Binomial Theorem and Binomial Expansion

It is possible to expand any nonnegative power of $(x+y)$ into a sum of the form

$$(x+y)^n = ({}^nC_0)\,x^n y^0 + ({}^nC_1)\,x^{n-1}y^1 + ({}^nC_2)\,x^{n-2}y^2$$

where $n \geq 0$ is an integer and each $({}^nC_r)$ is a positive integer known as binomial coefficient.

$$\boxed{(x+y)^n = \sum_{r=0}^{n} ({}^nC_r)\,x^{n-r}\cdot y^r}$$

Pascal's triangle

(Row 0) 1

(Row 1) 1 1

(Row 2) 1 2 1

⋮ 1 3 3 1

⋮ 1 4 6 4 1

1 5 10 10 5 1

1 6 15 20 15 6 1

(Row 7) 1 7 21 35 35 21 7 1

Pascal's triangle

(Row 0)	1							
(Row 1)	1	1						
(Row 2)	1	2	1					
⋮	1	3	3	1				
⋮	1	4	6	4	1			
	1	5	10	10	5	1		
	1	6	15	20	15	6	1	
(Row 7)	1	7	21	35	35	21	7	1

Combinations

$$^{n}C_{r} = \frac{n!}{r!\,(n-r)!}$$

$$^{2}C_{0} = \frac{2!}{0!\,(2-0)!} = \boxed{1}$$

$$^{2}C_{1} = \frac{2!}{1!\,(2-1)!} = \boxed{2}$$

$$^{2}C_{2} = \frac{2!}{2!\,(2-2)!} = \boxed{1}$$

Pascal's triangle

(Row 0) 1

(Row 1) 1 1

(Row 2) $\boxed{1 \quad 2 \quad 1}$

(Row 3) $\boxed{1 \quad 3 \quad 3 \quad 1}$

1 4 6 4 1

1 5 10 10 5 1

1 6 15 20 15 6 1

(Row 7) 1 7 21 35 35 21 7 1

combinations

$$^{n}C_{r} = \frac{n!}{r!\,(n-r)!}$$

$$^{2}C_{0} = \frac{2!}{0!\,(2-0)!} = \boxed{1}$$

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$$^{2}C_{2} = \frac{2!}{2!\,(2-2)!} = \boxed{1}$$

$^{3}C_{0} = \boxed{1}$

$^{3}C_{1} = \boxed{3}$

$^{3}C_{2} = \boxed{3}$

$^{3}C_{3} = \boxed{1}$

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$$^nC_r = \frac{n!}{r!(n-r)!}$$

$^nC_r \rightarrow$ 'n' items Taken 'r' at a time

$$\boxed{\begin{aligned}(a+b)^2 &= a^2 + 2ab + b^2 \\ (a+b)^3 &= a^3 + 3a^2b^1 + 3a^1b^2 + b^3\end{aligned}}$$

Power of 'a' decreases
Power of 'b' increases

$$(a+b)^n = {}^nC_0 + {}^nC_1 + {}^nC_2 + {}^nC_3 + \dots + {}^nC_n$$

$$(a+b)^n = {}^nC_0\, a^n + {}^nC_1\, a^{n-1} + {}^nC_2\, a^{n-2} + {}^nC_3\, a^{n-3} + \dots + {}^nC_n\, a^0$$

$$(a+b)^n = {}^nC_0\, a^n b^0 + {}^nC_1\, a^{n-1} b^1 + {}^nC_2\, a^{n-2} b^2 + {}^nC_3\, a^{n-3} b^3 + \cdots + {}^nC_n\, a^0 b^n$$

∴ General problem

$$(a+b)^n = {}^nC_0 a^n + {}^nC_1 a^{n-1} b + {}^nC_2 a^{n-2} b^2 + {}^nC_3 a^{n-3} b^3 + \cdots + {}^nC_n b^n a^0$$

Expand $(x+2)^4$

$= 1 \cdot x^4 2^0 + 4x^3 2^1 + 6x^2 2^2 + 4x^1 2^3 + 1 x^0 2^4$

$$\boxed{(x+2)^4 = x^4 + 8x^3 + 24x^2 + 32x + 16}$$