Recursion Trees

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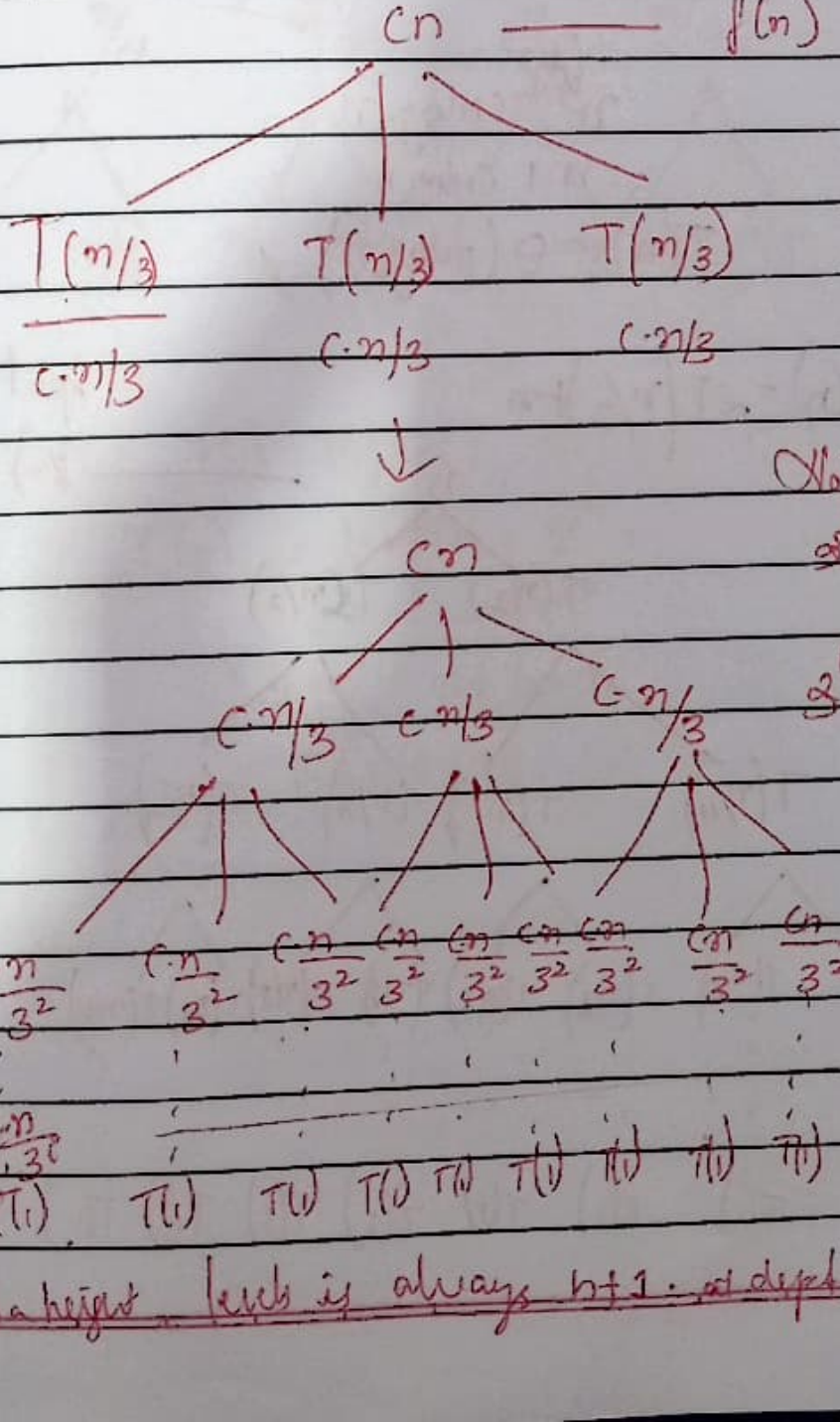
$$T(n) = 3T(n/3) + cn$$

$$T(n/3) = 3T(n/3^2) + c \cdot n/3$$

$$T(n/9) = 3T(n/3^3) + c \cdot n/3^2$$



$3^h \times T(1)$

Total cost $= cn + cn + cn$

$$\therefore T(n) = 3^h \times T(1) + \sum_{i=0}^{h-1} cn$$

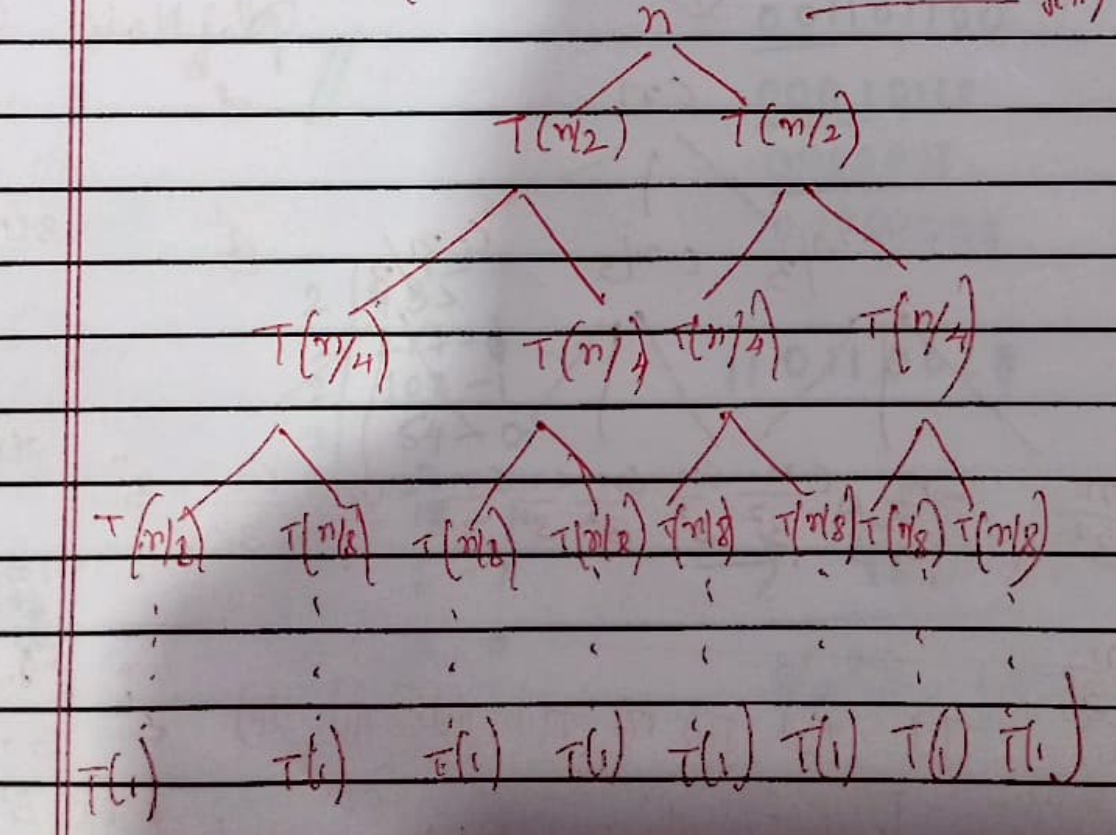
$$= 3^{\log_3 n} \cdot T(1) + cn \sum_{i=0}^{h-1} 1 \quad cn \times h \text{ times}$$

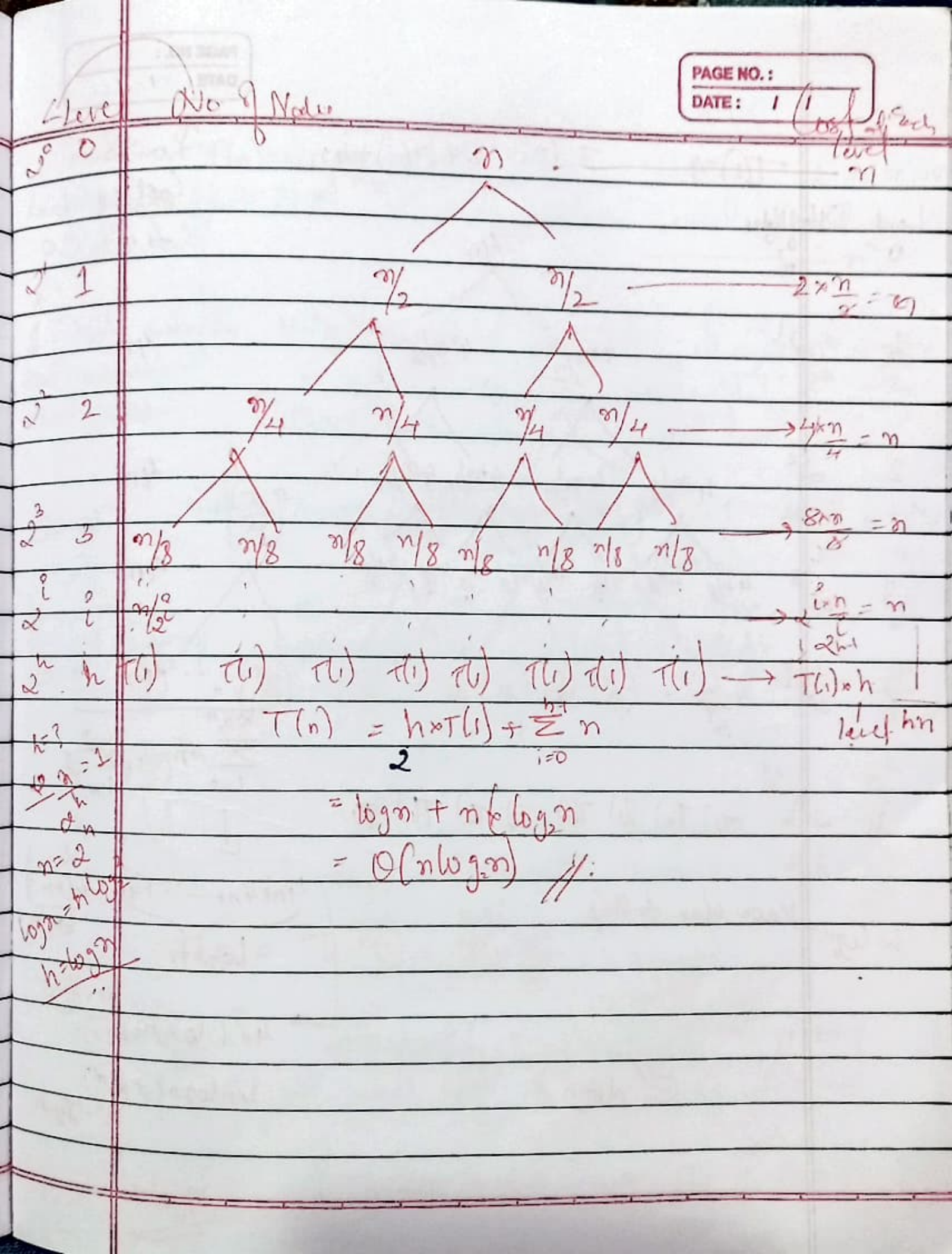
$$= n^{\log_3 3} + cn \log_3 n$$

$$= n + cn \log_3 n$$

$$T(n) = \Theta(n \log_3 n) \quad //$$

$\rightarrow T(n) = 2T(n/2) + n$ — root $f(n)$





$$T(n) = h \times T(1) + \sum_{i=0}^{h-1} n$$

$$= \log n + n \log_2 n$$

$$= O(n \log_2 n) \quad //$$

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$T(1) = 4$ $\quad T(n) = 2T(n/2) + 4n$

Level	No. of Nodes	Row Sum Cost
0	2^0	$4n$
1	2^1	$4n$
2	2^2	$4n$
3	2^3	$4n$
.	.	.
.	.	.
i	2^i	$\frac{4 \times n}{2^i}$ + - - - - - + - - - - + $4n$
.	.	.
.	.	.
h	2^h	$T(1)$ $T(1)$ $T(1)$ $T(1)$ $T(1)$ $T(1)$ $T(1)$ $T(1)$

$\log_2 n$

$\sum_{i=0}^{\log n} 4n = 4n \sum_{i=0}^{\log n} 1$

$4n + 4n + \underbrace{\text{---} + 4n}_{\log_2 n + 1} = 4n \underbrace{(1+1+\cdots)}_{\log_2 n + 1}$

$4n(\log_2 n + 1)$

$4n \log_2 n + 4n$

Ignore

$h = \log_2 n$

Know where to stop.

$\therefore O(n \log n)$

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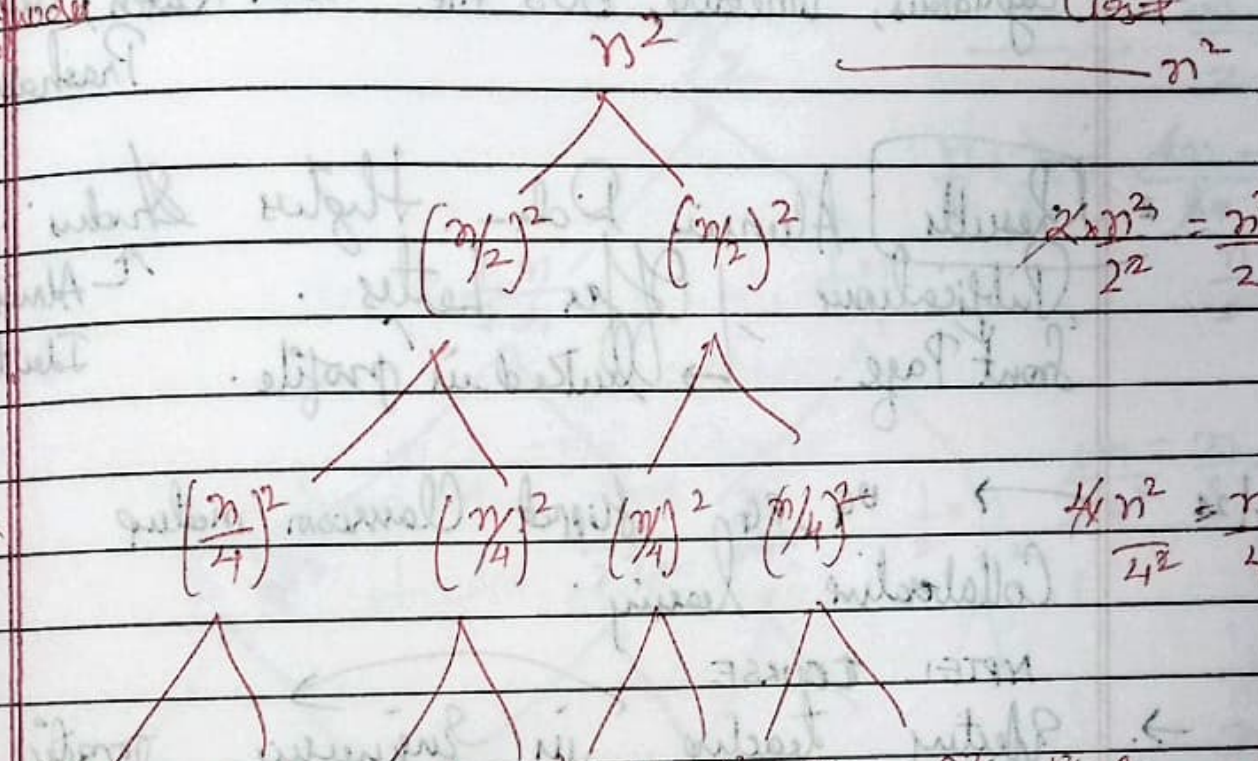
$T(n) = 2T\left(\frac{n}{2}\right) + n^2$



$T(n) = h \cdot T(1) + \sum_{i=0}^{\log_2 n} \frac{n^2}{2^i}$

$\log_2 n \times n^2$

$\log_2 n \times n$

$n \log_2 n$

$= n^2 \sum_{i=0}^{\log_2 n} \frac{1}{2^i}$

$= n^2 \left[\left(\frac{1}{2}\right)^0 + \left(\frac{1}{2}\right)^1 + \left(\frac{1}{2}\right)^2 + \cdots + \left(\frac{1}{2}\right)^{2+1} \right]$

$= n^2 \left[\frac{a}{1-r} \right] = 2n^2$

$\therefore \log n + 2n^2 \Rightarrow T(n) = O(n^2)$ //