

Recursion Trees

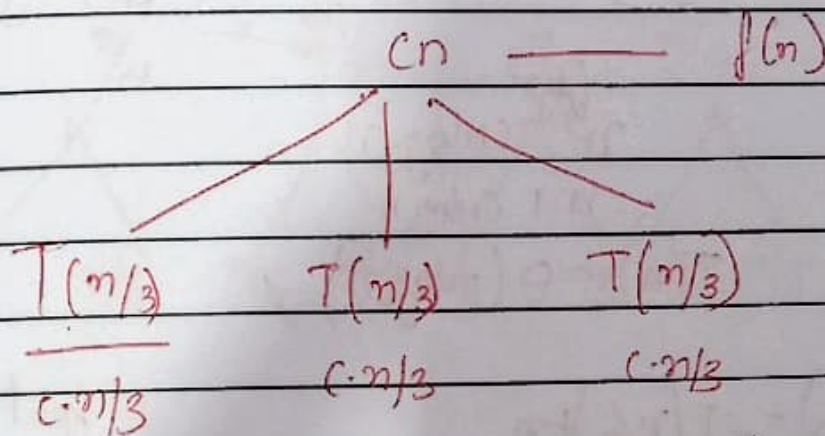
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$$T(n) = 3T(n/3) + cn$$

$$T(n/3) = 3T(n/9) + c \cdot n/3$$

$$T(n/9) = 3T(n/27) + c \cdot n/9$$

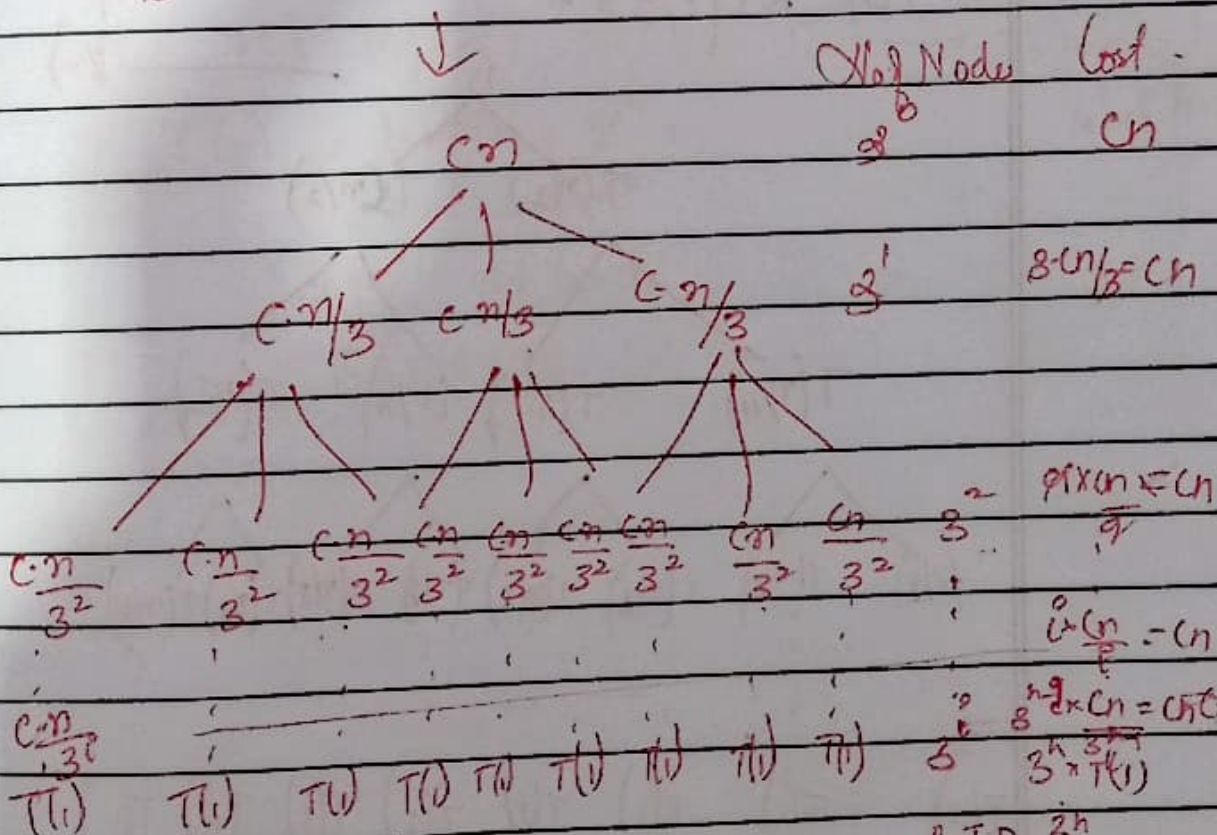


Levels

0

1

2



for a height levels is always $h+1$ and depth h

$$3^h \times T(1)$$

$$\text{Total cost} = cn + cn + cn$$

$$\therefore T(n) = 3^h \times T(1) + \sum_{i=0}^{h-1} cn$$

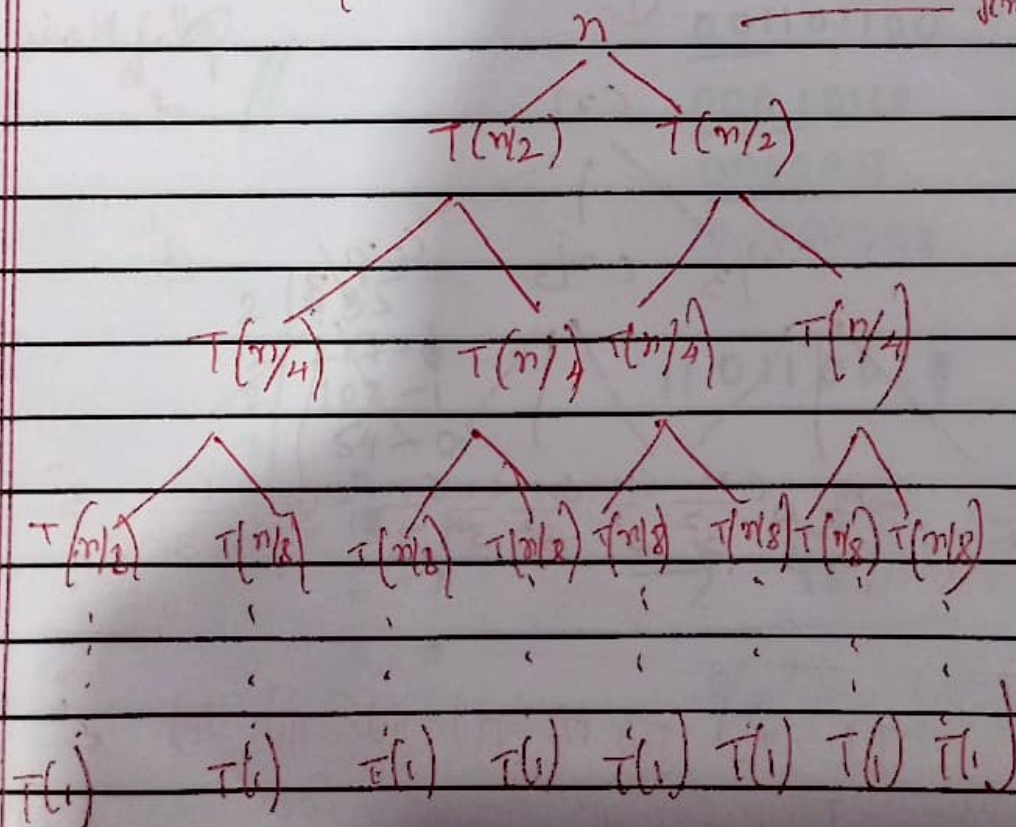
$$= 3^{\log_3 n} \cdot T(1) + cn \cdot \sum_{i=0}^{h-1} 1 \quad \text{cn} \times \text{height}$$

$$= n^3 + cn \log_3 n$$

$$= n + cn \log_3 n$$

$$T(n) = \Theta(n \log_3 n) //$$

$$\rightarrow T(n) = 2T(n/2) + n \quad \begin{matrix} \text{root} \\ \text{S(n)} \end{matrix}$$



Level No of Nodes

no 0

n

n

1

 $n/2$ $n/2$ $2 \times \frac{n}{2} = n$

2

 $n/4$ $n/4$ $n/4$ $n/4$ $4 \times \frac{n}{4} = n$

3

 $n/8$ $n/8$ $n/8$ $n/8$ $n/8$ $n/8$ $n/8$ $n/8$ $8 \times \frac{n}{8} = n$

i

 $n/2^i$ $2^i \times \frac{n}{2^i} = n$

h

 $T(1)$ $T(1)$ $T(1)$ $T(1)$ $T(1)$ $T(1)$ $T(1)$ $T(1)$ $T(1) \times h$

$$T(n) = h \times T(1) + \sum_{i=0}^{h-1} n$$

leaf hn

$$= \log_2 n + n \log_2 n$$

$$= O(n \log_2 n)$$

k?

$$\frac{n}{2^k} = 1$$

$$\frac{n}{2^k} = 1$$

$$n = 2^k$$

$$\log_2 n = k$$

$$h = \log_2 n$$

$$T(1) = 4$$

$$T(n) = 2T(n/2) + 4n$$

Row Sum
Cost.Level 0
Nodes 2

$$4n$$

$$4n$$

Level 1
Nodes 2¹

$$4n/2$$

$$4n/2$$

$$4n$$

Level 2
Nodes 2²

$$4n/4$$

$$4n/4$$

$$4n/4$$

$$4n/4$$

$$4n$$

Level 3
Nodes 2³

$$4n/8$$

$$4n/8$$

$$4n/8$$

$$4n/8$$

$$4n/8$$

$$4n/8$$

$$4n/8$$

$$4n/8$$

$$4n$$

Level 4
Nodes 2⁴

$$4n/16$$

$$4n/16$$

$$4n/16$$

$$4n/16$$

$$4n/16$$

$$4n/16$$

$$4n/16$$

$$4n/16$$

$$4n$$

Level h
Nodes 2^h

$$4n/2^h$$

$$4n/2^h$$

$$4n/2^h$$

$$4n/2^h$$

$$4n/2^h$$

$$4n/2^h$$

$$4n/2^h$$

$$4n/2^h$$

$$4n$$

$$\sum_{i=0}^{\log_2 n} 4n = 4n \sum_{i=0}^{\log_2 n} 1$$

↓

$$4n + 4n + \dots + 4n = 4n(\log_2 n + 1)$$

$$\log_2 n + 1$$

$$4n(\log_2 n + 1)$$

$$4n \log_2 n + 4n$$

Ignore

$$h = \log_2 n$$

Know where to stop.

$$\therefore O(n \log n)$$

$$T(n) = 2T\left(\frac{n}{2}\right) + n^2$$

Level # nodes

0 2^0 1 2^1 2 2^2 3 2^3 4 2^4 h 2^h n^2 $\left(\frac{n}{2}\right)^2$ $\left(\frac{n}{2}\right)^2$ $\left(\frac{n}{4}\right)^2$ $\left(\frac{n}{4}\right)^2$ $\left(\frac{n}{4}\right)^2$ $\left(\frac{n}{4}\right)^2$ $\left(\frac{n}{8}\right)^2$ $\left(\frac{n}{8}\right)^2$ $\left(\frac{n}{8}\right)^2$ $\left(\frac{n}{8}\right)^2$ $\left(\frac{n}{8}\right)^2$ $\left(\frac{n}{8}\right)^2$ $\left(\frac{n}{8}\right)^2$ $\left(\frac{n}{16}\right)^2$ $\left(\frac{n}{16}\right)^2$ $\left(\frac{n}{16}\right)^2$ $\left(\frac{n}{16}\right)^2$ $\left(\frac{n}{16}\right)^2$ $\left(\frac{n}{16}\right)^2$ $\left(\frac{n}{16}\right)^2$ $\left(\frac{n}{32}\right)^2$ $\left(\frac{n}{32}\right)^2$ $\left(\frac{n}{32}\right)^2$ $\left(\frac{n}{32}\right)^2$ $\left(\frac{n}{32}\right)^2$ $\left(\frac{n}{32}\right)^2$ $\left(\frac{n}{32}\right)^2$

(log)

 n^2

$$2 \times \frac{n^2}{2^2} = \frac{n^2}{2}$$

$$\frac{4 \times n^2}{4^2} = \frac{n^2}{4}$$

$$\frac{8 \times n^2}{8^2} = \frac{n^2}{8}$$

$$\frac{2 \times n^2}{2^2} = \frac{n^2}{2^2}$$

$$\frac{2 \times n^2}{2^h} = \frac{n^2}{2^h}$$

$$T(n) = n \cdot T(1) + \sum_{i=0}^{h-1} \frac{n^2}{2^i}$$

 $\log_2 n \times n^2$ $\frac{\log_2 n}{2}$ $\log_2 n \times n^2$ $\frac{\log_2 n}{2}$

$$= n^2 \left[\left(\frac{1}{2}\right)^0 + \left(\frac{1}{2}\right)^1 + \left(\frac{1}{2}\right)^2 + \dots + \left(\frac{1}{2}\right)^{h-1} \right]$$

$$= n^2 \left[\frac{1}{1-\frac{1}{2}} \right] = 2n^2$$

$$T(n) = \log_2 n + 2n^2$$