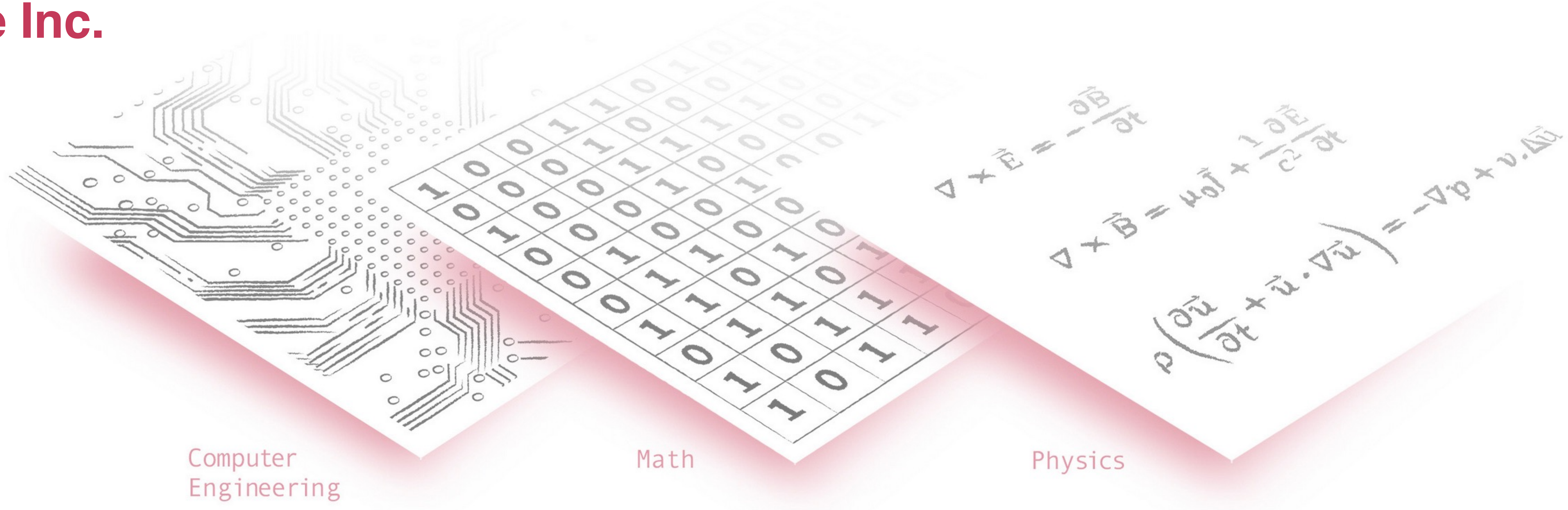
INTRO TO FDTD (7)

Flexcompute Inc.



Tidy3D

Maxwell's equation in 1D:

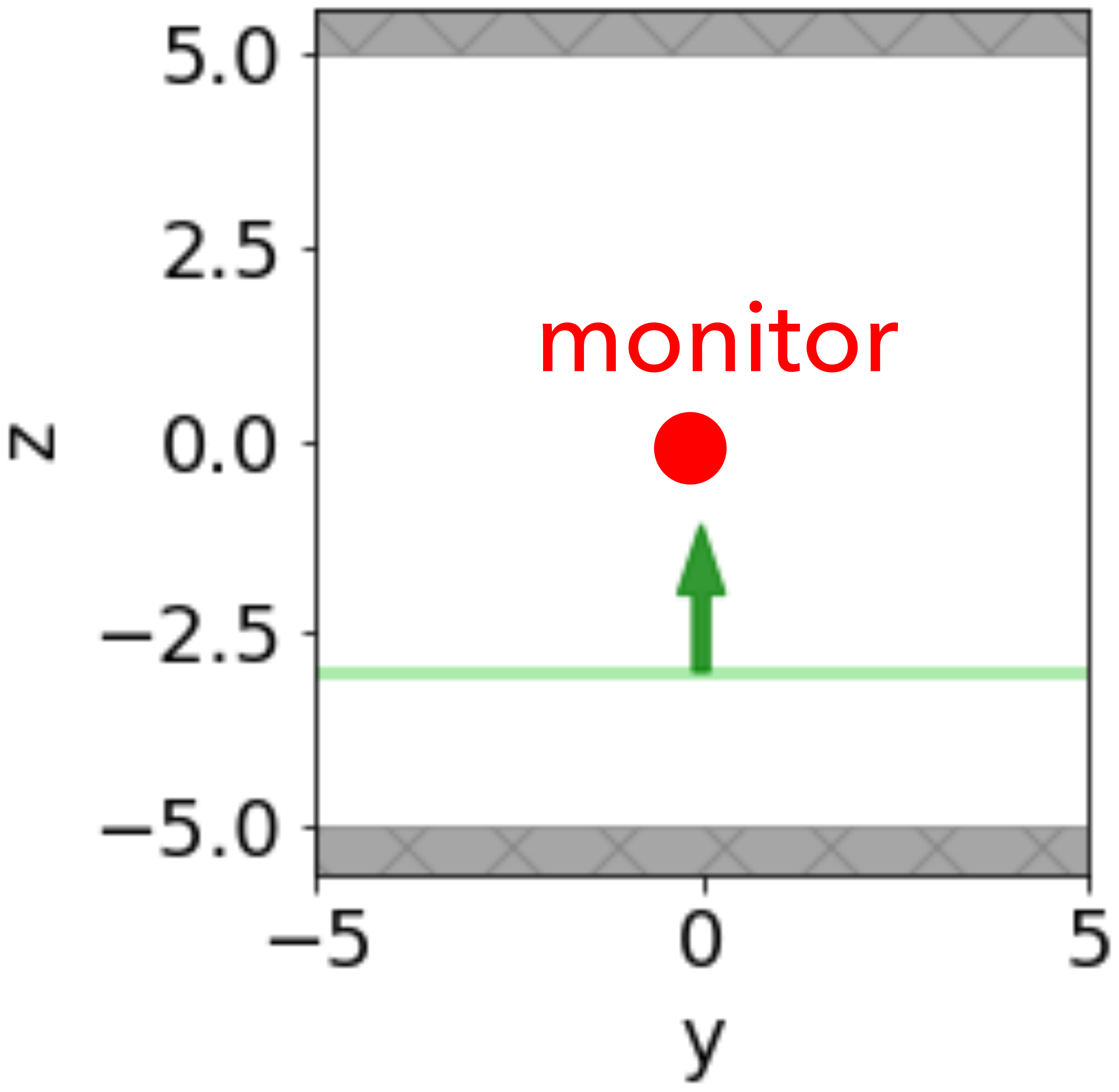
$$\varepsilon \frac{\partial E_z}{\partial t} = \frac{\partial H_y}{\partial x}$$

$$\mu \frac{\partial H_y}{\partial t} = \frac{\partial E_z}{\partial x}$$

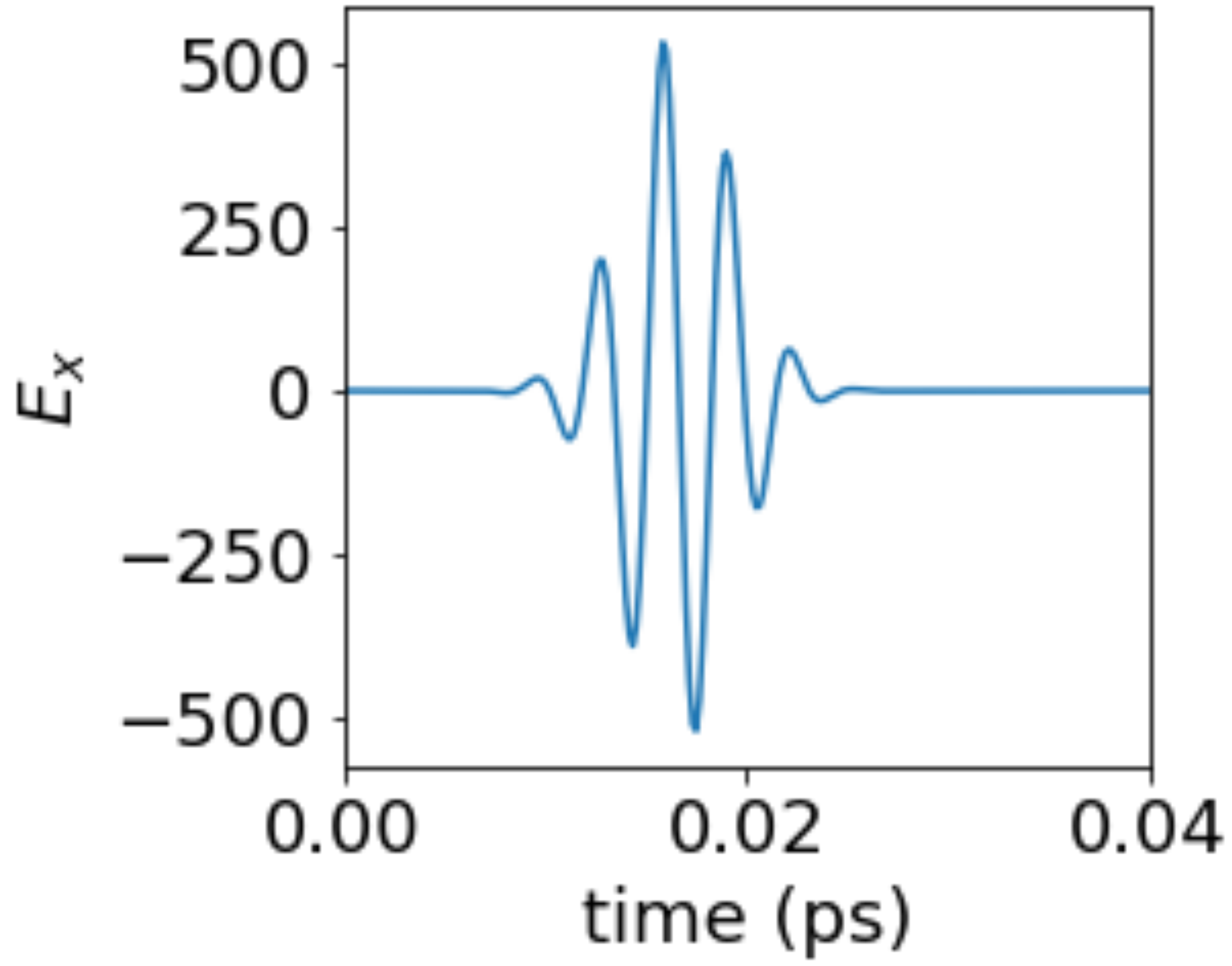
Convert the differential operator to difference operator, for example

$$\frac{\partial E_z}{\partial t} \rightarrow \frac{E_z^{(\alpha+1)\Delta t} - E_z^{\alpha\Delta t}}{\Delta t}$$

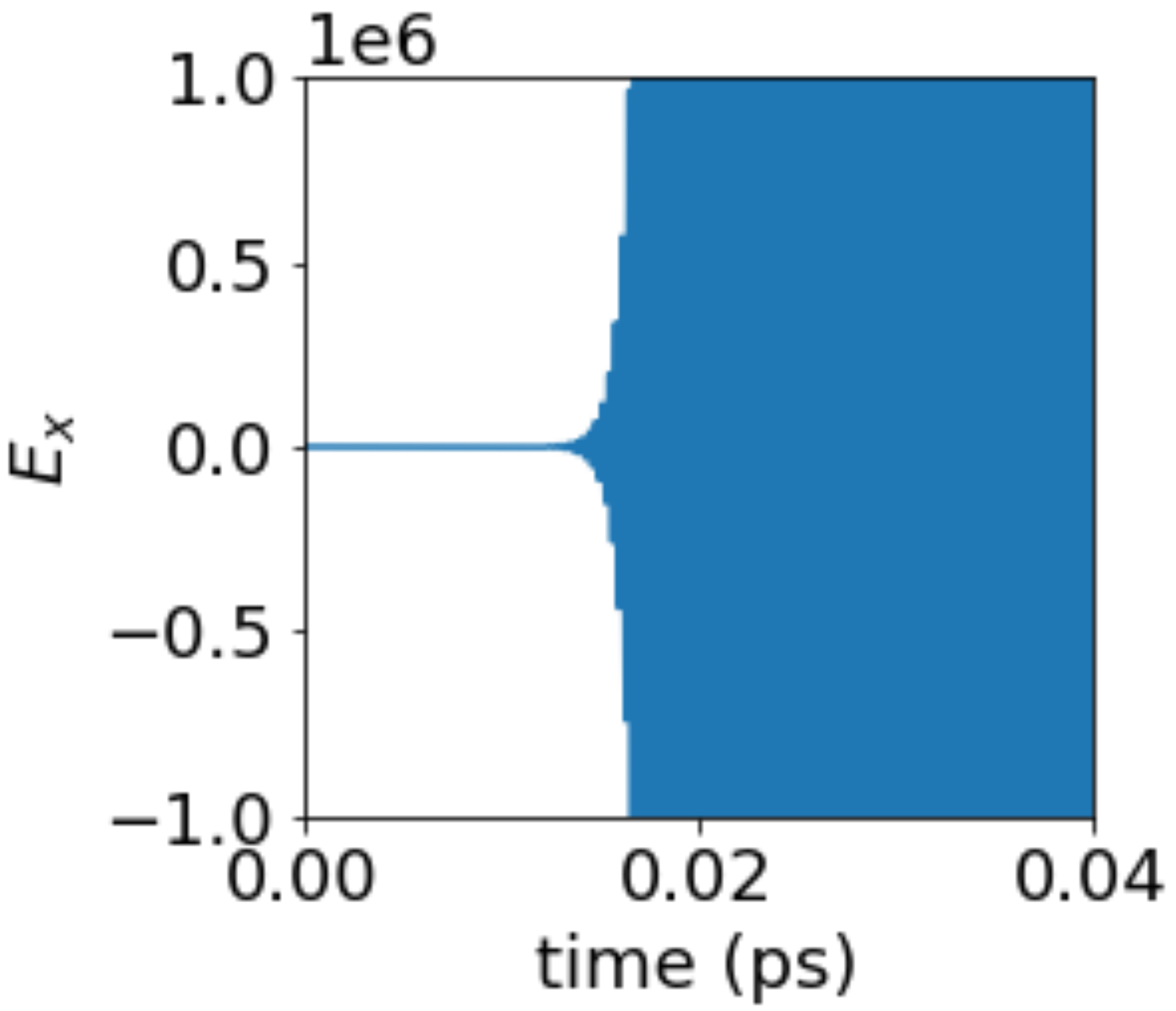
Today's subject: The choice of time step Δt in FDTD



$\Delta t = 0.99 \Delta x / c$



$\Delta t = 1.01 \Delta x / c$



Planewave propagation in vacuum

Only difference: increase time step size by 2%

→ Courant-Friedrichs-Lewy (CFL) condition

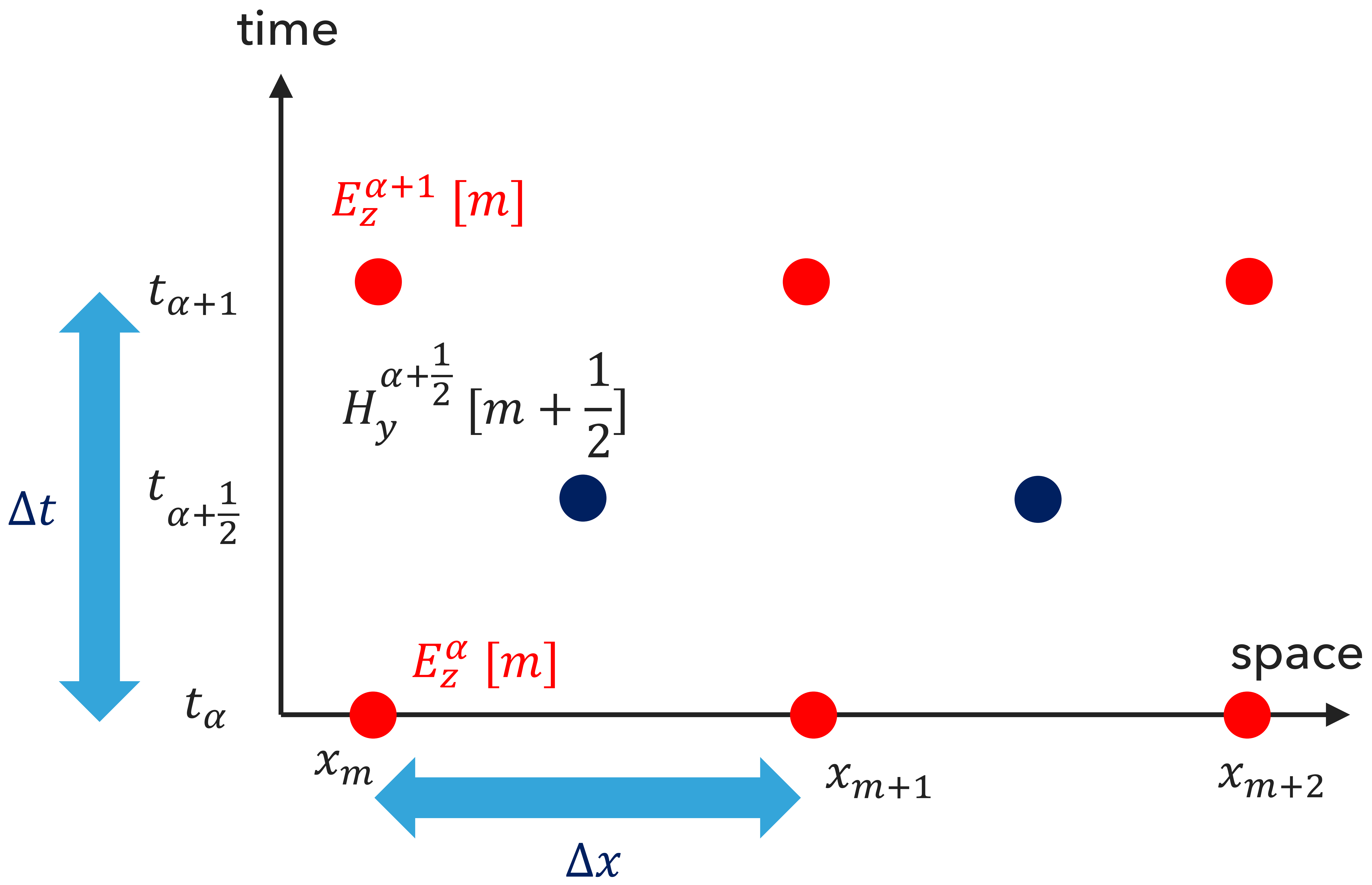
1D Yee grid in space and time

Maxwell's equation in 1D:

$$\varepsilon \frac{\partial E_z}{\partial t} = \frac{\partial H_y}{\partial x}$$
$$\mu \frac{\partial H_y}{\partial t} = \frac{\partial E_z}{\partial x}$$

Discretize:

- Spatial step size: Δx
- Temporal step size: Δt
- Staggered ***E*** and ***H*** in space and time

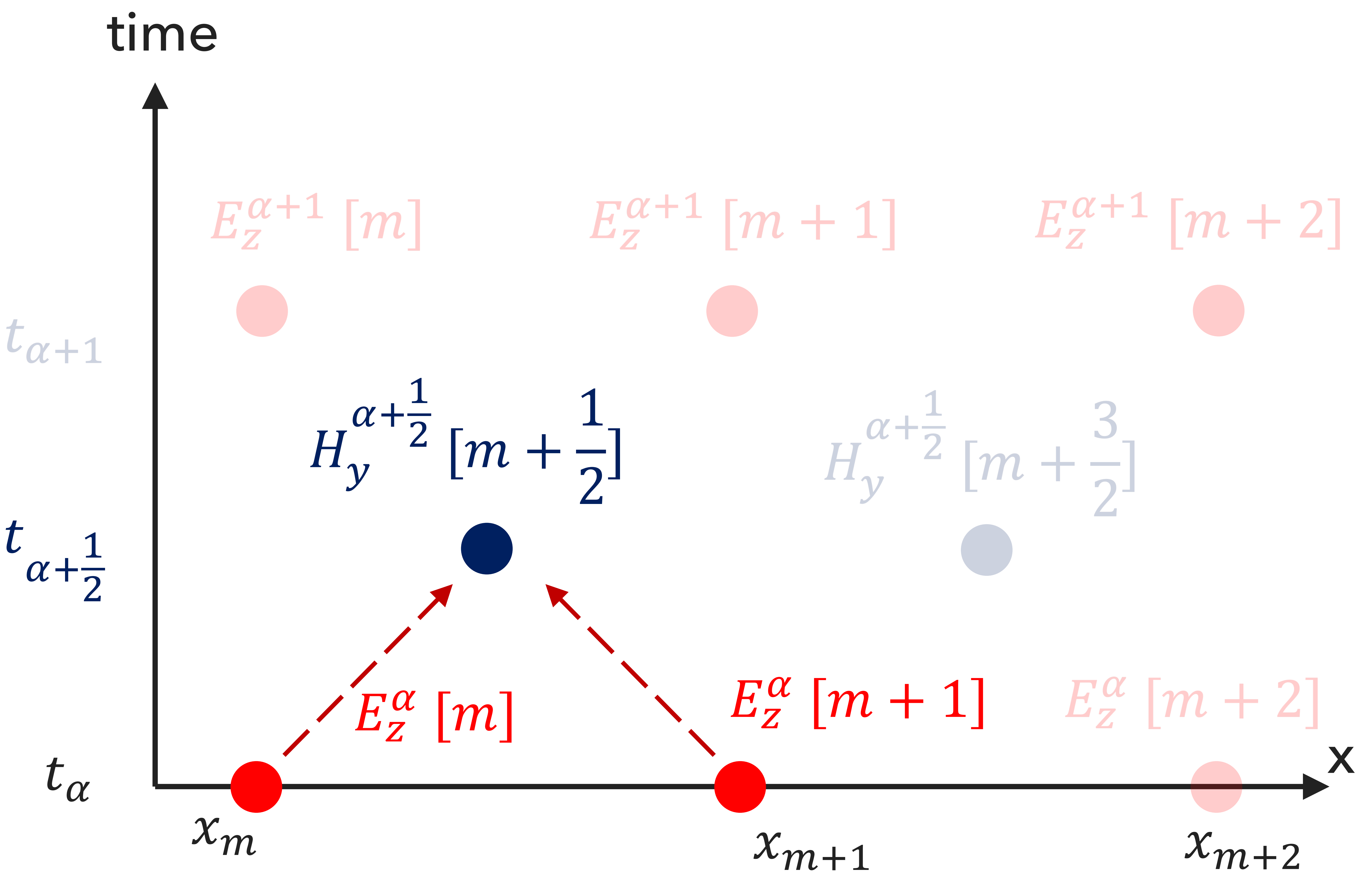


Time marching in 1D

Time update for H_y:

$$\mu \frac{\partial H_y}{\partial t} = \frac{\partial E_z}{\partial x}$$

$$H_y^{\alpha+\frac{1}{2}}\left[m+\frac{1}{2}\right]$$
$$= H_y^{\alpha-\frac{1}{2}}\left[m+\frac{1}{2}\right] + \frac{\Delta t}{\mu \Delta x}\left(E_z^{\alpha}[m+1]\right.$$
$$\left. - E_z^{\alpha}[m]\right)$$

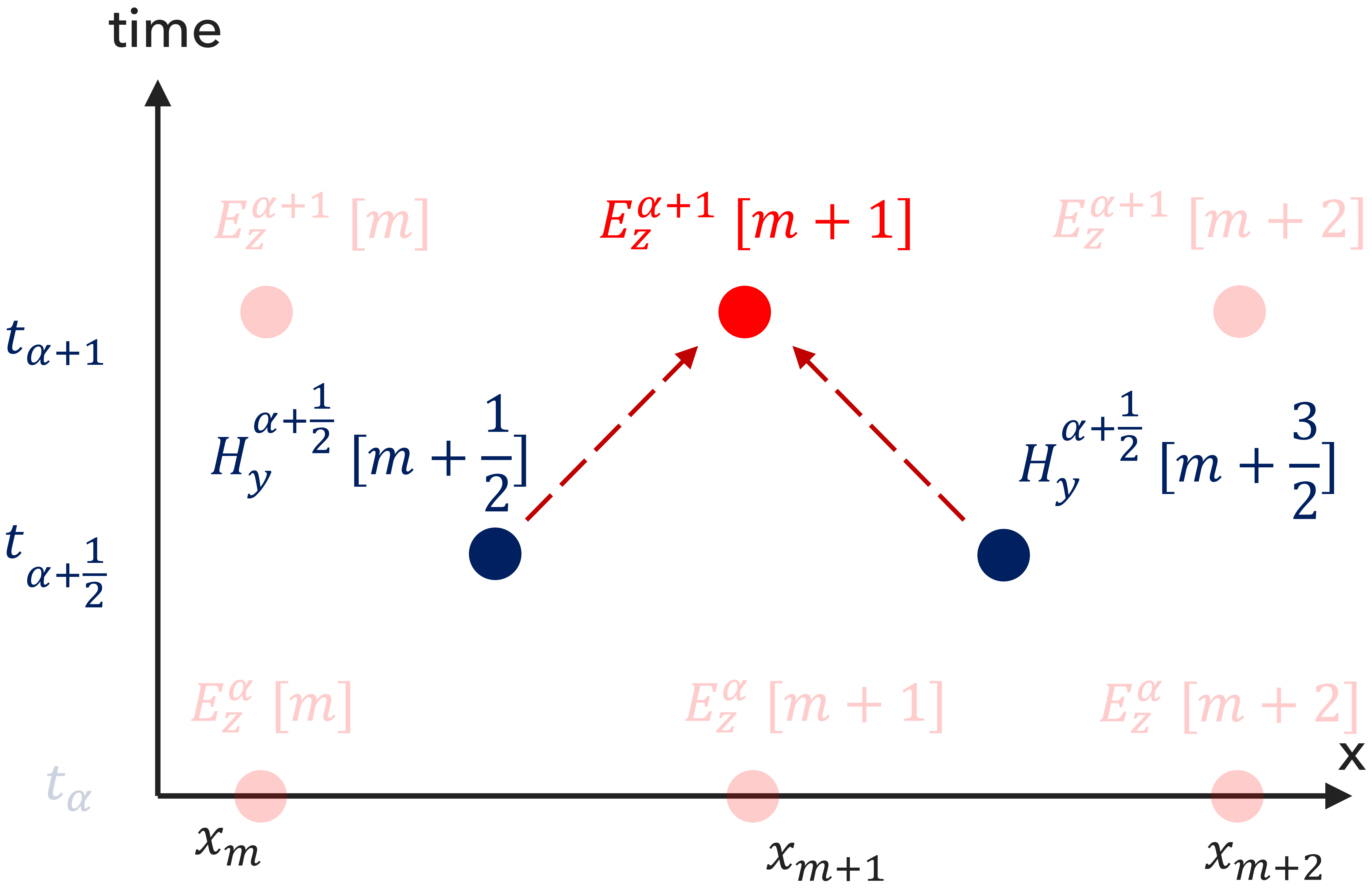


Time marching in 1D

Time update for E_z:

$$\varepsilon \frac{\partial E_z}{\partial t} = \frac{\partial H_y}{\partial x}$$

$$E_z^{\alpha+1}[m+1]$$
$$= E_z^{\alpha}[m+1] + \frac{\Delta t}{\varepsilon \Delta x}\left(H_y^{\alpha+\frac{1}{2}}\left[m+\frac{3}{2}\right]\right.$$
$$\left.- H_y^{\alpha+\frac{1}{2}}\left[m+\frac{1}{2}\right]\right)$$



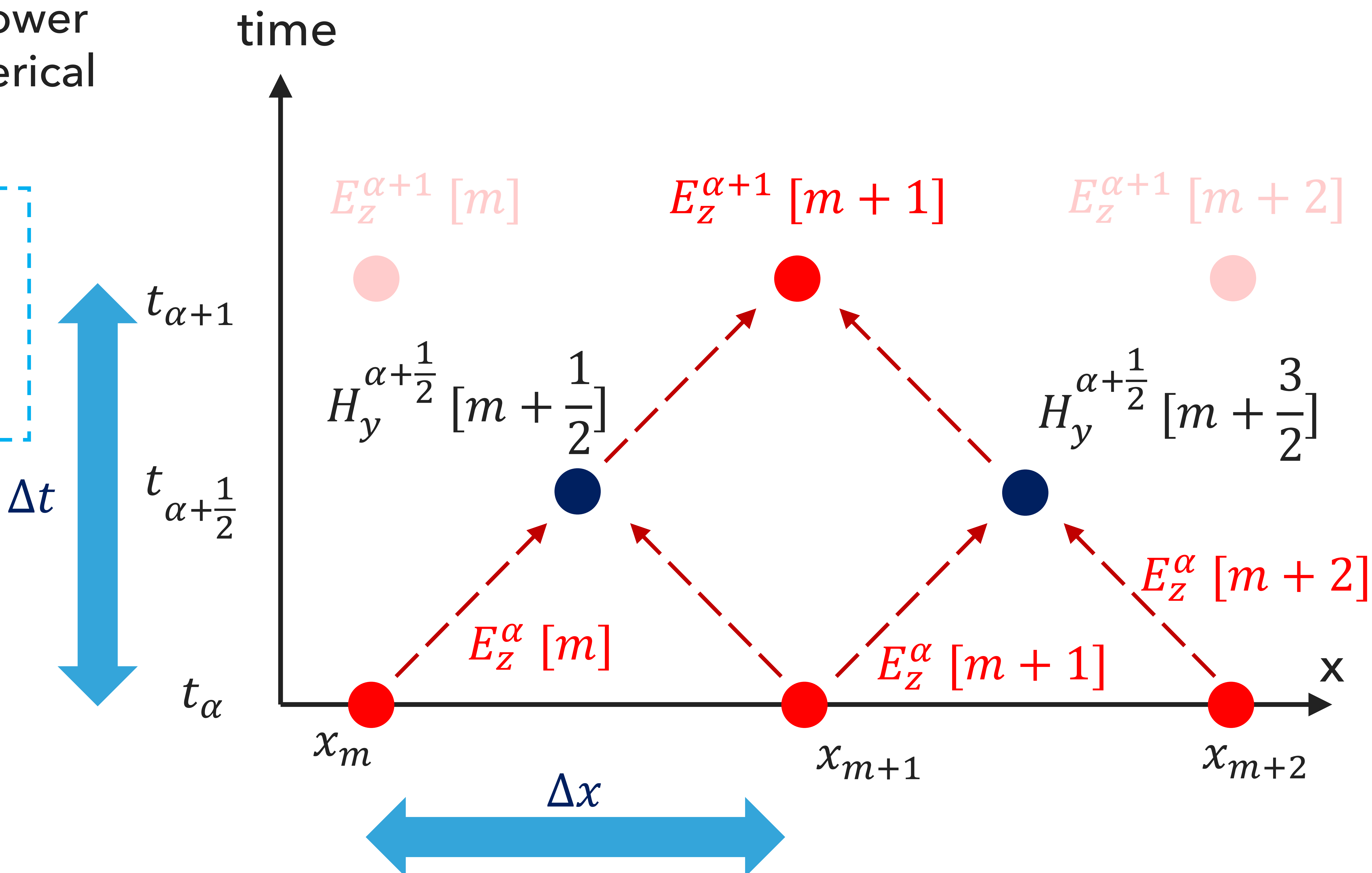
CFL condition in 1D

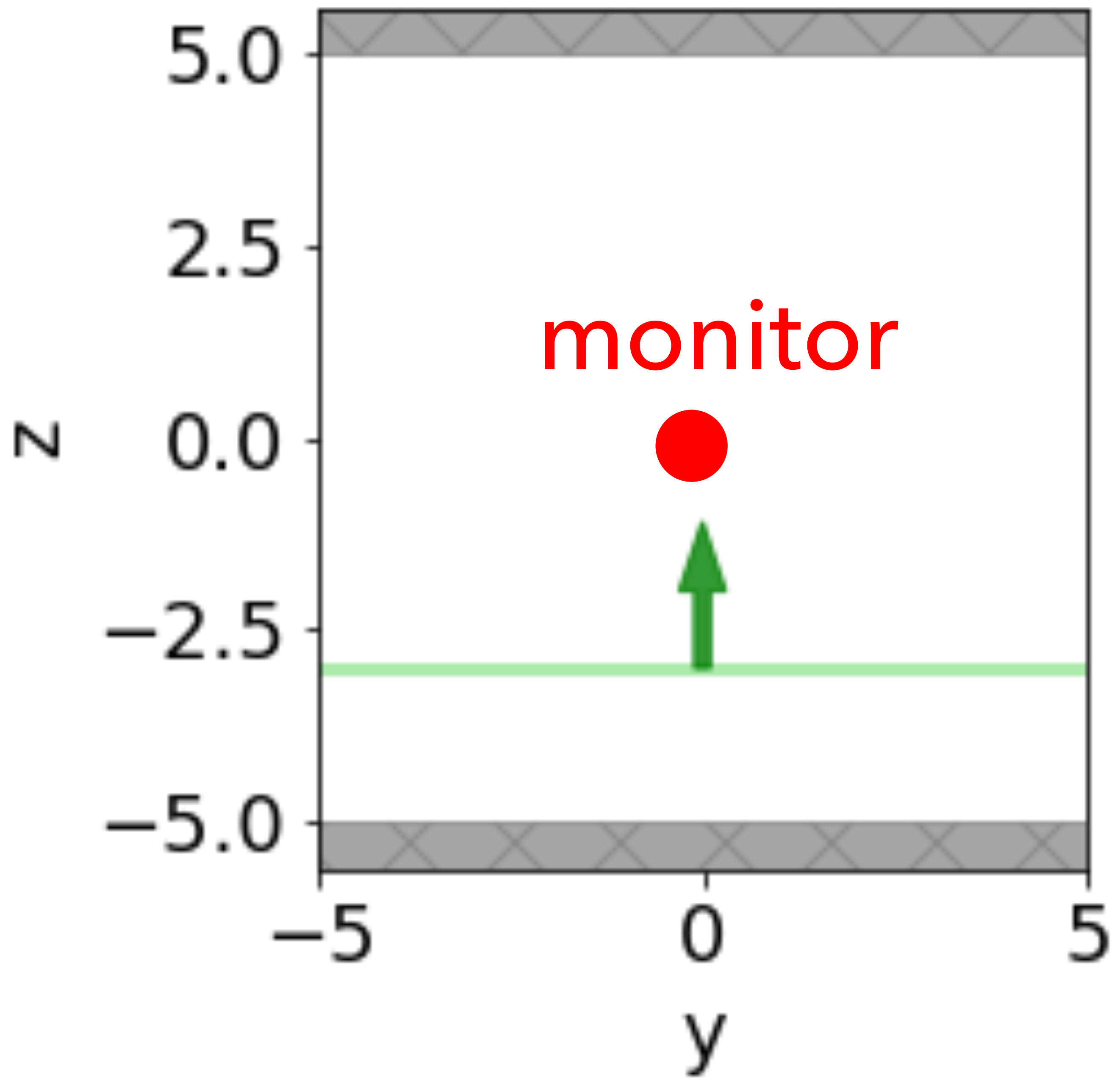
CFL condition:
Physical wave should propagate slower than the speed of the slowest numerical dependence.

physical		numerical
c	$\leq$	$\dfrac{\Delta x}{\Delta t}$

$$\Delta t \leq \frac{\Delta x}{c}$$

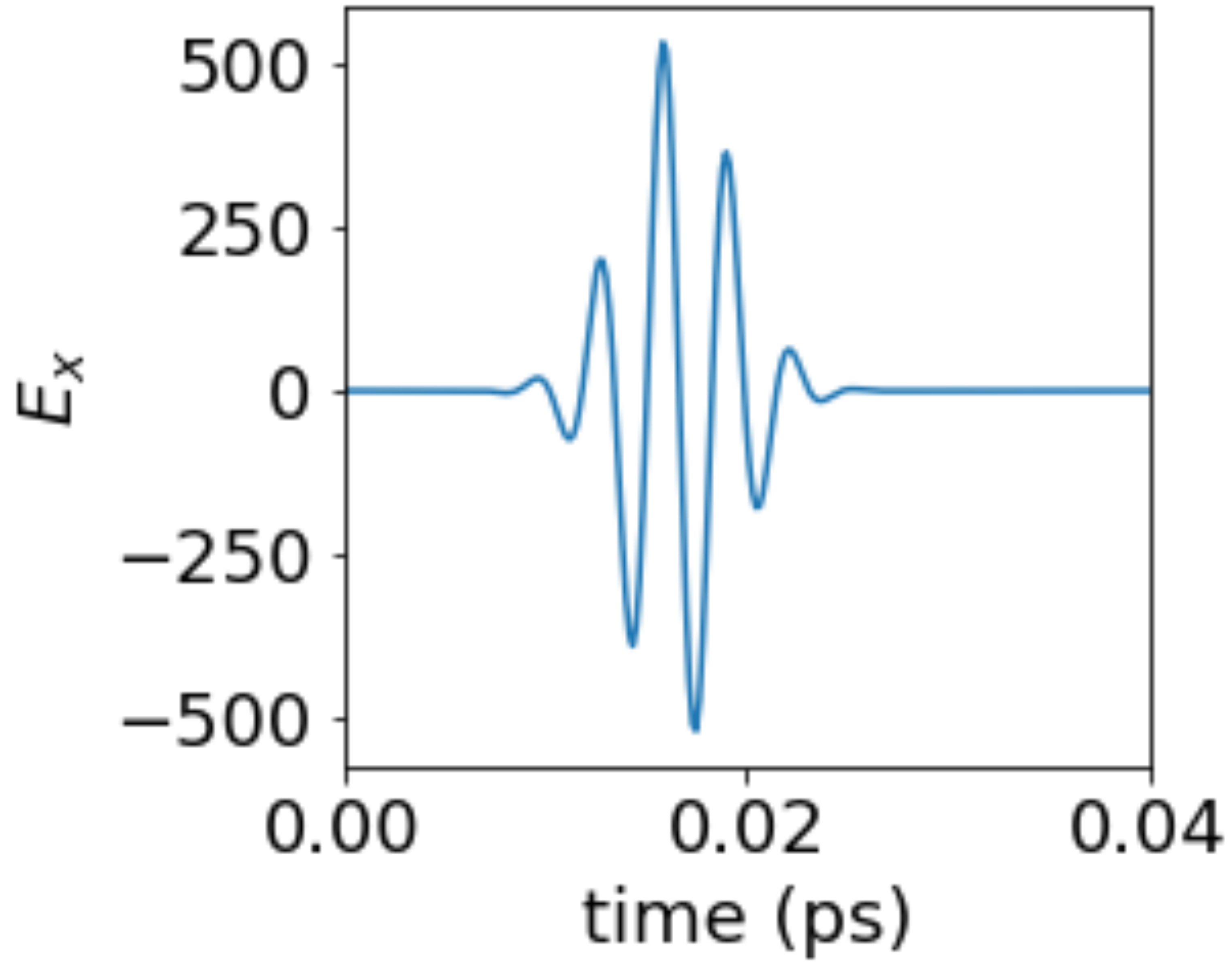
Courant number: $C = \frac{c\Delta t}{\Delta x} \leq 1$





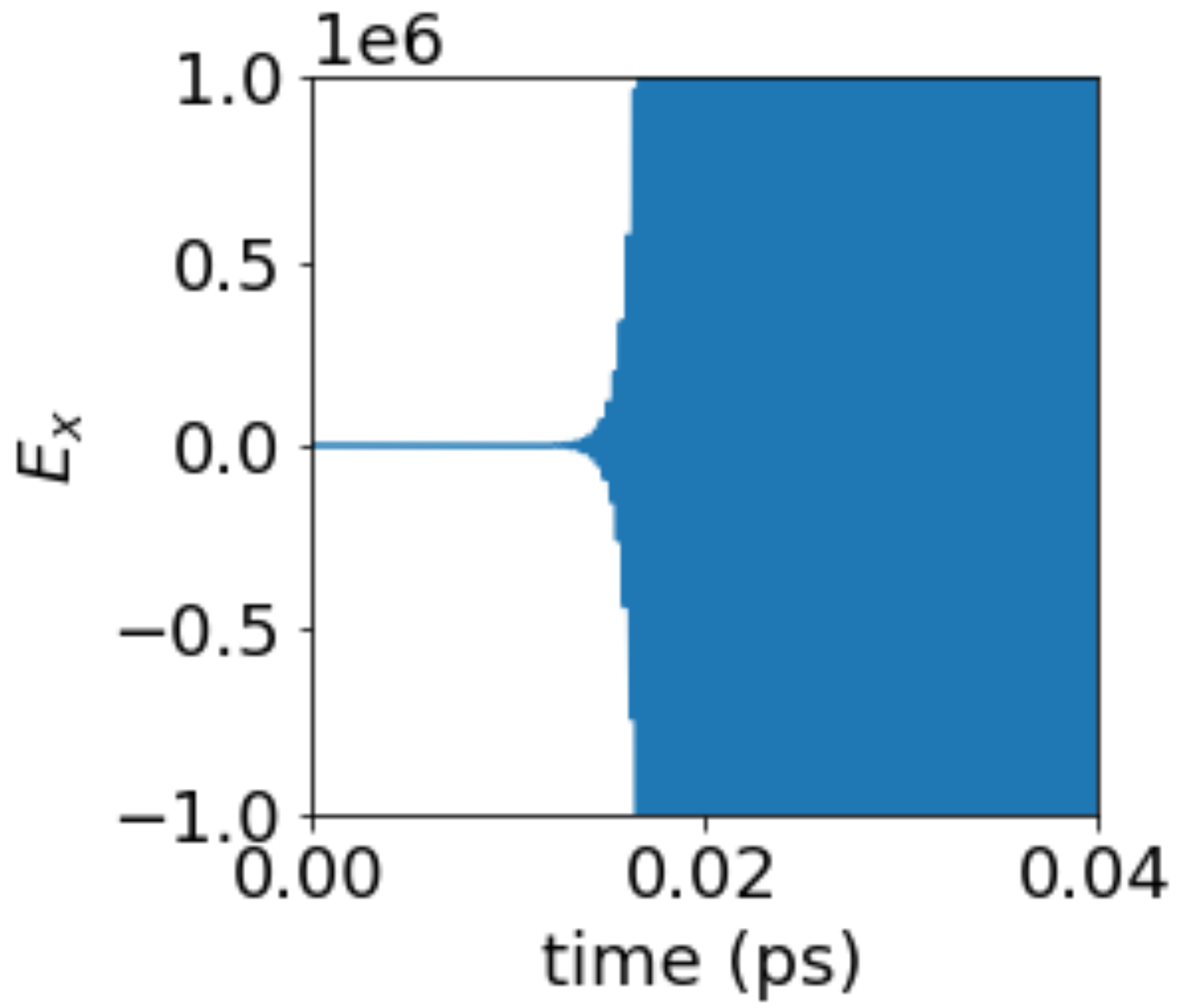
Planewave propagation in vacuum

$\Delta t = 0.99 \Delta x / c$



Courant number: $C = 0.99$

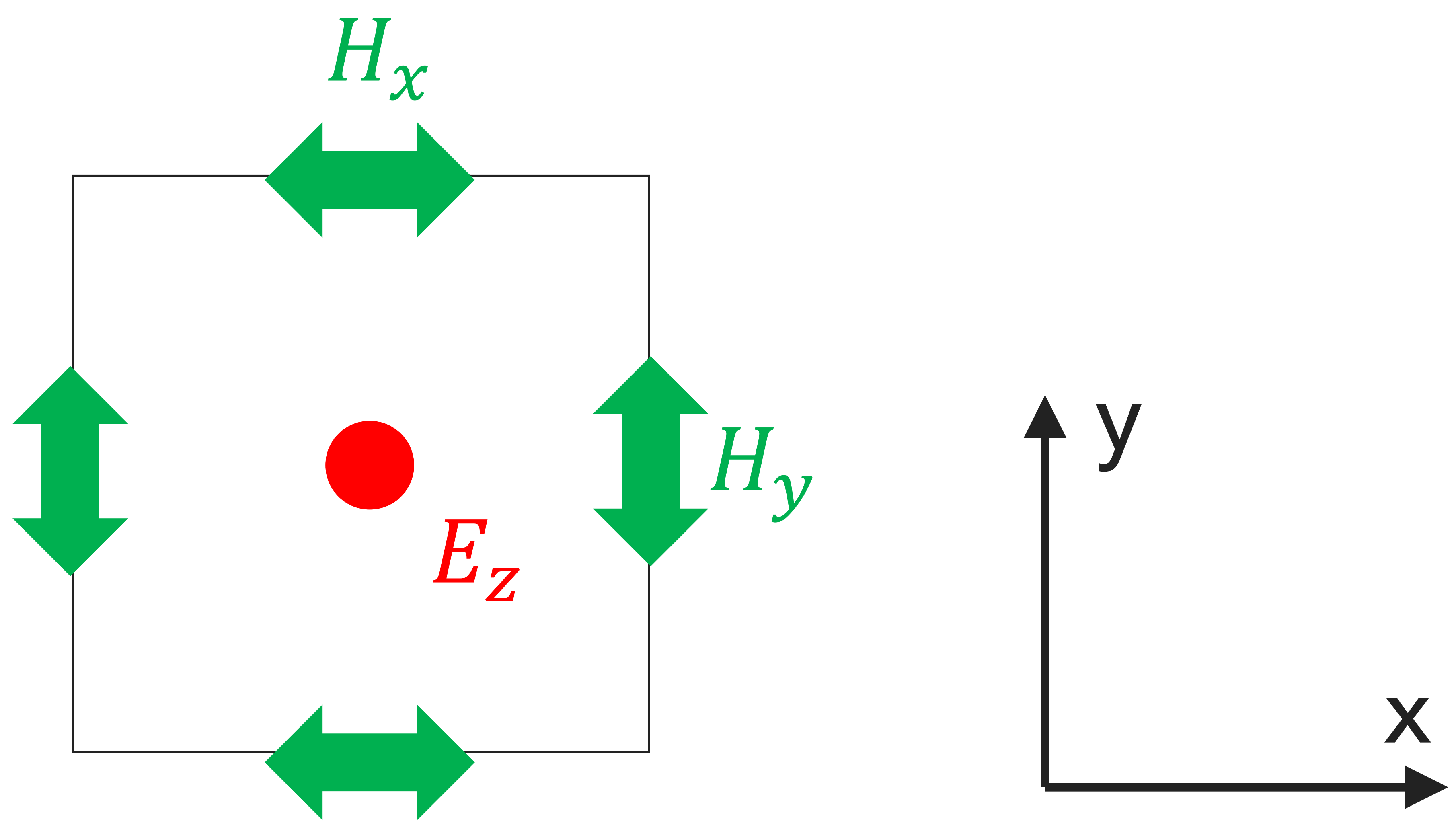
$\Delta t = 1.01 \Delta x / c$



Courant number: $C = 1.01$

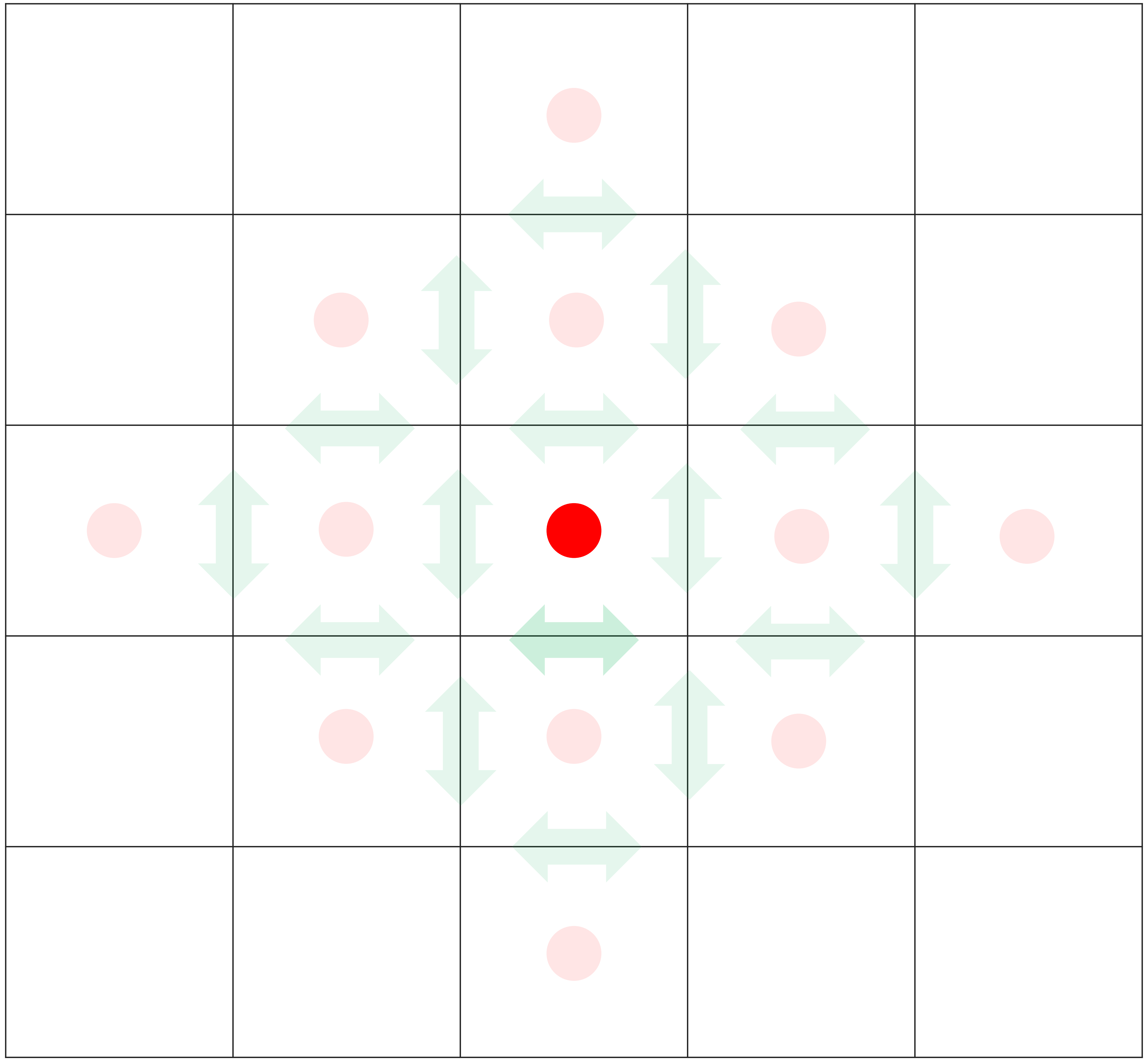
Extension to 2D

Yee-cell in 2D for TM polarization



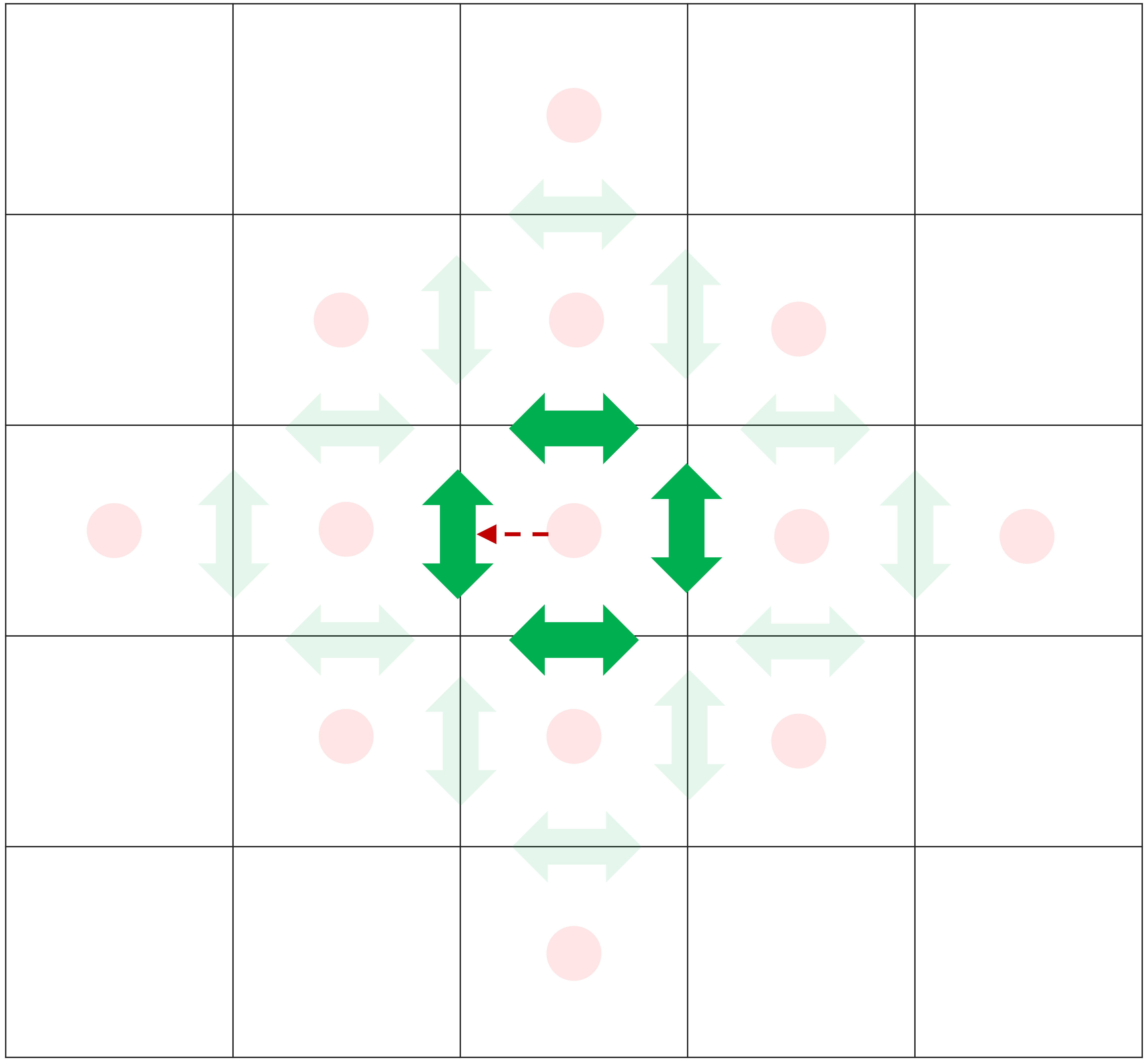
Time marching in 2D (isotropic)

$t = 0$



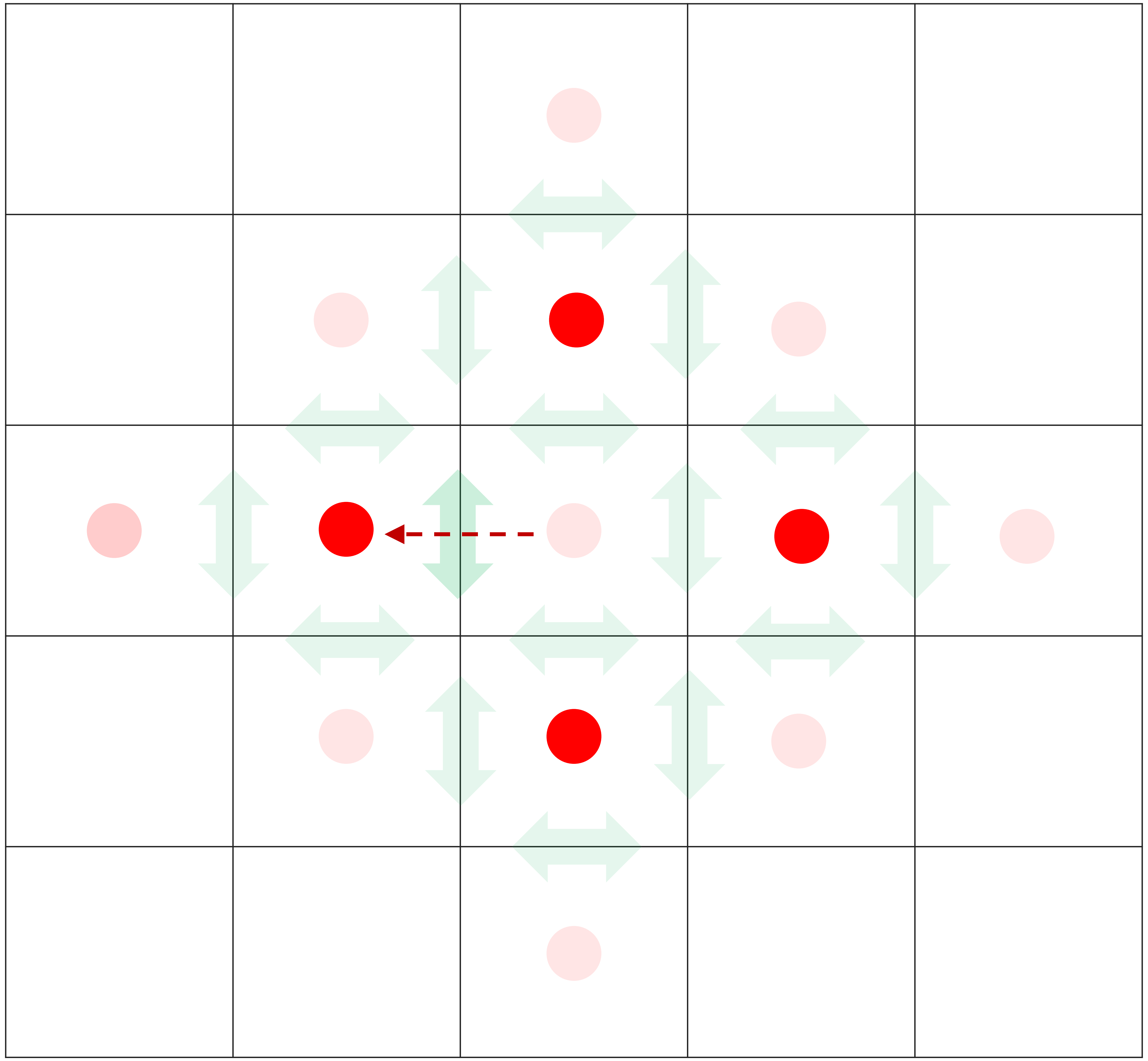
Time marching in 2D

$$t = \frac{\Delta t}{2}$$



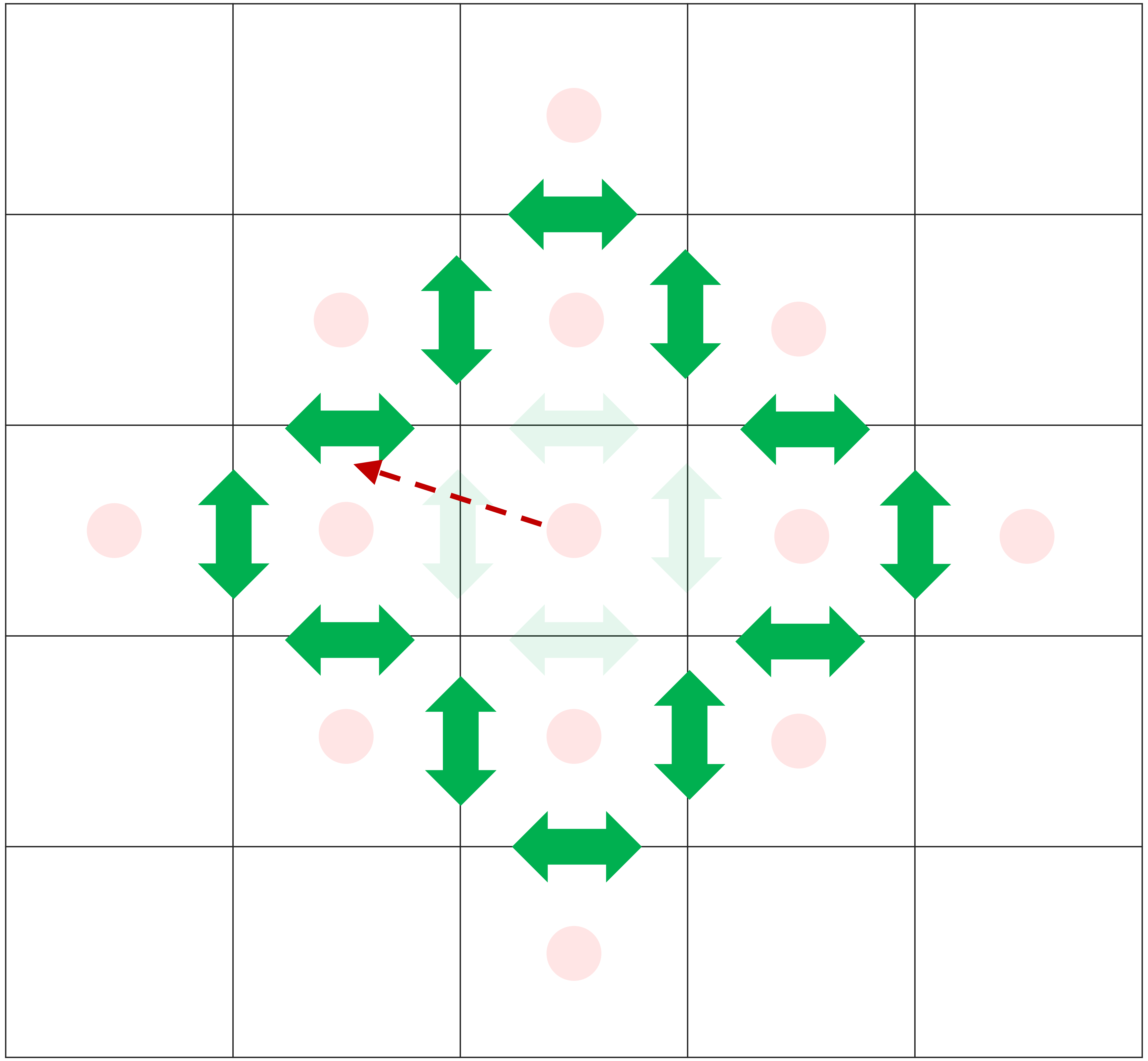
Time marching in 2D

$t = \Delta t$



Time marching in 2D

$$t = \frac{3\Delta t}{2}$$

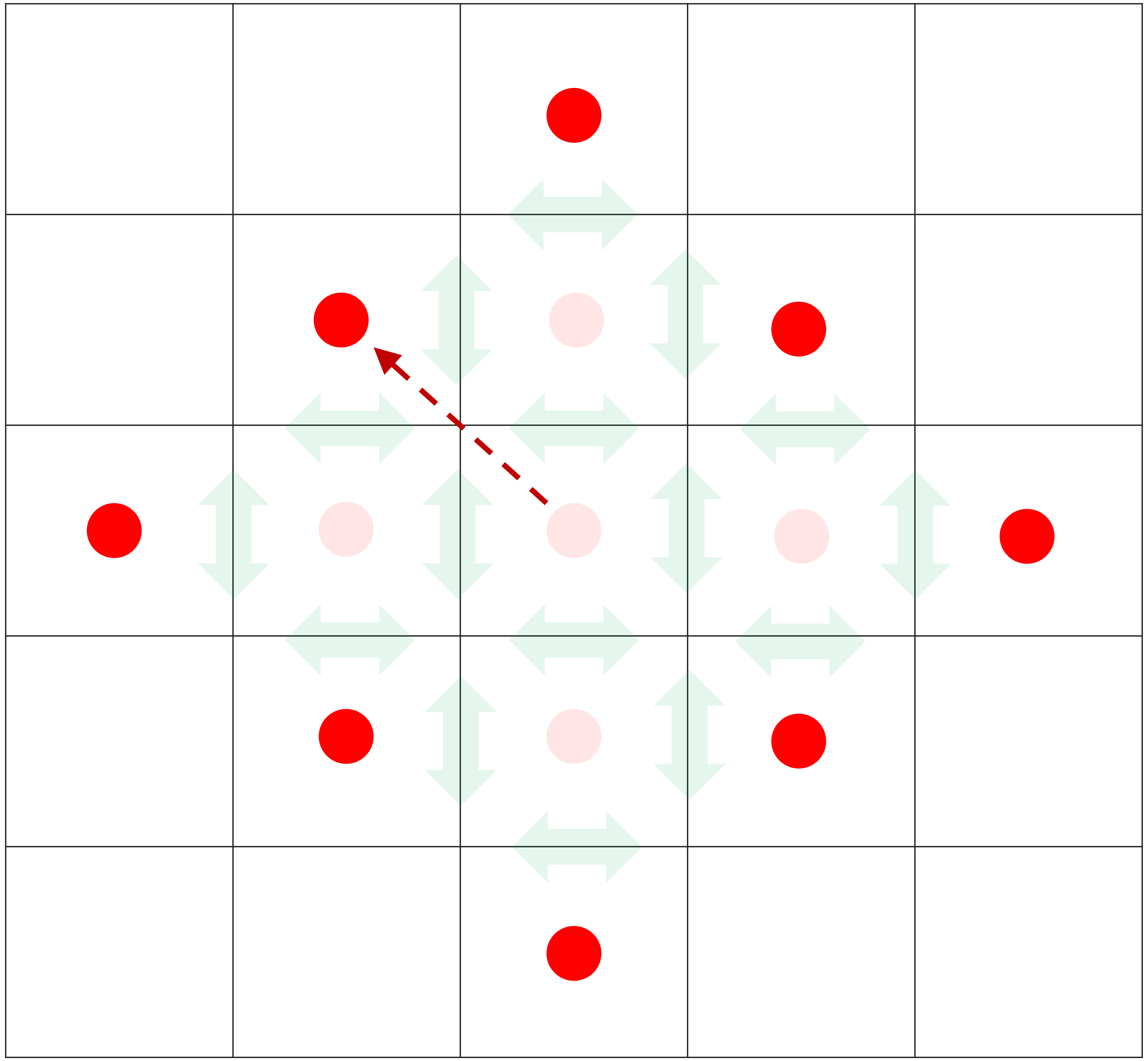


Time marching in 2D

$t = 2\Delta t$

Slowest numerical speed:

$$\frac{\sqrt{2}\Delta x}{2\Delta t}$$

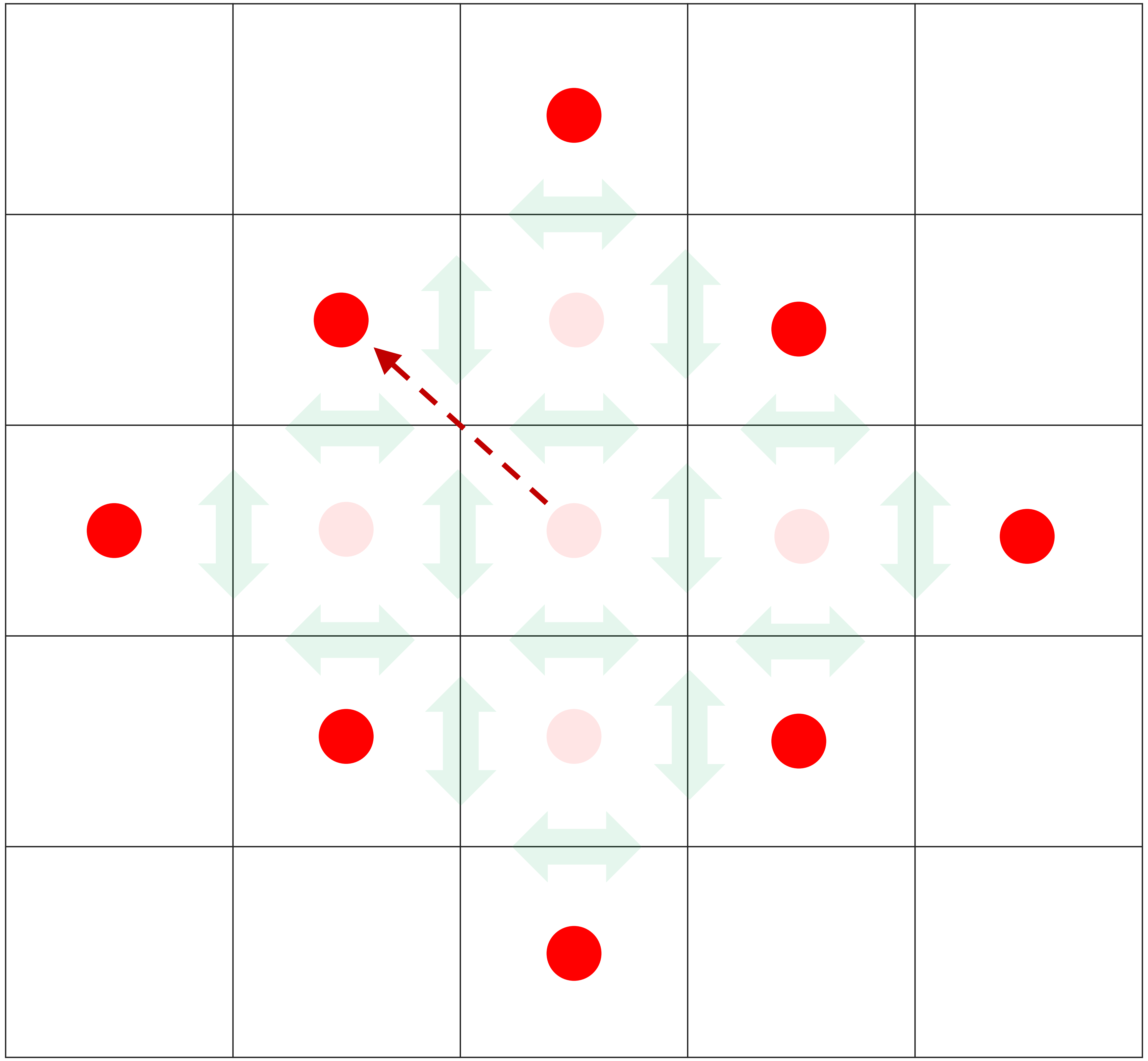


CFL condition in 2D

physical		Slowest numerical
c	$\leq$	$\dfrac{\sqrt{2}\Delta x}{2\Delta t}$

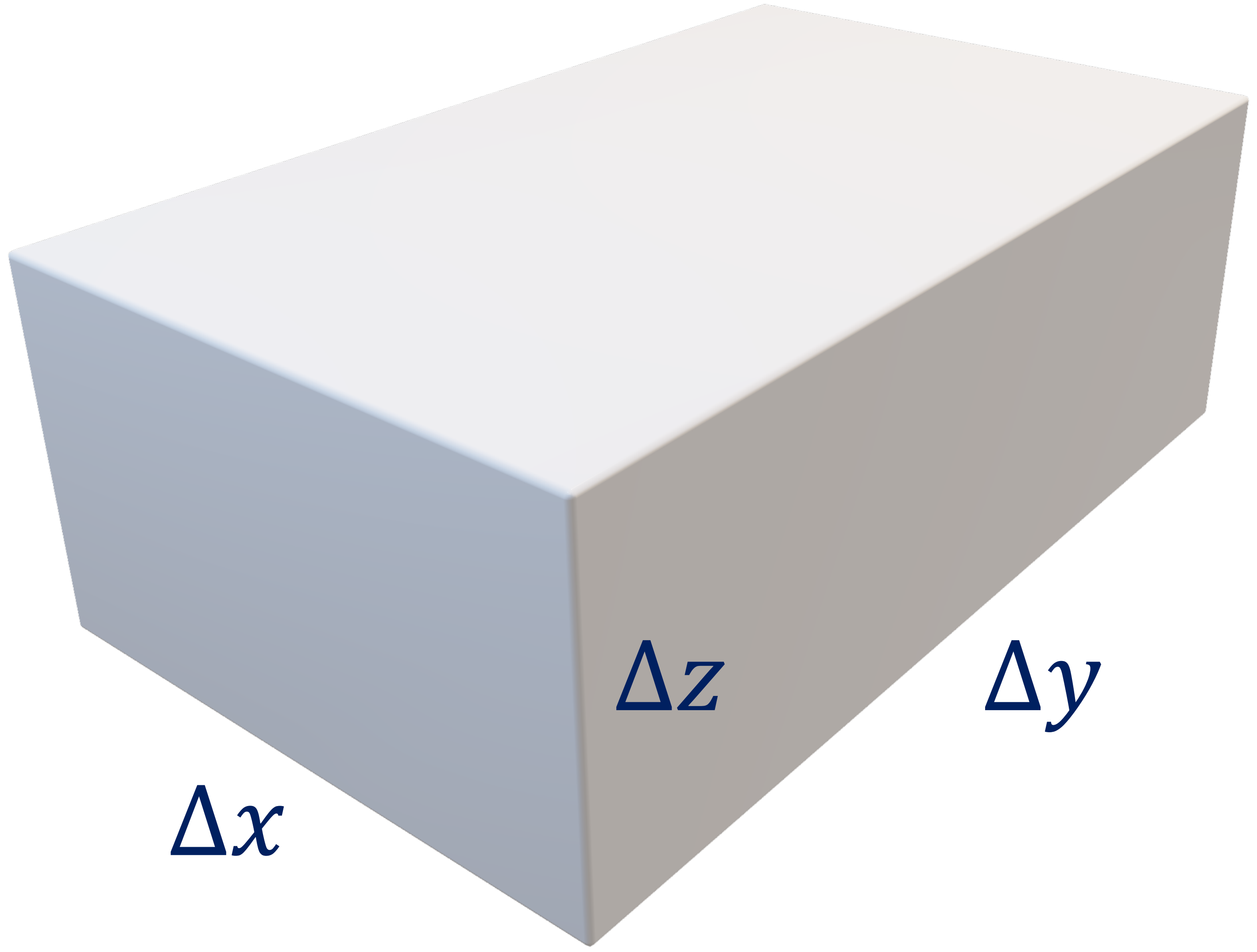
$$\Delta t \leq \frac{\Delta x}{\sqrt{2}c}$$

Courant number: $C = \frac{c\Delta t}{\Delta x} \leq \frac{1}{\sqrt{2}}$



General result for CFL condition in 3D

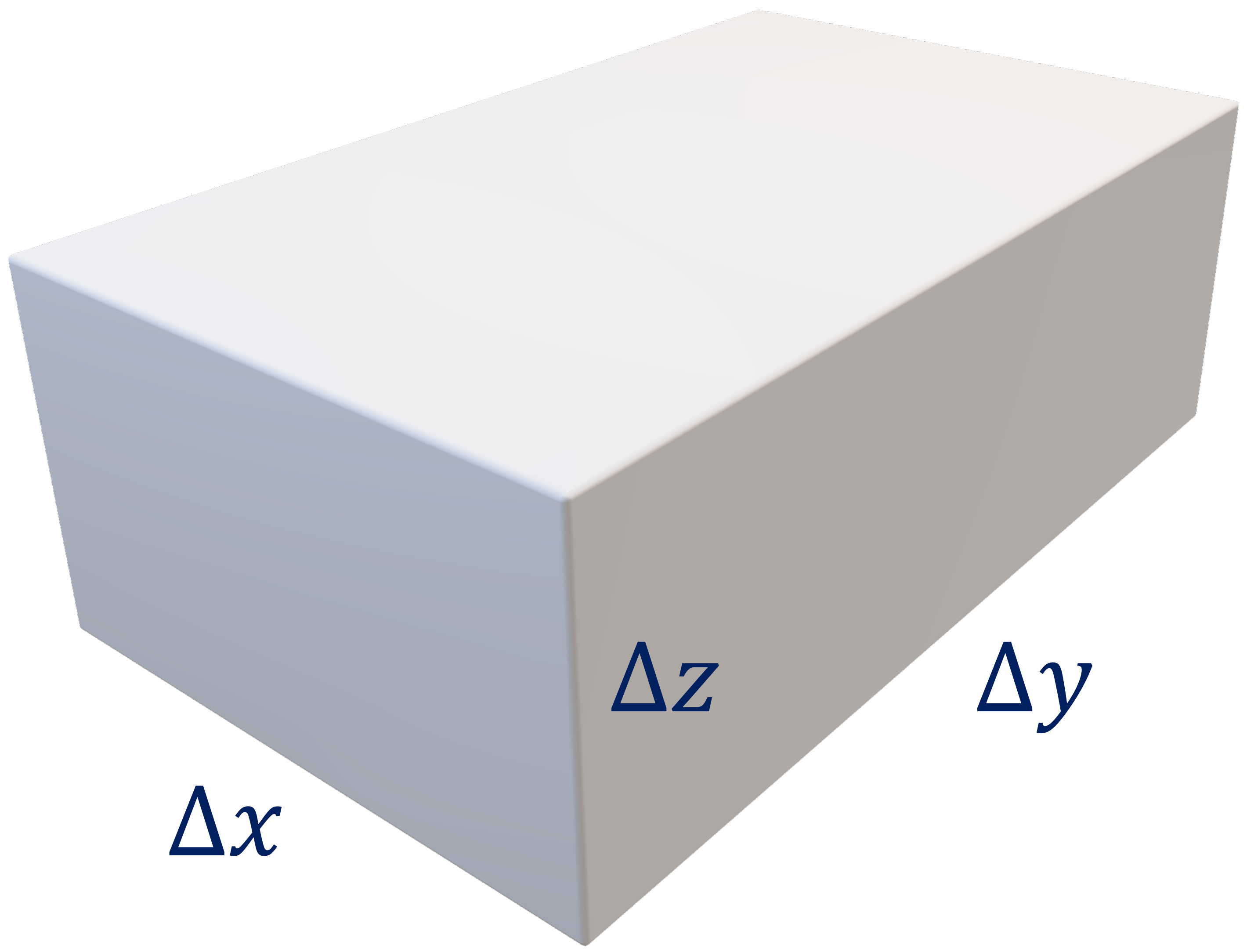
Index of refraction: n



$$\Delta t \leq \frac{n}{c\sqrt{\frac{1}{\Delta x^2}+\frac{1}{\Delta y^2}+\frac{1}{\Delta z^2}}}$$

Practical implementations

Index of refraction: n



$$\Delta t \leq \frac{n}{c\sqrt{\frac{1}{\Delta x^2}+\frac{1}{\Delta y^2}+\frac{1}{\Delta z^2}}}$$

- Nonuniform mesh: time step size is determined by the smallest grid size
- Computational cost for uniform isotropic mesh:
    - Number of grid: $\sim\frac{1}{\Delta x^3}$
    - Time marching steps: $\sim\frac{1}{\Delta x}$
    - In total: $\sim\frac{1}{\Delta x^4}$
    - As an example, reducing the grid size by half leads to increase of total cost by 16 times.