

$$n + \frac{n}{2} + \frac{n}{3} = \frac{5n + 2n + 2n}{6} = \frac{5n}{2}$$

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$$T(n) = T\left(\frac{n}{2}\right) + T\left(\frac{n}{3}\right) + n$$

$$= n + \frac{n}{2} + T\left(\frac{n}{6}\right) + T\left(\frac{n}{4}\right) + \frac{n}{3} + T\left(\frac{n}{6}\right) + T\left(\frac{n}{9}\right)$$

$$= n + \frac{5n}{6} + T\left(\frac{n}{4}\right) + T\left(\frac{n}{9}\right) + 2T\left(\frac{n}{6}\right)$$

$$= n + \frac{5n}{6} + \frac{n}{4} + T\left(\frac{n}{12}\right) + T\left(\frac{n}{8}\right) + \frac{n}{9} + T\left(\frac{n}{18}\right) + T\left(\frac{n}{27}\right) + 2T\left(\frac{n}{6}\right)$$

$$= n + \frac{5n}{6} + \frac{n}{4}$$

$$= n + \frac{5n}{6} + \frac{n}{4} + \frac{n}{9} + 2T\left(\frac{n}{6}\right) + 3T\left(\frac{n}{12}\right) + 3T\left(\frac{n}{18}\right) + T\left(\frac{n}{8}\right) + T\left(\frac{n}{27}\right)$$

$$= n + \frac{5n}{6} + \left(\frac{5}{6}\right)^2 n \dots \dots \dots \text{Continue}$$

$$\therefore O(n) //$$

$$T(n) = T\left(\frac{n}{2}\right) + T\left(\frac{n}{3}\right) + n$$

$$T\left(\frac{n}{2}\right) = T\left(\frac{n}{4}\right) + T\left(\frac{n}{6}\right) + \frac{n}{2}$$

$$T\left(\frac{n}{4}\right) = T\left(\frac{n}{8}\right) + T\left(\frac{n}{12}\right) + \frac{n}{4}$$

$$T\left(\frac{n}{8}\right) = T\left(\frac{n}{16}\right) + T\left(\frac{n}{24}\right) + \frac{n}{8}$$

$$T(n) = T\left(\frac{n}{2}\right) + T\left(\frac{n}{3}\right) + n$$

$$T\left(\frac{n}{3}\right) = T\left(\frac{n}{6}\right) + T\left(\frac{n}{9}\right) + \frac{n}{3}$$

$$T\left(\frac{n}{9}\right) = T\left(\frac{n}{18}\right) + T\left(\frac{n}{27}\right) + \frac{n}{9}$$

$$T\left(\frac{n}{27}\right) = T\left(\frac{n}{54}\right) + T\left(\frac{n}{81}\right) + \frac{n}{27}$$

$$\frac{n}{4} + \frac{n}{9} = \frac{9n+4n}{36} = \frac{11n}{36}$$

$$\frac{5n}{6} + \frac{n}{4} + \frac{n}{9} =$$

$$T\left(\frac{n}{6}\right) = T\left(\frac{n}{12}\right) + T\left(\frac{n}{18}\right) + \frac{n}{6}$$

$$\frac{n}{4} + \frac{n}{9} + \frac{2n}{6} = \frac{9n+4n+12n}{36} = \frac{25n}{36} = \left(\frac{5}{6}\right)^2 n$$

$$\begin{array}{r} 27 \\ \times 21 \\ \hline 54 \end{array}$$

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$$\begin{array}{r} 27 \\ 32191 \\ 321913 \\ 22131 \\ 3131 \\ \hline 1111 \end{array}$$

$$T(n) = n^2 + T(n/2) + T(n/4)$$

$$= n^2 + \left(\frac{n}{2}\right)^2 + T\left(\frac{n}{4}\right) + T\left(\frac{n}{8}\right) + \left(\frac{n}{4}\right)^2 + T\left(\frac{n}{8}\right) + T\left(\frac{n}{16}\right)$$

$$= n^2 + \frac{5n^2}{16} + T\left(\frac{n}{4}\right) + 2T\left(\frac{n}{8}\right) + T\left(\frac{n}{16}\right)$$

$$= n^2 + \frac{5n^2}{16} + \left(\frac{n}{4}\right)^2 + T\left(\frac{n}{8}\right) + T\left(\frac{n}{16}\right) + 2\left(\frac{n}{8}\right)^2 + 2T\left(\frac{n}{16}\right) + 2T\left(\frac{n}{32}\right)$$

$$+ \left(\frac{n}{16}\right)^2 + T\left(\frac{n}{32}\right) + T\left(\frac{n}{64}\right)$$

$$= n^2 + \frac{5n^2}{16} + \left(\frac{5}{16}\right)^2 n^2 + T\left(\frac{n}{8}\right) + 3T\left(\frac{n}{16}\right) + 3T\left(\frac{n}{32}\right) + T\left(\frac{n}{64}\right)$$

$$= n^2 + \frac{5n^2}{16} + \left(\frac{5}{16}\right)^2 n^2 + T\left(\frac{n}{16}\right) + T\left(\frac{n}{32}\right) + \left(\frac{n}{8}\right)^2 + 3T\left(\frac{n}{32}\right) + 3T\left(\frac{n}{64}\right)$$

$$+ 3\left(\frac{n}{16}\right)^2 + 3T\left(\frac{n}{64}\right) + 3T\left(\frac{n}{128}\right) + 3\left(\frac{n}{32}\right)^2 + T\left(\frac{n}{128}\right) + T\left(\frac{n}{256}\right) + \frac{n}{64}$$

$$O(n^2) //$$

$$\rightarrow T(n) = n^2 + T(n/2) + T(n/4)$$

$$T(n) = n^2 + T\left(\frac{n}{2}\right) + T\left(\frac{n}{4}\right)$$

$$T(n/2) = n^2 + T\left(\frac{n}{4}\right) + T\left(\frac{n}{8}\right)$$

$$T(n/4) = \left(\frac{n}{4}\right)^2 + T\left(\frac{n}{8}\right) + T\left(\frac{n}{16}\right)$$

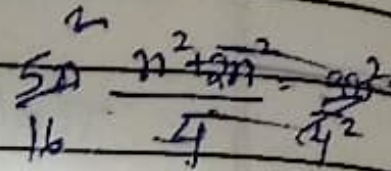
$$T\left(\frac{n}{8}\right) = \left(\frac{n}{8}\right)^2 + T\left(\frac{n}{16}\right) + T\left(\frac{n}{32}\right)$$

$$T(n/4) = \left(\frac{n}{4}\right)^2 + T\left(\frac{n}{8}\right) + T\left(\frac{n}{16}\right)$$

$$T\left(\frac{n}{16}\right) = \left(\frac{n}{16}\right)^2 + T\left(\frac{n}{32}\right) + T\left(\frac{n}{64}\right)$$

$$\frac{n^2}{4} + \frac{n^2}{16} = \frac{4n^2 + n^2}{16} = \frac{5n^2}{16}$$

~~ca^2~~



$$\left(\frac{5}{16}\right)^2 x^2$$

$$\frac{5}{16} \cdot \frac{16-5}{16} = \frac{11}{16} \cdot \frac{16}{11}$$

$$\begin{array}{r|l} 2 & 16, 8, 4 \\ 2 & 8, 1, 1, 2 \\ 4 & 4, 1, 1, 1 \end{array}$$

1/12/21 morning = evening

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$$T(n) = 2 + (\sqrt{n}) \log n$$

$$n = 2^k$$

$$T(n) = 2T(n/2) + \log n$$

$$T(2^k) = 2T(2^{k/2}) + \log 2^k$$

$$T(2^k) = 2T(2^{k/2}) + \log 2^k$$

$$S(k) = 2T(k/2) + \log k = k \log_2 k \quad T(2^k) = S(k)$$

$$a=2, b=2, f(n) = \log k = k$$

$$\text{only } T(2^k) \text{ not } \log 2^k$$

$$\therefore k \log k \therefore k = k$$

Case 2.

$$\therefore \Theta(k \log k) = \Theta(\log \log n)$$

$$S(k) = \Theta(k \log k)$$

$$T(2^k) = \Theta(\log \log \log n)$$

$$T(n) = \Theta(\log \log \log n)$$

$$\frac{n^2}{16}$$

$$\frac{n^2}{256} + \frac{n^2}{64} + \frac{n^2}{64} + \frac{n^2}{16} = \frac{n^2 + 4n^2 + 4n^2 + 16n^2}{256} + \frac{n^2}{16}$$

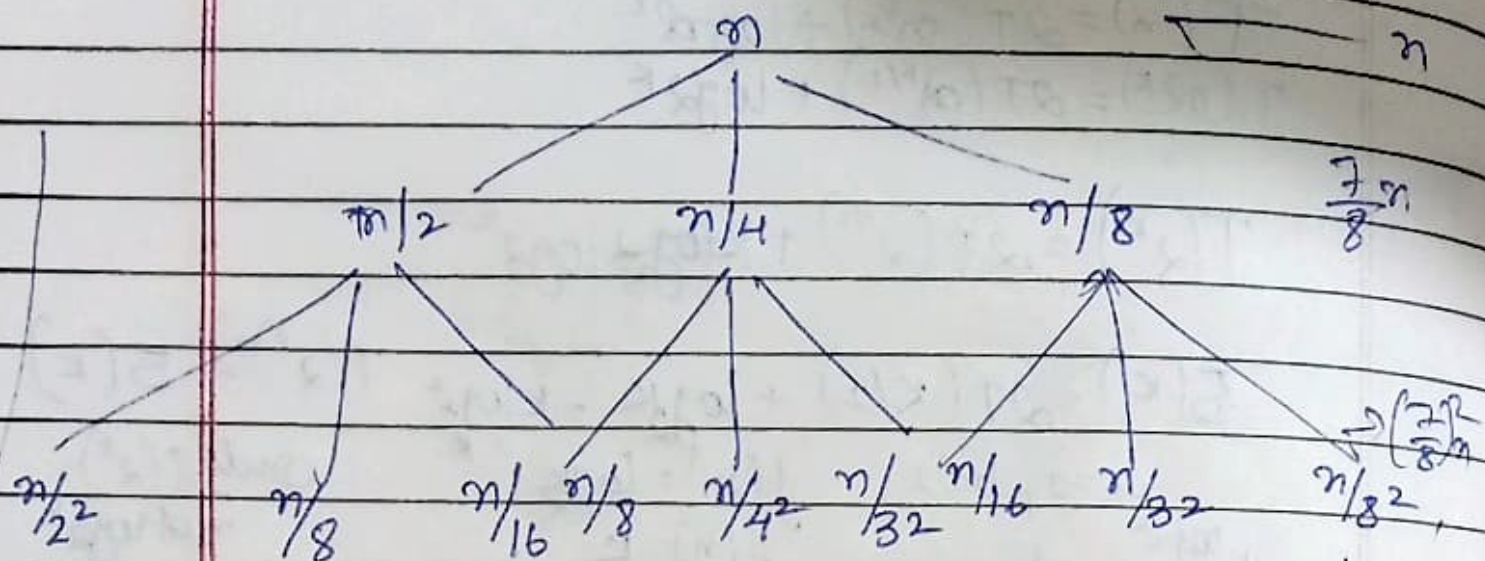
$$= \frac{25n^2}{256} = \left(\frac{5}{16}\right)^2 n^2$$

$$\frac{n^2 + 4n^2}{16} = \frac{5n^2}{16}$$

$$\begin{array}{r|rrrr} 2 & 256 & 64 & 64 & 16 \\ 2 & 128 & 32 & 32 & 8 \\ 2 & 64 & 16 & 16 & 4 \\ 2 & 32 & 8 & 8 & 2 \\ 4 & 16 & 4 & 4 & 1 \\ 4 & 4 & 1 & 1 & \end{array}$$

$$\frac{64}{4} = 16$$

$$T(n) = T(n/2) + T(n/4) + T(n/8) + n$$



$$\rightarrow T(n) = 2T(\sqrt{n}) + C$$

Assume: $n = 2^k$

$$n = 2^k$$

Taking log b.s

$$\log n = k \log 2$$

$$k = \log n$$

$$T(n) = 2T(\sqrt{n}) + C$$

$$T(2^k) = 2T(2^{k/2}) + C$$

$$T(2^k) = 2T(2^{k/2}) + C$$

$$S(k) = 2T(n/2) + C$$

$$a=2, b=2, f(n)=C$$

$$k = \log n \therefore n > C$$

Assume $T(2^k) = S(k)$

$$n^0 = 1$$

~~$T(n)$~~

$S(m) = K$ — Convert it back.

$$S(k) = T(2^k) = K$$

$$T(n) = \log n \therefore O(\log n)$$

$$2 \times 2 \times 2 \times 2 \times 2 \times 2$$

$$2^2 \times 16 \times 2$$

$$\frac{2^2}{2} + \frac{2^1}{4} + \frac{2^0}{8} = \frac{32n + 16n + 2n}{64}$$

$$= \frac{50n}{64} = \frac{25n}{32} = \left(\frac{5}{16}\right)n$$

$$= \frac{2n + n}{4} = \frac{3n + n}{4 \times 8} = \frac{6n + n}{8} = \frac{7n}{8}$$

$$\frac{16}{2 \times 2}$$

$$4 \mid 4, 8$$

$$2 \mid 1, 2$$

$$T(n) = T\left(\frac{n}{3}\right) + n$$

$$= T\left(\frac{n}{3^2}\right) + \frac{n}{3} + n$$

$$= T\left(\frac{n}{3^3}\right) + \frac{n}{3^2} + \frac{n}{3} + n$$

$$= T\left(\frac{n}{3^i}\right) + n \left[1 + \frac{1}{3} + \frac{1}{3^2} \right]$$

$$= T\left(\frac{n}{3^i}\right) + n \times \frac{3}{2}$$

$$= 1 + \frac{3n}{2}$$

$$O(n) //$$

$$T(n) = T\left(\frac{n}{2}\right) + T\left(\frac{n}{3}\right) + n$$

$$= T\left(\frac{n}{2^2}\right) + T\left(\frac{n}{6}\right) + \frac{n}{2} + T\left(\frac{n}{3^2}\right) + T\left(\frac{n}{6}\right) + \frac{n}{3} + n$$

$$= T\left(\frac{n}{2^2}\right) + T\left(\frac{n}{3^2}\right) + 2T\left(\frac{n}{6}\right) + \frac{n}{3} + \frac{n}{2} + n$$

$$= T\left(\frac{n}{2^i}\right) + T\left(\frac{n}{3^i}\right) + 2T\left(\frac{n}{6}\right) + \frac{11n}{6} \left[1 + \frac{1}{2} + \frac{1}{3} + \dots \right]$$

$$= T\left(\frac{n}{n}\right) + T\left(\frac{n}{3 \log_2 n}\right) + 2T\left(\frac{n}{3 \log_2 n}\right) + \frac{11n}{6}$$

$$= T(1) + T\left(\frac{n}{n \log_2 2}\right) + 2T\left(\frac{n}{3 \log_2 n}\right) + \frac{11n}{6}$$

$$= 1 + T\left(\frac{n}{n \log_2 2}\right) + 2T\left(\frac{n}{3 \log_2 n}\right) + \frac{11n}{6}$$

$$O(n) //$$

$$\frac{n^2 + 3n + 6}{3} = \frac{n^2}{3} + n + 2$$

$$\frac{2n + 3n + 6n}{6} = \frac{11n}{6}$$

$$= \frac{5n + 6n}{6} = \frac{11n}{6}$$

$$T(n) = T\left(\frac{n}{2}\right) + T\left(\frac{n}{2}\right) + n$$

$$T(n) = T\left(\frac{n}{2}\right) + n$$

$$= T\left(\frac{n}{4}\right) + \frac{n}{2} + n$$

$$= T\left(\frac{n}{2^3}\right) + \frac{n}{2^2} + \frac{n}{2} + n$$

$$= n \left[\frac{1}{2} + \frac{1}{2^2} + \frac{1}{2^3} \right]$$

$$= T\left(\frac{n}{2^p}\right) + \frac{n}{2^{p-1}} + \frac{n}{2^{p-2}} + \dots + n = n \frac{1 - \frac{1}{2^p}}{1 - \frac{1}{2}}$$

$$= T\left(\frac{n}{n}\right) + \frac{n}{2^{\log_2 n - 1}} + \frac{n}{2^{\log_2 n - 2}} + \dots + n$$

$$= 1 + \frac{n}{2^{\log_2 n - 1}} + \frac{n}{2^{\log_2 n - 2}} + \dots + n$$

$$= 1 + \frac{n}{n^1} + \frac{n}{n^2} + \dots + n$$

$$= 1 + \frac{n}{n} + \frac{n}{n} + \dots + n$$

$$\Theta(n) //$$

$$\frac{n}{3^2} + \frac{n}{2^2} + \frac{n}{3} + \frac{n}{2} + n$$

$$\frac{n^5}{3^2} + \frac{n^4}{3} = \frac{3n+9n}{27} = \frac{12n}{27} = \frac{4n}{9} = \frac{n}{2^2} + \frac{n}{2} + n$$

$$\frac{n^{x^2 \times 4}}{3^4} + \frac{n}{2} = \frac{2n+4n}{8} = \frac{6n}{8} = \frac{3n}{4} = \frac{3n}{4} + \frac{4n}{9} + n$$

$$= \frac{27n+16n+36n}{36} = \frac{79n}{36}$$

$$T(n) = T\left(\frac{n}{2}\right) + n$$

$$= T\left(\frac{n}{2^2}\right) + \frac{n}{2} + n$$

$$= T\left(\frac{n}{2^3}\right) + \frac{n}{2^2} + n$$

$$T(n) = T\left(\frac{n}{2}\right) + T\left(\frac{n}{3}\right) + n$$

$$= T\left(\frac{n}{2^2}\right) + T\left(\frac{n}{6}\right) + \frac{n}{2}$$

$$= T\left(\frac{n}{2^3}\right) + T\left(\frac{n}{12}\right) + \frac{n}{2^2}$$

$$T(n) = T\left(\frac{n}{2^3}\right) + T\left(\frac{n}{3^3}\right) + T\left(\frac{n}{18}\right) + T\left(\frac{n}{12}\right) + \frac{n}{3^2} + \frac{n}{2^2} + n$$

$$= T\left(\frac{n}{2^6}\right) + T\left(\frac{n}{3^6}\right) + T\left(\frac{n}{6^3}\right) + T\left(\frac{n}{4^3}\right) + n \left[\frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} \right]$$

$$= T\left(\frac{n}{2^{\log_2 n}}\right) + T\left(\frac{n}{3^{\log_3 n}}\right) + T\left(\frac{n}{6^{\log_6 n}}\right) + T\left(\frac{n}{4^{\log_4 n}}\right) + n$$

$$= T\left(\frac{n}{n \log_2 n}\right) + 1 + \Theta(n)$$

$$T(n) = T\left(\frac{n}{2}\right) + T\left(\frac{n}{3}\right) + n$$

$$\therefore T\left(\frac{n}{2}\right) = T\left(\frac{n}{2^2}\right) + T\left(\frac{n}{6}\right) + \frac{n}{2}$$

$$T(n) = T\left(\frac{n}{2}\right) + T\left(\frac{n}{3}\right) + n$$

$$T\left(\frac{n}{2}\right) = T\left(\frac{n}{2^2}\right) + T\left(\frac{n}{6}\right) + \frac{n}{2}$$

$$T\left(\frac{n}{2^2}\right) = T\left(\frac{n}{2^3}\right) + T\left(\frac{n}{12}\right) + \frac{n}{2^2}$$

$$T(n) = T\left(\frac{n}{2}\right) + T\left(\frac{n}{3}\right) + n$$

$$T\left(\frac{n}{3}\right) = T\left(\frac{n}{6}\right) + T\left(\frac{n}{3^2}\right) + \frac{n}{3}$$

$$T\left(\frac{n}{3^2}\right) = T\left(\frac{n}{18}\right) + T\left(\frac{n}{3^3}\right) + \frac{n}{3^2}$$

$$T(n) = T\left(\frac{n}{2^2}\right) + T\left(\frac{n}{6}\right) + \frac{n}{2} + T\left(\frac{n}{6}\right) + T\left(\frac{n}{3^2}\right) + \frac{n}{3} + n$$

$$= T\left(\frac{n}{2^2}\right) + T\left(\frac{n}{3^2}\right) + 2T\left(\frac{n}{6}\right) + \frac{n}{3} + \frac{n}{2} + n$$

$$= T\left(\frac{n}{2^3}\right) + T\left(\frac{n}{12}\right) + \frac{n}{2^2} + T\left(\frac{n}{18}\right) + T\left(\frac{n}{3^3}\right) + \frac{n}{3^2} + 2T\left(\frac{n}{6}\right) + \frac{n}{3} + \frac{n}{2} + n$$

$$= T\left(\frac{n}{2^3}\right) + T\left(\frac{n}{3^3}\right) + T\left(\frac{n}{3 \times 4}\right) + T\left(\frac{n}{3 \times 6}\right) + 2T\left(\frac{n}{3 \times 2}\right) + \frac{n}{3^2} + \frac{n}{2^2} + \frac{n}{3} + \frac{n}{2} + n$$

$$2^c = T\left(\frac{n}{2^{20}}\right) + T\left(\frac{n}{3^c}\right) +$$

$$T(n) = T\left(\frac{n}{2}\right) + T\left(\frac{2n}{5}\right) + 7n$$

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of nodes
 2^0

Level 0

$7n$

$7n$

2^1

1

$7\left(\frac{n}{2}\right)$

$7\left(\frac{2n}{5}\right)$

$$\begin{aligned} &\rightarrow \frac{7n}{2} + \frac{14n}{5} \\ &= \frac{35n + 28n}{10} \\ &= 7\left(\frac{9}{10}\right)n \end{aligned}$$

2^2

2

$7\left(\frac{n}{2^2}\right)$

$7\left(\frac{2n}{10}\right)$

$7\left(\frac{2n}{10}\right)$

$7\left(\left(\frac{2}{5}\right)^2 n\right)$

$$\rightarrow 7\left(\frac{9}{10}\right)^2 n$$

2^h

h

$$T(n) = 7n + 7n\left(\frac{1}{2} + \frac{2}{5}\right)$$

$$\left. 7 \cdot \left(\frac{2}{5}\right)^h n \right\}$$

$$+ 7n\left(\left(\frac{1}{2}\right)^2 + \frac{1}{5} + \frac{1}{5} + \left(\frac{2}{5}\right)^2\right) \quad \Pi(1)$$

$$+ 7n\left(\frac{1}{2} + \frac{1}{5}\right)^3 + 7n\left(\frac{1}{2} + \frac{2}{5}\right)^4 + \dots$$

$$7\left(\frac{2}{5}\right)^h n = 1$$

$$7n = \frac{1}{\left(\frac{2}{5}\right)^h}$$

$$7n = \left(\frac{5}{2}\right)^h$$

$$= 7n \left[\left(\frac{1}{2} + \frac{2}{5}\right)^0 + \left(\frac{1}{2} + \frac{2}{5}\right)^1 + \dots + \left(\frac{1}{2} + \frac{2}{5}\right)^h \right]$$

$$= 7n \left[\left(\frac{1}{10}\right)^0 + \frac{9}{10} + \frac{9^2}{10} + \frac{9^3}{10} + \dots \right]$$

$$= 7 \left[1 + \frac{9}{10} + \left(\frac{9}{10}\right)^2 + \dots + \left(\frac{9}{10}\right)^{\log n} \right]$$

$$\begin{aligned} \log n &= \log_{5/2} n \\ &= h = \log_{5/2} n \end{aligned}$$

$$T\left(\frac{n}{2}\right) \quad T\left(\frac{2n}{5}\right) = \frac{n+2n}{2 \cdot 5} = \frac{5n+4n}{10}$$

$$T\left(\frac{9n}{10}\right)$$

$$= 7n \cdot \frac{1 - \left(\frac{9}{10}\right)^{\log_{5/2} n}}{1 - \frac{9}{10}}$$

$$= 7n \cdot \left(1 - n^{\log_{5/2} 9/10}\right)$$

$$= 7n - 7n^{0.11}$$

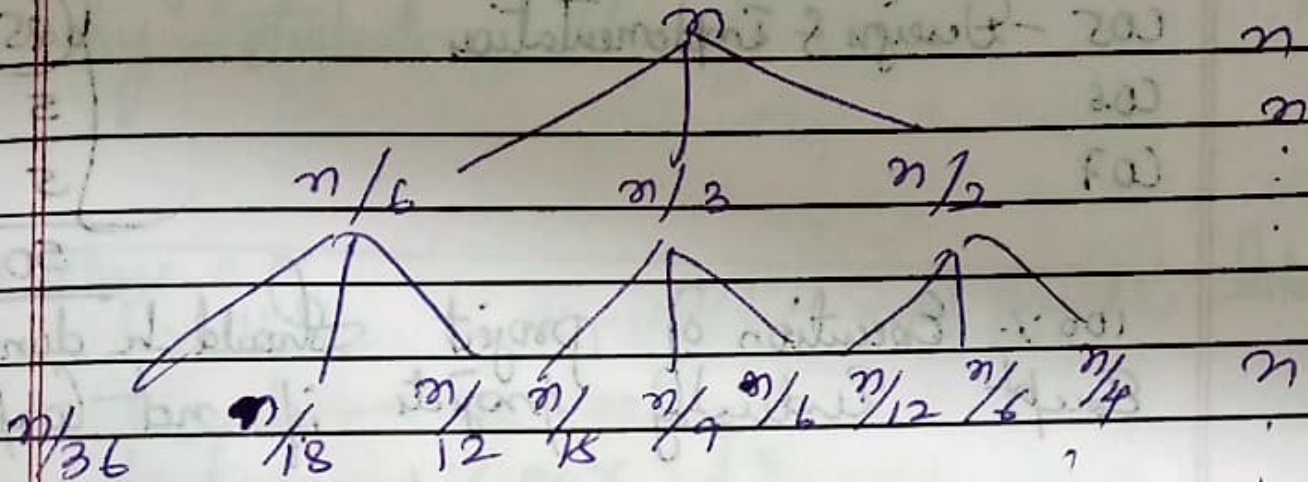
$$= O(n)$$

$$= \frac{n}{2^2} + \frac{2n}{10} + \frac{2n}{10} + \frac{2^2 n}{5^2} =$$

$$\frac{4n}{10} + \frac{4n}{25} = \frac{100n + 40n}{250} = 140n$$

$$\frac{14n}{25} + \frac{n}{4} = \frac{56n + 25n}{100} = \frac{81n}{100} = \left(\frac{9}{10}\right)n$$

Sum



log₂ 3

$T(n) = n \log n$

$$\frac{x^3}{2} + \frac{x^2}{3} = \frac{3x+2x}{6} = \frac{5x}{6} = \left(\frac{5}{6}\right)x.$$

$$\cancel{f(n)} \frac{n}{4} + \left[\frac{n}{6} + \frac{n}{6} \right] + \frac{n}{9} = \frac{6n + 4n}{24} = \frac{10n}{24} = \frac{5n}{12}$$

~~$$\frac{n+n}{6} = \frac{2n}{6} = \frac{n}{3} = \frac{n+2n}{3+6} = \frac{6n+3n}{18} = \frac{9n}{18} = \frac{n}{2}$$~~

$$= \frac{21n}{36} + \frac{n}{9} = \frac{18n + 36n}{324} = \frac{225}{324}$$