9.14: Kinetic Theory of Gases- The Total Molecular Kinetic Energy

Equation (1) from Postulates of the Kinetic Theory can tell us a lot more than this about gases, however. If both sides are multiplied by *V*, we have:

$$PV = \tfrac{1}{3}Nm(u^2)_{\text{ave}} \tag{9.14.1}$$

The kinetic energy of an individual molecule is ½ m $(u^2)_{\text{ave}}$, and so the average kinetic energy $(E_k)_{\text{ave}}$ of a collection of molecules, all of the same mass *m* is:

$$(E_k)_{\text{ave}} = (\frac{1}{2}mu^2)_{\text{ave}} = \frac{1}{2}m(u^2)_{\text{ave}}$$

The total kinetic energy E_k is just the number of molecules times this average:

$$E_k = N \times (E_k)_{\text{ave}} = N \times \frac{1}{2}m(u^2)_{\text{ave}}$$

or, multiplying both sides by 3/3 (i.e., by 1)

$$E_k = \tfrac{3}{3} \times \tfrac{1}{2}Nm(u^2)_{\text{ave}} = \tfrac{3}{2} \times \tfrac{1}{3}Nm(u^2)_{\text{ave}}$$

Substituting from Eq.

$$E_k = \tfrac{3}{2}PV$$

or

$$PV = \tfrac{2}{3}E_k$$

The product of the pressure and the volume of a gas is two-thirds the total kinetic energy of the molecules of the gas. Now we can understand why *PV* comes out in joules—it is indeed energy. According to postulate 4 of the kinetic theory, gas molecules have constant total kinetic energy. This is reflected on the macroscopic scale by the constancy of *PV*, or, in other words, by Boyle's law. The kinetic theory also gives an important insight into what the *temperature* of gas means on a microscopic level. We know from the ideal gas law that $PV = nRT$. Substituting this into Eq. 9.14.2,

$$nRT = \tfrac{2}{3}E_k \tag{9.14.2}$$

If we divide both sides of Eq. 9.14.3 by *n* and multiply by $\tfrac{3}{2}$,

$$\frac{E_k}{n} = \tfrac{3}{2}RT \tag{9.14.3}$$

The term E_k/n is the total kinetic energy divided by the amount of substance, that is, the **molar kinetic energy**. Representing molar kinetic energy E_m by we have:

$$E_{\text{m}} = \tfrac{3}{2}RT$$

The molar kinetic energy of a gas is proportional to its temperature, and the proportionality constant is $\tfrac{3}{2}$ times the gas constant *R*.

The video below demonstrates the relationship between molar kinetic energy and temperature. The demonstration highlights the fact that a higher temperature means a higher molar kinetic energy.

When food coloring is placed into water of different temperatures, it behaves differently. Food coloring in hot water is rapidly dispersed because of its high molar kinetic energy/temperature. The cold water on the other hand, has a low molar kinetic energy and therefore the food color spreads slowly through it.