Binomial coefficients

Binomial coefficients tell us how many ways there are to choose 'k' things out of large set. They appear as the coefficient of powers of 'x' in the expansion of $(1+x)^n$:

$$(1+x)^n = {}^nC_0 + {}^nC_1 x + {}^nC_2 x^2 + \cdots {}^nC_n x^n$$

here $\binom{n}{k}$ read as (n choose k), where $({}^nC_k)$ is the binomial coefficient of the x^k term of the polynomial.

For non-negative Integer values of n (number for the set) and k (no. of items to chose), every binomial coefficient ${}^nC_k$ is given by

$$\boxed{{}^nC_k = \frac{n!}{k!\,(n-k)!}}$$

Note: n and k are positive Integers with $k \leq n$.

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Example :-

(1) Calculate the value of $\binom{7}{c_3}$

→ we have. $^nC_k = \frac{n!}{k!(n-k)!}$

$^7C_3 = \frac{7!}{3!(7-3)!} = \frac{5040}{6\times24} = 35.$

(2) Imagine you have 5 elements {a, b, c, d, f}

Find out how many different subsets of 2 elements it has.

→ The binomial coefficient is $^5C_2$

$\therefore {}^5C_2 = \frac{5!}{2!(5-2)!} = 10.$

Pascal's Triangle

The coefficients of the successive powers of $(a+b)$ [or $(1+x)$] can be arranged in a triangular array of numbers, and is called pascal's Triangle.

$$(a+b)^0 = 1.$$
$$(a+b)^1 = a+b$$
$$(a+b)^2 = a^2+2ab+b^2$$
$$(a+b)^3 = a^3+3a^2b+3ab^2+b^3$$
$$(a+b)^4 = a^4+4a^3b+6a^2b^2+4ab^3+b^4.$$

$\vdots$

so on.

1

1 1

1 2 1

1 3 3 1

1 4 6 4 1

1 5 10 10 5 1