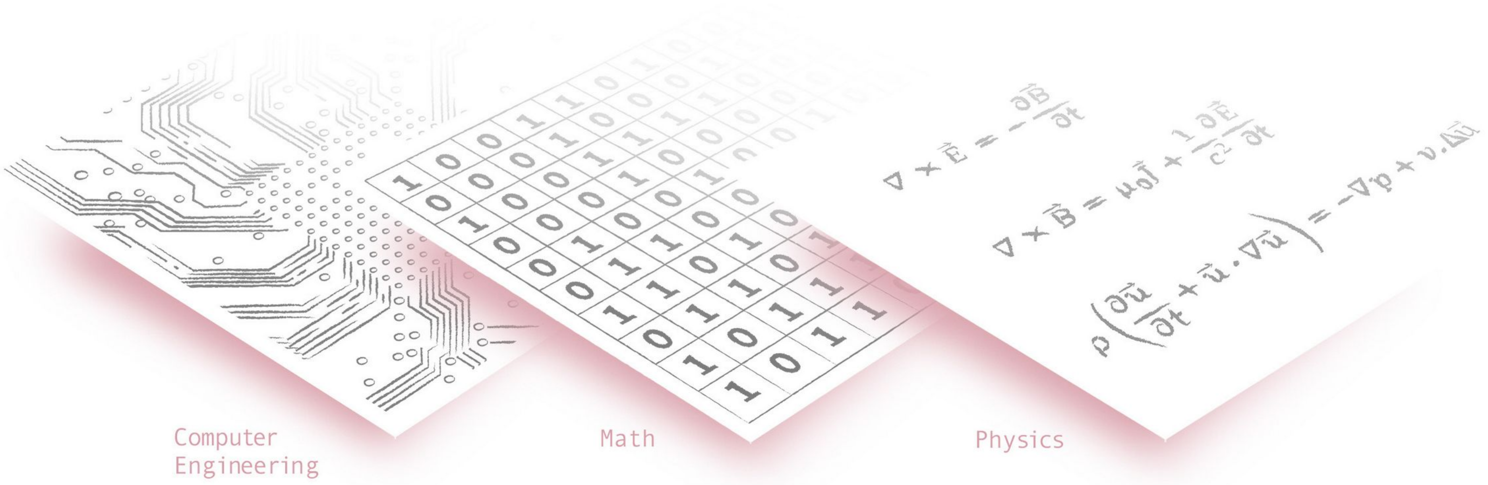

Inverse Design in Photonics

Tutorial 2: Adjoint Method

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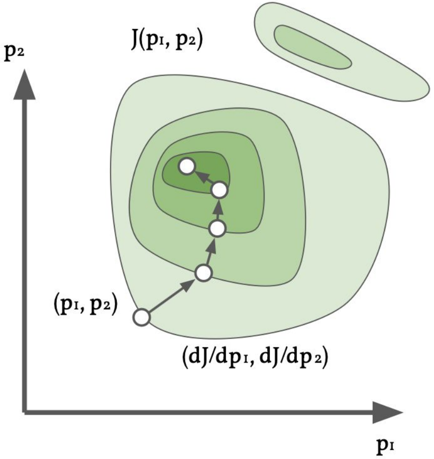
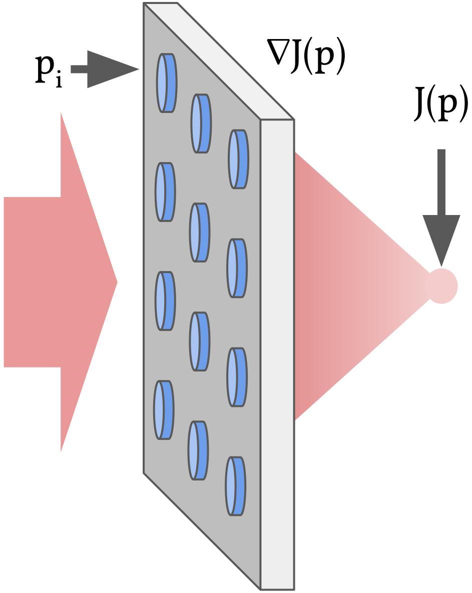
Review: Gradient & Optimization

- Device performance "J" is a function of design parameters "p".
- The goal of computational design is to maximize J(p).
- Strategy: compute "gradient" $\nabla J(p)$ and repeatedly update p.





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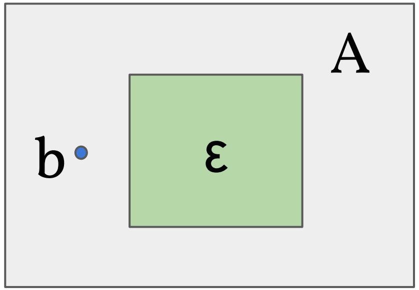
Simulation

First, let's review the math of a linear EM simulation.



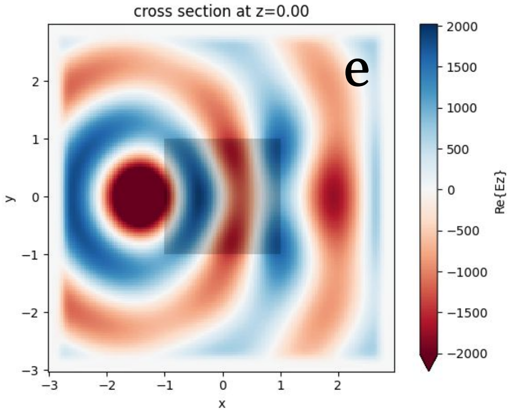
Maxwell's equations tell us how to solve for E(r) given permittivity distribution at frequency ω.

$$\left[\nabla \times \nabla \times -k_0^2 \epsilon_r(r)\right] E(r) = -i\mu_0 \omega P(r)$$

Write as linear system $$Ae = b$$

(e is a vector of the flattened electric field values).

Solve the system. $$e = A^{-1}b$$



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Objective Function

Now, introduce a figure of merit "J" for this device.

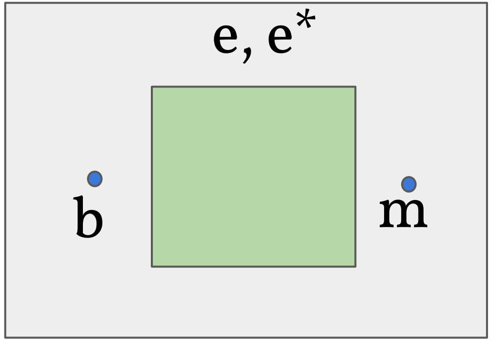
J depends explicitly on the field solution (e) and its complex conjugate (e^*).

As illustration, say we want to maximize the electric field intensity ($|E|^2$) at a point "m". Write J as:

$$J(e, e^*) = \underbrace{\left(m^T e\right)}_{E(m)} \underbrace{\left(m^T e^*\right)}_{E^*(m)}$$

Note: "m" is a vector that has a single non-zero entry at point m.

$$J = J(e, e^*)$$



Gradient (General Form)

Written in terms of the parameters, our objective function is:

$$J = J(e(p), e^*(p))$$

To compute the gradient of J w.r.t. p, first take derivative.

$$\frac{dJ}{dp} = \frac{\partial J}{\partial e}\frac{de}{dp} + \frac{\partial J}{\partial e^*}\frac{de^*}{dp} = 2\mathcal{R}\left\{\frac{\partial J}{\partial e}\frac{de}{dp}\right\}$$

Depends only on form of J

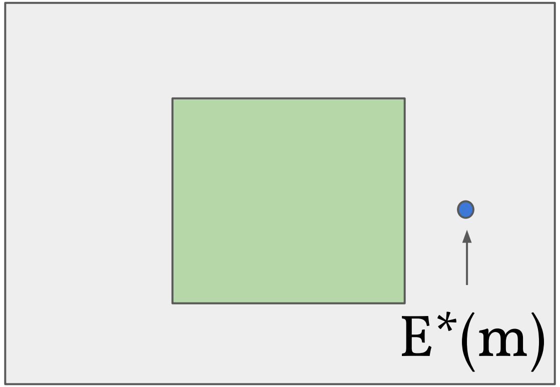
Depends only on simulation

Gradient (Example)

Consider the first term:

$$\boxed{\frac{dJ}{dp} = 2\mathcal{R}\left\{\boxed{\frac{\partial J}{\partial e}}\frac{de}{dp}\right\}}$$

Zero everywhere but “m”, with amplitude of $E^*(m)$



Our objective:

$$J(e, e^*) = \left(m^T e\right)\left(m^T e^*\right)$$

Take partial w.r.t e.

$$\frac{\partial J}{\partial e} = \left(m^T e^*\right) m^T = E^*(m)\; m^T$$

Complex conjugate of measured E field at m.

Vector of 0’s with 1 at m.

Gradient Field Dependence

We computed dJ/de, but what about de/dp? Return to our original system of equations, consider 2nd term.

$$\frac{dJ}{dp} = 2\mathcal{R}\left\{\frac{\partial J}{\partial e}\boxed{\frac{de}{dp}}\right\} \qquad (1)$$

$$Ae = b$$ → Apply d/dp

$$\frac{dA}{dp}e + A\frac{de}{dp} = 0$$ → Solve for de/dp

$$\frac{de}{dp} = -A^{-1}\frac{dA}{dp}e$$ → Plug into (1)

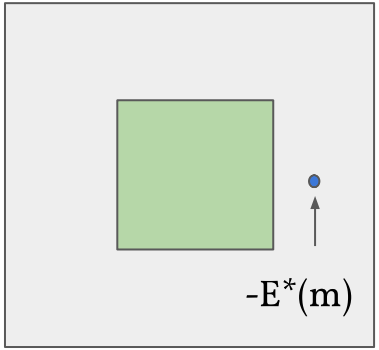
Full gradient equation:

$$\boxed{\frac{dJ}{dp} = 2\mathcal{R}\left\{-\frac{\partial J}{\partial e}A^{-1}\frac{dA}{dp}e\right\}}$$

Interpretation of Gradient Equation

$$\frac{dJ}{dp} = 2\mathcal{R}\left\{-\frac{\partial J}{\partial e} A^{-1} \frac{dA}{dp} e\right\}$$

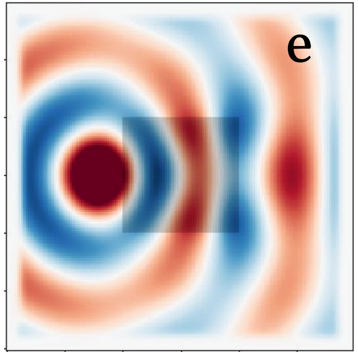
E*(m) at point “m”



Inverse of system matrix (Maxwell’s Eqs).

Derivative of system matrix w.r.t. “p”.

Original fields



Adjoint Simulation

Consider solving the first two terms:

$$\frac{dJ}{dp} = 2\mathcal{R}\left\{\boxed{-\frac{\partial J}{\partial e} A^{-1}}\frac{dA}{dp} e\right\}$$

$$\downarrow$$

$$e_{adj}^T = -\frac{\partial J}{\partial e} A^{-1}$$

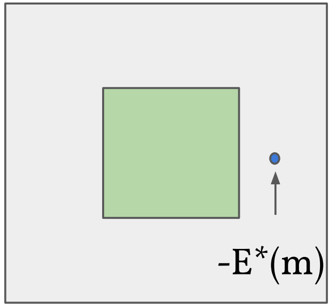
Transpose, multiply by A.

Note: A is symmetric

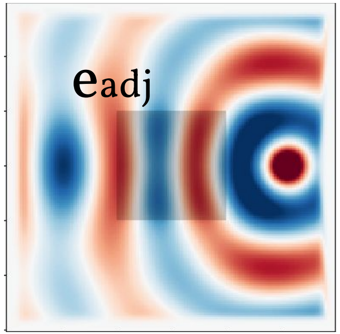
$$A \; e_{adj} = -\frac{\partial J}{\partial e}^T$$

“adjoint source”

We call this the “adjoint” simulation, note the source at m with amplitude $E^*(m)$



Solve the same way as original.



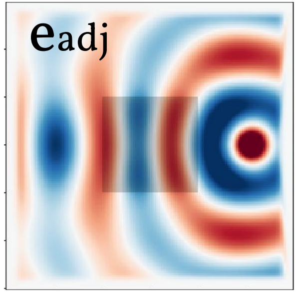
Evaluating the Gradient

With our adjoint field solution:

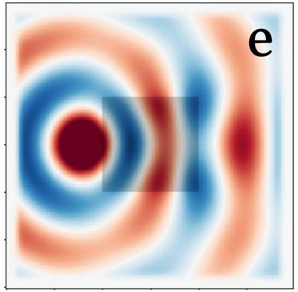
$$e_{adj} = A^{-1}\left(-\frac{\partial J}{\partial e}^{T}\right)$$

Write gradient in terms of field overlap over dA/dp

$$\boxed{\frac{dJ}{dp} = 2\mathcal{R}\left\{\, e_{adj}^{T}\frac{dA}{dp}e \,\right\}}$$



$$\cdot\ \frac{dA}{dp}\ \cdot$$



System Matrix Derivative

$$\frac{dJ}{dp} = 2\mathcal{R}\left\{ e_{adj}^T \boxed{\frac{dA}{dp}} e \right\}$$

How to interpret the dA/dp term?

From definition of A from Maxwell's Equations

$$A \equiv \nabla \times \nabla \times -k_0^2 \epsilon_r(r)$$

$$\frac{dA}{dp} = -k_0^2 \frac{d\epsilon_r(r)}{dp}$$

dA/dp is a diagonal matrix with each element relating how the permittivity at each point (r) depends on parameter "p".

$$\boxed{\frac{dJ}{dp} = -2k_0^2\mathcal{R}\left\{ e_{adj}^T \frac{d\epsilon_r(r)}{dp} e \right\}}$$

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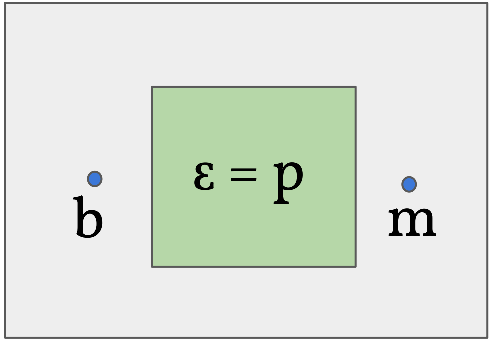
System Matrix derivative (example)

How to interpret this term for our example?

$$\frac{dJ}{dp} = -2k_0^2 \mathcal{R}\left\{ e_{adj}^T \boxed{\frac{d\epsilon_r(r)}{dp}} e \right\}$$

Let “p” be the permittivity of our box.



Diagonal matrix with 1 for points inside box, otherwise 0.

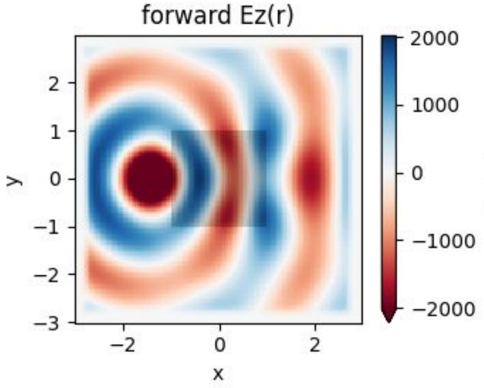
$$\frac{d\epsilon_r(r)}{dp} = \delta_{r \in p}$$

Plug into gradient equation

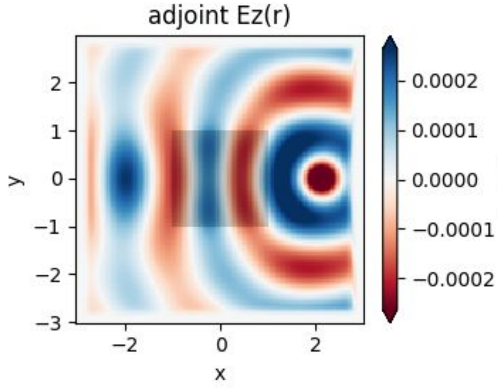
$$\frac{dJ}{dp} = -2k_0^2 \mathcal{R}\left\{ e_{adj}^T \delta_{r\in p} e \right\}$$
$$= -2k_0^2 \sum_{i \in p} \mathcal{R}\left\{ e_{adj}^{(i)} e^{(i)} \right\}$$

Interpretation: sum the product of original and adjoint fields over the Box location.

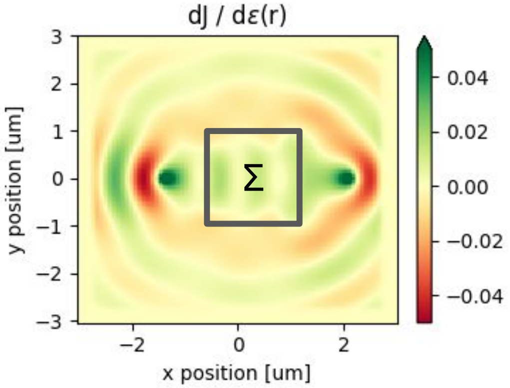
General Procedure



1. Compute the solution to the original simulation (e).



2. Construct the adjoint source and solve the adjoint solution (e_adj)

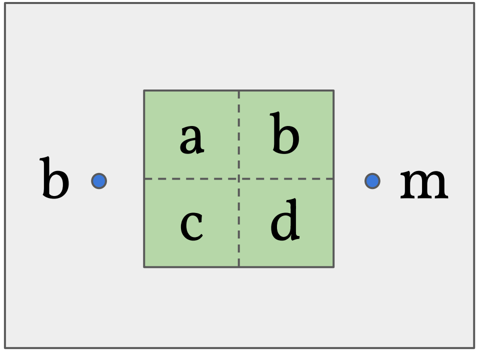


3. Multiply the result and sum over regions where the parameters have influence.

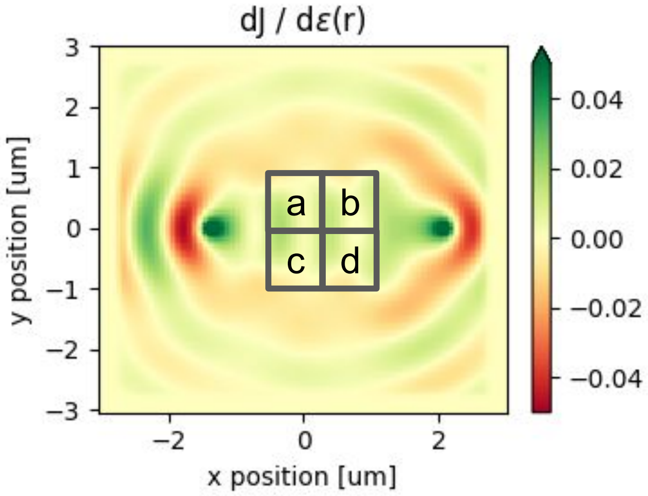
Multiple Parameters

Now, assume we have many parameters.

Example, 4 parameters describing the permittivity of each quadrant of the box.



To compute gradient, simply sum gradient map over each quadrant!



Note: only 2 simulations needed! Can reuse no matter how many parameters you have.
In inverse design, can be millions.