Implementation of a finite difference 1st order upwind scheme to solve the one-dimensional ideal hydrodynamic equations

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1 Introduction

In this report the one-dimensional hydrodynamic equations will be numerically solved by using a finite difference 1st order upwind scheme. The implementation will be checked by using two test cases. First, the sod shock problem, where the initial state has a discontinuity and second the isothermal acoustic wave, where the initial conditions are smooth.

2 Problem statement and preliminaries

The hydrodynamic equations in one-dimension, without source terms are written as:

$$\partial_t \left(\mathbf{U}\right) + \partial_x \mathbf{F}\left(\mathbf{U}\right) = 0$$

$$\partial_t \begin{pmatrix} \rho \\ \rho v \\ e \end{pmatrix} + \partial_x \begin{pmatrix} \rho v \\ \rho v^2 + p \\ ((e+P)v) \end{pmatrix} = 0$$

3 Numerical implementation

3.1 Results

3.2 Convergence Analysis