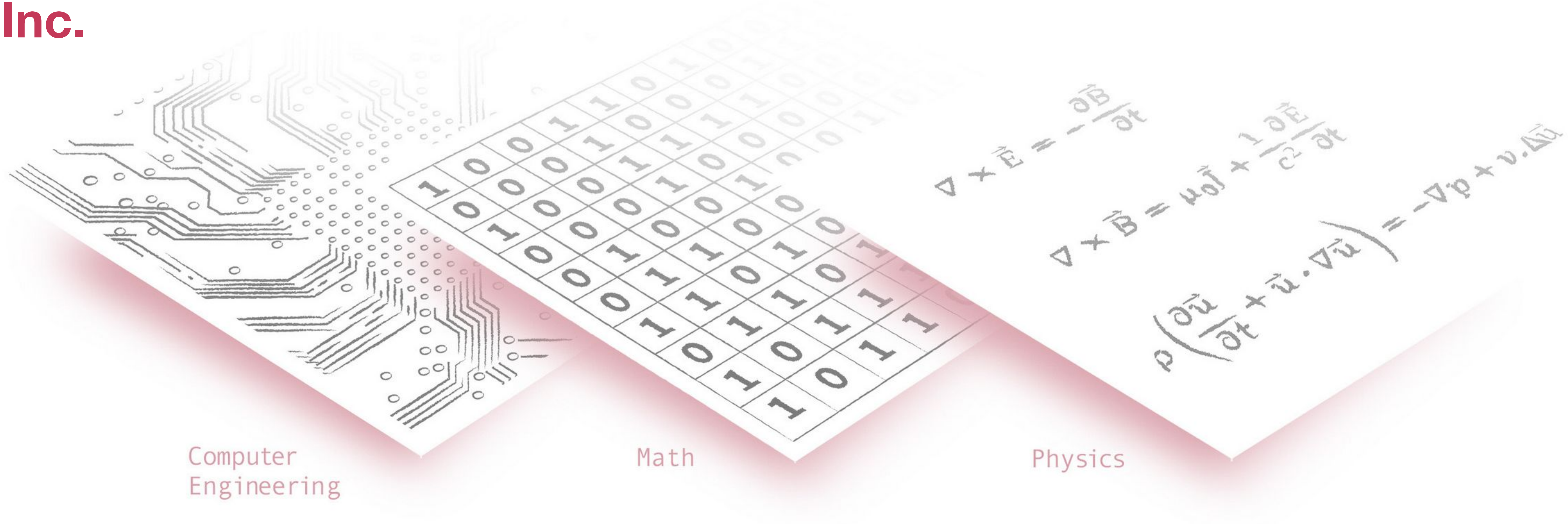
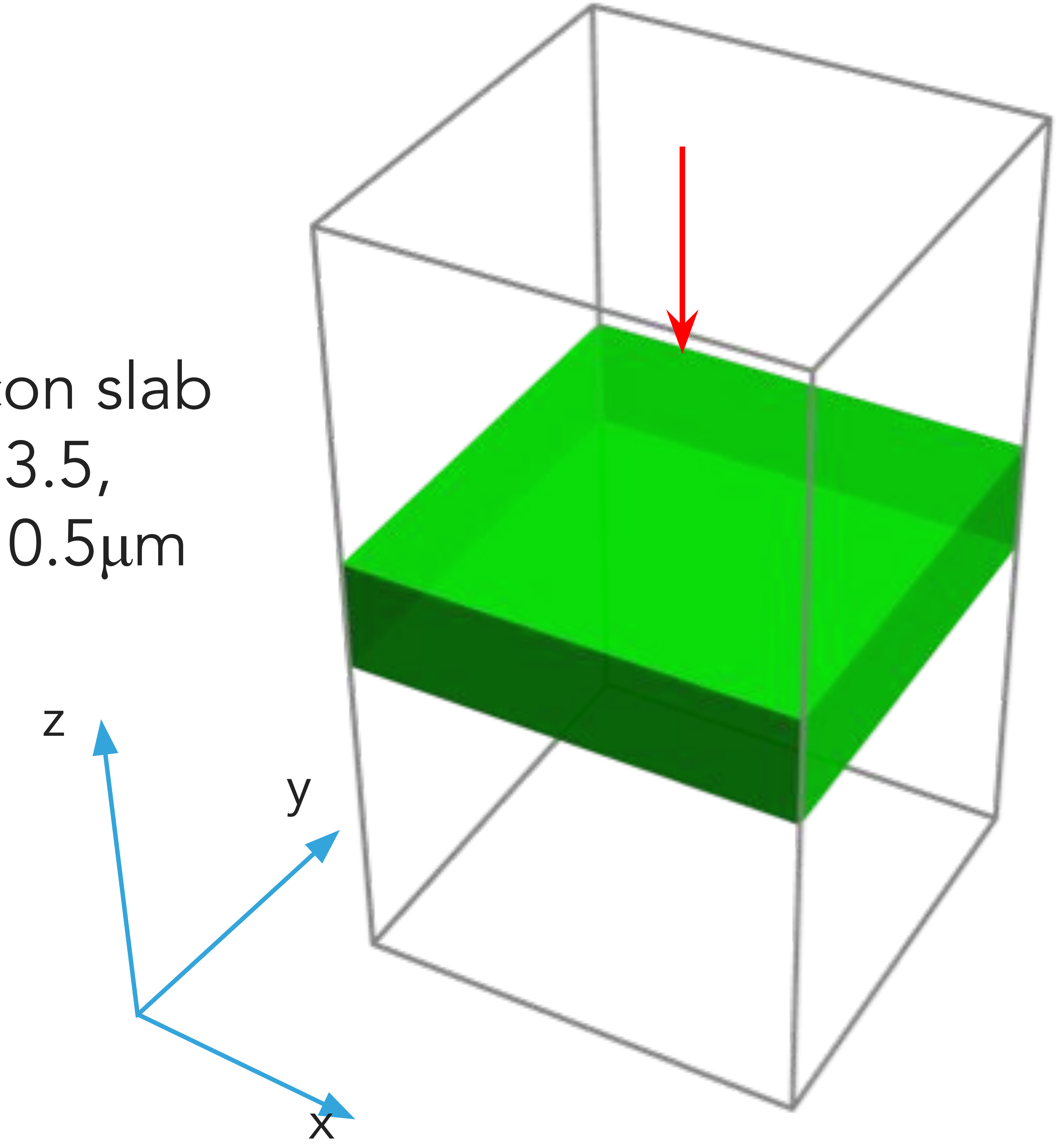
INTRO TO FDTD (8)

Flexcompute **Inc.**

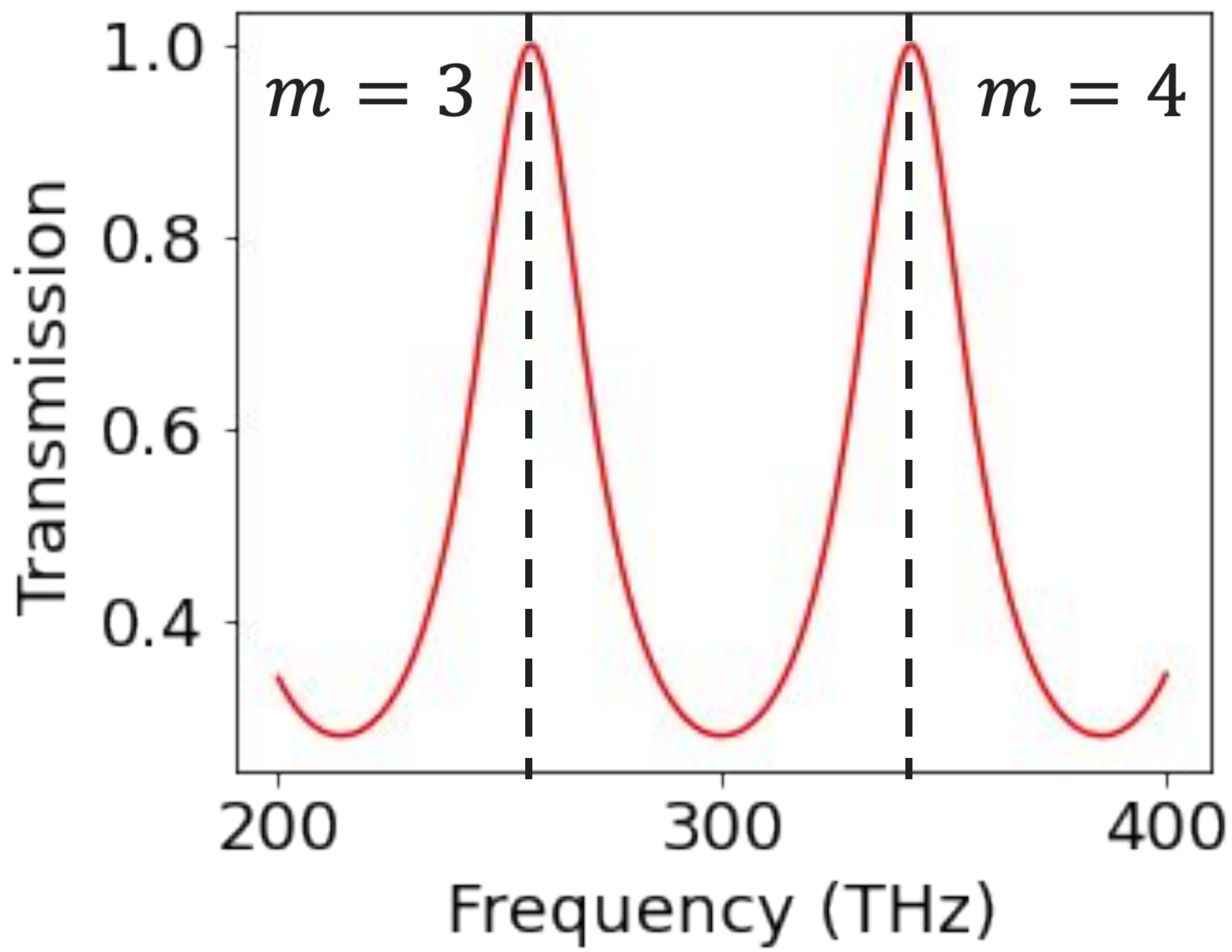


Tidy3D

Silicon slab
$n = 3.5$,
$d = 0.5\mu m$

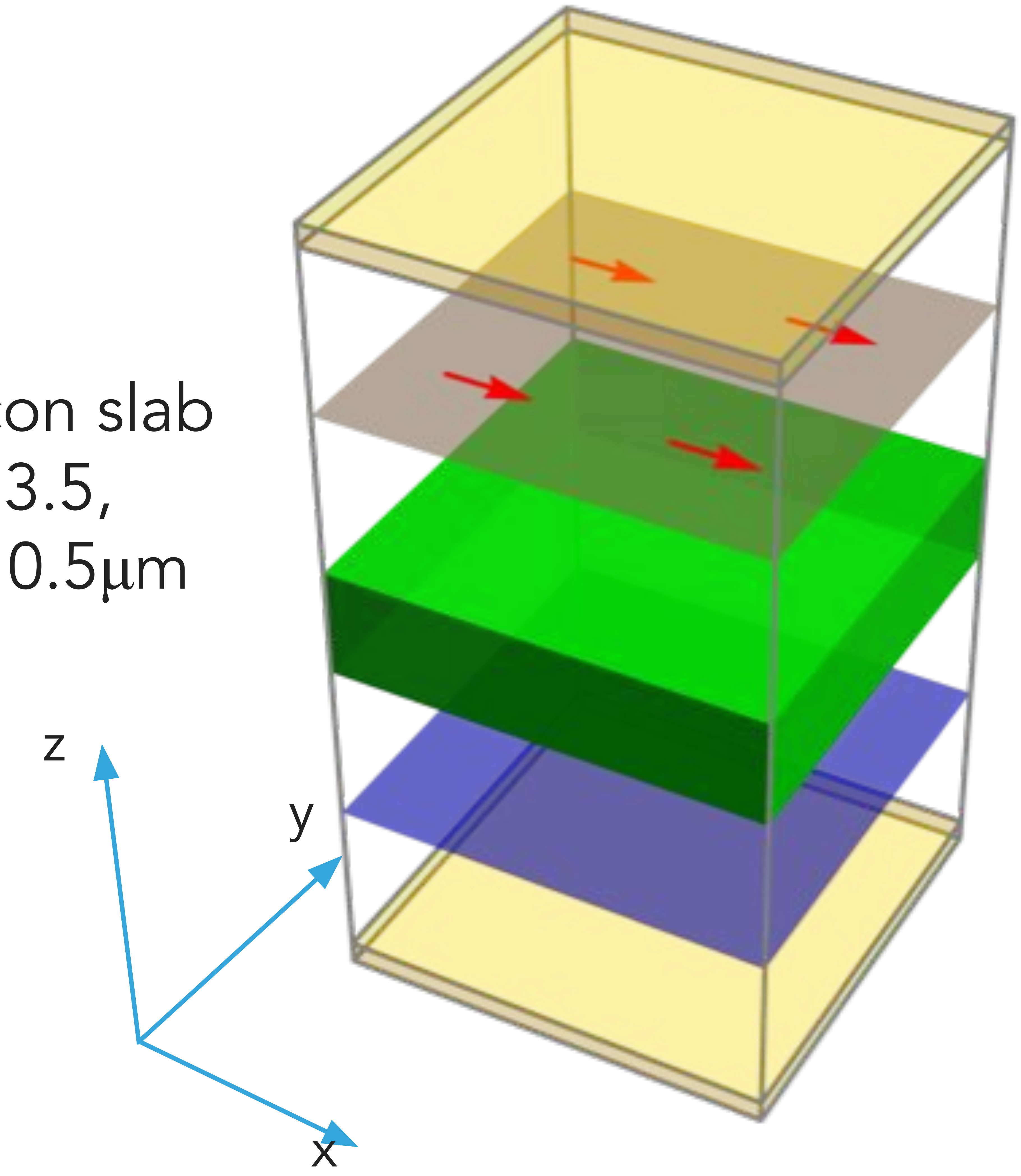


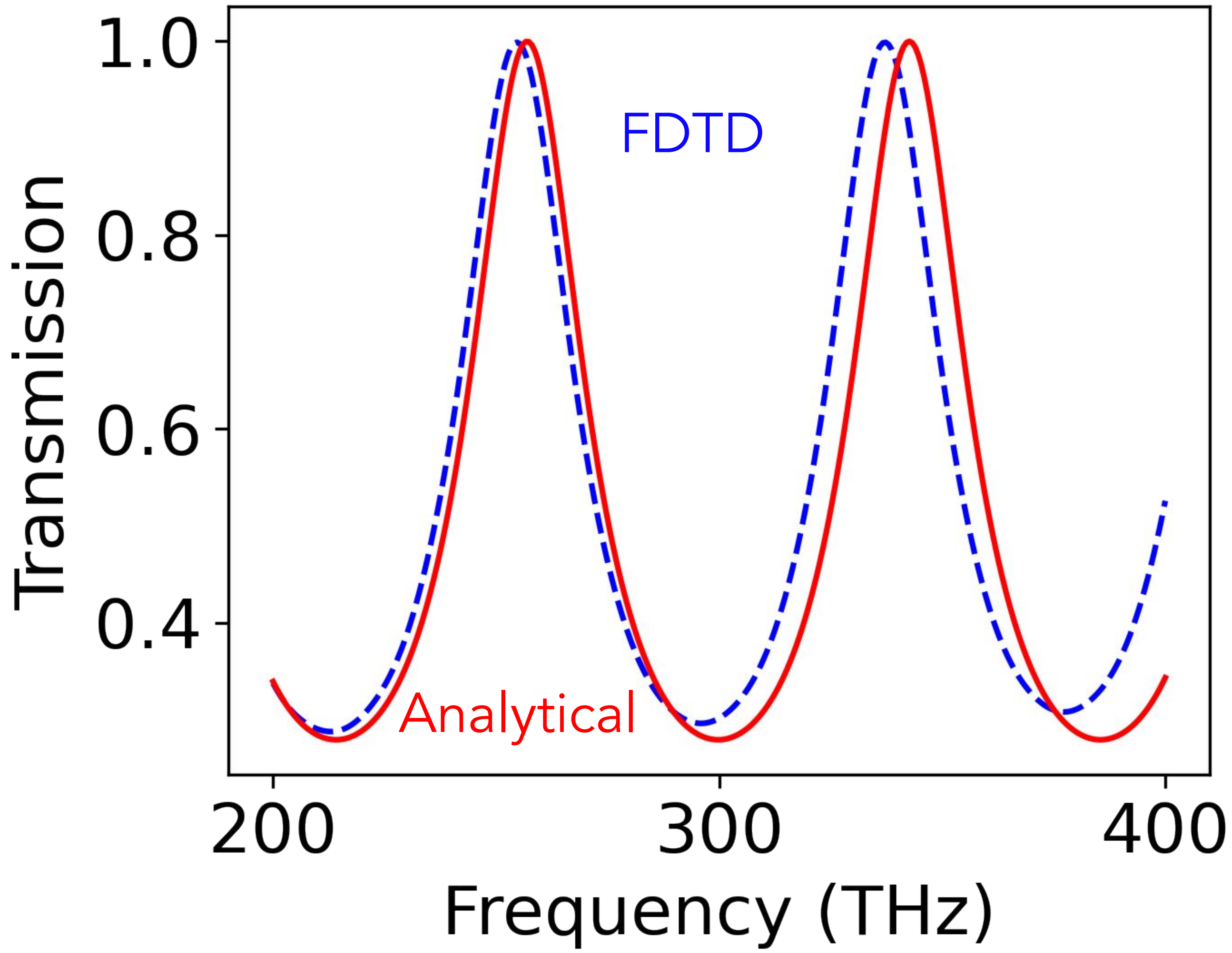
Analytical Results



Peak transmission occurs at $2k_z d = m \cdot 2\pi$,
where $m = 0,1,2,\cdots$

Silicon slab
n = 3.5,
d = 0.5μm





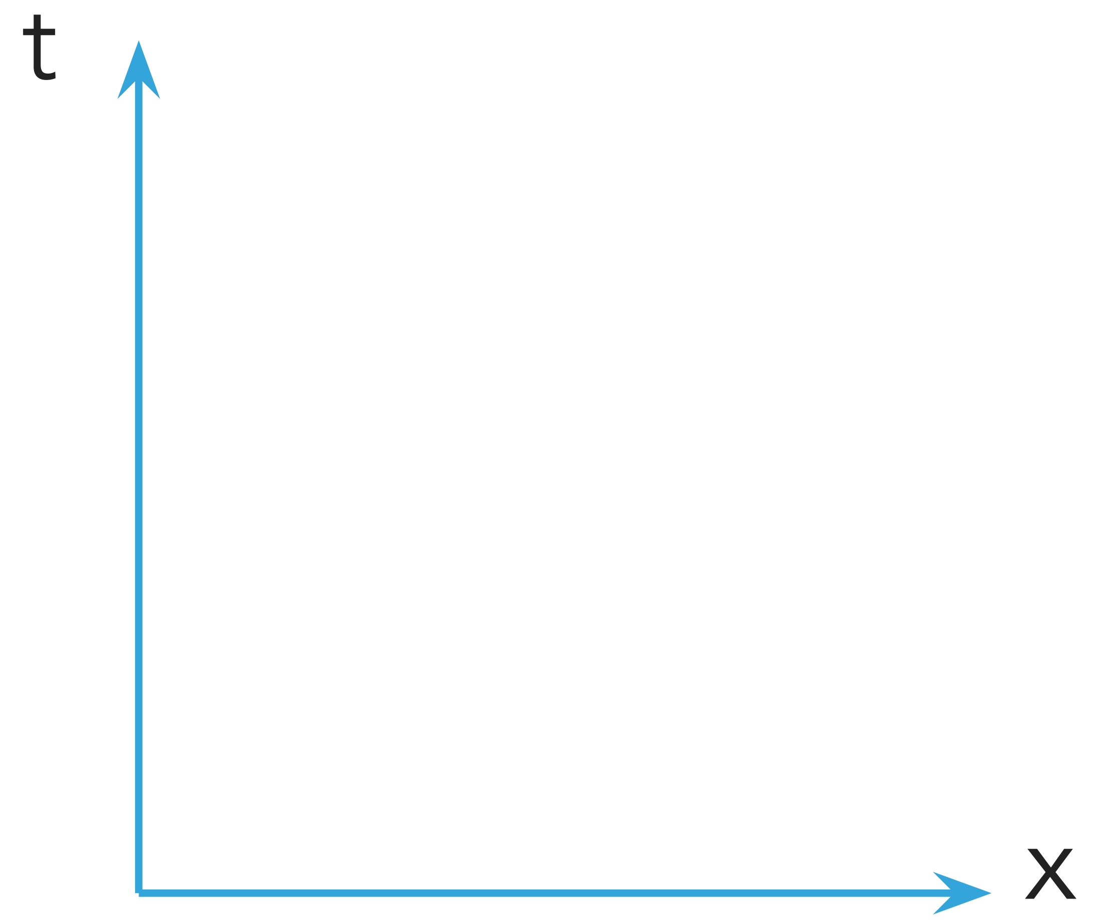
Spatial discretization: 25 nm; ~11 grids/λ_{Si}

See Tutorial Video 2

Physical dispersion in 1D

1D wave equation: $\left(\frac{\partial^2}{\partial x^2} - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right) E = 0$

- Monochromatic travelling wave: $E(x) = \; e^{i(kx-\omega t)}$



Dispersion relation between angular frequency ω and wavevector k:

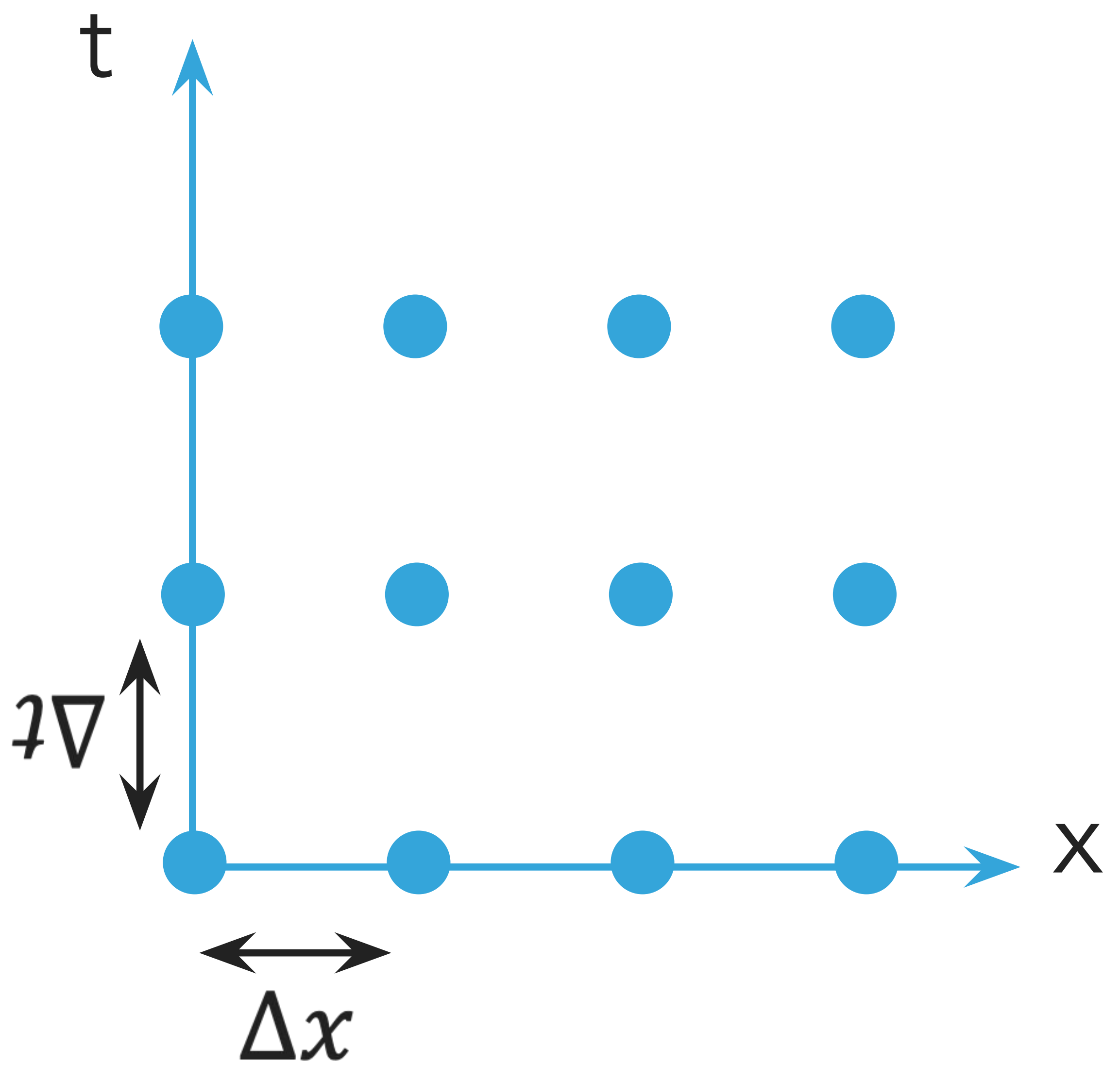
- Physical wave: $(\frac{\omega}{c})^2 = k^2$

Numerical dispersion in 1D

1D wave equation: $\left(\frac{\partial^2}{\partial x^2} - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)E = 0$

Applying finite difference in both space and time:

- $\frac{\partial^2}{\partial x^2}E(x_m) \approx \frac{1}{\Delta x^2}[E(x_m + \Delta x) + E(x_m - \Delta x) - 2E(x_m)]$
- $\frac{\partial^2}{\partial t^2}E(t_\alpha) \approx \frac{1}{\Delta t^2}[E(t_\alpha + \Delta t) + E(t_\alpha - \Delta t) - 2E(t_\alpha)]$



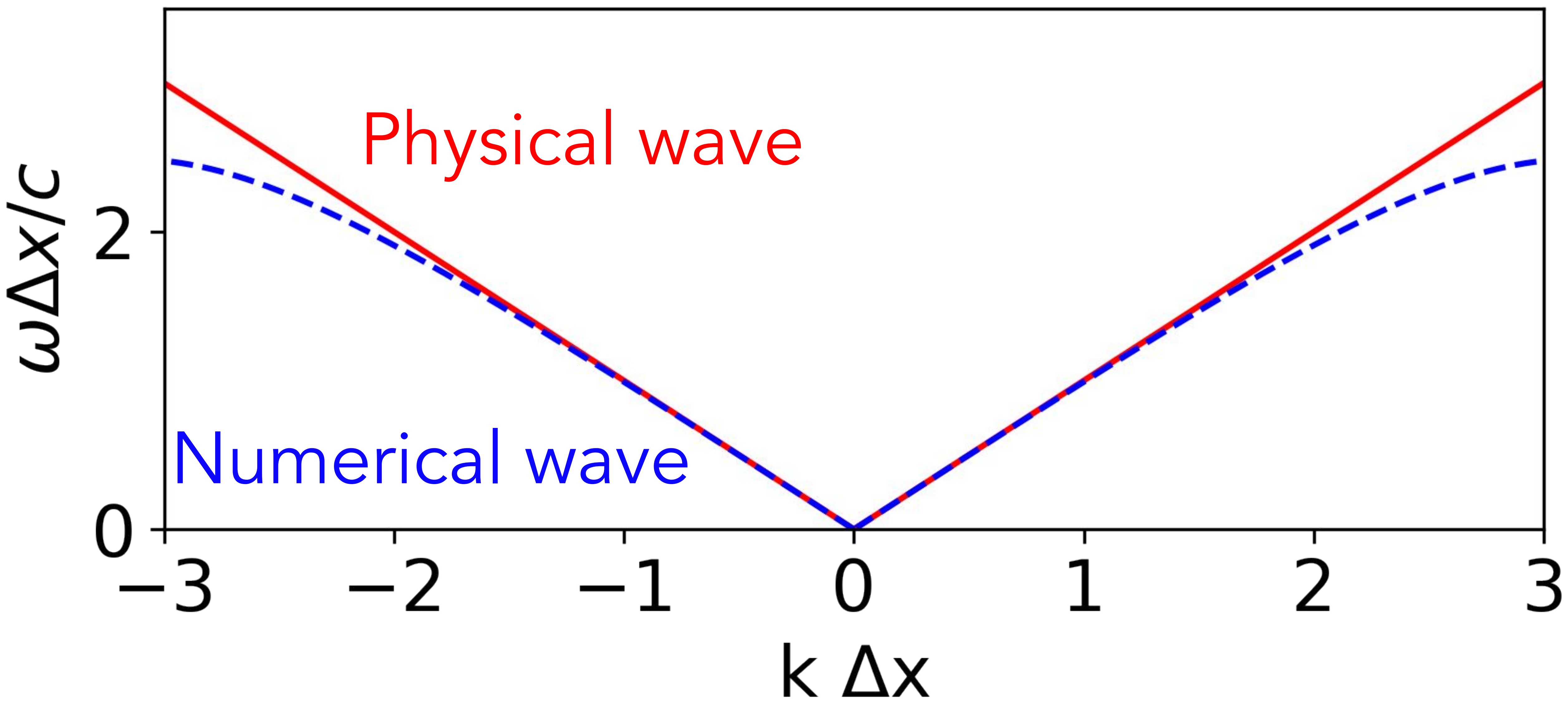
Consider monochromatic travelling wave: $E(x) = e^{i(kx-\omega t)}$, the dispersion relation is

- $(\frac{1}{c\Delta t}\sin\frac{\omega\Delta t}{2})^2 = (\frac{1}{\Delta x}\sin\frac{k\Delta x}{2})^2$

Numerical dispersion in 1D

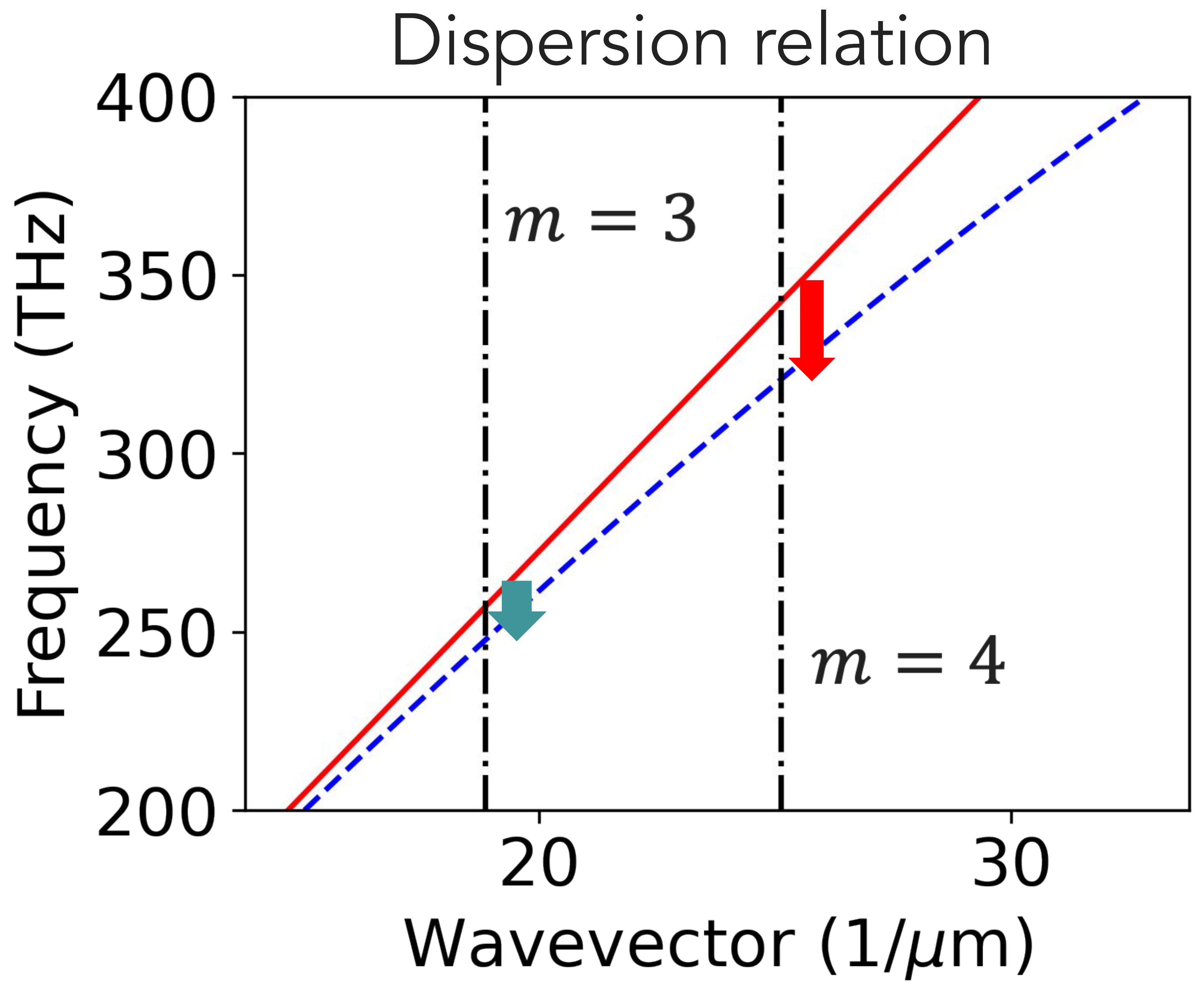
Dispersion relation:

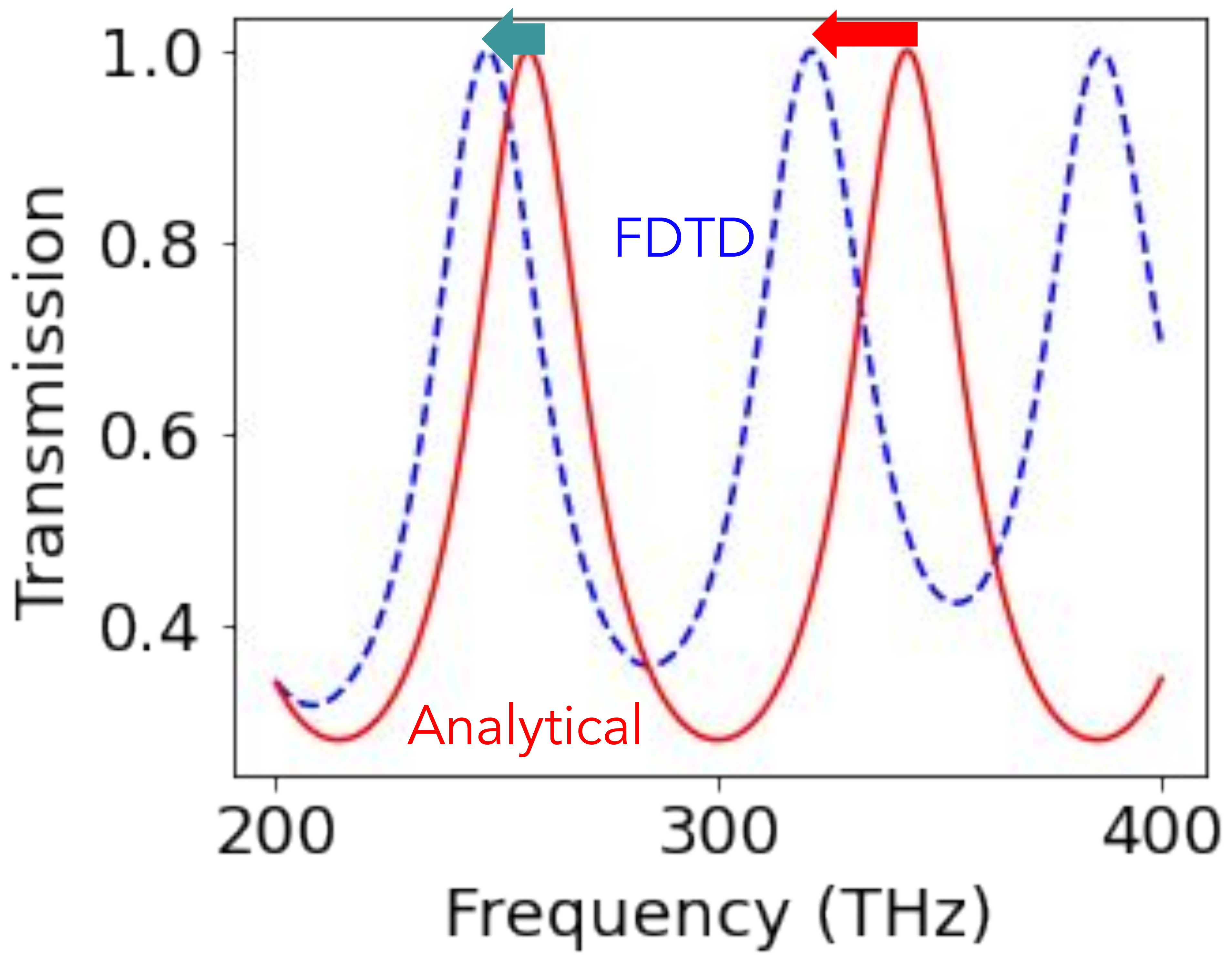
- Physical wave: $(\frac{\omega}{c})^2 = k^2$
- Numerical wave: $(\frac{1}{c\Delta t}\sin\frac{\omega\Delta t}{2})^2 = (\frac{1}{\Delta x}\sin\frac{k\Delta x}{2})^2$



- Courant number $\frac{c\Delta t}{\Delta x} = 0.9$
- Mismatch decreases with $\mathrm{k}\Delta x = \frac{2\pi}{\lambda_k}\Delta x$, requiring a minimal number of grids per effective wavelength.
- 2nd order convergence with Δx

Coarse spatial discretization

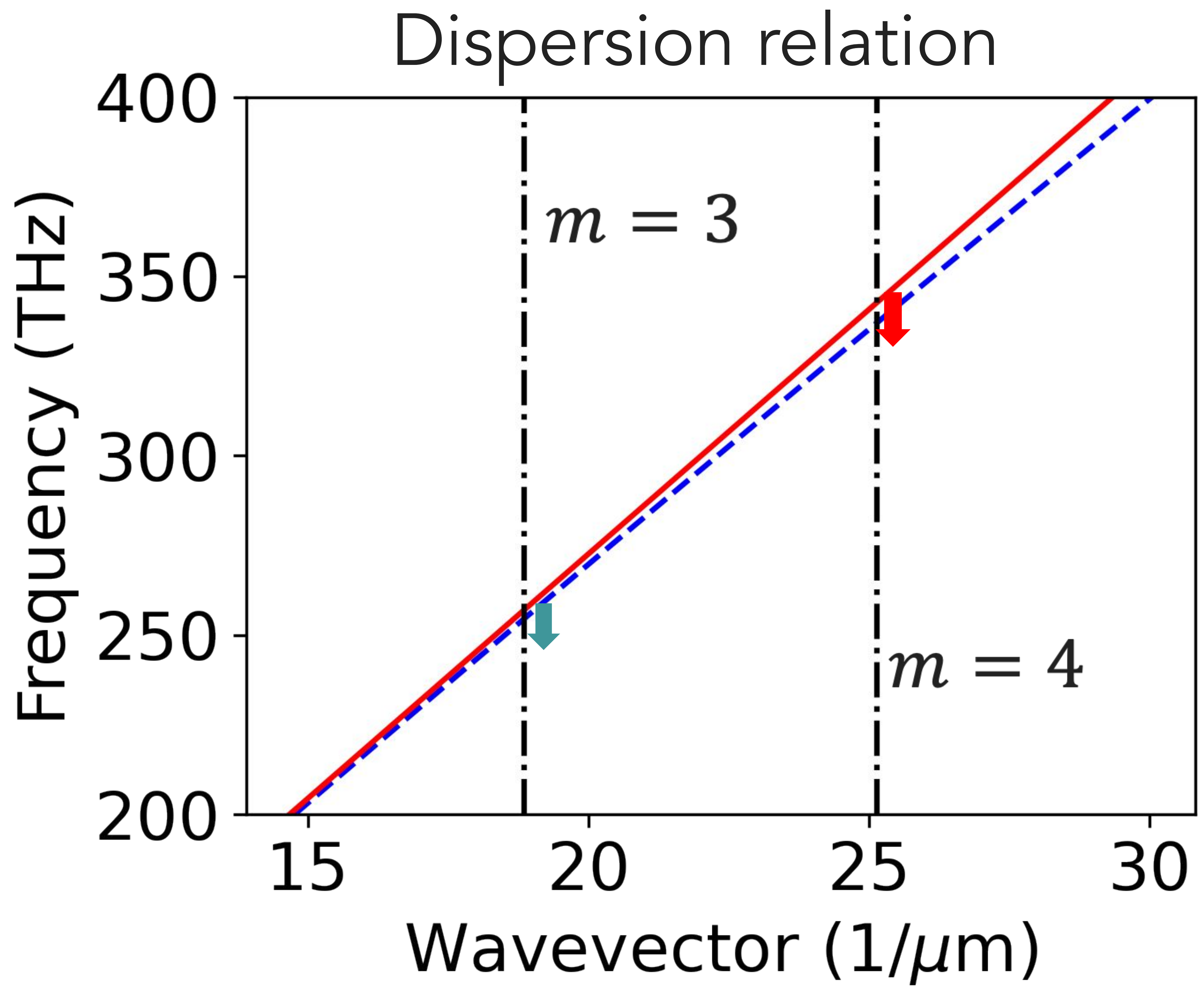


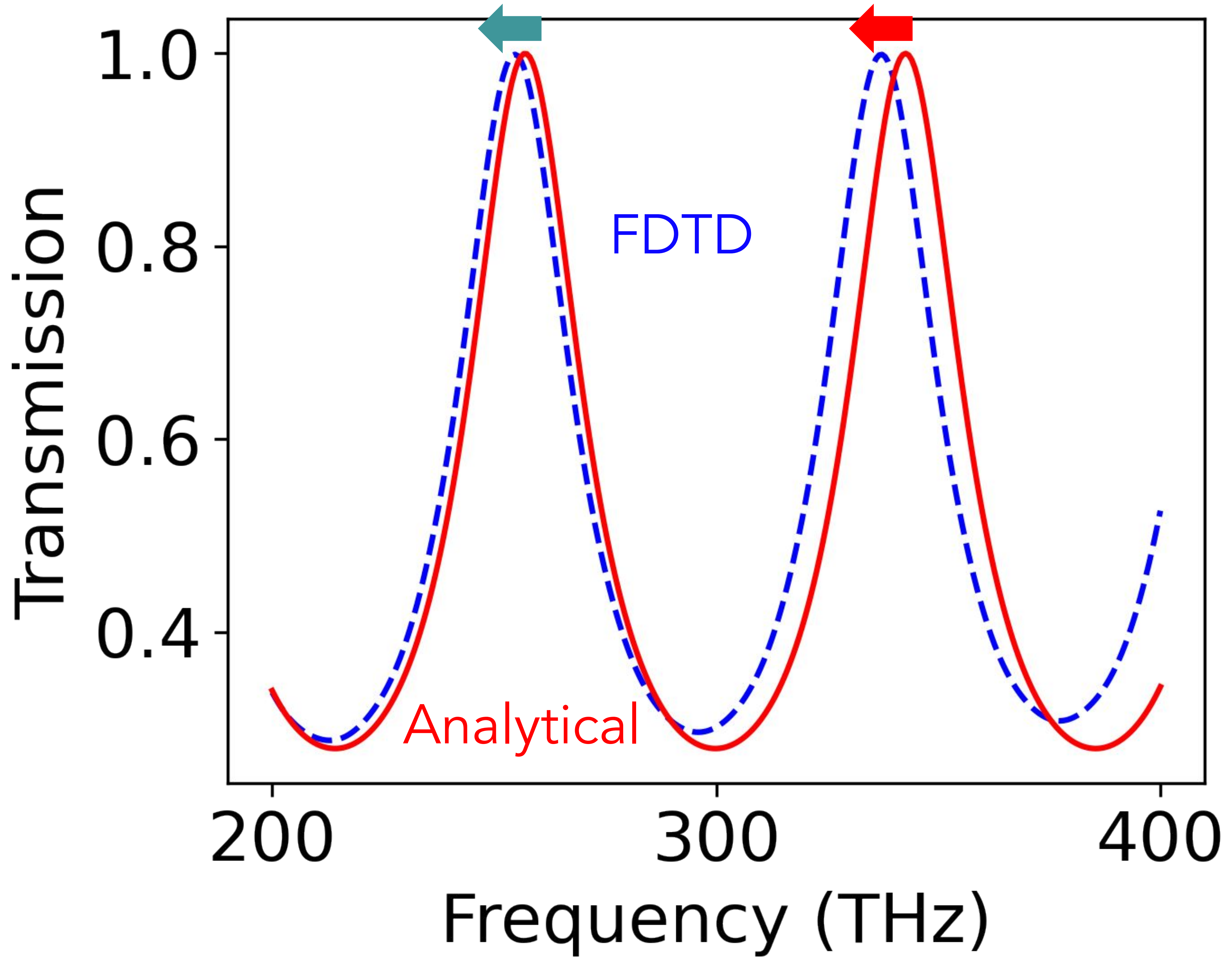


Peak transmission occurs at $2k_z d = m \cdot 2\pi$, where $m = 0,1,2,\cdots$

Spatial discretization: 50 nm; ~6 grids/λ_{Si}

Finer spatial discretization

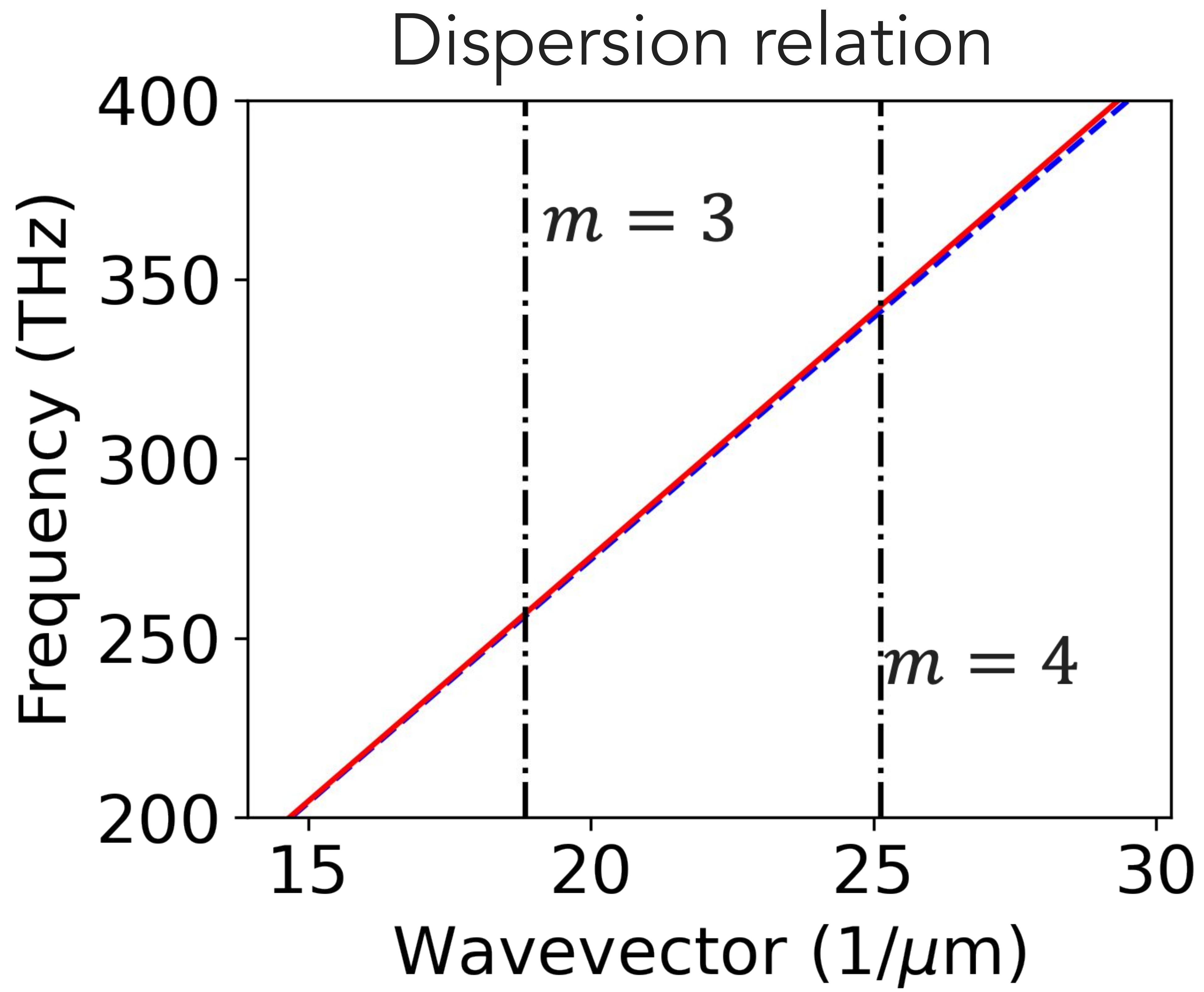


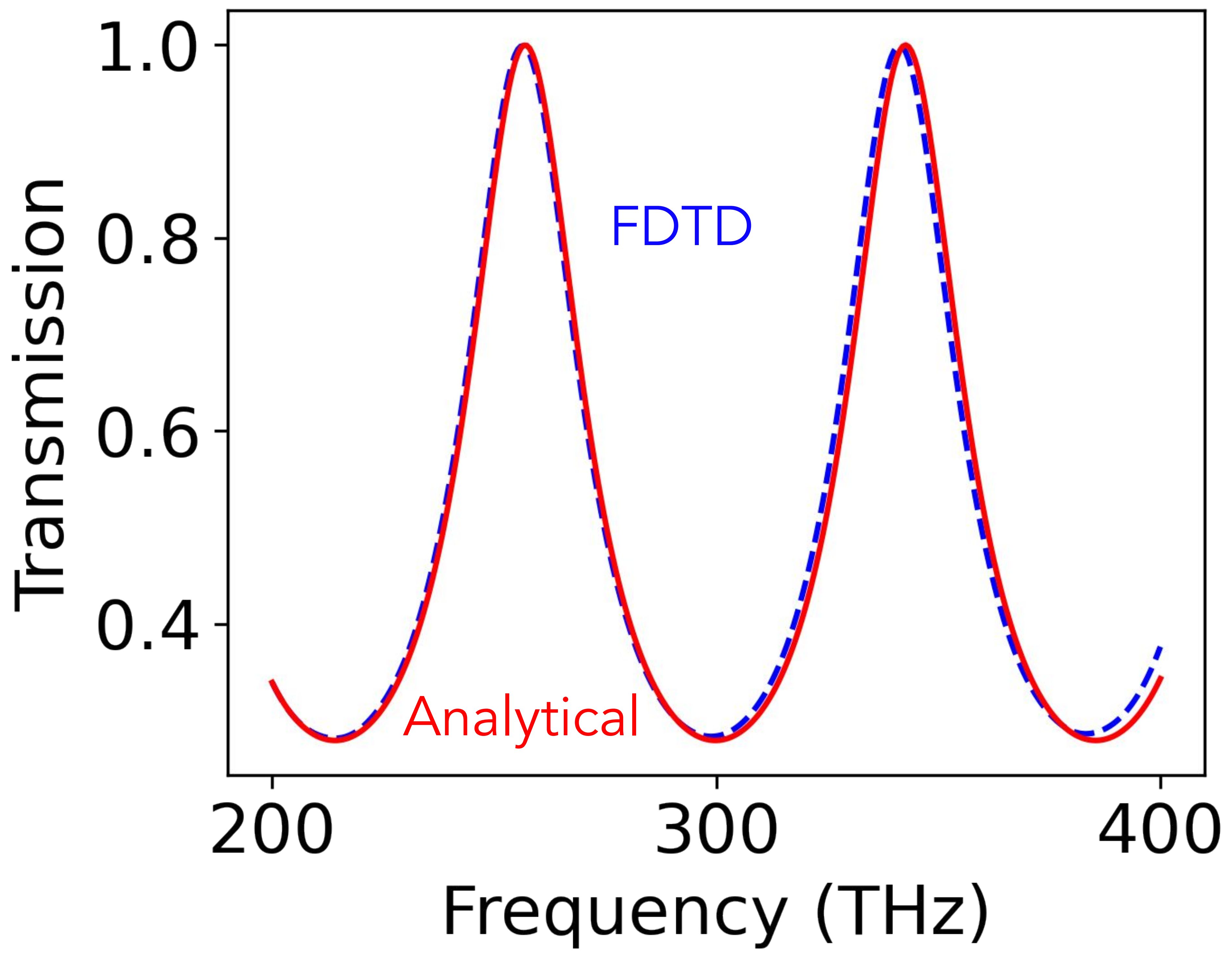


Peak transmission occurs at $2k_z d = m \cdot 2\pi$, where $m = 0,1,2,\cdots$

Spatial discretization: 25 nm; ~11 grids/λ_{Si}

Even finer spatial discretization

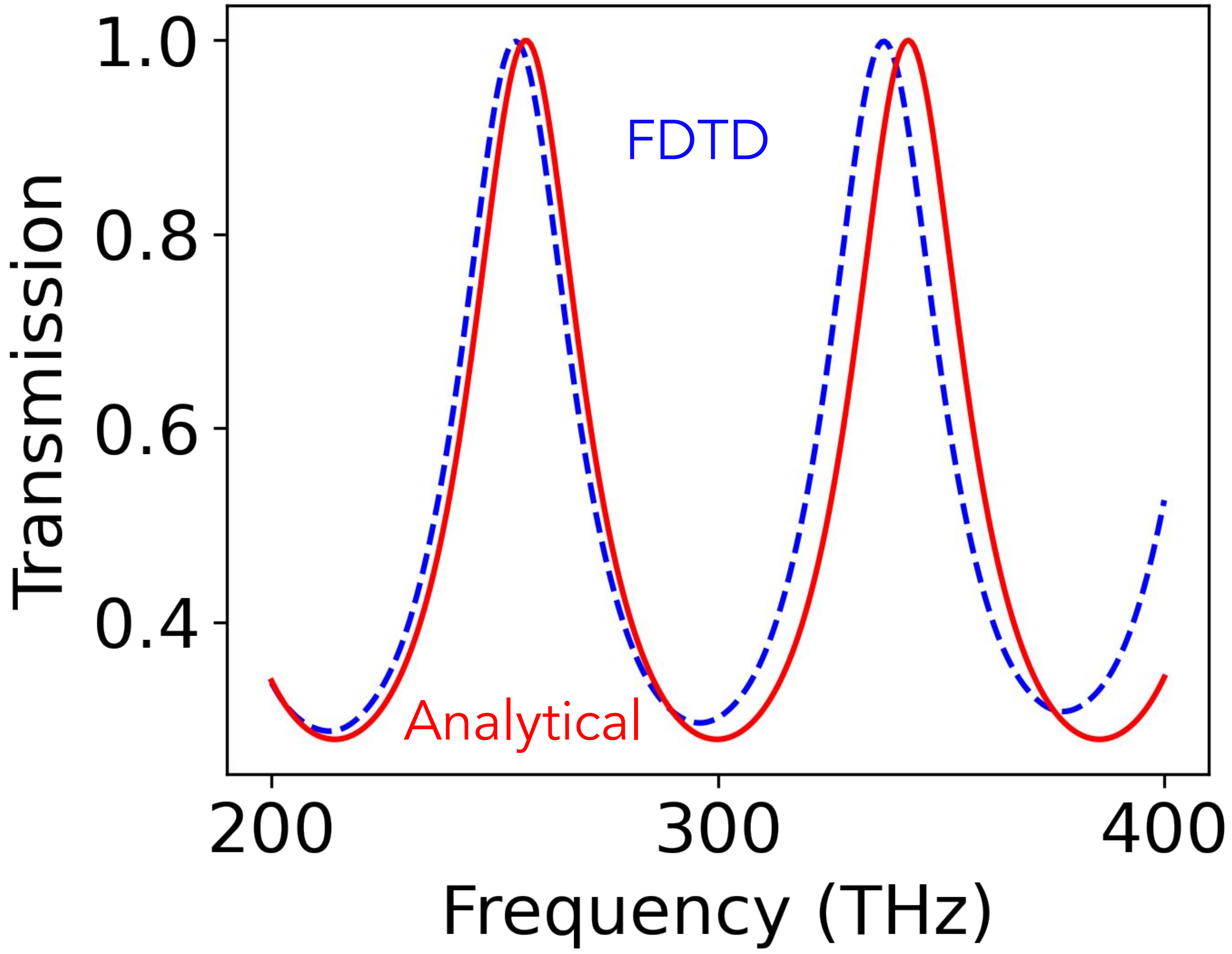




Peak transmission occurs at $2k_z d = m \cdot 2\pi$**, where** $m = 0,1,2,\cdots$

Spatial discretization: 12.5 nm; ~23 grids/λ_{Si}

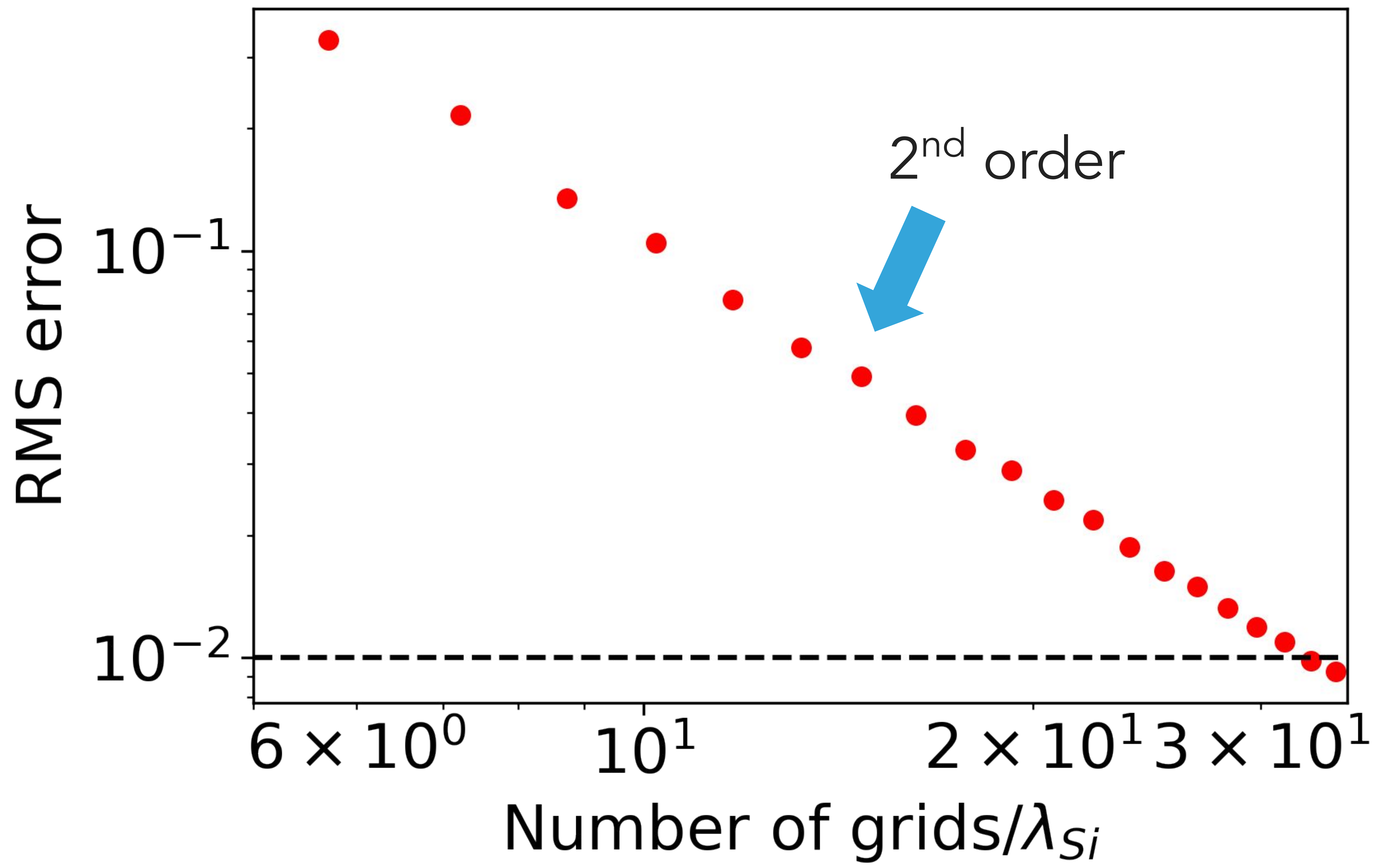
Numerical dispersion: convergence



RMS error $= \sqrt{\frac{\sum_{i=1}^{N}[T^F(f_i) - T^A(f_i)]^2}{N}}$

Let's sample at N=301 frequencies linearly in the range [200,400] THz

Numerical dispersion: convergence



To achieve RMS error < 1%, the number of grids per wavelength in medium $\gtrsim 30$

Numerical dispersion in 2D

Monochromatic travelling wave: $E(x) = e^{i(k_x x + k_y y - \omega t)}$

Dispersion relation:

- Physical wave: $(\frac{\omega}{c})^2 = k_x^2 + k_y^2$
- Numerical wave: $(\frac{1}{c\Delta t} \sin \frac{\omega \Delta t}{2})^2 = (\frac{1}{\Delta x} \sin \frac{k_x \Delta x}{2})^2 + (\frac{1}{\Delta y} \sin \frac{k_y \Delta y}{2})^2$

Numerical dispersion in 2D

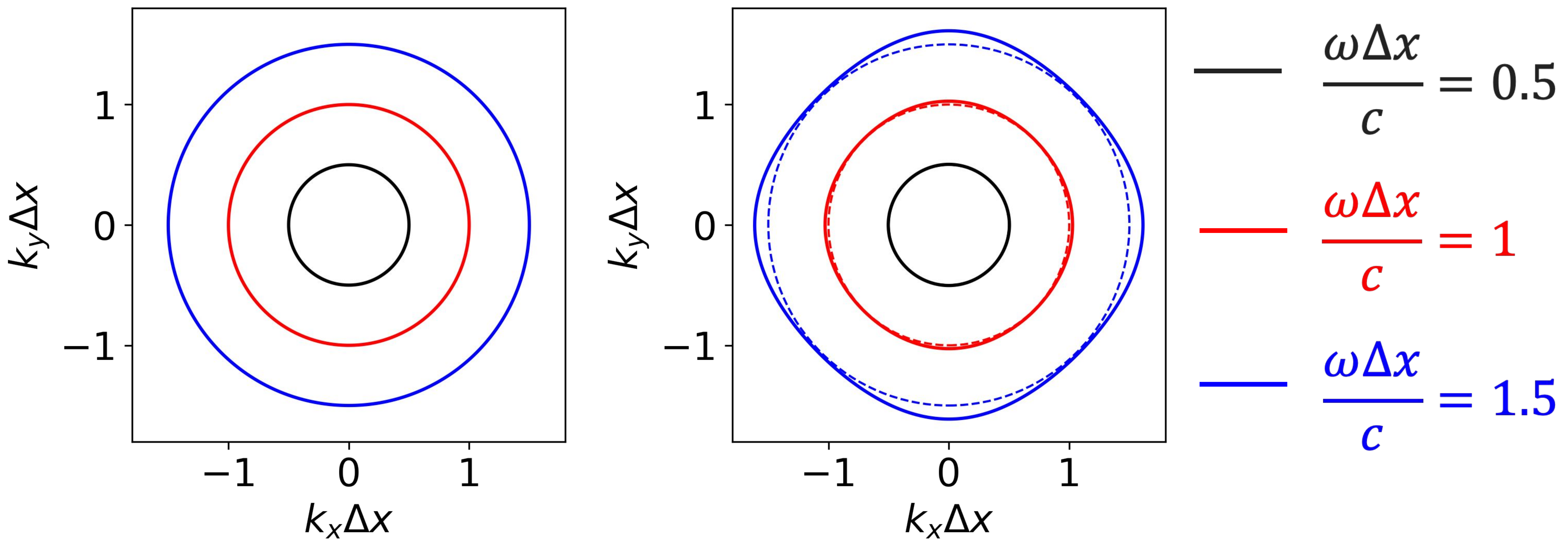
$$\left(\frac{\omega}{c}\right)^2 = k_x^2 + k_y^2$$

Physical

$$\left(\frac{1}{c\Delta t}\sin\frac{\omega\Delta t}{2}\right)^2 = \left(\frac{1}{\Delta x}\sin\frac{k_x\Delta x}{2}\right)^2 + \left(\frac{1}{\Delta y}\sin\frac{k_y\Delta y}{2}\right)^2$$

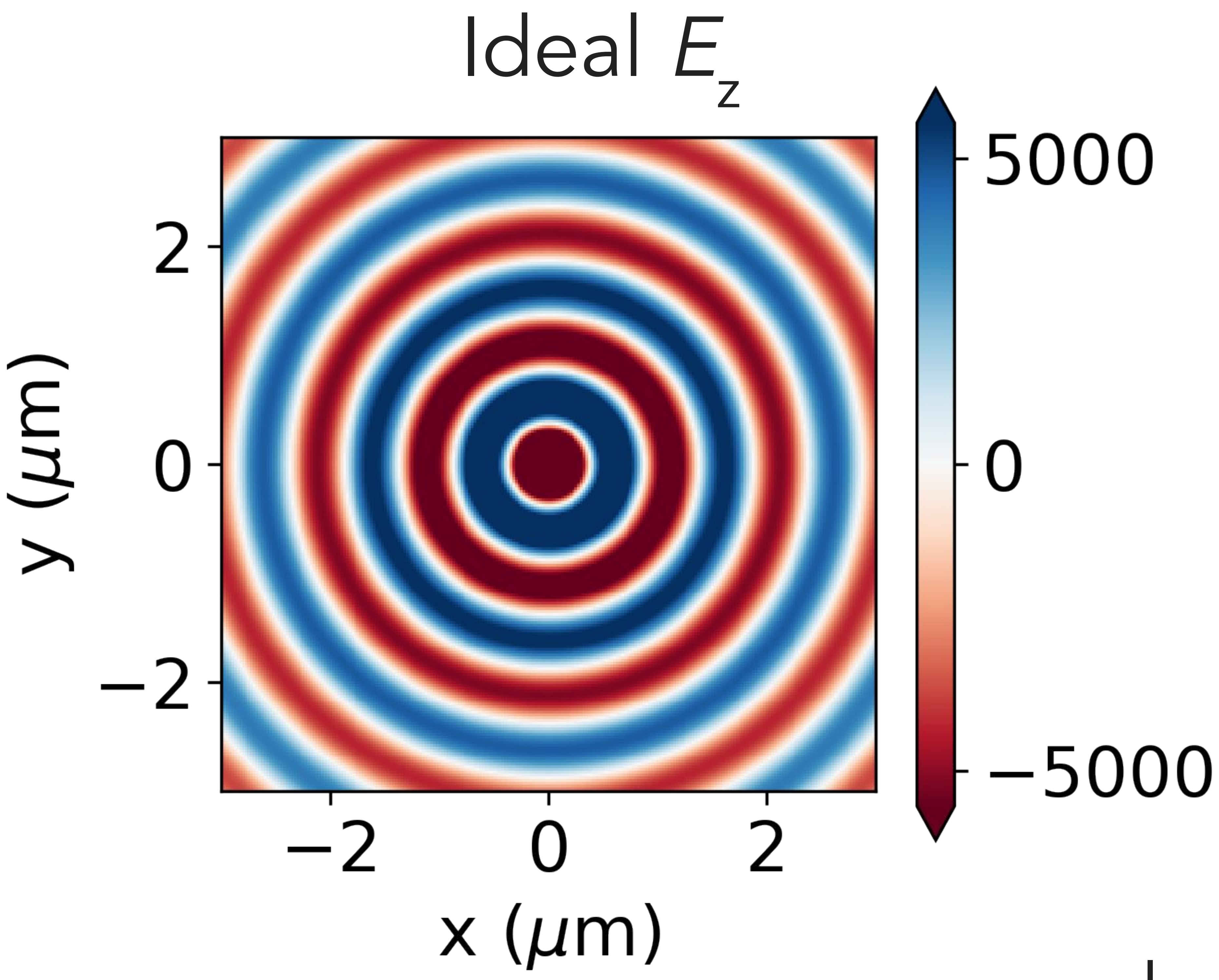
Numerical

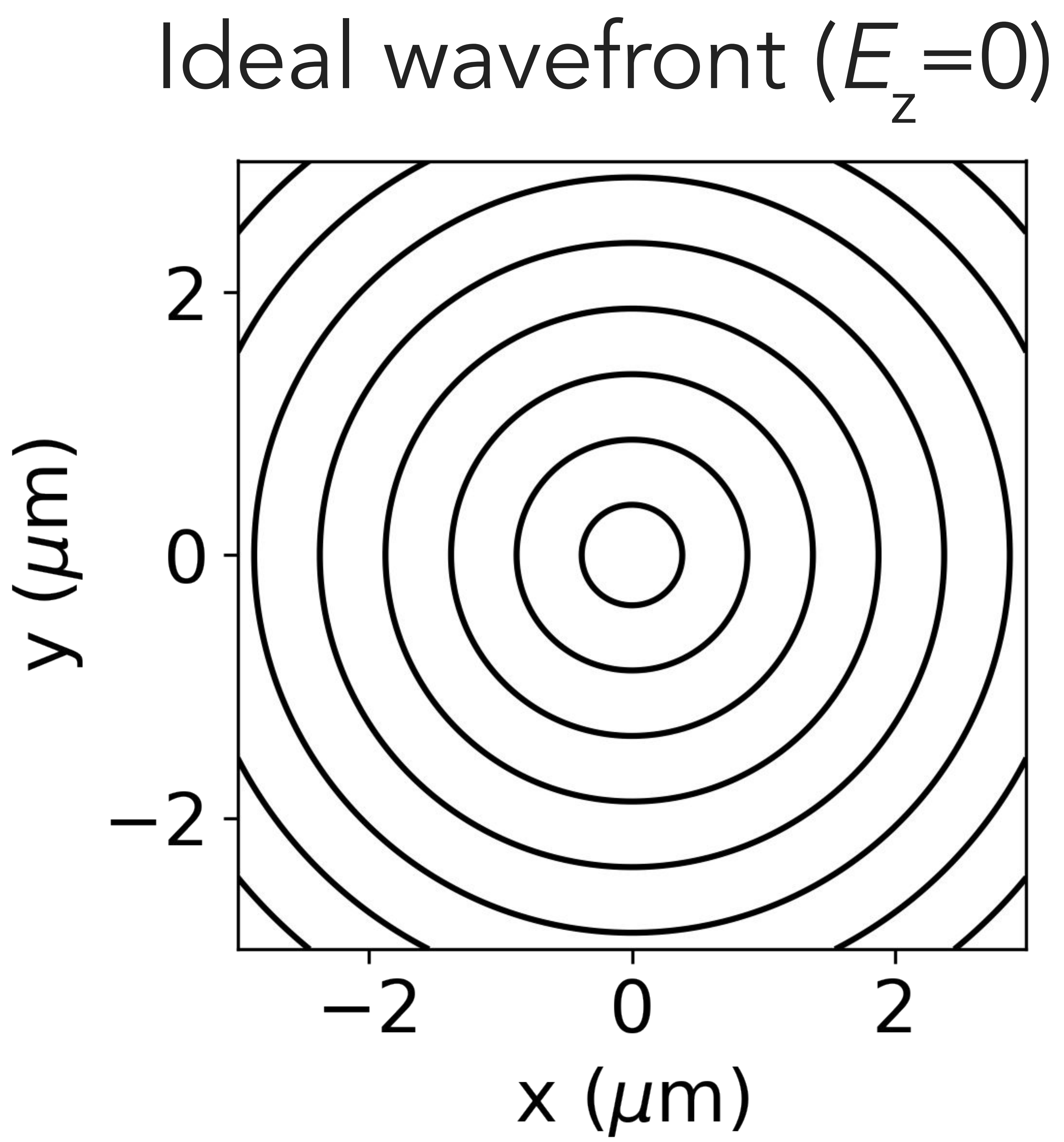
- **Isotropic grids:** $\Delta x = \Delta y$
- **Courant number:** $\frac{c\Delta t}{\Delta x} = 0.9/\sqrt{2}$



Numerical wave has anisotropic phase velocity

Example: dipole radiation in 2D

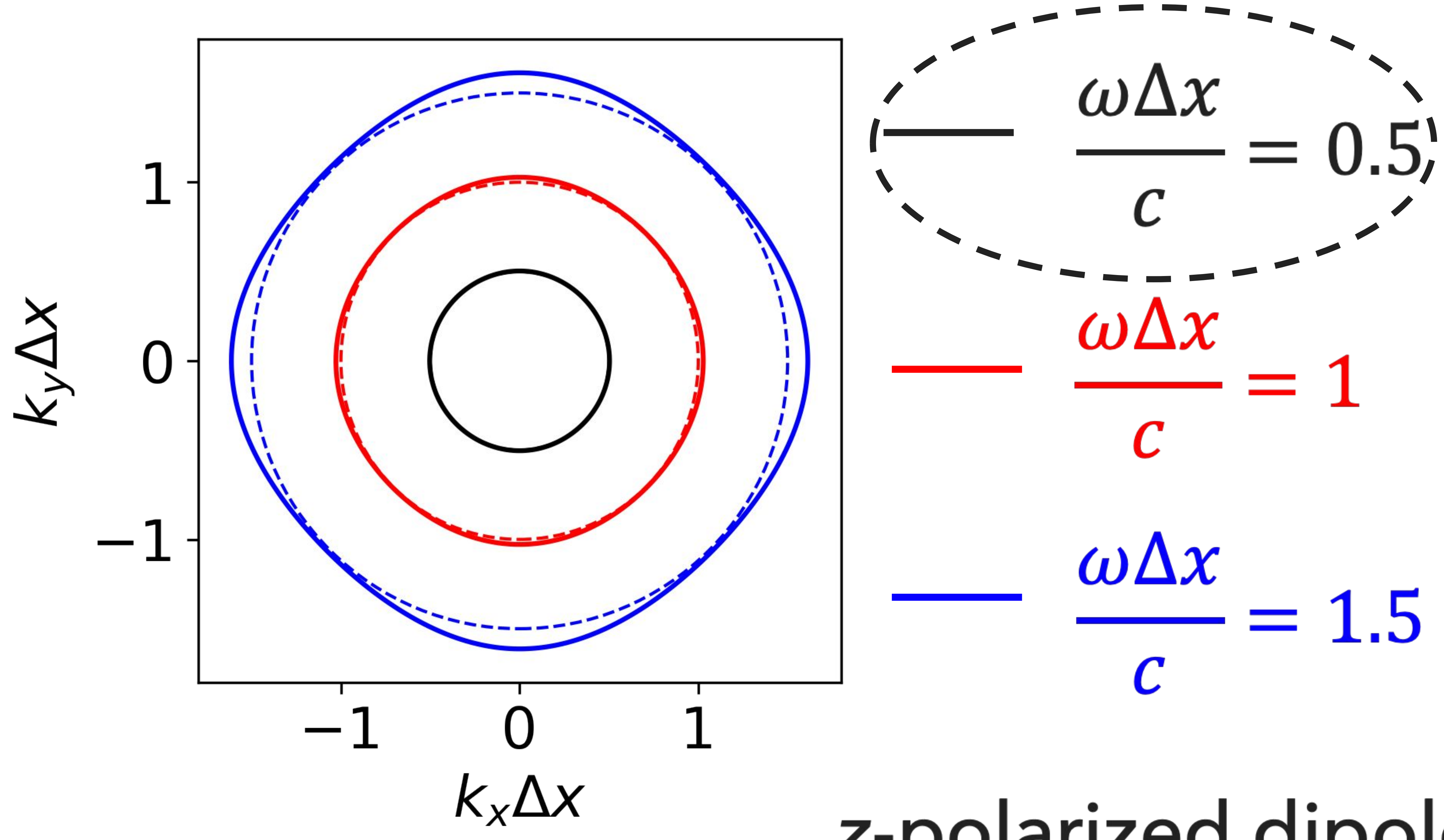


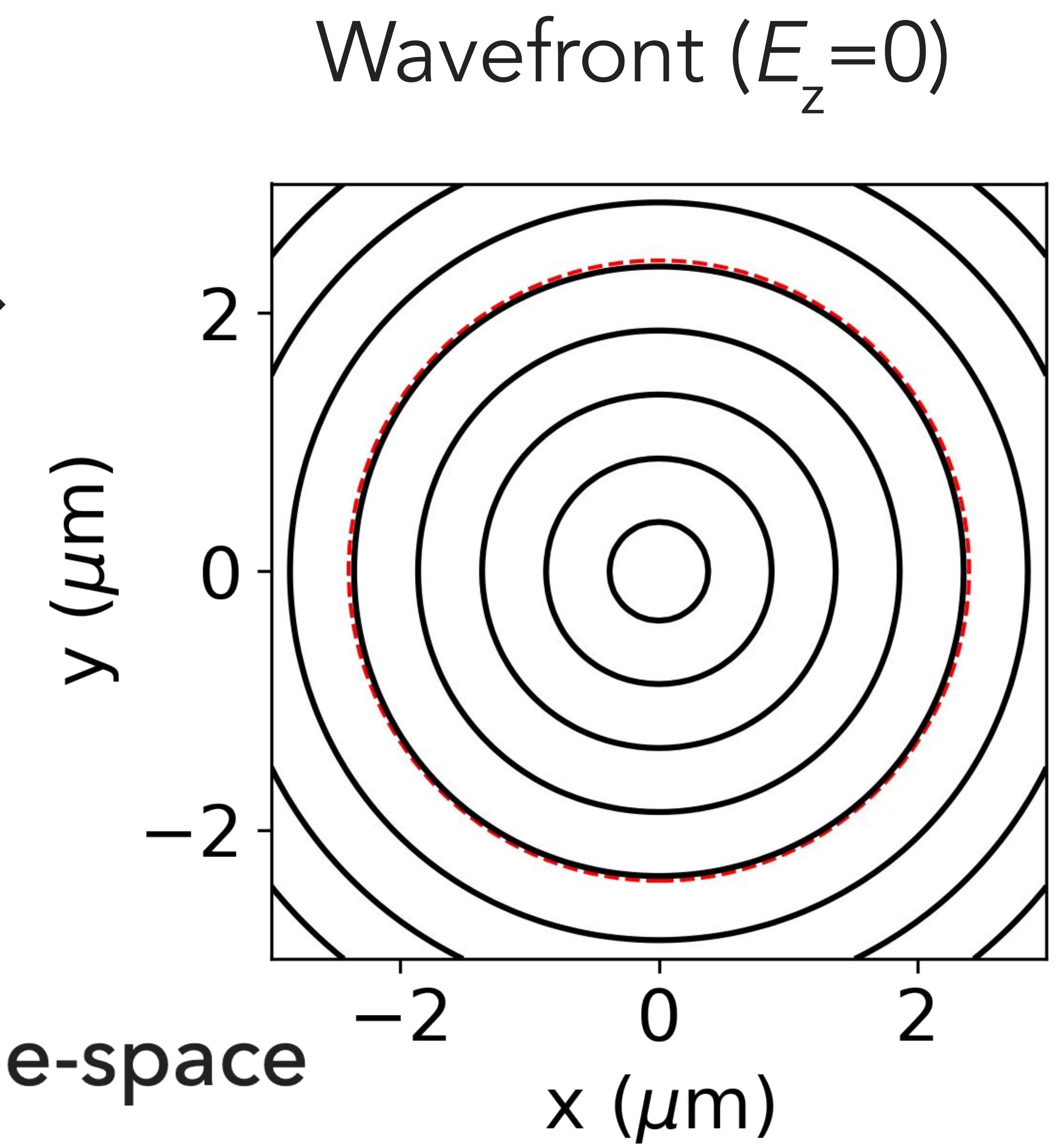


z-polarized dipole in free-space

- Wavelength: 1 μm

Example: dipole radiation in 2D

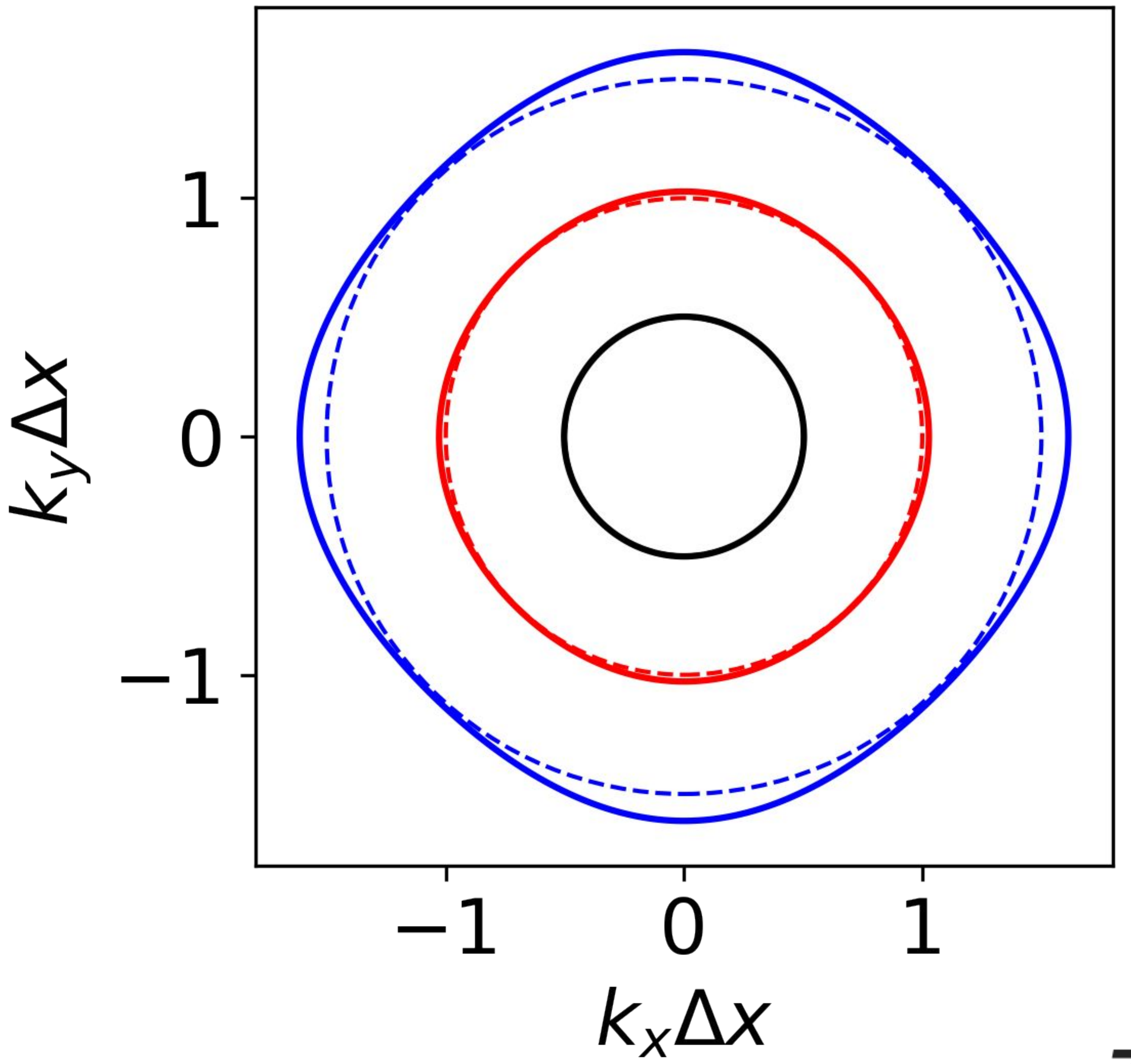




z-polarized dipole in free-space

- Wavelength: 1 μm
- Spatial discretization: $\Delta x = 0.080\ \mu m$

Example: dipole radiation in 2D

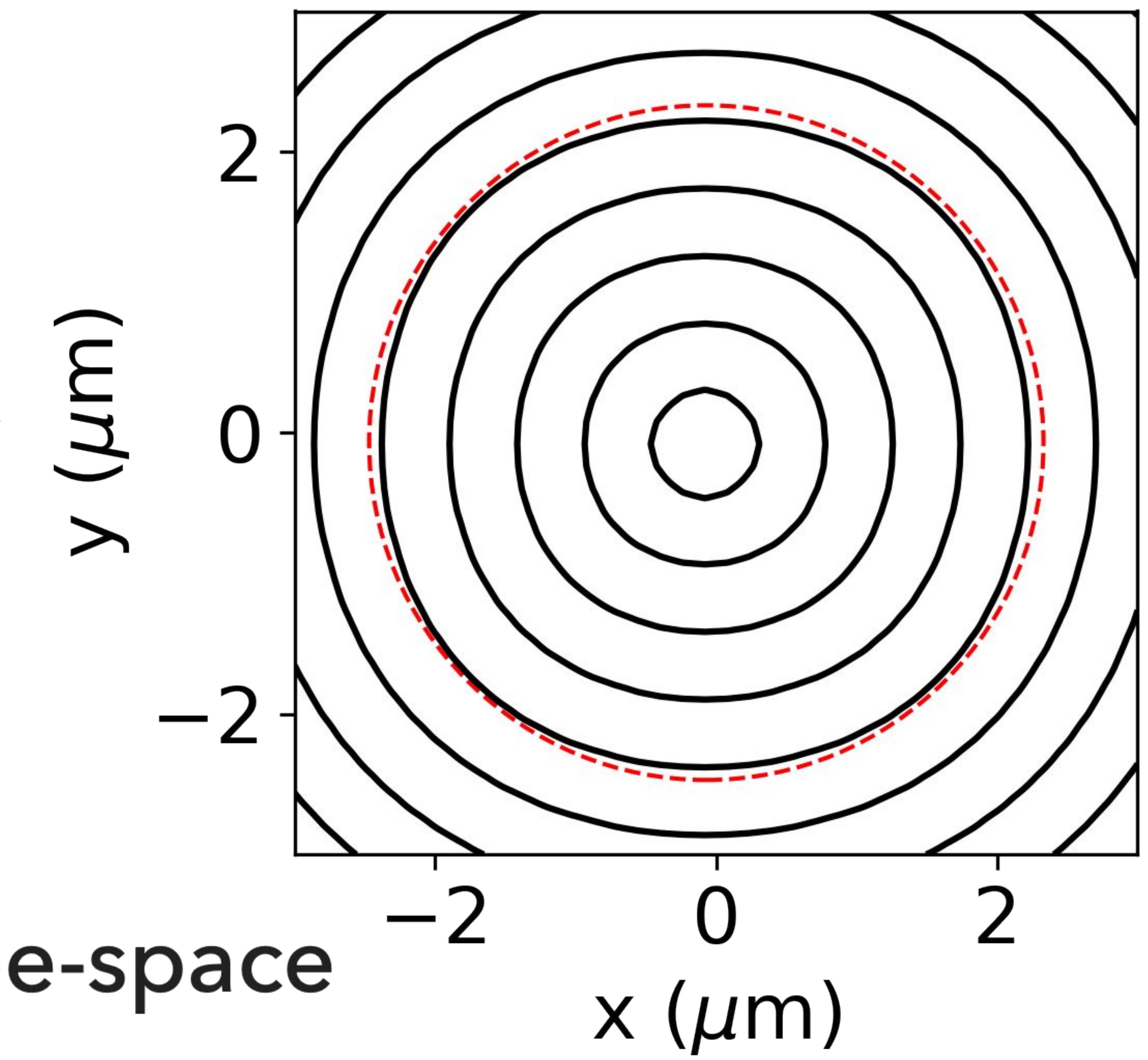


$\frac{\omega\Delta x}{c} = 0.5$

$\frac{\omega\Delta x}{c} = 1$

$\frac{\omega\Delta x}{c} = 1.5$

Wavefront (E_z=0)



z-polarized dipole in free-space

- Wavelength: 1 μm
- Spatial discretization: $\Delta x = 0.159\ \mu m$

Summary

A key source of error in FDTD is numerical dispersion.

To have small numerical dispersion error, a good rule of thumb is to have a spatial discretization less than $\lambda_{material}/20$.