3.2.5: Foods- Metabolism of Dietary Sugar

We eat a lot of sugars, and in the next few sections we'll explore some of the body chemistry that explains why they may (or may not) lead to weight gain, why they're a good source of energy, and why astronauts excrete more water than they drink. As we've seen, there are many sugars, but one of the most common is sucrose, $C_{12}H_{22}O_{11}$. A balanced overall chemical equation for the metabolism of sucrose will help us understand the questions above, and many others. A balanced chemical equation such as:

$2\ C_{12}H_{22}O_{11}(s) + 35\ O_2(g) \rightarrow 12\ CO_2(g) + 11\ H_2O(l)$ (1) not only tells how many molecules of each kind are involved in a reaction, it also indicates the *amount* of each substance that is involved. Equation (1) says that 2 $C_{12}H_{22}O_{11}$ *molecules* can react with 35 O_2 *molecules* to give 12 CO_2 *molecules* and 11 H_2O *molecules*. It also says that 2 *mol* $2C_{12}H_{22}O_{11}$ would react with 35 *mol* O_2 yielding 12 *mol* CO_2 and 11 *mol* H_2O.

In other words, the equation indicates that exactly 35 mol O_2 must react *for every* 2 mol $C_{12}H_{22}O_{11}$consumed. For the purpose of calculating how much O_2 is required to react with a certain amount of $C_{12}H_{22}O_{11}$ therefore, the significant information contained in Eq. (1) is the *ratio*

$\frac{35 \text{ mol } O_2}{2 \text{ mol } C_{12}H_{22}O_{11}}$ We shall call such a ratio derived from a balanced chemical equation a **stoichiometric ratio** and give it the symbol *S*. Thus, for Eq. (1), $S\left(\frac{O_2}{C_{12}H_{22}O_{11}}\right) = \frac{35 \text{ mol } O_2}{2 \text{ mol } C_{12}H_{22}O_{11}}$

The word *stoichiometric* comes from the Greek words *stoicheion*, “element,“ and *metron*, “measure.“ Hence the stoichiometric ratio measures one element (or compound) against another.

EXAMPLE 1 Derive all possible stoichiometric ratios from Eq. (1)

Solution

Any ratio of amounts of substance given by coefficients in the equation may be used, so in addition to (2) above, we have:

$$S\left(\frac{C_{12}H_{22}O_{11}}{O_2}\right) = \frac{2 \text{ mol } C_{12}H_{22}O_{11}}{35 \text{ mol } O_2}$$

$$S\left(\frac{C_{12}H_{22}O_{11}}{CO_2}\right) = \frac{2 \text{ mol } C_{12}H_{22}O_{11}}{12 \text{ mol } CO_2} = \frac{1 \text{ mol } C_{12}H_{22}O_{11}}{6 \text{ mol } CO_2}$$

$$S\left(\frac{CO_2}{C_{12}H_{22}O_{11}}\right) = \frac{12 \text{ mol } CO_2}{2 \text{ mol } C_{12}H_{22}O_{11}}$$

$$S\left(\frac{CO_2}{H_2O}\right) = \frac{12 \text{ mol } CO_2}{11 \text{ mol } H_2O}$$

$$S\left(\frac{CO_2}{O_2}\right) = \frac{12 \text{ mol } CO_2}{35 \text{ mol } O_2} \quad S\left(\frac{O_2}{H_2O}\right) = \frac{35 \text{ mol } O_2}{11 \text{ mol } H_2O}$$

$$S\left(\frac{C_{12}H_{22}O_{11}}{H_2O}\right) = \frac{2 \text{ mol } C_{12}H_{22}O_{11}}{11 \text{ mol } H_2O}$$

There are six more stoichiometric ratios, each of which is the reciprocal of one of these. [Eq. (2) gives one of them.]

When any chemical reaction occurs, the amounts of substances consumed or produced are related by the appropriate stoichiometric ratios. Using Eq. (1) as an example, this means that the ratio of the amount of O_2 consumed to the amount of NH_3 consumed must be the stoichiometric ratio $S(O_2/NH_3)$: $\frac{n_{C_{12}H_{22}O_{11} \text{ consumed}}}{n_{O_2 \text{ consumed}}} = S\left(\frac{C_{12}H_{22}O_{11}}{O_2}\right) = \frac{2 \text{ mol } C_{12}H_{22}O_{11}}{35 \text{ mol } O_2}$ Similarly, the ratio of the amount of H_2O produced to the amount of NH_3 consumed must be

$S(H_2O/C_{12}H_{22}O_{11})$:

$\frac{n_{H_2O \text{ produced}}}{n_{C_{12}H_{22}O_{11} \text{ consumed}}} = S\left(\frac{H_2O}{C_{12}H_{22}O_{11}}\right) = \frac{11 \text{ mol } H_2O}{2 \text{ mol } C_{12}H_{22}O_{11}}$ In general we can say that $\text{Stoichiometric ratio } \left(\frac{X}{Y}\right) = \frac{\text{amount of X consumed or produced}}{\text{amount of Y consumed or produced}}$ (3a) or, in symbols, $S\left(\frac{X}{Y}\right) = \frac{n_{X \text{ consumed or produced}}}{n_{Y \text{ consumed or produced}}}$ (3b)

Note that in the word Eq. (3a) and the symbolic Eq. (3b), X and Y may represent *any* reactant or *any* product in the balanced chemical equation from which the stoichiometric ratio was derived. No matter how much of each reactant we have, the amounts of reactants *consumed* and the amounts of products *produced* will be in appropriate stoichiometric ratios.

EXAMPLE 2 Find the amount of water produced when 1 cup (roughly 8 oz or 240 g) $C_{12}H_{22}O_{11}$ is consumed according to Eq. (1).

Solution

The amount of water produced must be in the stoichiometric ratio S($H_2O/C_{12}H_{22}O_{11}$) to the amount of sugar consumed, and the amount is n = m/M = 240 g /342.3 g mol^{-1} = .70 mol.

$\frac{n_{H_2O\text{ produced}}}{n_{C_{12}H_{22}O_{11}\text{ consumed}}} = S\left(\frac{H_2O}{C_{12}H_{22}O_{11}}\right) = \frac{11\text{ mol }H_2O}{2\text{ mol }C_{12}H_{22}O_{11}}$ Multiplying both sides by $n_{C_{12}H_{22}O_{11}}$ consumed, we have $n_{H_2O\text{ produced}} = n_{C_{12}H_{22}O_{11}\text{ consumed}} \times S\left(\frac{H_2O}{C_{12}H_{22}O_{11}}\right) = 0.70\text{ mol}C_{12}H_{22}O_{11} \times \frac{11\text{ mol }H_2O}{2\text{ mol }C_{12}H_{22}O_{11}} = 3.85\text{ mol }H_2O$

This calculation shows why an astronaut dirnks about 2 L of H_2 per day, but excretes about 2.4 L of H_2 per day! Think about it, and check your answer[1]

This is a typical illustration of the use of a stoichiometric ratio as a conversion factor. Example 2 is analogous to Examples 1 and 2 from Conversion Factors and Functions, where density was employed as a conversion factor between mass and volume. Example 2 is also analogous to Examples 2.4 and 2.6, in which the Avogadro constant and molar mass were used as conversion factors. As in these previous cases, there is no need to memorize or do algebraic manipulations with Eq. (3) when using the stoichiometric ratio. Simply remember that the coefficients in a balanced chemical equation give stoichiometric ratios, and that the proper choice results in cancellation of units. In road-map form

$\text{amount of X consumed or produced} \overset{\substack{\text{stoichiometric}\\ \text{ratio X/Y}}}{\longleftrightarrow} \text{amount of Y consumed or produced}$ or symbolically. &nbnbsp; $n_{\text{X consumed or produced}} \overset{S(X/Y)}{\longleftrightarrow} n_{\text{Y consumed or produced}}$

When using stoichiometric ratios, be sure you *always* indicate moles *of what*. You can only cancel moles of the same substance. In other words, 1 mol $C_{12}H_{22}O_{11}$ cancels 1 mol $C_{12}H_{22}O_{11}$ but does not cancel 1 mol H_2O.

The next example shows that stoichiometric ratios are also useful in problems involving the mass of a reactant or product.

EXAMPLE 3 It is estimated that each human exhales about 1 kg (2.2 lb)[2][3] of carbon dioxide per day. If that came entirely from glucose, what mass of glucose must be metabolized according to the equation:

$C_6H_{12}O_6 + 6\ O_2 \rightarrow 6\ CO_2 + 6\ H_2O$

Solution

The problem asks that we calculate the mass of $C_6H_{12}O_6$ consumed. As we learned in Example 2 of The Molar Mass, the molar mass can be used to convert from the amount of CO_2 to the mass of CO_2. Then we can calculate the amount of $C_6H_{12}O_6$ consumed from the amount of CO_2 produced with a stoichiometric ratio, just as in Example 2. Finally, we can convert the amount of glucose to its mass using the molar mass as a conversion factor. It requires the stoichiometric ratio.

$S\left(\frac{C_6H_{12}O_6}{CO_2}\right) = \frac{1\text{ mol }C_6H_{12}O_6}{6\text{ mol }CO_2}$

The *amount* of CO_2 produced is n = m/M = 1000 g/44 g mol^{-1} = 22.7 mol

The *amount* of $C_6H_{12}O_6$ consumed is then

$n_{C_6H_{12}O_6} = n_{CO_2\text{ produced}} \times \text{conversion factor} = \text{n mol }CO_2 \times \frac{1\text{ mol }C_6H_{12}O_6}{6\text{ mol }CO_2}$

$= 22.7\text{ mol }CO_2 \times \frac{1\text{ mol }C_6H_{12}O_6}{6\text{ mol }CO_2} = 3.79\text{ mol}$

The *mass* of $C_6H_{12}O_6$ is

$m_{C_6H_{12}O_6} = 3.79\text{ mol }C_6H_{12}O_6 \times \frac{180\text{ g }C_6H_{12}O_6}{1\text{ mol }C_6H_{12}O_6} = 682\text{ g }C_6H_{12}O_6$

By similar calculations, we could show that to produce 6 mol of carbon dioxide, it takes the same *amount* of oxygen (22.7 mol) and the same amount (22.7 mol)of water will be produced. That's 22.7 mol x 32 g/mol = 726 g of oxygen consumed and 22.7 mol x 18 g/mol = 409 g of water produced. The reactants weigh 682 + 726 = 1408 g and the products weigh the same (within error), 1000 + 409 = 1409 g.

We notice two things: First, both products are eliminated (CO_2 in breath, water in urine, so how do we gain weight? Only sugar that isn't metabolized goes into weight gain, and we'll see soon how to calculate that.

Second, this accounts for .409 kg of the total 2.4 L of water excreted per day. It's a rough estimate of the part that results from metabolism (not from ingested water).

With practice this kind of problem can be solved in one step by concentrating on the units. The appropriate stoichiometric ratio will convert moles of CO_2 to amount (in moles) of $C_6H_{12}O_6$ and the molar mass will convert amount of $C_6H_{12}O_6$ to mass (in grams) of SO_2. A schematic road map for the one-step calculation can be written as

$n_{CO_2} \xrightarrow{S(C_6H_{12}O_6/O_2)} n_{C_6H_{12}O_6} \xrightarrow{M_{C_6H_{12}O_6}} m_{C_6H_{12}O_6}$ Thus $m_{C_6H_{12}O_6} = \text{xxx mol } CO_2 \times \frac{1 \text{ mol } C_6H_{12}O_6}{11 \text{ mol } O_2} \times \frac{64.06 \text{ g}}{1 \text{ mol } C_6H_{12}O_6} = 179 \text{ g}$ The chemical reaction in this example is of environmental interest. If each person on Earth exhales 1000 g of CO_2/day, and there are 6.7 billion people, this accounts for 6.7 x 10^{12} (6.7 quadrillion) g or 6.2 Tg (teragrams) of the greenhouse gas CO_2. How does this compare with the amount produced by burning fuels in its effect on the atmosphere? In 2000, fossil fuel burning released 7 x 10^9 tons of carbon dioxide [4]. That's 7 x 10^9 tons x 907 kg/ton x 1000 g/kg or 6.3 x 10^{15}. Our breath contributes about 0.1%. The mass of atmospheric carbon dioxide comes from calculations like the following:

EXAMPLE 4 What mass of oxygen and carbon dioxide would be consumed when 3.3 × 10^{15} g, 3.3 Pg (petagrams), of octane (C_8H_{18}) is burned to produce CO_2 and H_2O?

Solution

First, write a balanced equation

$2C_8H_{18} + 25O_2 \rightarrow 16CO_2 + 18H_2O$ The problem gives the mass of C_8H_{18} burned and asks for the mass of O_2 required to combine with it. Thinking the problem through before trying to solve it, we realize that the molar mass of octane could be used to calculate the amount of octane consumed. Then we need a stoichiometric ratio to get the amount of O_2 consumed. Finally, the molar mass of O_2 permits calculation of the mass of O_2. Symbolically

$$m_{C_8H_{18}} \xrightarrow{M_{C_8H_{18}}} n_{C_8H_{18}} \xrightarrow{S(SO_2/C_8H_{18})} n_{O_2} \xrightarrow{M_{O_2}} m_{O_2}$$

$m_{O_2} = 3.3 \times 10^{15} \text{ g} \times \frac{1 \text{ mol } C_8H_{18}}{114 \text{ g}} \times \frac{25 \text{ mol } O_2}{2 \text{ mol } C_8H_{18}} \times \frac{32.00 \text{ g}}{1 \text{ mol } O_2} = 1.2 \times 10^{16} \text{ g}$ Thus 12 Pg (petagrams) of O_2 would be needed. By a similar calculation, we see that the amount of carbon dioxide would be 16/2 = 8 times the amount of octane consumed, or 2.3 x 10^{14} mol, which, multiplied by the molar mass 44 g/mol, is 1 x 10^{16} g or 10 Pg. The large mass of oxygen obtained in this example is an estimate of how much O_2 is removed from the earth's atmosphere each year by human activities. Octane, a component of gasoline, was chosen to represent coal, gas, and other fossil fuels. Fortunately, the total mass of oxygen in the air (1.2 × 10^{21} g) is much larger than the yearly consumption. If we were to go on burning fuel at the present rate, it would take about 100 000 years to use up all the O_2. Actually we will consume the fossil fuels long before that! One of the least of our environmental worries is running out of atmospheric oxygen.

References

1. Some of the extra 0.4 L is present as moisture in food, but a large amount is produced by metabolism, where it is a product of a chemical reaction like the one in Example 2
2. Harte, J. "Consider a Spherical Cow, A Course in Environmental Problme Solving", University Science Books, Sausalito, CA 1998, p. 263
3. Campbell, J.A. *J. Chem. Educ.* **49**, 181 (1972)
4. www.worldwatch.org/node/1811