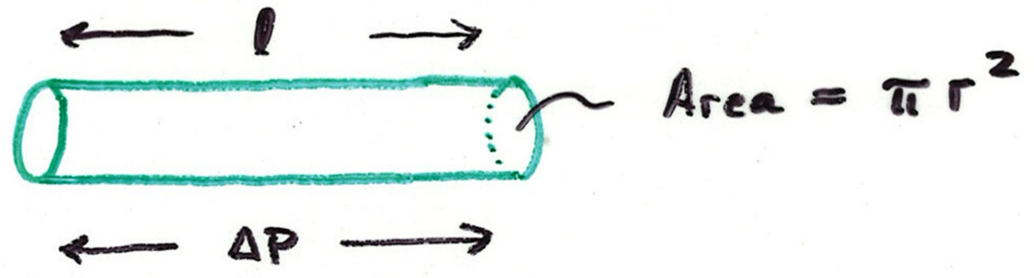
Model for Permeability

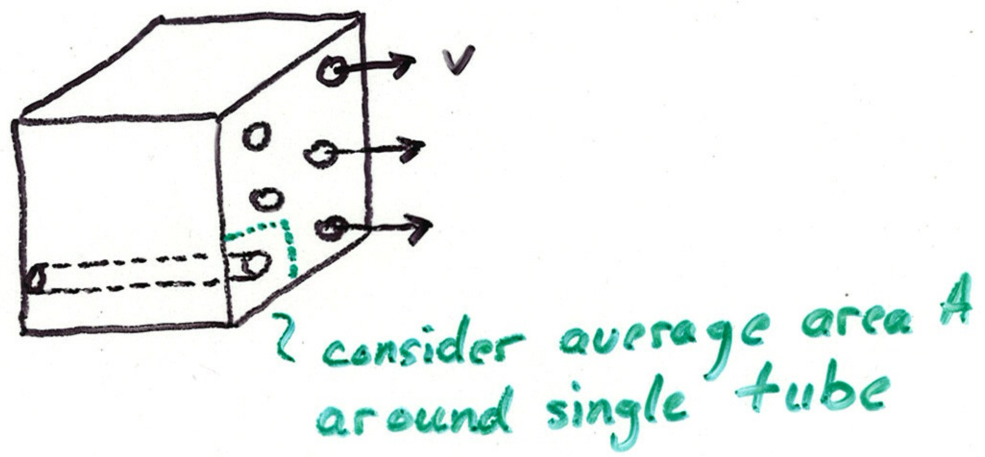
Poisseuille Flow



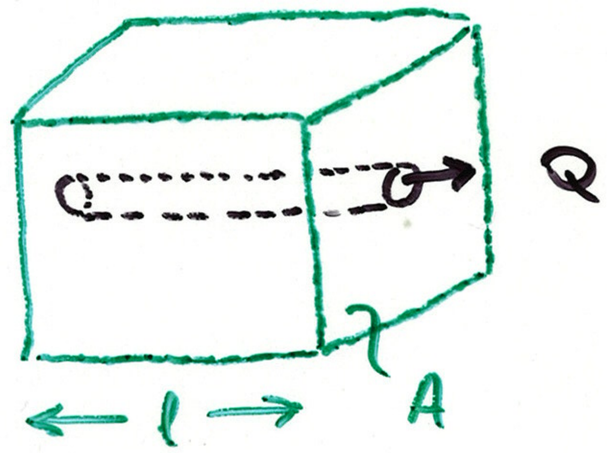
$$Q = -\frac{\pi r^4}{8\eta}\left(\frac{\Delta P}{l}\right)$$

Since $A_c = \pi r^2$

$$Q = -\frac{A_c r^2}{8\eta}\left(\frac{\Delta P}{l}\right)$$



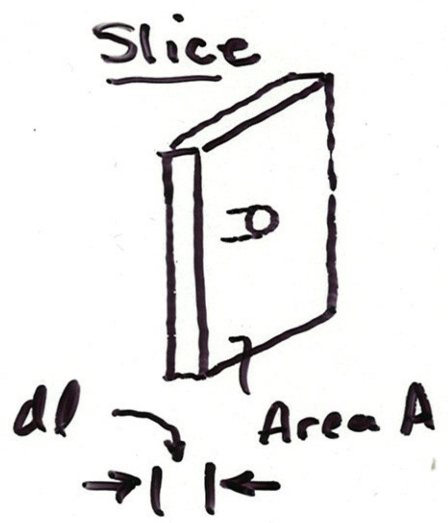
Model for Permeability (Con't)



$$q = \frac{Q}{A} = -\frac{A_c}{A}\frac{r^2}{8\eta}\left(\frac{\Delta P}{l}\right) .$$

Comparing with Darcy's Law

$$k = \frac{A_c}{A}\frac{r^2}{8} = \frac{\phi r^2}{8} \quad \text{units } m^2$$

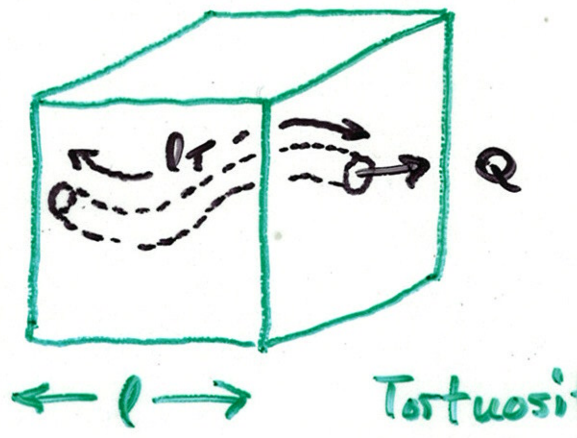


Volume of pore $= A_c \cdot dl$

Total volume $= A \cdot dl$

$$\therefore \phi = \frac{V_P}{V_T} = \frac{A_c}{A}$$

More General Case

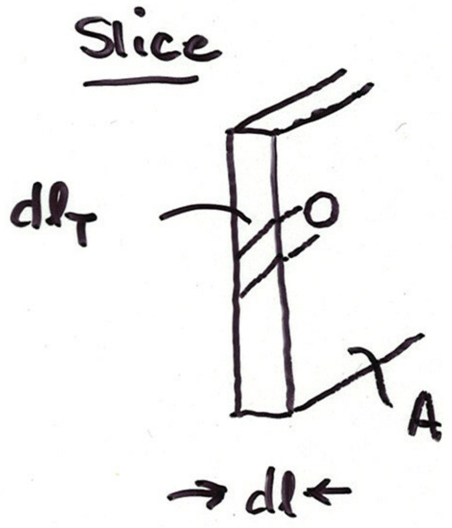


Tortuosity $(T) = \dfrac{l_T}{l}$

$$q \;=\; \frac{Q}{A} \;=\; -\frac{A_c}{A}\frac{r^2}{8\eta}\left(\frac{\Delta P}{l_T}\right)$$

Comparing with Darcy's Law

$$k \;=\; \frac{A_c}{A}\frac{r^2}{8}\frac{1}{T} \;=\; \frac{\phi\, r^2}{8T^2} \qquad \text{units } \; m^2$$



Volume of Pore $= A_c\, dl_T$

Total Volume $= A\, dl$

$$\phi = \frac{A_c}{A}\frac{dl_T}{dl} = \frac{A_c}{A}T$$

$$\therefore \frac{A_c}{A} = \frac{\phi}{T}$$

Further Generalizations

Kozeny - Carman Equation

$$k \;=\; \frac{1}{k_0 T^2}\frac{\phi^3}{S^2}$$

where

- $T \approx 2$ is tortuosity
- ϕ is porosity
- S is the specific surface area (surface area to volume ratio)
- $k_0 \approx 2.5$ is a constant

this relationship is commonly used in many applications