2.11.1: Biology- Water

It should now be clear that knowing the number of water (or other biological) molecules is central to many issues in biology, and it's an issue that chemistry can shed light on.

For example, photosynthesis involves the reaction of carbon dioxide and water to make sugars. Richard Feynman noted chemistry's contribution in a very poetic way:

“The world looks so different after learning science. For example, trees are made of air, primarily. When they are burned, they go back to air, and in the flaming, heat is released, the flaming heat of the sun which was bound in to convert the air into tree. And in the ash is the small remnant of the part which did not come from air, that came from the solid earth, instead.

Sugar (and other "carbohydrates" in plants) is made photosynthetically according to overall equations like

$$6CO_2 + 6H_2O + Light Energy \longrightarrow C_6H_{12}O_6 + 6O_2$$

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http://employees.csbsju.edu/hjakubowski/Jmol/Beta-D-Glucopyranose/betaglucose.pdb

This explains why plants die without water. But how much water is required to make a pound (457 grams) of sugar? The equation tells us 6 water molecules make 6 glucose (sugar) molecules. How do we get useful, macroscopic answers, like how much water (and CO_2) is needed for a pound of sugar? We start with molar masses.

It is often convenient to express physical quantities per *unit amount of substance* (per mole), because in this way equal numbers of atoms or molecules are being compared. Such **molar quantities** often tell us something about the atoms or molecules themselves. For example, if the molar volume of one solid is larger than that of another, it is reasonable to assume that the molecules of the first substance are larger than those of the second. (Comparing the molar volumes of liquids, and especially gases, would not necessarily give the same information since the molecules would not be as tightly packed.)

A molar quantity is one which has been divided by the amount of substance. For example, an extremely useful molar quantity is the molar mass *M*:

$$Molar\ mass = \frac{\text{mass}}{\text{amount of substance}}$$

It is almost trivial to obtain the molar mass, since atomic and molecular weights expressed in grams give us the masses of 1 mol of substance.

EXAMPLE 1 Obtain the molar mass of (a) C (carbon) and (b) H_2O.

Solution

a) The atomic weight of carbon is 12.01, and so 1 mol C weighs 12.01 g.

$$M_{\mathrm{C}}=\frac{\mathrm{m}_{\mathrm{C}}}{n_{\mathrm{C}}}=\frac{12.01\ \mathrm{g}}{1\ \mathrm{mol}}=12.01\ \mathrm{g\ mol}^{-1}$$

b) Similarly, for H_2O, the molecular weight is 18.02 (16.00 for O and 2.02 for 2 H), and so

$$M_{\mathrm{H_2O}}=\frac{\mathrm{m}_{\mathrm{H_2O}}}{n_{\mathrm{H_2O}}}=18.02\ \mathrm{g\ mol}^{-1}$$

The molar mass is numerically the same as the atomic or molecular weight, but it has units of grams per mole. The equation, which defines the molar mass, has the same form as those defining density, and the Avogadro constant. As in the case of density or the Avogadro constant, while it's always good to know a formula and be able to manipulate it, it's often only necessary to remember that mass and amount of substance are related *via* molar mass.

$$\text{Mass}\overset{\text{Molar mass}}{\longleftrightarrow}\text{amount of substance}m\overset{M}{\longleftrightarrow}n$$

The molar mass is easily obtained from atomic weights and may be used as a conversion factor, provided the units cancel.

EXAMPLE 2 Calculate the amount of glucose ($C_6H_{12}O_6$) in 457 g of this solid.

Solution Any problem involving interconversion of mass and amount of substance requires molar mass

$$M = (6 \text{ x } 12.01 + 12 \text{ x } 1.008 + 6 \text{ x } 16)\ \mathrm{g\ mol}^{-1} = 180\ \mathrm{g\ mol}^{-1}$$

The amount of substance will be the mass times a conversion factor which permits cancellation of units:

$$n=m\ \cdot\ \text{conversion factor}=m\ \cdot\frac{1}{M}=457\ \mathrm{g}\cdot\frac{1\ \mathrm{mol}}{180\ \mathrm{g}}=2.54\ \mathrm{mol}$$

In this case the reciprocal of the molar mass was the appropriate conversion factor. The Avogadro constant, molar mass, and density may be used in combination to solve more complicated problems.

EXAMPLE 3 The chemical equation above tells us that 6 glucose molecules require 6 water molecules, so 6 moles of glucose require 6 moles of water, and since they're equal, the 2.54 moles of glucose above would require 2.54 moles of water. What mass of water is that?

The mass of substance will be the amount times a conversion factor which permits cancellation of units:

$$m=n\cdot\text{conversion factor}=\mathrm{n}\cdot\mathrm{M}=2.54\ \mathrm{mol}\cdot 18.02\frac{\mathrm{g}}{\mathrm{mol}}=45.7\ \mathrm{g}$$

EXAMPLE 4 How many molecules would be present in 50 mL of pure water?

Solution In previous examples, we showed that the number of molecules may be obtained from the amount of substance by using the Avogadro constant. The amount of substance may be obtained from mass by using the molar mass, and mass from volume by means of density. A road map to the solution of this problem is

$$\text{Volume}\xrightarrow{\text{density}}\text{mass}\overset{\text{Molar mass}}{\longleftrightarrow}\text{amount}\overset{\text{Avogadro constant}}{\longleftrightarrow}\text{number of molecules}$$

or in shorthand notation

$$V\xrightarrow{\rho}m\xrightarrow{M}n\xrightarrow{\mathrm{N_A}}N$$

The road map tells us that we must look up the [[File:Chapter 1 page 23.jpg|density of H_2]O]:

$\rho = 1.0\ \mathrm{g\ cm}^{-3}$

The molar mass must be calculated from the Table of Atomic Weights.

$M = (2\ 1.008 + 1 \times 16.00)\ \mathrm{g\ mol}^{-1} = 18.02\ \mathrm{g\ mol}^{-1}$

and we recall that the Avogadro constant is

$N_A = 6.022 \times 10^{23}\ \text{mol}^{-1}$

The last quantity (N) in the road map can then be obtained by starting with the first (V) and applying successive conversion factors:

$$\mathrm{N} = 50.0\ \mathrm{cm}^3 \cdot \tfrac{1.00\ \mathrm{g}}{1\ \mathrm{cm}^3} \cdot \tfrac{1\ \mathrm{mol}}{18.02\ \mathrm{g}} \cdot \tfrac{6.022 \cdot 10^{23}\ \mathrm{molecules}}{1\ \mathrm{mol}} \tag{2.11.1.1}$$

$$= 1.67 \cdot 10^{24}\ \mathrm{molecules} \tag{2.11.1.2}$$

Notice that in this problem we had to *combine* techniques from previous examples. To do this you must remember relationships among quantities. For example, a volume was given, and we knew it could be converted to the corresponding mass by means of density, and so we looked up the density in a table. By writing a road map, or at least seeing it in your mind's eye, you can keep track of such relationships, determine what conversion factors are needed, and then use them to solve the problem.

Example 5 A student's body may contain 10^{27} water molecules and would need to accumulate about 10^{18} (ten million million million) water molecules *per second* over 18 years.

a. Show that the number of water molecules is about right, assuming a body weight of 150 lb and that water is about 70% of body weight.
b. Show that the rate of accumulation of water molecules per second is approximately right.

Solution

a.

$$150 lb \cdot \frac{0.453\ \mathrm{kg}}{1\ \mathrm{lb}} \cdot .70 = 47.6 kg water$$

$$\frac{47{,}600\ \mathrm{g\ water}}{18\,\frac{\mathrm{g}}{\mathrm{mol}}} = 2.6 \cdot 10^3 \mathrm{mol}$$

$$2.6 \cdot 10^3 \mathrm{mol} \cdot 6.02 \cdot 10^{23}\,\frac{\mathrm{molecules}}{\mathrm{mol}} = 1.6 \cdot 10^{27} \mathrm{molecules}$$

b.

$$\frac{10^{27} \mathrm{water\ molecules}}{18 \mathrm{y} \cdot \frac{365 \mathrm{d}}{\mathrm{y}} \cdot \frac{24 \mathrm{h}}{\mathrm{d}} \cdot \frac{60\ \mathrm{min}}{\mathrm{h}} \cdot \frac{60\ \mathrm{sec}}{\mathrm{min}}} = 4 \cdot 10^{18}\,\frac{\mathrm{molecules}}{\mathrm{sec}}$$