BAYE'S THEOREM

An event 'A' corresponds to a number of exhaustive events $B_1, B_2, \ldots, B_n$.
If $P(B_i)$ and $P(A|B_i)$ are given, then

$$P(B_i|A) = \frac{P(B_i)\,P(A|B_i)}{\sum P(B_i).\,P(A|B_i)}$$

By the multiplication law of conditional probability we have

$$P(A|B_i) = \frac{P(A \cap B_i)}{P(B_i)}$$

and $$P(B_i|A) = \frac{P(B_i \cap A)}{P(A)}$$

These can be written as

$$P(A \cap B_i) = P(A|B_i) \cdot P(B_i) \quad \text{--- (1)}$$

$$P(B_i \cap A) = P(B_i|A) \cdot P(A) \quad \text{--- (2)}$$

Since $P(A \cap B_i)$ and $P(B_i \cap A)$ are same

equating eqn (1) & (2)

$$P(A|B_i) \cdot P(B_i) = P(B_i|A) \cdot P(A)$$

$$P(B_i|A) = \frac{P(A|B_i) \cdot P(B_i)}{P(A)}$$

Re arranging, we get

$$\boxed{P(B_i|A) = \frac{P(B_i) \cdot P(A|B_i)}{P(A)}}$$

Since the event corresponds to $B_1, B_2 \ldots B_n$, by additional law of probability we have

$$P(A) = P(AB_1) + P(AB_2) + \ldots + P(AB_n) = \Sigma P(AB_i) = \Sigma P(B_i) P(A|B_i)$$

$$\boxed{\therefore P(B_i|A) = \frac{P(B_i)\, P(A|B_i)}{\Sigma P(B_i) \cdot P(A|B_i)}}$$

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