27.9 NORMAL DISTRIBUTION

Normal probability distribution or simply normal distribution is the probability distribution of a continuous random variable X, known as normal random variable or normal variate. It is given by

$$N(\overline{X}, \sigma) = f(X) = Y(X) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}(X-\overline{X})^2/\sigma^2} \quad (1)$$

Here $\overline{X}$ = Arithmetic mean, σ = standard deviation, are the two parameters of the continuous distribution (1). Normal distribution (N.D.) is also known as Gaussian distribution (due to Karl Friedrich Gauss and also credited to de Moivre and Laplace). This theoretical distribution (1) is most important, simple, useful and is the corner stone of modern statistics because (a) discrete probability distributions such as Binomial, Poisson, Hypergeometric can be approximated by N.D. (b) sampling distributions 't', F, χ^2 tend to be normal for large samples and (c) it is applicable in statistical quality control in industry.

Properties of Normal Distribution (N.D.)

1. The graph of the N.D. $y = f(X)$ in the XY-plane is known as normal curve (N.C.). N.C. is (a) symmetric about y-axis (b) it is bell shaped (c) the mean, median and mode coincide and therefore N.C. is unimodal (has only one maximum point). (d) N.C. has inflection points at $\overline{x} \pm \sigma$. (e) N.C. is asymptotic to both positive x-axis and negative x-axis (see Fig. 27.6).

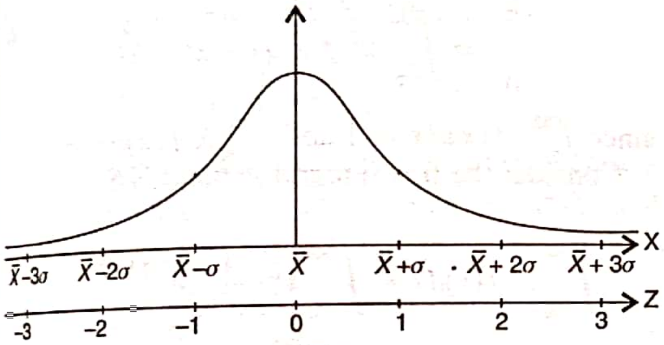

Fig. 27.6

2. Area under the normal curve is unity.
3. Probability that the continuous random variable X lies between X_1 and X_2 is denoted by probability $(X_1 \le X \le X_2)$ and is given by

$$P(X_1 \le X \le X_2) = \int_{X_1}^{X_2} \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\overline{x}}{\sigma}\right)^2} dx \tag{2}$$

Since (2) depends on the two parameters $\overline{x}$ and σ, we get different normal curves for different values of $\overline{x}$ and σ and it is an impractiable task to plot all such normal curves. Instead, by introducing

$$Z = \frac{x - \overline{x}}{\sigma}$$

the R.H.S. integral in (2) becomes independent (dimensionless) of the two parameters $\overline{x}$ and σ. Here Z is known standard (or standardized) variable (variate).

4. Change of scale from x-axis to z-axis.

$$P(X_1 \le X \le X_2) = \int_{X_1}^{X_2} \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\overline{x}}{\sigma}\right)^2} dx$$

$$P(Z_1 \le Z \le Z_2) = \int_{Z_1}^{Z_2} \frac{1}{\sigma\sqrt{2\pi}} e^{-Z^2/2} \sigma dZ$$

$$= \int_{Z_1}^{Z_2} \frac{1}{\sqrt{2\pi}} e^{-Z^2/2} dZ \tag{3}$$

where

$$Z_1 = \frac{X_1 - \overline{X}}{\sigma}, \quad Z_2 = \frac{X_2 - \overline{X}}{\sigma}.$$

5. Error function or probability integral is defined as

$$P(Z) = \frac{1}{\sqrt{2\pi}} \int_0^Z e^{-Z^2/2} dZ \tag{4}$$

Now (3) can be written using (4) as

$$P(Z_1 \le Z \le Z_2) = \int_{Z_1}^{Z_2} \frac{1}{\sqrt{2\pi}} e^{-Z^2/2} dZ$$

$$= P(Z_2) - P(Z_1) \tag{5}$$

Normal distribution $N(\overline{x}, \sigma)$ transformed by the standard variable Z is given by

$$N(0, 1) = Y(Z) = \frac{1}{\sqrt{2\pi}} e^{-Z^2/2}$$

with mean 0 and standard deviation 1. $N(0, 1)$ is known as "Standard Normal Distribution" and its normal curve as standard normal curve (Fig. 27.7). The probability integral (4) is tabulated for various values of Z varying from 0 to 3.9 and is known as normal table (see A12). Thus the entries in the normal table gives (represents) the area under the normal curve between the ordinates $Z = 0$ to Z (shaded in the figure). Since normal curve is symmetric about y-axis, the area from 0 to $-Z$ is same as the area from 0 to Z. For this reason, normal table is tabulated only for positive values of Z. Hence the determination of normal probabilities (3) reduce to the determination of areas under

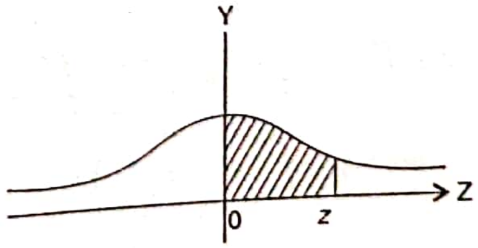

Fig. 27.7

Fig. 27.7). Therefore

$$P(X_1 \le X \le X_2) = P(Z_1 \le Z \le Z_2) = P(Z_2) - P(Z_1)$$
$$= (\text{Area under the N.C. from 0 to} Z_2)$$
$$- (\text{Area under the N.C. from 0 to} Z_1)$$

6. Area under the N.C. is distributed as follows:
   68.27% area lies between $\overline{X} - \sigma$ to $\overline{X} + \sigma$
   i.e., between $-1 \le Z \le 1$
   94.45% area lies between $\overline{X} - 2\sigma$ to $\overline{X} + 2\sigma$
   i.e., between $-2 \le Z \le 2$
   99.73% area lies between $\overline{X} - 3\sigma$ to $\overline{X} + 3\sigma$
   i.e., between $-3 \le Z \le 3$

Note: 50% area in the Z-interval (−.745, +.745)
99% area in the Z-interval (−2.58, +2.58)

Arithmetic Mean of Normal Distribution

By definition
the A.M. of a continuous distribution $f(x)$ is given by

$$\text{A.M.} = \frac{\int_{-\infty}^{\infty} x\, f(x)dx}{\int_{-\infty}^{\infty} f(x)dx}.$$

Consider the normal distribution with B, C as the parameters, i.e., $N(B, C) = f(x) = \frac{1}{c\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-B}{c}\right)^2}$. Then

$$\text{A.M.} = \overline{X} = \int_{-\infty}^{\infty} x \cdot \frac{1}{c\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-B}{c}\right)^2} dx$$

since $\int_{-\infty}^{\infty} f(x)dx =$ area under the normal curve $= 1$

Put $\frac{x-B}{c} = z$ so $x = B + cz$, $dx = cdz$

$$\text{So } \overline{X} = \int_{-\infty}^{\infty} (B + cz) \frac{1}{c\sqrt{2\pi}} e^{-\frac{1}{2}z^2} cdz$$
$$= B \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2}} dz + \frac{c}{\sqrt{2\pi}} \int_{-\infty}^{\infty} z e^{-\frac{z^2}{2}} dz$$
$$= B + \frac{c}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-\frac{z^2}{2}} d\left(\frac{z^2}{2}\right)$$

since $\int_{-\infty}^{\infty} f(x)dx = \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2}} dz = 1.$

$$= B + \frac{c}{\sqrt{2\pi}} \left.\frac{e^{-\frac{z^2}{2}}}{-1}\right|_{-\infty}^{\infty} = B + 0$$

So $\overline{X} = B.$

Variance for Normal Distribution

By definition

$$\text{Variance} = \int_{-\infty}^{\infty} (x - \overline{x})^2 f(x)dx$$
$$= \int_{-\infty}^{\infty} x^2 f(x)dx + \overline{x}^2 \int_{-\infty}^{\infty} f(x)dx$$
$$-2\overline{x} \int_{-\infty}^{\infty} x\, f(x)dx$$
$$= \int_{-\infty}^{\infty} x^2\, f(x)dx + \overline{x}^2 - 2\overline{x}\overline{x}$$

since $\int_{-\infty}^{\infty} f(x)dx = 1$ and $\int_{-\infty}^{\infty} x\, f(x)dx = \overline{x}$.
Consider the first integral in the R.H.S.

$$\int_{-\infty}^{\infty} x^2 f(x)dx = \int_{-\infty}^{\infty} x^2 \frac{1}{c\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\overline{x}}{c}\right)^2} dx$$

Put $\frac{x-\overline{x}}{c} = z$ so $x = \overline{x} + cz$, $dx = cdz$

$$\int_{-\infty}^{\infty} x^2 f(x)dx = \int_{-\infty}^{\infty} (\overline{x} + cz)^2 \frac{1}{c\sqrt{2\pi}} e^{-\frac{z^2}{2}} cdz$$
$$= \frac{1}{\sqrt{2\pi}} \left[c^2 \int_{-\infty}^{\infty} z^2 e^{-\frac{1}{2}z^2} dz \right.$$
$$\left. + \overline{x}^2 \int_{-\infty}^{\infty} e^{-\frac{z^2}{2}} dz + 2c\overline{x} \int_{-\infty}^{\infty} z e^{-\frac{z^2}{2}} dz \right]$$
$$= \frac{-c^2}{\sqrt{2\pi}} \int_{-\infty}^{\infty} z d\left(e^{-\frac{z^2}{2}}\right) + \overline{x}^2 \cdot 1 + 2c\overline{x} \cdot 0$$
$$= \frac{-c^2}{\sqrt{2\pi}} \left. z e^{-\frac{z^2}{2}} \right|_{-\infty}^{\infty} + \frac{c^2}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-\frac{z^2}{2}} dz + \overline{x}^2$$
$$= 0 + c^2 \cdot 1 + \overline{x}^2$$

Substituting this value

$$\text{Variance} = \int_{-\infty}^{\infty} x^2 f(x)dx - \overline{x}^2 = [c^2 + \overline{x}^2] - \overline{x}^2 = c^2$$

Thus the standard deviation (s.d.), of N.D. is c.

Book Work: Show that the area under the normal curve is unity.

Worked Out Examples

Normal distribution

Example 1: Find the area A under the normal curve:

a. to the left of $z = -1.78$

b. to the left of $z = 0.56$

c. to the right of $z = -1.45$

d. corresponding to $z \geq 2.16$

e. corresponding to $-0.80 \leq z \leq 1.53$

f. to the left of $z = -2.52$ and to right of $z = 1.83$

Solution: Refer to normal table (A12)

a. $A = 0.5 -$ Area (0 to $-$ 1.78) (Fig. 27.9)

$= 0.5 -$ Area (0 to 1.78) due to symmetry

$= 0.5 - 0.4625 = 0.0375$ (from table)

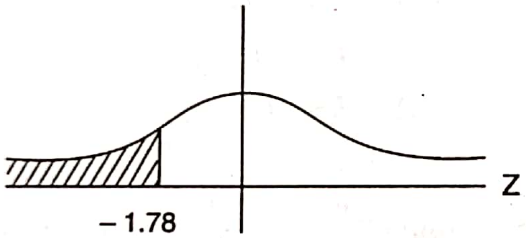

Fig. 27.9

b. $A = 0.5 +$ Area from 0 to 0.56 (Fig. 27.10)

$= 0.5 + 0.2123$ (from table)

$= 0.7123$

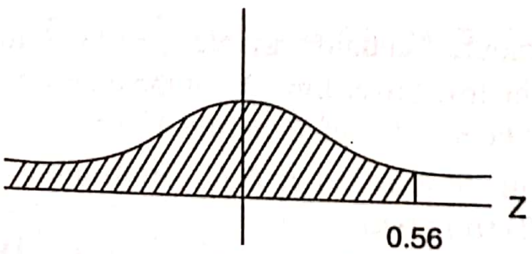

Fig. 27.10

c. $A = 0.5 +$ Area from 0 to $-$ 1.45 (Fig. 27.11)

$= 0.5 +$ Area from 0 to 1.45

$= 0.5 + 0.4265 = 0.9265$

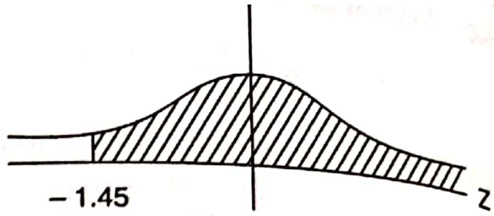

Fig. 27.11

d. $A = 0.5 - A$ (0 to 2.16)

$= 0.5 - 0.4846 = 0.0154$ (Fig. 27.12)

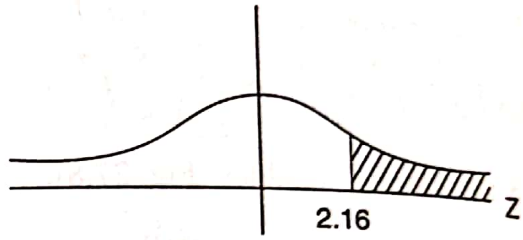

Fig. 27.12

e. $A =$ Area from (0 to $-$ 0.8)

$+$Area from (0 to 1.53)

$=$ Area from (0 to 0.8)

$+$Area from (0 to 1.53)

$= 0.4370 + 0.2881 = 0.7251$ (Fig. 27.13)

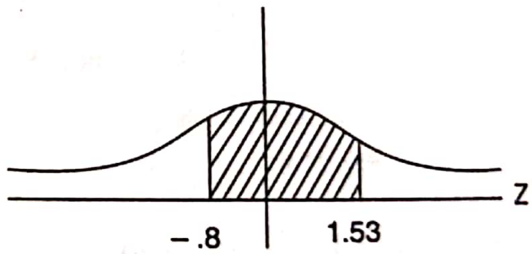

Fig. 27.13

f. $A = [0.5 - A(0 \text{ to } 2.52)] + [0.5 - A(0, 1.83)]$

$= (0.5 - 0.4941) + (0.5 - 0.4664)$

$= 0.0059 + 0.0336$

$= 0.0395$ (Fig. 27.14)

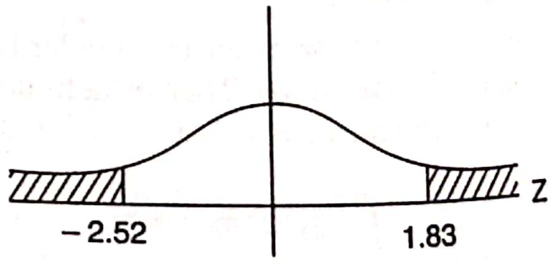

Fig. 27.14

Example 2: If z is normally distributed with mean 0 and variance 1, find

a. $P(z \geq -1.64)$
b. $P(-1.96 \leq z \leq 1.96)$
c. $P(z \leq 1)$
d. $P(z \geq 1)$

Solution:

a. $P(z \geq -1.64) = 0.5 + A(0 \text{ to } -1.64)$ (Fig. 27.15)

$= 0.5 + A(0 \text{ to } 1.64)$

$= 0.5 + 0.4495 = 0.9495$

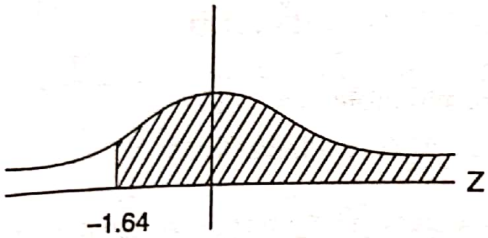

Fig. 27.15

b. $P(-1.96 \leq z \leq 1.96)$

$= 2A(0 \text{ to } 1.96)$ by symmetry

$= 2(0.4750) = 0.9500$ (Fig. 27.16)

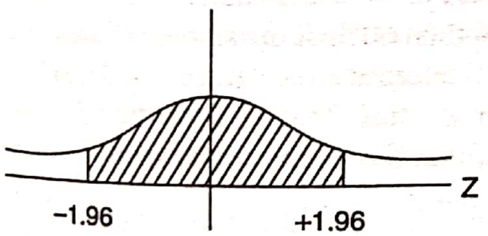

Fig. 27.16

c. $P(z \leq 1) = 0.5 + A(0 \text{ to } 1)$

$= 0.5 + 0.3413$

$= 0.8413$ (Fig. 27.17)

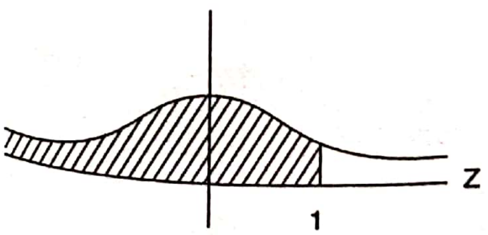

Fig. 27.17

d. $P(z \geq 1) = 0.5 - A(0 \text{ to } 1)$

$= 0.5 - 0.3413 = 0.1587$ (Fig. 27.18).

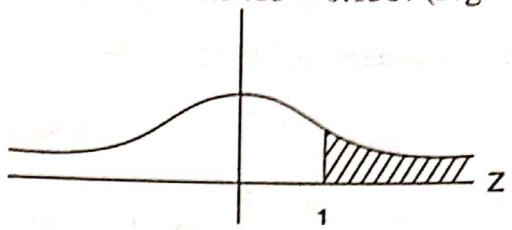

Fig. 27.18

Example 3: Determine the value of z such that (a) area to the right of z is 0.2266 (b) area to the left of z is 0.0314.

Solution: Here the areas (entries of the normal table) are given, the values of z (1st column) are determined.

a. Since area $0.2266 < \frac{1}{2}$ is to the right of z, z must be positive such that area from 0 to z is $0.5 - 0.2266 = 0.2734$. From normal table for area 0.2734, the value of z is 0.75 (Fig. 27.19).

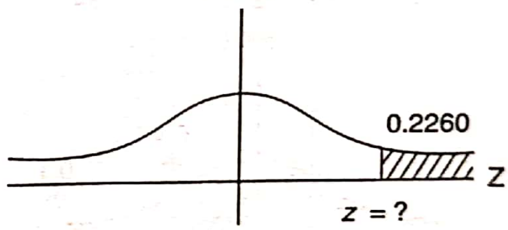

Fig. 27.19

b. Since area $0.134 < \frac{1}{2}$ is to the left of z, z must be negative. So determine z such that area from 0 to z is $0.5 - 0.134 = 0.4686$. From table A12, $z = -1.86$ (Fig. 27.20).

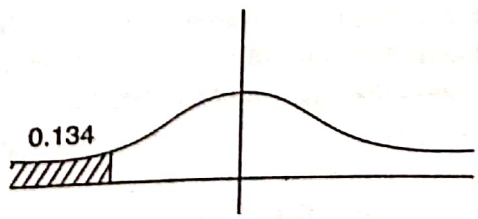

Fig. 27.20

Example 4: Find the (a) mean and (b) standard deviation of an examination in which grades 70 and

88 correspond to standard scores of -0.6 and 1.4 respectively.

Solution: Standard variable $z = \frac{X-\overline{X}}{\sigma}$.

Here $$-0.6 = \frac{70-\overline{X}}{\sigma} \quad \text{so} \quad \overline{X} - 0.6\sigma = 70$$

$$1.4 = \frac{88-\overline{X}}{\sigma} \quad \text{so} \quad \overline{X} + 1.4\sigma = 88$$

Solving $\overline{X} = 75.4$, $\sigma = 9$ are the mean and standard deviation.

Example 5: Determine the minimum mark a student must get in order to receive an A grade if the top 10% of the students are awarded A grades in an examination where the mean mark is 72 and standard deviation is 9.

Solution: The 0.1 area to the right of z corresponds to the top 10% of the students (see Fig. 27.21). From table if area from 0 to z is 0.4, then $z = 1.28$. Given $\overline{X}=72$, $\sigma=9$, we have $1.28 = z = \frac{X-\overline{X}}{\sigma} = \frac{X-72}{9}$,

$$X = 72 + 11.52 = 83.52 \simeq 84$$

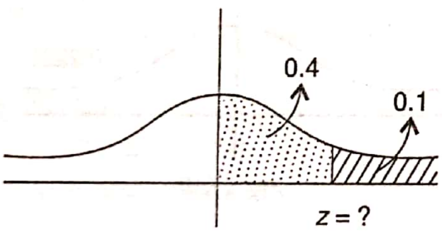

Fig. 27.21

So a student must get a minimum (or more) of 84 marks to get an A grade.

Example 6: Find the mean and standard deviation of a normal distribution in which 7% of the items are under 35 and 89% are under 63 (see Fig. 27.22).

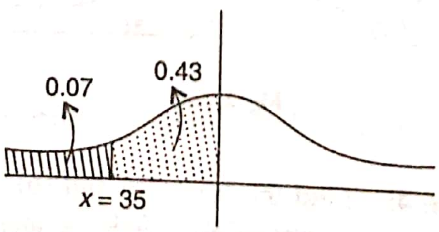

Fig. 27.22

Solution: Let X be the continuous random variable. Given that $P(X < 35) = 0.07 < \frac{1}{2}$. So z must be negative such that area from 0 to z is $0.5 - 0.07 = 0.43$. From normal table $z = -1.48$. Given that $P(X < 63) = 0.89 > \frac{1}{2}$. So z must be positive such that area from 0 to z is $0.89 - 0.5 = 0.39$ (Fig. 27.23).

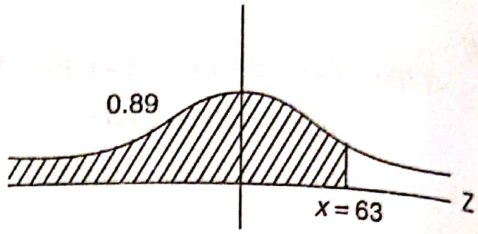

Fig. 27.23

From normal table $z = 1.23$.

Since $z = \frac{X-\overline{X}}{\sigma}$, we have

$$-1.48 = \frac{35-\overline{X}}{\sigma} \quad \text{or} \quad \overline{X} - 1.48\sigma = 35$$

$$1.23 = \frac{63-\overline{X}}{\sigma} \quad \text{or} \quad \overline{X} + 1.23\sigma = 63$$

Solving the arithmetic mean $\overline{X} = 50.3$ and standard deviation $\sigma = 10.33$.

Example 7: When the mean of marks was 50% and S.D. 5% then 60% of the students failed in an examination. Determine the 'grace' marks to be awarded in order to show that 70% of the students passed. Assume that the marks are normally distributed.

Solution: Let X be the marks obtained in the exam. Given $\overline{X} = 0.5$, $\sigma = \text{s.d.} = 0.05$.

Before grace marks were awarded, 60% failed. Since 60% failure corresponds 0.6 area, z_1 must be positive (Fig. 27.24). Determine z_1 such that the area to its left is 0.6. The value of z_1 for which the area is 0.1 is 0.25.

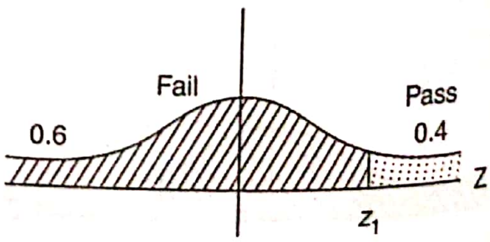

Fig. 27.24

$$0.25 = z_1 = \frac{X_1 - 0.5}{0.05} \quad \text{so} \quad X_1 = 0.5125$$

After grace marks were awarded, 70% passed examination. The area 0.70 ($> \frac{1}{2}$) corresponds pass students (Fig. 27.25). Determine z_2 such that the area to its right is 0.7. So z_2 must be negative and from table, $z_2 = -0.52$. Then

$$z_2 = -0.52 = \frac{X - 0.5}{0.05} \quad \text{or} \quad X_2 = 0.4740$$

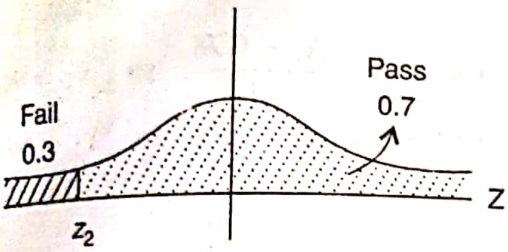

Fig. 27.25

Thus the minimum pass mark for a student is 51.25 before grace while the minimum pass mark is 47.40 after grace. So grace marks awarded is 51.25 − 47.40 = 3.85.

Example 8: Assume that the 'reduction' of a person's oxygen consumption during a period of Transcendenta Meditation (T.M.) is a continuous random variable X normally distributed with mean 37.6 cc/mt and s.d. 4.6 cc/mt. Determine the probability that during a period of T.M. a person's oxygen consumption will be reduced by (a) at least 44.5 cc/mt (b) at most 35.0 cc/mt (c) anywhere from 30.0 to 40.0 cc/mt.

Solution: $z = \frac{X - \bar{X}}{\sigma} = \frac{X - 37.6}{4.6}$

a. For $X = 44.5$, $z = \frac{44.5 - 37.6}{4.6} = 1.5$ (Fig. 27.26)

$P(X \geq 44.5) = P(z \geq 1.5) = 0.5 - 0.4332 = 0.068$

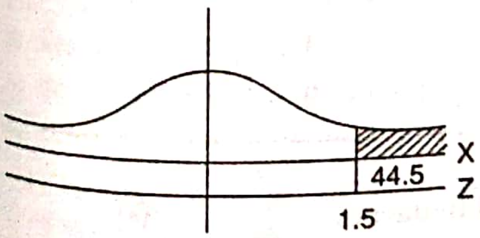

Fig. 27.26

b. For $X = 35.0$, $z = \frac{35.0 - 37.6}{4.6} = -0.5652$ (Fig. 27.26)

$$P(X \leq 35) = P(z \leq -0.5652)$$
$$= 0.5 - 0.2157$$
$$= 0.2843.$$

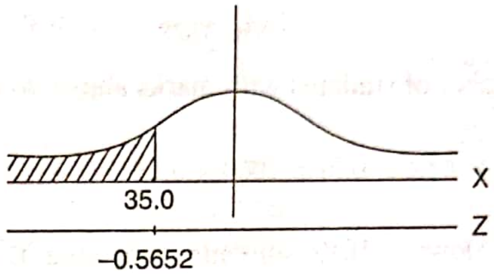

Fig. 27.27

c. For $X_1 = 30$, $z_1 = \frac{30 - 37.6}{4.6} = -1.6521$

For $X_2 = 40$, $z_2 = \frac{40 - 37.6}{4.6} = 0.52173$ (Fig. 27.26)

$$P(30 \leq X \leq 40) = P(-1.6521 \leq z \leq 0.52173)$$
$$= 0.4505 + 0.1985$$
$$= 0.6490.$$

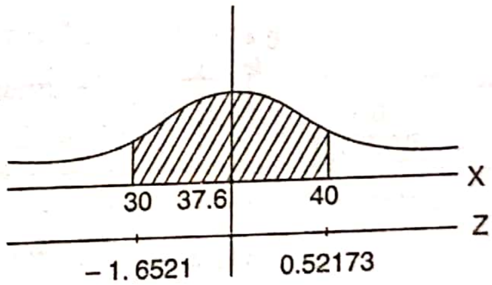

Fig. 27.28

Example 9: The marks X obtained in mathematics by 1000 students in normally distributed with mean 78% and s.d. 11% (Fig. 27.29). Determine (a) how many students got marks above 90%? (b) what was the highest mark obtained by the lowest 10% of students? (c) semi-inter quartile range (d) within what limits did the middle 90% of students lie?

Solution: Here $z = \frac{X - \bar{X}}{\sigma} = \frac{X - 0.78}{0.11}$

a. For $X = 0.9$, $z = \frac{0.9 - 0.78}{0.11} = 1.09$.

$P(X > 0.9) = P(z > 1.09) = 0.5 - 0.3621 = 0.1379$

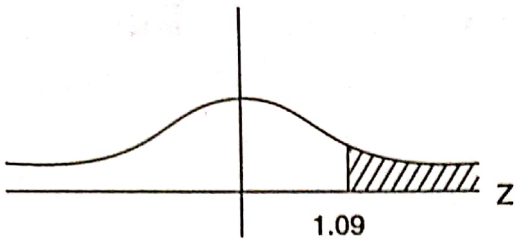

Fig. 27.29

Number of students with marks above 90%

$= 1000 \times P(X > 0.9) = 1000 \times 0.1379 = 137.9 \approx 138.$

b. The lowest 10% students constitute 0.1 area $(< \frac{1}{2})$ of extreme left tail. So z_1 must be negative. From table $0.4 = 0.5 - 0.1 =$ 0.5– Area 0.1 from 0 to z_1 so $z_1 = -1.28$.

Thus $\quad -1.28 = z_1 = \dfrac{X - 0.78}{0.11}$ or $X = 0.6392$

(see Fig. 27.30)

Thus the highest mark obtained by the lowest 10% of students is $63.92 \approx 64\%$.

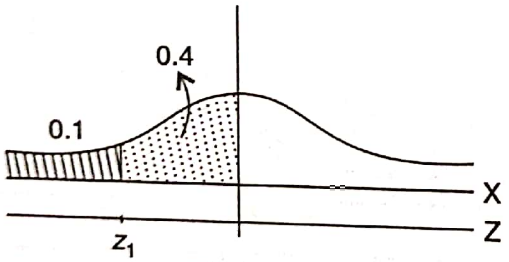

Fig. 27.30

c. Quartiles Q_1, Q_2, Q_3 divide the area into four equal parts. The value of z_1 corresponding to the first quartile Q_1 is such that the area to its left is 0.25. From table $z_1 = -0.67$. Similarly, $z_3 = 0.67$ corresponding to Q_3. Now $-0.67 = z_1 = \frac{X_1 - 0.78}{0.11}$. So the quartile mark is $X_1 = 0.7063 = 70.63\%$. Similarly, $X_3 = 85.37\%$. Thus the semi-inter quartile range $= \frac{Q_3 - Q_1}{2} = \frac{85.37 - 70.63}{2} = 7.37$ (Fig. 27.31).

d. Middle 90% correspond to 0.9 area, leaving 0.05 area on both sides. Then the corresponding z's are ± 1.64 (refer Fig. 27.32).

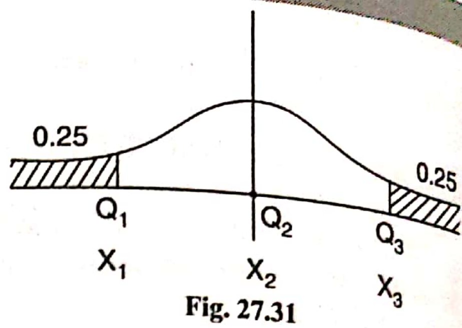

Fig. 27.31

$$1.64 = z_2 = \frac{X_2 - 0.78}{0.11} \quad \text{so} \quad X_2 = 96.04$$

$$-1.64 = z_1 = X_1 - 0.78 \quad \text{so} \quad X_1 = 59.96$$

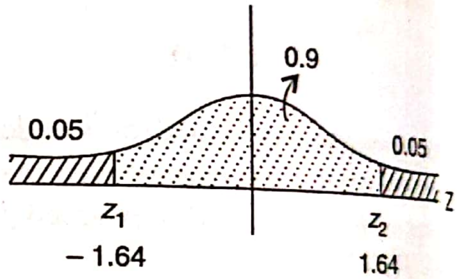

Fig. 27.32

Thus the middle 90% have marks in between 60 to 96.

Example 10: Fit a normal distribution to the following data (frequency distribution):

S. No.	Class	Observed frequency f_i
1	5–9	1
2	10–14	10
3	15–19	37
4	20–24	36
5	25–29	13
6	30–34	2
7	35–39	1

$$\text{Total frequency} = \sum_{i=1}^{7} f_i = 100$$

Solution:

1	2	3	4	5	6	7	8
S. No.	Class	Frequency f_i	True lower class limit X_i	Standard variate $z_i = \frac{X_i - 20}{5}$	Area from 0 to z_i	Area for class (Probability P)	Expected or theoretical frequency $= NP = 100P$
			4.5	−3.1	0.4990		
1	5–9	1	9.5	−2.1	0.4821	0.0169	$1.69 \approx 2$
2	10–14	10	14.5	−1.1	0.3643	0.1178	$11.78 \approx 12$
3	15–19	37	19.5	−0.1	0.0398	0.3245	$32.45 \approx 32$
4	20–24	36	24.5	0.9	0.3159	0.3557	$35.57 \approx 36$
5	25–29	13	29.5	1.9	0.4713	0.1554	$15.54 \approx 16$
6	30–34	2	34.5	2.9	0.4981	0.0268	$2.68 \approx 3$
7	35–39	1	39.5	3.9	0.5000	0.0019	$0.19 \approx 0$
		Total Frequency $N = 100$					

Note: Entries in column 7 are obtained by subtracting successive values in column 6 whenever they (in 6) are of the same sign. Add the values in column 6 when they are of opposite sign.

EXERCISE

Normal distribution

1. Determine the area under the normal curve
   a. between $z = -1.2$ and $z = 2.4$
   b. between $z = 1.23$ and $z = 1.87$
   c. between $z = -2.35$ and $z = -0.5$
   d. to the left of $z = -1.90$
   e. to the left of $z = 1.0$
   f. to the right of $z = -2.40$
   g. to the left of $z = -3.0$ and to the right of $z = 2.0$.

Ans. (a) 0.8767 (b) 0.0786 (c) 0.2991 (d) 0.0287 (e) 0.8413 (f) 0.9918 (g) 0.0241

2. Find the value of z such that
   a. area between -0.23 and z is 0.5722
   b. area between 1.15 and z is 0.0730
   c. area between $-z$ and z is 0.9.

Ans. (a) $z = 2.08$ (b) $z = 0.1625$
(c) $z = -1.65$ to $+1.65$

3. **a.** Calculate the standard marks of two students whose marks are 93 and 62 in an examination given that the mean mark is 78 and s.d. is 10.
   b. If the standard marks of two students are −0.6 and 1.2, determine their respective marks.

Ans. (a) $z = 1.5, -1.6$ (b) $X = 72, 90$

4. Determine the probability that the amount of cosmic radiation X a pilot of jet plane will be exposed is more than 5.20 m rem if X is normally distributed with mean 4.35 m rem and s.d. 0.59 m rem.

Ans. $P(X > 5.20) = P(z > 1.44)$
$= 0.5 - 0.4251 = 0.0749.$

5. Suppose the life span X of certain motors is normally distributed with mean 10 years and s.d. 2 years. If the manufacturer is ready to replace only 3% of motors that fail, how many years of guarantee can he offer (Fig. 27.33).

Ans. $-1.88 = z = \frac{X - \overline{X}}{\sigma} = \frac{X-10}{2}$, $X = 6.24$ years

6. Determine the expected number of boys whose weight is
   a. between 65 and 70 kg
   b. greater than or equal to 72 kg