2} + E_1^2 + E_2^2 + E_3^2 + \ldots\right]}$$

Same eqⁿˢ for PM & FM.

Date: / /
Page No.

NARROW BAND FREQUENCY MODULATION

→ Category of FM
→ improve spectrum η.
→ here we have narrow B.W.
→ used in voice communication & radio settings.
→ here $\boxed{m = \frac{\Delta f}{f_m} \leq 1}$

FM expression is given by :-

$$e(t) = E(t) \cos\left[\omega_c t + \boxed{\frac{K_f E_m}{\omega_m}} \sin \omega_m t\right]$$

here $m = \frac{K_f \cdot E_m}{\omega_m}$

$$e(t) = E_c \cos\left[\omega_c t + m \sin \omega_m t\right]$$

$$= E_c\left[\cos \omega_c t \cdot \underline{\cos(m \sin \omega_m t)} - \sin \omega_c t \cdot \underline{\sin(m \sin \omega_m t)}\right]$$

If m is very small :- $\cos(m \sin \omega_m t) \approx 1$

$\sin(m \sin \omega_m t) \approx m \sin \omega_m t$

$[\sin \alpha \approx \alpha]$

$$e(t) = E_c \cos \omega_c t - E_c\, m \underline{\sin \omega_c t} \cdot m \underline{\sin \omega_m t}$$

$$① \leftarrow \boxed{e(t) = E_c \cos \omega_c t + \frac{E_c m}{2} \cos(\omega_c + \omega_m)t - \frac{m E_c}{2} \cos(\omega_c - \omega_m)t}$$

Carrier

USB

LSB.

$\text{amp}^l = \left[\frac{E_c m}{2}\right]$

$\text{amp}^l = \left[\frac{E_c m}{2}\right]$

also, NBFM eqⁿ is :-

$$\boxed{\begin{aligned} e(t) &= E_c \cos \omega_c t - K_f\, a(t) \sin \omega_c t \\ \text{where} \quad a(t) &= \int m(t)\, dt \end{aligned}}$$

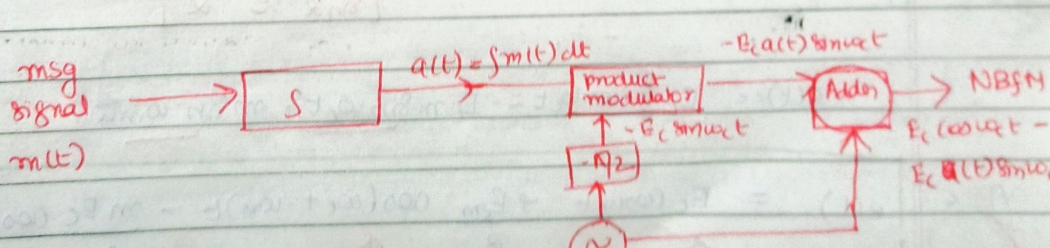
$|K_f \cdot a(t)| \ll 1$

from eqⁿ (1) :-

- → LSB is 180° out of phase than USB
- → $m \leq 1$
- → NBFM has only two side bands
- → Ampᵈ of LSB & USB is $\frac{E_c m}{2}$
- → Δf is limited with NBFM.
- → $\boxed{m = \frac{K_f \cdot E_m}{\omega_m} = \frac{\Delta f}{f_m}}$

Narrowband FM signal is :-

$$y_{FM}(t) = E_c\Big[\underbrace{\cos \omega_c t}_{\text{Carrier}} - K_f\, \underbrace{a(t) \sin \omega_c t}_{\text{Sideband}}\Big]$$



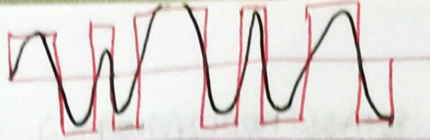
Carrier signal $c(t) = E_c \cos \omega_c t$

some issues:-

(1) During addition, variation in ampᵈ can happen. (E_c) bcoz of product modulator & adder.

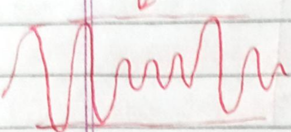
↓

To Resolve this issue, we can use BPF.

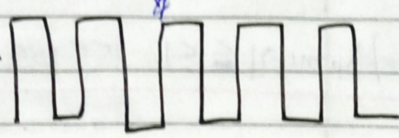


→ By BPF, we can resolve issue of amp^d variation.

$$V_{in}(t) \rightarrow \boxed{\text{hard limiter}} \xrightarrow{V'} \boxed{\text{BPF}} \rightarrow v(\theta).$$



(different amp^d)



[Square wave]

$$V' = \begin{cases} 1 & \cos\theta > 1 \\ -1 & \cos\theta < 1 \end{cases}$$

f^n of this square wave :-

$$V'(\theta) = \boxed{\frac{4}{\pi}}\left[\cos\theta - \frac{1}{3}\cos 3\theta + \frac{1}{5}\sin 3\theta - \ldots\ldots\right]$$

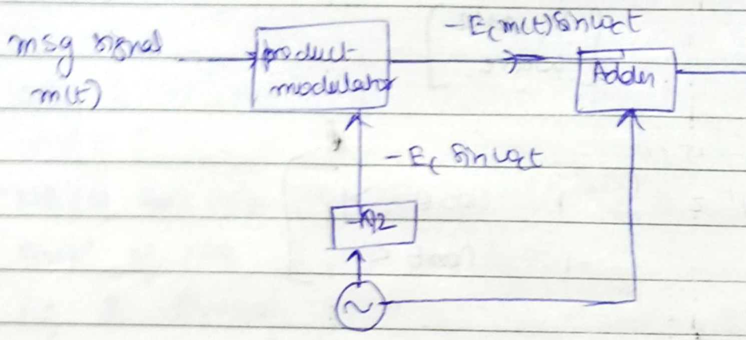
$\downarrow$ BPF

$$\boxed{V^{o}(\theta) = \frac{4}{\pi}\left[\cos\theta\right], \text{ where } \theta = \omega_c t + k_f 2\pi \int m(t)\,dt}$$

NARROW BAND PHASE MODULATION.

$$y_{PM} = E_c[\underbrace{\cos\omega_c t}_{\text{carrier.}} \underbrace{- k_p m(t)\sin\omega_c t}_{\text{sideband.}}]$$

where, $|k_p m(t)| \ll 1$



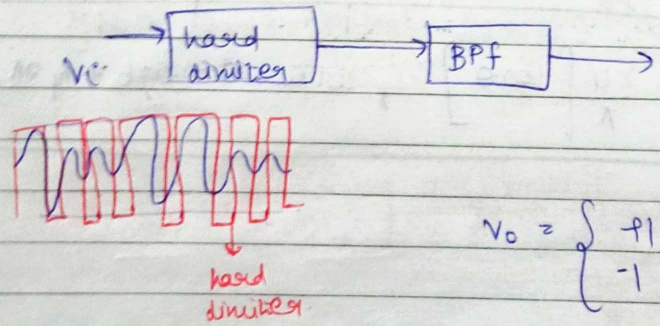
$\rightarrow E_c\cos\omega_c t - E_c m(t)\sin\omega_c t$

$= y_{PM}.$

Carrier signal

$c(t) = E_c\cos\omega_c t.$

→ same problem of A.M. → BP. limiter can used. to resolve the issue.



If variation of ampl. is high then only we use this.

$$V_o = \begin{cases} +1 & \cos\theta > 0 \\ -1 & \cos\theta < 0 \end{cases}$$

$$V_\theta = \frac{4}{\pi}\left[\cos\theta - \frac{1}{3}\cos 3\theta + \frac{1}{5}\cos 5\theta - \dots\right]$$

↓ BPF

$$\boxed{V_\theta = \frac{4}{\pi}\cos(\theta)}$$

Date : / /
Page No.

Performance Comparison of Narrow band & Wideband FM.

Parameters	WBFM	NBFM.
① Modulation index	$m_f >> 1$	less than 1 (or) slightly greater than 1
② max. deviation	75 KHz	5 KHz.
③ ~~freq~~ Range of modulating signal	30 Hz to 15 KHz	30 Hz to 3 KHz
④ Max. Modulation index.	5 to 2500	slightly greater than 1
⑤ BW.	15 times than NBFM.	Same as AM.
⑥ Applications.	→ can be used for ~~entertaining~~ entertainment broadcasting. → for high quality music transmission.	→ FM mobile communication for police wireless comm, for ambulance wireless comm. → for speech transmission.

FM GENERATION BY DIRECT METHOD.

FM signal :-

$$y_{FM}(t) = E_c \cos(\omega_c t + k_f \int m(t)\, dt)$$

where :-

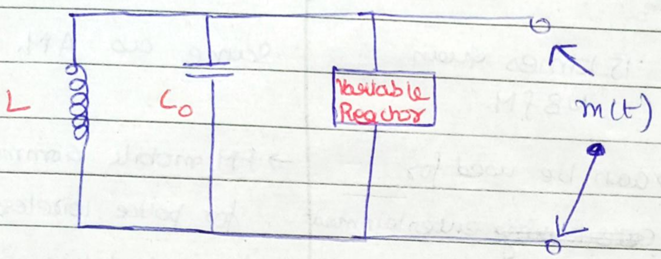
$$\omega(t) = \omega_c t + k_f m(t).$$

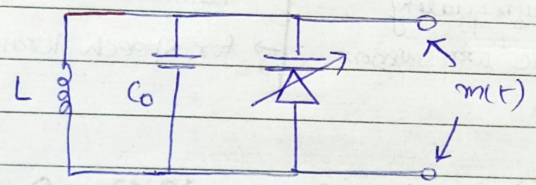
In Direct Method → VCO (Voltage Controlled Osillator), where the freq. is controlled by external voltage

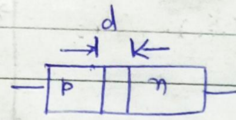
can be achieved by three ways :-

we can generate an FM wave by using the modulating signal m(t) as a control signal.

1. Use Hartley or Colpitt Oscillators.
2. Using opamp and hysteratic compensator [Schimtt trigger ckt.].
3. Using saturable core reader.







$$\boxed{C = \frac{\epsilon_0 \epsilon A}{d}}$$

→ capacitance depends on depletion region width

generated freq. :-

$$\omega = \frac{1}{\sqrt{LC}}$$

If C varies by modulating signal m(t).

$$C = C_0 - Km(t)$$

$$\omega = \frac{1}{\sqrt{L(C_0 - Km(t))}} = \frac{1}{\sqrt{LC_0\left[1 - \frac{Km(t)}{C_0}\right]}}$$

$$= \frac{1}{\sqrt{LC_0}} \left[1 - \frac{Km(t)}{C_0}\right]^{-1/2}$$

ω_c

Date : / /
Page No.

$$\rightarrow \omega = \omega_c \left[1 - \frac{k m(t)}{C_0}\right]^{-1/2}$$

If $\frac{k m(t)}{C_0} << 1$

$$\omega = \omega_c \left[1 + \frac{k m(t)}{2 C_0}\right]$$ {neglecting higher order terms}

{applying taylor series
$(1+x)^n \approx 1 + nx \quad |x| << 1$

$$\omega = \omega_c + \underbrace{\frac{k \omega_c}{2 C_0}}_{\to k_f} m(t)$$

$$\boxed{\omega = \omega_c + k_f m(t)} \qquad k_f = \frac{k \omega_c}{2 C_0}.$$

→ We can have hartley osillator as well insead of diode

Adv:- ① It generates sufficient freq. deviation.
② It requires little freq. multiplication.

Disad:- Poor freq. stability. FB is used to provide stability which generates error, that is used to correct the stability.

~~FM GENERATION BY INDIRECT METHOD Of ARMSTRONG.~~

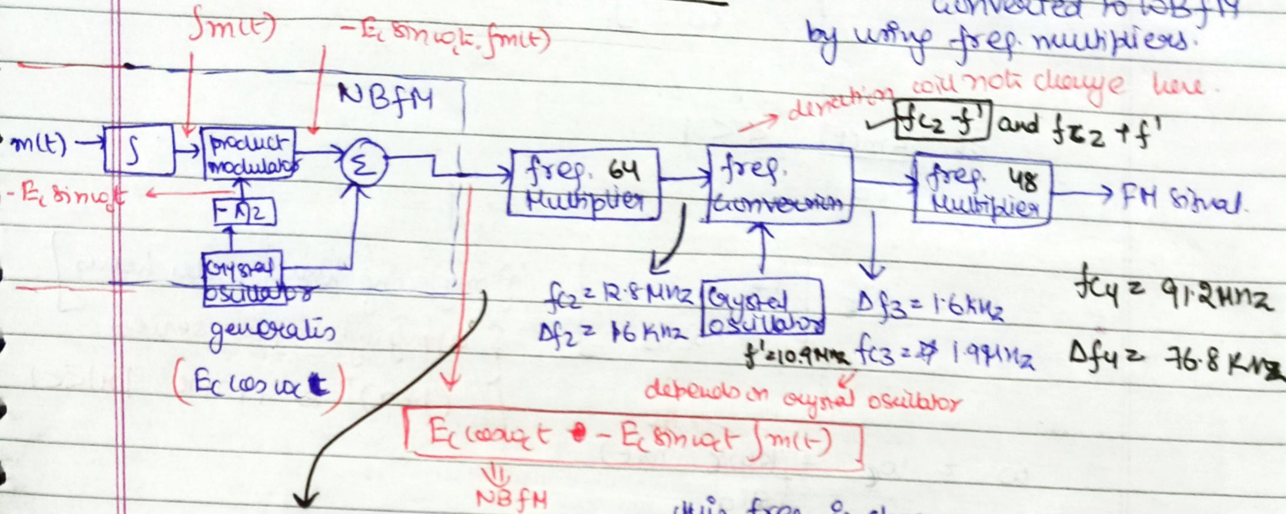
max. capacitance deviation is:-

$$\Delta C = k m_p = \frac{2 k_f C_0 m_p}{\omega_c}$$

$$\boxed{\frac{\Delta C}{C} = \frac{2 k_f m_p}{\omega_c}} = \frac{2 \Delta f}{f_c}$$

FM GENERATION BY INDIRECT METHOD OF ARMSTRONG.

→ In this NBFM is converted to WBFM by using freq. multipliers.



e.g. NBFM has $f_4 = 200\,KHz$ & $\Delta f_1 = 25\,Hz$ — this freq. is choosen as easy to make using crystal oscillator.

$$\frac{\Delta f_4}{\Delta f_1} \approx \frac{75K}{25Hz} = 3000 \Rightarrow \text{deviation factor}$$

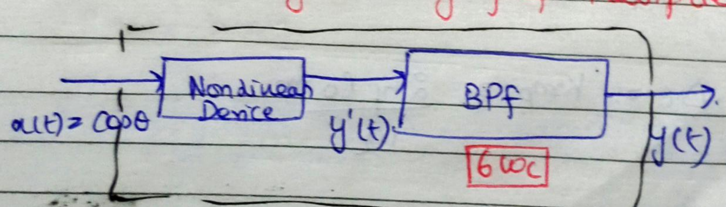
$f_4 = 200\,KHz \times 3000 = \underline{600\,MHz}$ but we want $91.2\,MHz$

so, we want to achieve $91.2\,MHz$.

→ { As per n^th law → freq. multiplier1 should multiply by 64
→ freq multiplier2 should multiply by 48 }.

$$f_{c3} = \frac{91.2M}{12.8M} = \underline{1.9\,MHz}.$$

Basic Block diagram of freq. Multiplier :-



O/P of non linear device. ← $y'(t) = a_0 + a_1 x(t) + a_2 x^2(t) + a_3 x^3(t) + \text{———}$

→ If we want 12 fold rise in freq. deviation, then use 12th order nonlinear device or 2 second order and one third order NLD.

$$y'(t) = a_0 + a_1\cos\theta + a_2\cos^2\theta + a_3\cos^3\theta + \cdots$$

$$\cos^2\theta = \frac{1}{2}\left[1+\cos 2\theta\right]$$

$$\cos^3\theta = \frac{1}{4}\left[\cos\theta + \cos 3\theta\right]$$

$$\cancel{y'(t) = a_0 + a_1\cos\theta + a_2}$$

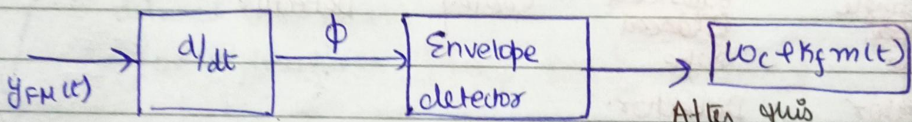
$$y'(t) = b_0 + b_1\cos\theta + b_2\cos 2\theta + b_3\cos 3\theta + \cdots$$

If BPF only allowing $6\omega_c$ so, o/p of BPF is:-

$$\boxed{y'(t) = b_6\cos 6\omega_c t}$$

→ freq. multiplier can rise both the carrier freq. & freq. deviation by the factor of n.

FM DEMODULATION & FM CLASSIFICATION OF DETECTION



After this we can use HPF or capacitor to extract m(t).

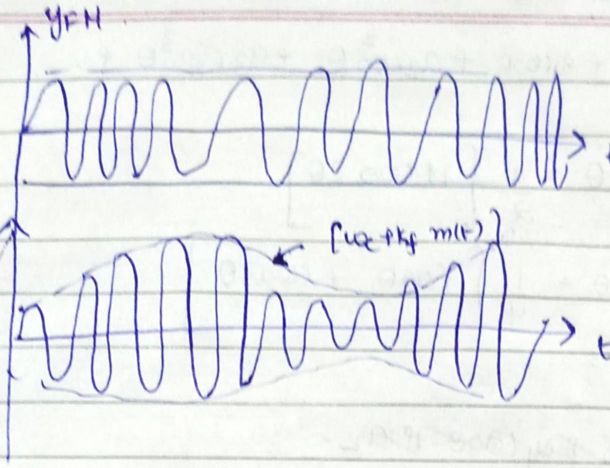
→ FM signal:-

$$y_{FM}(t) = E_c\cos\left(\omega_c t + K_f\int m(t)\,dt\right)$$

→ After differentiation:-

condition $\omega_c > K_f m(t)$.

$$\phi = \frac{dy_{FM}(t)}{dt} = \underbrace{-E_c\left[\omega_c + K_f m(t)\right]}_{\text{Similar to AM signal. [envelope]}}\sin\left(\omega_c t + K_f\int m(t)\,dt\right)$$



→ If freq. high → high amp.

→ If freq. low → low amp

FM Detection Classification

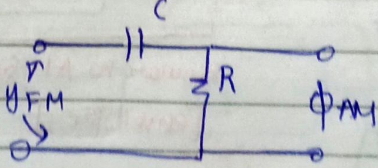
- Freq. Discriminator
  - Single slope detector
  - Balanced slope Detector
- Phase Discriminator
  - Foster Seeley
  - Ratio Detector
- PLL
- Zero Crossing Detector

SINGLE SLOPE METHOD (FM DEMODULATION)

→ for differentiator, transfer f^n $H(f) \propto f$

→ differentiator can be made by using HPF.

HPF.



$$H(f) = \frac{R}{R + \frac{1}{j\omega C}}$$

Date: / /
Page No.

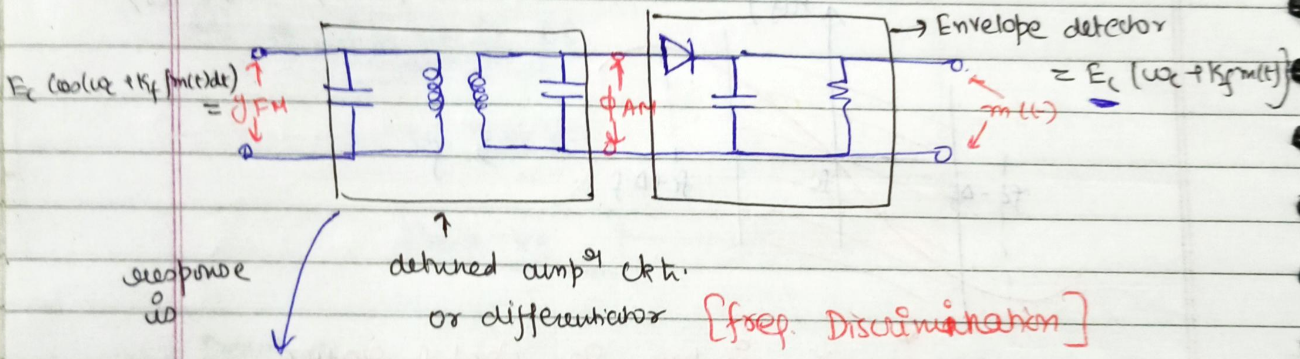
$$H(f) = \frac{jR\omega C}{jR\omega C + 1}$$

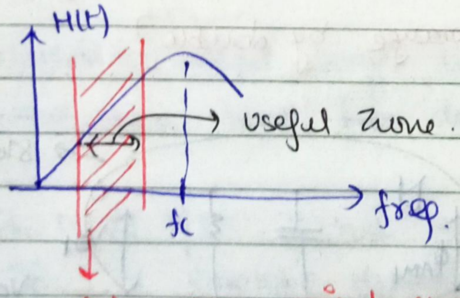
If $R\omega C \ll 1$

$$\boxed{H(f) \approx jR\omega C \propto \omega}$$

→ $d/dt \Rightarrow$ gives slope.

Here differentiation $\frac{dy_{FM}}{dt}$ = Slope of the msg signal.





Only utilize this portion for operation
as ① linearity of transfer f^n.
② donot use in non linear zone
③ Due to slope it f^n as differentiator.

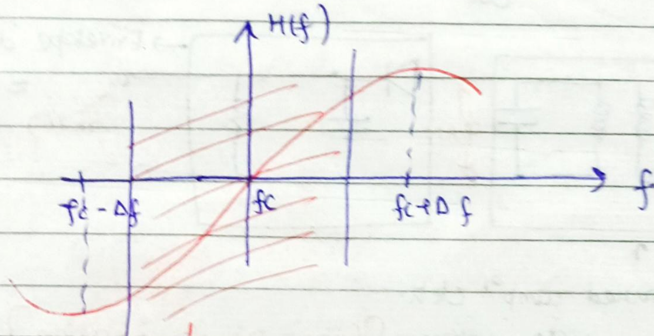
→ If the signal exceeds the useful zone, then E_c will change. Amp^d changes so, phase distortion will be there.

Drawbacks

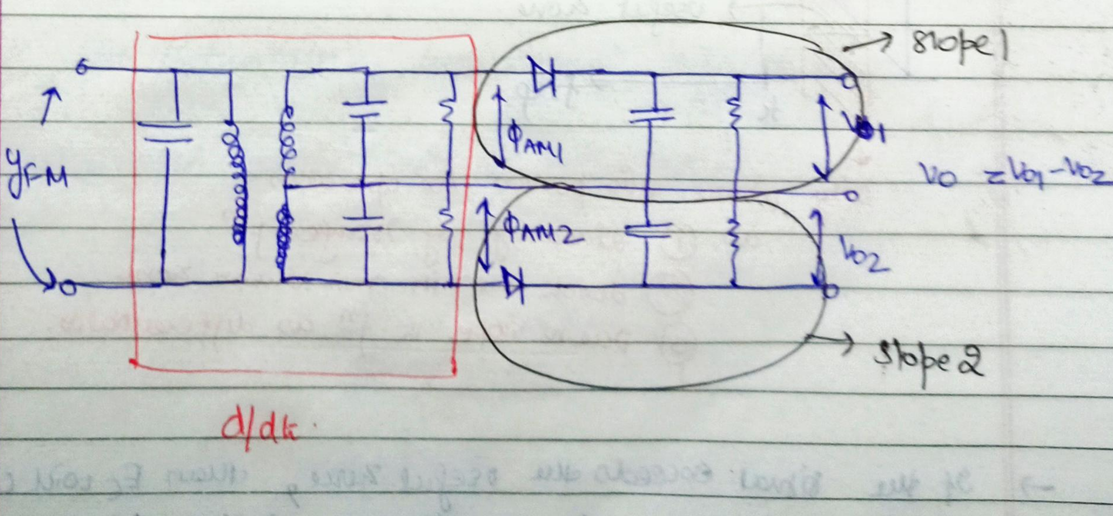
- → Useful range is limited.
- → doesnot eliminate any ampl variation and o/p is sensitive to any ampl variation.

FM DEMODULATION, FREQ. DISCRIMINATION BY BALANCED SLOPE DETECTION

→ In single slope → tuning range is limited. So, to rise tuning range. we provide staggered tuning. [provide tuning at 2 freqs $f_c+\Delta f$ & $f_c-\Delta f$)



linearity is here. So, Useful range
[rised this range by double]



Three cases of YFM :-

(1) Tuned at f_c $\Rightarrow$ $V_{01} = V_{02}$

($f_m \rightarrow f_c$) $\boxed{V_0 = 0}$ $\longrightarrow$ O/P is getting balance w.r.t. centre freq f_c where deviation can be +ve Δf and -ve Δf

(2) If ckt tuned at $f_m \rightarrow f_c + \Delta f$.

slope 1 o/p > slope 2 o/p

$V_{01} > V_{02}$

$\boxed{V_0 = +ve}$

(3) If $f_m \rightarrow f_c - \Delta f$

$V_{02} > V_{01}$

$\boxed{V_0 = -ve}$

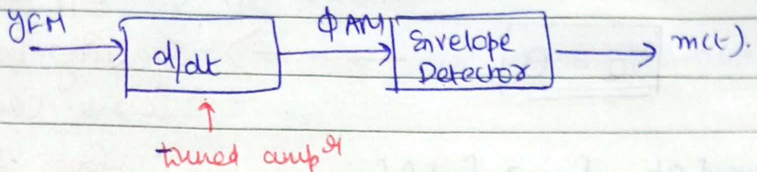
Drawback.

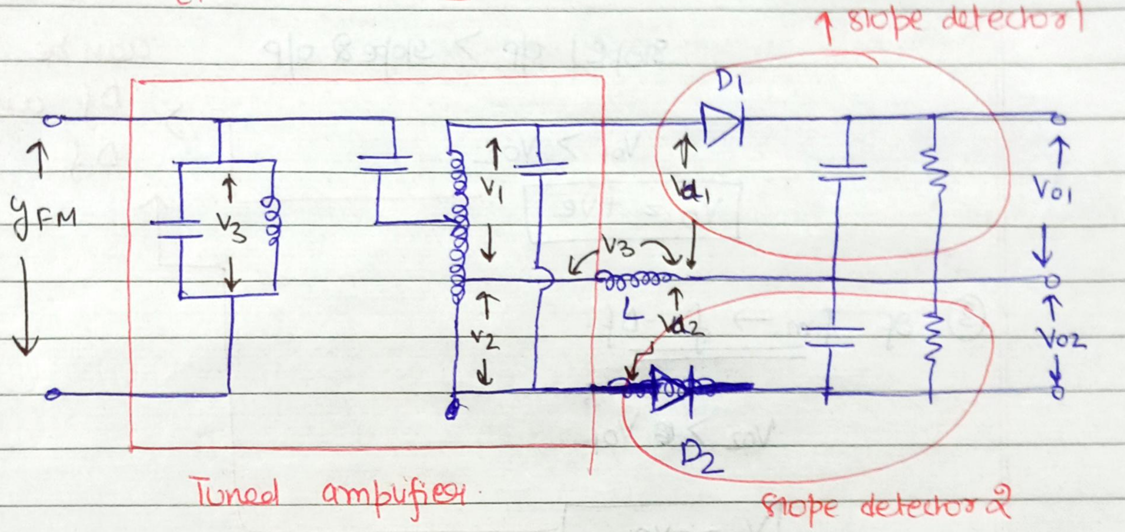
1) Limited to useful range i.e. linear characteristics is limited to a small freq deviation.

2.) The bund O/P is not purely band limited. Hence, the LPF of envelope detector introduce distortion.
[O/P goes beyond the useful range and O/P will be nonlinearity which will result into distortion]

3.) The discriminator characteristics depend critically on the amt. of detuning of resonant ckt.
[the differentiator is the heart of the ckt. which is balanced tuning. If the o/p goes out of the useful range then we won't be able to extract the original signal].

FOSTER SEELEY PHASE DISCRIMINATOR





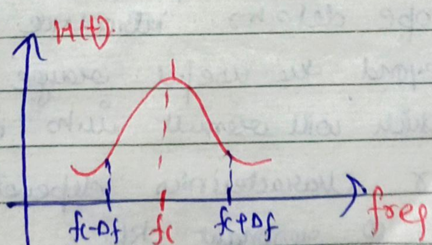
→ working is based on phase difference.

→ capacitor provides out of phase polarity to V_1 & V_2 [out of phase voltages]

→ o/p voltage $V_0 = |V_{o2}| - |V_{o1}|$

$V_{a1} = V_3 + V_1$

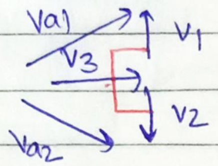
$V_{a2} = V_3 - V_2$



Date : / /
Page No.

Three cases.

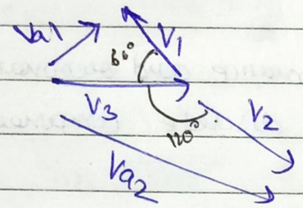
① If i/p $f_m \rightarrow$ tuned to f_c. {at resonance}



$$\boxed{|Va_1| = |Va_2|}$$

$V_0 = |V_{01}| - |V_{02}|$
$= \underline{\underline{0}}$.

② At off Resonance $f_m \rightarrow$ $f_c + \Delta f$.

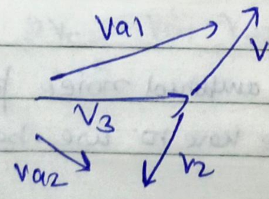


(phase b/w V_3 & V_1 ↓se
and phase b/w V_3 & V_2 ↑ses]

$|Va_1| < |Va_2|$.

$\left\{ \begin{aligned} V_0 &= |V_{02}| - |V_{01}| \\ &= +ve. \end{aligned} \right\}$

③ At $f_m \rightarrow$ $f_c - \Delta f$. (at off Resonance).

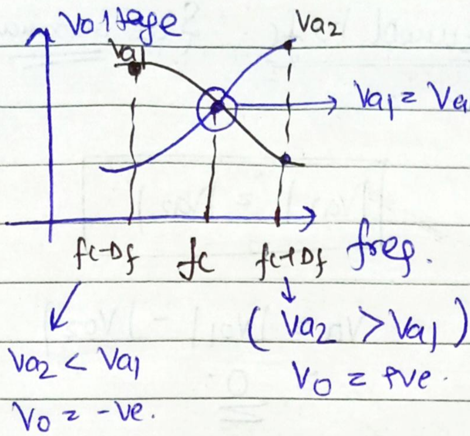


$|Va_1| > |Va_2|$.

$V_0 = |V_{02}| - |V_{01}|$
$= \underline{-ve}$.

Date : / /
Page No.

w.r.t. phase diagram



Adv :-

(1) offers good level of performance and resonance linearity.
(2) Easy to construct using discrete components.

Drawbacks :-

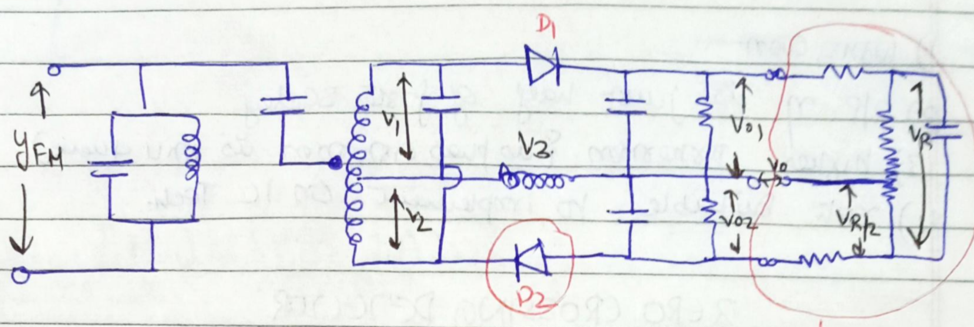
(1) Not suitable to use IC Tech.
(2) High cost
(3) Noise immunity.

$y_{FM} \longrightarrow \phi_{FM}$

tuned ampr [some amplified noise provide it so, we have to use band pass limiter]

Date : / /
Page No.

Radio Detector



→ 2 changes which will improve the noise.

→ O/P of Radio detector is 1/2 of O/P of foster seely.

$$V_o = \frac{1}{2}(V_{of's})$$

→ $V_R = V_{o1} + V_{o2}$ → (1)

→ $V_o = V_{o2} - \frac{V_R}{2}$ → (2)

from (1) & (2)

$$V_o = V_{o2} - \frac{(V_{o1} + V_{o2})}{2}$$

$$V_o = \frac{2V_{o2} - V_{o1} - V_{o2}}{2} = \frac{V_{o2} - V_{o1}}{2}$$ } O/P is halfed compared to foster seely. (Disadv.)

Adv:-

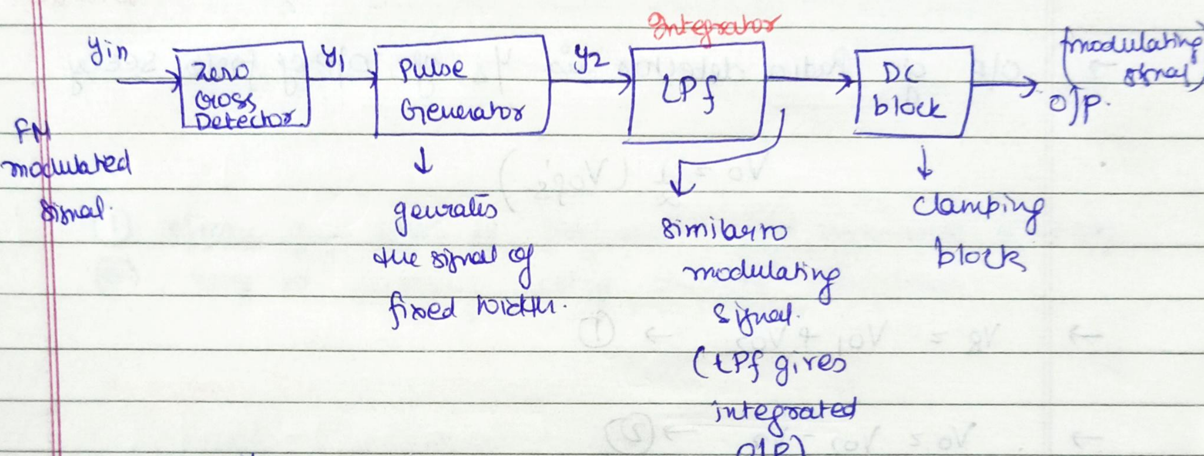
1.) good performance and reasonable linearity.
2.) Better immunity against amplitude noise. { b'coz of additional ckt. that acts as Band pass limiter)
3) Wider B.W.
4.) Easy to construct

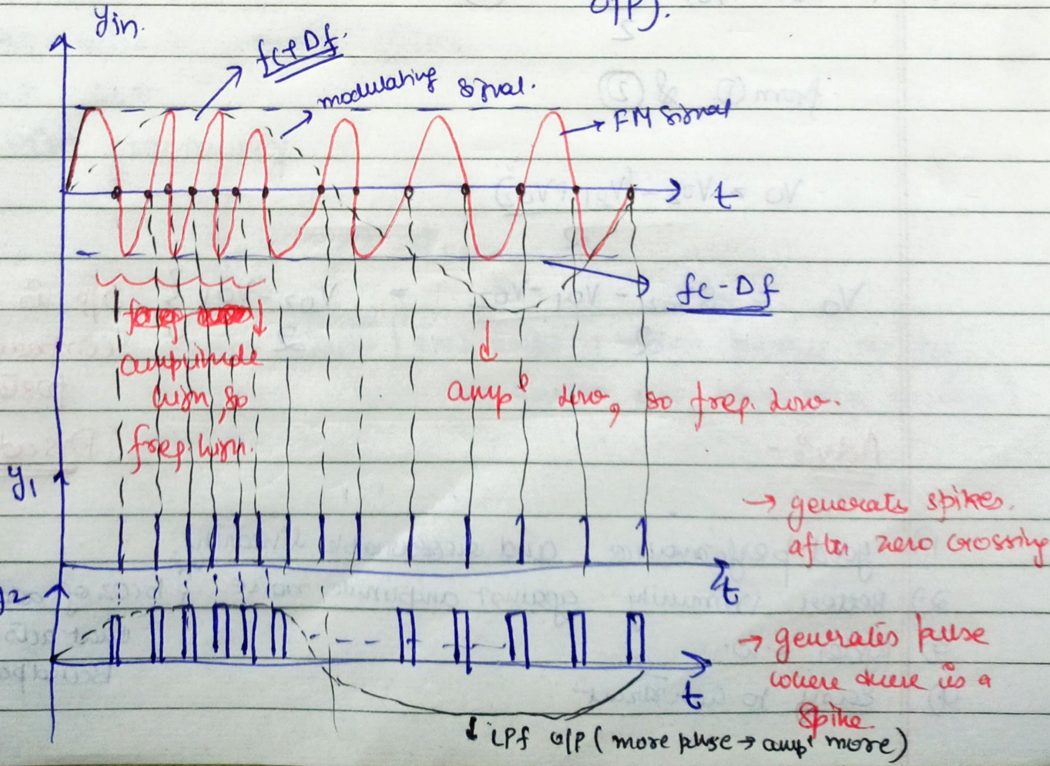
Date : / /
Page No.

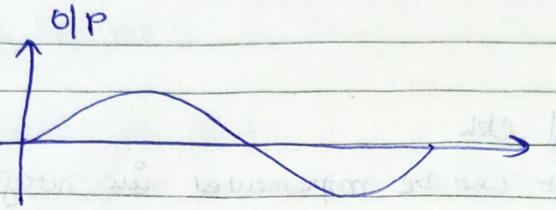
Drawbacks:-

1.) high cost.
2.) O/P. η is just half of foster seeley.
3.) higher Distortion. (as phase distortion is still there)
4.) not suitable to implement on IC Tech.

ZERO CROSSING DETECTOR





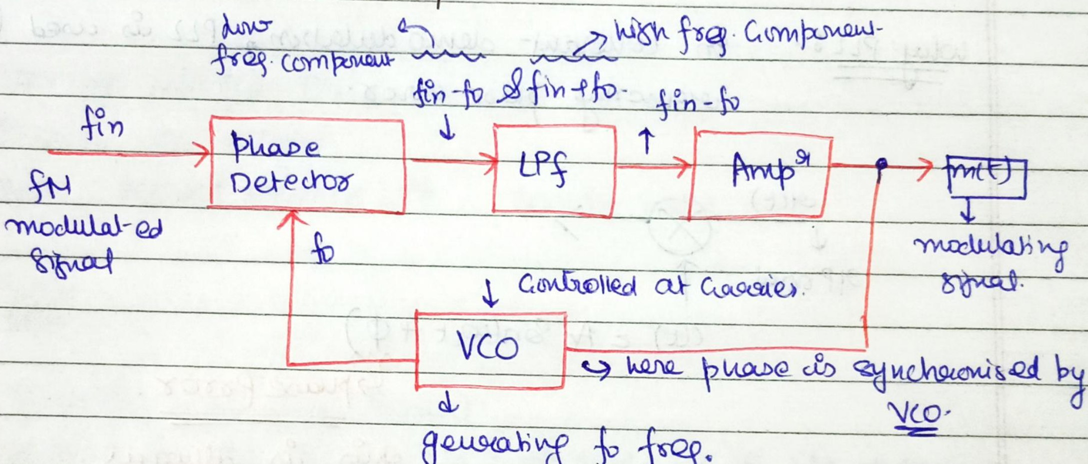


→ O/P of DC block.

PLL FM DEMODULATOR

↓

Phase Locked Loop.



→ The O/P freq. of VCO is equal to the freq. of unmodulated carrier.

→ The phase detector generates voltage proportional to difference b/w FM signal & VCO O/P.

→ The O/P of phase detector passes through LPF & ampr.

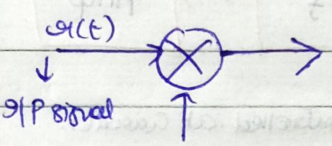
→ hence, freq. correlation is not required at VCO as, this is done at transmitter section.

Advantages :-

1. No need of tuned ckt.
2. Simple ckt. that can be implemented in integrated ckt.

PLL → A PLL is non linear f/b s/m that tracks the phase of i/p signal & minimize phase error at local oscillator.

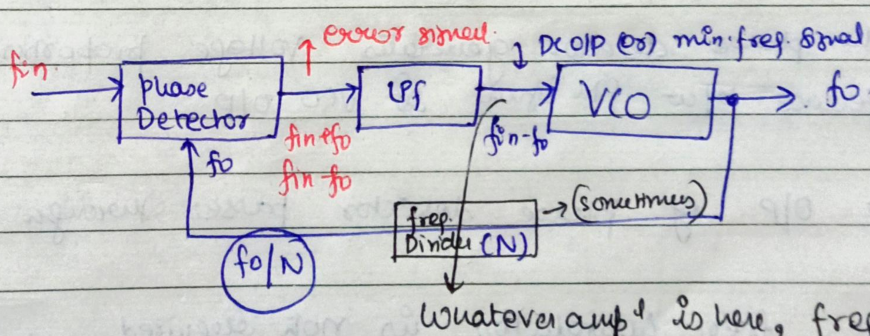
Why PLL :- In coherent demodulation, PLL is used for removing phase error.



$c(t) = A_c \sin(\omega_c t + \underbrace{\phi}_{\text{phase error}})$.

this is always generated and multiplied with i/P signal.

This can be removed by PLL



Whatever amp^n is here, freq. is generated in that manner.

If amp↑ → freq ↑

amp↓ → freq.↓

Date : / /
Page No.

Phase Detector

1. Compares fin & fo.
2. o/p of phase detector is proportional to phase difference b/w fin & fo
3. o/p of phase detector is DC or min. freq. signal. so, reflected as error voltage (e).

LPF.

1. Removes high freq. Noise.
2. Provide DC signal
3. Both Active / Passive LPF can be used

VCO:

1. generates high freq. Signal.
2. instaneous VCO freq. is controlled by its i/p voltage.

$$f_o = f + K\omega_m$$

↳ Based on i/p voltage.

3. freq. of VCO is directly controlled by DC i/p voltage

Operation of PLL

Three states :-

1. free running. → If no i/p is given, PLL is in free running state. VCO generates const. freq. $\boxed{f_o = f}$
2. Capture
3. Phase lock

→ Once the i/p is applied, VCO freq. starts to change and PLL is said to be in capture mode. {DC voltage at o/p of LPF changes which changes VCO freq.}.

→ Also called as "FREQUENCY PLL IN."

Date: / /
Page No.

3) Phase Lock. → The lock range is defined as the range of freq. over which its changes the i/p freq. f_{in} and f_o is zero, then it refers as Phase Lock.

o/p of LPF → $f_{in} - f_o$.

If o/p of VCO → $f_{in} - f_o = 0$. → PLL will be phase lock

APPLICATION

1. FM Coherent demodulation.
2. AM coherent demodulation.

Q. Determine the % modulation for an FM wave with a freq. deviation of 10KHz if the max. deviation allowed is 25KHz.

As.

$\Delta f = 10KHz$

$\Delta f_{max} = 25KHz$

$$\% \text{ Modulation} = \frac{\text{Actual deviation}}{\text{max. allowed deviation}} \times 100$$

$$= \frac{10}{25} \times 100 = 40\%.$$

Date : / /
Page No.

Q. In an FM S/M, if the max. value of deviation is 75 KHz and the max. modulating freq is 10KHz, calculate the deviation ratio & B.W of the S/M using Carson's rule.

As.

$f_{m\,max} = 10\,KHz$

$\Delta f_{max} = 75\,KHz$

$$\text{Deviation ratio} = \frac{\Delta f_{max}}{f_{m\,max}} = \frac{75}{10} = 0.75$$

$$\text{S/M B.W} = 2 \times \left[f_{m(max)} + \Delta f \right] \quad \{\text{by Carson's rule}\}$$

$$= 2[10 + 75] = 170\,KHz$$

Q. The eq^n of angle modulation is

$$e = 10\sin(10^8 t + 3\sin 10^3 t)$$

What form of angle modulation is this? Calculate the carrier and modulating freqs., the modulation index and deviation and power dissipated in 100 Ω resistor.

$$e = 10\sin(10^8 t + 3\sin 10^3 t).$$

Standard form :-

$$e = E_c \sin(\omega_c t + m_f \sin \omega_m t)$$

$E_c = 10V.$

$\omega_c = 10^8 \Rightarrow f_c = \dfrac{10^8}{2\pi} = 15.91\,MHz.$

$m_f = 3$