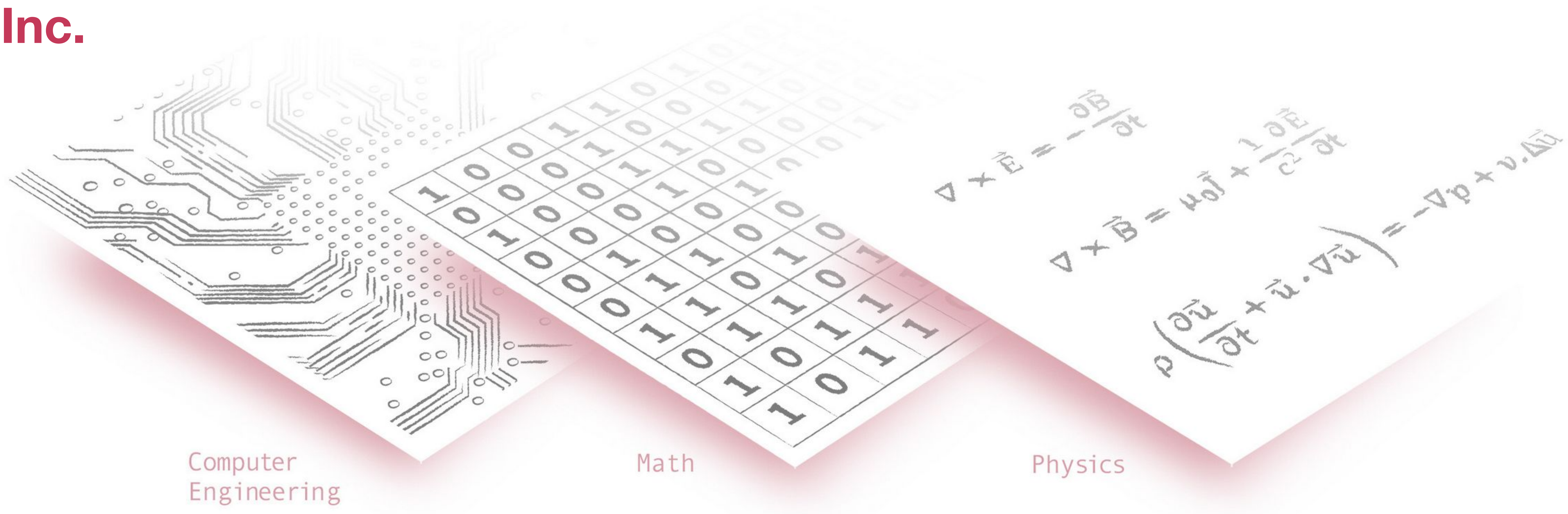
INTRO TO FDTD (6)

Flexcompute **Inc.**

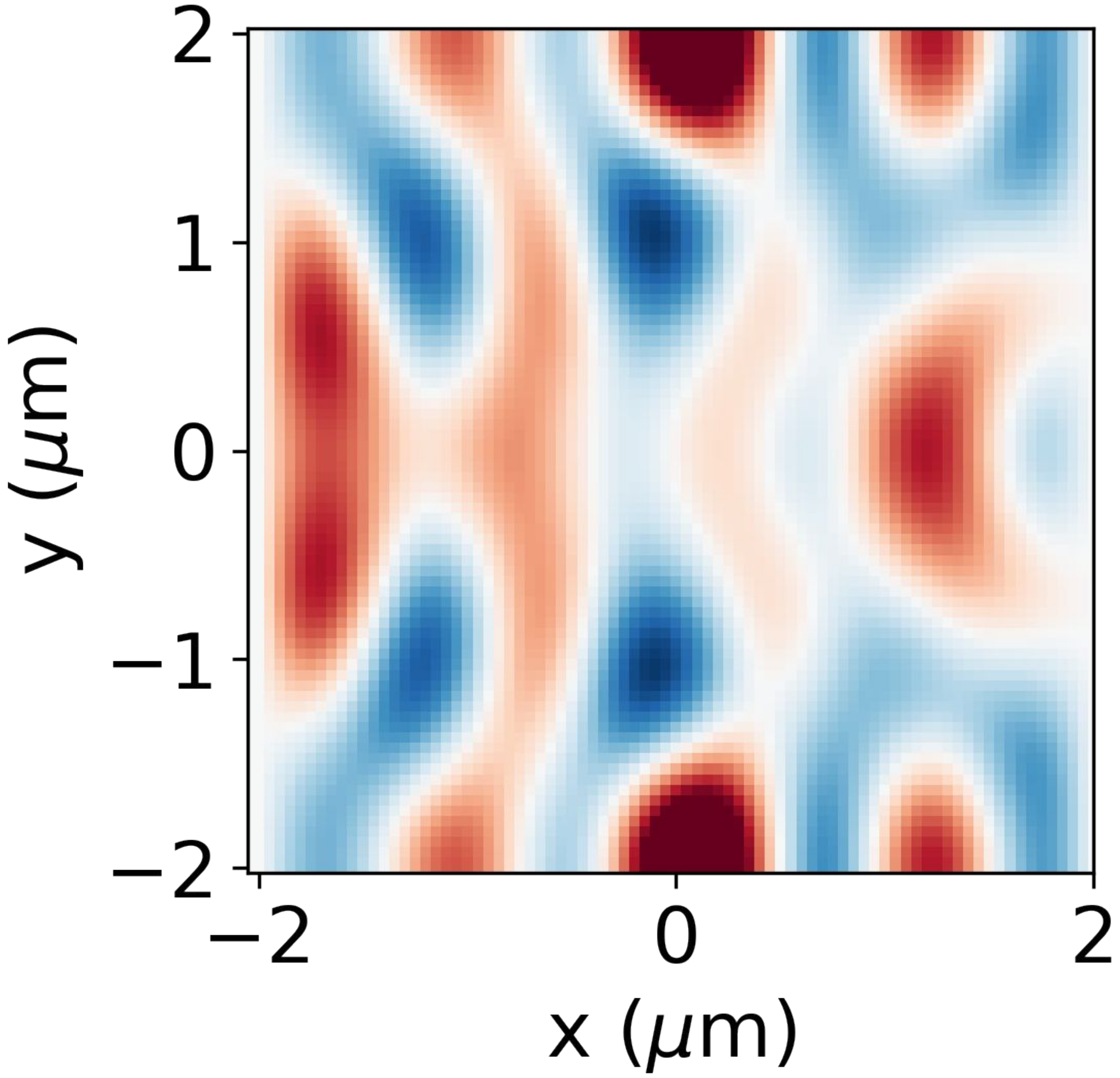


Tidy3D

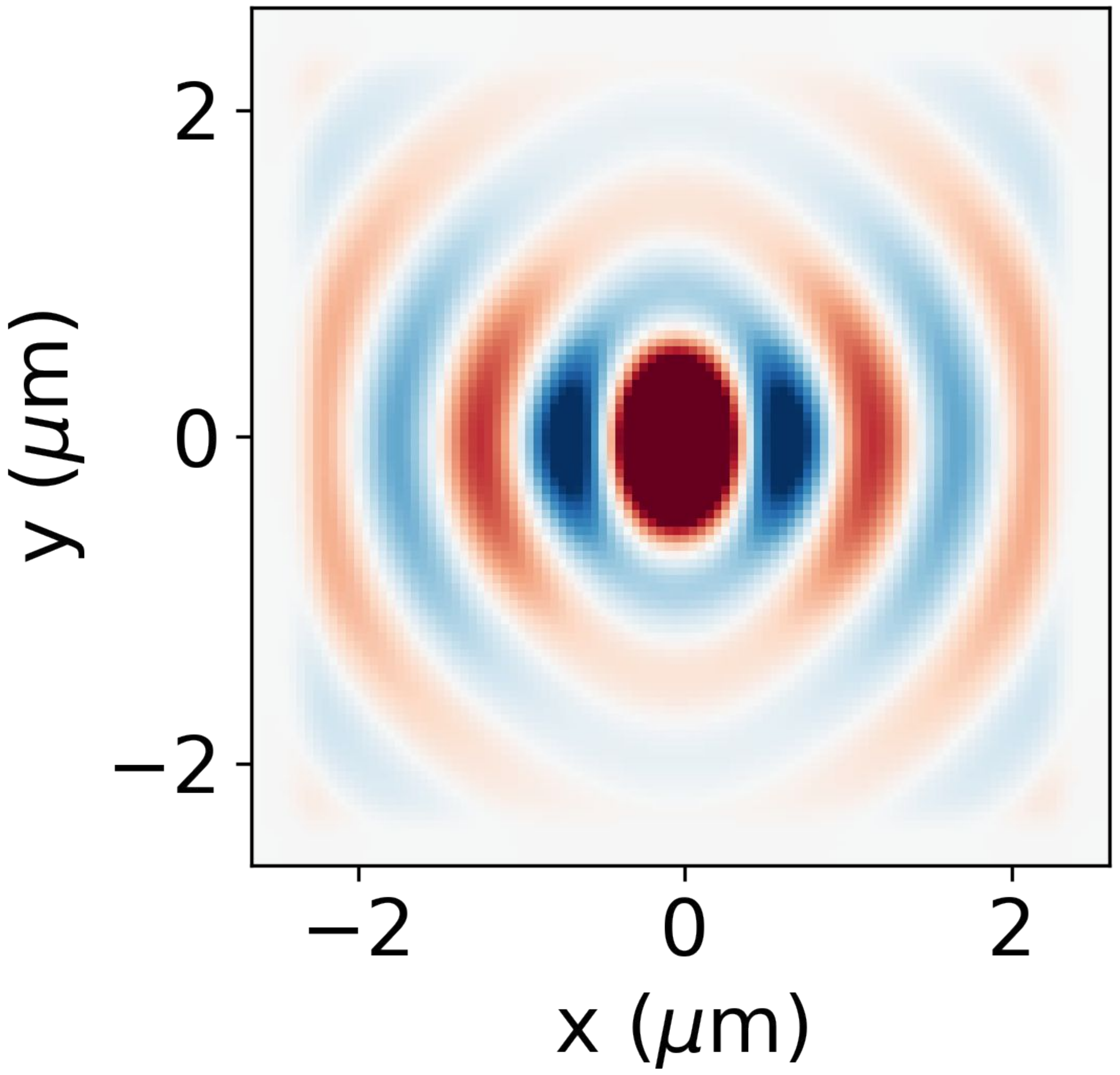
How to truncate unbounded spatial domain?

- Requirement: no reflection for incidence over all angles at the boundary.

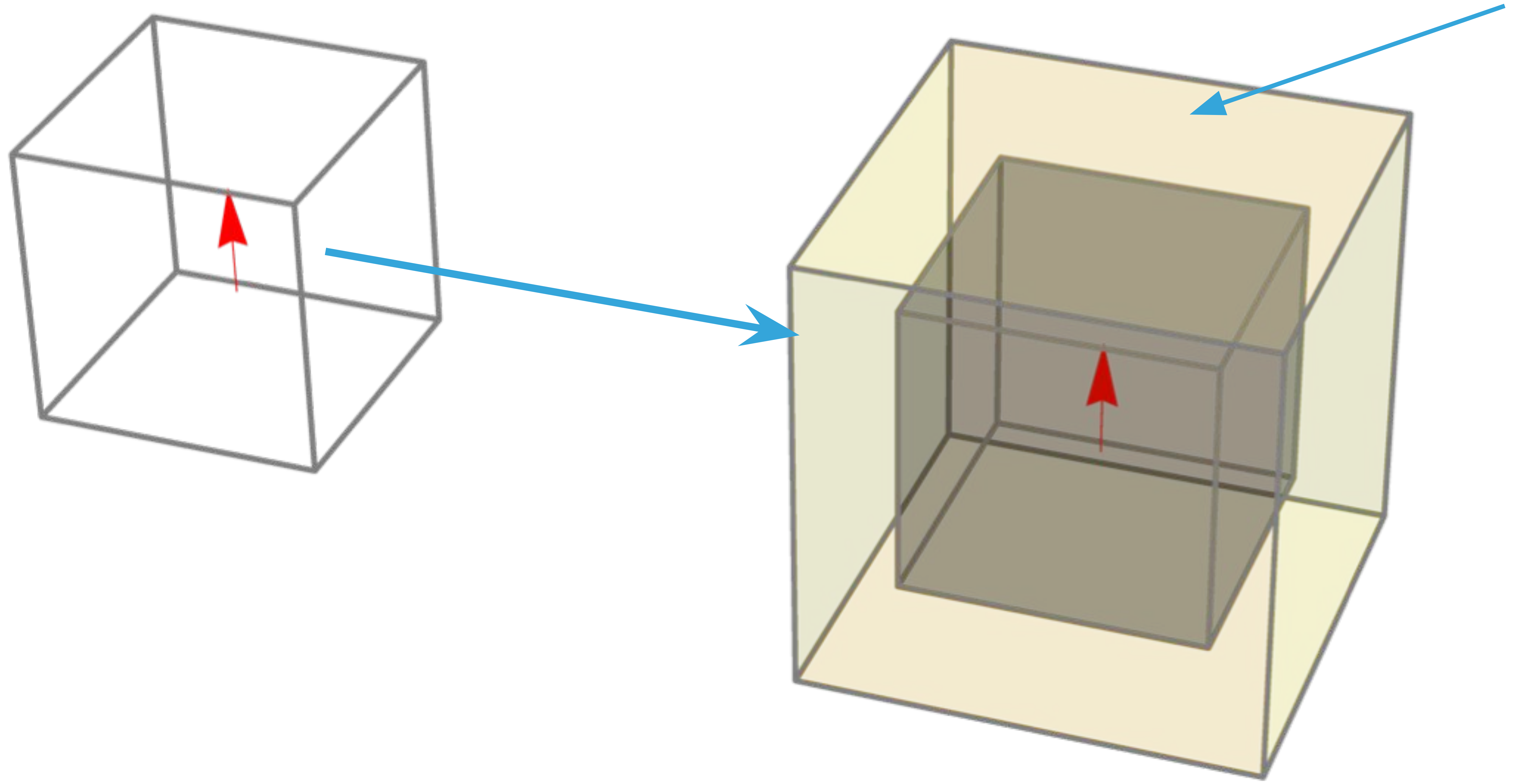
Dipole in free space



PEC boundary



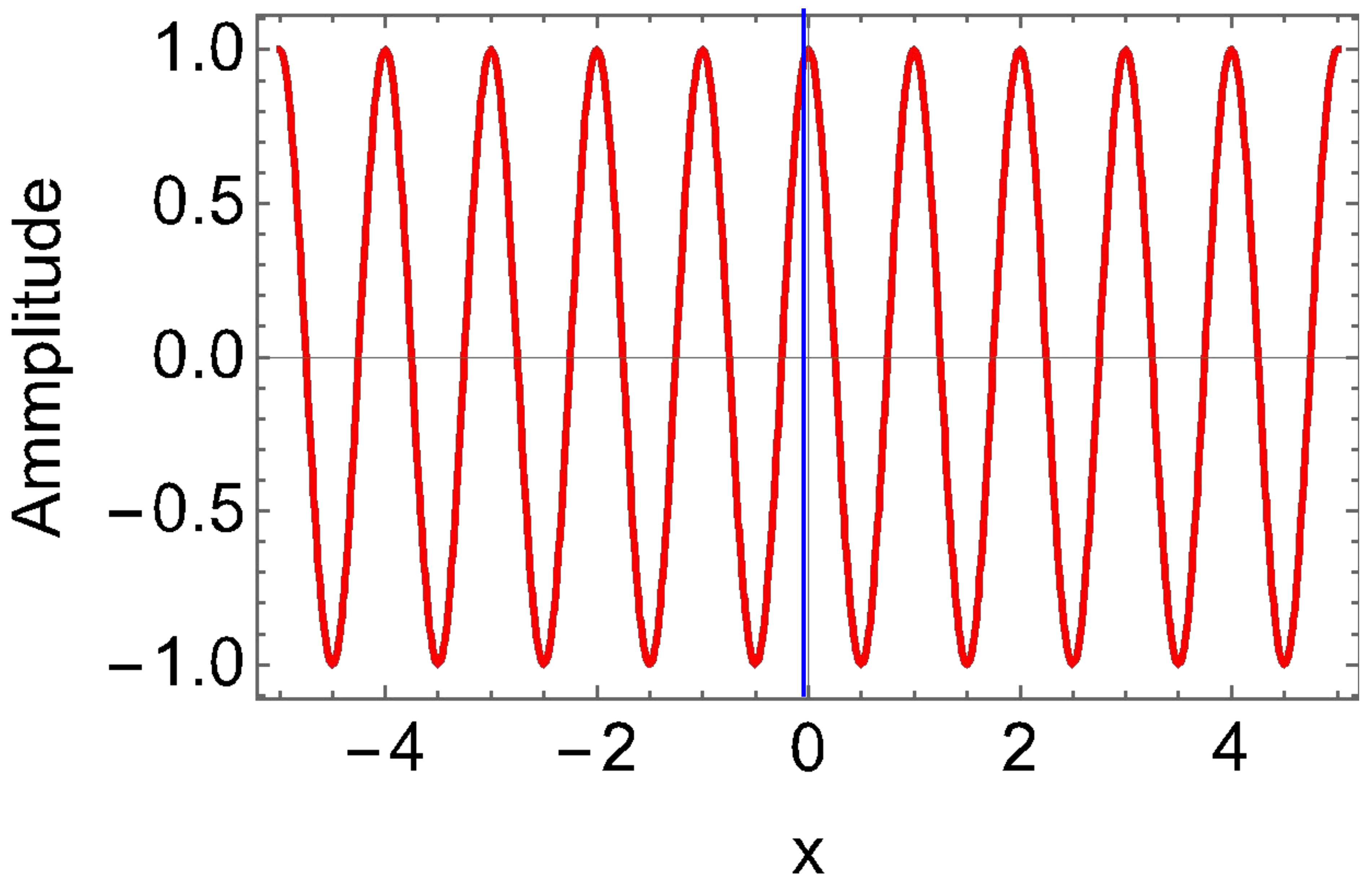
Expected field pattern



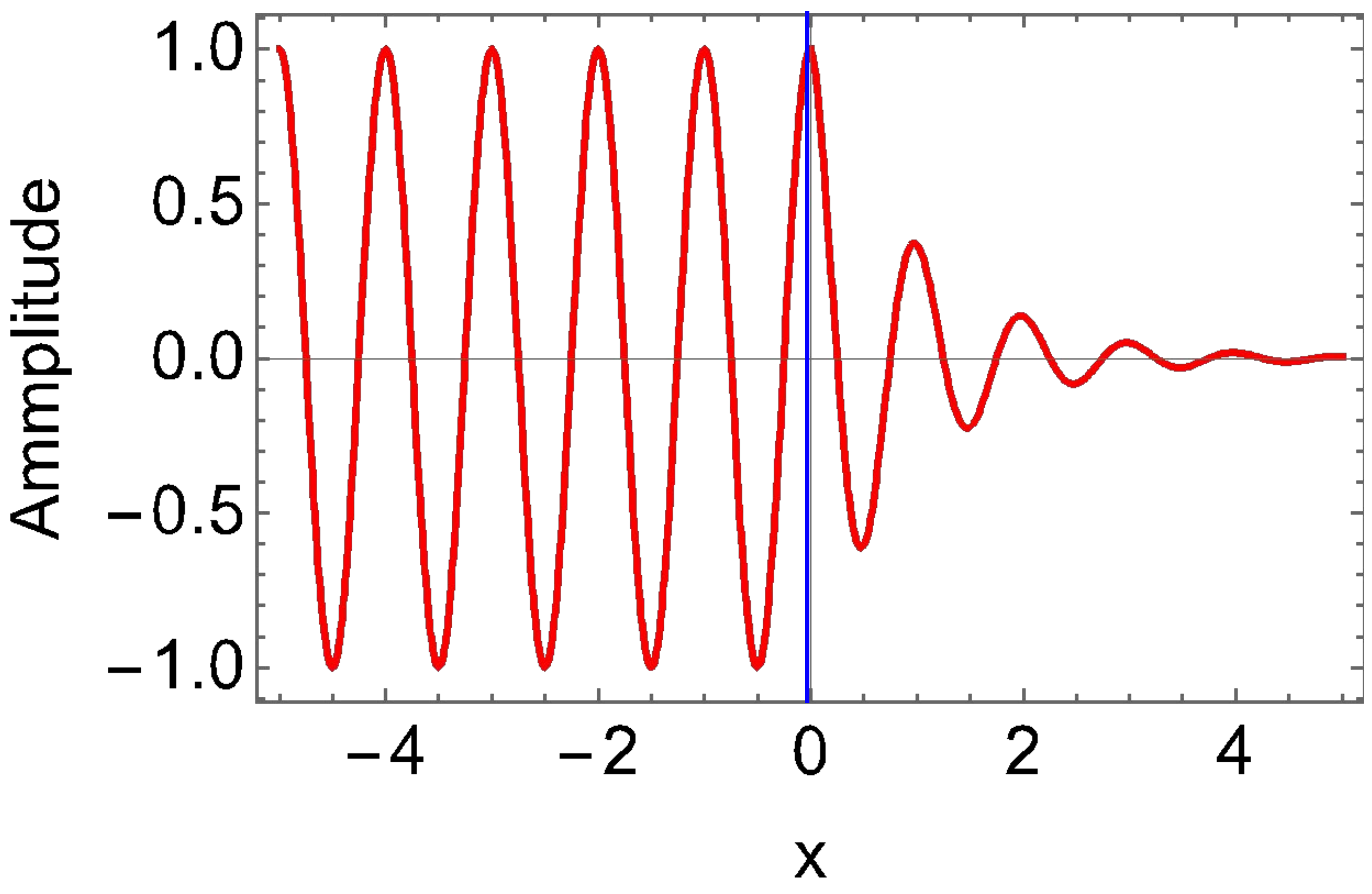
PML: A lossy material that complete absorbs incoming wave from all angles of the incidence without any reflection.

Idea for PML: Coordinate transformation

Vacuum



Vacuum PML



Use coordinate transformation to transform a propagating wave into an attenuating wave

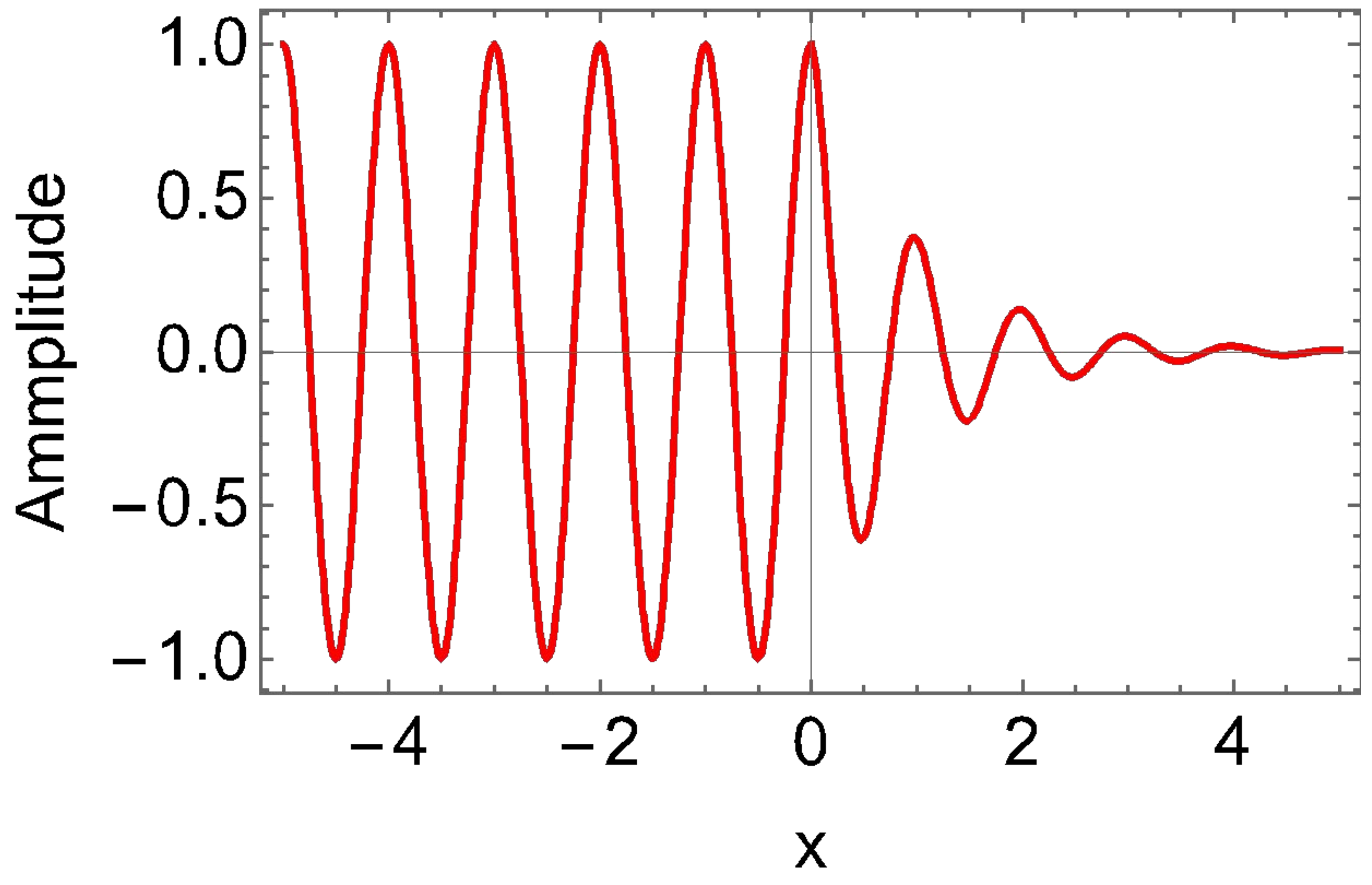
1D illustration:

- Consider a transformed wave equation: $\left(\left(\frac{1}{s(x)}\frac{\partial}{\partial x}\right)^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)E = 0$

  where $s(x) = \begin{cases} 1, x < 0 \\ 1 - \frac{\sigma}{i\omega\epsilon_0}, x > 0 \end{cases}$

- Solution:

$$E(x) = e^{i[k\int^x s(x')dx\prime - \omega t]} = \begin{cases} e^{i(kx-\omega t)}, x < 0 \\ e^{i(kx-\omega t)} \times e^{-\frac{\sigma}{c\varepsilon_0}x}, x > 0 \end{cases}$$



- No reflection
- Frequency-independent exponential attenuation

[W. C. Chew and W. H. Weedon, Microwave and Optical Tech. Lett., 7 (13), 599,1994; S. Johnson, arXiv 2108.05348, 2021]

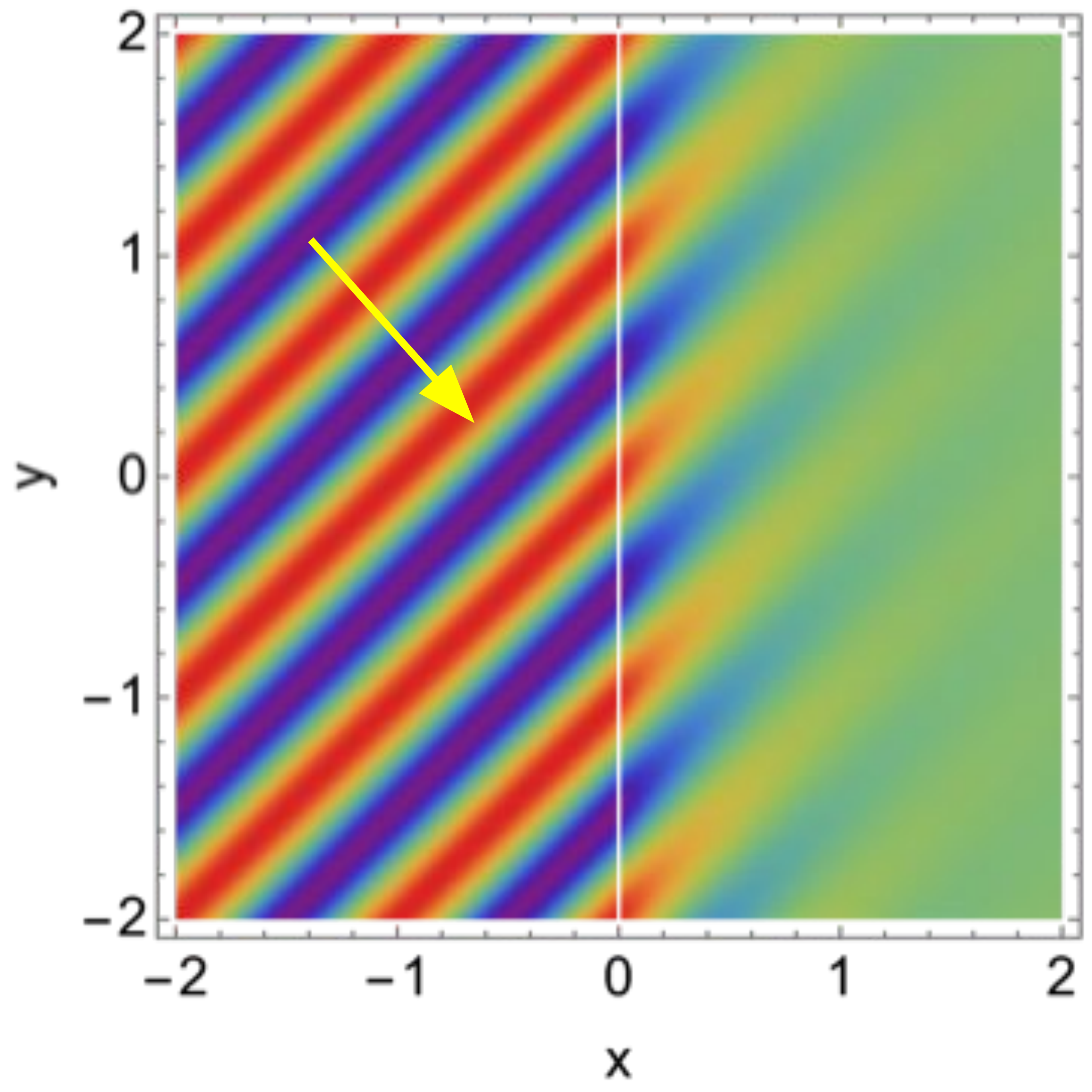
Higher dimensional case:

- Consider a transformed wave equation: $\left(\left(\frac{1}{s(x)}\frac{\partial}{\partial x}\right)^2 + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)E = 0$

  where $s(x) = \begin{cases} 1, x < 0 \\ 1 - \frac{\sigma}{i\omega\epsilon_0}, x > 0 \end{cases}$

- Solution:

  $E(x) = e^{i[k_x \int^x s(x')dx' + k_y y + k_z z - \omega t]}$



- No reflection for every angle of incidence
- Attenuation in the PML for every angle of incidence

Practical considerations:

- PML is reflectionless in the exact wave equation even with a discontinuous jump in conductivity, but reflection is introduced in solving for the discretized problem.
  Solution: tapering the conductivity profile in real space.

- Finite truncation of PML region: the exponential field tail can reflect off the boundary.
  Solution: more PML layers.

- Many different PML formulations: for example, complex-frequency shifted PML

$$s(x) = \kappa(x) + \frac{\sigma(x)}{\alpha(x) - j\omega\varepsilon_0}$$

❖ σ: attenuation
❖ κ: scaling factor.
❖ α: complex frequency shift for evanescent wave attenuation.

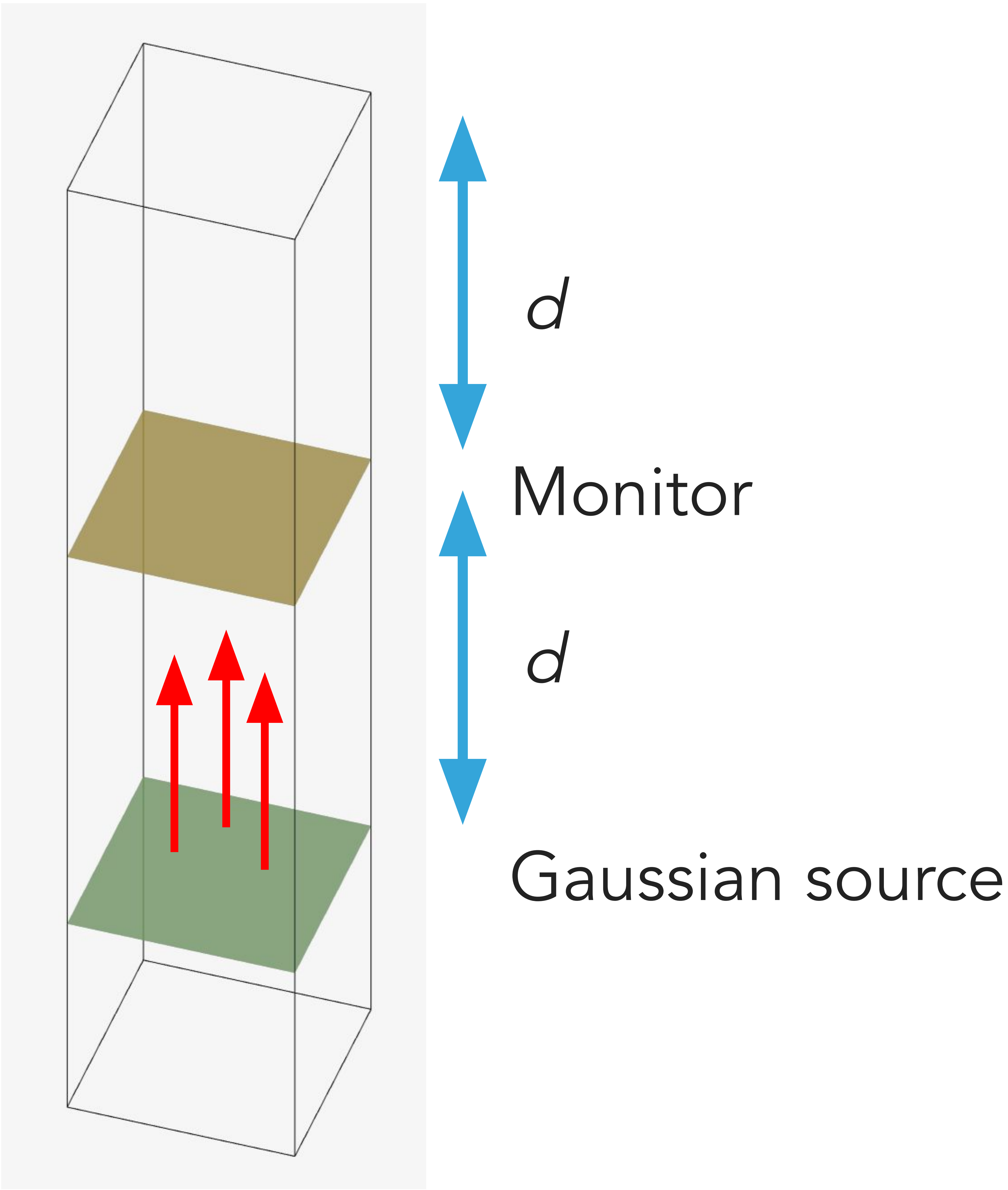
[Antonios Giannopoulos, IEEE Transactions on Antennas and Propagation, 56(9), 2995, 2008]

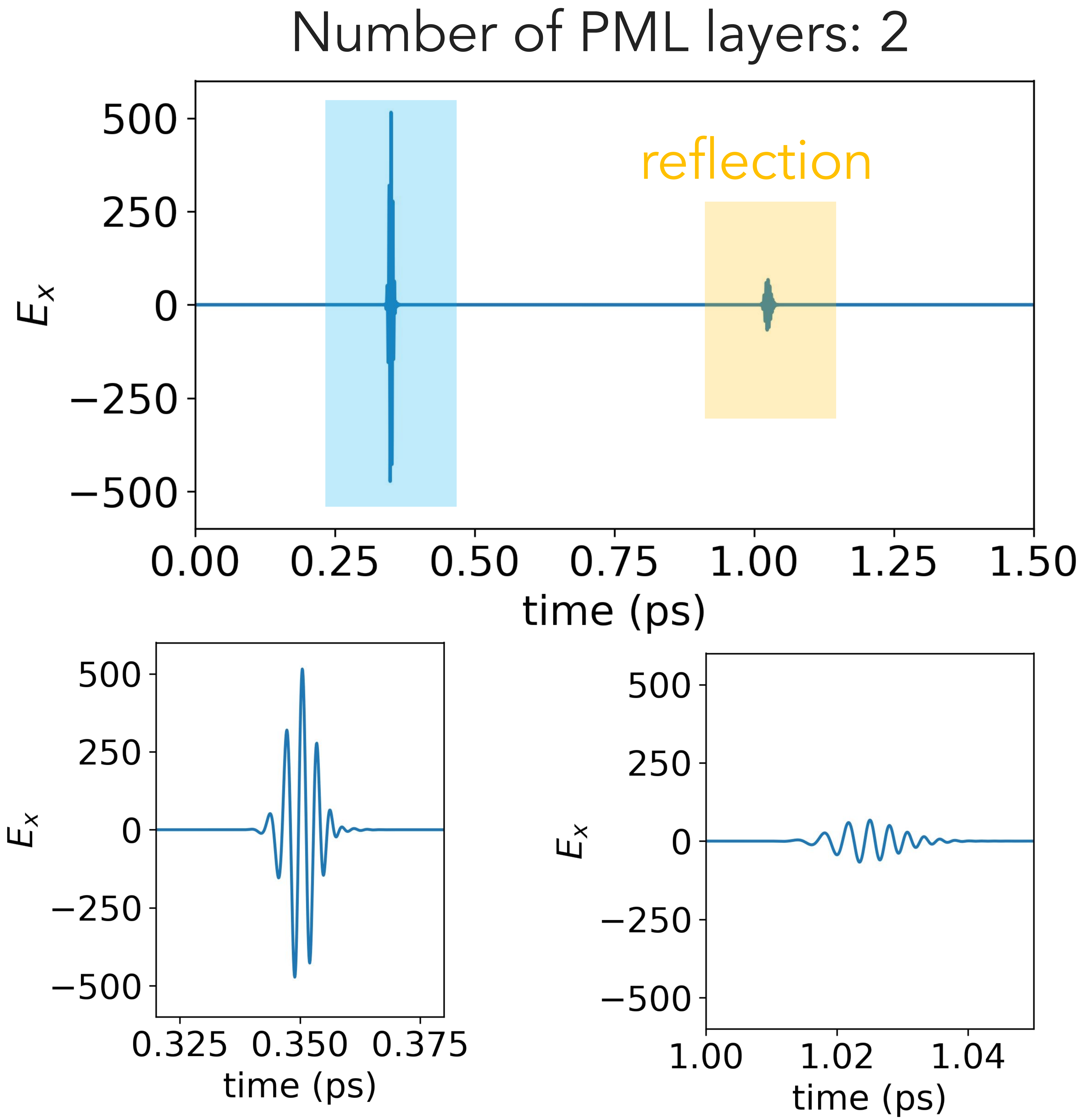
Impact of parasitic reflection on computation:

- $F \propto |\boldsymbol{E}|^2$
- $F' \propto |\boldsymbol{E} + \delta\boldsymbol{E}|^2 = F + \boxed{2Re[\boldsymbol{E}^* \cdot \delta\boldsymbol{E}]} + |\delta\boldsymbol{E}|^2$

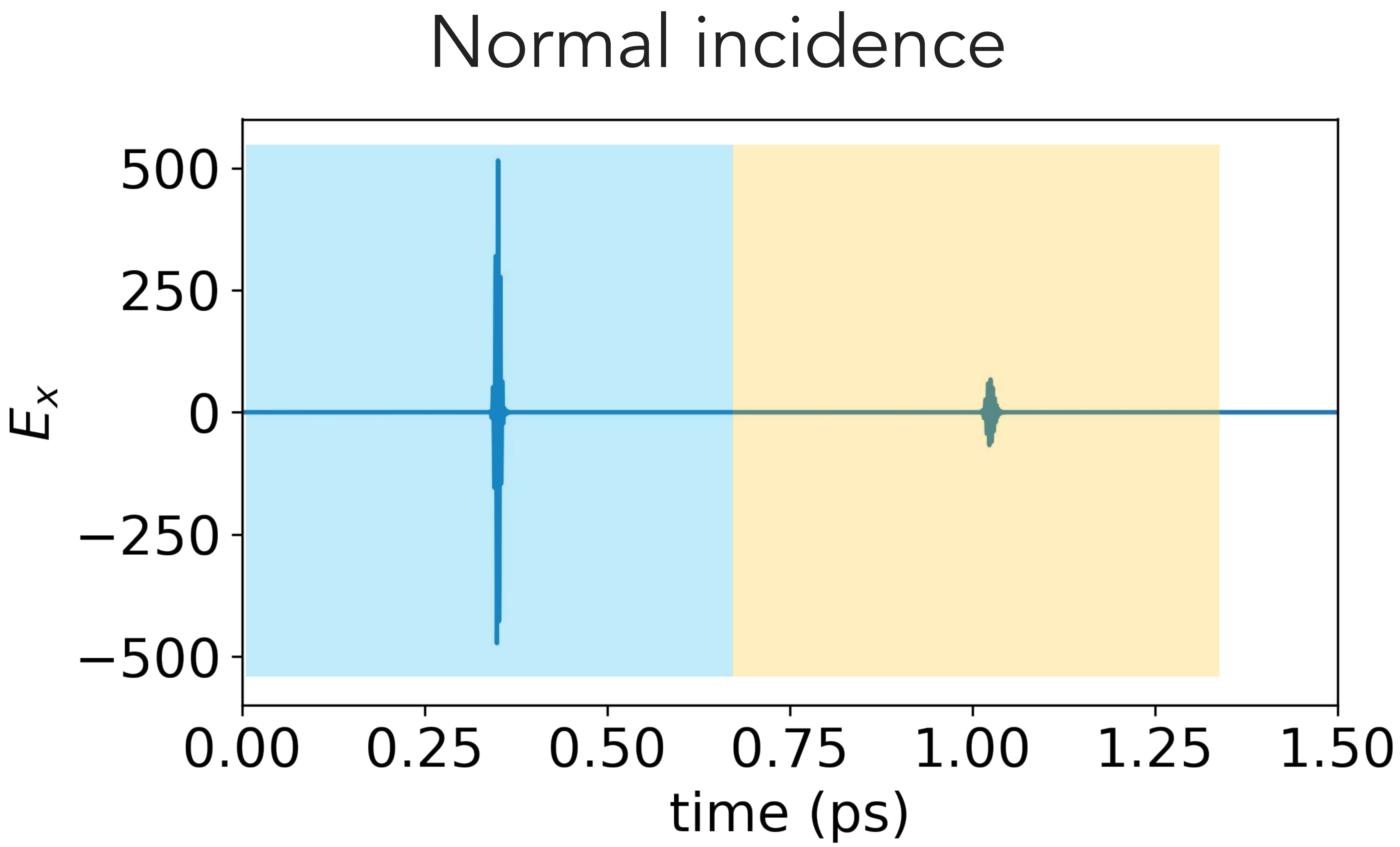
Error is proportional to reflection amplitude!

Measure reflection at the PML boundary

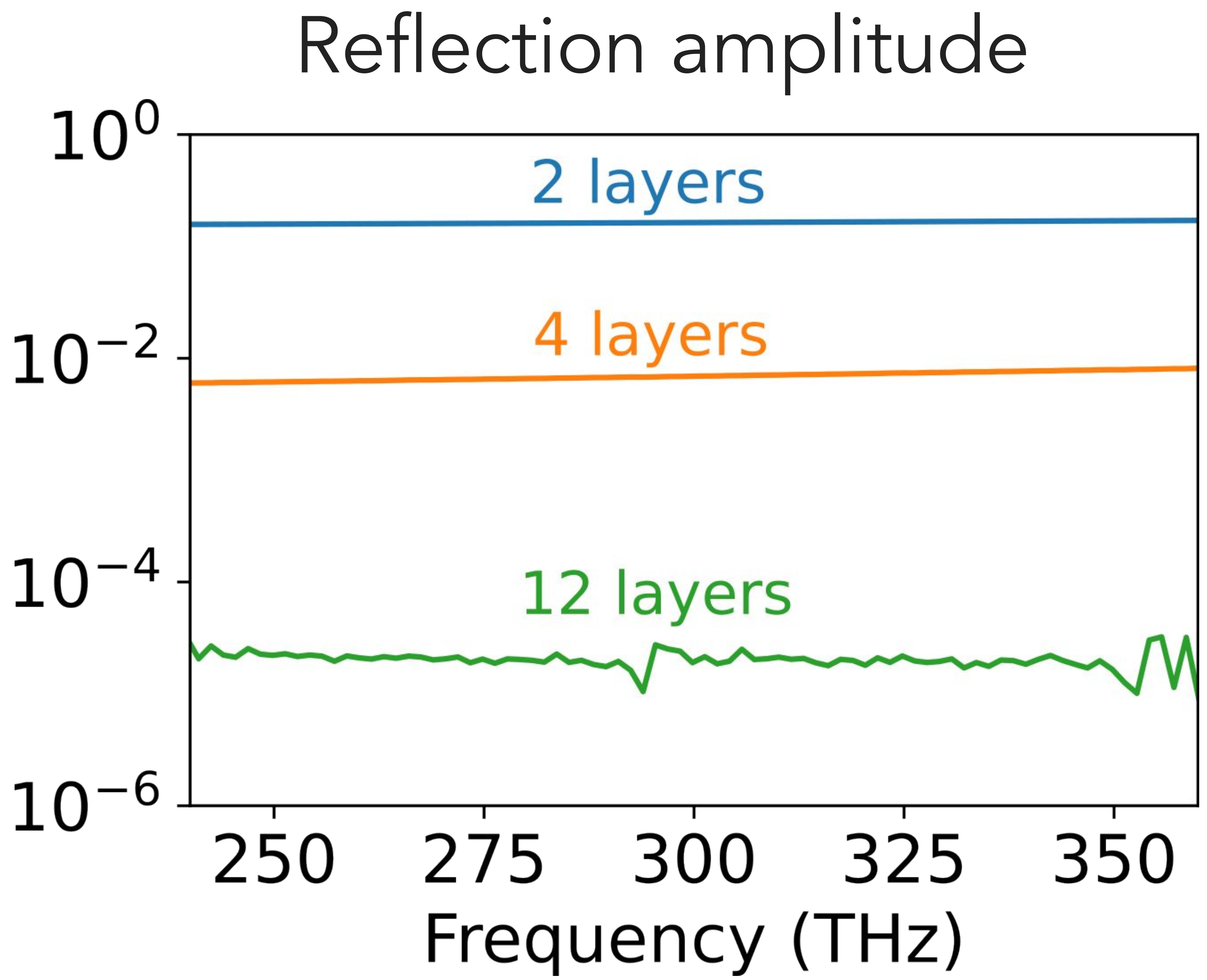




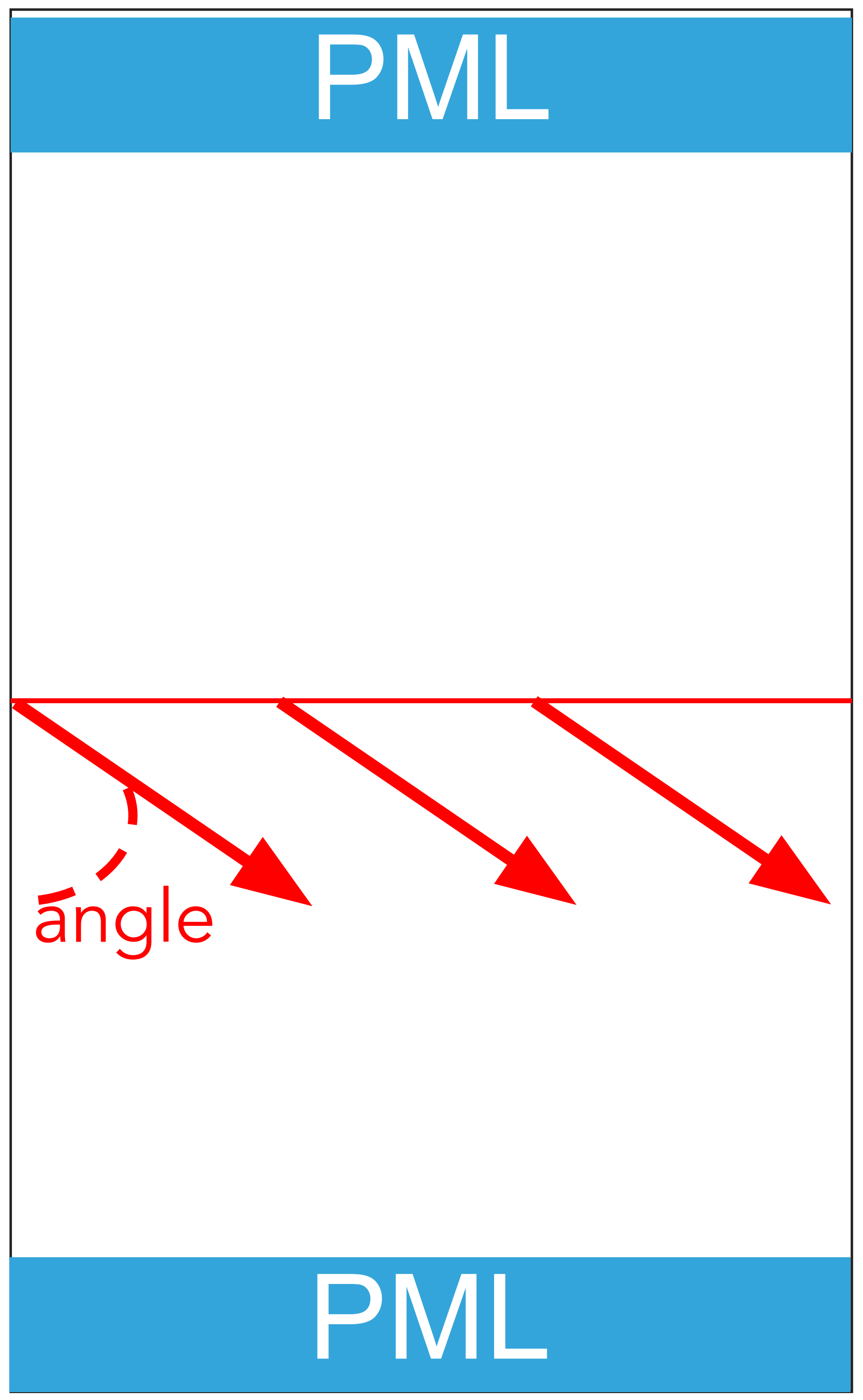
Reflection amplitude measurement

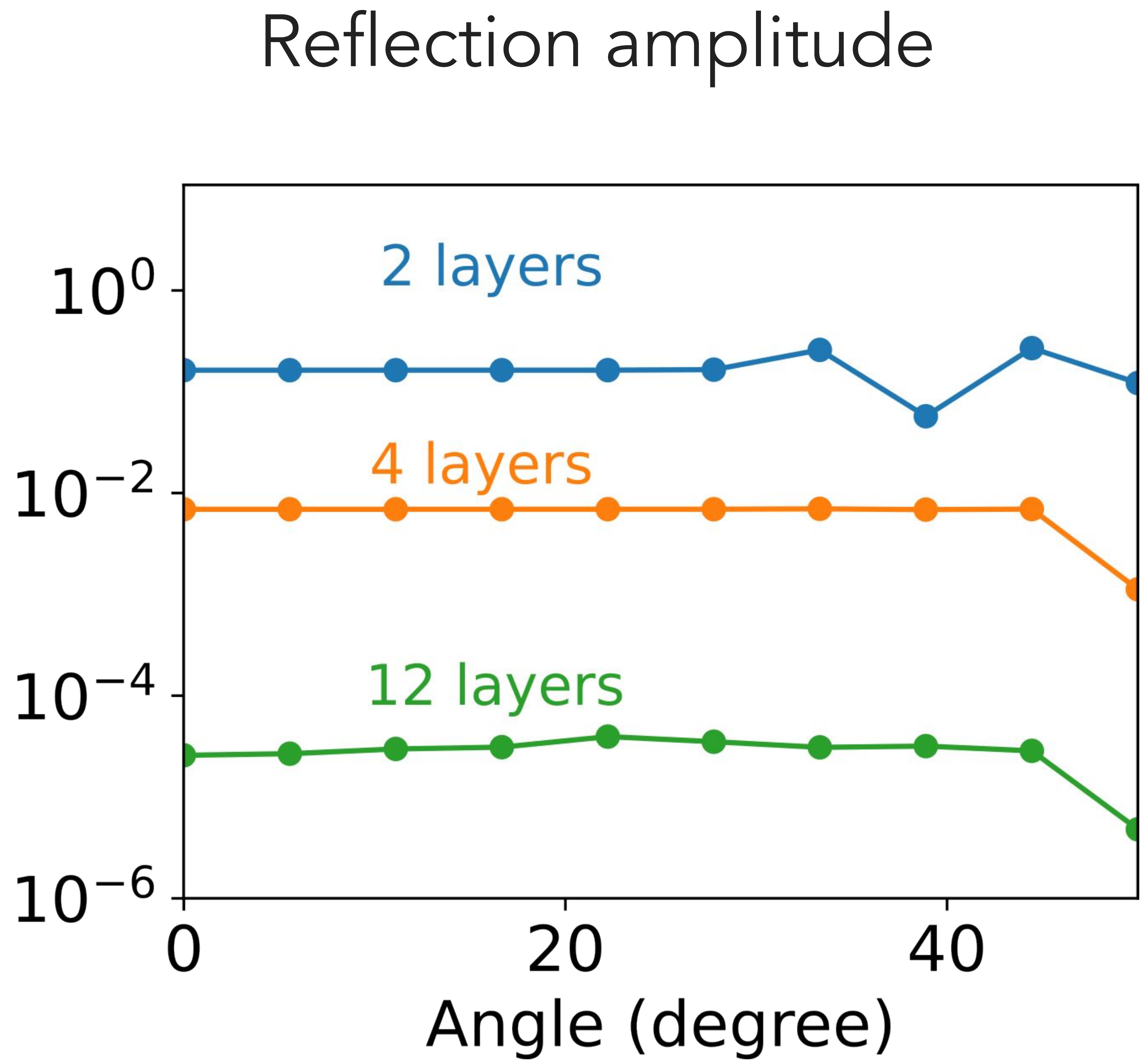


Fourier analysis



Angled incidence





Impact of reflection from PML in simulations of a Fabry-Perot cavity