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The Computer Implementation of Glimm's Method

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DISCLAIMER

I. INTRODUCTION

Glimm's method deals with the solution of the nonlinear hyperbolic equation

$$\partial_t \underline{U} + \partial_x (F(\underline{U})) = 0 \tag{1}$$

where $\underline{U}$ is the solution vector. The method is rooted in the constructive proof by Glimm [2] that such systems have weak solutions. Let $\underline{U}(x,o)$, the initial data, be close to constant (see Glimm [2]). Let time be divided into intervals of length k. Let h be a spatial increment. The solution is to be evaluated at time, $t = nk$, n an integer, at the points ih, $i = 0, \pm 1, \pm 2, \ldots$ and at time $t = (n + 1/2)k$, at the points $(i + 1/2)h$.

Consider the Riemann problem, where the initial data is given by

$$\underline{U}(x,o) = \begin{cases} \underline{U}_0, & x < 0 \\ \underline{U}_n, & x > 0 . \end{cases} \tag{2}$$

The following theorem is proved in [6]:

> <u>Theorem</u>: If the states $\underline{U}_0$ and $\underline{U}_n$ are sufficiently close (see [2] and [6]), the Riemann problem (1) and (2) has a solution. This solution consists of $n + 1$ constant states $\underline{U}_0, \underline{U}_1, \ldots \underline{U}_n$, separated by shocks or centered rarefaction waves.] for solving

The method developed by Glimm [2] for solving the initial value problem, with $\underline{U}(x,o)$ as initial data, where the oscillation of $\underline{U}(x,o)$ is small. The solution $\underline{U}$ is obtained in the limit as $h \to 0$ of approximate solutions $\underline{U}_i^n$, where $\underline{U}_i^n$ approximates $\underline{U}(ih,nh)$. At time $t = 0$, $\underline{U}_i^0$ is a piecewise constant approximation to $\underline{U}(x,o)$, i.e.,

$$\underline{U}_i^0 = \underline{U}_i, \quad ih < x < (i + 1)h, \quad i = 0, \pm 1 \ \ \pm 2, \ \ldots \tag{3}$$

where $\underline{U}_i$ is some mean value of $\underline{U}(x,o)$ over the interval $[ih,(i + 1)h]$. Consider $\frac{k}{h} < \frac{1}{|u| + C}$, where C is the local sound speed. For $0 \leq t < \frac{h}{2(|u| + C)}$, $\underline{U}_i^{1/2}$ is the exact solution of (1) and (3). This solution is constructed by solving the Riemann problem

$$\underline{U}(x,o) = \begin{cases} a_{i+1}, & x \geq (i + 1/2)h \\ a_i, & x < (i + 1/2)h. \end{cases} \qquad i = 0, \pm 1, \ldots \tag{4}$$

Since the oscillations of $\underline{U}(x,o)$ is small, $\underline{U}_i^0$, $\underline{U}_{i+1}^0$ are close and by the above theorem the initial value problem has a solution which consists of constant states separated by centered rarefaction waves and shocks originating from $((i + 1/2)h, 0)$, $i = 0, \pm 1, \pm 2, \ldots$ See Fig. 1.

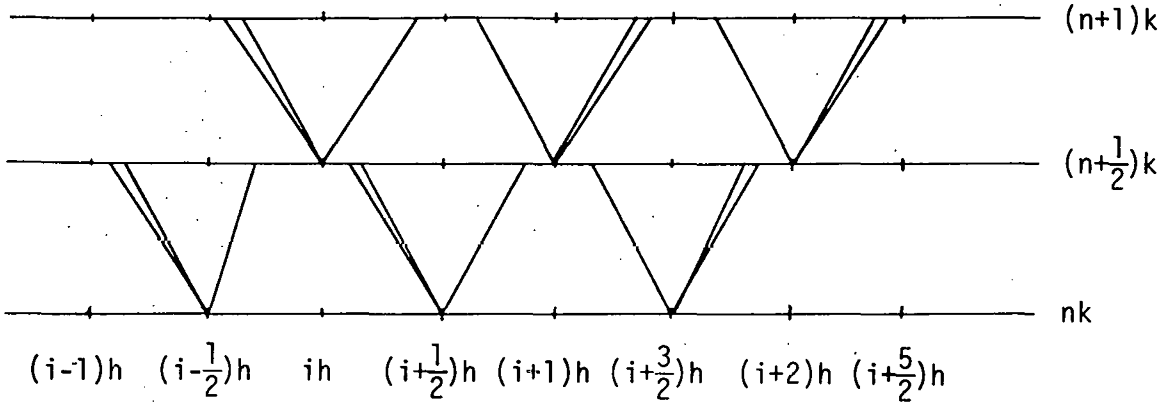


Fig. 1

For $t < \frac{h}{2(|u| + C)}$ these waves do not intersect each other. Hence the solution of the initial value problem (1) and (4) can be combined into a single exact solution $\underline{U}_{i+1/2}^{1/2}$. This process can be repeated for a time interval $[\frac{h}{2(|u| + C)}, \frac{h}{|u| + C}]$ and for the succeeding time intervals of width $\frac{h}{2(|u| + C)}$. Does this method yield an approximate solution

$\underline{U}_i^n$ which is defined by all time t? For the answer to be yes one must prove that the oscillation of $\underline{U}(x,t)$, $t = nh$ (and $t = (n + 1/2)h$) remain small, uniformly for $n = 1, 2, \ldots$, so that the Riemann problem (1) and (4) can be solved, and so that $|u| + C$ does not tend to infinity. This estimate critically depends on the way in which the average values a_i are selected. In the scheme introduced by Glimm [2] the quantity a_j are computed as follows: A sequence of random numbers, $\{\xi_j\}$, uniformly distributed in (-1/2, 1/2) is chosen. Then a_j^n, the average value of $\underline{U}(x, \frac{nh}{|u| + C})$ over $(jh,(j + 1)h)$ is taken to be

$$a_j^n = \underline{U}\left((j + 1/2)h + \xi_j^n h, \frac{nh}{|u| + C}\right) \tag{5}$$

where in the original scheme the random number ξ_j^n depends on x and t.

In [2] Glimm proved: (see [3] and [6])

(1) For all $\varepsilon > 0$, we can choose $\eta > 0$ so small that if the oscillation and total variation of $\underline{U}(x,o)$ are $< \eta$ then for any t, the oscillation and total variation of $\underline{U}(x,t)$ along any space-like line is $< \varepsilon$.

(2) A subsequence of $\underline{U}_i^n$ converges in the L_1-norm with respect to x, uniformly in t, to a limit $\underline{U}$.

(3) For almost all choices of random sequences $\{\xi_j\}$, this limit $\underline{U}$ is a weak solution.

In summary, given $\underline{U}_i^n$ and $\underline{U}_{i+1}^n$, the goal is to obtain $\underline{U}_{i+1/2}^{n+1/2}$. One begins by considering the Riemann problem (1) with the initial data

$$\underline{U}(x,o) = \begin{cases} \underline{U}_{i+1}^n & x \geq (i + 1/2)h \\ \underline{U}_i^n & x < (i + 1/2)h \end{cases} \tag{6}$$

Let $\hat{\underline{U}}(x,t)$ denote the solution of this Riemann problem. Let ξ_i be a value of a random variable ξ defined as above. Then we set $\underline{U}_{i+1/2}^{n+1/2} = \hat{\underline{U}}((i_{j+1/2} + \xi_i)h, \frac{(n + 1/2)h}{|u| + C})$. Similarly, one can proceed from $\underline{U}_{i+1/2}^{n+1/2}$ to $\underline{U}_i^{n+1}$.

Consider a shock wave with shock speed $\overline{U}$. Consider just one interval. See Fig. 2.

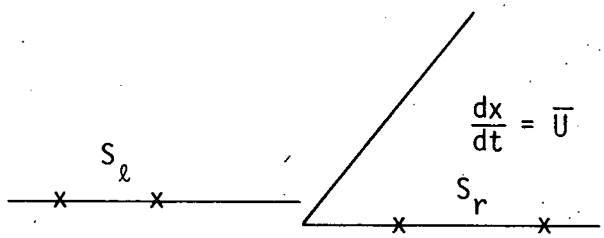


Fig. 2

This represents a Riemann problem with constant states S_ℓ and S_r. Sample at points "x" on this interval. If the point x lies to the left of the shock we take for its value at new time the value on the left of the shock and the shock moves over our grid point. If the point x lies to the right of the shock we take the value to the right of the shock and the shock does not move. This jiggling gives the shock and the average value of the velocity is $\overline{U}$. Clearly, in this way the shock remain sharp.

It should be noted that some randomness for the choices of a_i^n is needed For if we took $a_i^n = \underline{U}((i + 1/2)h, \frac{nh}{|u| + C})$, i.e. midpoints of the interval $(ih, (i + 1)h)$ and had a shock wave such that the slope is very steep, i.e. $\frac{dx}{dt} \sim 0$ but $\frac{dx}{dt} > 0$. In this case the value $(j + 1/2)h$ will always be to the left of the shock and hence by the above reasoning the shock will move to the right with speed $\frac{k}{h}$ independent of the above shock speed.

II. GODUNOV'S ITERATIVE METHOD

The one dimensional equations of gas dynamics may be written in the (conservation) form:

$$\partial_t \rho + \partial_x(\rho u) = 0 \tag{7}$$

$$\partial_t m + \partial_x\left(\frac{m^2}{\rho} + p\right) = 0 \tag{8}$$

$$\partial_t e + \partial_x\left((e + p)\frac{m}{\rho}\right) = 0 \tag{9}$$

where ρ is density, u is velocity, ρu is momentum, p is pressure, and e is energy per unit volume. We may write $e = \rho\varepsilon + 1/2\ \rho u^2$, where ε is the internal energy per unit mass. Assume the gas is polytropic, in which case

$$\varepsilon = \frac{p}{(\gamma - 1)\rho} \quad , \tag{10}$$

where γ is a constant greater than 1. Furthermore, from (10) we have

$$p = A(s)\rho^\gamma \tag{11}$$

where s denotes entropy.

Consider the initial data

$$\underline{U}(x,o) = \begin{cases} S_\ell = (\rho_\ell, u_\ell, p_\ell), & x < 0 \\ S_r = (\rho_r, u_r, p_r), & x \geq 0 \end{cases} \tag{12}$$

The solution at later times looks like (see [5] and [8]) Fig. 3,

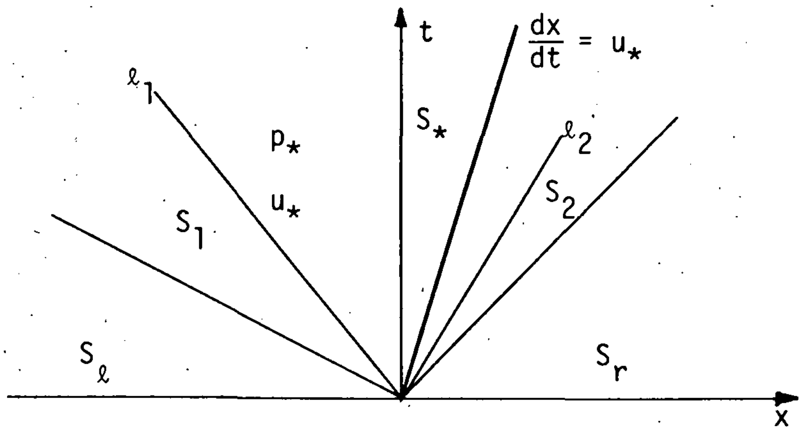


Fig. 3

where S_1 and S_2 are either a shock or a centered rarefaction wave. The region S_* is a steady state. The lines ℓ_1 and ℓ_2 are slip lines separating states. The slip line $\frac{dx}{dt} = u_*$ separates the state S_* into two parts with possibly different values of ρ_* , but equal constant values of u_* and p_*.

We wish to solve this Riemann problem. Using a method due to Godunov [4] (also [8]) we first evaluate p_* in the state S_*. Define the quantity

$$M_\ell = \frac{p_\ell - p_*}{u_\ell - u_*} . \tag{13}$$

If the left wave is a shock, using the jump condition $U_\ell[\rho] = [\rho u]$, we obtain

$$M_\ell = \rho_\ell(u_\ell - U_\ell) = \rho_*(u_* - U_\ell) \tag{14}$$

where U_ℓ is the velocity of the left shock and ρ_* is the density in the portion of S_* adjoining the left shock. Similarly, define the quantity

$$M_r = \frac{p_r - p_*}{u_r - u_*} \tag{15}$$

If the right wave is a shock, using the jump condition $U_r[\rho] = [\rho u]$, we obtain

$$M_r = -\rho_r(u_r - U_r) = -\rho_*(u_* - U_r) \tag{16}$$

where U_r is the velocity of the right shock and ρ_* is the density in the portion of S_* adjoining the right shock.

In either of the two cases ((13) or (14) for M_ℓ and (15) and (16) for M_r) we obtain

$$M_r = \sqrt{\rho_r p_r}\ \phi(p_*/p_r) \tag{17}$$

$$M = \sqrt{\rho_\ell p_\ell}\ \phi(p_*/p_\ell) \tag{18}$$

where

$$\phi(x) = \begin{cases} \sqrt{\dfrac{\gamma+1}{2}\ x + \dfrac{\gamma-1}{2}}\ , & x \geq 1 \\ \dfrac{\gamma-1}{2\sqrt{\gamma}}\ \dfrac{1-x}{1-x^{\frac{\gamma-1}{2\gamma}}}\ , & x \leq 1\ . \end{cases} \tag{19}$$

Upon the elimination of u_* from (13) and (15) we obtain

$$p_* = \frac{(u_\ell - u_r + \dfrac{p_r}{M_r} + \dfrac{p_\ell}{M_\ell})}{\dfrac{1}{M_\ell} + \dfrac{1}{M_r}}\ . \tag{20}$$

Equations (17), (18), and (20) represent three equations in three unknowns for which it can be seen that there exists a real solution. These three equations lead to the following iteration scheme:

Upon choosing a starting value of p_*^0 (or M^0 and M_r^0), we compute (see Chorin [1])

$$\hat{p}_*^q = \frac{u_\ell - u_r + \dfrac{p_\ell}{M_\ell^q} + \dfrac{p_r}{M_r^q}}{\dfrac{1}{M_\ell^q} + \dfrac{1}{M_r^q}} \tag{21}$$

$$p_*^{q+1} = \max\ (\varepsilon, \hat{p}_*^q) \tag{22}$$

$$M_\ell^{q+1} = \sqrt{\rho_\ell p_\ell}\ \phi\ (p_\star^{q+1}/p_\ell) \tag{23}$$

$$M_r^{q+1} = \sqrt{\rho_r p_r}\ \phi\ (p_\star^{q+1}/p_r)\ . \tag{24}$$

As there is no guarantee that in the course of the iteration $\hat{p}_\star^q$ will remain non-negative, we add equation (22). The iteration procedure is stopped when

$$\max\ (|M_\ell^{q+1} - M_\ell^q|,\ |M_r^{q+1} - M_r^q|) \le \varepsilon'. \tag{25}$$

Then one sets $M_\ell = M_\ell^{q+1}$, $M_r = M_r^{q+1}$, and $p_\star = p_\star^{q+1}$. We usually took $\varepsilon = \varepsilon' = 10^{-6}$. See Chorin [1]. However, the tolerance ε' can be reduced. Following Chorin, as a starting value we took

$$p_\star^0 = \frac{(p_\ell + p_r)}{2}\ .$$

Godunov [4] noted that the iteration may fail in the pressence of a strong rarefaction wave. This problem has been overcome by Chorin [1]: If the iteration has not conveyed after $\ell \cdot N$ iterations ($\ell = 1, 2, \ldots$), equation (22) may be replaced by

$$p_\star^{q+1} = \alpha_\ell \max(\varepsilon,\ \hat{p}_\star^q) + (1 - \alpha_\ell)p_\star^q \tag{22'}$$

where $\alpha_\ell = \alpha_{\ell-1}/2$, $\alpha_1 = 1/2$. In practice N was taken to be 20.

After $p_\star$, M_ℓ, and M_r have been determined we may obtain $u_\star$, by eliminating $p_\star$ from equations (13) and (15),

$$u_\star = \frac{p_\ell - p_r + M_\ell u_\ell + M_r u_r}{M_\ell + M_r} \tag{26}$$

III. SELECTION OF RANDOM NUMBERS

In Glimm's original construction a new value of ξ was chosen for each i and n. The practical effect of such a choice with finite h is disastrous, since our initial data is not close to constant. See Chorin [1]. In fact, if ξ is chosen for each i and n, it is possible that a state will propagate to the left and to the right and thus create a constant spurious state. This was observed by Moler & Smoller [7]. The first improvement (due to Chorin) is to chose a new ξ only once per time level. Under this choice, if we consider a shock moving at a constant speed U between two constant states, with $k < h/U$, the position of the shock after n half steps is

$$x = h \sum_{i=1}^{n} \eta_i ,$$

where

$$\eta_i = \begin{cases} 1/2, & h\xi_i < Uk/2 \\ -1/2, & h\xi_i \geq Uk/2 \end{cases} .$$

The standard deviation of x, which is a measure of the statistical error is

$$\text{s.d. } x = h\sqrt{n}\,[(1/2 + Uk)\,(1/2 - Uk)]^{1/2}$$

which attains its maximum value when $Uk = 0$, thus

$$\max \text{ s.d. } x = \frac{h\sqrt{n}}{2} .$$

The following improvement, due to Chorin, in the selection of the random numbers reduces the variance. The idea is to construct the sequence of samples ξ_i reach an approximate equidistribution over $(-1/2, 1/2)$ at a faster rate.

To this end pick two integers k_1 and k_2 such that $k_2 < k_1$ and k_1, k_2 are mutually prime. Construct a sequence of integers, when η_0 is given such that $\eta_0 < k_1$, by

$$\begin{aligned} \eta_{i+1} &= (\eta_i + k_2) \bmod k_1 \\ &= (\eta_i + k_2) - \mathrm{trunc}\,((\eta_i + k_2)/k_2)k_1 \, . \end{aligned} \tag{27}$$

Consider the sequence of samples ξ_1, ξ_2,...of ξ. We modify this sample in the following way

$$\xi_i' = \frac{1}{k_1} (\xi_i + \eta_i) \, . \tag{28}$$

The ξ_i' will be used in place of ξ_i. If in the above analysis of the standard deviation we replace ξ_i by ξ_i' we obtain

$$\max \text{ s.d. } x = \frac{h\sqrt{n}}{2\sqrt{k_1}} \, .$$

In practice we choose $k_1 = 11$, $k_2 = 7$, and $\eta_0 = 2$.

IV. IMPLEMENTATION OF GLIMM'S METHOD

The boundary conditions are easily handled with symmetry considerations. If there is a boundary at some point x with the fluid, say, to the right of the boundary the boundary condition will be imposed on the grid point jh, to the boundary at x. A false left state S_ℓ is created, using

$$\rho_{j-1/2} = \rho_{j+1/2} \tag{29}$$

$$u_{j-1/2} = -u_{j+1/2} \tag{30}$$

$$p_{j-1/2} = p_{j+1/2} \tag{31}$$

this will produce $u_x^n = 0$; i.e., the velocity will be zero at the wall. This will allow shock and rarefaction waves to reflect from the boundary which is on the average exact. In the case where the boundary is moving to the right with velocity, V, we replace equation (30) by

$$u_{j-1/2} = 2V - u_{j+1/2} \quad . \tag{30'}$$

Consider a passive quantity S being transported by the fluid, it can readily be seen that

$$S_{i+1/2}^{n+1/2} = \begin{cases} S_i^n , & \xi_n h < u_\star k \\ S_{i+1}^n , & \xi_n h \geq u_\star k \end{cases} \quad . \tag{32}$$

As a result, if S is sharply defined, S remains sharply defined. See Chorin [1].

With the use of the Godunov iteration, Glimm's method can readily deal with multifluid problems. Consider the Riemann problem with initial data

$$U(x,o) = \begin{cases} \bar{S}_\ell = (\rho_\ell, u_\ell, P_\ell, \gamma_\ell), & x < 0 \\ \bar{S}_r = (\rho_r, u_r, P_r, \gamma_r), & x \geq 0 \end{cases} \tag{33}$$

where γ_ℓ and γ_r refer to the γ's of γ-law relation (10) and (11) in the left and right states. As a result the Godunov iteration is modified, so that M_ℓ^{q+1} in (23) uses γ_ℓ in equation (19) and similarly M_r^{q+1} in (24) uses γ_r in equation (19). The motion of interface between the two fluids is treated as a passive quantity and is transported using (32).

As in Fig. 3, the fluid initially at $x \leq 0$ is separated from the fluid initially at $x > 0$ by a slip line $\frac{dx}{dt} = u_*$. There are a total of 10 cases to consider.

I. The sample point $\xi_n h$ lies to the left of the slip line ($\xi_n h < u_* k/2$).

(a) If the left wave is a shock wave ($p_* > p_\ell$) and (1) if $\xi_n h$ lies to the left of the shockline $\frac{dx}{dt} = U_\ell$, we have $\rho = \rho_\ell$, $u = u_\ell$, and $p = p_\ell$, (2) if $\xi_n h$ lies to the right of the shockline $\frac{dx}{dt} = U_\ell$, we have $\rho = \rho_*$, $u = u_*$, $p = p_*$, where ρ_* can be obtained from (14)

$$\rho_* = \frac{M_\ell}{U_\ell - u_*} \tag{34}$$

(b) If the left wave is a rarefaction wave ($p_* \leq p_\ell$). Define the sound speed to be $C = \sqrt{\frac{\gamma p}{\rho}}$. The rarefaction wave is bounded on the left by the line defined by $\frac{dx}{dt} = u_\ell - C_\ell$, where $C_\ell = \sqrt{\frac{\gamma p_\ell}{\rho_\ell}}$, and on the right by the line defined by $\frac{dx}{dt} = u_* - C_*$, where $C_* = \sqrt{\frac{\gamma p_*}{\rho_*}}$. The flow is adiabatic in smooth regions, so in this region A(s) in (11) is a constant, denoted by A, and we obtain the isentropic law $p = A\rho^\gamma$. ρ_* is obtained by using the isentropic law

$$p_\ell \rho_\ell^{-\gamma} = p_* \rho_*^{-\gamma} = A \quad . \tag{35}$$

Then we obtain from (34)

$$\rho_* = \left(\frac{p_*}{A}\right)^{1/\gamma} , \tag{36}$$

(1) if $\xi_n h$ lies to the left of the rarefaction wave, then $\rho_* = \rho_\ell$, $u = u_\ell$, and $p = p_\ell$.

(2) if $\xi_n h$ lies inside the left rarefaction wave, we equate the slope of the characteristic $\frac{dx}{dt} = u - C$ to the slope of the line through the origin and $(\xi_n h, k/2)$, obtaining

$$u - C = \frac{2\xi_n h}{k} . \tag{37}$$

With the constancy of the Riemann invariant

$$2C(\gamma - 1)^{-1} + u = 2C_\ell(\gamma - 1)^{-1} + u_\ell , \tag{38}$$

the isentropic law, and the definition of C, we can obtain ρ, u, and p. Using the isentropic law we obtain

$$p = p_\ell \rho^{-\gamma} \rho^\gamma = A\rho^\gamma . \tag{39}$$

Using equation (38) we obtain, by solving for C

$$C = C_\ell + \frac{\gamma - 1}{2}(u_\ell - u) . \tag{40}$$

By substitution of (40) into (37) and solving for u we obtain

$$u = \frac{2}{\gamma + 1}\left(\frac{2\xi_n h}{k} + C_\ell + \frac{(\gamma - 1)}{2} u_\ell\right) . \tag{41}$$

By substitution of (41) into (40) C is obtained; by substitution of (40) into the definition of C and solving for ρ we obtain

$$\rho = \left(\frac{C^2}{\gamma A} \right)^{\frac{1}{\gamma-1}} \tag{42}$$

(3) if $\xi_n h$ lies to the right of the left rarefaction wave we obtain $\rho = \rho_*$, $u = u_*$, and $p = p_*$.

II. The sample point $\xi_n h$ lies to the right of the slip line ($\xi_n h \geq u_* k/2$)

(a) if the right wave is a shock wave ($p_* > p_r$) and (1) if $\xi_n h$ lies to the left of the shockline defined by $\frac{dx}{dt} = U_r$, we have $\rho = \rho_*$, $u = u_*$, and $p = p_*$, where ρ_* is obtained from (15)

$$\rho_* = \frac{-M_r}{u_* - U_r} \quad . \tag{43}$$

(2) If $\xi_n h$ lies to the right of the shockline defined by $\frac{dx}{dt} = U_r$, we have $\rho = \rho_r$, $u = u_r$, and $p = p_r$.

(b) If the right wave is a rarefaction wave ($p_* \leq p_r$). The rarefaction wave is bounded on the left by the line defined by $\frac{dx}{dt} = u_* + C_*$, where $C_* = \sqrt{\frac{\gamma p_*}{\rho_*}}$ and ρ_* can be obtained from the isentropic law

$$p_r \rho_r^{-\gamma} = p_* \rho_*^{-\gamma} = A. \tag{44}$$

Then we obtain from (44)

$$\rho_* = \left(\frac{p_*}{A} \right)^{\frac{1}{\gamma}} \quad ; \tag{45}$$

and on the right by the line defined by $\frac{dx}{dt} = u_r + C_r$, $C_r = \sqrt{\frac{\gamma p_r}{\rho_r}}$.

(1) if $\xi_n h$ lies to the left of the rarefaction wave, then $\rho = \rho_*$, $u = u_*$, and $p = p_*$.

(2) if $\xi_n h$ lies inside the right rarefaction wave, we equate the slope of the characteristic $\frac{dx}{dt} = u + C$ to the slope of the line through the origin and $(\xi_n h,\ k/2)$, obtaining

$$u + C = \frac{2\xi_n h}{k} \quad . \tag{46}$$

With the constancy of the Riemann invariant

$$2C(\gamma - 1)^{-1} - u \qquad 2C_r(\gamma - 1)^{-1} - u_r \tag{47}$$

the isentropic law, and the definition of C, we can obtain ρ , u, and p. Using the isentropic law we obtain

$$p = p_r \rho_r^{-\gamma} \rho^{\gamma} = A\rho^{\gamma} \ . \tag{48}$$

Using equation (47) we obtain, by solving for C

$$C = C_r + \frac{\gamma - 1}{2}(u - u_r) \quad . \tag{49}$$

Substitution of (49) into (46) and solving for u we obtain

$$u = \frac{2}{\gamma + 1} \left(\frac{2\xi_n h}{k} - C_r + \frac{\gamma - 1}{2} u_r \right). \tag{50}$$

By substitution of (50) into (49) C is obtained; by substitution of (48) into the definition of C and solving for ρ we obtain

$$\rho = \left(\frac{C^2}{\gamma A} \right)^{\frac{1}{\gamma - 1}} \quad . \tag{51}$$

(3) if $\xi_n h$ lies to the right of the right rarefaction wave we obtain $\rho = \rho_r$, $u = u_r$, and $p = p_r$.

Equations (34) - (51) are the key to the programming of Glimm's method. For a summary see the flow chart, Fig. 4.

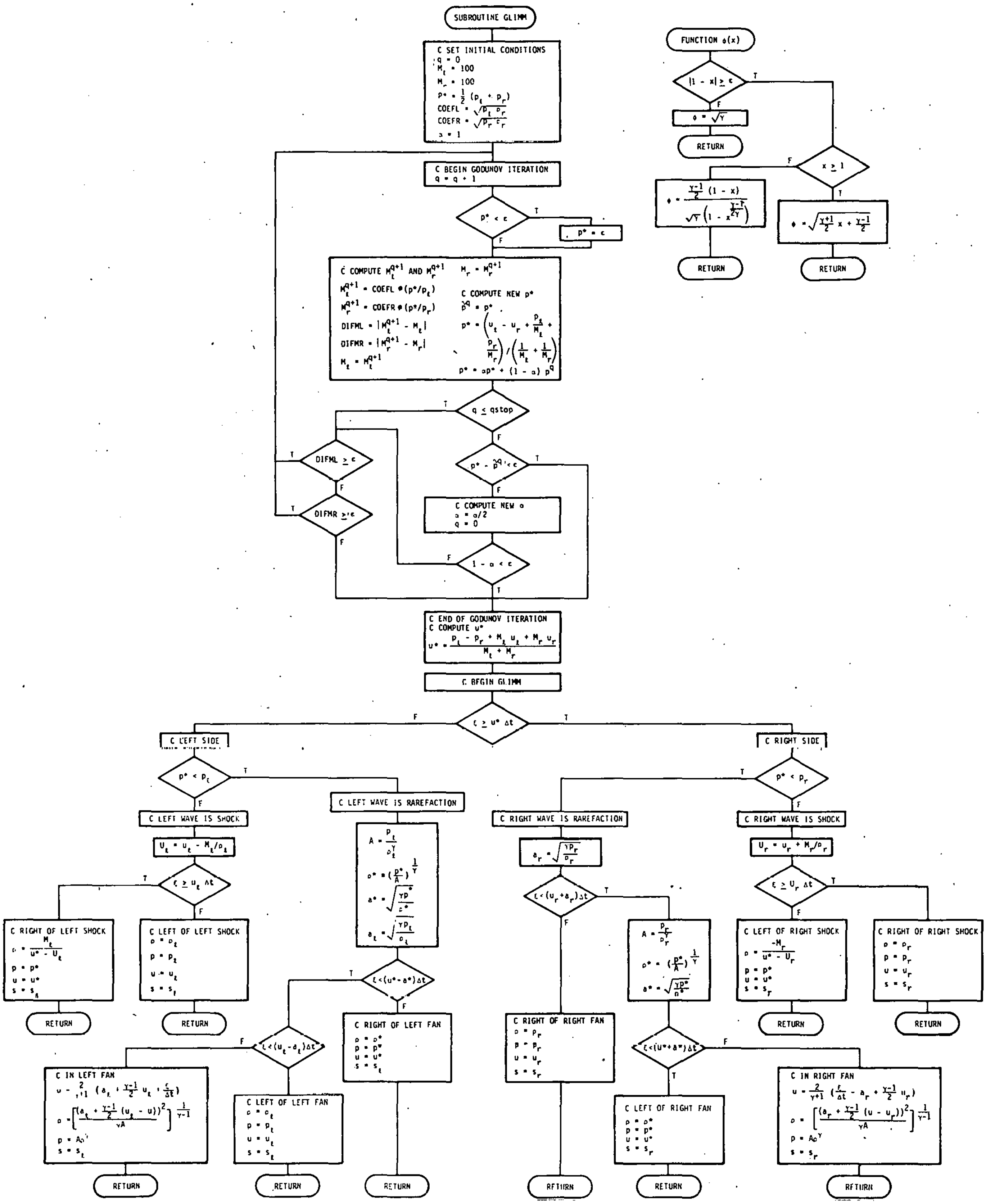

SUBROUTINE GLIMM
C SET INITIAL CONDITIONS
C BEGIN GODUNOV ITERATION
C COMPUTE NEW p*
C COMPUTE NEW α
C END OF GODUNOV ITERATION
C COMPUTE u*
C BEGIN GLIMM
C LEFT SIDE
C RIGHT SIDE
C LEFT WAVE IS SHOCK
C LEFT WAVE IS RAREFACTION
C RIGHT WAVE IS RAREFACTION
C RIGHT WAVE IS SHOCK
C RIGHT OF LEFT SHOCK
C LEFT OF LEFT SHOCK
C LEFT OF RIGHT SHOCK
C RIGHT OF RIGHT SHOCK
C RIGHT OF LEFT FAN
C RIGHT OF RIGHT FAN
C IN LEFT FAN
C LEFT OF LEFT FAN
C LEFT OF RIGHT FAN
C IN RIGHT FAN
FUNCTION φ(x)
RETURN

Fig. 4

V. USAGE OF THE SUBROUTINE GLIMM

In order to use the FORTRAN SUBROUTINE GLIMM the following COMMON statement which contains all of the necessary input parameters is required:

```
COMMON//DX,DT,RL,PL,UL,EL,GL,R,P,MOM,E,GAMMA,RR,PR,UR,ER,GR,XI
```

where

DX = The spatial increment (input parameter).

DT = The time step (input parameter).

RL = The density in the left state of the Riemann problem (input parameter).

PL = The pressure in the left state of the Riemann problem (input parameter).

UL = The velocity in the left state of the Riemann problem (input parameter).

EL = The total energy per unit volume in the left state of the Riemann problem (input parameter).

GL = The gamma in the left state of the Riemann problem (input parameter).

R = The density as computed by GLIMM (output parameter).

P = The pressure as computed by GLIMM (output parameter).

MOM = The momentum as computed by GLIMM (output parameter).

E = The total energy per unit volume as computed by GLIMM (output parameter).

GAMMA = Gamma as computed by GLIMM, i.e. the location of the interface (output parameter).

RR = The density in the right state of the Riemann problem (input parameter).

PR = The pressure in the right state of the Riemann problem (input parameter).

UR = The velocity in the right state of the Riemann problem (input parameter).

ER = The velocity in the right state of the Riemann problem (input parameter).

GR = The gamma in the right state of the Riemann problem (input parameter).

XI = The random number used by GLIMM (input parameter).

To use GLIMM one simply makes the following call once per spatial step and once per half-time step:

CALL GLIMM.

The time (using the CHAT compiler) on the CDC 7600 for a typical shock tube problem (see Section VI) with 100 spatial points for 250 time steps (or 500 half steps) is 10.38 seconds.

VI. NUMERICAL RESULTS

In Figures 5-13 graphical results are presented for three shock tube problems with 100 spatial grid points. After every five time steps we average the data. This averaging is for display purposes only, it is not used subsequently in GLIMM. In Figs. 5, 6, and 7 the initial conditions are presented, each with $\gamma = 5/3$. The boundary conditions are reflective (i.e. rigid walls).

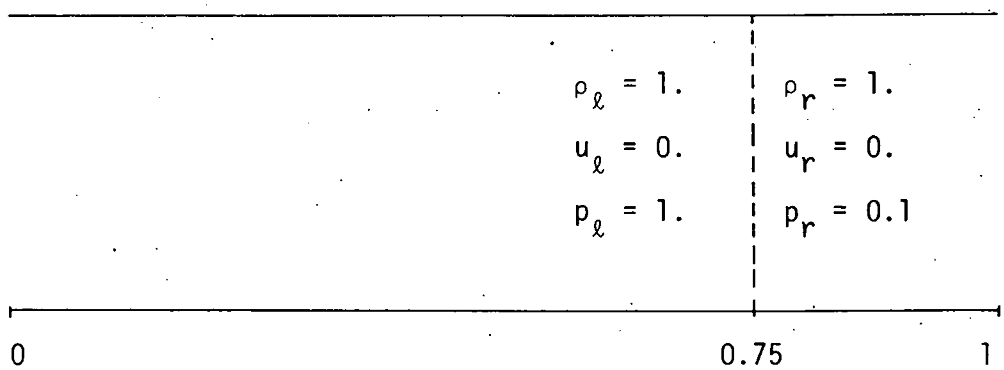


Fig. 5

For the first example the initial conditions are given in Fig. 5. In Fig. 8 at t = 1.780E-01 we see a rarefaction wave (1) moving to the left, a contact surface (2), and a shock wave (3) moving to the right. Notice the sharpness of the shock wave with the right boundary. This is just prior to the interaction of the shock wave and the contact surface. In Fig. 9 at t = 8.526E-01 we see the reflection of the shock wave with the right boundary. This is just prior to the interaction of the shock wave and the contact surface. In Fig. 10 at t = 1.021E+00 we see just after the interaction of the shock wave and the contact surface, a shock wave (2) has been transmitted through, moving to the left, the contact surface (3), and a rarefaction wave (4) has been reflected, moving to the right. The original rarefaction wave (1) has not yet reached the left boundary.

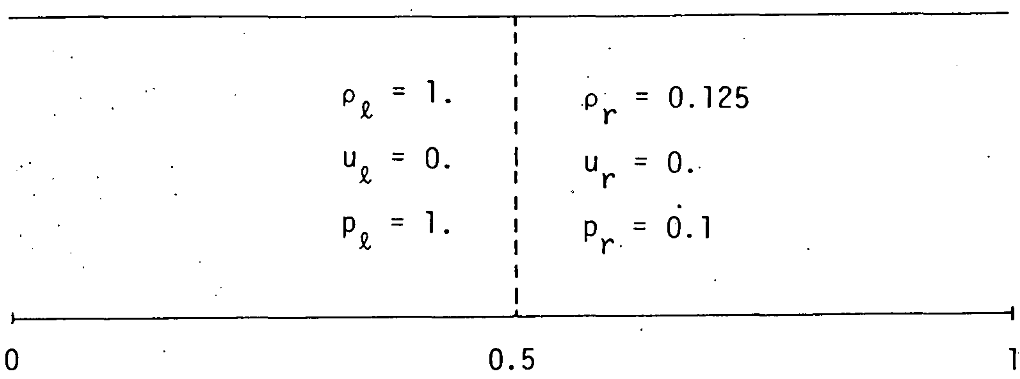


Fig. 6

For the second problem, the initial conditions are given in Fig. 6. The results of the experiment can be compared with those obtained with the BBC code. In Fig. 11 at t=2.746E-01 the density, velocity, internal energy per unit mass, and pressure profiles are displayed, prior to any interaction with the boundary. It is observed in the density profile that the rarefaction wave (1) is smooth while the contact (2) and the shock (3) are extremely sharp, as well as the states joining them. The results obtained from BBC exhibit an overshooting in the state between the contact and shock as well as much oscillation in the state joining the rarefaction wave and the contact.

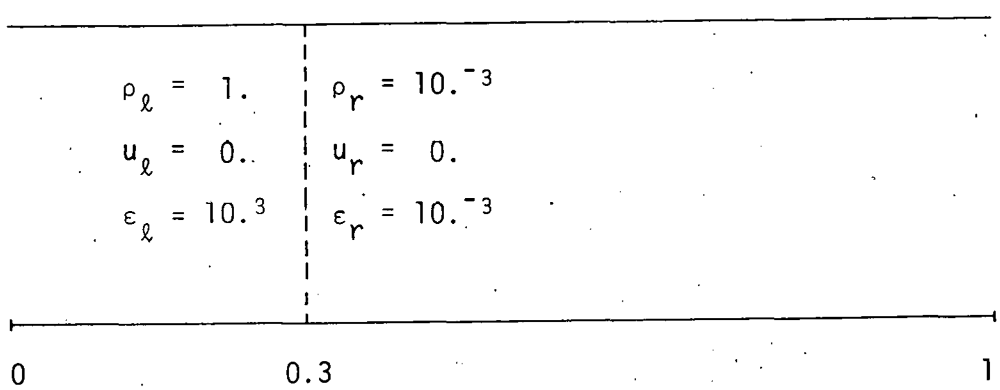
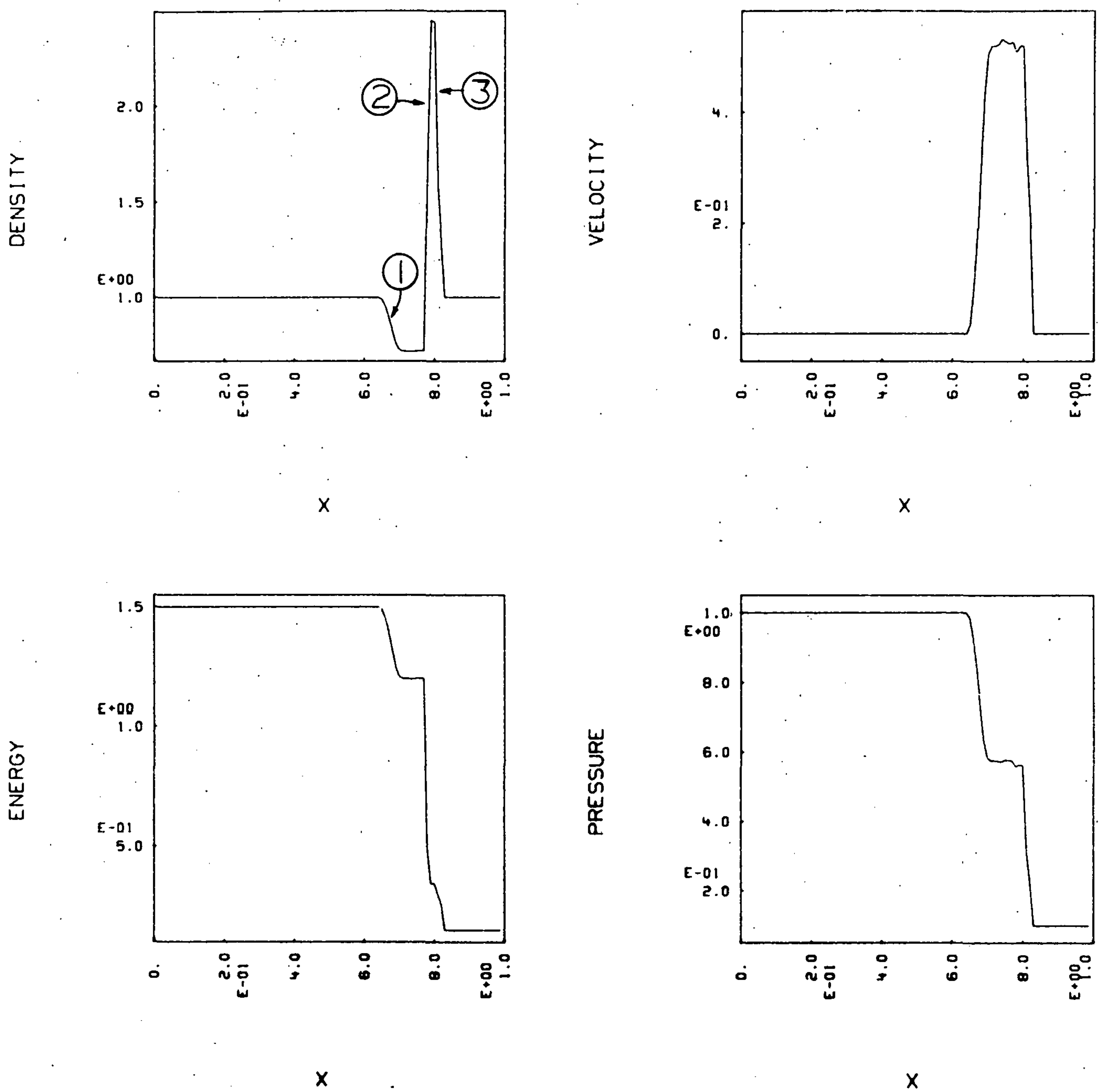


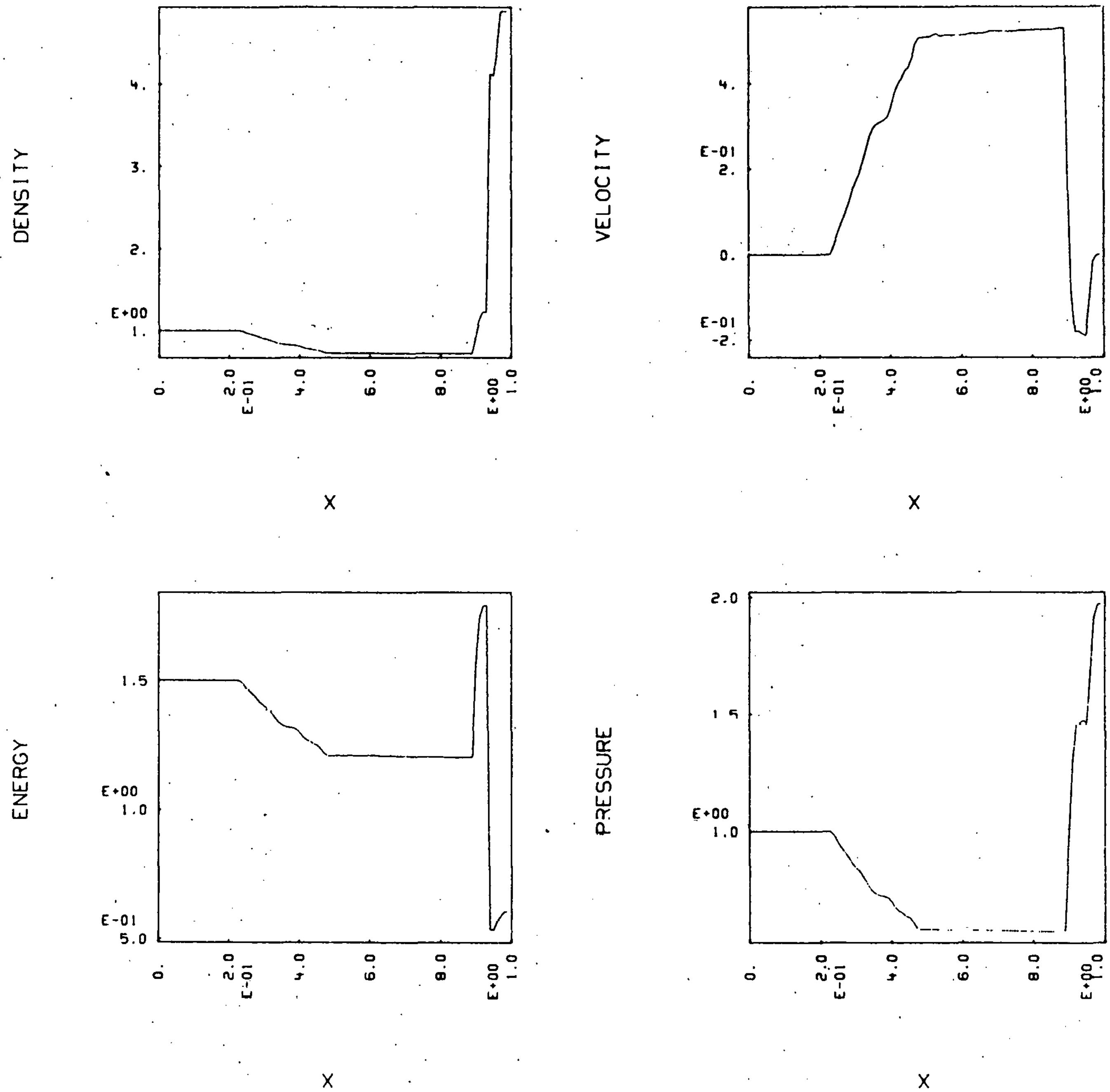
Fig. 7

For the final example (suggested by K. Trigger), the inital conditions are given in Fig. 7. In Fig. 12 at t=1.088E-02, a rarefaction wave (1) moving to the left, a contact surface (2) and a shock wave (3) moving to the right are observed prior to interaction with the boundaries. In Fig. 13 the profiles are presented after reflection at time t=4.966E-02.



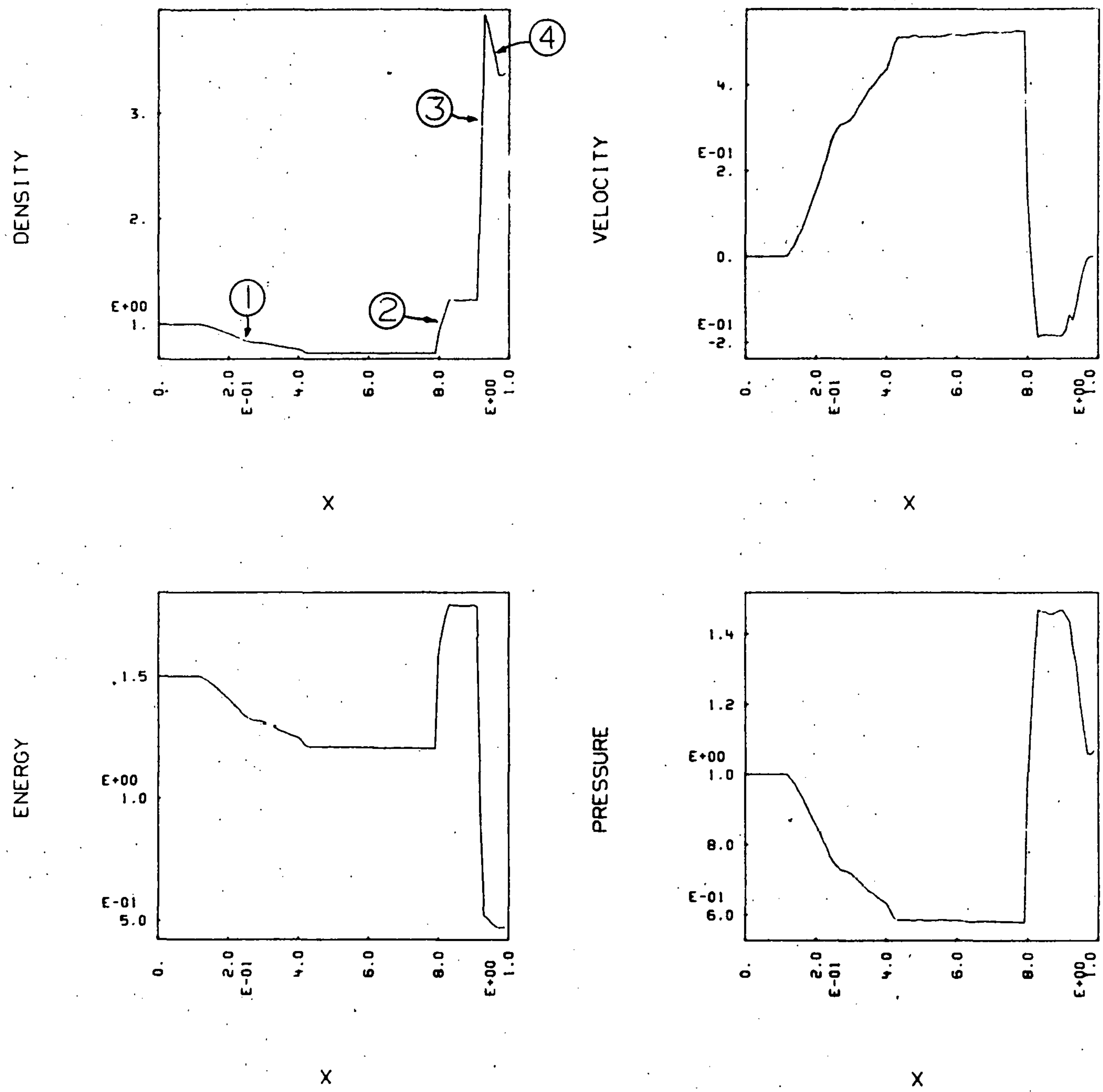
DX = 1.000E-02 TIME = 1.780E-01 DT = 5.622E-03

Fig. 8



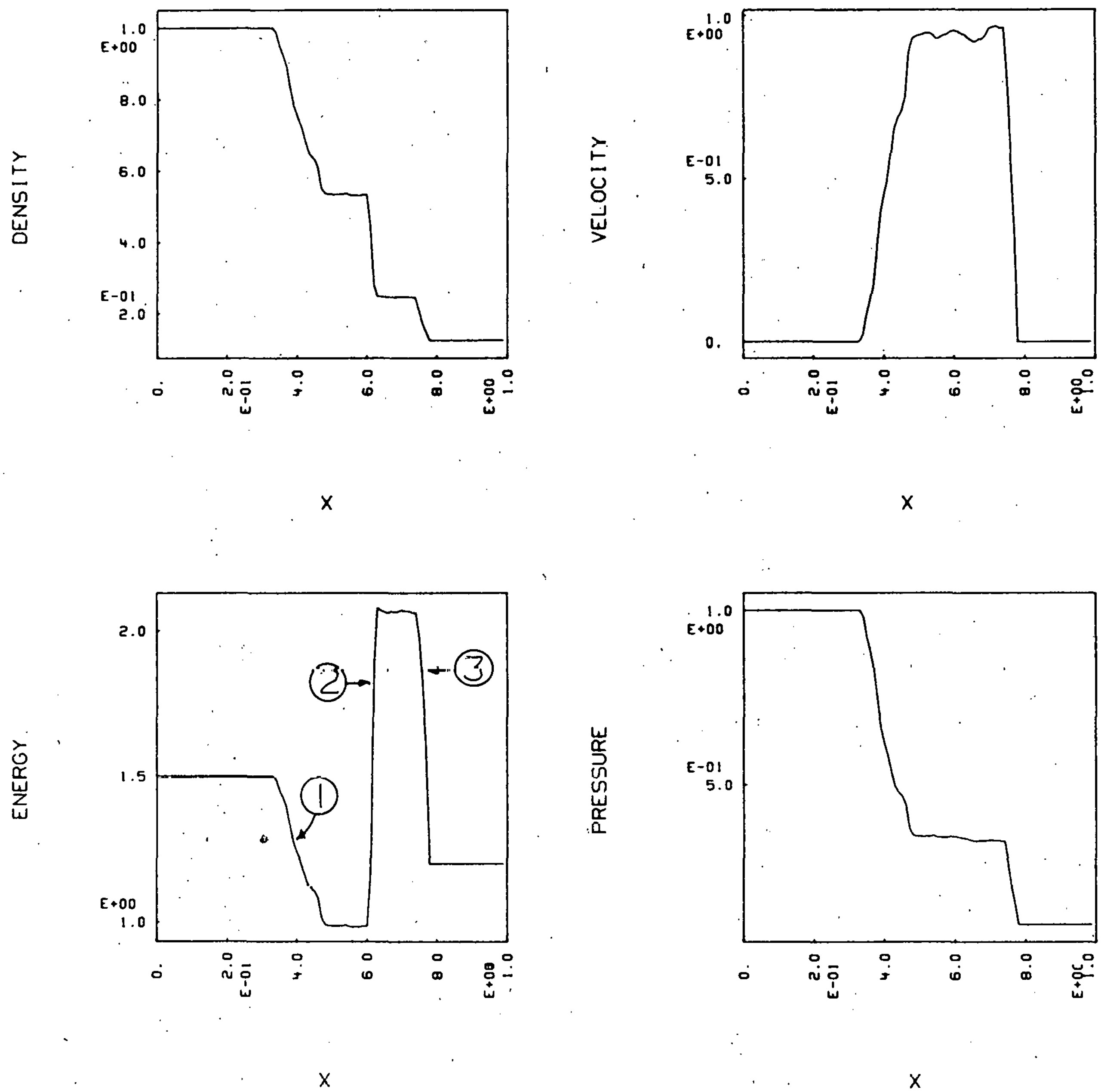
DX = 1.000E-02 TIME = 8.526E-01 DT = 5.622E-03

Fig. 9



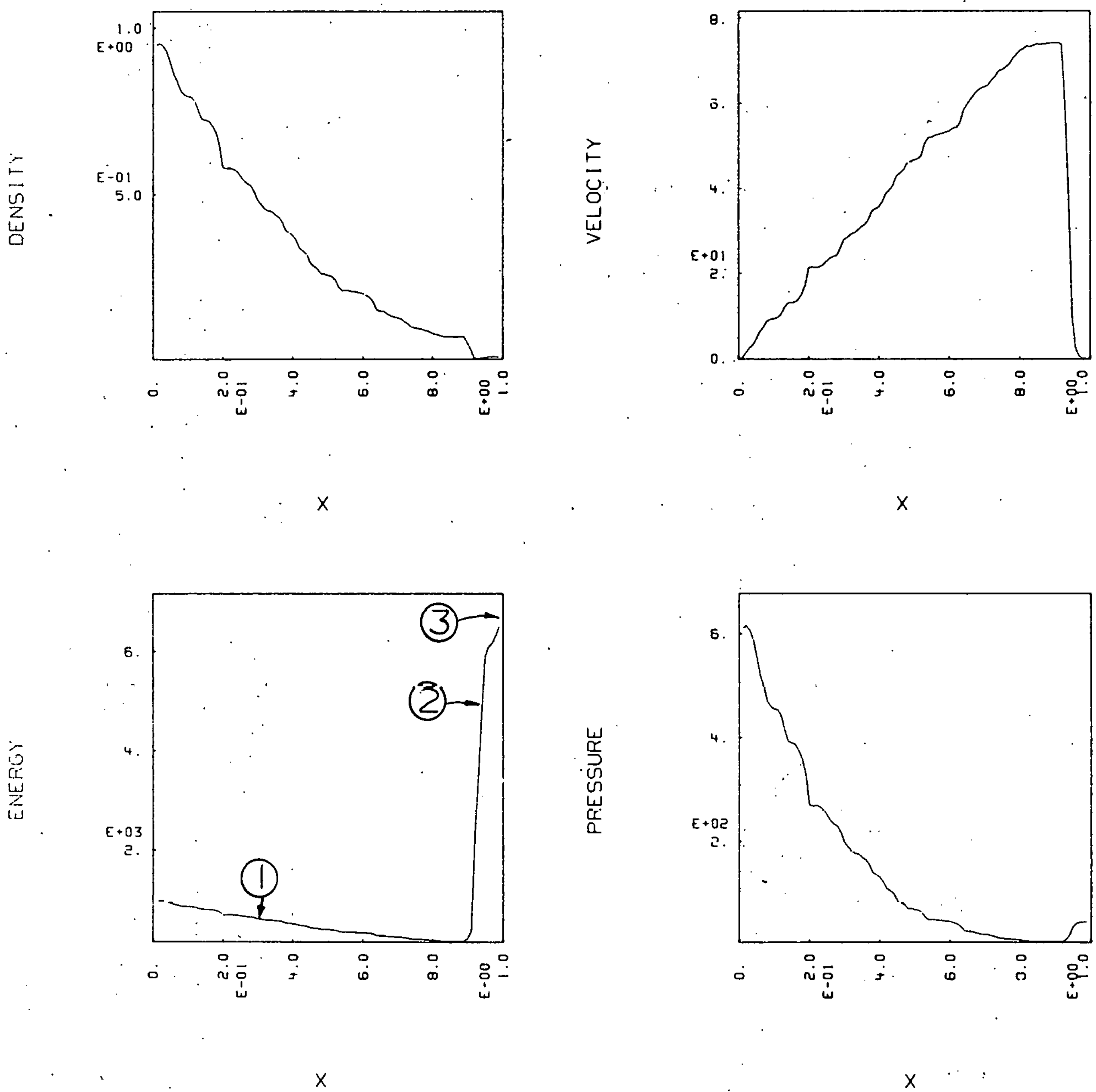
DX = 1.000E-02   TIME = 1.021E+00   DT = 5.622E-03

Fig. 10



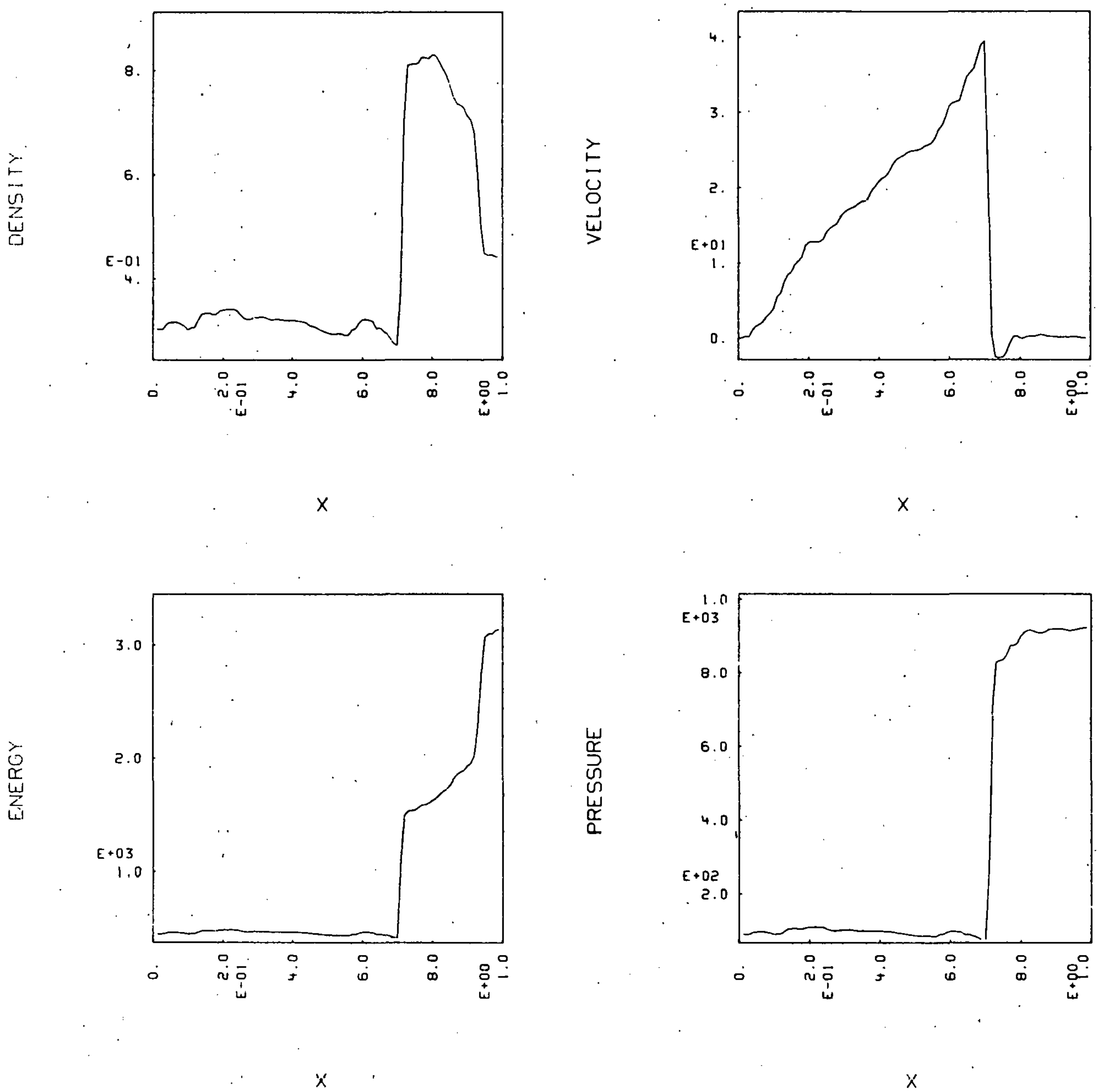
DX = 1.000E-02   TIME = 2.746E-01   DT = 3.812E-03

Fig. 11



DX = 1.000E-02   TIME = 1.812E-02   DT = 7.231E-05

Fig. 12



DX = 1.000E-02　　TIME = 4.966E-02　　DT = 6.274E-05

Fig. 13

VI. REFERENCES

[1] A. J. Chorin, "Random Choice Solution of Hyperbolic Systems," J. Comp. Phys., to appear.

[2] J. Glimm, Comm. Pure Appl. Math. 18, 697 (1965).

[3] ________ and P. D. Lax, "Decay of Solutions of Systems of Nonlinear Hyperbolic Conservation Laws," Mem. Amer. Math. Soc., 101, 1970.

[4] S. K. Godunov, Mat. Sbornik, 47, 271 (1959).

[5] P. D. Lax, Comm. Pure Appl. Math., 10, 537 (1957).

[6] ________, "Hyperbolic Systems of Conservation Laws and the Mathematical Theory of Shock Waves," SIAM CBMS, 11 (1973).

[7] C. Moler and J. Smoller, Arch. Rat. Mech. Anal., 37, 309 (1970).

[8] R. D. Richtmayer and K. Morton, Finite Difference Method for Initial Value Problems, Interscience Publishers, New York, 1967.

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