# 14.3: pH and pOH

The calculations we have just done show that the concentrations of hydronium and hydroxide ions in aqueous solution can vary from about 1 mol $L^{-1}$down to about 1 × $10^{-14}$ mol $L^{-1}$, and perhaps over an even wider range. The numbers used to express [$H_3O^+$] and [$OH^-$] in the units mole per liter will often include large negative powers of 10. Consequently it is convenient to define the following:

$$\mathrm{pH} = -\log \frac{[\,\mathrm{H_3O^+}\,]}{1 \text{ mol L}^{-1}}$$

$$\mathrm{pOH} = -\log \frac{[\,\mathrm{OH^-}\,]}{1 \text{ mol L}^{-1}}$$

Note carefully what these equations tell us to do. To obtain pH, for example, we divide [$H_3O^+$] by the units mole per liter. This gives a pure number, and so we can take its logarithm. (It does not make sense to take the logarithm of a unit, such as mole per cubic decimeter.) The minus sign insures that we will obtain a positive result most of the time.

The logarithm of a number is the power to which 10 must be raised to give the number itself. Therefore the definitions of pH and pOH mean that we can deal with powers of 10 rather than numerical values. Since the numbers needed to express [$H_3O^+$] and [$OH^-$] are usually between 1 and $10^{-14}$ pH and pOH values are usually between 0 and 14.

### Example 14.3.1: pH & pOH

Calculate the pH and the pOH of each of the following aqueous solutions: (a) 1.00 *M* $HNO_3$; 0.306 *M* $Ba(OH)_2$ .

**Solution**

**a)** Our previous discussion showed that for this solution [$H_3O^+$] = 1.00 mol/L and [$OH^-$] = 1.00 x $10^{-14}$. Applying the definitions of pH and pOH, we have

$$\mathrm{pH} = -\log \frac{1.00 \text{ mol L}^{-1}}{1 \text{ mol L}^{-1}} = -\log(10^{0}) \tag{14.3.1}$$

$$= -(0) = 0.00 \tag{14.3.2}$$

$$\tag{14.3.3}$$

$$\mathrm{pOH} = -\log \frac{1.00 \times 10^{-14} \text{ mol L}^{-1}}{1 \text{ mol L}^{-1}} = -\log(10^{-14}) \tag{14.3.4}$$

$$= -(-14) = 14.00 \tag{14.3.5}$$

**b)**In the example in the section on ionization of water, we found for this solution [$H_3O^+$] = 1.63 × $10^{-14}$ mol/L and [$OH^-$] = 6.12 × $10^{-1}$ mol/L. Thus

$$\mathrm{pH} = -\log 1.63 \times 10^{-14} = -(-13.788) = 13.788 \tag{14.3.6}$$

$$\tag{14.3.7}$$

$$\mathrm{pOH} = -\log 6.12 \times 10^{-1} = -(-0.213) = 0.213 \tag{14.3.8}$$

In the laboratory it is convenient to measure the pH of a solution using a pH meter. Such a device works on a different principle from the conductivity measurements we have already mentioned, and an accurate explanation of how it works is beyond the scope of the present discussion. While greater accuracy can be obtained when great care and special instruments are used, pH is usually measured to an accuracy of ± 0.01. Therefore pH values are usually rounded to the second decimal place; the results of Example 1*b* would commonly be rounded to pH = 13.79 and pOH = 0.21.


Figure 14.3.1 A pH meter. This instrument is used for measuring the pH of solutions in a laboratory. The electrode used for measuring, readout, and input are marked. (public domain; Datamax)

Because pH measurements are so easily made, it is essential that you be able to convert from pH to [$H_3O^+$]. This is the reverse of finding pH from [$H_3O^+$]. Consequently it involves antilogs instead of logs. From the definition

$$\mathrm{pH} = -\log \frac{[\mathrm{H_3O^+}]}{\mathrm{mol\ L^{-1}}}$$

we have

$$\mathrm{pH} = \log \frac{[\mathrm{H_3O^+}]}{\mathrm{mol\ L^{-1}}}$$

Taking the antilog of both sides, we have

$$\text{antilog}\,(-\mathrm{pH}) = \text{antilog} - \mathrm{pH} = \left\{ \log \frac{[\mathrm{H_3O^+}]}{\mathrm{mol\ L^{-1}}} \right\}$$

so that

$$\text{antilog}\,(-\mathrm{pH}) = \frac{[\mathrm{H_3O^+}]}{\mathrm{mol\ L^{-1}}}$$

remembering that antilog $x = 10^x$, we can write this expression as

$$10^{-\mathrm{pH}} = \frac{[\mathrm{H_3O^+}]}{\mathrm{mol\ L^{-1}}}$$

or

$$[\mathrm{H_3O^+}] = 10^{\text{-pH}}\ \mathrm{mol\ L^{-1}} \tag{14.3.9}$$

An alternative method of writing this equation is

$$[\mathrm{H_3O^+}] = \frac{1}{10^{\mathrm{pH}}}\ \mathrm{mol\ L^{-1}}$$

**Example 14.3.2: Hydronium Concentration**

The pH of a solution is found to be 3.40. Find the hydronium-ion concentration of the solution.

**Solution**

If you have a calculator which has an antilog or $10^x$ button, the problem is very simple. You enter – 3.40 and hit the button. The number thus obtained, 3.9822 x $10^{-4}$ is the number of moles of hydronium ion per liter. This follows from Eq. 14.3.9:

$$\left[H_3O^+\right] = 10^{-pH}\ \text{mol L}^{-1} = 10^{-3.4}\ \text{mol L}^{-1} = 3.98 \times 10^{-4}\ \text{mol L}^{-1}$$

The same result is almost as easy to find using Eq. (1*b*).

$$10^{-pH} = \text{antilog}\,(\text{pH}) = \text{antilog}\ 3.40 = \text{antilog}\ 3 \times \text{antilog}\ 0.40 = 10^3 \times 2.51$$

Thus

$$10^{-pH} = \frac{1}{10^{pH}} = \frac{1}{2.51 \times 10^3} = 3.98\ \times\ 10^{-4}$$

in other words,

$\left[H_3O^+\right] = 3.98 \times 10^{-4}\ \text{mol L}^{-1}$

There is a very simple relationship between the pH and the pOH of an aqueous solution at 25°C. We know that at this temperature

$$K_w = K_c\left(55.5\ \text{mol L}^{-1}\right)^2\ \left[H_3O^+\right]\ \left[OH^-\right] = K_w =\ 10^{-14}\text{mol}^2\text{L}^{-2}$$

Dividing both sides by $\text{mol}^2\ \text{L}^{-2}$, we obtain

$$\frac{[\ H_3O^+\ ]}{\text{mol L}^{-1}} \times \frac{[OH^-]}{\text{mol L}^{-1}} = 10^{-14}$$

Taking logs and multiplying both by – 1, we then have

$$\log\frac{[\ H_3O^+\ ]}{\text{mol L}^{-1}} - \log\ \frac{[OH^-]}{\text{mol L}^{-1}} = -\log(10^{-14})$$

or

$$\text{pH} + \text{pOH} = 14.00$$

This simple relationship is often useful in finding the pH of solutions containing bases, as the following example shows.

## ✓ Example 14.3.3: pH of a Solution

If 3.53 g of pure NaOH is dissolved in 10 L of $H_2O$ find the pH of the resulting solution.

**Solution**

We first calculate the concentration of the NaOH.

$n_{\text{NaOH}} = 3.53\ \times\ \frac{1\ \text{mol}^{-1}}{40.0\ \text{g}} = 0.088\ 25\ \text{mol}$

so that

$$c_{\text{NaOH}} = \frac{n_{\text{NaOH}}}{V} = \frac{0.088\ 25\ \text{mol}}{10\ \text{L}^1} = 8.82\ \times\ 10^{-3}\ \text{mol L}^{-1}$$

Since NaOH is a strong base, each mole of NaOH dissolved produces 1 mol $OH^-$ ions, so that

$\left[OH^-\right] = 8.82\ \times 10^{-3}\text{mol L}^{-1}$

Thus

$\text{pOH} = -\log\ \left(8.82 \times 10^{-3}\right) = -\,(0.95 - 3.00) = +2.05$

From which

$\text{pH} = 14.00\ \text{-}\ \text{pOH} = 11.95$

While the ability to calculate the pH of a solution from the hydronium-ion concentration and vice versa is useful, it is not the only thing we need to understand about pH. If someone gives you a solution whose pH is 14.74, it is true that the hydronium-ion concentration must be $1.82 \times 10^{-15}$ mol $L^{-1}$ but it is perhaps more important to know that the solution is corrosively basic. In general, then, we need not only to be able to calculate a pH but also to have some realization of what kind of solutions have what kind of pH, as displayed in the following table. This table is part of our collection of acid and base constants.

Table 14.3.2: The pH Scale

| Substance | pH | $[H_3O^+]$ | $[OH^-]$ | pOH | Strength |
|---|---|---|---|---|---|
| Battery acid | 0 | 1 | $10^{-14}$ | 14 | Strongly acidic |
| Stomach acid | 1 | $10^{-1}$ | $10^{-13}$ | 13 | |
| | 2 | $10^{-2}$ | $10^{-12}$ | 12 | |
| Lemon juice | 3 | $10^{-3}$ | $10^{-11}$ | 11 | Weakly acidic |
| Soda water | 4 | $10^{-4}$ | $10^{-10}$ | 10 | |
| Black coffee | 5 | $10^{-5}$ | $10^{-9}$ | 9 | Barely acidic |
| | 6 | $10^{-6}$ | $10^{-8}$ | 8 | |
| Pure water | 7 | $10^{-7}$ | $10^{-7}$ | 7 | Neutral |
| Seawater | 8 | $10^{-8}$ | $10^{-6}$ | 6 | Barely basic |
| Baking soda | 9 | $10^{-9}$ | $10^{-5}$ | 5 | |
| Toilette soap | 10 | $10^{-10}$ | $10^{-4}$ | 4 | Mildly basic |
| Laundry water | 11 | $10^{-11}$ | $10^{-3}$ | 3 | |
| Household ammonia | 12 | $10^{-12}$ | $10^{-2}$ | 2 | Very basic |
| | 13 | $10^{-13}$ | $10^{-1}$ | 1 | |
| Drain cleaner | 14 | $10^{-14}$ | 1 | 0 | |

In pure water at 25°C the hydronium-ion concentration is close to $1.00 \times 10^{-7}$ mol/L, so that the pH is 7. In consequence any solution, not only pure water, which has a *pH of 7* is described as being *neutral*. An *acidic* solution, as we know, is one in which the hydronium-ion concentration is greater than that of pure water, i.e., *greater than* $10^{-7}$ mol/L. In pH terms this translates into a pH which is *less than 7* (because the pH is a negative logarithm). Small pH values are thus characteristic of acidic solutions; the smaller the pH, the more acidic the solution.

By contrast, a *basic* solution is one in which the hydroxide-ion concentration is greater than $10^{-7}$ mol/L. In such a solution the hydronium-ion concentration is *less than* $10^{-7}$ mol/L, so that the pH of a basic solution is *greater than 7*. Large pH values are thus characteristic of basic solutions. The larger the pH, the more basic the solution.