(*i*) **Exhaustive events.** A set of events is said to be *exhaustive*, if it includes all the possible events. For example, in tossing a coin there are two exhaustive cases either head or tail and there is no third possibility.

(*ii*) **Mutually exclusive events.** If the occurrence of one of the events procludes the occurrence of all other, then such a set of events is said to be *mutually exclusive*. Just as tossing a coin, either head comes up or the tail and both can't happen at the same time, *i.e.*, these are two mutually exclusive cases.

(*iii*) **Equally likely events.** If one of the events cannot be expected to happen in preference to another then such events are said to be *equally likely*. For instance, in tossing a coin, the coming of the head or the tail is equally likely.

Thus when a *die** is thrown, the turning up of the six different faces of the die are exhaustive, mutually exclusive and equally likely.

(*iv*) **Odds in favour of an event.** If the number of ways favourable to an event $A$ is $m$ and the number of ways not favourable to $A$ is $n$ then *odds in favour of* $A = m/n$ and *odds against* $A = n/m$.

(2) **Definition of probability.** *If there are n exhaustive, mutually exclusive and equally likely cases of which m are favourable to an event A, then probability (p) of the happening of A is*

$$\mathbf{P(A) = m/n.}$$

As there are $n - m$ cases in which $A$ will not happen (denoted by $A'$), *the chance of A not happening is q or* $P(A')$ *so that*

$$q = \frac{n-m}{n} = 1 - \frac{m}{n} = 1 - p$$

$$P(A') = 1 - P(A) \text{ so that } \mathbf{P(A) + P(A') = 1,}$$

*i.e.*, *if an event is certain to happen then its probability is unity, while if it is certain not to happen, its probability is zero.*

**Obs.** This definitions of probability fails when

(*i*) number of outcomes is infinite (not exhaustive) and (*ii*) outcomes are not equally likely.

(3) **Statistical (or Empirical) definition of probability.** *If in n trials, an event A happens m times, then the probability (p) of happening of A is given by*

$$p = P(A) = \underset{n \to \infty}{Lt} \frac{m}{n}$$

**Example 26.4.** *Find the chance of throwing (a) four, (b) an even number with an ordinary six faced die.*

**Solution.** (*a*) There are six possible ways in which the die can fall and of these there is only one way of throwing 4. Thus the required chance $= \frac{1}{6}$.

(*b*) There are six possible ways in which the die can fall. Of these there are only 3 ways of getting 2, 4 or 6. Thus the required chance $= 3/6 = \frac{1}{2}$.

**Example 26.5.** *What is the chance that a leap year selected at random will contain 53 Sundays ?* (*Madras, 2003*)

**Solution.** A leap year consists of 366 days, so that there are 52 full weeks (and hence 52 Sundays) and two extra days. These two days can be (*i*) Monday, Tuesday (*ii*) Tuesday, Wednesday, (*iii*) Wednesday, Thursday (*iv*) Thursday, Friday (*v*) Friday, Saturday (*vi*) Saturday, Sunday (*vii*) Sunday, Monday.

Of these 7 cases, the last two are favourable and hence the required probability $= \frac{2}{7}$.

**Example 26.6.** *A five figure number is formed by the digits 0, 1, 2, 3, 4 without repetition. Find the probability that the number formed is divisible by 4.*

**Solution.** The five digits can be arranged in 5! ways, out of which 4! will begin with zero.

$\therefore$ total number of 5-figure numbers formed $= 5! - 4! = 96$.

Those numbers formed will be divisible by 4 which will have two extreme right digits divisible by 4, *i.e.*, numbers ending in 04, 12, 20, 24, 32, 40.

Now numbers ending in $04 = 3! = 6$, numbers ending in $12 = 3! - 2! = 4$,
numbers ending in $20 = 3! = 6$, numbers ending in $24 = 3! - 2! = 4$,
numbers ending in $32 = 3! - 2! = 4$, and numbers ending in $40 = 3! = 6$.

[The numbers having 12, 24, 32 in the extreme right are $(3! - 2!)$ since the numbers having zero on the extreme left are to excluded.]

*Die is a small cube. Dots 1, 2, 3, 4, 5, 6 are marked on its six faces. The outcome of throwing a die is the number of dots on its upper face.

$\therefore$ total number of favourable ways $= 6 + 4 + 6 + 4 + 4 + 6 = 30$.

Hence the required probability $= \dfrac{30}{96} = \dfrac{5}{16}$.

... 40 tickets numbered 1, 2, 3 ... 40 ...

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## 26.3 PROBABILITY AND SET NOTATIONS

**(1) Random experiment.** Experiments which are performed essentially under the same conditions and whose results cannot be predicted are known as *random experiments. e.g.,* Tossing a coin or rolling a die are random experiments.

**(2) Sample space.** The set of all possible outcomes of a random experiment is called *sample space* for that experiment and is denoted by $S$.

The elements of the sample space $S$ are called the *sample points.*

*e.g.,* On tossing a coin, the possible outcomes are the head ($H$) and the tail ($T$). Thus $S = \{H, T\}$.

**(3) Event.** The outcome of a random experiment is called an *event*. Thus every subset of a sample space $S$ is an *event*.

*The null set* $\phi$ is also an event and is called an *impossible event. Probability of an impossible event* is zero *i.e.,* $P(\phi) = 0$.

**(4) Axioms**

(*i*) *The numerical value of probability lies between 0 and 1.*

*i.e.,* for any event $A$ of $S$, $0 \le P(A) \le 1$.

(*ii*) *The sum of probabilities of all sample events is unity i.e.,* $P(S) = 1$.

(*iii*) *Probability of an event made of two or more sample events is the sum of their probabilities.*

**(5) Notations**

(*i*) *Probability of happening of events* **A** *or* **B** is written as $P(A + B)$ or $P(A \cup B)$.

(*ii*) *Probability of happening of* **both the events A and B** is written as $P(AB)$ or $P(A \cap B)$.

(*iii*) *'Event A implies* ($\Rightarrow$) *event B'* is expressed as $A \subset B$.

(*iv*) *'Events A and B are mutually exclusive'* is expressed as $A \cap B = \phi$.

**(6)** *For any two events A and B,*

$$P(A \cap B') = P(A) - P(A \cap B)$$

*Proof.* From Fig. 26.1,

$$(A \cap B') \cup (A \cap B) = A$$

$$\therefore \quad P[(A \cap B') \cup (A \cap B)] = P(A)$$

or $$P(A \cap B') + P(A \cap B) = P(A)$$

or $$P(A \cap B') = P(A) - P(A \cap B)$$

Similarly, $P(A' \cap B) = P(B) - P(A \cap B)$

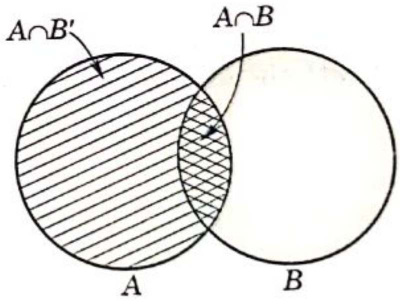


Fig. 26.1

## 26.4 ADDITION LAW OF PROBABILITY *or* THEOREM OF TOTAL PROBABILITY

(1) *If the probability of an event A happening as a result of a trial is P(A) and the probability of a* **mutually exclusive** *event B happening is P(B), then the probability of* **either** *of the events happening as a result of the trial is* **$P(A + B)$ or $P(A \cup B) = P(A) + P(B)$.**

*Proof.* Let $n$ be the total number of equally likely cases and let $m_1$ be favourable to the event $A$ and $m_2$ be favourable to the event $B$. Then the number of cases favourable to $A$ or $B$ is $m_1 + m_2$. Hence the probability of $A$ or $B$ happening as a result of the trial

$$= \frac{m_1 + m_2}{n} = \frac{m_1}{n} + \frac{m_2}{n} = P(A) + P(B).$$

(2) *If A, B, are any two events* **(not mutually exclusive),** *then*

$$\mathbf{P(A + B) = P(A) + P(B) - P(AB)}$$

or

$$\mathbf{P(A \cup B) = P(A) + P(B) - P(A \cap B)}$$

If the events $A$ and $B$ are any two events then, there are some outcomes which favour both $A$ and $B$. If $m_3$ be their number, then these are included in both $m_1$ and $m_2$. Hence the total number of outcomes favouring either $A$ or $B$ or both is

$$m_1 + m_2 - m_3.$$

Thus the probability of occurrence of $A$ or $B$ or both

$$= \frac{m_1 + m_2 - m_3}{n} = \frac{m_1}{n} + \frac{m_2}{n} - \frac{m_3}{n}$$

Hence $\quad P(A + B) = P(A) + P(B) - P(AB)$

or $\quad P(A \cup B) = P(A) + P(B) - P(A \cap B)$ (V.T.U., 2012 S)

**Obs.** *When A and B are mutually exclusive* $P(AB)$ or $P(A \cap B) = 0$ and we get

$$P(A + B) \text{ or } P(A \cup B) = P(A) + P(B).$$

In general, for a number of **mutually exclusive** events $A_1, A_2, \ldots A_n$, we have

$$P(A_1 + A_2 + \ldots + A_n) \text{ or } P(A_1 \cup A_2 \cup \ldots \cup A_n) = P(A_1) + P(A_2) + \ldots + P(A_n).$$

(3) *If A, B, C are any three events, then*

$$\mathbf{P(A + B + C) = P(A) + P(B) + P(C) - P(AB) - P(BC) - P(CA) + P(ABC).}$$

or

$$\mathbf{P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(C \cap A) + P(A \cap B \cap C)}$$

*Proof.* Using the above result for any two events, we have

$$\begin{aligned}
P(A \cup B \cup C) &= P[(A \cup B) \cup C] \\
&= P(A \cup B) + P(C) - P[(A \cup B) \cap C] \\
&= [P(A) + P(B) - P(A \cap B)] + P(C) - P[(A \cap C) \cup (B \cap C)] && \text{(Distributive Law)} \\
&= P(A) + P(B) + P(C) - P(A \cap B) - \{P(A \cap C) + P(B \cap C) - P(A \cap B \cap C)\} \\
& && [\because (A \cap C) \cap (B \cap C) = A \cap B \cap C] \\
&= P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(C \cap A) + P(A \cap B \cap C) && [\because A \cap C = C \cap A.]
\end{aligned}$$

**Example 26.11.** *In a race, the odds in favour of the four horses $H_1$, $H_2$, $H_3$, $H_4$ are 1 : 4, 1 : 5, 1 : 6, 1 : 7 respectively. Assuming that a dead heat is not possible, find the chance that one of them wins the race.*

**Solution.** Since it is not possible for all the horses to cover the same distance in the same time (a dead heat), the events are mutually exclusive.

If $p_1, p_2, p_3, p_4$ be the probabilities of winning of the horses $H_1, H_2, H_3, H_4$ respectively, then

$$p_1 = \frac{1}{1+4} = \frac{1}{5} \qquad [\because \text{ Odds in favour of } H_1 \text{ are } 1:4]$$

and

$$p_2 = \frac{1}{6}, \; p_3 = \frac{1}{7}, \; p_4 = \frac{1}{8}.$$

Hence the chance that one of them wins $= p_1 + p_2 + p_3 + p_4$

$$= \frac{1}{5} + \frac{1}{6} + \frac{1}{7} + \frac{1}{8} = \frac{533}{840}.$$

**Example 26.12.** *A bag contains 8 white and 6 red balls. Find the probability of drawing two balls of the same colour.*

**Solution.** Two balls out of 14 can be drawn in ${}^{14}C_2$ ways which is the total number of outcomes. Two white balls out of 8 can be drawn in ${}^{8}C_2$ ways. Thus the probability of drawing 2 white balls

$$= \frac{{}^{8}C_2}{{}^{14}C_2} = \frac{28}{91}$$

Similarly 2 red balls out of 6 can be drawn in ${}^{6}C_2$ ways. Thus the probability of drawing 2 red balls

$$= \frac{{}^{6}C_2}{{}^{14}C_2} = \frac{15}{91}.$$

Hence the probability of drawing 2 balls of the same colour (either both white or both red)

$$= \frac{28}{91} + \frac{15}{91} = \frac{43}{91}.$$

**Example 26.13.** *Find the probability of drawing an ace or a spade or both from a deck of cards** ?

**Solution.** The probability of drawing an ace from a deck of 52 cards = 4/52.

Similarly the probability of drawing a card of spades = 13/52, and the probability of drawing an ace of spades = 1/52.

Since the two events (*i.e.*, a card being an ace and a card being of spades) are not mutually exclusive, therefore, the probability of drawing an ace or a spade

$$= \frac{4}{52} + \frac{13}{52} - \frac{1}{52} = \frac{4}{13}.$$

## 26.5 (1) INDEPENDENT EVENTS

Two events are said to be *independent,* if happening or failure of one does not affect the happening or failure of the other. Otherwise the events are said to be *dependent.*

*For two dependent events A and B, the symbol P(B / A) denotes the probability of occurrence of B, when A has already occurred.* It is known as the **conditional probability** and is read as a *'probability of B given A'.*

**(2) Multiplication law of probability** *or* **Theorem of compound probability.** *If the probability of an event A happening as a result of trial is P(A) and after A has happened the probability of an event B happening as a result of another trial* (*i.e.*, **conditional probability of B given A**) *is P(B / A), then the probability of* **both** *the events A and B happening as a result of two trials is* $\mathbf{P(AB)}$ **or** $\mathbf{P(A \cap B) = P(A) \,.\, P(B/A)}$.

*Proof.* Let $n$ be the total number of outcomes in the first trial and $m$ be favourable to the event $A$ so that $P(A) = m/n$.

Let $n_1$ be the total number of outcomes in the second trial of which $m_1$ are favourable to the event $B$ so that $P(B/A) = m_1/n_1$.

Now each of the $n$ outcomes can be associated with each of the $n_1$ outcomes. So the total number of outcomes in the combined trial is $nn_1$. Of these $mm_1$ are favourable to both the events $A$ and $B$. Hence

$$P(AB) \text{ or } P(A \cap B) = \frac{mm_1}{nn_1} = P(A) \,.\, P(B/A).$$

Similarly, the *conditional probability of A given B is P(A/B).*

$\therefore \quad P(AB) \text{ or } P(A \cap B) = P(B) \,.\, P(A/B)$

Thus $\quad P(A \cap B) = P(A) \,.\, P(B/A) = P(B) \,.\, P(A/B).$

**(3)** *If the events A and B are* **independent,** *i.e.*, if the happening of $B$ does not depend on whether $A$ has happened or not, then $P(B/A) = P(B)$ and $P(A/B) = P(A)$.

$\therefore \quad \mathbf{P(AB)}$ **or** $\mathbf{P(A \cap B) = P(A) \,.\, P(B).}$

In general, $\quad P(A_1 A_2 ... A_n)$ or $P(A_1 \cap A_2 \cap ... \cap A_n) = P(A_1) \,.\, P(A_2) .... \,.\, P(A_n).$

**Cards* : A pack of cards consists of four suits *i.e.*, Hearts, Diamonds, Spades and Clubs. Each suit has 13 cards : an Ace, a King, a Queen, a Jack and nine cards numbered 2, 3, 4, ..., 10. Hearts and Diamonds are *red* while Spades and Clubs are *black.*

**Cor.** If $p_1, p_2$ be the probabilities of happening of two independent events, then

(*i*) the probability that the first event happens and the second fails is $p_1(1 - p_2)$.

(*ii*) the probability that both events fail to happen is $(1 - p_1)(1 - p_2)$.

(*iii*) the probability that at least one of the events happens is

$1 - (1 - p_1)(1 - p_2)$. This is commonly known as their **cumulative probability.**

*In general, if $p_1, p_2, p_3, \ldots p_n$ be the chances of happening of n independent events, then their cumulative probability (i.e., the chance that at least one of the events will happen) is*

$$1 - (1 - p_1)(1 - p_2)(1 - p_3) \ldots (1 - p_n).$$

**Example 26.14.** *Two cards are drawn in succession from a pack of 52 cards. Find the chance that the first is a king and the second a queen if the first card is (i) replaced, (ii) not replaced.*

**Solution.** (*i*) The probability of drawing a king $= \frac{4}{52} = \frac{1}{13}$.

If the card is replaced, the pack will again have 52 cards so that the probability of drawing a queen is 1/13.

The two events being independent, the probability of drawing both cards in succession $= \frac{1}{13} \times \frac{1}{13} = \frac{1}{169}$.

(*ii*) The probability of drawing a king $= \frac{1}{13}$.

If the card is not replaced, the pack will have 51 cards only so that the chance of drawing a queen is 4/51.

Hence the probability of drawing both cards $= \frac{1}{13} \times \frac{4}{51} = \frac{4}{663}$.

**Example 26.15.** *A pair of dice is tossed twice. Find the probability of scoring 7 points (a) once, (b) at least once (c) twice.* *(Kurukshetra, 2009 S ; V.T.U., 2004)*

**Solution.** In a single toss of two dice, the sum 7 can be obtained as (1, 6), (2, 5), (3, 4), (4, 3), (5, 2), (6, 1) *i.e.*, in 6 ways, so that the probability of getting 7 = 6/36 = 1/6.

Also the probability of not getting 7 = 1 – 1/6 = 5/6.

(*a*) The probability of getting 7 in the first toss and not getting 7 in the second toss = 1/6 × 5/6 = 5/36.

Similarly, the probability of not getting 7 in the first toss and getting 7 in the second toss = 5/6 × 1/6 = 5/36.

Since these are mutually exclusive events, addition law of probability applies.

$\therefore$ required probability $= \frac{5}{36} + \frac{5}{36} = \frac{5}{18}$.

(*b*) The probability of not getting 7 in either toss $= \frac{5}{6} \times \frac{5}{6} = \frac{25}{36}$

$\therefore$ the probability of getting 7 at least once $= 1 - \frac{25}{36} = \frac{11}{36}$.

(*c*) The probability of getting 7 twice $= \frac{1}{6} \times \frac{1}{6} = \frac{1}{36}$.

**Example 26.16.** *There are two groups of objects : one of which consists of 5 science and 3 engineering subjects, and the other consists of 3 science and 5 engineering subjects. An unbiased die is cast. If the number 3 or number 5 turns up, a subject is selected at random from the first group, otherwise the subject is selected at random from the second group. Find the probability that an engineering subject is selected ultimately*

**Solution.** Prob. of turning up 3 or 5 $= \frac{2}{6} = \frac{1}{3}$.

Prob. of selecting an engg. subject from first group $= \frac{3}{8}$

$\therefore$ Prob of selecting an engg. subject from first group on turning up 3 or 5

$$= \frac{1}{3} \times \frac{3}{8} = \frac{1}{8} \qquad \ldots(i)$$

Now prob. of not turning 3 or 5 $= 1 - \frac{1}{3} = \frac{2}{3}$.

Prob. of selecting an engg. subject from second group $= \frac{5}{8}$

$\therefore$ prob. of selecting an engg. subject from second group on turning up 3 or 5

$$= \frac{2}{3} \times \frac{5}{8} = \frac{5}{12} \quad ...(ii)$$

Thus the prob. of selecting an engg. subject

$$= \frac{1}{8} + \frac{5}{12} = \frac{13}{24}. \quad \text{[From } (i) \text{ and } (ii)\text{]}$$

**Example 26.17.** *A box A contains 2 white and 4 black balls. Another box B contains 5 white and 7 black balls. A ball is transferred from the box A to the box B. Then a ball is drawn from the box B. Find the probability that it is white.* (*Rohtak, 2011 S ; V.T.U., 2004*)

**Solution.** The probability of drawing a white ball from box $B$ will depend on whether the transferred ball is black or white.

If a black ball is transferred, its probability is 4/6. There are now 5 white and 8 black balls in the box $B$. Then the probability of drawing white ball from box $B$ is $\frac{5}{13}$.

Thus the probability of drawing a white ball from urn $B$, if the transferred ball is black

$$= \frac{4}{6} \times \frac{5}{13} = \frac{10}{39}.$$

Similarly the probability of drawing a white ball from urn $B$, if the transferred ball is white

$$= \frac{2}{6} \times \frac{6}{13} = \frac{2}{13}.$$

Hence required probability $= \frac{10}{39} + \frac{2}{13} = \frac{16}{39}$.

**Example 26.18.** *(a) A biased coin is tossed till a head appears for the first time. What is the probability that the number of required tosses is odd.* (*Mumbai, 2006*)

*(b) Two persons A and B toss an unbiased coin alternately on the understanding that the first who gets the head wins. If A starts the game, find their respective chances of winning.*

**Solution.** (*a*) Let $p$ be the probability of getting a head and $q$ the probability of getting a tail in a single toss, so that $p + q = 1$.

Then probability of getting head on an odd toss

= Probability of getting head in the 1st toss
+ Probability of getting head in the 3rd toss
+ Probability of getting head in the 5th toss + ... $\infty$

$$= p + qqp + qqqqp + \ldots \infty$$

$$= p\,(1 + q^2 + q^4 + \ldots) = p \cdot \frac{1}{1-q^2} \quad (q < 1)$$

$$= p \cdot \frac{1}{(1-q)(1+q)} = p \cdot \frac{1}{p\,(1+q)} = \frac{1}{1+q}.$$

(*b*) Probability of getting a head = 1/2. Then $A$ can win in 1st, 3rd, 5th, ... throws.

$\therefore$ the chances of $A$'s winning $= \frac{1}{2} + \left(\frac{1}{2}\right)^2 \frac{1}{2} + \left(\frac{1}{2}\right)^4 \frac{1}{2} + \left(\frac{1}{2}\right)^6 \frac{1}{2} + \cdots$

$$= \frac{1/2}{1-(1/2)^2} = \frac{1}{2} \cdot \frac{4}{3} = \frac{2}{3}.$$

Hence the chance of $B$'s winning = 1 – 2/3 = 1/3.

**Example 26.19.** *Two cards are selected at random from 10 cards numbered 1 to 10. Find the probability $p$ that the sum is odd, if*

*(i) the two cards are drawn together.*

*(ii) the two cards are drawn one after the other without replacement.*

*(iii) the two cards are drawn one after the other with replacement.* (J.N.T.U., 2003)

**Solution.** (*i*) Two cards out of 10 can be selected in ${}^{10}C_2 = 45$ ways. The sum is odd if one number is odd and the other number is even. There being 5 odd numbers (1, 3, 5, 7, 9) and 5 even numbers (2, 4, 6, 8, 10), an odd and an even number is chosen in $5 \times 5 = 25$ ways.

Thus $$p = \frac{25}{45} = \frac{5}{9}.$$

(*ii*) Two cards out of 10 can be selected one after the other *without replacement* in $10 \times 9 = 90$ ways.

An odd number is selected in $5 \times 5 = 25$ ways and an even number in $5 \times 5 = 25$ ways

Thus $$p = \frac{25 + 25}{90} = \frac{5}{9}.$$

(*iii*) Two cards can be selected one after the other *with replacement* in $10 \times 10 = 100$ ways.

An odd number in selected in $5 \times 5 = 25$ ways and an even number in $5 \times 5 = 25$ ways.

Thus $$p = \frac{25 + 25}{100} = \frac{1}{2}.$$

**Example 26.20.** *Given $P(A) = 1/4$, $P(B) = 1/3$ and $P(A \cup B) = 1/2$, evaluate $P(A/B)$, $P(B/A)$, $P(A \cap B')$* and *$P(A/B')$.*

**Solution.** (*i*) Since $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$$\therefore \quad \frac{1}{2} = \frac{1}{4} + \frac{1}{3} - P(A \cap B) \text{ or } P(A \cap B) = \frac{1}{12}.$$

Thus $$P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{1/12}{1/3} = \frac{1}{4}.$$

(*ii*) $$P(B/A) = \frac{P(A \cap B)}{P(A)} = \frac{1/12}{1/4} = \frac{1}{3}.$$

(*iii*) $$P(A \cap B') = P(A) - P(A \cap B) = \frac{1}{4} - \frac{1}{12} = \frac{1}{6}.$$

(*iv*) $$P(A/B') = \frac{P(A \cap B')}{P(B')} = \frac{1/6}{1 - P(B)} = \frac{1/6}{1 - 1/3} = \frac{1}{4}.$$

**Example 26.21.** *The odds that a book will be reviewed favourably by three independent critics are 5 to 2, 4 to 3 and 3 to 4. What is the probability that of the three reviews, a majority will be favourable.* (V.T.U., 2003 S)

**Solution.** The probability that the book shall be reviewed favourably by first critic is 5/7, by second 4/7 and by third 3/7.

A majority of the three reviews will be favourable when two or three are favourable.

$\therefore$ prob. that the first two are favourable and the third unfavourable

$$= \frac{5}{7} \times \frac{4}{7} \times \left(1 - \frac{3}{7}\right) = \frac{80}{343}$$

Prob. that the first and third are favourable and second unfavourable

$$= \frac{5}{7} \times \frac{3}{7} \times \left(1 - \frac{4}{7}\right) = \frac{45}{343}$$

Prob. that the second and third are favourable and the first unfavourable

$$= \frac{4}{7} \times \frac{3}{7} \times \left(1 - \frac{5}{7}\right) = \frac{24}{343}$$

Finally, prob. that all the three are favourable $= \frac{5}{7} \times \frac{4}{7} \times \frac{3}{7} = \frac{60}{343}$

Since they are mutually exclusive events, the required prob.

$$= \frac{80}{343} + \frac{45}{343} + \frac{24}{343} + \frac{60}{343} = \frac{209}{343}.$$

**Example 26.22.** *I can hit a target 3 times in 5 shots, B 2 times in 5 shots and C 3 times in 4 shots. They fire a volley. What is the probability that (i) two shots hit, (ii) atleast two shots hit ?* (A.M.I.E.T.E., 2003)

**Solution.** Prob. of $A$ hitting the target = 3/5, prob. of $B$ hitting the target = 2/5

Prob. of $C$ hitting the target = 3/4.

(*i*) In order that two shots may hit the target, the following cases must be considered :

$$p_1 = \text{Chance that } A \text{ and } B \text{ hit and } C \text{ fails to hit} = \frac{3}{5} \times \frac{2}{3} \times \left(1 - \frac{3}{4}\right) = \frac{6}{100}$$

$$p_2 = \text{Chance that } B \text{ and } C \text{ hit and } A \text{ fails to hit} = \frac{2}{5} \times \frac{3}{4} \times \left(1 - \frac{3}{5}\right) = \frac{12}{100}$$

$$p_3 = \text{Chance that } C \text{ and } A \text{ hit and } B \text{ fails to hit} = \frac{3}{4} \times \frac{3}{5} \times \left(1 - \frac{2}{5}\right) = \frac{27}{100}$$

Since these are mutually exclusive events, the probability that any 2 shots hit

$$= p_1 + p_2 + p_3 = \frac{6}{100} + \frac{12}{100} + \frac{27}{100} = 0.45.$$

(*ii*) In order that at least two shots may hit the target, we must also consider the case of all $A$, $B$, $C$ hitting the target [in addition to the three cases of (*i*)] for which

$$p_4 = \text{chance that } A, B, C \text{ all hit} = \frac{3}{5} \times \frac{2}{5} \times \frac{3}{4} = \frac{18}{100}$$

Since all these are mutually exclusive events, the probability of *atleast* two shots hit

$$= p_1 + p_2 + p_3 + p_4 = \frac{6}{100} + \frac{12}{100} + \frac{27}{100} + \frac{18}{100} = 0.63.$$

**Example 26.23.** *A problem in mechanics is given to three students A, B, and C whose chances of solving it are $\frac{1}{2}, \frac{1}{3}$ and $\frac{1}{4}$ respectively. What is the probability that the problem will be solved.* (V.T.U., 2004)

**Solution.** The probability that $A$ can solve the problem is 1/2.

The probability that $A$ cannot solve the problem is $1 - \frac{1}{2}$.

Similarly the probabilities that $B$ and $C$ cannot solve the probiem are $1 - \frac{1}{3}$ and $1 - \frac{1}{4}$.

$\therefore$ the probability that $A$, $B$ and $C$ cannot solve the problem is $\left(1 - \frac{1}{2}\right)\left(1 - \frac{1}{3}\right)\left(1 - \frac{1}{4}\right)$.

Hence the probability that the problem will be solved, *i.e.*, at least one student will solve it

$$= 1 - \left(1 - \frac{1}{2}\right)\left(1 - \frac{1}{3}\right)\left(1 - \frac{1}{4}\right) = \frac{3}{4}.$$

**Example 26.24.** *The students in a class are selected at random, one after the other, for an examination. Find the probability p that the boys and girls in the class alternate if*

(*i*) *the class consists of 4 boys and 3 girls.*

(*ii*) *the class consists of 3 boys and 3 girls.* (J.N.T.U., 2003)

**Solution.** (*i*) As there are 7 students in the class, the first examined must be a boy.

$\therefore$ prob. that first is a boy $= \frac{4}{7}$

Then the prob. that the second is a girl $= \frac{3}{6}$.

$\therefore$ prob. of the next boy $= \frac{3}{5}$

Similarly the prob. that the fourth is a girl $= \frac{2}{4}$,

the prob. that the fifth is a boy $= \frac{2}{3}$,

the prob. that the sixth is a girl $= \frac{1}{2}$

and the last is a boy $= \frac{1}{1}$.

Thus $$p = \frac{4}{7}\cdot\frac{3}{6}\cdot\frac{3}{5}\cdot\frac{2}{4}\cdot\frac{2}{3}\cdot\frac{1}{2}\cdot\frac{1}{1} = \frac{1}{35}.$$

(*ii*) The first student is a boy and the first student is a girl are two mutually exclusive cases. If the first student is a boy, then the probability $p_1$ that the students alternate is

$$p_1 = \frac{3}{6}\cdot\frac{3}{5}\cdot\frac{2}{4}\cdot\frac{2}{3}\cdot\frac{1}{2}\cdot\frac{1}{1} = \frac{1}{20}.$$

If the first student is a girl, then the probability $p_2$ that the students alternate is

$$p_2 = \frac{3}{6}\cdot\frac{3}{5}\cdot\frac{2}{4}\cdot\frac{2}{3}\cdot\frac{1}{2}\cdot\frac{1}{1} = \frac{1}{20}.$$

Thus the required prob. $p = p_1 + p_2 = \frac{1}{20} + \frac{1}{20} = \frac{1}{10}$.

**Example 26.25.** (*Huyghen's problem*) *A and B throw alternately with a pair of dice. A wins if he throws 6 before B throws 7 and B wins if he throws 7 before A throws 6. If A begins, find his chance of winning.*

(*Madras, 2006 ; J.N.T.U., 2003*)

**Solution.** The sum 6 can be obtained as follows : (1, 5), (2, 4), (3, 3), (4, 2), (5, 1), *i.e.*, in 5 ways.

The probability of $A$'s throwing 6 with 2 dice is $\frac{5}{36}$.

$\therefore$ the probability of $A$'s not throwing 6 is 31/36.

Similarly the probability of $B$'s throwing 7 is 6/36, *i.e.*, $\frac{1}{6}$.

$\therefore$ the probability of $B$'s not throwing 7 is 5/6.

Now $A$ can win if he throws 6 in the first, third, fifth, seventh etc. throws.

$\therefore$ the chance of $A$'s wining

$$= \frac{5}{36} + \frac{31}{36}\times\frac{5}{6}\times\frac{5}{36} + \frac{31}{36}\times\frac{5}{6}\times\frac{31}{36}\times\frac{5}{6}\times\frac{5}{36} + \cdots$$

$$= \frac{5}{36}\left[1 + \left(\frac{31}{36}\times\frac{5}{6}\right) + \left(\frac{31}{36}\times\frac{5}{6}\right)^2 + \left(\frac{31}{36}\times\frac{5}{6}\right)^3 + \cdots\right]$$

$$= \frac{5}{36}\cdot\frac{1}{1-(31/36)\times(5/6)} = \frac{5}{36}\times\frac{36\times 6}{61} = \frac{30}{61}.$$