Andean Cosmology School, June 2015
Practice session: lecture 1

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1. Deflection angle by a point mass

(a) Consider a light ray passing by a point with mass M, with an impact parameter b. What is the deflection angle in this configuration? The potential is given by:

$$\Phi = -\frac{GM}{r} \tag{1}$$

with $r = \sqrt{x^2 + y^2 + z^2}$ and $b = \sqrt{x^2 + y^2}$.

(b) Use your result (the magnitude of the $\hat{\vec{\alpha}}$ vector, α) to give the deflection angle for the Sun in arcseconds, assuming that the impact parameter is equal to the radius of the Sun. Pay attention to the units. You can write a short `python` function if you want. Moreover, if you have `astropy`, use it to get the constants needed: `from astropy import constants`. Compare your results with the Newtonian result of 0.875 arcsec.

2. Convergence map of a lens through the Kaiser-Squires algorithm

In this exercise you will get some practice with `python` by writing a short program that uses the Kaiser and Squires algorithm (Kaiser & Squires (1993)) to reconstruct the convergence of a lens system from shear maps.

(a) Open the files `gamma1.dat` and `gamma2.dat` that contain a matrix of the values (γ_1, γ_2) for a particular system on a grid. Use a routine such as `numpy.loadtxt` or `numpy.genfromtxt`.

(b) Calculate the discrete Fourier transform of `gamma1.dat` and `gamma2.dat`. A possible useful routine is `numpy.fft.fft2`. Check out the documentation on this to understand its input and output.

(c) Use the Kaiser and Squires algorithm to calculate the discrete Fourier transform of the convergence map, κ:

$$\tilde{\kappa}(\vec{k}) = \frac{1}{||\vec{k}||^2}\left((k_1^2 - k_2^2)\tilde{\gamma}_1(\vec{k}) + 2k_1k_2\tilde{\gamma}_2(\vec{k})\right) \tag{2}$$

where $\vec{k} = (k_1, k_2)$ and tildes denote Fourier transforms.

(d) Take the inverse discrete Fourier transform of the $\tilde{\kappa}$ map to recover the convergence map in real space

(e) Use `matplotlib.pylab.matshow` to plot and see your result.

Recently, scientists from the Dark Energy Survey used early survey data to calculate the largest convergence map to date. See V. Vikram et al. (2015) and C. Chang et al (2015).

Useful commands

- `numpy` arrays: `numpy.zeros`, `numpy.ones`, `numpy.linspace`
- FFT: `numpy.real`, `numpy.imag`, `numpy.fft.fft2`, `numpy.fft.fftfreq`, `numpy.fft.ifft2`
- Plotting 2D color maps: `matplotlib.pylab.matshow`, `matplotlib.pylab.tittle`, `matplotlib.pylab.colorbar`, `matplotlib.pylab.plot`

3. Singular Isothermal Sphere

One of the most widely used axially symmetric lens model is the Singular Isothermal Sphere (SIS). The density profile of this model can be derived assuming that the matter content of the lens behaves as an ideal gas confined by a spherically symmetric gravitational potential. This gas is taken to be in thermal and hydrostatic equilibrium.

(a) An isothermal sphere with velocity dispersion sigma has

$$\rho(r) = \frac{\sigma^2}{2\pi G r^2}, \tag{3}$$

where σ is the velocity dispersion. What is the shear γ vs angular radius θ?
Hint: Use Poisson's equation for the lensing potential, and write the latter in polar coordinates in terms of the angular radius θ, taking into account the axial symmetry: $\nabla^2_\theta \psi = \frac{1}{\theta}\frac{\partial}{\partial\theta}(\theta\frac{\partial\psi}{\partial\theta})$. Then find a solution of the potential in terms of θ just by inspection of the resulting equation.

(b) There are n resolved background galaxies per square arcminute in the background, with a shape noise of σ_γ. What is the uncertainty in the mean shear in an annulus at radius θ and width $\Delta\theta$? What is the S/N ratio of the shear measurement in this annulus in terms of $\frac{\Delta\theta}{\theta}$?

(c) Give a numerical value to the expression found in (b), by assuming that $\sigma = 1000$km/s, $n = \frac{20}{\text{arcmin}^2}$, $\sigma_\gamma = 0.3$, $z_l = 0.3$ (lens), $z = 1$(sources), and $h_0 = 0.7$, $\Omega_m = 0.3$, and $\Omega_\Lambda = 0.7$ to calculate cosmology-dependent quantities such as D_L and D_{SL}.