液晶物理重点

一、

1、液晶：介于各向异性晶体和各向同性的液体之间的取向有序的流体。

Liquid Crystal

具：液体的流动性 晶体的各向异性.

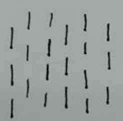
Liquid　Crystal　anisotropy (anisotropic)

☆2、液晶的分类

相变随温度变化 { 热致液晶 Thermotropic { 胆甾相 cholesteric；向列相 Nematic (NLC)；近晶相 ~~sem~~ smectic

随溶剂浓度 — 溶致液晶 Lyotropic

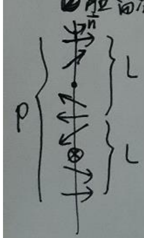
①向列相液晶 Nematic



centroial position disorder

{ 分子质心位置无序；取向有序 orientational order } { 对称；$\vec{n}$与$-\vec{n}$等概率 }

②胆甾相液晶 cholesteric



{ 质心位置无序；优先取向均匀螺旋旋转 } { P=2L；$\vec{n}$⊥螺旋轴；$\vec{n}$与$-\vec{n}$等概率 }

③近晶相液晶 Smectic

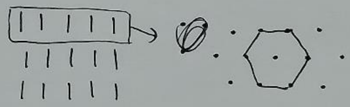
近晶A相

{ 层面分子质心位置无序；优先取向$\vec{n}$⊥层面 }

近晶C相

{ 层面质心位置无序；分子优先取向$\vec{n}$不垂直层面. }

近晶B相



{ 层面分子质心位置有序(正六边形)；分子优先取向$\vec{n}$⊥或∥层面；质心不能构成三维有序 }

3、序参数(S) Orientational order

向列相液晶分子场理论 $V(\theta_i,\varphi_i)=-\varepsilon n S P_2(\cos\theta_i)$

平均场近似：分子间的相互作用被平均势场代替，忽略了分子行为、随温度场的激荡，分子矢量程有序，势场的表达形式是近似结果.

4、矢量变换 vectors

正变换 $\vec{e_i}'=M_{ij}\vec{e_j}$　　$\vec{A_i}'=M_{ij}\vec{A_j}$

反变换 $\vec{e_i}=M_{ji}\vec{e_j}'$　　$\vec{A_i}=M_{ji}\vec{A_j}'$

$M_{ij}=\vec{e_i}'\cdot\vec{e_j}$

5、张量转换 tensor

$M^T=M^{-1}$　　$\vec{B}=M^T\vec{B}'M$

$\vec{B}'=M\vec{B}M^T$

6、序参数张量 $Q_{\alpha\beta}$：定域坐标系中分子优先取向方向

$\overline{Q}$：分子坐标系中序参数张量

$Q(\vec{r})=M^T(\vec{r})\overline{Q}M(\vec{r})$

空间固定坐标系内一点的序参数张量 $Q_{\alpha\beta}(\vec{r})=S(\vec{r})[\frac{3}{2}n_\alpha(\vec{r})n_\beta(\vec{r})-\frac{1}{2}\delta_{\alpha\beta}]$

二、

1、相转变：一级相变→序参数数值从某一有限值突然变为0

二级相变→序参数连续的变为0，而不是突变.

朗道理论：二级相转变点时，系统自由能可用序参数及导数表述

☆2、相变点序参数 S_c transformation orientation order

$$\begin{cases}\frac{\partial f}{\partial S}=0 & \text{平衡条件，自由能极小} \\ F_1=f=0 & \text{相变条件，自由能为0}\end{cases}$$

$$F=f=\frac{3}{4}a(T-T^*)S^2+\frac{1}{4}BS^3+\frac{9}{16}CS^4$$

$\frac{\partial f}{\partial S}=0$　$S_{c_1}=0$　$\frac{3}{2}a(T-T^*)+\frac{3}{4}BS_{c_2}+\frac{9}{4}CS_{c_2}^2=0$

$F=f=0$　$S_{c_1}=0$　$\frac{3}{4}a(T-T^*)+\frac{1}{4}BS_{c_3}+\frac{9}{16}CS_{c_3}^2=0$

同时满足，$S_{c_1}=0$　$S_{c_2}=-\frac{2B}{9C}$ 相转变为各向同性相时的序参数

代入S_{c_2}得 $T_c=\frac{B^2}{27aC}+T^*$

☆3、最高极限温度 T^{**} Maximum limit of NLC

稳定的向列相，只需满足平衡条件，只考虑 $\frac{\partial f}{\partial S}=0$

需使 ~~除$S_c=0$外 $\frac{\partial f}{\partial S}$有解~~ $\frac{\partial f}{\partial S}=0$有非零解，$\Delta\geq0$　$T\leq\frac{B^2}{24aC}+T^*$　$T^{**}=\frac{B^2}{24aC}+T^*$

☆4、最低过冷温度 T^* Minimum undercooling temperature

向列相液晶在各向同性相时自由能的极大与极小值的临界

各向同性相，S=0，在T^*，各向同性相即将转变为不稳定状态.

$$\left.\frac{\partial^2 f}{\partial S^2}\right|_{S=0}=0.$$

三、

1、介电各向异性 dielectric anisotropic

$\overleftrightarrow{\varepsilon}=\begin{pmatrix}\varepsilon_\perp & 0 & 0\\ 0 & \varepsilon_\perp & 0\\ 0 & 0 & \varepsilon_\parallel\end{pmatrix}$　$\Delta\varepsilon=\varepsilon_\parallel-\varepsilon_\perp$　$\vec{D}=\overleftrightarrow{\varepsilon}\vec{E}$

磁化率各向异性 magnetic susceptibility anisotropic

$\overleftrightarrow{\chi}=\begin{pmatrix}\chi_\perp & 0 & 0\\ 0 & \chi_\perp & 0\\ 0 & 0 & \chi_\parallel\end{pmatrix}$　$\Delta\chi=\chi_\parallel-\chi_\perp$　$\vec{M}=\overleftrightarrow{\chi}\vec{H}$

☆2、磁场引起的自由能密度

$F_m=-\int\vec{M}\cdot d\vec{H}$

$F_m=-\frac{1}{2}[\chi_\perp H^2+\Delta\chi(\vec{H}\cdot\vec{n})^2]$

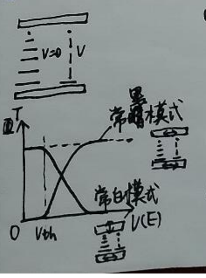
☆3、电场引起的自由能密度

$F_e=-\frac{1}{2}\vec{D}\cdot\vec{E}$

$F_e=-\frac{1}{2}[\varepsilon_\perp E^2+\Delta\varepsilon(\vec{E}\cdot\vec{n})^2]$

四、

☆1、扭曲向列相液晶显示器 (TN) Twist-Nematic LCD

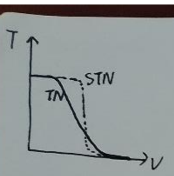


(1) 扭曲(TN)常白 $\Delta\varepsilon>0$

①未加电时，使用正性液晶，液晶分子沿面排列，液晶分子在上下玻璃基板间连续扭曲90°，上下偏光片正交设置. 常黑模式则平行设置

②当$\Delta n\cdot d$满足>>λ时，线偏振光入射，偏振光方向沿液晶分子指向矢扭曲逐步旋转90°，因而可以透过检偏器，为亮态.

③给液晶盒施加大于阈值电压V_{th}的电压，液晶分子扭曲结构被破坏，除上下基板表面分子外，其它分子趋于沿电场排列，旋光效应消失，无法通过检偏器，为暗态.



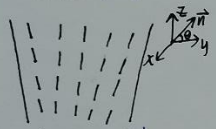
(2) 超扭曲(STN)

①未加电压，液晶分子在上下基板间连续扭曲 180°-270°

上下基板处液晶分子与偏振方向成一定角度。经起偏器获得的线偏振光在射入液晶层时会发生双折射现象，在上极板重新合成，变成椭圆偏振光，最终有一部分光射出。

②加大于 V_{th} 的电压，液晶分子扭曲结构被破坏，沿电场排列，偏振片(上下)正交，光不能通过，呈现暗态。

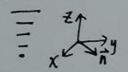
☆2、展曲 splay



$\vec{n}=(0,n_y,n_z)$　$\theta(y,z)$

$\nabla\cdot\vec{n}\neq0$　$\nabla\times\vec{n}\neq0$

扭曲 twist



$\vec{n}=(n_x,n_y,0)$　$\theta=\theta(z)$

$\nabla\cdot\vec{n}=0$　$\nabla\times\vec{n}\neq0$　$\vec{n}$ 与 $\nabla\times\vec{n}$ 平行

$\vec{n}\cdot(\nabla\times\vec{n})\neq0$　$\vec{n}\times(\nabla\times\vec{n})=0$

弯曲 bend

$\vec{n}=(0,n_y,n_z)$　$\theta=\theta(y,z)$

$\nabla\cdot\vec{n}\neq0$　$\nabla\times\vec{n}\neq0$　$\vec{n}$ 与 $\nabla\times\vec{n}$ 垂直.

$\vec{n}\cdot(\nabla\times\vec{n})=0$　$\vec{n}\times(\nabla\times\vec{n})\neq0$

弯曲与展曲同时存在

3. Frank's 理论

自由能密度 $\boxed{F=\frac{1}{2}k_{11}(\nabla\cdot\vec{n})^2+\frac{1}{2}k_{22}(\vec{n}\cdot\nabla\times\vec{n})^2+\frac{1}{2}k_{33}(\vec{n}\times\nabla\times\vec{n})^2}$

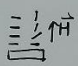
4. 变分法

欧拉方程 $\boxed{\frac{\partial G_1}{\partial\theta}-\frac{d}{dz}\left(\frac{\partial G_1}{\partial(\frac{d\theta}{dz})}\right)=0}$　强锚定条件 strong anchoring boundary conditions

指向矢分布计算：①建坐标系 ②边界条件 ③平衡态方程 ④求解

五.

1、磁场中的指向矢分布



☆2、Freedericksz 转变

①建坐标、边界条件

②求自由能密度

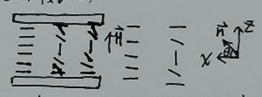
③平衡态方程

④求解 (同乘 $2\frac{d\theta}{dz}$)

积分变换 ($\sin\theta=\sin\theta_m\sin\lambda$)

⑤临界磁场 $\theta_m\to0$.

第一种(展曲)

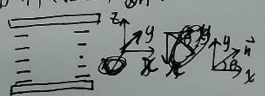


$F=\frac{1}{2}k_{11}(\nabla\cdot\vec{n})^2+\frac{1}{2}k_{22}(\vec{n}\cdot\nabla\times\vec{n})^2+\frac{1}{2}k_{33}(\vec{n}\times\nabla\times\vec{n})^2$　$F_m=-\frac{1}{2}\Delta\chi(\vec{H}\cdot\vec{n})^2$

$\frac{\partial G}{\partial\theta}-\frac{d}{dz}\left(\frac{\partial G}{\partial(\frac{d\theta}{dz})}\right)=0$　$\times2\frac{d\theta}{dz}$　$\frac{d\theta}{dz}=\sqrt{\quad}$　$\int d\theta=\int dz$

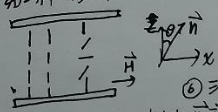
$\sin\theta=\sin\theta_m\sin\lambda$　$\theta_m\to0$　$H_c=\frac{\pi}{d}\sqrt{\frac{k_{11}}{\Delta\chi}}$

第二种(扭曲) $\otimes\vec{H}$.



$H_c=\frac{\pi}{d}\sqrt{\frac{k_{22}}{\Delta\chi}}$

第三种(弯曲)　(向哪转把θ角设在哪)



$H_c=\frac{\pi}{d}\sqrt{\frac{k_{33}}{\Delta\chi}}$

⑥三种Freedericks转变.

大题：① NLC，取z轴为 $\vec{n}$ 方向. 证明介电常数张量 $\overleftrightarrow{\varepsilon}=\begin{pmatrix}\varepsilon_{xx}&\varepsilon_{xy}&\varepsilon_{xz}\\\varepsilon_{yx}&\varepsilon_{yy}&\varepsilon_{yz}\\\varepsilon_{zx}&\varepsilon_{zy}&\varepsilon_{zz}\end{pmatrix}=\begin{pmatrix}\varepsilon_{\perp}&0&0\\0&\varepsilon_{\perp}&0\\0&0&\varepsilon_{\parallel}\end{pmatrix}$

② NLC，$f=\frac{3a}{4}(T-T^*)S^2+\frac{3}{4}BS^3+\frac{9}{16}CS^4$. 计算 T^{**}, S^{**}

③ $F=\frac{n}{2}N\varepsilon S^2-NKT\ln Z$，$Z=\int_0^{2\pi}\int_0^{\pi}\exp\left[\frac{n\varepsilon}{KT}SP_2(\cos\theta)\right]\sin\theta d\theta d\varphi$.

求 T^*, S^* ($\int_0^{\pi}P_m(\cos\theta)P_n(\cos\theta)\sin\theta d\theta=\frac{2}{2l+1}\delta_{mn}$)

④ 指向矢分布 (HAN)(TN)　⑤ 电场磁场自由能密度，$u=\frac{E}{d}$　$u=1V$ $d=5um$，证明 $\Delta F_e=\frac{1}{2}\Delta\varepsilon\vec{E}$　ΔF_e.

液晶物理中常用英文表述

Liquid Crystal	液晶	分子	molecular
Thermotropic	热致	矢量	vector
Lyotropic	溶致	张量	tensor
Nematic	向列相	标量	scalar
cholesteric	胆甾相	曲线	curve
smectic	近晶相	分布	distribution
magnetic	磁场	计算	calculate

electric displacement vector 电位移矢量 $\vec{D}$

dielectric tensor 介电张量 $\overleftrightarrow{\varepsilon}$

electric intensity 电场强度 $\vec{E}$

各向同性相 isotropic medium

各向异性 anisotropic

展曲 splay

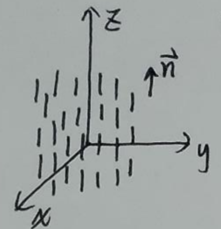
弯曲 bend

扭曲 twist

混合液晶盒 hybrid LC cell

自由能 free energy density

1. NLC



证明

$$\overleftrightarrow{\varepsilon}=\begin{pmatrix}\varepsilon_{xx}&\varepsilon_{xy}&\varepsilon_{xz}\\ \varepsilon_{yx}&\varepsilon_{yy}&\varepsilon_{yz}\\ \varepsilon_{zx}&\varepsilon_{zy}&\varepsilon_{zz}\end{pmatrix}=\begin{pmatrix}\varepsilon_\perp&0&0\\0&\varepsilon_\perp&0\\0&0&\varepsilon_\parallel\end{pmatrix}$$

证明：NLC 轴对称关于$\vec{n}$即z轴方向，

$\vec{n}$与$-\vec{n}$等概率

① 绕z轴旋转90°得到坐标系x'y'z'

转换矩阵 $\cancel{A=\begin{pmatrix}-1&0&0\\0&-1&0\\0&0&1\end{pmatrix}}$ $A^T=A$ $\quad A=\begin{pmatrix}\cos\theta&-\sin\theta&0\\ \sin\theta&\cos\theta&0\\0&0&1\end{pmatrix}$

在x'y'z'系下，$A=\begin{pmatrix}0&-1&0\\1&0&0\\0&0&1\end{pmatrix}$ $A^T=\begin{pmatrix}0&1&0\\-1&0&0\\0&0&1\end{pmatrix}$

$$\cancel{\overleftrightarrow{\varepsilon}'=A\overleftrightarrow{\varepsilon}A^T}$$

$$\cancel{=\begin{pmatrix}-1&0&0\\0&-1&0\\0&0&1\end{pmatrix}\begin{pmatrix}\varepsilon_{11}&\varepsilon_{12}&\varepsilon_{13}\\ \varepsilon_{21}&\varepsilon_{22}&\varepsilon_{23}\\ \varepsilon_{31}&\varepsilon_{32}&\varepsilon_{33}\end{pmatrix}\begin{pmatrix}-1&0&0\\0&-1&0\\0&0&1\end{pmatrix}}$$

$$\cancel{=\begin{pmatrix}-\varepsilon_{11}&-\varepsilon_{12}&-\varepsilon_{13}\\ -\varepsilon_{21}&-\varepsilon_{22}&-\varepsilon_{23}\\ \varepsilon_{31}&\varepsilon_{32}&\varepsilon_{33}\end{pmatrix}\begin{pmatrix}-1&0&0\\0&-1&0\\0&0&1\end{pmatrix}}$$

$$\cancel{=\begin{pmatrix}\varepsilon_{11}&\varepsilon_{12}&-\varepsilon_{13}\\ \varepsilon_{21}&\varepsilon_{22}&-\varepsilon_{23}\\ -\varepsilon_{31}&-\varepsilon_{32}&\varepsilon_{33}\end{pmatrix}}$$

~~由对称性知，~~ $\cancel{\overleftrightarrow{\varepsilon}'=\overleftrightarrow{\varepsilon}}$

$\cancel{\therefore -\varepsilon_{13}=\varepsilon_{13}=0 \quad -\varepsilon_{23}=\varepsilon_{23}=0}$

$\cancel{-\varepsilon_{31}=\varepsilon_{31}=0 \quad -\varepsilon_{32}=\varepsilon_{32}=0}$

$$\overleftrightarrow{\varepsilon}'=A\overleftrightarrow{\varepsilon}A^T$$

$$=\begin{pmatrix}0&-1&0\\1&0&0\\0&0&1\end{pmatrix}\begin{pmatrix}\varepsilon_{11}&\varepsilon_{12}&\varepsilon_{13}\\ \varepsilon_{21}&\varepsilon_{22}&\varepsilon_{23}\\ \varepsilon_{31}&\varepsilon_{32}&\varepsilon_{33}\end{pmatrix}\begin{pmatrix}0&1&0\\-1&0&0\\0&0&1\end{pmatrix}$$

$$=\begin{pmatrix}-\varepsilon_{21}&-\varepsilon_{22}&-\varepsilon_{23}\\ \varepsilon_{11}&\varepsilon_{12}&\varepsilon_{13}\\ \varepsilon_{31}&\varepsilon_{32}&\varepsilon_{33}\end{pmatrix}\begin{pmatrix}0&1&0\\-1&0&0\\0&0&1\end{pmatrix}=\begin{pmatrix}\varepsilon_{22}&-\varepsilon_{21}&-\varepsilon_{23}\\ -\varepsilon_{12}&\varepsilon_{11}&\varepsilon_{13}\\ -\varepsilon_{32}&\varepsilon_{31}&\varepsilon_{33}\end{pmatrix}$$

由对称性，$\overleftrightarrow{\varepsilon}'=\overleftrightarrow{\varepsilon}$

$\therefore \varepsilon_{22}=\varepsilon_{11} \quad -\varepsilon_{21}=\varepsilon_{12} \quad -\varepsilon_{23}=\varepsilon_{13}$

$-\varepsilon_{12}=\varepsilon_{21} \quad \varepsilon_{13}=\varepsilon_{23}$

$-\varepsilon_{32}=\varepsilon_{31} \quad \varepsilon_{31}=\varepsilon_{32}$

$\therefore \varepsilon_{13}=\varepsilon_{23}=0 \quad \varepsilon_{31}=\varepsilon_{32}=0 \quad \varepsilon_{11}=\varepsilon_{22}$

$$\therefore \overleftrightarrow{\varepsilon}=\begin{pmatrix}\varepsilon_{11}&\varepsilon_{12}&0\\ -\varepsilon_{12}&\varepsilon_{11}&0\\ 0&0&\varepsilon_{33}\end{pmatrix}$$

② 绕x轴旋转180°得x''y''z''

旋转矩阵 $B=\begin{pmatrix}1&0&0\\0&-1&0\\0&0&-1\end{pmatrix}$ $\quad B^T=B$

$$\overleftrightarrow{\varepsilon}''=B\overleftrightarrow{\varepsilon}B^T$$

$$=\begin{pmatrix}1&0&0\\0&-1&0\\0&0&-1\end{pmatrix}\begin{pmatrix}\varepsilon_{11}&\varepsilon_{12}&0\\ -\varepsilon_{12}&\varepsilon_{11}&0\\ 0&0&\varepsilon_{33}\end{pmatrix}\begin{pmatrix}1&0&0\\0&-1&0\\0&0&-1\end{pmatrix}$$

$$=\begin{pmatrix}\varepsilon_{11}&\varepsilon_{12}&0\\ \varepsilon_{12}&-\varepsilon_{11}&0\\ 0&0&-\varepsilon_{33}\end{pmatrix}\begin{pmatrix}1&0&0\\0&-1&0\\0&0&-1\end{pmatrix}=\begin{pmatrix}\varepsilon_{11}&-\varepsilon_{12}&0\\ \varepsilon_{12}&\varepsilon_{11}&0\\ 0&0&\varepsilon_{33}\end{pmatrix}$$

由$\vec{n}$与$-\vec{n}$等概率 $\overleftrightarrow{\varepsilon}''=\overleftrightarrow{\varepsilon}$

$\therefore \varepsilon_{12}=-\varepsilon_{12}=0$

$\therefore \overleftrightarrow{\varepsilon}=\begin{pmatrix}\varepsilon_{11}&0&0\\0&\varepsilon_{11}&0\\0&0&\varepsilon_{33}\end{pmatrix}$ $\quad \varepsilon_{11}=\varepsilon_\perp \quad \varepsilon_{33}=\varepsilon_\parallel$

$\therefore \overleftrightarrow{\varepsilon}=\begin{pmatrix}\varepsilon_\perp&0&0\\0&\varepsilon_\perp&0\\0&0&\varepsilon_\parallel\end{pmatrix}$ 得证.

2. $f=\frac{3a}{4}(T-T^*)s^2+\frac{3}{4}Bs^3+\frac{9}{16}Cs^4$，求$T^{**}$，$S^{**}$

T^{**}为最高极限温度.

向列相液晶稳定存在时 $\frac{\partial f}{\partial s}=0$

需有非零解 $\therefore \Delta\geq 0$

$$\frac{\partial f}{\partial s}=\frac{3a}{2}(T-T^*)S+\frac{9}{4}BS^2+\frac{9}{4}CS^3=0$$

$$\frac{3a}{2}(T-T^*)+\frac{9}{4}BS+\frac{9}{4}CS^2=0$$

$$\Delta=\left(\frac{9}{4}B\right)^2-4\times\frac{9}{4}C\times\frac{3a}{2}(T-T^*)$$

$$=\frac{81}{16}B^2-\frac{27aC}{2}(T-T^*)\geq 0$$

$$T\leq\frac{9B^2}{24aC}+T^*$$

$$\therefore T^{**}=\frac{9B^2}{24aC}+T^*$$

代入方程中，$\frac{3a}{2}\times\frac{9B^2}{24aC}+\frac{9}{4}BS^{**}+\frac{9}{4}CS^{**2}=0$

$$S^{**}=-\frac{B}{2C}$$

6.续

$$\int_0^{\frac{\pi}{2}}\sqrt{\frac{1}{\Delta\chi H^2\sin^2\theta_m\cos^2\lambda}}\sqrt{\frac{\cancel{\sin^2\theta_m\cos^2\lambda}}{1-\sin^2\theta_m\sin^2\lambda}}\,d\lambda=\int_0^{\frac{d}{2}}\sqrt{\frac{1}{K_{22}}}\,dz$$

$$\int_0^{\frac{\pi}{2}}\sqrt{\frac{1}{\Delta\chi H^2(1-\sin^2\theta_m\sin^2\lambda)}}\,d\lambda=\int_0^{\frac{d}{2}}\sqrt{\frac{1}{K_{22}}}\,dz$$

$\theta_m\to 0$ 临界磁场Hc 有 $\frac{\pi}{2}\cdot\frac{1}{\sqrt{\Delta\chi}H_c}=\frac{1}{\sqrt{K_{22}}}\cdot\frac{d}{2}$

$$H_c=\frac{\pi}{d}\sqrt{\frac{K_{22}}{\Delta\chi}}$$

3. $F=\frac{n}{2}N\varepsilon S^2 - NKT\ln Z$

$$Z=\int_0^{2\pi}\int_0^{\pi}\exp\left[\frac{n\varepsilon}{KT}SP_2(\cos\theta)\right]\sin\theta\, d\theta\, d\varphi$$

$$\left(\int_0^{\pi}P_m(\cos\theta)P_n(\cos\theta)\sin\theta\, d\theta=\frac{2}{2l+1}\delta_{mn}\right)$$

求 T^*, S^*

T^* 最低过冷温度

$$\left.\frac{\partial^2 f}{\partial S^2}\right|_{S=0}=0 \qquad f=F$$

$$\therefore \frac{\partial^2 f}{\partial S^2}=nN\varepsilon - NKT\frac{\partial}{\partial S}\left(\frac{1}{Z}\frac{\partial Z}{\partial S}\right)$$

$$=nN\varepsilon - NKT\left(-\frac{1}{Z^2}\left(\frac{\partial Z}{\partial S}\right)^2+\frac{1}{Z}\frac{\partial^2 Z}{\partial S^2}\right)$$

$$\frac{\partial Z}{\partial S}=\int_0^{2\pi}\int_0^{\pi}\frac{n\varepsilon}{KT}P_2(\cos\theta)\exp\left[\frac{n\varepsilon}{KT}SP_2(\cos\theta)\right]\sin\theta\, d\theta\, d\varphi$$

$$\left.\frac{\partial Z}{\partial S}\right|_{S=0}=\frac{n\varepsilon}{KT}\int_0^{2\pi}\int_0^{\pi}P_2(\cos\theta)\sin\theta\, d\theta\, d\varphi$$

$$=\frac{n\varepsilon}{KT}\int_0^{2\pi}\int_0^{\pi}P_2(\cos\theta)P_0(\cos\theta)\sin\theta\, d\theta\, d\varphi$$

$\because m\neq n \quad \therefore \delta_{mn}=0 \quad \therefore \left.\frac{\partial Z}{\partial S}\right|_{S=0}=0$

$$\frac{\partial^2 Z}{\partial S^2}=\int_0^{2\pi}\int_0^{\pi}\left[\frac{n\varepsilon}{KT}P_2(\cos\theta)\right]^2\exp\left[\frac{n\varepsilon}{KT}SP_2(\cos\theta)\right]\sin\theta\, d\theta\, d\varphi$$

$$\left.\frac{\partial^2 Z}{\partial S^2}\right|_{S=0}=\left(\frac{n\varepsilon}{KT}\right)^2\int_0^{2\pi}\int_0^{\pi}P_2(\cos\theta)P_2(\cos\theta)\sin\theta\, d\theta\, d\varphi$$

$m=n=l=2 \quad \therefore \left.\frac{\partial^2 Z}{\partial S^2}\right|_{S=0}=\frac{2}{2\times2+1}\left(\frac{n\varepsilon}{KT}\right)^2\cdot 2\pi=\frac{2}{5}\left(\frac{n\varepsilon}{KT}\right)^2\cdot2\pi=\frac{4\pi}{5}\left(\frac{n\varepsilon}{KT}\right)^2$

$$Z|_{S=0}=\int_0^{2\pi}\int_0^{\pi}\sin\theta\, d\theta\, d\varphi=4\pi$$

$$\therefore \left.\frac{\partial^2 f}{\partial S^2}\right|_{S=0}=nN\varepsilon - NKT\cdot\frac{1}{4\pi}\cdot\frac{4\pi}{5}\left(\frac{n\varepsilon}{KT}\right)^2=0$$

$$1-\frac{n\varepsilon}{5KT}=0$$

$$T^*=+\frac{n\varepsilon}{5K} \qquad S^*=0.$$

5. 续电场

$F_e=-\frac{1}{2}\varepsilon_\perp E^2-\frac{1}{2}\Delta\varepsilon(\vec{E}\cdot\vec{n})^2$

$\Delta\varepsilon>0$, $\vec{E}\parallel\vec{n}$, $F_e=-\frac{1}{2}\varepsilon_\perp E^2-\frac{1}{2}\Delta\varepsilon E^2$

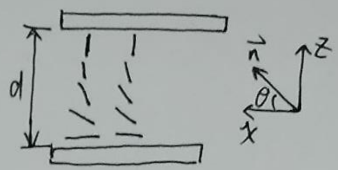
$\Delta\varepsilon<0$, $\vec{E}\perp\vec{n}$, $F_e=-\frac{1}{2}\varepsilon_\perp E^2$

$\Delta F_e=\frac{1}{2}\Delta\varepsilon E^2$

$E=\frac{U}{d}=\frac{1V}{5\mu m}=0.2\times10^6=2\times10^5\, V/m$

$\therefore \Delta F_e=2\times10^{10}\Delta\varepsilon$

4.(1) HAN 的 order distribution



① 建坐标系如图.

$\vec{n}=(\cos\theta, 0, \sin\theta) \qquad \theta=\theta(z)$

边界条件 $\theta|_{z=0}=0 \qquad \theta|_{z=d}=\frac{\pi}{2}$

② 自由能密度

$$F=\frac{1}{2}K_{11}(\nabla\cdot\vec{n})^2+\frac{1}{2}K_{22}(\vec{n}\cdot\nabla\times\vec{n})^2+\frac{1}{2}K_{33}(\vec{n}\times\nabla\times\vec{n})^2$$

$$\nabla\cdot\vec{n}=\cos\theta\frac{d\theta}{dz}$$

$$\nabla\times\vec{n}=\begin{vmatrix}\vec{i} & \vec{j} & \vec{k}\\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z}\\ \cos\theta & 0 & \sin\theta\end{vmatrix}=-\sin\theta\frac{d\theta}{dz}\vec{j}$$

$$\vec{n}\cdot\nabla\times\vec{n}=0 \qquad \vec{n}\times\nabla\times\vec{n}=\begin{vmatrix}\vec{i} & \vec{j} & \vec{k}\\ \cos\theta & 0 & \sin\theta\\ 0 & -\sin\theta\frac{d\theta}{dz} & 0\end{vmatrix}=-\sin^2\theta\frac{d\theta}{dz}\vec{i}-\sin\theta\cos\theta\frac{d\theta}{dz}\vec{k}$$

$$\therefore F=\frac{1}{2}K_{11}\cos^2\theta\left(\frac{d\theta}{dz}\right)^2+\frac{1}{2}K_{33}\sin^2\theta\left(\frac{d\theta}{dz}\right)^2$$

③ 平衡态方程

单位面积自由能 $A=\int_0^d F\,dz=\int_0^d\left[\frac{1}{2}K_{11}\cos^2\theta\left(\frac{d\theta}{dz}\right)^2+\frac{1}{2}K_{33}\sin^2\theta\left(\frac{d\theta}{dz}\right)^2\right]dz$

欧拉公式 $\frac{\partial G}{\partial\theta}-\frac{d}{dz}\left(\frac{\partial G}{\partial(\frac{d\theta}{dz})}\right)=0$

$$\frac{\partial G}{\partial\theta}=-K_{11}\cos\theta\sin\theta\left(\frac{d\theta}{dz}\right)^2+K_{33}\sin\theta\cos\theta\left(\frac{d\theta}{dz}\right)^2$$

$$\frac{\partial G}{\partial(\frac{d\theta}{dz})}=K_{11}\cos^2\theta\left(\frac{d\theta}{dz}\right)+K_{33}\sin^2\theta\left(\frac{d\theta}{dz}\right)$$

$$\frac{d}{dz}\left(\frac{\partial G}{\partial(\frac{d\theta}{dz})}\right)=-2K_{11}\cos\theta\sin\theta\left(\frac{d\theta}{dz}\right)^2+K_{11}\cos^2\theta\frac{d^2\theta}{dz^2}+2K_{33}\sin\theta\cos\theta\left(\frac{d\theta}{dz}\right)^2+K_{33}\sin^2\theta\frac{d^2\theta}{dz^2}$$

$$\therefore K_{11}\cos\theta\sin\theta\left(\frac{d\theta}{dz}\right)^2-K_{33}\sin\theta\cos\theta\left(\frac{d\theta}{dz}\right)^2-K_{11}\cos^2\theta\frac{d^2\theta}{dz^2}-K_{33}\sin^2\theta\frac{d^2\theta}{dz^2}=0$$

$K_{11}=K_{33}$ 时,

$$\frac{d^2\theta}{dz^2}=0 \qquad \theta=C_1z+C_2$$

由边界条件 $\theta|_{z=0}=C_2=0 \quad \theta|_{z=d}=C_1d=\frac{\pi}{2} \quad C_1=\frac{\pi}{2d}$

$$\therefore \theta=\frac{\pi z}{2d}$$

(2) TN 指向矢分布

不想推了.

5. 电场自由能密度

$F_e = -\frac{1}{2}\vec D\cdot\vec E$

$\vec D = \vec{\vec\varepsilon}\vec E = \begin{pmatrix}\varepsilon_\perp & 0 & 0\\ 0 & \varepsilon_\perp & 0\\ 0 & 0 & \varepsilon_\parallel\end{pmatrix}\vec E$

$D_x = \varepsilon_\perp E_x$ $D_y = \varepsilon_\perp E_y$ $D_z = \varepsilon_\parallel E_z$

$F_e = -\frac{1}{2}\vec D\cdot\vec E = -\frac{1}{2}(D_xE_x + D_yE_y + D_zE_z)$

$= -\frac{1}{2}(\varepsilon_\perp E_x^2 + \varepsilon_\perp E_y^2 + \varepsilon_\parallel E_z^2)$

$= -\frac{1}{2}\varepsilon_\perp(E_x^2+E_y^2+E_z^2) - \frac{1}{2}(\varepsilon_\parallel - \varepsilon_\perp)E_z^2$

$= -\frac{1}{2}\varepsilon_\perp E^2 - \frac{1}{2}\Delta\varepsilon E_z^2$

$= -\frac{1}{2}\varepsilon_\perp E^2 - \frac{1}{2}\Delta\varepsilon(\vec E\cdot\vec n)^2$

$\Delta\varepsilon>0$, $\vec E\parallel\vec n$, $F_e = -\frac{1}{2}\varepsilon_\perp E^2$

$\Delta\varepsilon<0$, $\vec E\perp\vec n$, $F_e = -\frac{1}{2}\varepsilon_\perp E^2$

磁场自由能密度

$F_m = -\int\vec M\cdot d\vec H$

$\vec M = \vec{\vec\chi}\vec H = \begin{pmatrix}\chi_\perp & 0 & 0\\ 0 & \chi_\perp & 0\\ 0 & 0 & \chi_\parallel\end{pmatrix}\vec H$

$M_x = \chi_\perp H_x$ $M_y = \chi_\perp H_y$ $M_z = \chi_\parallel H_z$

$F_m = -\int(\chi_\perp H_x^2 + \chi_\perp H_y^2 + \chi_\parallel H_z^2)$

$= -\int\chi_\perp(H_x^2 + H_y^2 + H_z^2)$

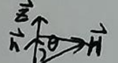
$F_m = -\int M_x dH_x - \int M_y dH_y - \int M_z dH_z$

$= -\int\chi_\perp H_x dH_x - \int\chi_\perp H_y dH_y - \int\chi_\parallel H_z dH_z$

$= -\frac{1}{2}\chi_\perp H_x^2 - \frac{1}{2}\chi_\perp H_y^2 - \frac{1}{2}\chi_\parallel H_z^2$

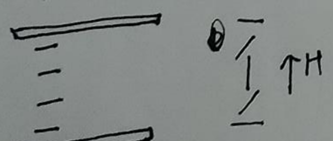
$= -\frac{1}{2}\chi_\perp(H_x^2+H_y^2+H_z^2) - \frac{1}{2}(\chi_\parallel - \chi_\perp)H_z^2$

$= -\frac{1}{2}\chi_\perp H^2 - \frac{1}{2}\Delta\chi H_z^2$

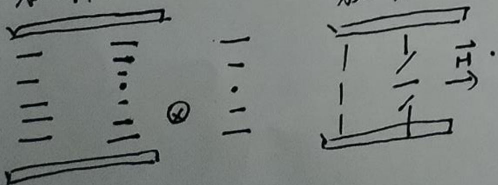


$= -\frac{1}{2}\chi_\perp H^2 - \frac{1}{2}\Delta\chi(\vec H\cdot\vec n)^2$

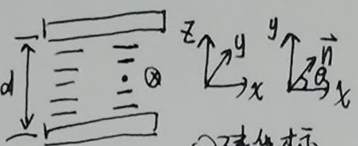
6. (1) 第一种Freedericksz转变



第二种Freedericksz转变 第三种



6. 第二种Freedericksz转变



① 建坐标

$\vec n = (\cos\theta, \sin\theta, 0)$ $\theta = \theta(z)$

边界条件 $\theta|_{z=0} = 0$ $\theta|_{z=d} = 0$ $\theta|_{z=\frac{d}{2}} = \frac{\pi}{2} = \theta_{max}$

$\frac{d\theta}{dz}\Big|_{z=\frac{d}{2}} = 0$ $\vec H = (0, H, 0)$

② 自由能密度

$F = \frac{1}{2}K_{11}(\nabla\cdot\vec n)^2 + \frac{1}{2}K_{22}(\vec n\cdot\nabla\times\vec n)^2 + \frac{1}{2}K_{33}(\vec n\times\nabla\times\vec n)^2$

$F_m = -\frac{1}{2}\Delta\chi(\vec H\cdot\vec n)^2$

$\nabla\cdot\vec n = 0$ $\nabla\times\vec n = \begin{vmatrix}\vec i & \vec j & \vec k\\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z}\\ \cos\theta & \sin\theta & 0\end{vmatrix} = -\cos\theta\frac{d\theta}{dz}\vec i - \sin\theta\frac{d\theta}{dz}\vec j$

$\vec n\cdot\nabla\times\vec n = -\cos^2\theta\frac{d\theta}{dz} - \sin^2\theta\frac{d\theta}{dz} = -\frac{d\theta}{dz}$

$\vec n\times\nabla\times\vec n = \begin{vmatrix}\vec i & \vec j & \vec k\\ \cos\theta & \sin\theta & 0\\ -\cos\theta\frac{d\theta}{dz} & -\sin\theta\frac{d\theta}{dz} & 0\end{vmatrix} = 0$ $\cancel{= -\sin\theta\cos\theta\frac{d\theta}{dz}}$

$\therefore F = \frac{1}{2}K_{22}\left(\frac{d\theta}{dz}\right)^2$

$\vec H\cdot\vec n = H\sin\theta$ $F_m = -\frac{1}{2}\Delta\chi H^2\sin^2\theta$

③ 平衡态方程

单位面积 $A = \int_0^d F + F_m\,dz = \int_0^d \frac{1}{2}K_{22}\left(\frac{d\theta}{dz}\right)^2 - \frac{1}{2}\Delta\chi H^2\sin^2\theta\,dz$

$\frac{\partial g}{\partial\theta} - \frac{d}{dz}\left(\frac{\partial g}{\partial(\frac{d\theta}{dz})}\right) = 0$

$\frac{\partial g}{\partial\theta} = -\Delta\chi H^2\sin\theta\cos\theta$ $\frac{\partial g}{\partial(\frac{d\theta}{dz})} = K_{22}\frac{d\theta}{dz}$

$\frac{d}{dz}\left(\frac{\partial g}{\partial(\frac{d\theta}{dz})}\right) = K_{22}\frac{d^2\theta}{dz^2}$ $\therefore -\Delta\chi H^2\sin\theta\cos\theta - K_{22}\frac{d^2\theta}{dz^2} = 0$.

④ 求解

$\times 2\frac{d\theta}{dz}$ $-2\Delta\chi H^2\sin\theta\cos\theta\frac{d\theta}{dz} - 2K_{22}\frac{d\theta}{dz}\cdot\frac{d^2\theta}{dz^2} = 0$

$\frac{d(-\Delta\chi H^2\sin^2\theta)}{dz} - \frac{d[K_{22}(\frac{d\theta}{dz})^2]}{dz} = 0$

$\therefore -\Delta\chi H^2\sin^2\theta - K_{22}\left(\frac{d\theta}{dz}\right)^2 = C$ $z = \frac{d}{2}$ $\frac{d\theta}{dz} = 0$ $\therefore C = -\Delta\chi H^2\sin^2\theta_m$

$\cancel{\frac{d\theta}{dz} = \sqrt{\Delta\chi H^2\sin^2\theta}}$

$\therefore \frac{d\theta}{dz} = \pm\sqrt{\frac{\Delta\chi H^2(\sin^2\theta_m - \sin^2\theta)}{K_{22}}}$

$z\in[0, \frac{d}{2}]$

$\theta\in[0, \frac{\pi}{2}]$时, $\frac{d\theta}{dz} = \sqrt{\frac{\Delta\chi H^2(\sin^2\theta_m - \sin^2\theta)}{K_{22}}}$

$\int_0^{\frac{\pi}{2}}\sqrt{\frac{1}{\Delta\chi H^2(\sin^2\theta_m - \sin^2\theta)}}\,d\theta = \int_0^{\frac{d}{2}}\sqrt{\frac{1}{K_{22}}}\,dz$

积分变换 $\sin\theta = \sin\theta_m\sin\lambda$ $\lambda\in[0, \frac{\pi}{2}]$ $d\theta = \frac{\sin\theta_m\cos\lambda}{\cos\theta}d\lambda$

$\therefore \int_0^{\frac{\pi}{2}}\sqrt{\frac{1}{\Delta\chi H^2\sin^2\theta_m\cos^2\lambda}}\cdot\frac{\sin\theta_m\cos\lambda}{\sqrt{\cos^2\lambda\sin^2\theta_m}}$