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Computational Methods.

$f(x) \rightarrow$ Algebraic $x^2+2x+1=0$.

$f(x) \rightarrow$ Transcedental :- alg. eq which involves log, exp, trigo.

eg:- $x \log x = 1.7$

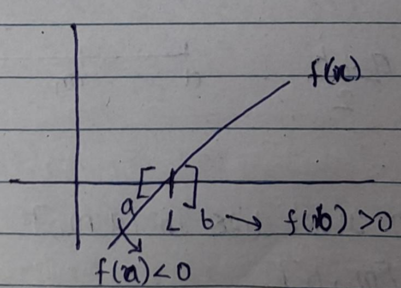
Unit - I

Solving $f(x)=0$ through numerical methods.

General procedure to solving iteratively.

1. get the initial approximation to the root of $f(x)=0$.
2. Apply the method on the initial approx to real, towards the root.
3. Stop the iterations as per one of the stopping criteria.

Intermediate Value Property (IVP).



for a continous function $f(x)$ defined on $[a,b]$ if $f(a).f(b)<0$ then there exist atleast one root of $f(x)=0$ in (a,b)

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Qn) show that $f(x) = x^3 + 4x^2 - 10 = 0$ has a root in $[1, 2]$

$$x^3 + 4x^2 - 10 = 0$$

$$x^3 + 8x^2 + 2x^2 + 10$$

$$f(1) = -5$$

$$f(2) = 14$$

$f(x)$ has a root in $[1, 2]$ as per IVP

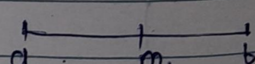
Bisection Method.

It involves a series of step to find the approximate root of $f(x) = 0$. Let the exact root (zero or solution) of $f(x) = 0$ 'α'.

procedure of Bisection method.

1) find the initial approximation in the form of interval $[a, b]$ i.e $[a, b]$ should have a root of $f(x) = 0$

2) find mid point, $m_1 = \dfrac{a + b}{2}$.



check the sign of $f(m_1)$

If $f(a) f(m_1) < 0$, then root lies in $[a, m_1]$.

otherwise root lies in $[m_1, b]$

3) Calculate m_2, m_3, ... so on as per the step ②. stop the iteration as per the desired level of accuracy as per the one of the following stopping criteria.

show that $f(x) = x^3 + 4x^2 - 10 = 0$ has a root in [1,2]

$f(1) = -5$

$f(2) = 14$

Initial interval [1 2]

Initial I $m_1 = \frac{1+2}{2} = \frac{3}{2} = 1.5$

1	1.5	2
-ve	+ve	+ve

$f(1.5) = +2.375$

Root lies in [1, 1.5]

II $m_2 = \frac{1+1.5}{2} = 1.25$

1	1.25	1.5
−	−	+

$f(1.25) = -1.796$

Root lies in [1.25, 1.5]

III $m_3 = \frac{1.25+1.5}{2} = 1.37$

1.25	1.37	1.5
−	+	+

$f(1.37) = +0.162$

Root lies in [1.25, 1.37]

IV $m_4 = \frac{1.25+1.37}{2} = 1.3125$

1.25	1.31	1.37
−	−	+

$f(1.31) = -0.887$

Root lies in ~~[1.25, 1.37]~~ [1.31, 1.37]

V $m_5 = \frac{1.31+1.37}{2} = 1.34$

1.31	1.34	1.37
−	+	+

$f(1.34) = 0.411$

Root lies in [1.31, 1.34]

VI $m_6 = \frac{1.31+1.34}{2} = 1.325$

1.31	1.32	1.34
−	−	+

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$f(1.325) = -0.651$

Root lies in $[1.32, 1.34]$

VII $m_7 = \frac{1.32 + 1.34}{2} = 1.33$

1.32 1.33 1.34

− + +

$f(1.33) = +0.571$

Root lies in $[1.32, 1.33]$

VIII $m_8 = \frac{1.32 + 1.33}{2} = 1.325$

1.32 1.325 1.33

− − +

$f(1.325) = -$

Root lies in $[1.325, 1.33]$

IX $m_9 = \frac{1.325 + 1.33}{2} = 1.327$

1.325 1.327 1.33

− + +

$f(1.327) = +$

X $m_{10} = \frac{1.325 + 1.327}{2} = 1.326$

1.325 1.326 1.327

− − +

$f(1.326) = -$

condition

→ use bisection method to determine an approximate root that is accurate within 10^{-3}

→ $|f(m_n)| < \epsilon$, for $\epsilon > 0$ (desire level of accuracy)

→ $|m_n - m_{n-1}| < 0$

→ $m_{11} = 1.3647461$

$m_{12} = 1.36499$

$\epsilon = 10^{-3} \Rightarrow 0.001$

∴ Ans 1.365

Stopping criterias

① for a given $\epsilon > 0$ (desired level of accuracy)
• if $|f(m_n)| < \epsilon$ then we may stop at the n^{th} iteration.

② $|m_n - m_{n-1}| < \epsilon$, the we may stop at the n^{th} iteration.

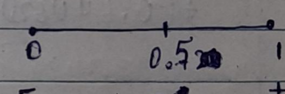
③ perform the no. of iteration as asked in the question.

use the bisection method to find the sol. of $\sqrt{x} - \cos x$ accurate with an 10^{-2} in $[0, 1]$ interval.

$f(0) = -1$

$f(1) = 0.459$

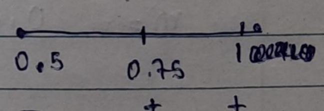
$m_1 \Rightarrow \frac{1 + 0}{2} = 0.5$



$f(0.729) = 0.17974069$

Root lies in [0, 0.729]

$m_2 = \frac{0 + 0.729}{2} = 0.3645$



Root lies in [0.5, 1]

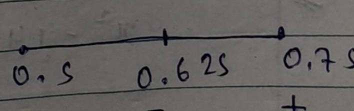
$m_2 \quad \frac{0.5 + 1}{2} = 0.75$

$f(0.75) \Rightarrow 0.134$

Root lies [0.5, 0.75]

$m_3 = \frac{0.5 + 0.75}{2} = 0.625$

$f(0.625) = -0.020$



PRACTICAL NOTE BOOK

Root lies $[0.625, 0.75)$

$$m_4 = \frac{0.625 + 0.75}{2} = 0.6875$$

0.625 (−) 0.6875 (+) 0.75 (+)

$f(0.6875) = \cancel{+ve}\ 0.056$

$$m_5 = \frac{0.625 + 0.6875}{2} = 0.65625$$

0.625 (−) 0.65625 (+) 0.6875 (+)

$f(0.65625) = +ve$

$$m_6 = \frac{0.625 + 0.65625}{2} = 0.640625$$

$f(0.640625) = -$

0.625 (−) 0.640625 (−) 0.65625 (+)

$$m_7 = \frac{0.640625 + 0.65625}{2} = 0.6484375$$

$f($

0.64065 (−) 0.6484 (+) 0.656 (+)

$$m_8 \Rightarrow \frac{0.64065 + 0.6484375}{2} = 0.6445437s$$

Qu calculate the $x = \sqrt{14}$ using bisectional method correct to 3 decimal places. interval $[3, 4]$ $(\epsilon = 0.001$

Sol

$x = \sqrt{14}$

$x^2 - 14 = 0$

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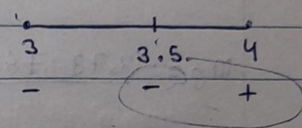
Sol^n $f(3) = -5$

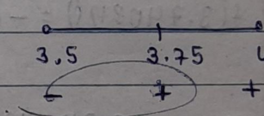
$f(4) = 2$

$m_1 = \frac{3+4}{2} \Rightarrow 3.5$



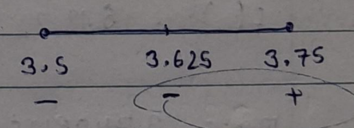
$f(3.5) = -1.75$

$m_2 = \frac{3.5+4}{2} = 3.75$



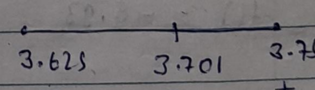
$f(3.75) = 0.0625$

$m_3 = \frac{3.5+3.75}{2} = 3.625$



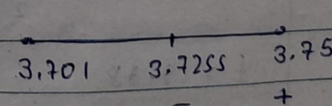
$f(3.625) = -0.859$

$m_4 = \frac{3.625 + 3.75}{2} = 3.6875$



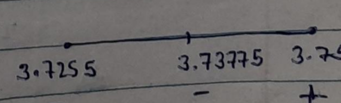
$f(3.701) = -0.302$

$m_5 = \frac{3.6875 + 3.75}{2} = 3.71875$



$f(3.7255) = -0.120$

$m_6 = \frac{3.71875 + 3.75}{2} = 3.734375$



$f(3.73775) = -0.029$

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$m_7 = \frac{3.73775 + 3.75}{2} = 3.743875$

3.73775   3.743875   3.75
−   +   +

$f(3.743875) = 0.016$

$m_8 = \frac{3.73775 + 3.743875}{2} = 3.7408125$

3.73775   3.7408125   3.74
−   −   +

$f(3.7408125) = -6.321$

$m_9 = \frac{3.7408125 + 3.743875}{2} = 3.74234375$

$f(\,) = 5.13$

3.7408125   3.74234375   3.74
−   +   +

$m_{10} = \frac{3.7408125 + 3.74234375}{2} = \cancel{3.741578125}\ 3.739257813$

$f(\,) = -5.93$

3.7408125   3.7415781   3.7423
−   −   +

$m_{11} = \frac{3.7415781 + 3.742375}{2} = \cancel{3.74197655}\ 3.739746094$

$f = 2.38$

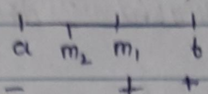
3.7415781   3.7419765   3.74237
−   +   +

$m_{12} = \frac{3.7415781 + 3.7419765}{2} = \cancel{3.7417773}\ 3.739990235$

Ans $\underline{3.740}$

Imp

Bisection methods will certainly converge !; Justify !.



$$(m_2 - m_1) < \frac{1}{2}(b-a)$$

$$(m_3 - m_2) < \frac{1}{2^2}(b-a)$$

$$\vdots$$

$$(m_n - m_{n-1}) < \frac{1}{2^{n-1}}(b-a)$$

~~Bisection method will cert~~

let $[a,b]$ the interval in which the root of $f(x) = 0$ lies (α).
let $m_1, m_2, \ldots m_n$ be the n approximations to the root 'α'. calculate using B.M.

$$|(m_2 - m_1)| < \frac{1}{2}(b-a).$$

and similarly $|m_3 - m_2| < \frac{1}{2^2}(b-a)$.

$$|m_n - m_{n-1}| < \frac{1}{2^n}(b-a).$$

let $E_n = |m_n - \alpha|$ = error at the n^{th} iteration
then

$$\boxed{E_n < \frac{1}{2^n}(b-a)}$$ error eqn.

Thus $\lim_{n \to \infty} E_n = 0$

which prove that B.M will converge to the root.

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Qu Determine the no. of iteration required to solve $f(x)=0$ using bysectional method to achieved 3 decimal accuracy on $a=0$, & $b=1$.

$$\epsilon = 0.001 = 10^{-3}$$

Error at n^{th} iteration $< \frac{1}{2^n}(b-a) < 10^{-3}$

$$\frac{1}{2^n} < 10^{-3}$$

$$-n\log 2 < -3$$

$$n > \frac{3}{\log 2}$$

$$n \geq 9.96$$

$$\boxed{n \simeq 10}$$

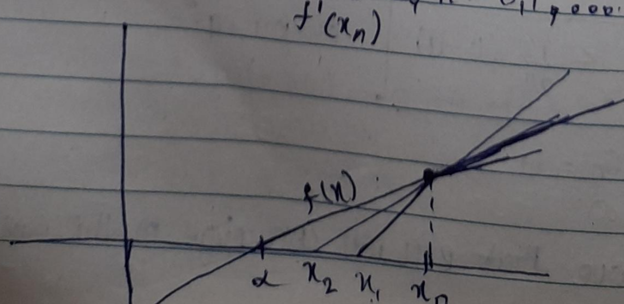
Newton's Raphson (N-R) method / Newton's method.

- To solve $f(x)=0$

** - It is the one of the most powerful numerical method to calculate the approx root of $f(x)=0$.

- The iteration scheme of N-R method with initial approximation x_0 is

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}, \quad n = 0, 1, \ldots \ldots$$



- Newton's method is also know as method of tangent
- Since the initial approximation is in the form of the point x_0 (Not in a b interval) therefore N-R method is an open method (Bisection method is called close / bracketing method)
- N-R method may or may not converge always but Bisection converg always.

Que perform 4 iteration to calculate the $x = \sqrt{14}$ using N-R method.

Soln

$$f(x) = x^2 - 14 = 0$$

$$\boxed{x_0 = 4} \quad \because \ (4^2 \text{ is close to } 14).$$

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}.$$

$$\because \quad f'(x) = 2x$$

$$x_n - \frac{x_n^2 - 14}{2x_n}.$$

$$x_{n+1} = \frac{x_n^2 + 14}{2x_n}, \quad n = 0, 1, \ldots$$

$$x_1 = \frac{x_0^2 + 14}{2x_0} = \frac{16 + 14}{8} \Rightarrow \frac{30}{8} = 3.75.$$

$$x_2 = \frac{x_1^2 + 14}{2x_1} = 3.74167$$

$$x_3 = 3.741657$$

$$x_4 = 3.7416577774 \approx 3.742$$

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Que Use N-R method to find the sol of $x = \cos x$ in the interval $[0, \frac{\pi}{2}]$, with 3 decimal digit accuracy.

$$x - \cos x = 0$$

$$x_0 = \frac{\pi}{2}.$$

$$f'(x) = 1 + \sin x.$$

$$x_{n+1} = x_n - \frac{x_n - \cos x_n}{1 + \sin x_n}.$$

$$= \frac{\cancel{x_n} + x_n \sin x_n - \cancel{x_n} - \cos x_n}{1 + \sin x_n}.$$

$$\Rightarrow \frac{x_n \sin x_n + \cos x_n}{1 + \sin x_n}.$$

$x_1 \Rightarrow 0.5$ 0.93970

$x_2 = \cancel{0.75522}\ 0.7395361 \approx \underline{0.73954}.$

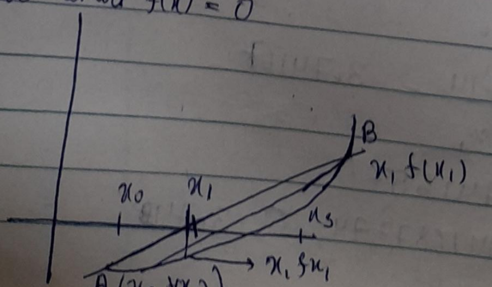
$x_3 = 0.739085$

$x_4 = 0.7390851$

$x_5 = 0.7390851332.$

Secant Method

To solve method $f(x) = 0$



	Interval [a b]	Type
Bisection	[a b]	Bracketing method
N-R method	x_0	
fixed point , Secant method	x_0, x_1	open method
graphical ; Regular false	x_0, x_1	open method / bracketing method

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Take two initial approximation x_0 & x_1 for the point $A(x_1, f_0)$, $B(x, f_1)$ join the line AB and where it cuts the x axis will give the nex approximation x_2. and the value of x_2 is defined by

$$x_2 = \frac{x_1 f_0 - x_0 f_1}{f_0 - f_1}$$

similarly x_3 can be obtained from x_1 and x_2.
x_4 can be obtained from x_2 and x_3.

Que Use secant method to find the solution of. $xe^x - 2 = 0$ ~~$2x - \cos x = 0$~~. correct to 2 decimal places starting with $x_0 = 0.9$ and $x_1 = 1$

Sol $x_0 = 0.9$

$x_1 = 1$

$f_0 = f(x_0) = f(0.9) = 0.2136428 \approx 0.2136$

$f_1 = f(x_1) = 0.718281 \approx 0.7183$

$$x_2 = \frac{0.2136 - 0.9(0.7183)}{0.2136 - 0.7183} = 0.857677 \approx 0.8577 \approx 0.86$$

$$\left\{ x_2 = \frac{x_1 f_1 - x_0 f_1}{f_0 - f_1} \right\}$$

f_2 ~~x_{3e}~~ $= f_0(x_2) = 0.02222$

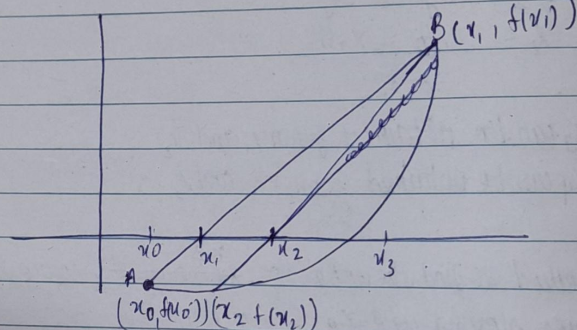
~~$x_{3e} = x_0 f_0 e^{-x}$~~

$$x_3 = \frac{(0.8577)(0.7183) - 1(0.0222)}{0.7183 - 0.0222} = 0.8532 \approx 0.85$$

$f(x_3) = 0.0026$

$$x_4 = \frac{(0.8532)(0.0222) - (0.8577)(0.0026)}{0.0222 - 0.0026} = 0.8526 \approx 0.85.$$

Regular-false position or method of false position rer Regular-false ~~po~~ to solve $f(x)=0$



The method of false position generates approximation in a same manner as the secent method but it includes a test to insure that the root is always bracketed btw successive iterations.

$$x_2 = \frac{x_1 f_0 - x_0 f_1}{f_0 - f_1}$$

Take two initial approximation x_0 and x_1 such that $f(x_0).f(x_1) < 0$

que $x = \cos x$ in interval $[\underset{x_0}{0}, \underset{x_1}{\frac{\pi}{2}}]$ solve using method false position

$x - \cos x = 0$

$f_0 = f0 \Rightarrow -1$

$f_1 = f\frac{\pi}{2} \Rightarrow 1.570796327$

er Regular-false

on in a same manner
e that the root is

$f(x_0).f(x_1) < 0$

false position

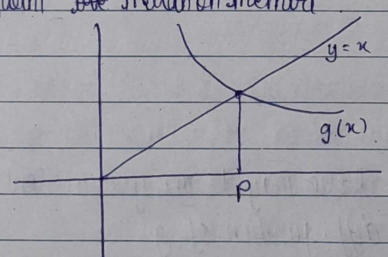
Ans $\Rightarrow \boxed{0.3908}$

$$x_2 = \frac{\frac{\pi}{2}(-1) - 0(1.57)}{-1 - 1.57079632} = 0.611015472.$$

$$f_2 = f(x_2) = -0.2080503925.$$

$$x_3 = \frac{x_2 f_1 - x_1 f_2}{f_1 - f_2} \Rightarrow$$

fixed point ~~ite~~ iteration method



$g(x) = x.$

$g(p) = p$

$p \to$ fixed point.

- for an ~~function~~ equation $f(x) = 0$, we seek a point where the curve g intersect the line $y = x$.
- A value of x such that $x = g(x)$ is called a fixed point of g.
- A fixed point iteration ~~ste~~ scheme is defined as. $x_{n+1} = g(x_n)$ where g has fixed point that are sol. of $f(x) = 0$.
- An inital starting point x_0 is selected and the iteration method is applied. repeatedly until it converges into a root.

Apply the fixed point procedure where $g(x) = 1 + \frac{2}{x}$ with $x_0 = 1$ compute 4 approximate.

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Sol $g(x) = 1 + \frac{2}{x}$.

$x_0 = 1$

$x_1 = g(x_0)$

$1 + \frac{2}{1} = 3$

$x_2 = g(x_1)$

$= 1 + \frac{2}{3} = 1.667$

$x_3 = g(x_2)$

$= 1 + \frac{2}{1.667} =$

Note # for an equation $f(x) = 0$, there may be many equivalent fixed point problems ($x = g(x)$) with diff. function of g.

If mod of $g'(x)$ is less than one then the fix point methods converge for any starting values to near ~~the~~ ~~root~~ x.

$$|g'(x)| < k \text{ where } x \in I$$

Here $x_0 \in I$,

Q evaluate root 5 using the eq $x^2 - 5 = 0$ by applying the fixed point method.

$$x^2 - 5 = 0$$

$$x = \frac{5}{x}.$$

$$g(x) = \frac{5}{x}.$$

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$x_0 = 2$

$x_1 = \frac{5}{2} = 2.5$

$x_2 = \frac{5}{2.5} =$

$x_3 = 2.5$

$x_4 = 2$

Brent's method :- It follows ~~a~~ hybrid approach that combines the reliability of bracketing method with the speed of the opening methods. It include the technique of the bisection methods with the secant method.

Let x_0, x_1 be the initial approximation obtained using the Intermediate value property and then appling the techniques of the secant method to find the next approximation also instead of applying the secant method we can also apply inverse quadratic interpolation.

Rate of conversion

Let $\mathcal{E}_0$ denotes an error introduce at some state and $\mathcal{E}_n$ represent the magnitude of the error after n subsequent iteration then.

i) If $\mathcal{E}_n \approx C\mathcal{E}_0$ where C is a constant then the growth of error is linear

ii) $\mathcal{E}_n \approx C^n \mathcal{E}_0$ when $C>1$ then the growth of the error is exponential

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Let $\{x_n\}_{n=0}^{\infty}$ let x_n is a sequence of approximate that convergous to ~~root α is the sequence~~ · root of α where $x_n \neq \alpha$ for all n then if their exit +ve constant λ & p

$$\lim_{n\to\infty} \frac{|x_{n+1} - \alpha|}{|x_n - \alpha|^p} = \lambda$$

then $\{x_n\}_{n=0}^{\infty}$ converges to the root α with order p and error constant λ.

$$\lim_{n\to\infty} \frac{\varepsilon_{n+1}}{\varepsilon_n^{p}} = \lambda$$

p → order of conversion

for bisection if $p = 1$, then the sequence is linearly convergent

for neutral if $p = 2$, then sequence is quadratic convergent.
Algent

for secant $p = 1.6$

Types of errors.

Let x_n be the approximate value of α.

1.) Absolute error :- $|x_n - \alpha|$

2.) Relative error :- $\frac{|x_n - \alpha|}{\alpha}$.

3.) percentage error :- ~~x~~ Relative error × 100% =

Qn find the relative error when p is approximate p^* when $p = 0.3 \times 10^{-3}$ & $p^* = 0.31 \times 10^{-3}$

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Sol: $\dfrac{0.31\times10^{-3} - 0.3\times10^{-3}}{0.3\times10^{-3}} = \cancel{0.0333}\ 0.033\times10^{-1}$

Q. $\pi = 3.14159265$ determine the five digit by rounding and choping.

$\pi = 3.1416$

$\pi = 3.1415$

Q. suppose the $x = \frac{5}{7}$ & $y = \frac{1}{3}$ use five digit choping for calculating $x+y$ & $x-y$

Sol: $\frac{5}{7} \Rightarrow 0.\overline{714284}$ $\quad y = \frac{1}{3} = 0.\overline{33333}$

$0.71428 + 0.33333 = 1.04761 \Rightarrow 1.0476$

$\frac{5}{7} + \frac{1}{3} \Rightarrow \frac{22}{21}$

$\frac{22}{21} - 1.0476 = 1.904$

PRACTICAL NOTE BOOK

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Questions

Q1 Use Newton, secant, ~~fato~~ method of false position to find the sol. correct to 3 decimal digit for the following problem.

i) $x^3+3x^2-1=0$, $[-3,-2]$

(ii) $x^3-2x^2-5=0$ $[1,4]$

(iii) $\ln(x-1)+\cos(x-1)=0$ $[1.3, 2]$

(iv) $(x-2)^2-\ln x=0$ $[1,2]$ & $[e,4]$

(v) $e^x-3x^2=0$ $[0,1]$

Qu Use the fixed point iteration method to determine the sol. for 2 decimal digit for the following problem

(i) $x^3-x-1=0$ on $[1,2]$, $x_0=1$

(ii) $x^4-3x^2-3=0$ on $[1,2]$ $x_0=1$

(iii) $25^{1/3}=0$

1.

*

Que 1

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Unit - 2

Solving the system of equations

1. **Gauss elimenation method** : convert the system to a triangular form and the solve it by substitution method

* Elementary operations (Row / column)

1.) $R_i \rightarrow R_i \pm R_j$

2.) $R_i \leftrightarrow R_j$

3.) $R_i \rightarrow R_i \pm k R_j$ / $R_i \rightarrow k R_i$

Que 1 $4x + y + 2z = 9$

$2x + 4y - z = -5$

$x + y - 3z = -9$.

using gauss elimination

$$\begin{bmatrix} 4 & 1 & 2 \\ 2 & 4 & -1 \\ 1 & 1 & -3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 9 \\ -5 \\ -9 \end{bmatrix}$$

A | b

$$\left[\begin{array}{ccc|c} 4 & 1 & 2 & 9 \\ 2 & 4 & -1 & -5 \\ 1 & 1 & -3 & -9 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 4 & 1 & 2 & 9 \\ 0 & 7/2 & -2 & -1/2 \\ 0 & 3/4 & -7/2 & -27/4 \end{array}\right] \quad \begin{array}{l} R_2 \rightarrow R_2 - \frac{R_1}{2} \\ R_3 \rightarrow R_3 - \frac{R_1}{4} \end{array}$$

$$R_3 \to R_3 - \frac{3}{4} \times \frac{2}{7} R_2$$

$$R_3 \to R_3 - \frac{3}{14} R_2 .$$

$$\left[\begin{array}{ccc|c} 4 & 1 & 2 & 9 \\ 0 & 7/2 & -2 & -1/2 \\ 0 & 0 & -43/14 & -186/28 \end{array}\right]$$

backward substitution (firstly find z then y & then x)

$$-\frac{43}{14} z = \frac{-186}{28}$$

$$z = \frac{186}{28} \times \frac{14}{43} \Rightarrow \frac{93}{43} .$$

$$\frac{7}{2} y - 2z = \frac{-1}{2} .$$

$$\frac{7}{2} y - \frac{186}{43} = \frac{-1}{2} .$$

$$\frac{7}{2} y = \frac{-1}{2} + \frac{186}{43}$$

$$\frac{7}{2} y = \frac{329}{86}$$

$$y = \frac{329}{86} \times \frac{2}{7} \Rightarrow \frac{47}{43}$$

$$4x + y + 2z = 9$$

$$4x + \frac{47}{43} - \frac{186}{43} = 9$$

$$4x - 3.2325 = 9 \qquad x = 3.058 =$$

$$4x - y + z = 8$$
$$2x + 5y + 2z = 3$$
$$x + 2y + 4z = 11$$

$$\left[\begin{array}{ccc|c} 4 & -1 & 1 & 8 \\ 2 & 5 & 2 & 3 \\ 1 & 2 & 4 & 11 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 4 & -1 & 1 & 8 \\ 0 & 11/2 & 3/2 & -1 \\ 0 & 9/4 & 15/4 & 9 \end{array}\right]$$ $R_2 \rightarrow R_2 - \frac{R_1}{2}$ $R_3 \rightarrow R_3 - \frac{R_1}{4}$

$$\left[\begin{array}{ccc|c} 4 & -1 & 1 & 8 \\ 0 & 11/2 & 3/2 & -1 \\ 0 & 0 & 69/22 & 207/22 \end{array}\right]$$ $R_3 \rightarrow R_3 - \frac{9}{\cancel{4}_2} \times \frac{\cancel{2}^1}{11} R_2$ $R_3 \rightarrow R_3 - \frac{9}{22} R_2$

$$\frac{69}{22} z = \frac{207}{22}$$

$$z = \frac{207}{\cancel{22}} \times \frac{\cancel{22}}{69} \Rightarrow 3$$

$$\frac{11}{2} y + \frac{3}{2} \times 3 = -1$$

$$\frac{11}{2} y = -1 - \frac{9}{2}$$

$$y = -\frac{11}{2} \times \frac{2}{11} \Rightarrow -1$$

$$4x + 1 + 3 = 8$$
$$\boxed{x = 1}$$

$$\boxed{\begin{array}{c} z = 3 \\ y = -1 \\ x = 1 \end{array}}$$

$-\frac{1}{2}$

$2 - \frac{1}{2}$

$3 - \frac{\cancel{8}\,9}{2}$

$2 + \frac{1}{4}$

$4 - \frac{1}{4}$

$11 - 2\frac{\cancel{8}}{4}$

$\frac{9}{4} \times \frac{9}{\cancel{22}} \times \frac{\cancel{11}}{2}$
2

$\frac{9}{4} \times \frac{9}{4}$

$\frac{81}{16}$

$\frac{15}{4} - \frac{9}{22} \times \frac{3}{2}$

$\frac{15}{4} - \frac{27}{44}$

$9 + \frac{9}{22}$

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Pivoting strategies

To overcome the zero entries at ~~piv~~ pivot positions, we apply the pivoting techniques:

partial   complete.

Qu $x - 5y + z = 7$

$10x + 20z = 6$

$5x - z = 4$.

$$\begin{bmatrix} 1 & -5 & 1 \\ 10 & 0 & 20 \\ 5 & 0 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 7 \\ 6 \\ 4 \end{bmatrix}$$

$(R_2 \leftrightarrow R_1)$

$$\begin{bmatrix} 10 & 0 & 20 \\ 1 & -5 & 1 \\ 5 & 0 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ 7 \\ 4 \end{bmatrix}$$

$$\left[\begin{array}{ccc|c} 10 & 0 & 20 & 6 \\ 0 & -5 & -1 & 6.4 \\ 0 & 0 & -11 & 1 \end{array}\right]$$

$R_2 \rightarrow R_2 - \frac{R_1}{10}$

$R_3 \rightarrow R_3 - \frac{R_1}{2}$

$$\left[\begin{array}{ccc|c} 10 & 0 & 20 & 6 \\ 0 & -5 & -1 & 6.4 \\ 0 & 0 & -11 & 1 \end{array}\right]$$

$6 - \frac{7}{10}$

$5 - \frac{10}{2}$

$0 - 0$

$$\begin{bmatrix} 10 & 0 & 20 \\ 0 & -5 & -1 \\ 0 & 0 & -11 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ 6.4 \\ 1 \end{bmatrix}$$

$$10x + 20z = 6$$

$$-5y - z = 6.4$$

$$-11z = 1$$

$$\boxed{z = -\frac{1}{11}}$$

$$-5y + \frac{1}{11} = 6.4$$

$$-5y = 6.4 - \frac{1}{11}$$

$$\boxed{y = -1.261}$$

$$10x - \frac{20}{11} = 6$$

$$\boxed{x = 0.781}$$

Q4 Apply gauss elimination to the system.

$$0.003000x + 59.14y = 59.17$$

$$5.291x - 6.130y = 46.78$$

using partial pivoting & 4 digit arethematic with ~~rand~~ rounding and compare result to exact solution $x = 10$ & $y = 1$

$$\begin{bmatrix} 0.003000 & 59.14 \\ 5.291 & -6.130 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 59.17 \\ 46.78 \end{bmatrix}$$

$R_1 \longleftrightarrow R_2$

$$\left[\begin{array}{cc|c} 5.291 & -6.130 & 46.78 \\ 0.003000 & 59.14 & 59.17 \end{array}\right]$$

$$\left[\begin{array}{cc|c} 5.291x & -6.130y & 46.78 \\ 0 & 59.14y & 59.14 \end{array}\right] \quad R_2 \to R_2 - \frac{0.003000}{5.291} R_1$$

$$5.291x - 6.130y = 46.78$$

$$59.14y = 59.14$$

$$\boxed{y = 1.}$$

$$\boxed{x = 10}$$

$59.14y = 59.$

$\boxed{y = 1}$

$5.291x - 6.130(1)$

46

$$\begin{bmatrix} 2 & -3 & 1 \\ 1 & 1 & -1 \\ -1 & 1 & -3 \end{bmatrix} \begin{bmatrix} 2 & 6 & 0 \\ -1 & 4 & 1 \\ 0 & 5 & -3 \end{bmatrix}$$

LU - Decomposition

Qu Solve
$$\begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 3 & 0 & 1 \end{bmatrix} \underbrace{\begin{bmatrix} 2 & 1 & -1 \\ 0 & 4 & 2 \\ 0 & 0 & 5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}}_{\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}} = \begin{bmatrix} 1 \\ 0 \\ -5 \end{bmatrix}$$

$$\boxed{y_1 = 1}$$

$$-2y_1 + y_2 = 0$$

$$\boxed{y_2 = 2}$$

$$-3y_1 - y_3 = -5$$

$$\boxed{y_3 = -8}$$

$$\begin{bmatrix} 2 & 1 & -1 \\ 0 & 4 & 2 \\ 0 & 0 & 5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} > \begin{bmatrix} 1 \\ 2 \\ -8 \end{bmatrix}$$

$$x_3 = -\frac{8}{5}$$

$$4x_2 - 2x_3 = 2$$

$$4x_2 - \frac{16}{5} = 2$$

$$\boxed{x_2 = 1.3}$$

$$2x_1 + x_2 - x_3 = 1$$

$$2x_1 + 1.3 + \frac{8}{5} = 1$$

$$\boxed{x_1 \Rightarrow -0.95}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ -3 & 2 & 1 \end{bmatrix} \underbrace{\begin{bmatrix} 1 & 2 & -3 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}}_{y} = \begin{bmatrix} 4 \\ 6 \\ 8 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ -3 & 2 & 1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 4 \\ 6 \\ 8 \end{bmatrix}$$

$$\boxed{y_1 = 4}$$

$$8 + y_2 = 6$$

$$\boxed{y_2 = -2}$$

$$-12 \oplus -4 + y_3 = 8$$

$$\boxed{y_3 \Rightarrow 24}$$

$$\begin{bmatrix} 1 & 2 & -3 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 4 \\ -2 \\ 24 \end{bmatrix}$$

$$\boxed{x_3 \Rightarrow 24}$$

$$x_2 - 2x_3 = -2$$

$$x_2 + 48 = -2$$

$$\boxed{x_2 = -50}$$

$$x_1 + 2x_2 - 3x_3 = 4$$

$$x_1 + -100 - 3(24) = 4$$

$$x + -100 - 72 = 4$$

$$\boxed{x = 176}$$

Q. Solve the system.

$$\begin{bmatrix} 1 & 0 & 0 \\ l_{21} & 1 & 0 \\ l_{31} & l_{32} & 1 \end{bmatrix} \begin{bmatrix} u_{11} & u_{12} & u_{13} \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{bmatrix} \Rightarrow \text{Dolittle factorization}$$

$$\begin{bmatrix} l_{11} & 0 & 0 \\ l_{21} & l_{22} & 0 \\ l_{31} & l_{32} & l_{33} \end{bmatrix} \begin{bmatrix} 1 & u_{12} & u_{13} \\ 0 & 1 & u_{23} \\ 0 & 0 & 1 \end{bmatrix} \Rightarrow \text{Crout factorization}$$

Solve the system using dolittle factorization.

$$3x + 2y + 7z = 4$$
$$2x + 3y + z = 5$$
$$3x + 4y + z = 7$$

$$\begin{bmatrix} 3 & 2 & 7 \\ 2 & 3 & 1 \\ 3 & 4 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ 5 \\ 7 \end{bmatrix}$$

$$\begin{bmatrix} 3 & 2 & 7 \\ 0 & 5/3 & 11/3 \\ 0 & 2 & -6 \end{bmatrix} \qquad R_2 \to R_2 - \frac{2R_1}{3}$$

$$R_3 \to R_3 - R_1 \qquad 2 - \frac{4}{3} \qquad 3 - \frac{21}{3}$$

$$U = \begin{bmatrix} 3 & 2 & 7 \\ 0 & -5/3 & -11/3 \\ 0 & 0 & -8/5 \end{bmatrix} \qquad R_3 \to R_3 - 2 \times \frac{3}{5} R_2 \qquad 9 - \frac{4}{1} \qquad 1 - \frac{14}{}$$

$$L = \begin{bmatrix} 1 & 0 & 0 \\ 2/3 & 1 & 0 \\ 1 & 6/5 & 1 \end{bmatrix} \quad U = \begin{bmatrix} 3 & 2 & 7 \\ 0 & 5/3 & -11/3 \\ 0 & 0 & -8/5 \end{bmatrix}$$

$$\begin{bmatrix} \ \end{bmatrix}\underbrace{\begin{bmatrix} \ \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix}}_{y} = \begin{bmatrix} \ \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 2/3 & 1 & 0 \\ 1 & 6/5 & 1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 4 \\ 5 \\ 7 \end{bmatrix}$$

$\frac{8}{3} + y_2 =$

$\boxed{y_1 = 4}$

$\boxed{y_2 = \frac{7}{3}}$

$\boxed{y_3 = \frac{1}{5}}$

$$\begin{bmatrix} 3 & 2 & 7 \\ 0 & 5/3 & -11/3 \\ 0 & 0 & -8/5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 4 \\ 7/3 \\ 1/5 \end{bmatrix}$$

$-\frac{8}{5} x_3 \Rightarrow \frac{1}{5}$

$\boxed{x_3 \Rightarrow \cancel{8} -\frac{1}{8}}$

$\frac{2}{3} y_1 + y_2 = 5$

$\frac{2}{3} \times 4$

$\frac{8}{3}$

$\frac{5}{3} x_2 - \frac{11}{3} x_3 = \frac{7}{3}$

$\cancel{x_2} \Rightarrow$

$\frac{5}{3} x_2 + \frac{11}{24} = \frac{7}{3}$

$\boxed{x_2 = \frac{9}{8}}$

$$3x_1 + 2x_2 + 7x_3 = 4$$

$$3x_1 + 2\left(\frac{9}{8}\right) + 7\left(-\frac{1}{8}\right) = 4$$

$$\boxed{x_1 = \frac{7}{8}}$$

Q solve the system of eq using gauss elimination with partial pivoting.

$$x_1 + 10y - z = 3$$
$$2x + 3y + 20z = 7$$
$$10x - y + 2z = 4$$

$$\begin{bmatrix} 1 & 10 & -1 \\ 2 & 3 & 20 \\ 10 & -1 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ 7 \\ 4 \end{bmatrix}$$

$$\left[\begin{array}{ccc|c} 1 & 10 & -1 & 3 \\ 2 & 3 & 20 & 7 \\ 10 & -1 & 2 & 4 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 10 & -1 & 2 & 4 \\ 2 & 3 & 20 & 7 \\ 1 & 10 & -1 & 3 \end{array}\right]$$

$3 + \frac{2}{10} \times -1$

$20 + \frac{4}{10}$

$$R_2 \to R_2 - \frac{2R_1}{10}$$

$$R_3 \to R_3 - \frac{R_1}{10}$$

$$\left[\begin{array}{ccc|c} 10 & -1 & 2 & 4 \\ 0 & 32/10 & 98/5 & 31/5 \\ 0 & \frac{101}{10} & \frac{-12}{10} & \frac{26}{10} \end{array}\right]$$

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LU Decomposition using gauss elimination

$$\begin{array}{ccc} & A & \\ \text{Raw oper} & \downarrow & \xrightarrow[\text{multiplication}]{} L \\ & \text{upper} & \end{array}$$

Do little

$$\begin{array}{ccc} & A & \\ \text{col oper} & \downarrow & \longrightarrow \text{upper} \\ & \text{Lower } \Delta & \end{array}$$

Apply crout's factorization to solve

$2x - 4y + z = 4$

$6x + 2y - z = 10$

$-2x + 6y - 2z = -6$

$$\begin{bmatrix} 2 & -4 & 1 \\ 6 & 2 & -1 \\ -2 & 6 & -2 \end{bmatrix}$$

$C_2 \to C_2 + 2C_1$

$C_3 \to C_3 - \frac{1}{2}C_1$

$$\begin{bmatrix} 2 & 0 & 0 \\ 6 & 14 & -4 \\ -2 & 2 & -1 \end{bmatrix}$$

$-4+4$

$1+4$

$1-1$

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$C_3 \to C_3 + \frac{\cancel{4}^{2}C_2}{\cancel{14}\,7}$

$$L \Rightarrow \begin{bmatrix} 2 & 0 & 0 \\ 6 & 14 & 0 \\ -2 & 2 & -3/7 \end{bmatrix} \qquad U = \begin{bmatrix} 1 & -2 & 1/2 \\ 0 & 1 & -2/7 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 0 & 0 \\ 6 & 14 & 0 \\ -2 & 2 & -3/7 \end{bmatrix} \underbrace{\begin{bmatrix} 1 & -2 & 1/2 \\ 0 & 1 & -2/7 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}}_{y} = \begin{bmatrix} 4 \\ 10 \\ -6 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 0 & 0 \\ 6 & 14 & 0 \\ -2 & 2 & -3/7 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 4 \\ 10 \\ -6 \end{bmatrix}$$

$2y_1 = 4$

$y_1 \Rightarrow \frac{\cancel{4}}{\cancel{2}} \Rightarrow 2$

$6y_1 + 14y_2 = 10$

$12 + 14y_2 = 10$

$y_2 = \frac{-\cancel{2}}{\cancel{14}\,7} \Rightarrow -\frac{1}{7}$

$-4 - \frac{1}{7} - \frac{3}{7}y_3 = -6$

$\frac{-28 - 1 - 3y_3}{7} = -6 \qquad \boxed{y_3 \Rightarrow 4}$

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$$\begin{bmatrix} 1 & -2 & 1/2 \\ 0 & 1 & -2/7 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} \cancel{y_1} \\ \cancel{y_2} \\ \cancel{y_3} \end{bmatrix} \begin{bmatrix} 2 \\ -1/7 \\ 4 \end{bmatrix}$$

$$\boxed{x_3 \Rightarrow 4}$$

$$x_2 - \frac{4}{7} \cancel{x_3} \Rightarrow -\frac{1}{7}$$

$$\boxed{x_2 = 1}$$

$$x_1 - 2 + \frac{\cancel{4}}{\cancel{2}} 2 \Rightarrow 2$$

$$\boxed{x_1 = 2}$$

$x + 10y - z = 3$
$2x + 3y + 20z = 7$
$10x - y + 2z = 4$

$$\begin{bmatrix} 1 & 10 & -1 \\ 2 & 3 & 20 \\ 10 & -1 & 2 \end{bmatrix}$$

$C_2 \longrightarrow C_2 - 10C_1$
$C_3 \longrightarrow C_3 + C_1$

$$\begin{bmatrix} 1 & 0 & 0 \\ 2 & -17 & 22 \\ 10 & -101 & 12 \end{bmatrix}$$

$12 + \frac{22}{7} \times 101$

$-\frac{300}{7}$ -20

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$$C_3 \rightarrow C_3 + \frac{22}{17} C_2$$

$$L \Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 2 & -17 & 0 \\ 10 & -101 & -\frac{2078}{17} \end{bmatrix}$$

$$U = \begin{bmatrix} 1 & 10 & -1 \\ 0 & 1 & 22/17 \\ 0 & 0 & 1 \end{bmatrix}$$

Q. Determine the inverse of the matrix (3×3) using gauss elimination.

$$\left[\begin{array}{ccc} 1 & 2 & -1 \\ 2 & 1 & 0 \\ -1 & 1 & 2 \end{array}\right] \left[\begin{array}{ccc} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{array}\right]$$

$R_2 \rightarrow R_2 - 2R_1$
$R_3 \rightarrow R_3 + R_1$

$$\left[\begin{array}{ccc|ccc} 1 & 2 & -1 & 1 & 0 & 0 \\ 0 & -3 & 2 & -2 & 1 & 0 \\ 0 & 3 & 1 & 1 & 0 & 1 \end{array}\right]$$

$R_3 \rightarrow R_3 + R_2$

$$\left[\begin{array}{ccc|ccc} 1 & 2 & -1 & 1 & 0 & 0 \\ 0 & -3 & 2 & -2 & 1 & 0 \\ 0 & 0 & 3 & -1 & 1 & 1 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & 2 & -1 & 1 \\ 0 & -3 & 2 & -2 \\ 0 & 0 & 3 & -1 \end{array}\right], \quad \left[\begin{array}{ccc|c} 1 & 2 & -1 & 0 \\ 0 & -3 & 2 & 1 \\ 0 & 0 & 3 & 1 \end{array}\right] \quad \left[\begin{array}{ccc|c} 1 & 2 & -1 & 0 \\ 0 & -3 & 2 & 0 \\ 0 & 0 & 3 & 1 \end{array}\right]$$

$3y_3 = -1$
$y_3 = -\frac{1}{3}$

$$\begin{bmatrix} x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \\ x_3 & y_3 & z_3 \end{bmatrix}$$

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$y_3' \Rightarrow -\frac{1}{3}$

$-3y_2 - \frac{2}{3} = -2$

$-3y_2 \Rightarrow -2 + \frac{2}{3}$

$y_2 \Rightarrow \frac{4}{9}$

$\boxed{y_2' = \frac{4}{9}}$

$y_1 + \frac{8}{9} + \frac{1}{3} \Rightarrow 1$

$\frac{9y_1 + 8 + 3}{9} \Rightarrow 1$

$9y_1 + 11 \Rightarrow 9$

$\boxed{y_1' \Rightarrow \frac{-2}{9}}$

$\boxed{y_3'' \Rightarrow \frac{1}{3}}$

$-3y_2 + \frac{2}{3} \Rightarrow 1$

$-3y_2 \Rightarrow 1 - \frac{2}{3}$

$\boxed{y_2'' \Rightarrow -\frac{1}{9}}$

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$$y_1 - \frac{2}{9} - \frac{1}{3} = 0$$

$$y_1 \quad \frac{-2-3}{9} = 0$$

$$\boxed{\cancel{9l \Rightarrow l4}} \quad y_1 - \frac{4}{9} \Rightarrow 0$$

$$\boxed{y_1'' \Rightarrow \frac{5}{9}}$$

$$\boxed{y'''_3 \Rightarrow 1}$$

$$-3y_2 + 2 \Rightarrow 0$$

$$\boxed{y_2 \Rightarrow \frac{2}{3}}$$

$$y_1 \quad \frac{4}{3} - 1 = 0$$

$$\boxed{y_1''' = -\frac{1}{3}}$$

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Diagonally Dominant matrix

The $n \times n$ matrix is said to be diagonally dominant if

$$|a_{ii}| \geq \sum_{\substack{j=1 \\ j \neq i}}^{n} |a_{ij}| \text{ for each } i$$

positive Definite matrix

A matrix A is positive definite if it is symmetrical and $x^T A x > 0$ for every non zero n dimensional vector.

cholesky factorisation

A symmetric positive definite matrix A can be written as $A = LL^T$ where L is a lower Δ matrix.

Qn Determine the cholesky factorisation of $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 8 & 22 \\ 3 & 22 & 82 \end{bmatrix}$

$A = LL^T$

$$\begin{bmatrix} 1 & 2 & 3 \\ 2 & 8 & 22 \\ 3 & 22 & 82 \end{bmatrix}$$

$C_2 \to C_2 - 2C_1$
$C_3 \to C_3 - 3C_1$

$$\begin{bmatrix} 1 & 0 & 0 \\ 2 & 2 & 16 \\ 3 & 16 & 73 \end{bmatrix}$$

22
82
-9

$C_3 \to C_3 - 8C_2$

73

L =

L^T =

1
2
3

1
2
3

...inant if

... $x^T A x > 0$

...n as $A = LL^T$

$$\begin{matrix} 3 \\ 22 \\ 82 \end{matrix}$$

$$L = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 2 & 0 \\ 3 & 16 & -55 \end{bmatrix}$$

$$L^T = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 2 & 16 \\ 0 & 0 & -55 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & 3 \\ 2 & 8 & 22 \\ 3 & 22 & 82 \end{bmatrix} = \begin{bmatrix} l_{11} & 0 & 0 \\ l_{21} & l_{22} & 0 \\ l_{31} & l_{32} & l_{33} \end{bmatrix} \begin{bmatrix} l_{11} & l_{21} & l_{31} \\ 0 & l_{22} & l_{32} \\ 0 & 0 & l_{33} \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & 3 \\ 2 & 8 & 22 \\ 3 & 22 & 82 \end{bmatrix} = \begin{bmatrix} l_{11}^2 & l_{11} l_{21} & l_{11} l_{31} \\ l_{21} l_{11} & l_{21}^2 + l_{22}^2 & l_{21} l_{31} + l_{22} l_{32} \\ l_{31} l_{11} & l_{31} l_{21} + l_{32} l_{22} & l_{31}^2 + l_{32}^2 + l_{33}^2 \end{bmatrix}$$

$l_{11}^2 = 1$.

$l_{11} l_{21} = 2$

$\boxed{l_{21} \Rightarrow 2}$

$l_{11} l_{31} \Rightarrow 3$

$\boxed{l_{31} = 3}$

$l_{21} l_{11} \Rightarrow 2$.

$\boxed{l_{21} = 2}$

$l_{21}^2 + l_{22}^2 \Rightarrow 8$

$\boxed{l_{22} \Rightarrow 2}$

$l_{31}(2) + l_{32}(2) \Rightarrow 22$.

$\boxed{l_{32} = 8}$

$\boxed{l_{33} \Rightarrow 3}$

$$\begin{bmatrix} 1 & 0 & 0 \\ 2 & 2 & 0 \\ 3 & 8 & 3 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 0 & 2 & 8 \\ 0 & 0 & 3 \end{bmatrix}$$

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Qus
$10x + y + z = 12$
$x + 10y + z = 12$
$x + y + 10z = 12$
using cholesky.

$$\begin{bmatrix} 10 & 1 & 1 \\ 1 & 10 & 1 \\ 1 & 1 & 10 \end{bmatrix} = \begin{bmatrix} l_{11} & 0 & 0 \\ l_{21} & l_{22} & 0 \\ l_{31} & l_{32} & l_{33} \end{bmatrix} \begin{bmatrix} l_{11} & l_{21} & l_{31} \\ 0 & l_{22} & l_{32} \\ 0 & 0 & l_{33} \end{bmatrix}$$

$$\begin{bmatrix} 10 & 1 & 1 \\ 1 & 10 & 1 \\ 1 & 1 & 10 \end{bmatrix} = \begin{bmatrix} l_{11}^2 & l_{11}l_{21} & l_{11}l_{31} \\ l_{21}l_{11} & l_{21}^2 + l_{22}^2 & l_{21}l_{31} + l_{22}l_{32} \\ l_{31}l_{11} & l_{31}l_{21} + l_{32}l_{22} & l_{31}^2 + l_{32}^2 + l_{33}^2 \end{bmatrix}$$

$l_{11}^2 \Rightarrow 10$

$\boxed{l_{11} \Rightarrow \sqrt{10}}$

$\sqrt{10}\, l_{21} \Rightarrow 1$

$\boxed{l_{21} = \frac{1}{\sqrt{10}}}$

$\boxed{l_{31} \Rightarrow \frac{1}{\sqrt{10}}}$

$l_{21} l_{11} \Rightarrow \frac{1}{\sqrt{10}} \sqrt{10} \Rightarrow 1$

$\frac{1}{10} + l_{22}^2 \Rightarrow 10$

$l_{22} \Rightarrow 10 - \frac{1}{10} \Rightarrow \frac{\sqrt{99}}{\sqrt{100}} \Rightarrow \boxed{\sqrt{9.9} \Rightarrow l_{22}}$

$\frac{1}{\sqrt{10}} \cdot \frac{1}{\sqrt{10}} + l_{32} \frac{\sqrt{99}}{\sqrt{10}} \Rightarrow 1$

$\frac{1}{\sqrt{10}}\left(\frac{1}{\sqrt{10}} + l_{32}\sqrt{99}\right) = 1$

$\frac{1}{\sqrt{10}} + l_{32}\sqrt{99} \Rightarrow \sqrt{10}$

$l_{32}\sqrt{99} \Rightarrow \sqrt{10} - \frac{1}{\sqrt{10}} \Rightarrow \frac{9}{\sqrt{10}}$

$\boxed{l_{32} \Rightarrow \frac{9}{\sqrt{10}\sqrt{99}}} \Rightarrow 3\sqrt{\frac{11}{10}}$

$\frac{1}{10} + \frac{9}{} \quad l_{33} \Rightarrow \sqrt{\frac{108}{11}}$

$$\begin{bmatrix} \sqrt{10} & \frac{1}{\sqrt{10}} & \frac{1}{\sqrt{10}} \\ \frac{1}{\sqrt{10}} & 3\sqrt{\frac{11}{10}} & \frac{3}{\sqrt{100}} \\ \frac{1}{\sqrt{10}} & 0 & \sqrt{\frac{108}{11}} \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 12 \\ 12 \\ 12 \end{bmatrix}$$

$$\begin{bmatrix} \sqrt{10} & \frac{1}{\sqrt{10}} & \frac{1}{\sqrt{10}} \\ \frac{1}{\sqrt{10}} & 3\sqrt{\frac{11}{10}} & \frac{3}{\sqrt{100}} \\ \frac{1}{\sqrt{10}} & 0 & \sqrt{\frac{108}{11}} \end{bmatrix} \underbrace{\begin{bmatrix} \sqrt{10} & \frac{1}{\sqrt{10}} & \frac{1}{\sqrt{10}} \\ \frac{1}{\sqrt{10}} & 3\sqrt{\frac{11}{10}} & 0 \\ \frac{1}{\sqrt{10}} & \frac{3}{\sqrt{100}} & \sqrt{\frac{108}{11}} \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}}_{y} = \begin{bmatrix} 12 \\ 12 \\ 12 \end{bmatrix}$$

$$\begin{bmatrix} \sqrt{10} & \frac{1}{\sqrt{10}} & \frac{1}{\sqrt{10}} \\ \frac{1}{\sqrt{10}} & 3\sqrt{\frac{11}{10}} & \frac{3}{\sqrt{100}} \\ \frac{1}{\sqrt{10}} & 0 & \sqrt{\frac{108}{11}} \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 12 \\ 12 \\ 12 \end{bmatrix}$$

$$y_1\sqrt{10} + \frac{y_2}{\sqrt{10}} + \frac{y_3}{\sqrt{10}} \Rightarrow 12$$

$$\cancel{\sqrt{10}(y_1\sqrt{10} + y_2 + (y_3) = 12)}$$

$$y_1(10) + y_2 + y_3 = 12 \quad \text{— (1)}$$

$$\frac{y_1}{\sqrt{10}} + 3\sqrt{\frac{11}{10}}\, y_2 + \frac{3}{\sqrt{100}}\, y_3 \Rightarrow 12$$

$$\frac{1}{\sqrt{10}}\left(y_1 + 3\sqrt{11}\, y_2\right) + \frac{3}{\sqrt{110}}\, y_3 = 12 \quad \text{— (2)}$$

$$\frac{y_1}{\sqrt{10}} + \frac{\sqrt{108}}{\sqrt{11}}\, y_3 \Rightarrow 12 \quad \text{— (3)}$$

$$\cancel{\frac{1}{\sqrt{10}}}\; y_1 + 3\sqrt{11}\, y_2 \cancel{+ \frac{3y_3}{\sqrt{110}}} \Rightarrow \left(12 - \frac{3}{\sqrt{110}}\, y_3\right)\sqrt{10}$$

$$y_1 + 3\sqrt{11}\, y_2 = 12\sqrt{10} - \frac{3\sqrt{10}}{\sqrt{110}}\, y_3$$

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$$\left(y_1 + 3\sqrt{11}\,y_2 + \frac{3\sqrt{10}}{\sqrt{110}}\,y_3 = 12\sqrt{10}\right) \times 10$$

$$10y_1 + 30\sqrt{11}\,y_2 + \frac{300\sqrt{10}}{\sqrt{110}}\,y_3 \Rightarrow 120\sqrt{10}.$$

Q. ~~using factorization find the inverse of~~

Q. using factorization find the inverse of.

$$A = \begin{bmatrix} 2 & -2 & 4 \\ 2 & 3 & 2 \\ -1 & 1 & -1 \end{bmatrix}$$

$A = LU$

$A^{-1} = U^{-1}L^{-1}$

$$U = \begin{bmatrix} 2 & -2 & 4 \\ 0 & 5 & -2 \\ 0 & 0 & 1 \end{bmatrix} \quad \begin{array}{l} R_2 \to R_2 - R_1 \\ R_3 \to R_3 + \frac{1}{2}R_1 \end{array}$$

$$L = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ -\frac{1}{2} & 0 & 1 \end{bmatrix}$$

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$$U^{-1} \Rightarrow \left[\begin{array}{ccc|ccc} 2 & -2 & 4 & 1 & 0 & 0 \\ 0 & 5 & -2 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right] \quad \begin{array}{l} R_2 \to R_2 + 2R_3 \\ R_1 \to R_1 - 4R_3 \end{array}$$

$$\left[\begin{array}{ccc|ccc} 2 & -2 & 0 & 1 & 0 & -4 \\ 0 & 5 & 0 & 0 & 1 & 2 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right] \quad R_1 \to R_1 + \frac{2}{5} R_2$$

$$\left[\begin{array}{ccc|ccc} 2 & 0 & 0 & 1 & 2/5 & -16/5 \\ 0 & 5 & 0 & 0 & 1 & 2 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right]$$

$$U^{-1} = \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1/2 & 1/5 & -8/5 \\ 0 & 1 & 0 & 0 & 1/5 & 2/5 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right]$$

$$L^{-1} \Rightarrow \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 & 0 \\ -1/2 & 0 & 1 & 0 & 0 & 1 \end{array}\right] \quad \begin{array}{l} R_2 \to R_2 - R_1 \\ R_3 \to R_3 + \frac{1}{2} R_1 \end{array}$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & -1 & 1 & 0 \\ 0 & 0 & 1 & 1/2 & 0 & 1 \end{array}\right]$$

power method to calculate the largest eigen value corresponding the eigen vector we apply the power method on the matrix A starting with any initial non-zero eigen vector

Q1 $A = \begin{bmatrix} 2 & 3 \\ 5 & 4 \end{bmatrix}$ let X_0 is $[1, 1]$

$$AX_0 \Rightarrow \begin{bmatrix} 2 & 3 \\ 5 & 4 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} \Rightarrow \begin{bmatrix} 2+3 \\ 5+4 \end{bmatrix} \Rightarrow \begin{bmatrix} 5 \\ 9 \end{bmatrix}$$

$\Rightarrow 0.56$

$$\lambda_0 X_1 \Rightarrow \begin{bmatrix} 0.56 \\ 1 \end{bmatrix}$$

$$AX_1 = \begin{bmatrix} 2 & 3 \\ 5 & 4 \end{bmatrix} \begin{bmatrix} 0.56 \\ 1 \end{bmatrix} \Rightarrow \begin{bmatrix} 2 \times 0.56 + 3 \\ 5 \times 0.56 + 4 \end{bmatrix} \Rightarrow \begin{bmatrix} 1.12 + 3 \\ 2.80 + 4 \end{bmatrix}$$

$$= \begin{bmatrix} 4.12 \\ 6.80 \end{bmatrix}$$

$$\lambda_1 X_2 \Rightarrow \begin{bmatrix} 0.605 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 3 \\ 5 & 4 \end{bmatrix} \begin{bmatrix} 0.60 \\ 1 \end{bmatrix} \Rightarrow \begin{bmatrix} 2 \times 0.60 + 3 \\ 5 \times 0.60 + 4 \end{bmatrix} \Rightarrow \begin{bmatrix} 1.2 + 3 \\ 3.0 + 4 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 4.2 \\ 7 \end{bmatrix}$$

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$$\lambda_2 X_3 = \begin{bmatrix} 0,6 \\ 1 \end{bmatrix}$$

Qu2 $\begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}$ $\quad x_0 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$.

$$A X_0 = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}$$

Interpolation

It is a technique to find an interpolating polynomial of degree 'n', $P_n(x)$ satisfying the given data points (x_i, y_i)

$\because$ $n \rightarrow$ intervals
$n+1 \rightarrow$ points

$$0 \le i \le n$$

i.e $P_n(x_i) = y_i$

Data points

Let $(n+1)$ values of y for $(n+1)$ values of x are given i.e (x_0, y_0), (x_1, y_1), (x_2, y_2) . . . , (x_n, y_n) are given data points.

(1) If $x_{i+1} - x_i = h$ (constant) $0 \le i \le n$ then data set is said to be equi-spaced otherwise unequi-spaced.

Interpolation Techniques

- for equi-spaced
- for unequi-spaced.
  * Lagrange interpolation.

The Lagrange interpolating polynomial for the data points (x_0, y_0) (x_1, y_1) is given by

$$P_1(x) = \frac{(x-x_1)}{(x_0-x_1)} y_0 + \frac{(x-x_0)}{(x_1-x_0)} y_1$$

for three points (x_0, y_0), (x_1, y_1) (x_2, y_2) is

$$P_2(x) = \frac{(x-x_1)(x-x_2)}{(x_0-x_1)(x_0-x_2)} y_0 + \frac{(x-x_0)(x-x_2)}{(x_1-x_0)(x_1-x_2)} y_1 + \frac{(x-x_0)(x-x_1)}{(x_2-x_0)(x_2-x_1)} y_2$$

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for four pts $(x_0 y_0)$ $(x_1 y_1)$ $(x_2 y_2)$ $(x_3 y_3)$

$$P_3(x) = \underbrace{\frac{(x-x_1)(x-x_2)(x-x_3)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)}}_{L_0(x)} y_0 + \underbrace{\frac{(x-x_0)(x-x_2)(x-x_3)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)}}_{L_1(x_1)} y_1 +$$

$$\frac{(x-x_0)(x-x_1)(x-x_3)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)} y_3 + \frac{(x-x_0)(x-x_1)(x-x_2)(x-x_4)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)(x_3-x_4)} y_3$$

~~$L_0(x_0) \Rightarrow$~~

$$l_i(x_j) = \begin{cases} 1 & i = j \\ 0 & i \neq j \end{cases}$$

Q. Determine linear lagrange interpolating polynomial passes to the point $\underset{x_0\ y_0}{(2,4)}, \underset{x_1\ y_1}{(5,1)}$.

$$P_1(x) = \frac{(x-5)}{(2-5)} 4 + \frac{(x-2)}{(5-2)} 1$$

$$P_1(x) \Rightarrow -\frac{4x+20}{3} + \frac{x-2}{3} \Rightarrow \frac{-3x + 18}{3}$$

$$P_1(x) \Rightarrow \frac{18-3x}{3} \Rightarrow \frac{\cancel{3}(6-x)}{\cancel{3}}$$

$$P_1(x) \Rightarrow 6 - x$$

Qu use the nodes $x_0 = 2$, $x_1 = 2.75$, $x_2 = 4$ ~~you~~ find the 2nd degree polynomial for $f(x) = \frac{1}{x}$.

$y_0 \Rightarrow 0.5$

$y_1 \Rightarrow \frac{1}{2.75} \Rightarrow 0.364$

$y_2 \Rightarrow \frac{1}{4} = 0.25$

$P_2(x) \Rightarrow \cancel{\frac{x-2.75}{2-2.75} 0.8 + (x-2)(x}$

$\Rightarrow \frac{(x-2.75)(x-4)}{(2-2.75)(2-4)}(0.5) + \frac{(x-2)(x-4)}{(2.75-2)(2.75-4)}(0.364) + \frac{(x-2)(x-2.75)}{(4-2)(4-2.75)}$

$\Rightarrow \cancel{(x-2.75)(x-4)}$

$\Rightarrow \frac{(x-2.75)(0.5x-2)}{(-0.75)\ (-2)} + \frac{(x^2-2x-4x+8)(0.364)}{(0.75)(-1.25)} +$

$\frac{(x^2 - 2.75x - 2x + 5.5)(0.21)}{2(1.25)}$

$\Rightarrow \frac{0.5x^2 - 1.375x \cancel{-} -2x + 5.5}{1.5}$

$\frac{0.5x^2 - 3.375x + 5.5}{1.5} + \frac{0.364x^2 - 0.728x - 1.456x + 2.912}{-0.9375}$

Cubic spline
given a funct
.... $< x_n = b$,
approximates

1. S is a cubic sp
$j = 0, 1 \ldots n$
2. $S_j(x_j) = f(x_j)$
3. $S_{j+1}(x_{j+1}) =$
4. $S_{j+1}'(x_{j+1}) =$
5. $S_{j+1}''(x$

Qu Determine a
[1,2], [2,3

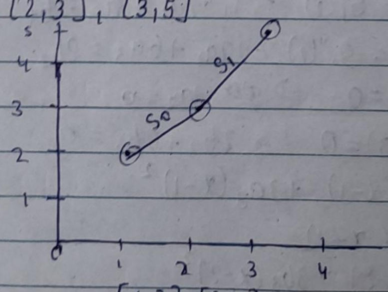
This spline conn
[1,2], [2,3]

Cubic spline interpolation.

given a function of defined on $[a,b]$ with a set of nodes $a = x_0 < x_1 \ldots < x_n = b$, a cubic spline interpolant is a function that approximates f and satisfies the following condition.

1. S is a cubic spline, denoted by $S_j(x)$ defined on sub interval $[x_j, x_{j+1}]$, $j = 0,1 \ldots, n-1$
2. $S_j(x_j) = f(x_j)$ and $S_{j+1}(x_{j+1}) = f(x_{j+1})$, $j = 0,1, n-1$
3. $S_{j+1}(x_{j+1}) = S_j'(x_{j+1})$ + $j = 0,1 \ldots n-2$
4. $S_{j+1}'(x_{j+1}) = S_j'(x_{j+1})$; $j = 0,1, n-2$.
5. $S_{j+1}''(x$

Q. Determine a natural cubic spline that passes through the point $[1,2]$, $[2,3]$, $[3,5]$



This Spline consist of 2 cubic $S_0(x)$ and $S_1(x)$ defined on the intervals $[1,2]$, $[2,3]$ respectively.
$[3,5]$

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Let $S_0(x) = a_0 + a_1(x-1) + a_2(x-1)^2 + a_3(x-1)^3$.

$S_1(x) = b_0 + b_1(x-2) + b_2(x-2)^2 + b_3(x-2)^3$.

There are 8 constants to be defined, while ~~requests~~ requires 8 conditions.

$S_0(x_0) = f(x_0)$

$S_0(x_1) = f(x_1)$

$S_1(x_1) = f(x_1)$

$S_1(x_2) = f(x_2)$.

$S_0(1) = f(1) = 2 \Rightarrow a_0 = 2$.

$S_0(2) = f(2) = 3 \Rightarrow a_0 + a_1 + a_2 + a_3 = 3 \Rightarrow a_1 + a_2 + a_3 = 1$ — (1)

$S_1(2) = f(2) = 3 \Rightarrow b_0 = 3$.

$S_1(3) = f(3) = 5 \Rightarrow b_0 + b_1 + b_2 + b_3 = 5 \Rightarrow b_1 + b_2 + b_3 = 2$.

$S_0'(2) = S_1'(2) \Rightarrow a_1 + 2a_2 + 3a_3 = b_1$

$S_0''(2) = S_1''(2) \Rightarrow 2a_2 + 6a_3 = 2b_2$.

$S''(1) = 0 \Rightarrow a_2 = 0$

$S''(3) = 0 \Rightarrow 2b_2 + 6b_3 = 0$

$S_0'(x) = a_1 + 2a_2(x-1) + 3a_3(x-1)^2$.

$S_0''(x) = 2a_2 + 6a_3(x-1)$

$S_1'(x) = b_1 + 2b_2(x-2) + 3b_3(x-2)^2$

$S_1''(x) = 2b_2 + 6b_3(x-2)$

$a_0 = 2$ $b_0 = 3$

$a_2 = 0$

$a_0 = 2$

$a_2 = 0$

$b_0 = 3$.

$a_1 + a_3 = 1$

$b_1 + 2a_3 = 2$

$a_1 + 3a_3 = b_1$ −

Qu Determine the

[3, 5] with b

$s'(1)$

Ans $a_0 \Rightarrow 2$, $a_1 =$

$b_0 = 3$, b_1

Qu Fit a natural

x	0	
y	1	

x_0

$S_0(x) = a_0 + a_1$

$S_1(x) = b_0 + b_1($

$S_2(x) = c_0 + c_1($

$S_0(0) = 1$

$S_0(1) = 2$

$S_1(1) = 2$

$S_1(2) = 33$

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$a_0 = 2$ $\quad a_3 = 1/4$ $\quad a_1 \Rightarrow 3/4$

$a_2 = 0$ $\quad b_2 \Rightarrow 3/4$ $\quad b_1 \Rightarrow 3/2$

$b_0 = 3$ $\quad b_3 \Rightarrow -1/4$

$a_1 + a_3 = 1$ — (1)

$b_1 + 2a_3 = 2$

$a_1 + 3a_3 = b_1$ — (2)

Qu Determine the plank spline passing through the points [1, 2] [2, 3], [3, 5] with boundary condition

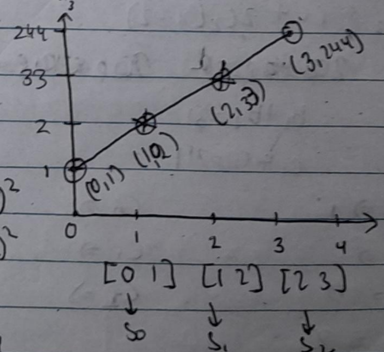
$s'(1) = 2$ and $s'(3) = 1$

Ans $a_0 \Rightarrow 2$, $a_1 = 2$, $a_2 = -5/2$, $a_3 \Rightarrow 3/2$.

$b_0 = 3$, $b_1 = 3/2$, $b_2 = 2$, $b_3 = -3/2$.

Qu fit a natural quadratic spline for the data.

x	0	1	2	3
y	1	2	33	244
	x_0	x_1	x_2	x_3



$S_0(x) = a_0 + a_1x + a_2x^2$

$S_1(x) = b_0 + b_1(x-1) + b_2(x-1)^2$

$S_2(x) = c_0 + c_1(x-2) + c_2(x-2)^2$

$S_0(0) = 1$ $\quad S_2(2) = 33$

$S_0(1) = 2$ $\quad S_2(3) = 244$

$S_1(1) = 2$

$S_1(2) = 33$

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$S_0'(1) = S_1'(1)$
$S_1'(2) = S_2'(2)$
$S''(0) = y''(0) = 0$

$\boxed{a_1 + 2a_2 \Rightarrow b_1}$
$\boxed{b_1 + 2b_2 \Rightarrow c_1}$

$a_0 + a_1 x + a_2 x^2 = 1.$

$\boxed{a_0 = 1}$

$\boxed{a_0 + 2a_1 + 4a_2 \Rightarrow 2}$

$\boxed{b_0 \Rightarrow 2}$

$\boxed{b_0 + b_1 + b_2 \Rightarrow 33}$

$\boxed{c_0 \Rightarrow 33}$

$\boxed{c_0 + c_1 + c_2 \Rightarrow 244}$

$S_0'(x) \Rightarrow a_1 + 2a_2 x.$
$S_1'(x) \Rightarrow b_1 + 2b_2(x-1)$
$S_2' \Rightarrow c_1 + 2c_2(x-2)$

$a_1 + 2a_2 \Rightarrow 1$
$b_1 + b_2 = 31$
$c_1 + c_2 = 211$

$a_0 \Rightarrow 1$
$a_1 \Rightarrow$
$a_2 = 0$
$b_0 \Rightarrow 2$
$b_1 \Rightarrow$
$b_2 \Rightarrow$

$c_0 \Rightarrow$
$c_1 \Rightarrow$
$c_2 \Rightarrow$

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from the following data compute x corresponding to $y = 7$

x	1	3	4
y	4	12	19

$$P_n(y) \Rightarrow \frac{(y-y_1)(y-y_2)}{(y_0-y_1)(y_0-y_2)} x_0 + \frac{(y-y_0)(y-y_2)}{(y_1-y_0)(y_1-y_2)} x_1 + \frac{(y-y_0)(y-y_1)}{(y_2-y_0)(y_2-y_1)} x_2$$

$$\Rightarrow \frac{(y-12)(y-19)}{(4-12)(4-19)}(1) + \frac{(y-4)(y-19)}{(12-4)(12-19)}(3) + \frac{(y-4)(y-12)}{(19-4)(19-12)}(4)$$

$$\Rightarrow \frac{(y-12)(y-19)}{(-8)(-15)} + \frac{(3y-12)(y-19)}{(8)(-7)} + \frac{(4y-16)(y-12)}{(15)(7)}$$

$$\Rightarrow \frac{y^2-12y-19y+228}{120} - \frac{3y^2-57y-12y+228}{56} + \frac{4y^2-48y-16y-192}{105}$$

$$\frac{49-217+228}{120} = \frac{147-399-84+228}{56} + \frac{196-448-192}{105}$$

$$P_n(y) \Rightarrow \frac{13}{7}$$ Ans

Unit-3
Numerical Differentiation & Numerical integration.

* finite difference operators.

(1) ~~formate~~ forward Difference operator (Δ).
Def^n $\Delta f(x) = f(x+h) - f(x)$.

(2) ~~Backard~~ Backward Diff operator (∇).
Def^n $\nabla f(x) = f(x) - f(x-h)$.

(3) Shift operator (E).
Def $E(f(x)) = f(x+h)$.

(4) Average operator (μ)
$\mu(f x) = \frac{1}{2}\left(f(x+h/2) + f(x-h/2)\right)$

(5) central operator (δ).
$\delta(f(x)) = f(x+h/2) - f(x-h/2)$

Dataset
(x_i, y_i), $i = 0$ to n.

x	$y = f(x)$
x_0	$y_0 = f(x_0) = f_0$
x_1	$y_1 = f(x_1) = f_1$
⋮	⋮ ⋮
⋮	⋮ ⋮

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Second order forward difference

$$\Delta^2 f(x) = \Delta(\Delta f(x))$$
$$= \Delta(f(x+h) - f(x))$$
$$= \Delta f(x+h) - \Delta f(x)$$
$$= f(x+2h) - f(x+h) - f(x+h) + f(x)$$
$$= f(x+2h) - 2f(x+h) + f(x)$$

$$\nabla^2 f(x) = \nabla(\nabla f(x)) = \nabla f(x) = \nabla(f(x) - f(x-h))$$
$$= \nabla f(x) - \nabla f(x-h)$$
$$= f(x) - f(x-h) - f(x-h) + f(x-2h)$$
$$= \boxed{f(x) - 2f(x-h) + f(x-2h)}$$

Q calculate $\Delta^2\left(\frac{1}{x}\right), h = 1$.

$\Rightarrow f\left(\frac{1}{x} + 2\right) - 2f\left(\frac{1}{x} + 1\right) + f\left(\frac{1}{x}\right)$.

$\Rightarrow f\left(\frac{1+2x}{x}\right) - 2f\left(\frac{1+x}{x}\right) + f\frac{1}{x}$

$\Rightarrow \frac{1}{x}\left[\cancel{f(1)} + f(2x) - \cancel{2f(1)} - 2f(x) + \cancel{f(1)}\right)$

$\Rightarrow \cancel{\frac{1}{x}} \qquad f\left(\frac{1}{x}\right)(f(2x) - 2f(x))$

Calculate $\Delta^2\left(\frac{1}{x}\right)$, $h = 1$.

$$\Delta f(x) = f(x+h) - f(x)$$
$$\Rightarrow f(x+1) - f(x)$$
$$\Rightarrow \frac{1}{x+1} - \frac{1}{x}$$

$$\Delta^2 (f x) = \Delta\left(\Delta \frac{1}{x}\right)$$
$$= \Delta\left(\frac{1}{x+1} - \frac{1}{x}\right)$$
$$\Rightarrow \Delta\left(\frac{1}{(x+1)}\right) - \Delta\left(\frac{1}{x}\right)$$
$$\Rightarrow \left(\frac{1}{x+2} - \frac{1}{x+1}\right) - \left(\frac{1}{x+1} - \frac{1}{x}\right)$$
$$\Rightarrow \left(\frac{\cancel{x}+1 - \cancel{x}-2}{(x+2)(x+1)}\right) - \left(\frac{\cancel{x} - \cancel{x}-1}{(x+1)(x)}\right)$$
$$\Rightarrow \frac{-1}{(x^2+x+2x+2)} + \frac{1}{x^2+x}$$
$$\Rightarrow \frac{-1}{(x^2+3x+2)} + \frac{1}{x^2+x}$$
$$\frac{-(x^2+x) + (x^2+3x+2)}{(x^2+3x+2)(x^2+x)}$$
$$\frac{2}{x(x+1)(x+2)}$$
$$\Rightarrow \frac{\cancel{-x^2} - x + \cancel{x^2} + 3x + 2}{x^4 + 3x^3 + 2x^2 + x^3 + 3x^2 + 2x} \Rightarrow \frac{2x+2}{x^4+4x^3+5x^2+2x}$$

Relationship

(1) $\Delta f(x) = f$
$= E$
$= E$
$\Delta = E$

$\nabla f(x) =$
$=$

$\boxed{\Delta + \nabla =}$

(2) $\delta f(x) = f(x$
$= E$
$\delta = E$

(3) $\mu (f(x) =$

$\mu = \frac{1}{2}$

Que calculate $\left(\frac{\Delta}{E}\right)(\sin x)$

(CAE

Relationship b/w forward (Δ) backward (∇) and shift (E).

(1) $\Delta f(x) = f(x+h) - f(x)$

$= E(f(x) - f(x)$

$= (E - I)\, f(x)$

$\Delta = E - I$
↳ identity operator.

$\nabla f(x) = f(x) - f(x-h)$

$= f(x) - E^{-1}(f(x))$

$= (I - E^{-1})\, f(x)$

$\nabla = I - E^{-1}$

$$\boxed{\Delta + \nabla = E - E^{-1}}$$

(2) $\delta f(x) = f(x + h/2) - f(x - h/2)$

$= E^{1/2} f(x) - E^{1/2} f(x)$

$\delta = E^{1/2} - E^{-1/2}$

(3) $\mu(f(x) = \frac{1}{2}\left(f(x + h/2) + f(x - h/2)\right)$

$\mu = \frac{1}{2}\left(E^{1/2} + E^{-1/2}\right)$

Que calculate $\left(\frac{\Delta}{E}\right)(\sin x)$ where $h = 1$.

~~$(\Delta E^{-1}) \sin(x)$~~ $\frac{(E - I)}{E}(\sin x) = (I - E^{-1}) \sin x$

$\Rightarrow \sin x - \sin(x-1)$

forward diff table .

x	$y = f(x)$	Δ	Δ^2	Δ^3
x_0	$y_0 = f(x_0)$	$f_1 - f_0 = \Delta f_0$	$\Delta^2 f_0$	$\Delta^3 f_0$
x_1	$y_1 \to f_1$	$f_2 - f_1 = \Delta f_1$	$\Delta^2 f_1$	
x_2	$y_2 \to f_2$	$f_3 - f_2 = \Delta f_2$		
x_3	$y_3 = f_3$			

x	y			
x_0	7	6 Δy_0	2 $\Delta^2 y_0$	6 $\Delta^3 y_0$
x_1	13	8	8	
x_2	21	16		
x_3	37			

construct a forward diff table for the data set.

x	y	Δ	Δ^2	Δ^3	Δ^4	Δ^5
x_0	1	7	12	6	0	0
x_1	8	19	18	6	0	
x_2	27	37	24	6		
x_3	64	61	30			
x_4	125	91				
x_5	216					

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Q. find the missing entry for the following data.

x	1	2	3	4	5
y	7	-	13	21	37

we have.

$$\Delta^4 y_0 = 0$$

$$(E - I)^4 y_0 = 0$$

$$(E^4 - {}^4C_1 E^3 + {}^4C_2 E^2 - {}^4C_3 E^1 + {}^4C_4) y_0 = 0$$

$$\left(E^4 - \frac{4!}{1!3!}E^3 + \frac{4!}{2!2!}E^2 - \frac{4!}{3!}E^1 + 1!\right) y_0 = 0$$

$$(E^4 - 4E^3 + 6E^2 - 4E + 1) y_0 = 0$$

$$y_4 - 4y_3 + 6y_2 - 4y_1 + y_0 = 0$$

$$37 - 4(21) + 6(13) - 4y_1 + 7 = 0$$

$$37 - 84 + 78 - 4y_1 + 7 = 0$$

$$38 \Rightarrow 4y_1$$

$$\boxed{y_1 \Rightarrow 9.5}$$

x	0	1	2	3	4	5
y	0	0	a	24	60	b

x	y	Δ	Δ^2	Δ^3	Δ^4
0	0	0	a	$24-3a$	$6a-36$
1	0	a	$24-2a$	$3a-12$	$b-4a-96$
2	a	$24-a$	$12+a$	$b-a-108$	
3	24	36	$b-96$		
4	60	$b-60$			
5	b				

$$6a - 36 = 0$$

$$6a \Rightarrow 36$$

$$\boxed{a \Rightarrow 6}$$

$$b - 4a - 96 \Rightarrow 0$$

$$\boxed{b = 120}$$

60 36 24 36 - (24 - a)

x	10	11	12	13	14	15
y	a	b	9	29	67	129

x	y	Δ	Δ^2	Δ^3	Δ^4
10	a	$b-a$	$9-2b+a$	$2+3b-a$	$5-4b+a$
11	b	$9-b$	$11+b$	$7-b$	$b-1$
12	9	20	18	6	
13	29	38	24		
14	67	62			
15	129				

$11+b-9+2b-a$

$2+3b-a-7+b$

$5-4b+a=0$

$\boxed{b=1}$

$5-4+a=0$

$\boxed{a \Rightarrow -1}$

prove that $\Delta\nabla = \Delta - \nabla$

$\because (\Delta \Rightarrow E - I)$

$(\nabla \Rightarrow I - E^{-1})$

$\Rightarrow (E-I)(I-E^{-1})$

$\Rightarrow \cancel{EI - EE^{-1} - I^2 + IE^{-1}}$

$\Rightarrow E(I-E^{-1}) - I(I-E^{-1})$

$\Rightarrow E - I - I + E^{-1}$

$\Rightarrow E + E^{-1} - 2I$

$\nabla - \Delta = E - I - I + E^{-1}$

$\Rightarrow E + E^{-1} - 2I$

$(\nabla E)(x^3)$

$\nabla(E(x^3))$

$\nabla(x+1)^3 \Rightarrow (x+1)^3 - x^3$

$\Rightarrow ((x+1)^2 + x^2 + x(x+1))$