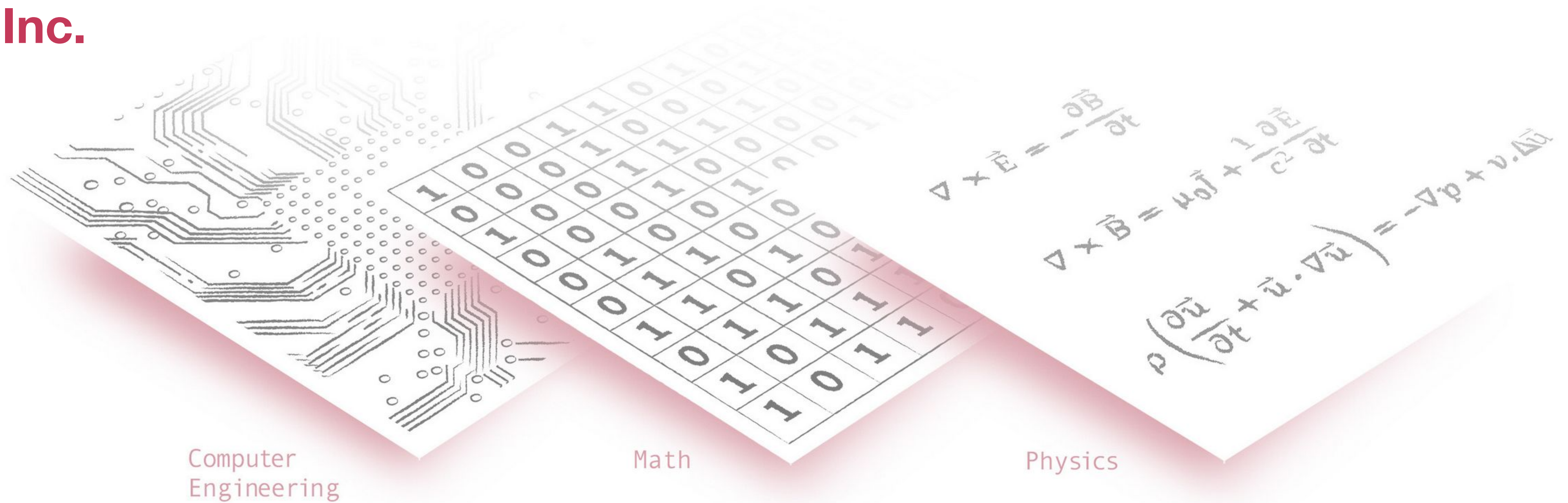


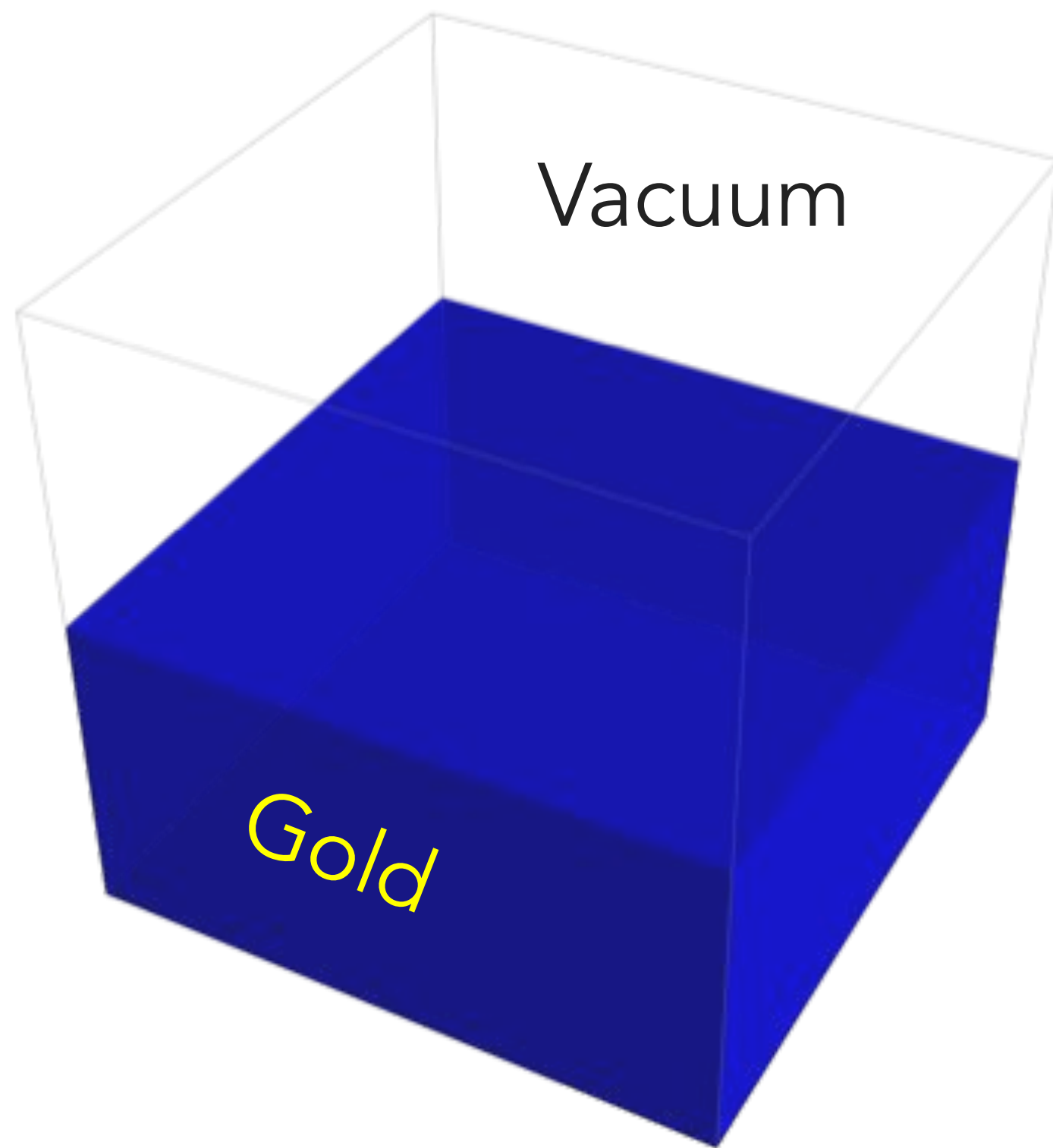


# INTRO TO FDTD (5)

Flexcompute Inc.

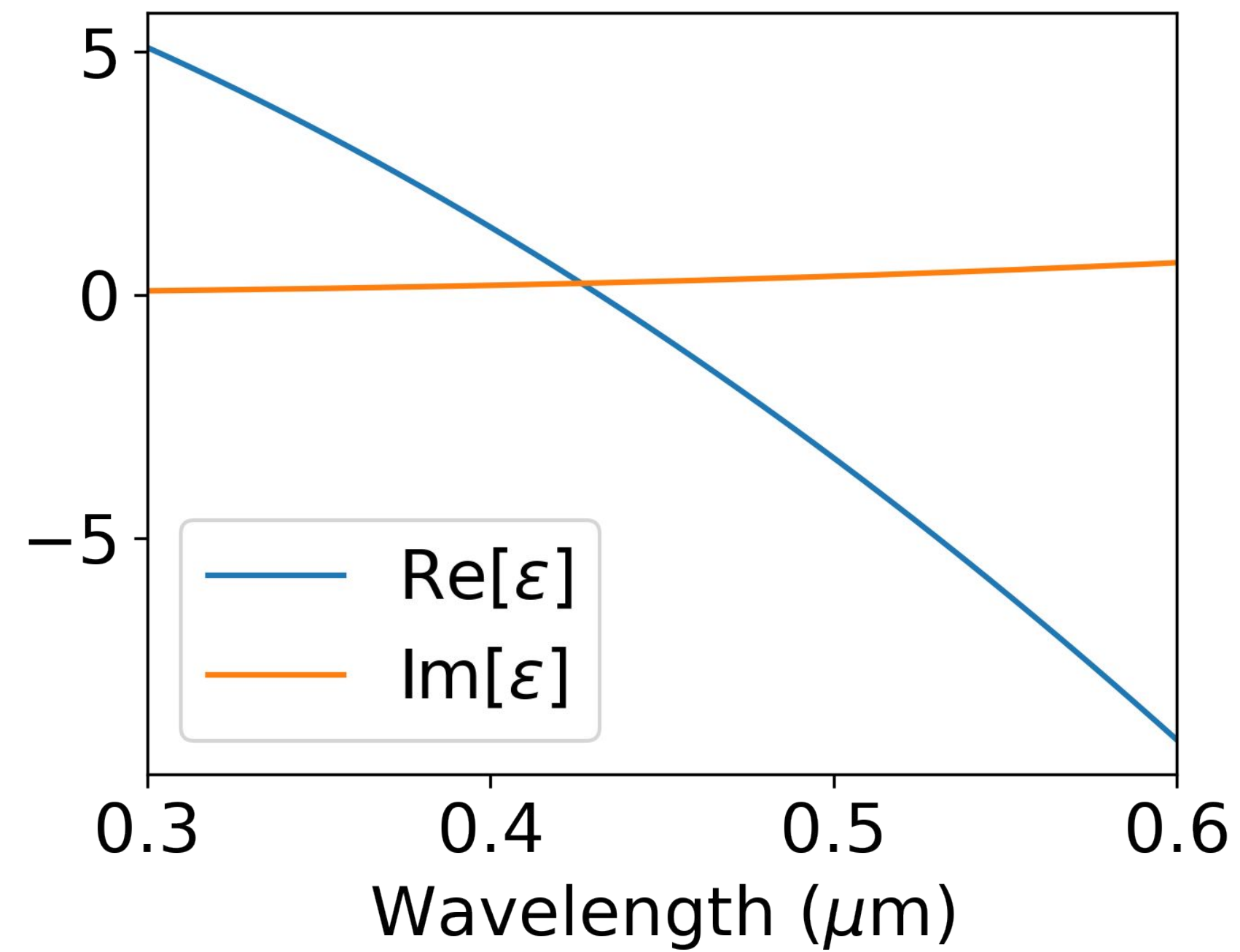


Material dispersion is common



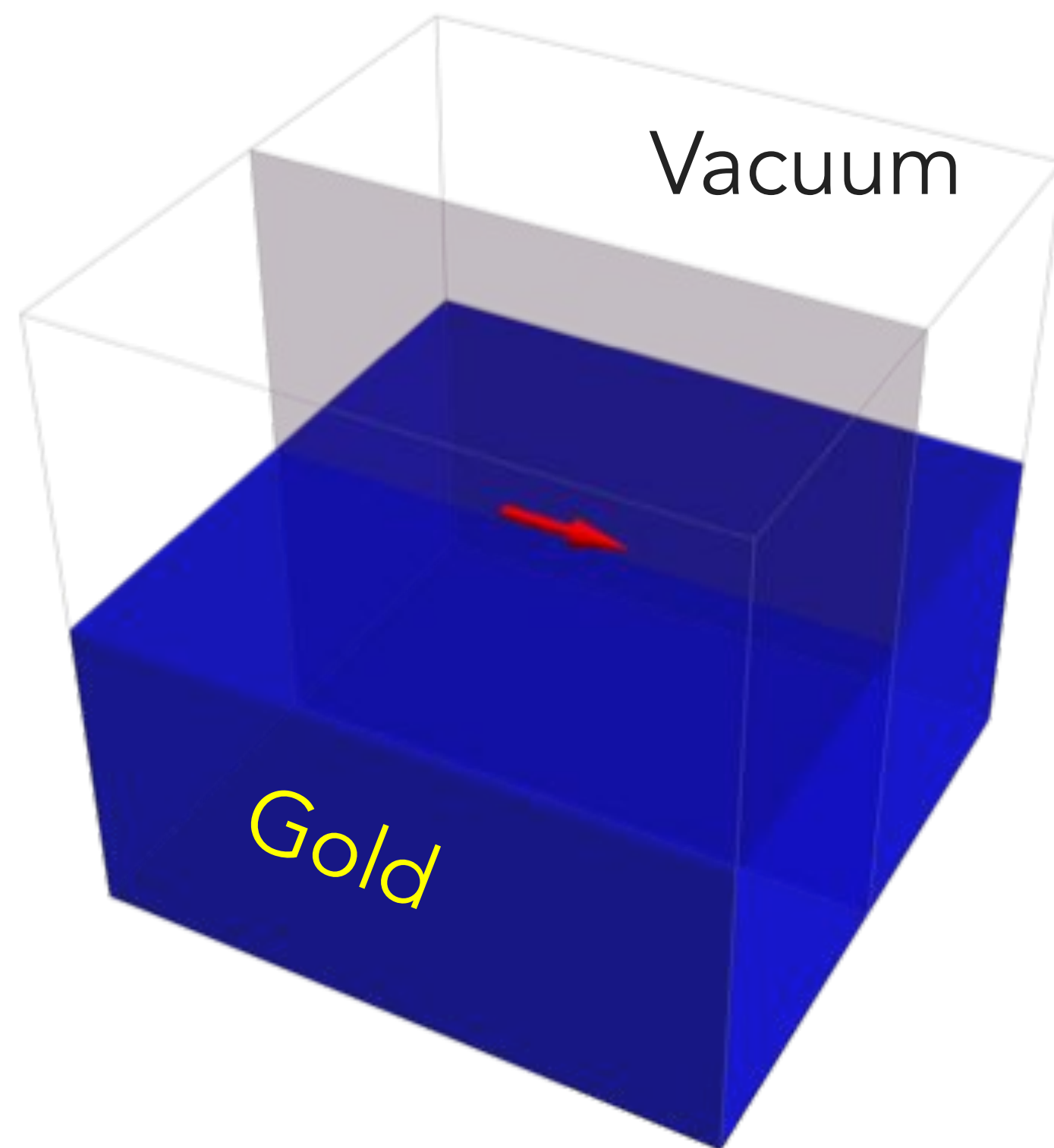
Gold dispersion with Drude model

$$\varepsilon(\omega) = \varepsilon_{\infty} - \frac{\omega_p^2}{\omega^2 + i\omega\gamma}$$



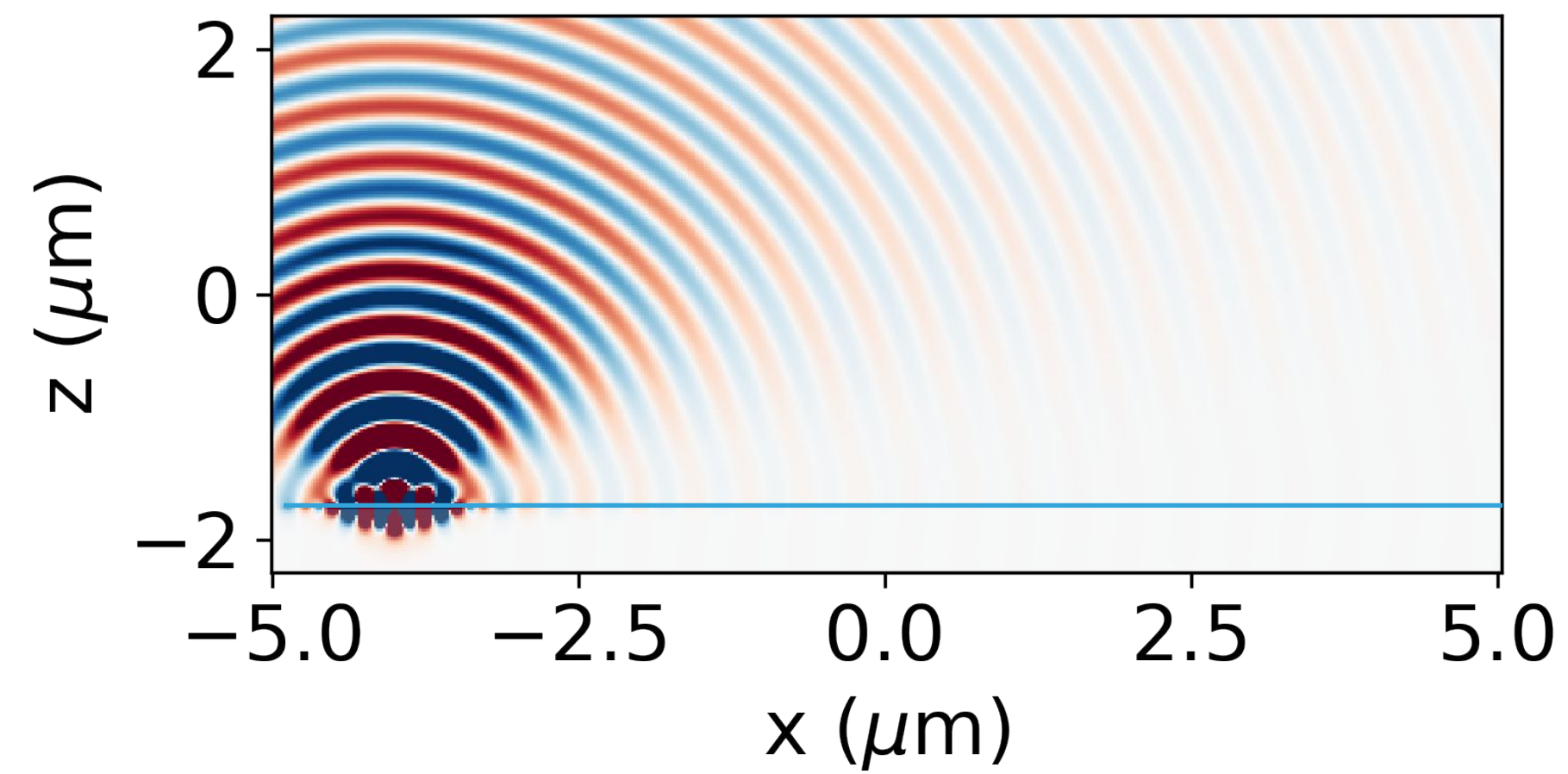
$$\varepsilon_{\infty} = 9.84, \omega_p = 9.01 \text{ eV}, \gamma = 0.072 \text{ eV}$$



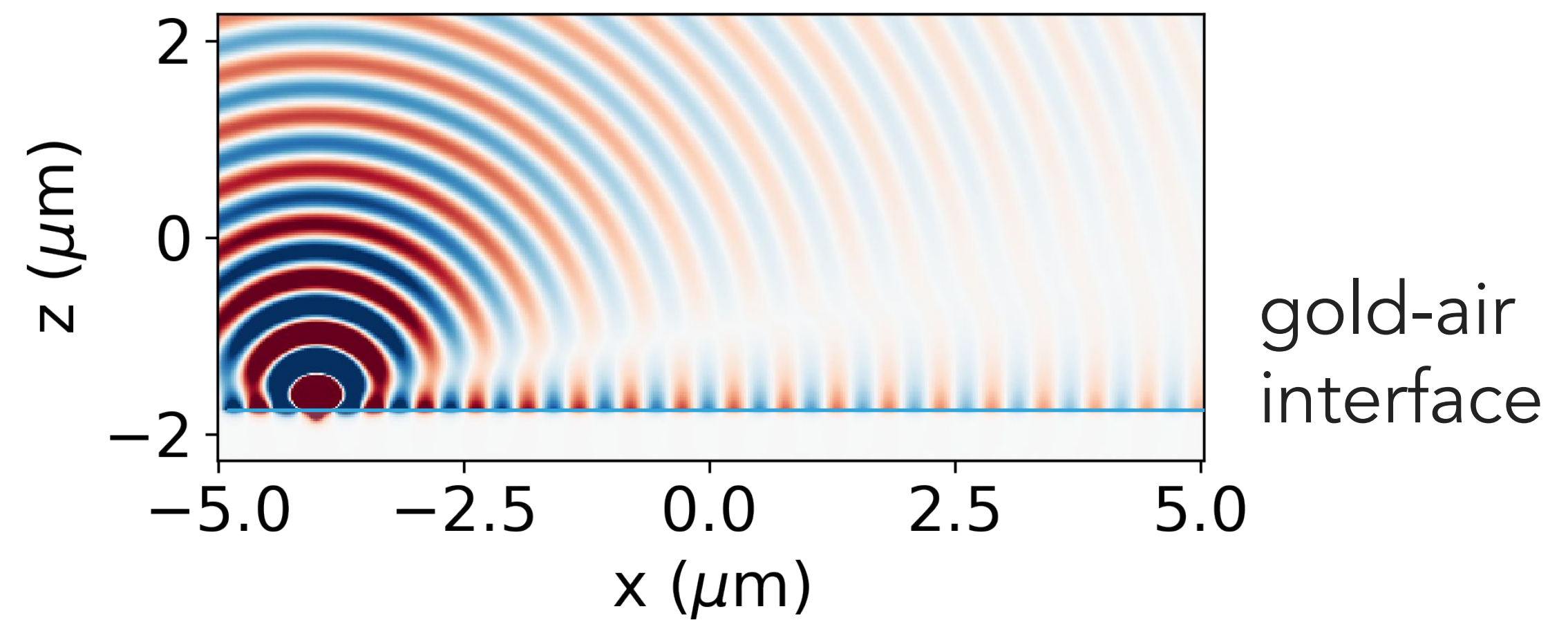


Excite surface plasmonic polariton (SPP) on gold surface

Wavelength 450 nm



Wavelength 550 nm



Without dispersion:

- $\mathbf{D}(t) = \varepsilon \mathbf{E}(t)$
- The displacement field reacts instantaneously to the applied electric field.

With dispersion:

- In frequency domain:  $\mathbf{D}(\omega) = \varepsilon(\omega) \mathbf{E}(\omega)$
- In time domain:  $\mathbf{D}(t) = \int dt' \epsilon(t - t') \mathbf{E}(t')$
- A dispersive material has a "memory". Its displacement field depends on the electric field in the past.

The method of complex-conjugate pole-residue pairs: fit  $\epsilon(\omega)$  as:

$$\epsilon(\omega) = \epsilon_{\infty} - \sum_m \left[ \frac{c_m}{i\omega + a_m} + \frac{c_m^*}{i\omega + a_m^*} \right]$$

M. Han, R. W. Dutton and S. Fan, IEEE Microwave and Wireless Component Letters, 16, 119 (2006).

### EXAMPLE:

For the Drude model

$$\epsilon(\omega) = \epsilon_{\infty} - \frac{\omega_p^2}{\omega^2 + i\omega\gamma}$$

Use two pairs

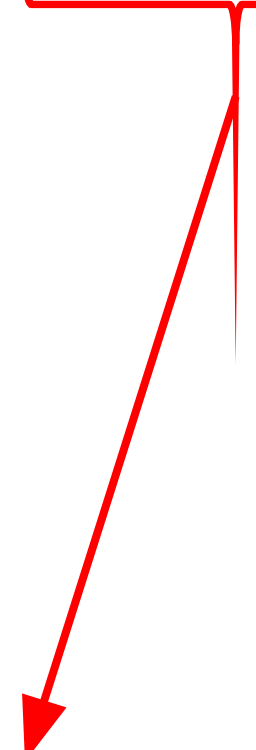
$$c_1 = \frac{\omega_p^2}{2\gamma}, a_1 = 0 \text{ and } c_2 = -\frac{\omega_p^2}{2\gamma}, a_2 = -\gamma$$



The method of complex-conjugate pole-residue pairs: fit  $\epsilon(\omega)$  as:

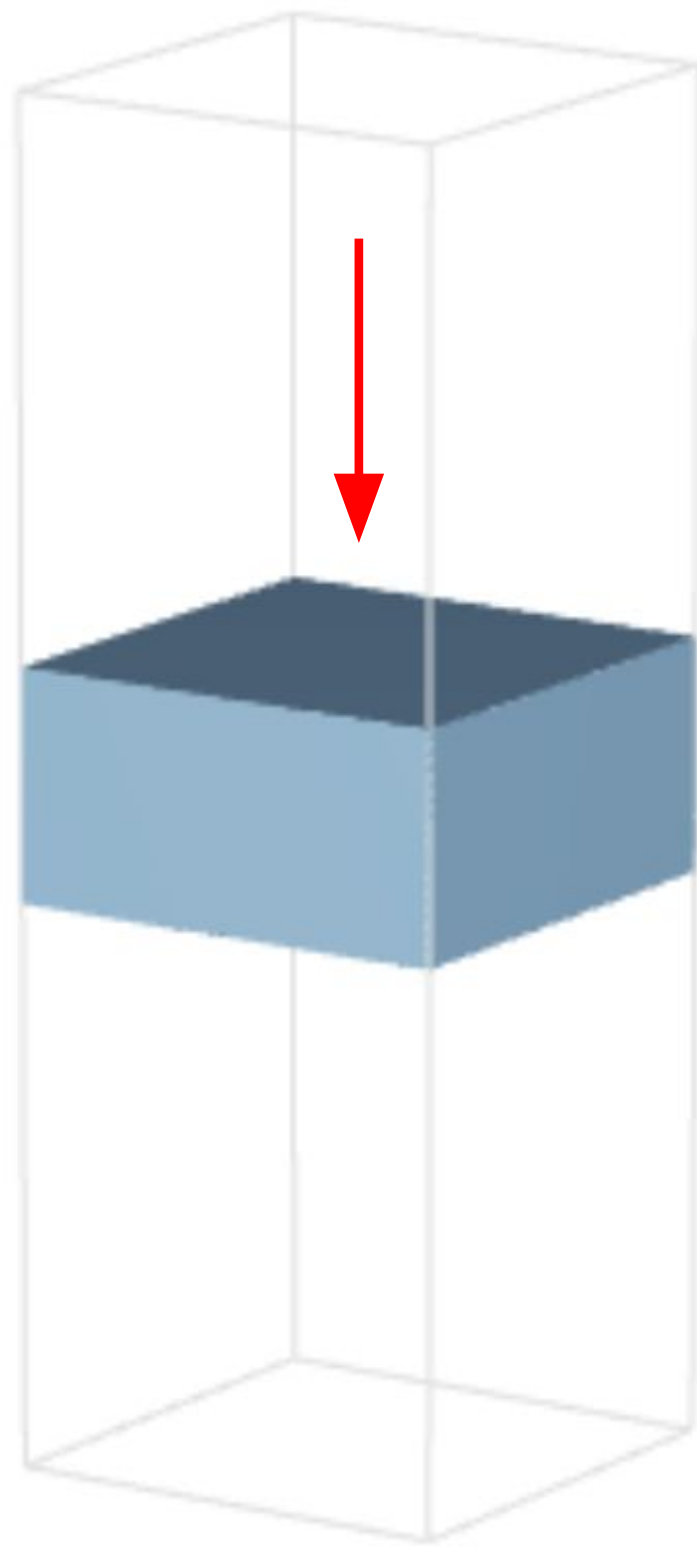
$$\epsilon(\omega) = \epsilon_{\infty} - \sum_m \left[ \underbrace{\frac{c_m}{i\omega + a_m}} + \frac{c_m^*}{i\omega + a_m^*} \right]$$

Auxiliary differential equation

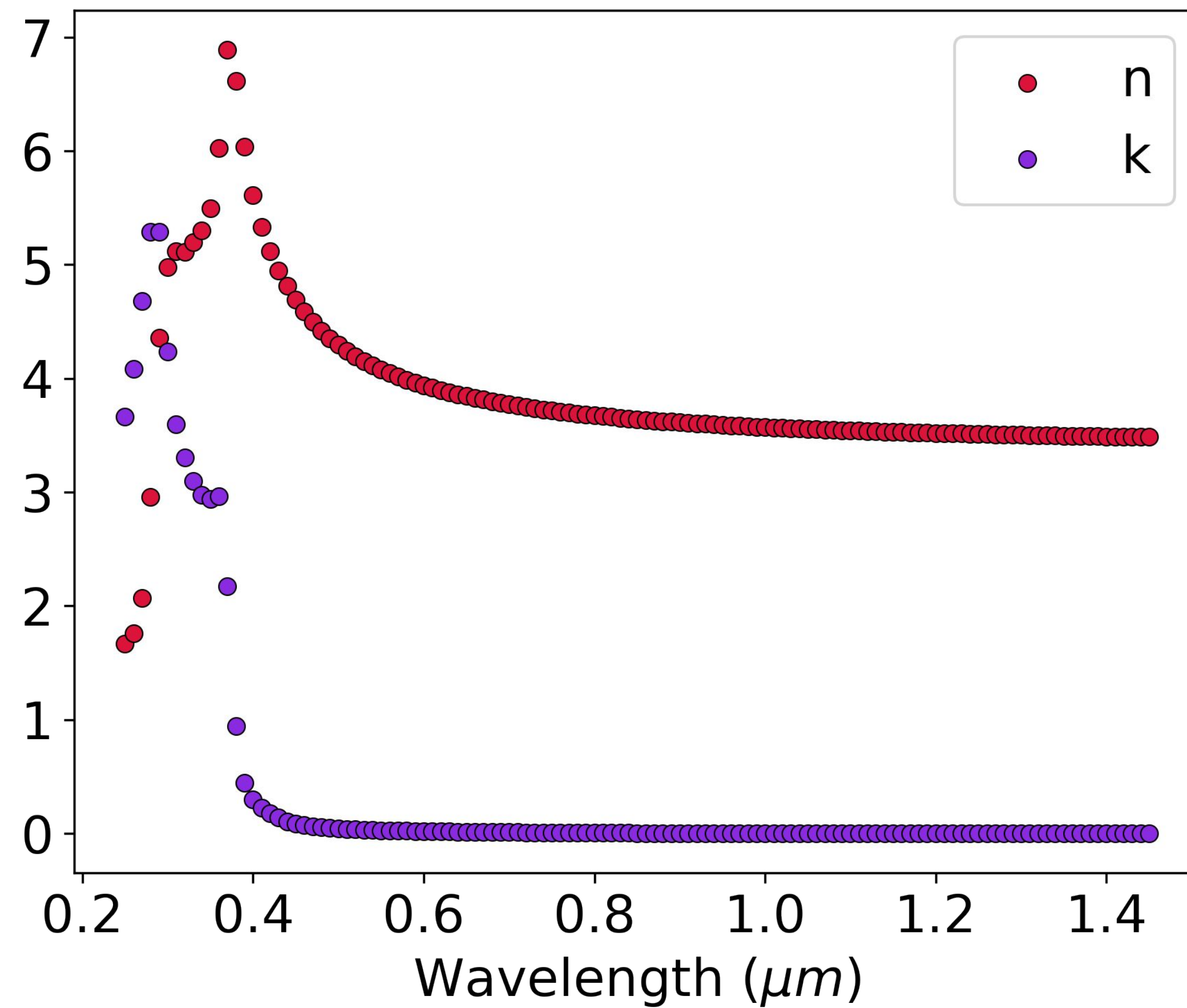

$$\frac{d}{dt} J_m(t) - a_m J_m(t) = \epsilon_0 c_m \frac{d}{dt} E(t)$$

Computational cost increases with the number of poles.

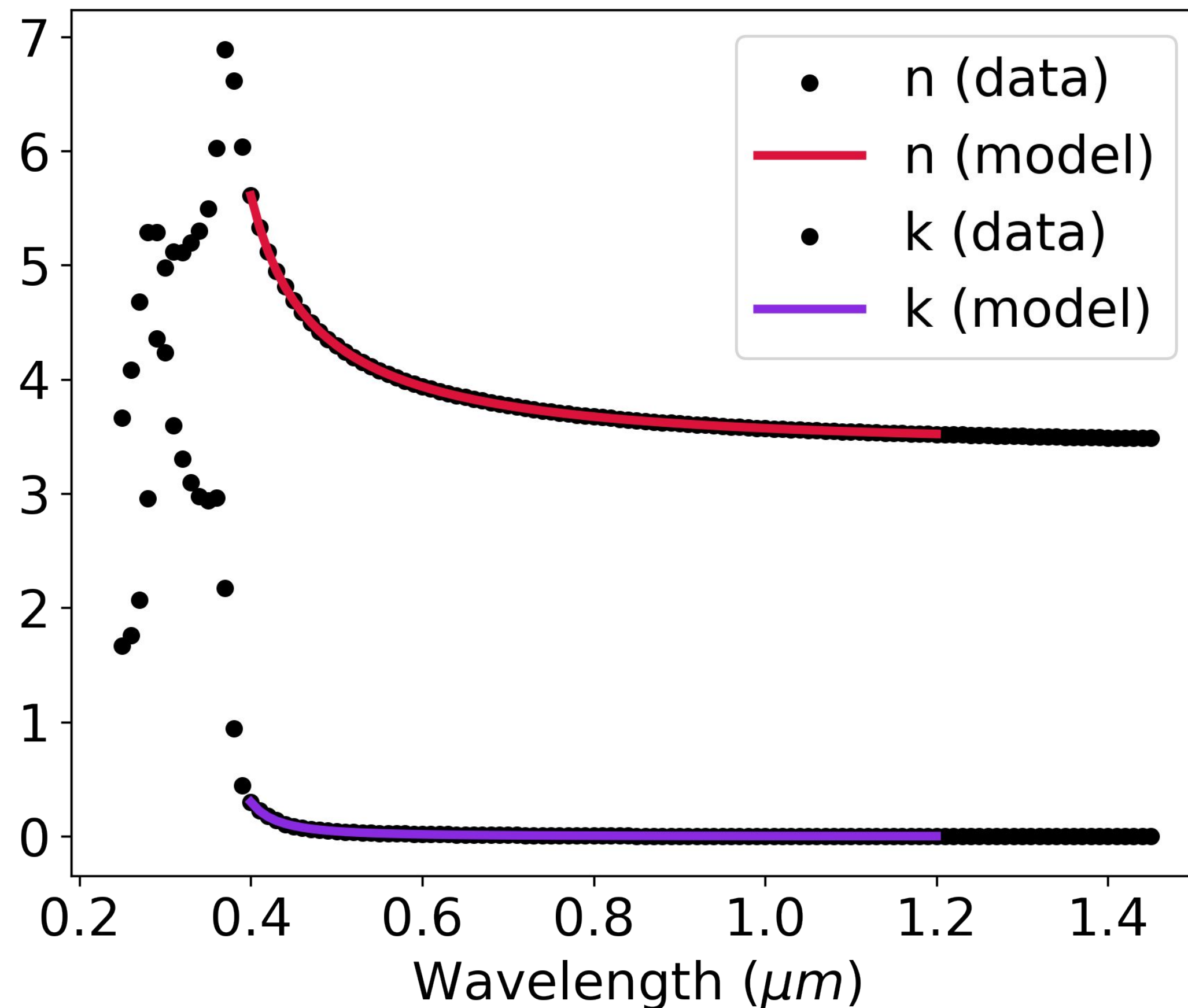
## Crystalline silicon slab transmission



Dispersion data [Green, 2008]



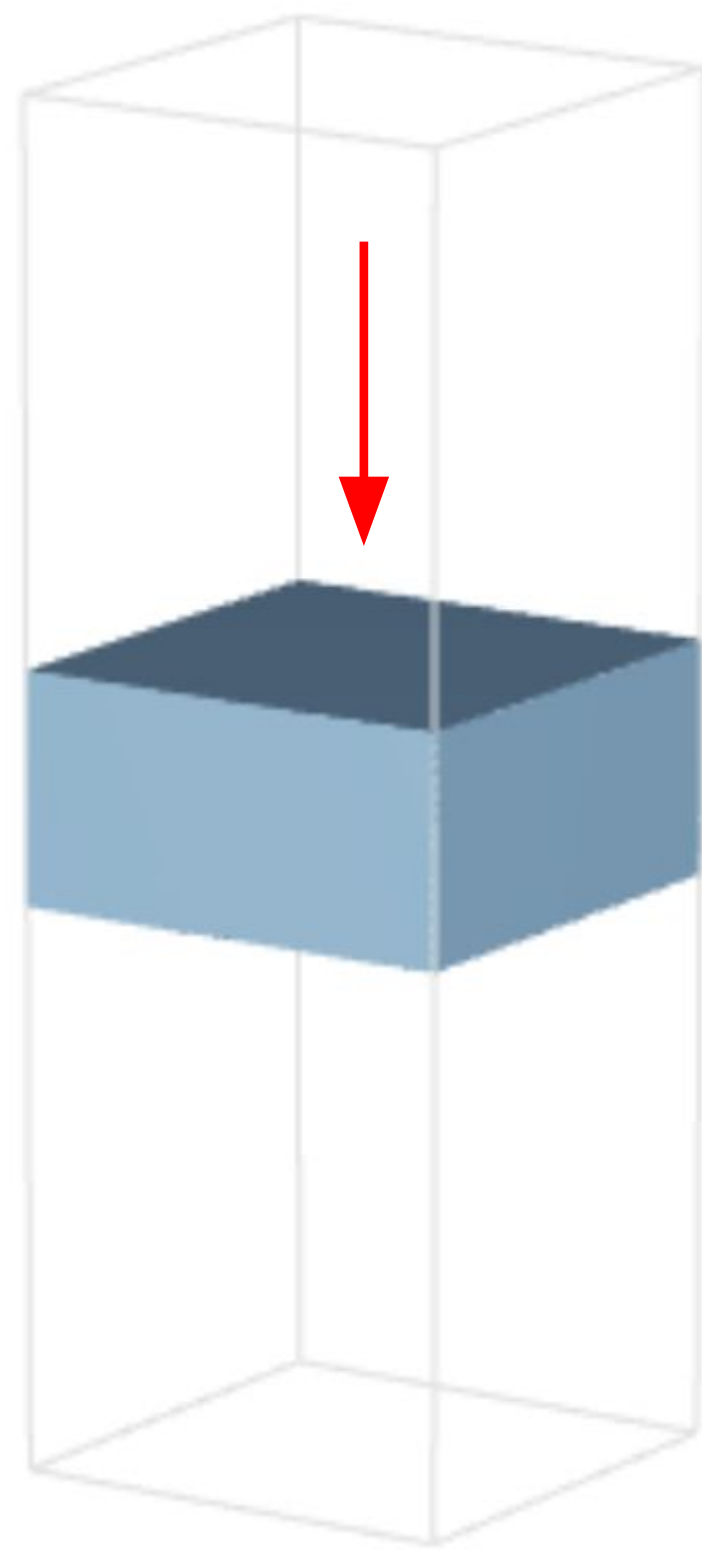
Fit dispersion data with CCPR model in wavelength range  $[0.4, 1.2] \mu\text{m}$



- 2 poles for RMS error  $\sim 2.6\%$
- Pole frequency:  $0.26 \mu\text{m}$  and  $0.35 \mu\text{m}$



Broadband transmission computation in a single simulation



Thickness:  $1\mu\text{m}$

