PROPOSITIONAL LOGIC

LEARNING OBJECTIVES

- Determine the truth values of an expression in propositional logic
- Construct truth table for logical connectives
- Write an expression in propositional logic in a sentence

PROPOSITIONAL LOGIC

- the area of logic that deals with propositions
- A ***formal propositional*** written using propositional logic notation, p, q, and r are used to represent statements. Also called as ***propositional variables*.**
  - i.e. Proposition p
  - "Today is Monday."
- **True Values** refer to the attribute assigned to a proposition in respect of its truth or falsehood, which in classical logic has only two possible values.
  - True or T
  - False or F

COMPOUND PROPOSITIONS (STATEMENTS)

- a statement composed of two or more simple statements connected by logical connectives
- a simple statement could be used to build a compound statement
- Examples:
  - “2 + 6 = 9” and “man is a mammal”
  - “The ocean is blue” or “It is cold today”
  - “Discrete Structure is **not** a difficult course to me”

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OPERATORS / CONNECTIVES

- an operator or connective combines one or more ***operand expressions*** into larger expression.
  - i.e. “+” in numeric expressions
- **Unary operators** take one operand
  - i.e. -4
- **Binary operators** take two operands
  - i.e. 3 x 5
- **Ternary operators** take three or more operands
  - i.e. p? a:b in C and Java
- ***Propositional or Boolean Operators*** operate in propositions or truth values instead of on numbers.

LOGICAL CONNECTIVES

- connectives are use to create a compound proposition from two or more propositions

- there are six main logical connectives
  - Negation
  - Conjunction
  - Disjunction
  - Exclusive-or
  - Conditional (Implication)
  - Biconditional

LOGICAL CONNECTIVES

Operator Formal Name	Nickname	Operand Type	Symbol
Negation Operator	NOT	Unary	¬, ~, −, !
Conjunction Operator	AND	Binary	∧
Disjunction Operator	OR	Binary	∨
Exclusive –OR	XOR	Binary	⊕
Implication Operator	IMPLIES	Binary	→
Biconditional Operator	“if and only if”	Binary	↔

LOGICAL CONNECTIVES

- ***Operator precedence*** is an ordering of logical operators designed to allow the dropping of parentheses in logical expressions.

Operator	Precedence
¬	1
∧	2
∨	3
→	4
↔	5

LOGICAL CONNECTIVES - NEGATION

- The negation of the statement p is written as ¬p, or sometimes ~p, ⁻p, !p or $\overline{p}$ where the symbol for "NOT"
- It has the property that it is false when p is true, and true when p is false
- The truth value of the negation of p, ¬p is the opposite of the truth value of p
- The Truth Table for the Negation of a Proposition

p	~p
T	F
F	T

LOGICAL CONNECTIVES - NEGATION

- Example.

  The following are statements for p, find the corresponding ~p.

1. 3 + 5 = 8.	3 + 5 ≠ 8.
2. Sofia is a girl.	Sofia is not a girl or Sofia is a boy.
3. Laisa is not here.	Laisa is here.

LOGICAL CONNECTIVES - NEGATION

- common words associated for the Negation of a Proposition

Logical Connectives	Logical Expression	Word
Negation	¬p, ~p, −p, !p, p', $\overline{p}$	not p It is false that p . . . It is not the case that p . . .

LOGICAL CONNECTIVES - NEGATION

- Example.
  Find the negation of the proposition "Today is Friday". And express this in simple English.

- Solution.
  "It is not the case that today is Friday."
  "It is false that today is Friday."

- Simple English.
  "Today is not Friday."
  "It is not Friday today."

LOGICAL CONNECTIVES - NEGATION

- Example.
  Find the negation of the proposition "At least 10 inches of rain fell today in Miami". And express this in simple English.

- Solution.
  "It is not the case that at least 10 inches of rain fell today in Siniloan, Laguna."

- Simple English.
  "Less than 10 inches of rain fell today in Siniloan, Laguna."

LOGICAL CONNECTIVES - NEGATION

❑ **Enrichment Exercise**.

The following are statements for p, find the corresponding ~p.

1. 23 is a composite number.
2. Earth has one moon.
3. Hexagon has five sides.

Find the negation of p (~p) and also negation of q (~q).

Let p = “It is not hot.”

q = “It is sunny.”

LOGICAL CONNECTIVES - CONJUNCTION

- The conjunction of the statement p and q is the compound statement “p and q”.
- Symbolically, p ^ q, where ^ is the symbol for “and”
- It has the property that If p is true and q is true, then p ^ q is true, otherwise p ^ q is false.
- The Truth Table for the Conjunction of Two Propositions

p	q	p ^ q
T	T	T
T	F	F
F	T	F
F	F	F

LOGICAL CONNECTIVES - CONJUNCTION

- Example:

  Determine the truth value of each of the following conjunction.

- 1. 2 + 6 = 9 and man is a mammal.

  Answer: False

- 2. Manny Pacquiao is a boxing champion and Gloria Macapagal Arroyo is the first female Philippine President.

  Answer: False

- 3. Ferdinand Marcos is the only three-term Philippine President and Joseph Estrada is the only Philippine President who resigns.

  Answer: True

LOGICAL CONNECTIVES - NEGATION

- common words associated for the Conjunction of Two Propositions

Logical Connectives	Logical Expression	Word
Conjunction	p ^ q	p and q p but q p also q p in addition q p moreover q

LOGICAL CONNECTIVES - CONJUNCTION

- Example:

Find the conjunction of the propositions p and q where p is the proposition “Today is Friday.” and q is the proposition “It is raining today.” and the truth value of the conjunction.

Solution:

“Today is Friday and it is raining today.”

The proposition is true on rainy Fridays.

LOGICAL CONNECTIVES - CONJUNCTION

- Example:

  If p = “I will have salad for lunch.” and

  q = “I will have steak for dinner.”

  $p \wedge q$ = ?

Solution:

“I will have salad for lunch and I will have steak for dinner”.

“I will have salad for lunch and steak for dinner.”

LOGICAL CONNECTIVES - CONJUNCTION

- **Enrichment Exercise**.

Determine the truth values of each of the following conjunction.

1. The earth is triangle and the moon is square.
2. Square has four sides and snakes are mammals.
3. Apple is a fruit and butterflies are insects.

Let p = “It is hot.” and q = “It is sunny.”
Find p and q ($p \wedge q$)

LOGICAL CONNECTIVES - DISJUNCTION

- The ***disjunction*** of the statement p and q is the compound statement "p or q".
- Symbolically, p v q, where v is the symbol for "or"
- It has the property that if p is true and q is true or both p and q are true, then p v q is true, otherwise p v q is false.
- The Truth Table for the Disjunction of Two Propositions

p	q	p v q
T	T	T
T	F	T
F	T	T
F	F	F

LOGICAL CONNECTIVES - DISJUNCTION

- Example:

Determine the truth value of each of the following disjunction.

1. 2 + 6 = 9 or Manny Pacquiao is a boxing champion.

Answer: True

2. Joseph Estrada is the only Philippine President who resigns or Gloria Macapagal Arroyo is the first female Philippine President.

Answer: False

3. Ferdinand Marcos is the only three-term Philippine President or man is a mammal.

Answer: True

LOGICAL CONNECTIVES - DISJUNCTION

- common words associated for the Disjunction of Two Propositions

Logical Connectives	Logical Expression	Word
Disjunction	p v q	p or q

LOGICAL CONNECTIVES - DISJUNCTION

- Example:

  p = “Today is Friday.”

  q = “It is raining today.”

  p v q = ?

Solution:

“Today is Friday or it is raining today.”

- Example:

  p = “My car has a bad engine.”

  q = “My car has a bad carburetor.”

  p v q = “My car has a bad engine or my car has a bad carburetor.”

LOGICAL CONNECTIVES - DISJUNCTION

❑ **Enrichment Exercises**

Determine the truth values of each of the following disjunction.

1. COBOL is a programming language or 2 + 5 = 7.

Answer: True

2. Windows 2010 is an operating system or 15 is a prime number.

Answer: True

3. 5 is less than 3 or 6 – 1 = 6.

Answer: False

LOGICAL CONNECTIVES – EXCLUSIVE OR

- The exclusive-or of the statement p and q is the compound statement "p exclusive-or q".
- Symbolically, $p \oplus q$, where $\oplus$ is the symbol for "if and only if".
- If p and q are true or both false, then $p \oplus q$ is false; if p and q have opposite truth values, then $p \oplus q$ is true.
- It can be noted that the truth values of $p \oplus q$ is the negation of the truth values of $p \leftrightarrow q$.
- It has nice reversibility properties (e.g. $p \oplus (p \oplus q)$ always has the same truth-value as q).

LOGICAL CONNECTIVES – EXCLUSIVE OR

- The Truth Table for the Exclusive-or of Two Propositions

p	q	$p \oplus q$
T	T	F
T	F	T
F	T	T
F	F	F

LOGICAL CONNECTIVES – EXCLUSIVE-OR

- common words associated for the Exclusive-or of Two Propositions

Logical Connectives	Logical Expression	Word
Exclusive -or	$p \oplus q$ $p \veebar q$	p exclusive-or q

LOGICAL CONNECTIVES – EXCLUSIVE OR

- Example

  p = “Today is Friday.”

  q = “It is raining today.”

  p ⊕ *q*: = “Today is Friday exclusive or in it’s raining “today”.

  The circuit is either on or off but not both.

  Let ab<0, then either a<0 or b<0 but not both.

  You may have cake or ice cream but not both.

LOGICAL CONNECTIVES – CONDITIONAL

- The conditional (or implication) of the statement p and q is the compound statement “if p then q”.
- Symbolically, $p \rightarrow q$, where $\rightarrow$ is the symbol for “if then”.
- p is called hypothesis (or antecedent or premise) and q is called conclusion (consequent or consequences).
- The conditional statement $p \rightarrow q$ is false only when p is true and q is false; otherwise $p \rightarrow q$ is true.

LOGICAL CONNECTIVES – CONDITIONAL

- The Truth Table for the Conditional of Two Propositions

p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

LOGICAL CONNECTIVES – CONDITIONAL

- Example:

  Obtain the truth values of each of the following conditional statements.

1. If vinegar is sweet, then sugar is sour.

   Answer: True

2. 2 + 5 = 7 is a sufficient condition for 5 + 6 = 1.

   Answer: False

3. 14 – 8 = 4 is a necessary condition that 6 ÷ 3 = 2.

   Answer: True

LOGICAL CONNECTIVES – CONDITIONAL

- common words associated for the Implications of Two Propositions

Logical Connectives	Logical Expression	Word
Conditional	$p \rightarrow q$	If p, then *q* If p, *q* p *is sufficient for q* p implies *q* p only if *q* p therefore *q* p is stronger than *q* p is sufficient condition for *q*

LOGICAL CONNECTIVES – CONDITIONAL

- common words associated for the Implications of Two Propositions

Logical Connectives	Logical Expression	Word
Conditional	$p \rightarrow q$	*q* if p *q* when p *q* follows from p *q* whenever p *q* is necessary for p *q* is weaker than p *q* is a necessary condition for p *q* unless ~p *a* sufficient condition for *q* is p

LOGICAL CONNECTIVES – CONDITIONAL

- Example

  Let p = "You study hard."

  q = "You will get an excellent grade."

  $p \rightarrow q$ = "If you study hard, then you will get an excellent grade."

  = "You will get an excellent grade when you study hard."

If you buy air ticket in advance, it is cheaper.
If it rains, the grass gets wet.
If 3+3=7, then all carabaos are white.

LOGICAL CONNECTIVES – CONDITIONAL

- Other conditional statements:
- **Converse** of $p \rightarrow q$: $q \rightarrow p$

  $p \rightarrow q$: "If it is noon, then I am hungry."

  $q \rightarrow p$: "If I am hungry, then it is noon."
- **Contrapositive** of $p \rightarrow q$: $\sim q \rightarrow \sim p$

  $p \rightarrow q$: "If it is noon, then I am hungry."

  $\sim q \rightarrow \sim p$: "If I am not hungry, then it is not noon."
- **Inverse** of $p \rightarrow q$: $\sim p \rightarrow \sim q$

  $p \rightarrow q$: "If it is noon, then I am hungry."

  $\sim p \rightarrow \sim q$: "If it is not noon, then I am not hungry."

LOGICAL CONNECTIVES – BICONDITIONAL

- The biconditional of the statement p and q is the compound statement “p if and only if q”.
- Symbolically, $p \leftrightarrow q$, where $\leftrightarrow$ is the symbol for “if and only if”.
- It has the property of if p and q are true or both false, then $p \leftrightarrow q$ is true; if p and q have opposite truth values, then $p \leftrightarrow q$ is false.
- Biconditional statements are also called bi-implications.

LOGICAL CONNECTIVES – BICONDITIONAL

- The Truth Table for the Biconditional of Two Propositions

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

LOGICAL CONNECTIVES – BICONDITIONAL

- Example:

  Determine the truth values of each of the following biconditional statements.

  1. 2 + 8 = 10 if and only if 6 – 3 = 3.

     Answer: True

  2. Manila is the capital of the Philippines is equivalent to fish live in moon.

     Answer: False

  3. 8 – 2 = 5 is a necessary and sufficient for 4 + 2 = 7.

     Answer: True

LOGICAL CONNECTIVES – CONDITIONAL

- common words associated for the Biconditional of Two Propositions

Logical Connectives	Logical Expression	Word
Biconditional	$p \leftrightarrow q$	p if and only if q p is equivalent to q p is necessary and sufficient condition for q

LOGICAL CONNECTIVES – BICONDITIONAL

- Example

Let p = “You can take the flight.”
q = “You buy a ticket.”
$p \leftrightarrow q$ = “You can take the flight if and only if you buy a ticket.”

Let p = “Pacquiao wins the senatorial election.”
q = “Pacquiao will be Senate President for all.”
$p \leftrightarrow q$ = “If, and only if, Pacquiao wins the senatorial election, Pacquiao will be Senate President for all.”

LOGICAL CONNECTIVES – BICONDITIONAL

- Enrichment Exercise

Determine the truth values of each of the following biconditional statements.

1. 8 + 4 = 5 if and only if 7 is less than 13.
2. 3 is a composite number is equivalent to 11 is a prime number.
3. 6 is divisible by 3 is a necessary and sufficient for 2 + 7 = 9.

LOGICAL CONNECTIVES

- For any two proposition p and q

p	q	$p \vee q$	$p \wedge q$	$p \oplus q$	$p \leftrightarrow q$	$p \rightarrow q$	$\neg p$	$\neg q$
T	T	T	T	F	T	T	F	F
T	F	T	F	T	F	F	F	T
F	T	T	F	T	F	T	T	F
F	F	F	F	F	T	T	T	T

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