The Basics of Turbulence Modeling

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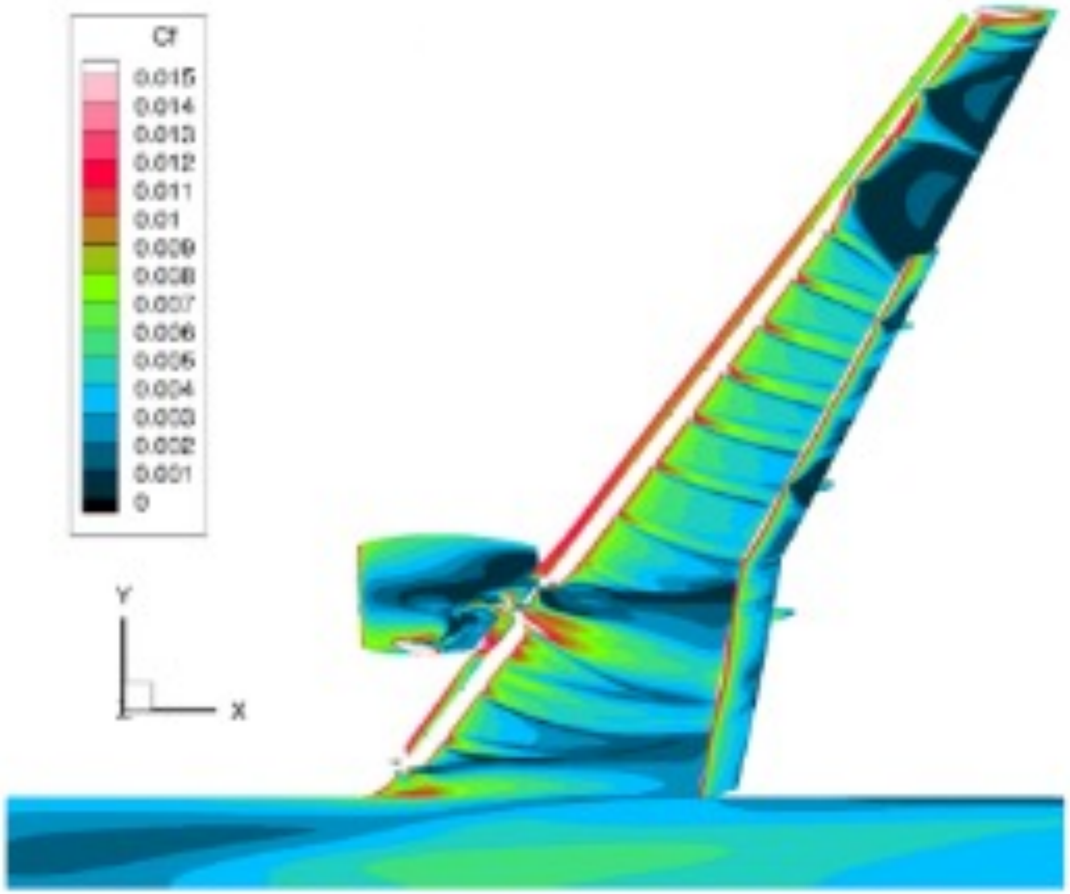
Outline

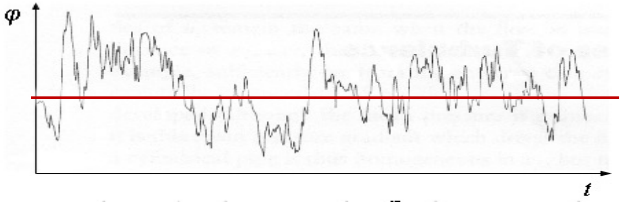
- The Reynolds-Averaged Navier-Stokes (RANS) equations
- The power and the paradox of Reynolds-Averaging and Turbulence Modeling
- The nature of a turbulence model

State of the Art on a Complex Geometry



The Reynolds-Averaged Navier-Stokes Equations

- The true velocity field is $u(x, y, z, t)$
- For simplicity, assume we average it in time, giving $U(x, y, z)$, also called $\overline{u}$
- Define the fluctuation $u'(x, y, z, t) \equiv u(x, y, z, t) - U(x, y, z)$
  - So, " $u = U + u'$ " and $\overline{u'} = 0$



- Take the incompressible Navier-Stokes equations
- The continuity condition is $\partial u_i / \partial x_i$ =0, and its average is $\partial U_i / \partial x_i$ =0: no change
- The momentum equation is (setting $\rho = 1$):

$$\frac{Du_i}{Dt} = \frac{\partial u_i}{\partial t} + u_j \frac{\partial u_i}{\partial x_j} = -\frac{\partial p}{\partial x_i} + \nu \frac{\partial^2 u_i}{\partial x_j \partial x_j}$$

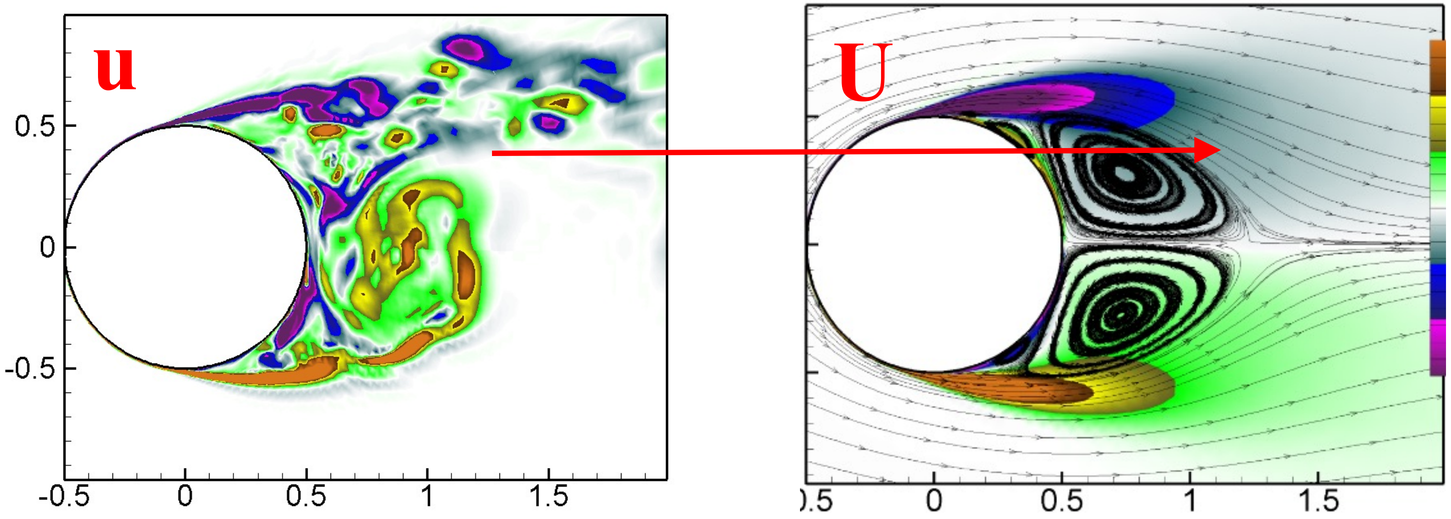
- Average this equation in time:

$$\frac{\overline{D}U_i}{Dt} = \qquad U_j \frac{\partial U_i}{\partial x_j} = -\frac{\partial \overline{p}}{\partial x_i} + \nu \frac{\partial^2 U_i}{\partial x_j \partial x_j} - \frac{\partial \left(\overline{u_i' u_j'}\right)}{\partial x_j}$$

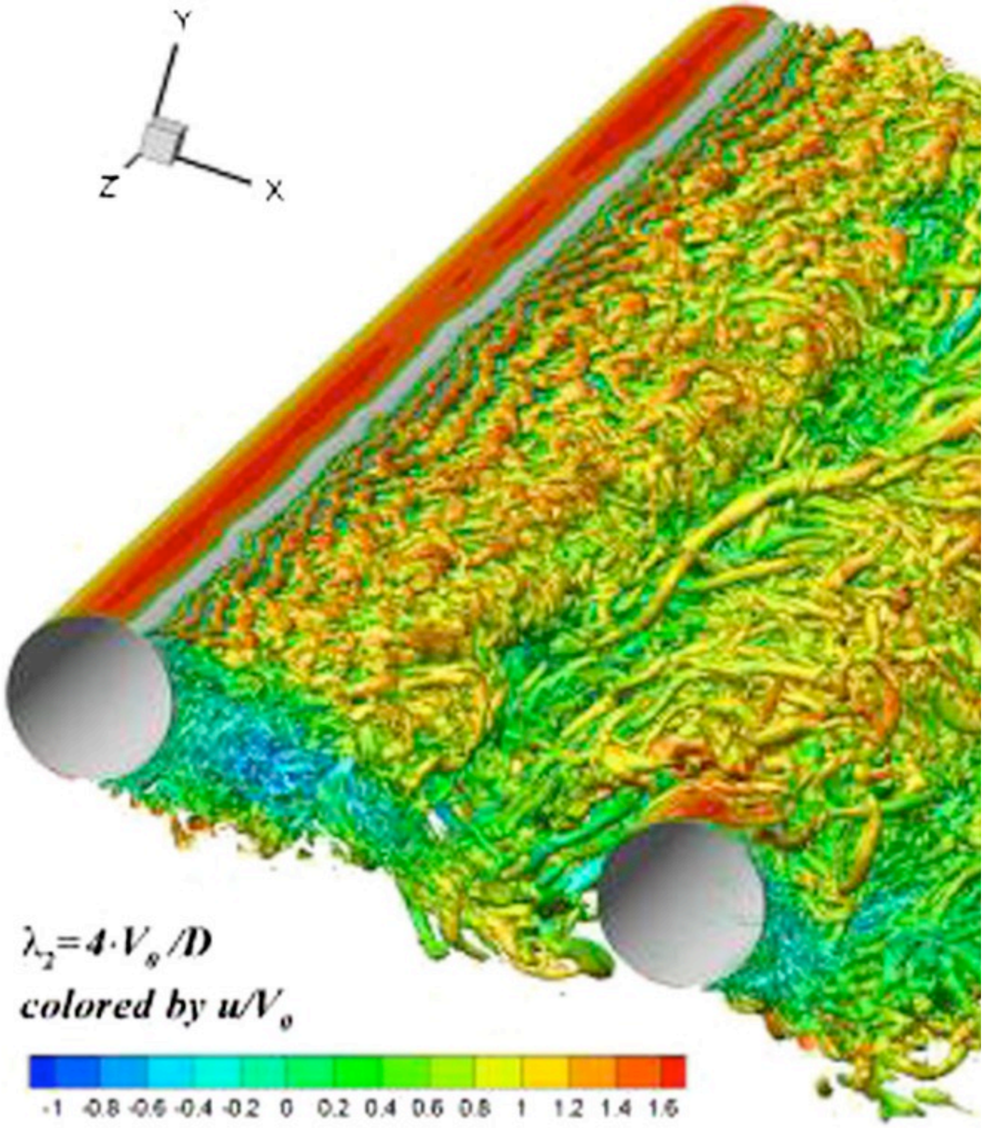
- The quantity $\overline{u_i' u_j'}$ created by the nonlinearity is the "Reynolds Stress"

The Power and Paradox of Turbulence Modeling

- Averaging is a valid procedure. However…



- Some very different *u(x,y,z,t)* conditions enter the single time-average at a given point, *U(x,y)*
- You could describe the Reynolds-averaged flow as the flow of a fluid with "very strange" internal stresses
  - These stresses are not a property of the fluid, unlike the viscous stresses; they depend on the entire history of the flow
  - The job of a turbulence model is to give the stresses, knowing only *U*! *Hoping for a rigorous approximation is not reasonable*



Strelets work

The Nature of a Turbulence Model

- The clear need of CFD is to have a simple and local mathematical model of the "very strange" Reynolds stresses!
- Most common is the artificial concept of eddy viscosity ν_t, due to Boussinesq in 1877
  - $-\overline{u_i' u_j'} = \nu_t(\partial U_i/\partial x_j + \partial U_j/\partial x_i)$
  - Very often, ν_t is larger than ν by orders of magnitude. It is different in every flow
- An example: the Spalart-Allmaras model of 1992 (basic form in boundary layer)

$$\frac{D\nu_t}{Dt} = c_{b1}\left|\frac{\partial U}{\partial y}\right|\nu_t - c_{w1}\left(\frac{\nu_t}{y}\right)^2 + \frac{1}{\sigma}\left[\frac{\partial}{\partial y}\left(\nu_t\frac{\partial \nu_t}{\partial y}\right) + c_{b2}\left(\frac{\partial \nu_t}{\partial y}\right)^2\right]$$

  - Where y is the distance from the wall
  - Every term is arbitrary!
  - They express "Evolution = Production - Wall Confinement + Diffusion"
  - The c and σ constants were adjusted to work in a few simple cases
  - The model was inspired by Baldwin and Barth at NASA
  - It shares much with the ν_t-92 model of Secundov at TsAGI in Moscow
- Other common models have two equations, seven equations…
  - The more elaborate models have equations for six Reynolds stresses + dissipation. No ν_t!