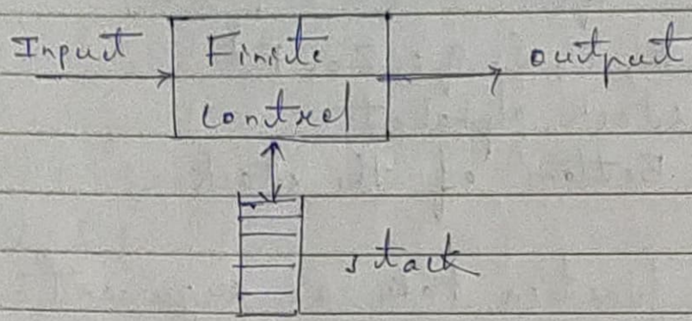
Unit-4

20/1/2022 Date PUSHDOWN AUTOMATA (PDA)



Pushdown automata is ϵ-NFA with addition of stack

Accepts CFL [CFG, PDA

Pushdown automata is ϵ-NFA with addition of stack, on which it can store a string of stack symbols.

→ In one transition, the pushdown automata:

i) Consumes from the input, the symbol that it uses in the transition. If ϵ is used for the input, then no input symbol is consumed.

ii) Goes to a new state, which may or may not be the same as previous state.

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iii) Replaces the symbol at the top of the stack by any string. a) The string could be ϵ, which corresponds to pop of the stack. b) It could be the same symbol that appeared at the top of the stack previously i.e, no change to the stack. c) It could also replace the top stack symbol by one other symbol, which in effect changes the top of the stack, but does not push or pop.
d) Finally the top stack symbol could be replaced by two or more symbols which has the effect of changing the

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top stack symbol and then pushing one or more new symbols onto the stack

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Q) Construct PDA for $L = \{a^n b^n \mid n \geq 1\}$

Sol:→ $P = (Q, \Sigma, \Gamma, \delta, q_0, Z_0, F)$

$\Gamma \rightarrow$ stack alphabet

$Z_0 \rightarrow$ Bottom of the stack

δ transition function takes as argument a triple tuple (q, a, X) where

$q \rightarrow$ state (Q)

$a \rightarrow$ input symbol / ϵ

$X \rightarrow$ a stack symbol, a member of Γ

- The output of δ is a finite set of pairs (p, γ) where p is a new state / or same same state as that of q and γ is string of stack symbols that replaces x at the top of the stack.

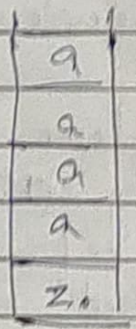
If $\gamma = \epsilon$, then the stack is popped

If $\gamma = X$, then the stack is unchanged

If $\gamma = YZ$, then X is replaced by Z and Y is pushed onto the stack.

Q) $L = \{a^n b^n \mid n \geq 1\}$

Sol:→ aaaabbbb



$\delta(q, a, X) = (p, \gamma)$

$\delta(q_0, a, z_0) = (q_0, a z_0)$

$\delta(q_0, a, a) = (q_0, aa)$

$\delta(q_0, b, a) = (q_1, \epsilon)$

$\delta(q_1, b, a) = (q_1, \epsilon)$

$\delta(q_1, \epsilon, Z_0) = (q_2, Z_0)$

Q2) $L = \{a^n b^n \mid n \geq 0\}$

Sol:- $\delta(q, a, X) = (P, D)$

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$\delta(q_0, \epsilon, z_0) = (q_2, z_0)$

$\delta(q_0, a, z_0) = (q_0, a z_0)$

$\delta(q_0, a, a) = (q_0, a a)$

$\delta(q_0, b, a) = (q_1, \epsilon)$

$\delta(q_1, b, a) = (q_1, \epsilon)$

$\delta(q_1, \epsilon, z_0) = (q_2, z_0)$

Q3) $L = \{a^m b^n \mid m > n,\ n \geq 1\}$

Sol:- $\delta(q_0, a, z_0) = (q_0, a z_0)$

$\delta(q_0, a, a) = (q_0, a a)$

$\delta(q_0, b, a) = (q_1, \epsilon)$

$\delta(q_1, b, a) = (q_1, \epsilon)$

$\delta(q_1, \epsilon, \cancel{z_0}\, a) = (q_2, \cancel{z_0}\, a)$

Q4) $L = \{a^n b^{2n} \mid n \geq 1\}$

Sol:→ $\delta(q_0, a, z_0) = (q_0, a a z_0)$

$\delta(q_0, a, a) = (q_0, a a a)$

$\delta(q_0, b, a) = (q_1, \epsilon)$

$\delta(q_1, b, a) = (q_1, \epsilon)$

$\delta(q_1, \epsilon, z_0) = (q_2, z_0)$

If $n \geq 0$ ~~then add 1 step~~

$\delta(q_0, a, z_0) = (q_2, z_0)$

$P = (Q, \Sigma, \Gamma, \delta, q_0, z_0, F)$

$Q = \{q_0, q_1, q_2\}$

$\Sigma = \{a, b\}$

$\Gamma = \{a, b, z_0\}$

$q_0 \rightarrow$ start state

$z_0 \rightarrow$ start symbol

$F \rightarrow \{q_2\}$

PDA accepts in 2 ways:

1) Acceptance by final state
2) Acceptance by empty stack

The above four examples are based on "Acceptance by final state"

Q5) Construct PDA for ~~L = {wcw^R |~~ ~~w ∈ {a, b}*}~~ equal no. of a's and b's

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~~δ(a~~

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Sol:→ $\delta(q_0, a, z_0) = (q_0, az_0)$

$\delta(q_0, b, z_0) = (q_0, bz_0)$

$\delta(q_0, a, a) = (q_0, aa)$

$\delta(q_0, b, b) = (q_0, bb)$

$\delta(q_0, a, b) = (q_0, \epsilon)$

$\delta(q_0, b, a) = (q_0, \epsilon)$

$\delta(q_0, \epsilon, z_0) = (q_f, z_0)$

Q6) Construct PDA for $L = \{wcw^R \mid w \in \{a, b\}^*\}$

Sol:→ $w = abb$

~~w~~ cw^R

abbcbba

$\delta(q_0, a, z_0) = (q_0, az_0)$

$\delta(q_0, b, z_0) = (q_0, bz_0)$

$\delta(q_0, a, a) = (q_0, aa)$

$\delta(q_0, b, a) = (q_0, ba)$

$\delta(q_0, a, b) = (q_0, ab)$

$\delta(q_0, b, b) = (q_0, bb)$

$\delta(q_0, c, a) = (q_1, a)$

$\delta(q_0, c, b) = (q_1, b)$ → $\delta(q_0, c, z_0) = (q_1, z_0)$

$\delta(q_1, a, a) = (q_1, \epsilon)$

$\delta(q_1, b, b) = (q_1, \epsilon)$

$\delta(q_1, \epsilon, z_0) = (q_2, z_0)$

Deterministic PDA :→ PDA $P = (Q, \Sigma, \Gamma, \delta, q_0, z_0, F)$ to be deterministic (a deterministic PDA or DPDA) if and only if the following condition are met:

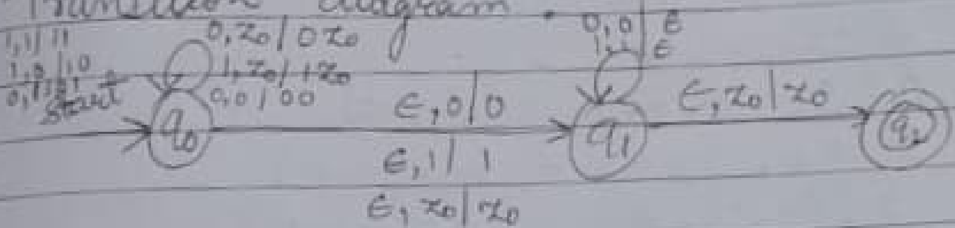
1. SC

(5) $L = \{ ww^R \mid w \in \{0,1\}^* \}$ → even length Palindrome

$\delta(q_0, 0, z_0) = (q_0, 0z_0)$
$\delta(q_0, 1, z_0) = (q_0, 1z_0)$
$\delta(q_0, 0, 0) = (q_0, 00)$
$\delta(q_0, 0, 1) = (q_0, 01)$
$\delta(q_0, 1, 1) = (q_0, 11)$
$\delta(q_0, 1, 0) = (q_0, 10)$
$\delta(q_0, \epsilon, 0) = (q_1, 0)$
$\delta(q_0, \epsilon, 1) = (q_1, 1)$
$\delta(q_0, \epsilon, z_0) = (q_1, z_0)$
$\delta(q_1, 0, 0) = (q_1, \epsilon)$
$\delta(q_1, 1, 1) = (q_1, \epsilon)$
$\delta(q_1, \epsilon, z_0) = (q_2, z_0)$.

5/4 Marks

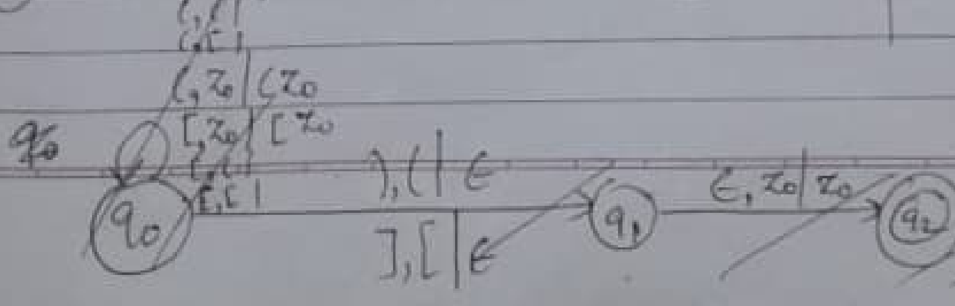
Transition diagram.



(6) Construct PDA for balanced parantheses.

① $\delta(q_0, (, z_0) = (q_0, (z_0)$
$\delta(q_0,), () = (q_1, \epsilon)$
② $\delta(q_0, [, z_0) = (q_0, [z_0)$
③ $\delta(q_0,), () = (q_0, \epsilon)$
④ $\delta(q_0,], [) = (q_0, \epsilon)$
⑤ $\delta(q_0, \epsilon, z_0) = (q_1, z_0)$

⑥ $\delta(q_0, (, () = (q_0, (()$
⑦ $\delta(q_0, (, [) = (q_0, ([)$
⑧ $\delta(q_0, [, () = (q_0, [()$
⑨ $\delta(q_0, [, [) = (q_0, [[)$



Definition 2/3 Marks

* Instantaneous Description. (IDs)

We shall represent the configuration of PDA by a triple q, w, γ where q is the state, w is the remaining input and γ is the stack content. Such a triple is called an instantaneous description (IDs). Let $P = (Q, \Sigma, \Gamma, q_0, \delta, F, Z_0)$ be a PDA define $\vdash$ as follows.

suppose $\delta(q, a, x)$ contains (p, α) then for all strings w in Σ^* and β in Γ^*

$$(q, aw, X\beta) \vdash (p, w, \alpha\beta)$$

$L = \{a^n b^n \mid n \geq 0\}$

$\cancel{\delta(q_0, \epsilon, z_0)}$

$\delta(q_0, a, z_0) = (q_0, az_0)$

$\delta(q_0, a, a) = (q_0, aa)$

$\delta(q_0, b, a) = \cancel{q_0}(q_1, \epsilon)$

$\delta(q_1, \epsilon, z_0) = (q_2, z_0)$

marks $\delta(q_1, b, a) = (q_1, \epsilon)$

Write IDs for the string aaabbb

$(q_0, aaabbb, z_0) \vdash (q_0, aabbb, az_0)$

$\vdash (q_0, aabbb, az_0) \vdash (q_0, abbb, aaz_0)$

$\vdash \cancel{(q_0, abbb, aaz_0)} \vdash (q_0, bbb, aaaz_0)$

$\vdash (q_1, bb, aaz_0) \vdash (q_1, b, az_0) \vdash (q_1, \epsilon, z_0)$

$\vdash (q_2, \epsilon, z_0)$

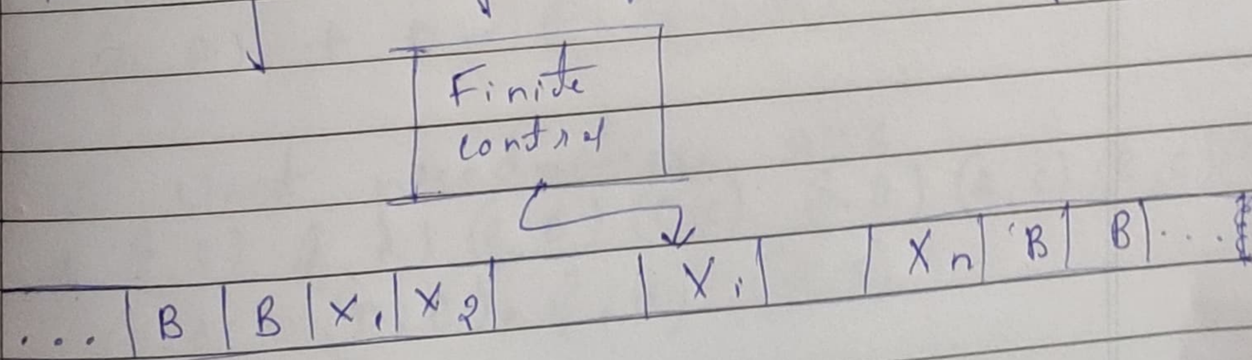
Deterministic PDA :→ PDA $P = (Q, \Sigma, \Gamma, \delta, q_0, Z_0, F)$ to be deterministic (a deterministic PDA or DPDA) if and only if the following conditions are met:

1. $\delta(q, a, X)$ has at most one member for any q in Q, a in Σ or $a = \epsilon$, and X in Γ.
2. If $\delta(q, a, X)$ is nonempty, for some a in Σ, then $\delta(q, \epsilon, X)$ must be empty.

Turing Machine :→

Notation for the Turing Machine

We may visualize a Turing machine as below diagram. The machine consists of a finite control, which can be in any of a finite set of states. There is a tape divided into squares or cells; each cell can hold any one of a finite number of symbols



A move of the Turing machine is a function of the state of the finite control and the tape symbol scanned. In one move, the turing machine will

1. Change state. The next state optionally may be the same as the current state.
2. Write a tape symbol in the cell scanned. This tape symbol replaces whatever symbol was in that cell. Optionally, the symbol written may be the same as the symbol currently there
3. Move the tape head left or right. In our formalism we require a move, and do not allow the head to remain stationary. This restriction does not constrain what a Turing machine can compute, since any sequence of moves with a stationary head could be condensed, along with the next tape-head move, into a single state change, a new tape symbol, and a move left or right

Def: Turing machine (TM) is similar to that used for finite automata or PDA's. We describe a TM by the 7 tuple:

$$M = (Q, \Sigma, \Gamma, \delta, q_0, B, F)$$

Q : The finite set of states of the finite control.
Σ : The finite set of input symbols.
Γ : The complete set of tape symbols; Σ is always a subset of Γ.
δ : The transition function. The arguments of $\delta(q, X)$

are a state q and a tape symbol X. The value of $\delta(q, X)$, if it is defined, is a triple (p, Y, D) where:

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Date: / /

1. p is the next state in Q.
2. Y is the symbol, in Γ, written in the cell being scanned, replacing whatever symbol was there.
3. D is a direction, either L or R standing for "left" or "right", respectively, and telling us the direction in which the head moves.

q_0: The start state, a member of Q, in which the finite control is found initially.

B: The blank symbol. This symbol is in Γ but not in Σ; i.e., it is not an input symbol. The blank appears initially in all but the finite number of initial cells that hold input symbols.

F: The set of final or accepting states, a subset of Q.

each cell can hold any one of finite no of symbol

Disad : tape head moves in both direc (left & right)

→ Blank is a tape symbol but not inp symbol

→ A move of TM (turing machine)

1) change state

2) write a tape symb in the cell scanned

3) Move

Ex: $L = \{ 0^n 1^n \mid n \geq 1 \}$

Finite control

0 0 0 0 1 1 1 1

B | B | B | B | B | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | B | B | B | B | B | B

x x x x y y y y

Replace 0 with x & 1 with y.

1) q_0 (encounter zero) q_1 0 → x move to Right direction (If y & zero's are found skip)

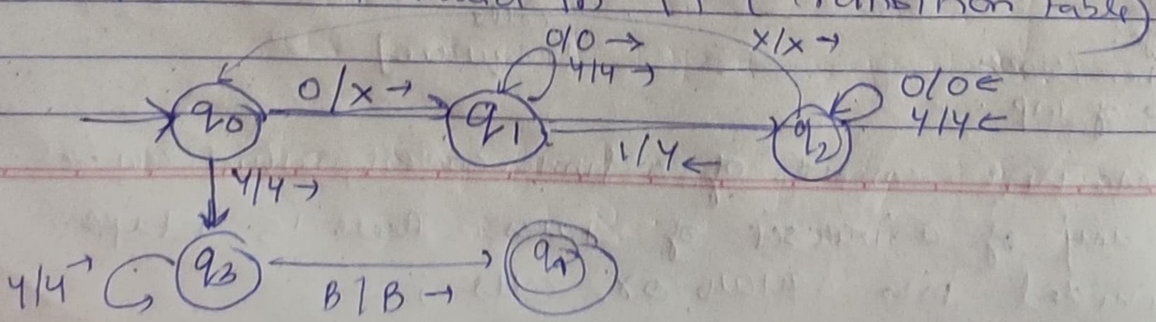
0 → x q_1

q_2 1 → y ~~move to left~~ (skip 0 & y go to x)

T. T. Right

State	0	1	X	Y	B
q_0	(q_1, X, R)	-	-	q_3, Y, R	-
q_1	$(q_1, 0, R)$	(q_2, Y, L)	-	q_1, Y, R	-
q_2	$(q_2, 0, L)$	-	q_0, X, R	q_2, Y, L	-
q_3	-	-	-	q_3 Y R	q_4, B, R
q_4	-	-	-	-	

- indicates invalid in TT (Transition table)



0011   Instantaneous Description (ID)

$q_0 0011 \vdash X q_1 011 \vdash X0q_1 11 \vdash Xq_2 0Y1$

$\vdash q_2 X0Y1 \vdash Xq_0 0Y1$

$\vdash XXq_1 Y1 \vdash XXYq_1 1$

$\vdash XXq_2 YY \vdash Xq_2 XYY$

$\vdash XXq_0 YY \vdash XXYq_3 Y$

$\vdash XXYYq_3 B \vdash XXYYBq_4 B$

** $L = \{ wcw^R \mid w \in \{a,b\}^* \}$

~~if w = abb~~

abbc bba

B	B	a	b	b	c	b	b	a	B	B

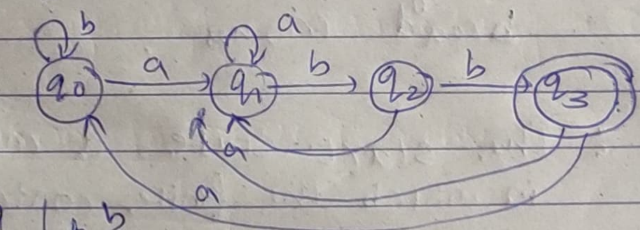
q_0 $a \to X, R, (\text{skip } b \,\&\, c \xrightarrow{q_1}$ encounter blank then stop

b

Blank ⇒ left replace x, encounter $X \to q_0$

$L = \{ (a+b)^* abb \}$

DFA



TT

	a	b
$\to q_0$	q_1	q_0
q_1	q_1	q_2
q_2	q_1	q_3
$* q_3$	q_1	q_0

TM table

	a	b	B
q_0	(q_1, a, R)	(q_0, b, R)	-
q_1	(q_1, a, R)	(q_2, b, R)	-
q_2	(q_1, a, R)	(q_3, b, R)	-
q_3	(q_1, a, R)	(q_0, b, R)	(q_4, B, R)
q_4	-	-	-

ababb

FC

B	B	a	b	a	b	b	B	B