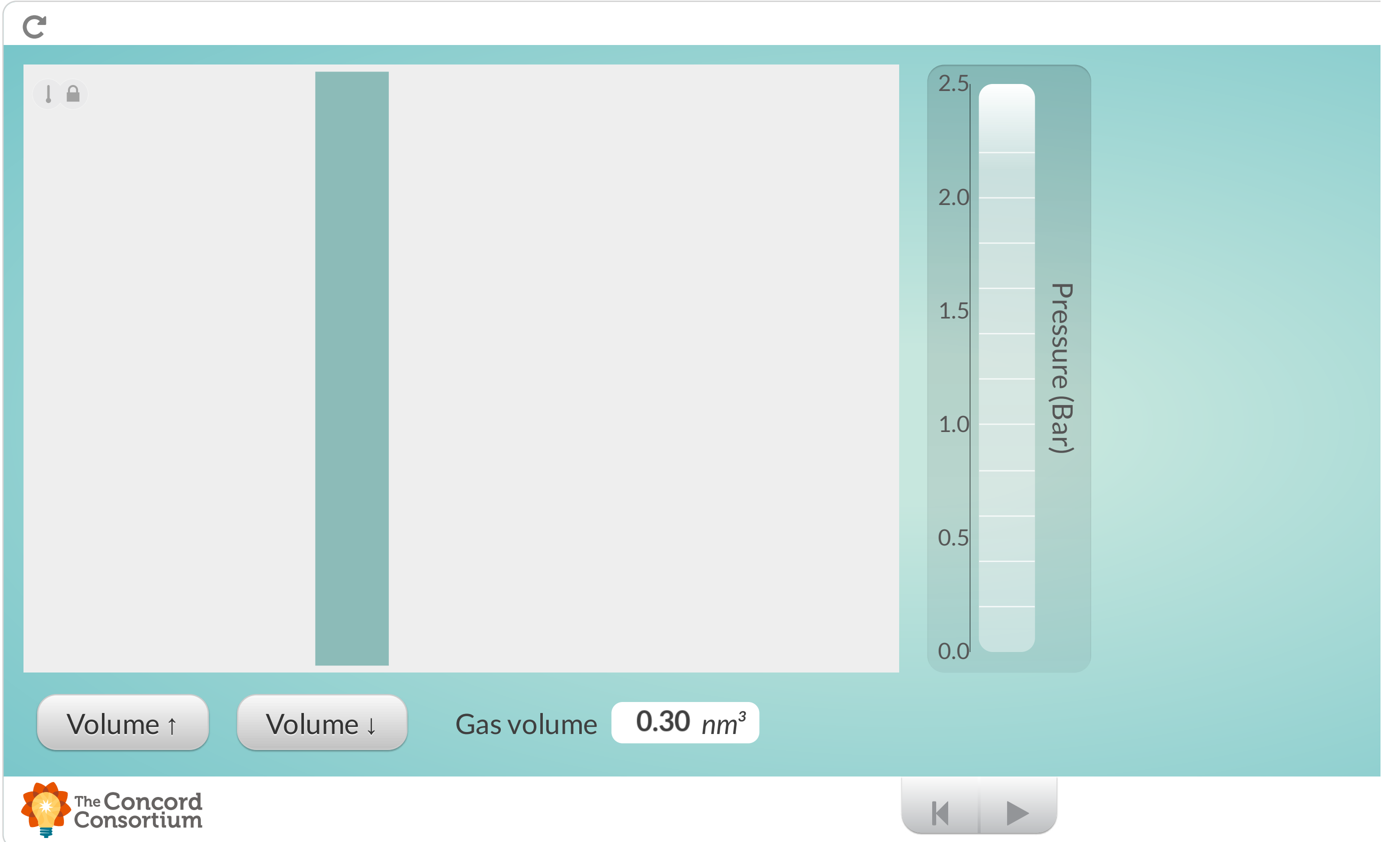
9.7: Boyle's Law

You are probably already familiar with the fact that when you squeeze a gas, it will take up less space. In formal terms, *increasing* the pressure on a gas will *decrease* its volume. English scientist Robert Boyle studied this phenomenon and eventually came up with Boyle's Law as a result. Below is an animation from the Concord Consortium that allows you to do your own experimentation to determine the relationship between volume and pressure. Try increasing and decreasing the volume and observe how this affects the pressure. What does doubling the volume do? Halving it?



While the simulation above is just that, a simulation, it should give you a feel for the type of results that Boyle obtained from his own experiments. Below are results similar to those Boyle himself would have gotten in Table 9.7.1. Careful, study of such data reveals that if we double the pressure, we halve the volume (as you discovered in the simulation):

Table 9.7.1 Variation in the Volume of 0.0446 mol H2(g) with Pressure at 0°C.

Trial	Pressure/kpa	Pressure/atm	Volume/liter
1	152.0	1.50	0.666
2	126.7	1.25	0.800
3	101.3	1.00	1.00
4	76.0	0.750	1.333
5	50.7	0.500	2.00
6	25.3	0.250	4.00
7	10.1	0.100	10.00

If we triple the pressure, the volume is reduced to one-third; and so on. In general, if we *multiply* the pressure by some factor *x*, then we *divide* the volume by the same factor *x*. Such a relationship, in which the increase in one quantity produces a proportional decrease in another, is called **inverse proportionality**.

The results of Boyle's experiments with gases are summarized in **Boyle's law**—*for a given amount of gas at constant temperature, the volume is proportional to the pressure*. In mathematical terms:

$$V \propto \frac{1}{P} \tag{9.7.1}$$

The reciprocal of P indicates the inverse nature of the proportionality. Using the proportionality constant k_A to convert relationship to an equation, we have

$$V = k_{\mathrm{B}} \times \frac{1}{P} = \frac{k_{\mathrm{B}}}{P} \tag{9.7.2}$$

Multiplying both sides of Eq.9.7.2 by P, we have

$$PV = k_B$$

where k_A represents a constant value for any given temperature and amount (or mass) of gas.

For a more tangible demonstration of Boyle's and an idea of what it looks like in real life, check out the following link: How Marshmallows, Balloons, and Shaving Cream Demonstrate Boyle's Law

Example 9.7.1: Boyle's Law

Using the data in red in Table 9.7.1, confirm that Boyle's law is obeyed.

Solution

Since the data apply to the same amount of gas at the same temperature, *PV* should be constant [Eq. (2*b*)] if Boyle's law holds.

$P_1V_1 = 1.50 \text{ atm} \times 0.666 \text{ liter} = 0.999 \text{ atm liter}$

$P_4V_4 = 0.750 \text{ atm} \times 1.333 \text{ liter} = 1.000 \text{ atm liter}$

$P_6V_6 = 0.250 \text{ atm} \times 4.00 \text{ liter} = 1.00 \text{ atm liter}$

The first product differs from the last two in the fourth significant digit. Since some data are reported only to three significant figures, *PV* is constant within the limits of the measurements.

If the units atmosphere liter, in which *PV* was expressed in Example 9.7.1, are changed to SI base units, an interesting result arises:

$$1 \text{ atm } \times 1 \text{ liter} = 101.3 \text{ kPa } \times 1 \text{ dm}^3 = 101.3 \times 10^3 \text{ Pa } \times 1 \text{ dm}^3 \times \left(\tfrac{1 \text{ m}}{10 \text{ dm}}\right)^3 = 101.3 \times 10^3 \tfrac{\text{N}}{\text{m}^2} \times 1 \text{ dm}^3$$
$$\times \tfrac{1 \text{ m}^3}{10^3 \text{ dm}^3} = 101.3 \text{ N m} = 101.3 \text{ kg m s}^{-2} \text{ m} = 101.3 \text{ J}$$

In other words, *PV has the same units (joules) as an energy*. While this does not guarantee that *PV is* an energy (be careful about relying on cancellation of units unless you *know* that a relationship between quantities exists), it does suggest that we should explore the possibility, covered in the section on Kinetic Theory of Gases The above argument also shows that the product of the units kilopascals times cubic decimeters is the unit joules. Boyle's law enables us to calculate the pressure or volume of a gas under one set of conditions, provided we know the pressure and volume under a previous set of circumstances.

Example 9.7.2 : Volume of a Gas

The volume of a gas is 0.657 liter under a pressure of 729.8 mmHg. What volume would the gas occupy at atmospheric pressure (760 mmHg)? Assume constant temperature and amount of gas.

Solution Two methods of solution will be given.

a) Since *PV* must be constant,

$P_1V_1 = k_B = P_2V_2$

Initial conditions: P_1 = 729.8 mmHgV_1 = 0657 liter

Final conditions: P_2 = 760 mmHgV_2 = ?

Solving for V_2, we have $V_2 = \dfrac{P_1V_1}{P_2} = \dfrac{729.8 \text{ mmHg } \times 0.657 \text{ liter}}{760 \text{ mmHg}} = 0.631 \text{ liter}$ **b)** Note that in method *a* the original volume was multiplied by a ratio of pressures (P_1/P_2): V_2 = 0.657 liter × ratio of pressures Rather than solving algebraically, we can use common sense to decide which of the two possible ratios $\dfrac{729.8 \text{ mmHg}}{760 \text{ mmHg}}$ or $\dfrac{760 \text{ mmHg}}{729.8 \text{ mmHg}}$ should be used. The units cancel in either case, and so units are no help. However, if you reread the problem, you will see that we are asked to find the new volume (V_2) produced by an *increase* in pressure. Therefore there must be a *decrease* in volume, and we multiply the original volume by a ratio which is *less than* 1: $V_2 = 0.657 \text{ liter } \times \dfrac{729.8 \text{ mmHg}}{760 \text{ mmHg}} = 0.631 \text{ liter}$ It is reassuring that both common sense and algebra produce the same answer.