UNIT - II

* Partial Pivoting - Method used to solve linear eqns.

$x_1 + x_2 + x_3 = 3$
$4x_1 + 3x_2 + 4x_3 = 8$
$9x_1 + 3x_2 + 4x_3 = 7$.

$$[A:B] = \left[\begin{array}{ccc:c} 1 & 1 & 1 & 3 \\ 4 & 3 & 4 & 8 \\ 9 & 3 & 4 & 7 \end{array}\right]$$

Step 1 highest no. to above upr (first ~~ro~~ column)

$\Rightarrow R_3 \longleftrightarrow R_1$

$$\Rightarrow \left[\begin{array}{ccc:c} 9 & 3 & 4 & 7 \\ 4 & 3 & 4 & 8 \\ 1 & 1 & 1 & 3 \end{array}\right]$$

Step 2: baaki ko zero banana hai (first ~~row~~ column)

$R_2 \to R_2 - 4R_3$, $R_3 \to R_1 - 9R_3$

$$\Rightarrow \left[\begin{array}{ccc:c} 9 & 3 & 4 & 7 \\ 0 & -1 & 0 & -4 \\ 0 & -6 & -5 & -20 \end{array}\right]$$

next square matrix ko uthayenge same steps repeat

$$\Rightarrow \left[\begin{array}{ccc:c} 9 & 3 & 4 & 7 \\ 0 & -1 & 0 & -4 \\ 0 & -6 & -5 & -20 \end{array}\right].$$

zero banyge.

$-20+24$

$\Rightarrow R_3 \longrightarrow R_3 - 6R_2$

$$\Rightarrow \left[\begin{array}{ccc:c} 9 & 3 & 4 & 7 \\ 0 & -1 & 0 & -4 \\ 0 & 0 & -5 & 4 \end{array}\right].$$

$\Rightarrow 9x_1 + 3x_2 + 4x_3 = 7$
$-x_2 = -4$. $\Rightarrow$ $\boxed{x_2 = 4}$

$-5x_3 = 4$ $\Rightarrow$ $\boxed{x_3 = 4/5}$

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$9x_1 + 3x_2 + 4x_3 = 7$

$\Rightarrow 9x_1 + 3(4) + 4\left(\frac{-4}{5}\right) = 7$

$\Rightarrow \boxed{x_1 = \frac{-1}{5}}$

* Completing Pivoting - Another method to solve linear eq^n's.

$x_1 + x_2 + x_3 = 3$

$4x_1 + 3x_2 + 4x_3 = 8$

$9x_1 + 3x_2 + 14x_3 = 7$

$$[A:B] = \left[\begin{array}{ccc:c} 1 & 1 & 1 & 3 \\ 4 & 3 & 4 & 8 \\ 9 & 3 & 14 & 7 \end{array}\right]$$

Step 1: A me sbse bada element a_{11} pe laana hai
(modulus of largest element)

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$\Rightarrow$ ~~C_3~~ $C_3 \leftrightarrow C_1$

$$\Rightarrow \left[\begin{array}{ccc:c} 1 & 1 & 1 & 3 \\ 4 & 3 & 4 & 8 \\ 14 & 3 & 9 & 7 \end{array}\right]$$

$R_3 \leftrightarrow R_1$

$$\Rightarrow \left[\begin{array}{ccc:c} 14 & 3 & 9 & 7 \\ 4 & 3 & 4 & 8 \\ 1 & 1 & 1 & 3 \end{array}\right]$$

zero kro.

$R_2 \rightarrow R_2 - 4R_3$, $R_3 \rightarrow R_1 - 14R_3$

$$\Rightarrow \left[\begin{array}{ccc:c} 14 & 3 & 9 & 7 \\ 0 & -1 & 0 & -4 \\ 0 & -11 & -5 & -35 \end{array}\right]$$

iss square matrix ko consider krenge.

repeat step 1.

$|-11| = +11 \rightarrow$ highest

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$\Rightarrow C_3 \leftrightarrow C_2$

$$\Rightarrow \left[\begin{array}{ccc:c} 14 & 9 & 3 & 7 \\ 0 & 0 & -1 & -4 \\ 0 & -5 & -11 & -35 \end{array}\right]$$

$R_3 \leftrightarrow R_2$

$$\Rightarrow \left[\begin{array}{ccc:c} 14 & 3 & 9 & 7 \\ 0 & -11 & -5 & -35 \\ 0 & \boxed{-1} & 0 & -4 \end{array}\right]$$

isko zero

$R_3 \to R_2 - 11R_3$

$$\Rightarrow \left[\begin{array}{ccc:c} 14 & 3 & 9 & 7 \\ 0 & -11 & -5 & -35 \\ 0 & 0 & -5 & +11 \end{array}\right]$$

-5 $+44$
$-35+44$

$\Rightarrow -5x_3 = 11 \Rightarrow \boxed{x_3 = -11/5}$

$-11x_2 - 5x_3 = -35$

$-11x_2 + 11 = -35$

$\cancel{-}11x_2 = \cancel{-}46$

$\boxed{x_2 = \frac{46}{11}} \Rightarrow x_1 = \_\_\_$

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★ Thomas algorithm for Tri-diagonal systems

$$\left[\begin{array}{cccccc|c} a_{11} & a_{12} & 0 & \cdots & 0 & 0 & b_1 \\ a_{21} & a_{22} & a_{23} & \cdots & 0 & 0 & b_2 \\ 0 & a_{32} & \cdots & \cdots & \cdots & 0 & b_3 \\ \vdots & \vdots & \vdots & & \vdots & & \vdots \\ 0 & 0 & 0 & \cdots & a_{n-1,n} & a_{nn} & b_n \end{array}\right]$$

Step 1 (i) Divide R_1 by a_{11}

Step 2 (ii) Use the pivot row R_1 to ~~get zeroes on C_1~~ convert $a_{12} = 0$ by

$R_2 \to R_2 - a_{21}R_1$

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$\boxed{AX = D}$

Tri-diagonal matrix

$$a_1\begin{bmatrix} b_1 & c_1 & 0 & 0 \\ a_2 & b_2 & c_2 & 0 \\ 0 & a_3 & b_3 & c_3 \\ 0 & 0 & a_4 & b_4 \end{bmatrix} c_4 \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} d_1 \\ d_2 \\ d_3 \\ d_4 \end{bmatrix}$$

3rd diagonal, 2nd diagonal, 1st diagonal

1st step :- Make all $b = 1$.

2nd step :- Make all $a = 0$.

3rd step :- Also change d values correspondingly

(All using row & column matrix operations)

$$\begin{bmatrix} b_1/b_1 & c_1/b_1 & 0 & 0 \\ a_2 & b_2 & c_2 & 0 \\ 0 & a_3 & b_3 & c_3 \\ 0 & 0 & a_4 & b_4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} d_1/b_1 \\ d_2 \\ d_3 \\ d_4 \end{bmatrix}$$

$$\begin{bmatrix} 1 & c_1/b_1 & 0 & 0 \\ a_2 & b_2 & c_2 & 0 \\ 0 & a_3 & b_3 & c_3 \\ 0 & 0 & a_4 & b_4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} d_1/b_1 \\ d_2 \\ d_3 \\ d_4 \end{bmatrix}.$$

$\boxed{\text{let } c_1/b_1 = c'}$ — (1)

$R_2 \rightarrow R_2 - a_2 R_1$

$$\begin{bmatrix} 1 & c_1/b_1 & 0 & 0 \\ 0 & b_2 - a_2 c' & c_2 & 0 \\ 0 & a_3 & b_3 & c_3 \\ 0 & 0 & a_4 & b_4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} d_1/b_1 \\ d_2 - a_2 d_1/b_1 \\ d_3 \\ d_4 \end{bmatrix}.$$

Ily isko 1, fir isko 0 and so on until step 1 & step 2 are satisfied

On generalising,

$$\boxed{c_n' = \frac{c_n}{b_n - a_n c_{n-1}'}} \quad \&$$

$$\boxed{d_n' = \frac{d_n - a_n d_{n-1}'}{b_n - a_n \times c_{n-1}'}}$$

At last, the matrix reduces to

$$\begin{bmatrix} 1 & c_1' & 0 & 0 \\ 0 & 1 & c_2' & 0 \\ 0 & 0 & 1 & c_3' \\ 0 & 0 & 0 & b_4' \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} d_1' \\ d_2' \\ d_3' \\ d_4' \end{bmatrix}.$$

~~b_4 remains b_4 only~~ ⇒ $\boxed{b_4' x_4 = d_4'}$

The eqn.

Final,

LU Decomposition same as 3rd sch

$AX = B$

where $A = LU$

~~$L = \begin{bmatrix} 1 & l_{12} & l_{13} \\ 0 & 1 & l_{23} \\ 0 & 0 & 1 \end{bmatrix}, \quad U = \begin{bmatrix} u_{11} & u_{12} & u_{13} \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{bmatrix}.$~~

$$L = \begin{bmatrix} 1 & 0 & 0 \\ l_{21} & 1 & 0 \\ l_{31} & l_{32} & 1 \end{bmatrix}, \quad U = \begin{bmatrix} u_{11} & u_{12} & u_{13} \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{bmatrix}$$

⇒ $LUX = B$

$X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$ → has to found out

let $\boxed{UX = Y}$

⇒ $\boxed{LY = B}$ $\quad Y = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}$ using this X will be calculated.

$l_{21} = \dfrac{a_{21}}{a_{11}}, \quad l_{31} = \dfrac{a_{31}}{a_{11}}$ & so on

$\ldots \; l_{n1} = \dfrac{a_{n1}}{a_{11}}$

using Gauss' eliminatⁿ in LU decomposition method,

A pr row ya ~~col~~ operatⁿs krke usko L, U me decompose krna hai.

* **Cholsky method** - method to solve linear eq^ns.

Working rule -

$AX = B$ — (1)

let $A = LL^T$, where, — (2)

$$L = \begin{bmatrix} l_{11} & 0 & 0 \\ l_{21} & l_{22} & 0 \\ l_{31} & l_{32} & l_{33} \end{bmatrix}$$

$$L^T = \begin{bmatrix} l_{11} & l_{21} & l_{31} \\ 0 & l_{22} & l_{32} \\ 0 & 0 & l_{33} \end{bmatrix}$$

Then,

find values of $l_{11}, l_{21}, l_{31}, l_{33}, l_{22}, l_{32}$

By (1) & (2),

$LL^T X = B$

let $\boxed{L^T X = Y}$ (3) where, $Y = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}$.

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$\Rightarrow \boxed{LY = B}$ — (4).

Solve & find Y, Put Y in (3) to find X.

Q. Solve the system of eq^n by cholsky method.

$x + 2y + 3z = 5$
$2x + 8y + 22z = 6$
$3x + 22y + 82z = -10.$

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 8 & 22 \\ 3 & 22 & 82 \end{bmatrix}, B = \begin{bmatrix} 5 \\ 6 \\ -10 \end{bmatrix}.$$

$A = LL^T$

$$\Rightarrow \begin{bmatrix} 1 & 2 & 3 \\ 2 & 8 & 22 \\ 3 & 22 & 82 \end{bmatrix} = \begin{bmatrix} l_{11} & 0 & 0 \\ l_{21} & l_{22} & 0 \\ l_{31} & l_{32} & l_{33} \end{bmatrix} \begin{bmatrix} l_{11} & l_{21} & l_{31} \\ 0 & l_{22} & l_{32} \\ 0 & 0 & l_{33} \end{bmatrix}$$

$\Rightarrow$ On solving, we get, $\boxed{l_{31} = 3}$ $\boxed{l_{22} = 2}$, $\boxed{l_{32} = 8}$, $\boxed{l_{33} = 3}$ $\boxed{l_{11} = 1}$, $\boxed{l_{21} = 2}$

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$\Rightarrow L = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 2 & 0 \\ 3 & 8 & 3 \end{bmatrix} \Rightarrow L^T = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 2 & 8 \\ 0 & 0 & 3 \end{bmatrix}$

$\Rightarrow LL^T X = B$

Let $L^T X = Y$

$\Rightarrow LY = B$

$\Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 2 & 2 & 0 \\ 3 & 8 & 3 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 5 \\ 6 \\ -10 \end{bmatrix}$

$\Rightarrow \boxed{y_1 = 5}$.

$2y_1 + 2y_2 = 6 \Rightarrow y_1 + y_2 = 3$

$5 + y_2 = 3$

$\boxed{y_2 = -2}$.

$3y_1 + 8y_2 + 3y_3 = -10$

$15 - 16 + 3y_3 = -10$

$3y_3 = -9$

$\Rightarrow \boxed{y_3 = -3}$.

$\Rightarrow \begin{bmatrix} 1 & 2 & 3 \\ 0 & 2 & 8 \\ 0 & 0 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 5 \\ -2 \\ -3 \end{bmatrix}$

$\Rightarrow 3z = -3 \Rightarrow \boxed{z = -1}$.

$2y + 8z = -2$

$2y - 8 = -2 \Rightarrow 2y = +6$

$\boxed{y = 3}$.

$x + 2y + 3z = 5$

$x + 6 - 3 = 5 \Rightarrow \boxed{x = 2}$.

$\Rightarrow X = \begin{bmatrix} 2 \\ 3 \\ -1 \end{bmatrix}$. $\rightarrow$ sol^n.

* <u>Matrix inverse using LU decomposition</u>-

To find A^{-1}, find matrices L & U. then

$\boxed{A^{-1} = U^{-1} L^{-1}}$ *

where, $\boxed{U^{-1} = \frac{1}{|U|} \text{adj } U}$, $\boxed{L^{-1} = \frac{1}{|L|} \text{adj } L}$

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$A A^{-1} = I$

$\boxed{LL^T} A^{-1} = I$

* Matrix inverse using cholesky method -

$\boxed{A = LL^T}$ → cholesky : - (1)

$L = \begin{bmatrix} l_{11} & 0 & 0 \\ l_{21} & l_{22} & 0 \\ l_{31} & l_{32} & l_{33} \end{bmatrix}$

find values of L & L^T.

then we know for any matrix B

$BB^{-1} = I$

$\therefore$ $\boxed{LL^{-1} = I}$ - (2) → yaha se L^{-1} nikaalo

let $L^{-1} = \begin{bmatrix} L_{11} & 0 & 0 \\ L_{21} & L_{22} & 0 \\ L_{31} & L_{32} & L_{33} \end{bmatrix}$

~~Also~~ Now

$A^{-1} = L^{-1}(L^T)^{-1}$ (from (1))

$\therefore$ $\boxed{A^{-1} = L^{-1}(L^{-1})^T}$ → yaha se A^{-1} niklega

L^{-1} (2) se aayega

iterative method.

(*) Gauss seidel method -

Q. Solve the system of eq^n by using Gauss-seidal iterative method.

$2x_1 - x_2 + 0x_3 = 7$ — (1)
$-x_1 + 2x_2 - x_3 = 1$ — (2)
$0x_1 - x_2 + 2x_3 = 1$ — (3)

→ Using (1), (2) & (3) to find values of x_1, x_2 & x_3 resp.

$\therefore$ $x_1 = \frac{7 + x_2}{2}$

$x_2 = \frac{1 + x_3 + x_1}{2}$

$x_3 = \frac{1 + x_2}{2}$

Now, initial approximation:

$x_1^{(0)} = 0, \; x_2^{(0)} = 0, \; x_3^{(0)} = 0.$

Will put values (initial) till next iterat^n values are not ~~fix~~ calculating.

1st iterath –

$$x_1^{(1)} = \frac{7+x_2^0}{2} = \frac{7}{2} = 3.5$$

$$x_2^{(1)} = \frac{1+x_3^0+x_1^1}{2} = \frac{1+0+3.5}{2} = \frac{4.5}{2} = 2.25.$$

$$x_3^{(1)} = \frac{1+x_2^1}{2} = \frac{1+2.25}{2} = \frac{3.25}{2} = 1.625$$

2nd iterath –

$$x_1^{(2)} = \frac{1}{2}(7+x_2^1) = \frac{7+2.25}{2} = \frac{9.25}{2} = 4.625.$$

$$x_2^{(2)} = \frac{1+x_3^1+x_1^2}{2} = \frac{1+1.625+4.625}{2} = \frac{7.25}{2} = 3.625$$

$$x_3^{(2)} = \frac{1+x_2^2}{2} = \frac{1+3.625}{2} = \frac{4.625}{2} = 2.1325$$

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jaisi-jaisi values nikalti jayngi
vaisi-vaisi use krte jayenge.

3rd iterath –

$$x_1^{(3)} = \frac{7+x_2^2}{2} = \frac{7+4.625}{2} = \frac{11.625}{2} = 5.3125$$

$$x_2^{(3)} = \frac{1+x_3^2+x_1^3}{2} = \frac{1+\cancel{3.6}\ 2.1325+5.8125}{2} = 4.3125$$

$$x_3^{(3)} = \frac{1+x_2^3}{2} = \frac{1+4.3125}{2} = \frac{5.3125}{2} = 2.65625$$

4th iterath –

$$x_1^{(4)} = \frac{7+x_2^3}{2} = 5.65625$$

$$x_2^{(4)} = 4.65625$$

$$x_3^{(4)} = 2.828125$$

5th iterath –

$$x_1^{(5)} = 5.8281, \quad x_2^{(5)} = 4.8281, \quad x_3^{(5)} = 2.9806$$

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On ~~1st~~ 9th iteration -

$x_1^{(7)} = 5.9570,\ x_2^{(7)} = 4.9570,$

$x_3^{(7)} = 2.9785$

$\Rightarrow$ $x_1 \approxeq 6$
$x_2 \approxeq 5$
$x_3 \approxeq 3.$

$\Rightarrow$ $\boxed{\begin{matrix} x_1 = 6 \\ x_2 = 5 \\ x_3 = 3 \end{matrix}}$.

* **Power method for eigenvectors and eigenvalue**

~~Q. Determine the largest eigen value and corresponding eigen vector of the matrix.~~

$$~~A = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}~~$$

→

Power method is particularly used for estimating numerically largest or smallest eigen value and its corresponding eigen vector.

The Power method which is iterative method can be used when

- The matrix A of order n has n linearly independent eigen vector.
- The eigen values can be ordered in magnitude as

$|\lambda_1| > |\lambda_2| \geq |\lambda_3| \geq |\lambda_4| \geq \dots \geq |\lambda_n|$

Q. Find the numerically larger eigen value of the matrix by the Power method.

$$A = \begin{bmatrix} 1 & -3 & 2 \\ 4 & 4 & -1 \\ 6 & 3 & 5 \end{bmatrix}.$$

Take initial approx as: $X_0 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

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→ Initially, $X_0 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ (Given)

Iterat^n 1:

$$AX_0 = \begin{bmatrix} 1 & -3 & 2 \\ 4 & 4 & -1 \\ 6 & 3 & 5 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1-3+2 \\ 4+4-1 \\ 6+3+5 \end{bmatrix} = \begin{bmatrix} 0 \\ 7 \\ 14 \end{bmatrix}$$

yaha se largest value ko nikaal dekhenge or use EV consider karenge for 1st iterat^n.

∴ 14 is the largest

$$\Rightarrow 14 \begin{bmatrix} 0 \\ 0.5 \\ 1 \end{bmatrix}$$

Iterat^n 2: Now, $X_0 = \begin{bmatrix} 0 \\ 0.5 \\ 1 \end{bmatrix}$

$$AX_0 = \begin{bmatrix} 1 & -3 & 2 \\ 4 & 4 & -1 \\ 6 & 3 & 5 \end{bmatrix} \begin{bmatrix} 0 \\ 0.5 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0.5 \\ 1 \\ 6.5 \end{bmatrix}$$ (Largest is 6.5)

$$\Rightarrow 6.5 \begin{bmatrix} 0.076 \\ 0.153 \\ 1 \end{bmatrix}$$

At the end of 2^nd iterat^n, largest eigenvalue = 6.5

Iterat^n 3 – $X_0 = \begin{bmatrix} 0.076 \\ 0.153 \\ 1 \end{bmatrix}$

$$AX_0 = \begin{bmatrix} 1 & -3 & 2 \\ 4 & 4 & -1 \\ 6 & 3 & 5 \end{bmatrix} \begin{bmatrix} 0.076 \\ 0.153 \\ 1 \end{bmatrix} = \begin{bmatrix} 1.6454 \\ -0.07691 \\ 5.92304 \end{bmatrix}$$

Highest = 5.92304

$$\Rightarrow 5.92304 \begin{bmatrix} 0.07692 \\ 0.15384 \\ 1 \end{bmatrix}.$$

At the end of the iteration, eigenvalue (largest)
= 5.92304

**If no. of iterations are given in the ques, go upto that iteration.
If not given, go upto no. of iteration till the value of the largest eigenvalue repeats.**

INTERPOLATION -

The technique or method of estimating unknown values from given set of observation is known as interpolation.

for equal interval — method used
→ Newton forward → upr wale ke liye
→ Newton Backward → niche wale
→ Gauss Forward/Backward
→ Stirling
→ Bessel

(Gauss Forward/Backward, Stirling, Bessel) → central difference

for unequal interval — method used
→ Lagrange interpolation formula
→ Newton divided difference.

• **Interpolation for equal interval -**

Q. Estimate the population in 1895 & 1925 from following stats -

Year	1891	1901	1911	1921	1931
Population (in 1000)	46	66	81	93	101

x	$f(x)$ (in 1000)	$\Delta f(x)$	$\Delta^2 f(x)$	$\Delta^3 f(x)$	$\Delta^4 f(x)$
1891	46				
		20			
1901	66		-5		
		15		2	
1911	81		-3		-3
		12		-1	
1921	93		-4		
		8			
1931	101				

$$\boxed{\Delta f(x) = f(x+h) - f(x)}$$

Δ — del operator. $f(x+h)$ — aagi wali term

→ aage waali term − phle waali term
(dono values ke beech me likha hai)

Newton forward formula–

$$f(a+hu) = f(a) + \frac{u}{1!}\Delta f(a) + \frac{u(u-1)}{2!}\Delta^2 f(a) + \frac{u(u-1)(u-2)}{3!}\Delta^3 f(a) + \frac{u(u-1)(u-2)(u-3)}{4!}\Delta^4 (a) + \ldots$$

~~New~~ NF is used when niche ke trend ko dekhke upr ki koi value nikaalni ho.

Here, $f(1895) = ?$

$a+hu = 1895$

here, $a = 1891$ $\quad hu = 4 \quad (h=10)$

$u = \frac{4}{10} = 0.4$

$\boxed{u = 0.4}$.

FEBRUARY							2024
WK	M	T	W	T	F	S	S
5				1	2	3	4
6	5	6	7	8	9	10	11
7	12	13	14	15	16	17	18
8	19	20	21	22	23	24	25
9	26	27	28	29			

$$f(1895) = f(1891) + \frac{0.4}{1!}\Delta f(1891) + \frac{0.4(0.4-1)}{2!}\Delta^2 f(1891) + \frac{0.4(0.4-1)(0.4-2)}{3!}\Delta^3 f(1891) + \frac{0.4(0.4-1)(0.4-2)(0.4-3)}{4!}\Delta^4 f(1891)$$

$= 46000 + 0.4(20000) + 0.12(5000) - 0.149(2000) + 0.11(3000)$

$= 46000 + 8000 + 600 - 298 + 330$

$= \underline{54632}$.

~~Newton backward~~

$$\boxed{\nabla f(x) = f(x) - f(x-h)}$$

nabla operator

MARCH							2024
WK	M	T	W	T	F	S	S
9					1	2	3
10	4	5	6	7	8	9	10
11	11	12	13	14	15	16	17
12	18	19	20	21	22	23	24
13	25	26	27	28	29	30	31

Newton backward formula -

$$f(a+hu) = f(a) + \frac{u}{1!}\nabla f(a) + \frac{u(u+1)}{2!}\nabla^2 f(a) + \frac{u(u+1)(u+2)}{3!}\nabla^3 f(a) + \frac{u(u+1)(u+2)(u+3)}{4!}\nabla^4 f(a) + \cdots$$

Used in opp. condn of NF.

Here, $f(1925)$ ⇒ $a+hu = 1925$

Here, $a = 1931$ ⇒ $1931 + 10u = 1925$

⇒ $\boxed{u = -0.6}$

On solving & putting value in formula

⇒ $f(1925) = 96836$.

$\nabla f(a)$ ki jgh $\Delta f(a)$ hu dalegi

NF me a upr waali value
NB me a neeche waali value.

Q. Find no. of men getting wages b/w Rs 10 & Rs 15 from following data

wages	0-10	10-20	20-30	30-40
freq.	9	30	35	42

→

x	$f(x)$	$\Delta f(x)$	$\Delta^2 f(x)$	$\Delta^3 f(x)$
below 10	9			
		30		
below 20	39		5	
		35		2
below 30	74		7	
		42		
below 40	116			

→ NF will be used.

$a = 10,\ h = 10$.

⇒ $15 = 10 + 10u$ ⇒ $\boxed{u = 0.5}$

$$f(15) = 9 + 0.5(30) + \frac{(0.5)(-0.5)}{2}(5) + \frac{(0.5)(-0.5)(-1.5)}{6}.(2)$$

$$= 9 + 15 - 0.625 + 0.125$$

$$= 23.5 \simeq 24.$$

Q. No. of men getting wages b/w Rs 10 & Rs 15

$= 24 - 9 = 15$

Q. Find the lowest degree polynomial $y(x)$ that fit the data, find $y(5)$.

x	0	2	4	6	8
y	5	9	61	209	501

x	$f(x)$	$\Delta f(x)$	$\Delta^2 f(x)$	$\Delta^3 f(x)$	$\Delta^4 f(x)$
0	5				
		4			
2	9		48		
		52		48.	
4	61		96		0
		148.		48	
6	209		144		
		292			
8	501				

NF –

$f(a+hu)$ – f. using NF –

$f(x) = ?$ & $x = a+hu$

$a=0,\ h=2$

$\Rightarrow 2u = x$ & $\boxed{u = \frac{x}{2}}$

$$f(x) = y(x) = 5 + \frac{x}{2}(4) + \frac{1}{2}\left(\frac{x}{2}\right)\left(\frac{x}{2}-1\right)(48)$$

$$+ \frac{1}{6}\left(\frac{x}{2}\right)\left(\frac{x}{2}-1\right)\left(\frac{x}{2}-2\right)(48).$$

$$\Rightarrow y(5) = 5 + 10 + \frac{1}{2}(2.5)(1.5)(48)$$

$$+ \frac{1}{6}(2.5)(1.5)(0.5)(48)$$

$$= 15 + 90 + 15 = \underline{120}.$$