Elements of Civil Engineering

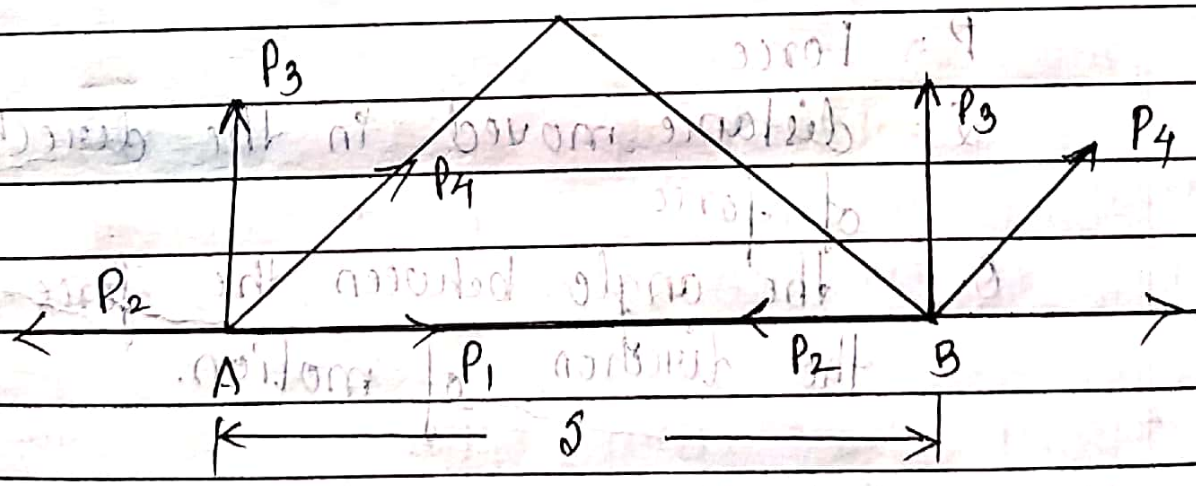
Unit : 5 Engineering Dynamics

I. Work, Power and Energy :

1. Work :

Definition : Work done by a force on a moving body is defined as the product of the force and distance moved in the direction of the force.

★ Interpretation :



Consider a system in which various forces are acting on a particle as shown.
If the particle moves a distance "s" in x-direction, from point A to B, then the work done by various forces are given below;

Force	Work Done = Force x Distance
P_1	$P_1 \times s$
P_2	$P_2 \times -s = -P_2 \times s$
P_3	$P_3 \times 0 = 0$
P_4	$P_4 \cos\theta \times s$.

∴ The general expression for work done by a force 'P' is :

$$\text{Work Done} = P \times s \times \cos\theta$$

where,

P = Force

s = distance moved in the direction of force

θ = the angle between the force and the direction of motion.

* **Alternate Definition :**

Work done by a force may be defined as the product of component of force in the direction of motion and the distance moved.

* Units of Work :

* Work is the product of force and distance. In S.I. system, force is expressed in Newton (N) where as the displacement is expressed in metre (m). Hence the unit of work is N-m. But work done is generally expressed in Joules.

* The work done by a force of 1 N on a body which moves a distance of 1m, is called 1 N-m. This is known as one Joule (J). Hence,
One Joule is the work done by a force of 1 N on a body whose displacement is 1m.

* Other commonly used units are : kJ (or kN-m) or milli Joules (mJ) or (N-mm).

2. Energy :

The capacity of doing work is known as energy.
It exists in many forms i.e., mechanical,

electrical, heat, chemical, light etc.
In engineering mechanics, we are only concerned with mechanical energy.

(A) Mechanical Energy :

It consists of the following two types:
(i) Potential energy.
(ii) Kinetic energy.

(i) Potential Energy :

Potential Energy is also known as position energy or datum energy.
It is the energy by virtue of position of a body with respect to any given reference or datum. It is represented by P.E.
A body of weight 'W' held at a height 'h' possesses an energy 'Wh' as the weight 'W' is capable of doing W × h work if it falls on the ground.

Examples:
i) A compressed spring has potential energy,

because it can do work in recovering its original shape.
ii) Compressed air also possesses potential energy because it is capable of doing work when allowed to expand.

(ii) Kinetic Energy :

The energy possessed by a body by virtue of its velocity (or its motion) is known as kinetic energy. It is represented by K.E.

(*) Expression for Kinetic Energy :

Consider a body of mass 'm' starting from rest. Let it be subjected to an accelerating force 'F' and after covering a distance S, its velocity becomes v.

Initial velocity, $u = 0$.

Work done on the body $=$ Force $\times$ Distance

$$= F \times S \longrightarrow (i)$$

But Force $=$ Mass $\times$ Acceleration

i.e $F = ma$

Substituting the value of F in eqn (i)

$$\text{Work done} = m \times a \times s$$
$$= m \times (as) \longrightarrow \text{(ii)}.$$

But from equation of motion, we have

$$v^2 - u^2 = 2as \qquad (\because u = 0)$$
$$v^2 - 0 = 2as$$
$$as = \frac{v^2}{2}$$

Substituting the value of as in eqn (ii)

$$\text{Work done} = m \times \frac{v^2}{2}$$

But work done on the body is equal to K.E. possessed by the body.

$$K.E = \frac{1}{2} m \cdot v^2.$$

3. Power:

The rate of doing work is known as power. Hence, power can be obtained by dividing the total work done by time.

In other words, power is the work done per second.

The S.I. unit of power is N.m/s or Watt. It is denoted by W.

One Watt is defined as one Joule of work done in one second.

In practice, kilowatt is commonly used unit which is equal to 1000 Watt.

Horse power is the unit in MKS system

1 metric H.P = 735.75 Watts.

Power = Work done per second.

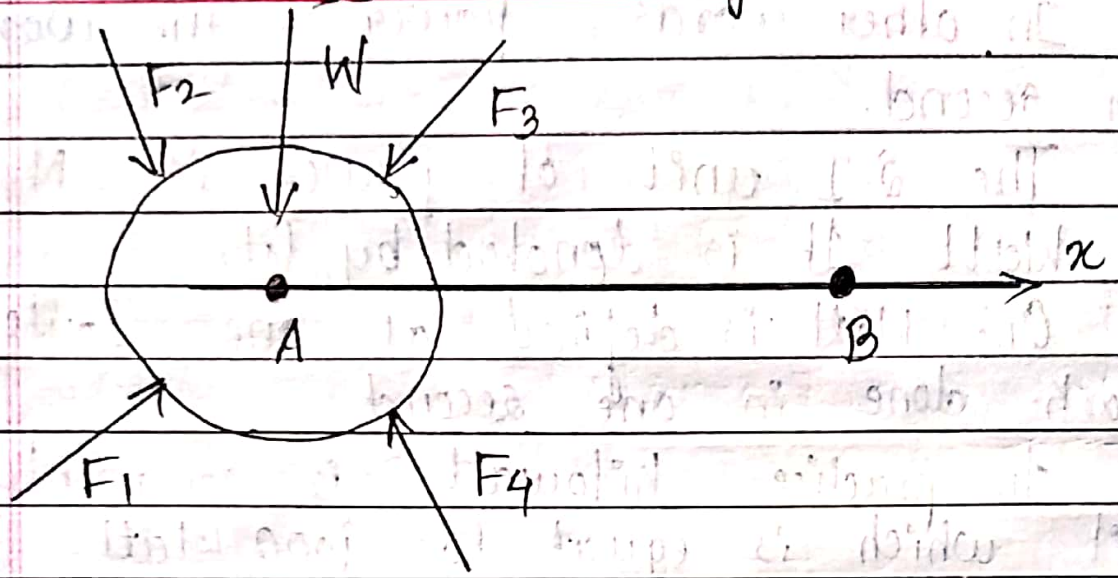
$= \frac{\text{Force} \times \text{Distance}}{\text{Time}}$

$= F \times \frac{\text{Distance}}{\text{Time}}$

= Force x Velocity

i.e The force and velocity should be in the same direction.

4. Work Energy Equation for Translation :



Consider a body as shown in the figure. subject to a system of forces. F_1, F_2..... and moving with an acceleration 'a' in x- direction. Let its initial velocity at A be 'u' and final velocity when it moves a distance AB = S be v. Then the resultant of the system of forces must be in x- direction.

From Newton's second law of motion,

$$F = ma \longrightarrow (i)$$

F = Resultant of all forces acting on a body.

m = Mass of the body.

$$WD = FKE - IKE$$

$$F \times S = \frac{1}{2}mv^2 - \frac{1}{2}mu^2$$

a = Acceleration in the direction of the resultant force.

$$a = v.\frac{dv}{ds}$$

Substituting the value of 'a' in eqn (i), we get.

$$F = m \times v.\frac{dv}{ds}$$

or

$$F.ds = m \times v.dv \longrightarrow (ii)$$

But F. ds is the work done by the resultant force F in displacing the body by a small distance ds. The total work done by the resultant force F in displacing the body by a distance S is obtained by integrating the above equation (ii).

Hence integrating equation (ii) on both sides, we get.

$$\int_0^S F \cdot ds = \int_u^V m \times v \times dv$$

where u is the initial velocity when distance is zero and v is the final velocity when distance is S.

$$F \cdot S = m \left[\frac{v^2}{2} \right]_u^V$$

$$= \frac{m}{2} (v^2 - u^2)$$

$$= \frac{mv^2}{2} - \frac{mu^2}{2}$$

or

Work done by resultant force

= change in kinetic energy.

Note:

Acceleration $a = \frac{dv}{dt}$

But $v = \frac{ds}{dt}$

Hence $a = \frac{dv}{dt}$

$$= \frac{dv}{ds} \cdot \frac{ds}{dt}$$

$$= \frac{dv}{ds} \cdot V$$

$$a = \frac{dv}{dt} = \frac{dv}{ds} \times \frac{ds}{dt} = \frac{dv}{ds} \times V$$

or

$a = V \cdot \frac{dv}{ds}$ this expression is used in work-energy principle.

★ Work Energy Principle Statement:

Work energy principle may be stated as the work done by a system of forces acting on a body during a displacement is equal to change in kinetic energy of the body during the same displacement.

Numericals:

(1.) Find the work done in drawing a body:

(i) Weighing 500 N through a distance of 5 m along a horizontal surface by a horizontal force of 250 N.

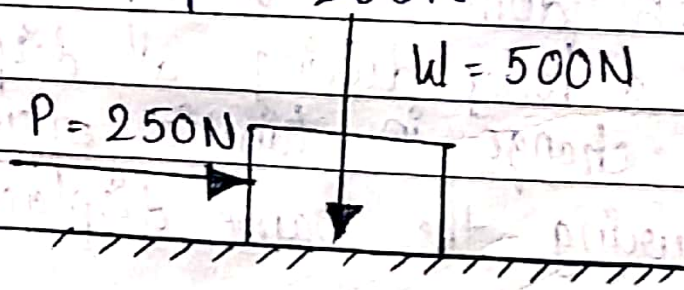
(ii) Weighing 500 N through a distance of 5 m along a horizontal surface by a force of 200 N whose line of action makes an angle of 30° with the horizontal.

Soln: Given:

Weight $W = 500\ N$

Distance $S = 5m$

Force $P = 250N$

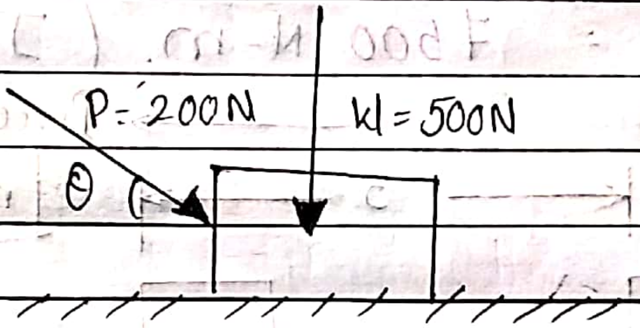


(a)

i) The forces acting on the body are shown in figure (a).

Work done = Force x Distance
$= 250 \times 5$
$= 1250$ N-m. (J)

(ii) $W = 500N$, $P = 200N$, $\theta = 30^\circ$



Work done = F x distance
$= P\cos\theta \times 5$
$= (200 \cos 30^\circ \times 5)$
$= 866$ N-m. (J)

(2) A body of weight 1500 N moves on a level horizontal road for a distance of 500 m. The resistance of the road is 10 N per 1000 N weight of the body. Find the work done on the body by the resistance.

Soln: Given: Weight, $W = 1500$ N
Distance $S = 500$ m
Resistance force, $P = 10$ N per 1000 N weight
$= 10/1000 \times 1500 = 15$ N

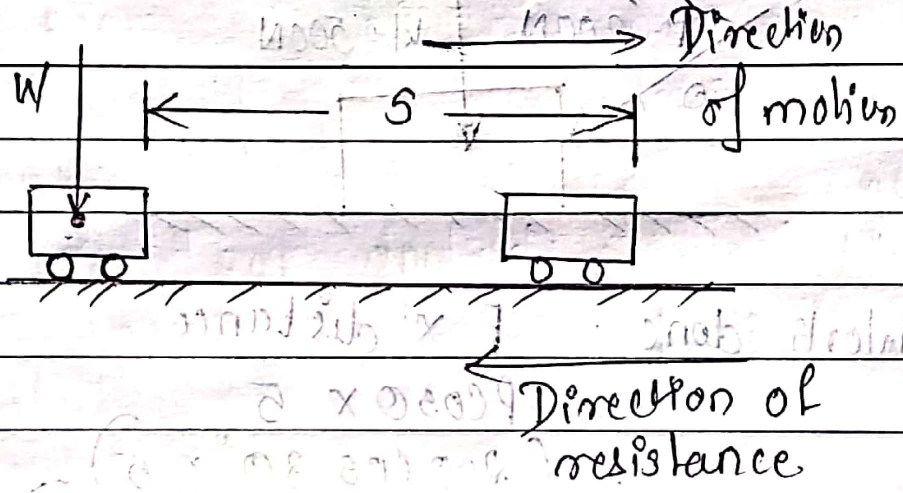
Work done by resistance

= Resistance force × distance

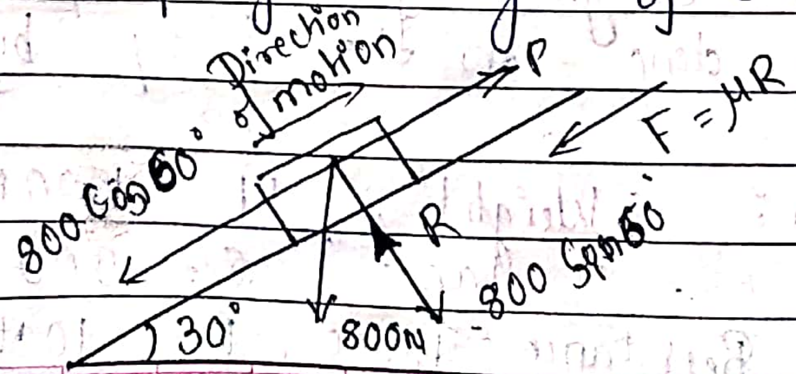
= $P \times S$

= 15×500

= 7500 N-m. (J)



(3) A block of wood of weight 800 N is placed on a smooth inclined plane which makes an angle of 30° with the horizontal. Find the work done in pulling the block up for a length of 5 m.



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Soln: Given :

Weight $W = 800\,N$

Angle $\theta = 30^\circ$ or 60° with x-axis

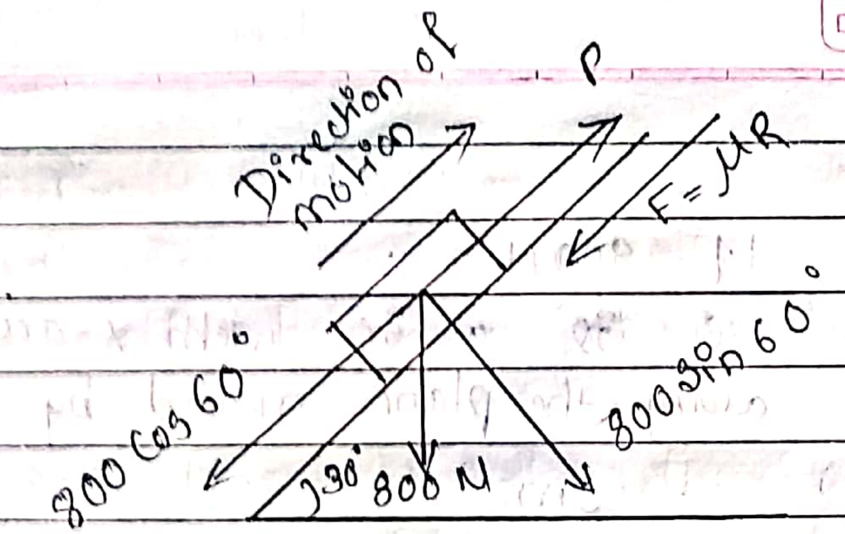
Distance along the plane moved by block
$= 5m$.

As the inclined surface is smooth, the frictional force is zero.

$$P = W\cos\theta$$
$$= 800\cos 60^\circ$$
$$= 400N.$$

Work done in pulling the block up
$= P \times$ distance moved
$= 400 \times 5$
$= 2000$ N-m (J)

(4) If in problem (3), the inclined plane is rough and the coefficient of friction between the block and plane is 0.3, then find the work done in pulling the block up for a length of 5m.



Soln: W = 800N , $\theta = 60^\circ$, $s = 5m$

Coefficient of friction $\mu = 0.3$.

Let P = Force applied so that the block starts moving up the plane.

For equilibrium of the block,

$R = 800 \sin 60^\circ$

$= 692.82$ N.

Frictional force $F = \mu R$

$= 0.3 \times 692.82$

$F = 207.846$ N

Consider forces along the plane,

$P = 800 \cos 60 + F$

$= 400 + 207.846$

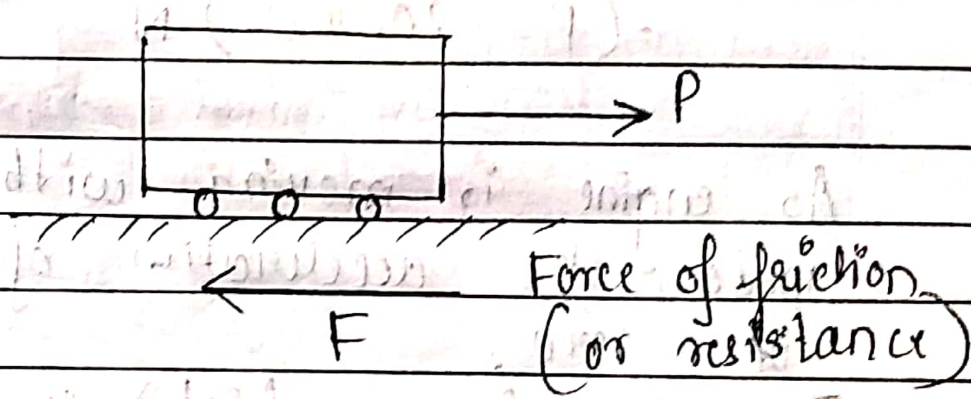
$= 607.846$ N.

Work done $= P \times S$

$= 607.846 \times 5$

$= 3039.23$ N-m (J)

(5) A train of weight 2000 kN is pulled by an engine on a level track at a constant speed of 36 kmph. The resistance due to friction is 10 N per kN of trains weight. Find the power of the engine.



Soln: Given :

Weight of train, $W = 2000\ kN$

Speed of train, $v = 36\ km/hr$

$= \dfrac{36 \times 1000}{60 \times 60}$

$= 10\ m/s$

Resistance due to friction

$F = 10\ N$ per $10\ kN$ weigh of train

$= 10 \times 2000 = 20,000N$

Let
P = Force created by the engine to pull the train at constant speed.

Net force in the direction of motion
$= P - F$
$= (P - 20,000)$ N.

As engine is moving with uniform speed, the acceleration of engine will be zero.
The net force which is equal to mass x acceleration = 0.
$P - 20,000 = 0$
$P = 20,000$ N.

Power = Work done per second
$= \frac{F \times \text{Distance}}{\text{Time}}$
$= F \times \frac{\text{Distance}}{\text{Time}}$
$= F \times \text{velocity}$
$= 20,000 \times 10$ Nm/s
Power = 200 KW.

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(6) If in problem (5), the train is to move with an acceleration of 0.5 m/s^2 on the level track after attaining a speed of 36 km/hr, then find the power of the engine.

Soln: Given:

$W = 2000$ kN

Resistance force $= F = 20,000$ N

velocity $V = 10$ m/s

Acceleration, $a = 0.5$ m/s^2

Now, mass of the train

$$m = \frac{W}{g} = \frac{2000}{9.81}$$

$$= \frac{2000 \times 1000}{9.81}$$

Let P = force exerted by engine, when train is moving with an acceleration of 0.5 m/s^2.

Net force = Mass x Acceleration

$(P - F) = m \times a$

$$F - 20,000 = \left(\frac{2000 \times 1000}{9.81}\right) \times 0.5$$

$F = 121936.8$ N

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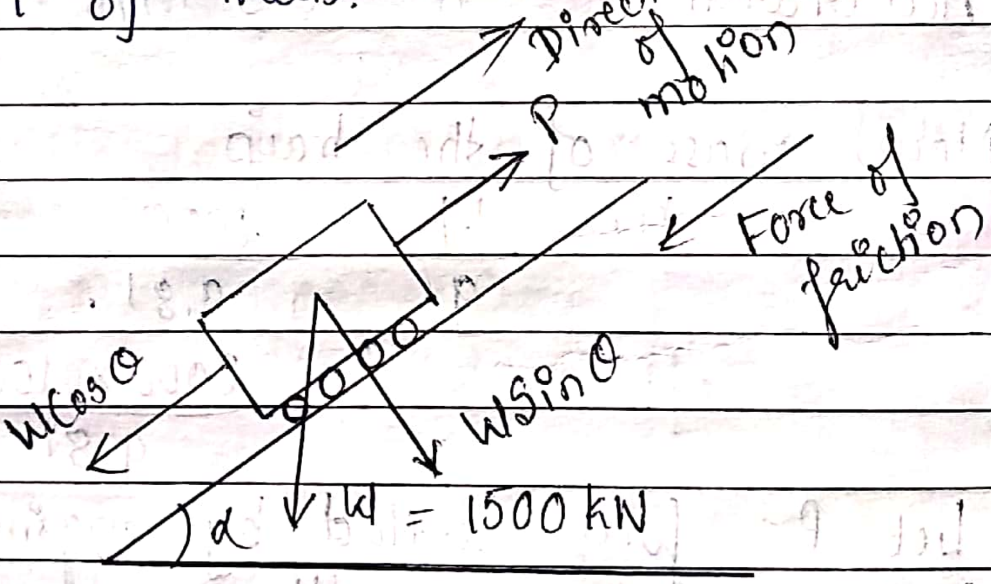
Power = Force exerted by engine × Velocity
$= 121936.8 \times 10$
$= 1219.368$ kW.

(7) A train of weight 1500 kN, is ascending (i.e going upwards) a slope of 1 in 100 with a uniform speed of 36 km/hr. Find the power exerted by the engine, if the road resistance is 5 N per kN weight of train.

Soln:



W = Weight of train. W = 1500 kN
$= 1500 \times 10^3$ N.

Slope α = 1 in 100

$\tan\alpha$ i.e $\frac{1}{100}$ $\alpha = 0.57^\circ$
$\theta = 89.43^\circ$

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Speed $v = 36 \text{ km/hr}$

$= \frac{36 \times 1000}{60 \times 60}$

$= 10 \text{ m/s}.$

Resistance force (or Force of friction)

$F = 5 \text{ N}$ per kN weight of train

$= 5 \times 1500$

$F = 7500 \text{ N}$

Net force $= P - W\cos\theta - F$

$= P - W \times \frac{1}{100} - 7500$

$= P - 1500 \times 1000 \times \frac{1}{100} - 7500$

$= P - 15000 - 7500$

As the train is moving with uniform speed acceleration is zero.

i.e Net force $=$ mass $\times$ acceleration

$P - 15000 - 7500 = \text{mass} \times 0$

$= 0$

$P = 22500 \text{N}$

Power exerted by engine

= Force exerted by engine × velocity

= 22500 × 10

= 225000 Nm/s

= 225 kW.

(9) A body of mass 2 kg is moving with a velocity of 50 m/s. What will be the kinetic energy of the body?

Soln Kinetic energy is given by $\frac{1}{2}mv^2$

$$KE = \frac{1}{2}mv^2$$

$$= \frac{1}{2} \times 2 \times (50)^2 = 2500 \text{ N-m}$$

(12) A body weighing 300 N is pushed up a
30° plane by a 400 N force acting

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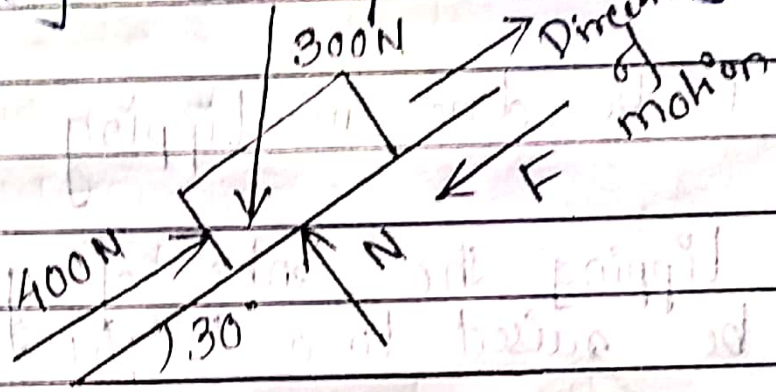
parallel to the plane. If the initial velocity of the body is 1.5 m/s and coefficient of friction $\mu = 0.2$, what velocity will the body have after moving 6m.



Soln: Consider the Free Body Diagram

$N - 300 \sin 60^\circ = 0$
$N = 259.81 N.$

The frictional force

$F = \mu N$
$= 0.2 \times 259.81$
$F = 51.96 N$

Initial velocity $u = 1.5 m/s$
Displacement $s = 6m$

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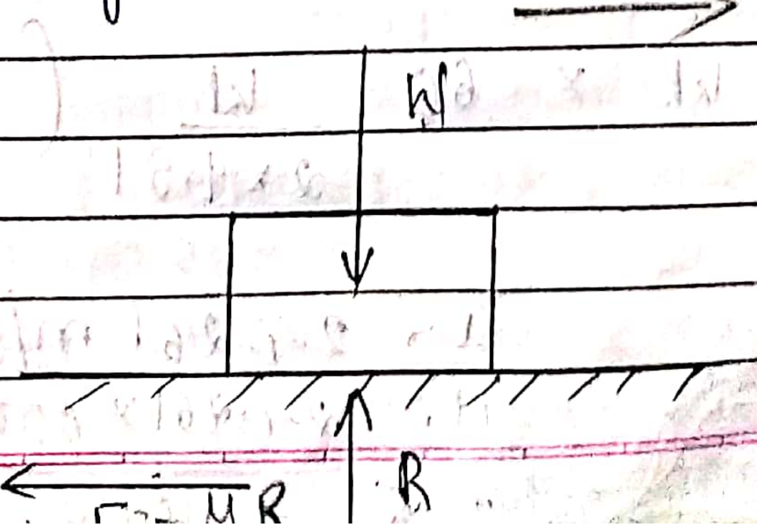
$Ex's = \frac{1}{2}mv^2 - \frac{1}{2}mu^2$

Let the final velocity be 'V' m/s

$$(400 - 51.96 - 300\cos 60^\circ) \times 6 = \frac{1}{2}\left(\frac{300}{9.81}\right)(V^2 - 1.5^2)$$

$$V = 8.942 \text{ m/s}$$

(13) * In a police investigation of tyre marks, it was concluded that a car while in motion along a straight level road skidded for a total of 60 m after the brakes were applied. If the coefficient of friction between the tyres and the pavement estimated as 0.5, what was the probable speed of the car just before the brakes were applied.



$S = 60$ m
$\mu = 0.5$
$V = 0$
$u = ?$

Let the probable speed of the car just before brakes were applied be "u" m/s.

Resolving forces along vertical.

$\Sigma V = 0$

$R - W = 0$

$R = W$

Also, the frictional force

$F = \mu R$

$F = 0.5 W$

Using work energy principle,

$-F \times S = \frac{W}{2g}(v^2 - u^2)$

$-0.5 \times W \times 60 = \frac{W}{2 \times 9.81}(v^2 - u^2)$

$v = 0$

$u = 24.261$ m/s

$u = \frac{24.261 \times 60 \times 60}{1000}$

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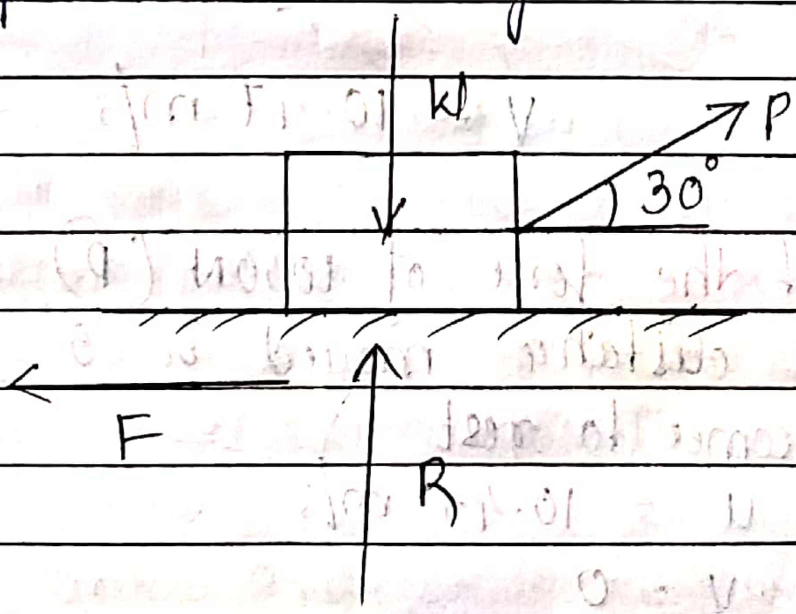
$u = 87.34$ kmph.

(14) A block weighing 2500 N rests on a level horizontal plane for which coefficient of friction is 0.2. This block is pulled by a force of 1000N acting at an angle of 30° to the horizontal. Find the velocity of the block after it moves 30 m starting from rest. If the force of 1000 N is then removed, how much further will it move?

Soln: When pull P is acting.



Resolving forces along vertical,

$\Sigma V = 0$

$-W + R + P\sin 30^\circ = 0$

$R = 2500 - 1000 \times \sin 30^\circ$

$R = 2000N$

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$F = \mu R$
$= 0.2 \times 2000$
$F = 400N$

$u = 0$ — initial velocity
v — final velocity
$s = 30m$
Applying work energy principle for the horizontal motion,

$$(P\cos\theta - F) \times s = \frac{W}{2g}(v^2 - u^2)$$

$$v = 10.47 \ m/s.$$

Now if the force of 1000N (P) is removed, let the distance moved be 's' before the body comes to rest

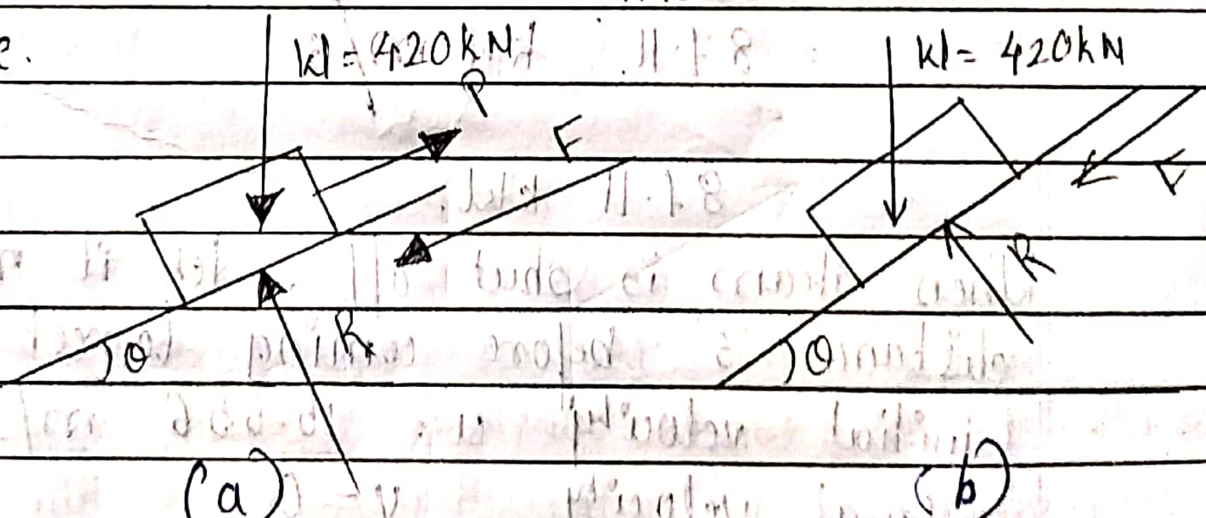
$u = 10.47 \ m/s$
$v = 0$

Using work energy principle,

$$-F \times s = \frac{1}{2} \times \frac{W}{g} \times (v^2 - u^2)$$

$s = 27.96 m$

(15) Find the power of a locomotive, drawing a train whose weight including that of engine is 420 kN up an incline 1 in 120 at a steady speed of 56 kmph, the frictional resistance being 5 N/kN.
While the train is ascending the incline, the steam is shut off. Find how far it will move before coming to rest, assuming that the resistance to motion remains the same.



Soln: $V = 56$ kmph

$$= \frac{56 \times 1000}{60 \times 60}$$

$$= 15.556 \text{ m/s}$$

$$F = 5 \times 420 = 2100 \text{ N} = 2.1 \text{ kN}$$

$P = F + W \cos(90° - \theta)$

$= F + W \sin\theta$

$= 2.1 + 420 \times \frac{1}{120}$

$= 5.6$ kN

Power of locomotive

= Work done by P per second

= P x distance moved per second.

= 5.6×15.556

= 87.11 kN.m/s

= 87.11 kW.

When steam is shut off, let it move a distance 's' before coming to rest

Initial velocity $u = 15.556$ m/s

Final velocity $v = 0$

In figure (b), shows the system of forces acting in this motion. Resultant force parallel to the plane is

$$= F + kl \cos(90^\circ - \theta)$$
$$= F + kl \sin\theta$$
$$= 2.1 + 420 \times \left(\frac{1}{120}\right)$$

$$= 5.6 \text{ kN} \text{ (down the plane)}$$

Using Work - Energy principle for motion up the plane, we have,

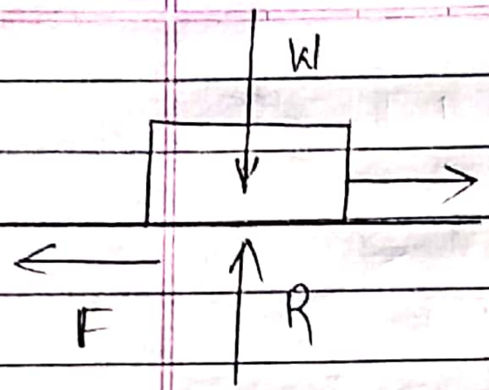
$$-5.6 \times S = \frac{1}{2} \times \frac{420}{9.81} \times (0 - 15.556^2)$$

$$S = 924.98 m.$$

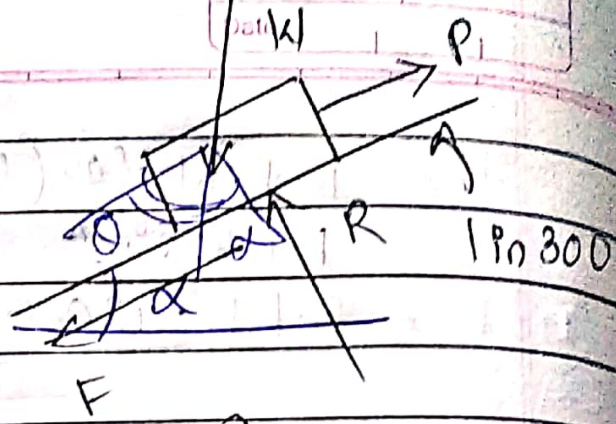
(16) A tram car weighs 120 kN, the tractive resistance being 5 N/kN. What power will be required to propel the car at a uniform speed of 20 kmph?

a) On level surface
b) up an incline of 1 in 300
c) down an inclination of 1 in 300?

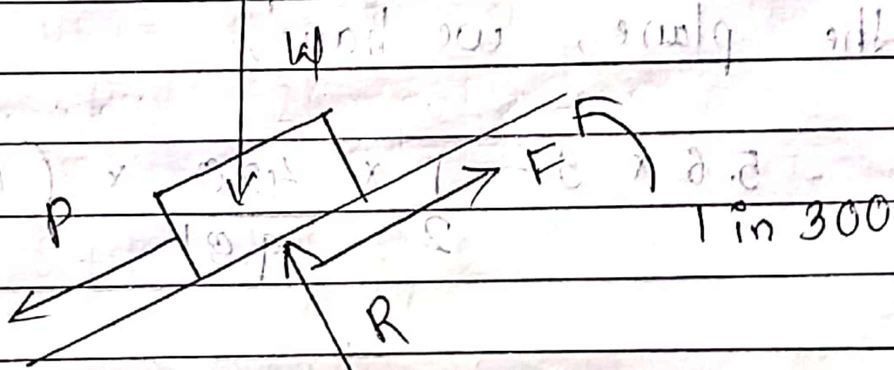
Take efficiency of motor as 80%.



(a)
On level track



(b)
Up the incline



(c) Down the Incline

Figure (a), (b) and (c) show the free body diagram of the locomotive in the three given cases.

In all the cases, the frictional resistance

$F = 5 N/kN = 5 \times 120$

$= 600 N = 0.6 kN$

The locomotive is moving with uniform velocity. Hence it is in equilibrium.

$V = 20 kmph$
$= \frac{20 \times 1000}{60 \times 60}$
$= 5.556\ m/s$

(a) On level track
$P = F = 0.6\ kN$
Output power $=$ Force $\times$ velocity
$= 0.6 \times 5.556$
$= 3.33\ kW$

$\eta = 80\% = 0.80$

Input power $= \frac{\text{Output power}}{\eta}$
$= \frac{3.33}{0.8}$
$= 4.167\ kW.$

(b) Up the plane:
The component of weight $W\cos(90-\theta)$ or $W\sin\theta$ acts down the plane and $\sin\theta \approx \tan\theta$
$= \frac{1}{300}$

$P = F + W \sin\theta$
$= 0.6 + 120 \times \frac{1}{300}$
$= 1 \text{ kN.}$

Output power required
$= P \times \text{velocity}$
$= 1 \times 5.556$
$= 5.506 \text{ kW}$

Input power $= \frac{\text{Output power}}{\eta}$

$= \frac{5.556}{0.8}$
$= 6.94 \text{ kW.}$

(c) Down the incline plane.

$P = F - W\sin\theta$
$= 0.6 - 120 \times \frac{1}{300}$
$P = 0.2 \text{ kN}$
Output power $= P \times \text{velocity}$
$= 0.2 \times 5.556$
$= 1.11 \text{ kW}$

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Input power $= \dfrac{1.11}{0.8}$

$= 1.389$ kW.

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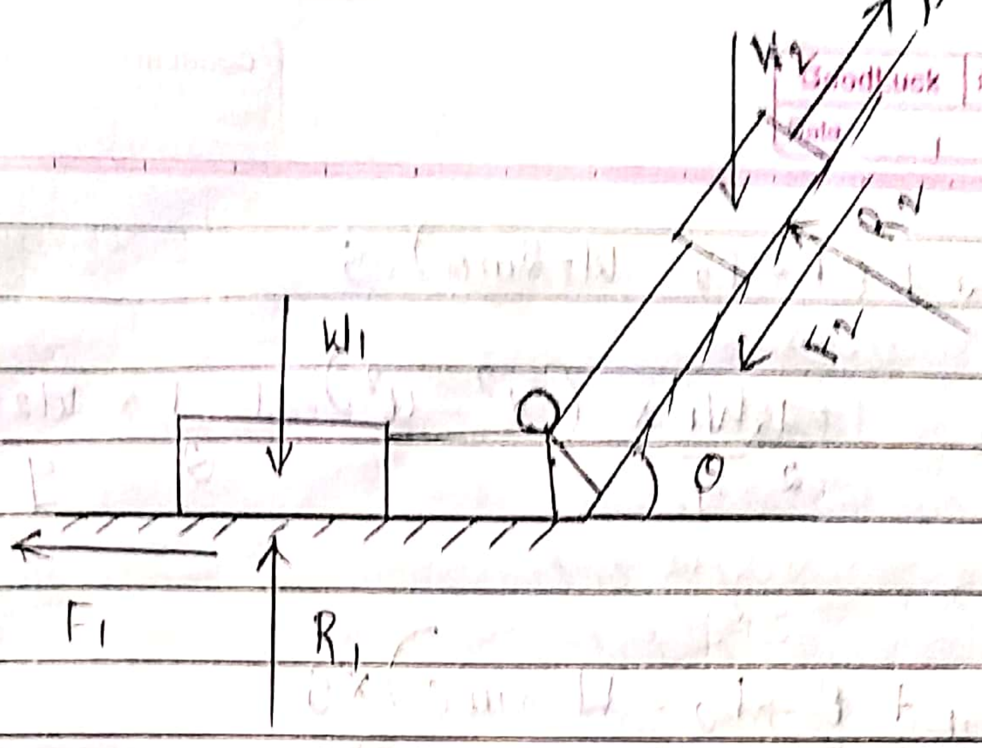
$s = 6m$.

Motion of Connected Bodies:

* Work energy equation may be applied to the connected bodies also.
* There is no need to separate connected bodies and work out for forces in the connecting member.
* Note down the various forces acting on connected bodies.
* Equate summation of work done by forces acting on bodies to the summation of change in kinetic energy of the bodies.
* While writing down work done note that force components in the direction of motion are to be multiplied by the distance moved.

Consider the two connected bodies shown in figure.



* Under the pull P, both bodies move the same distance and with the same velocity.
* Hence, initial velocity, final velocity and displacement are the same for the two bodies.
* Out of the three forces W_1, R_1 and F_1 acting on first body on the frictional force F_1 will do the work.
* Among the various forces acting on the second body, the applied force P, frictional force F_2 and down the plane component of weight, $W_2 \sin\theta$ (i.e $W_2 \cos(90^\circ-\theta)$) will do the work.

Hence the work energy equation for that system will be:

$$-F_1 S + (P - F_2 - W_2 \sin\theta) \cdot S$$

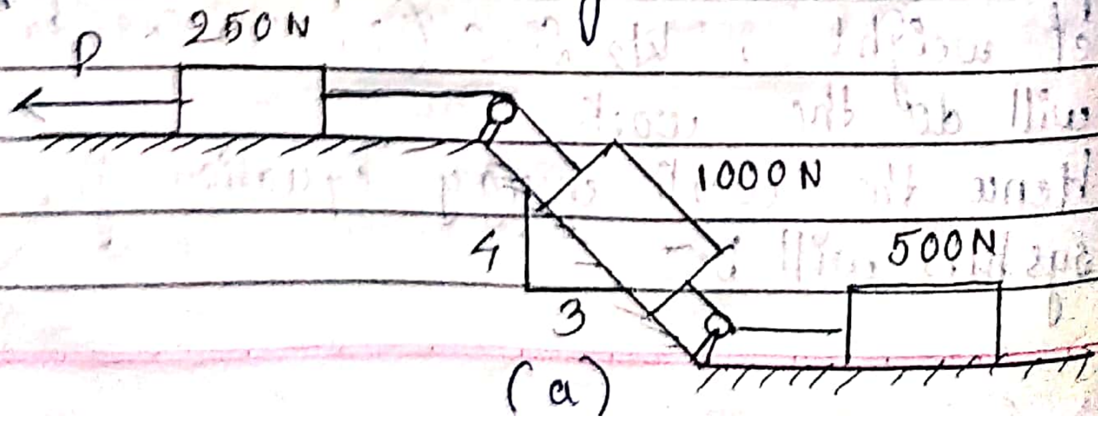
$$= \frac{1}{2} \times \frac{W_1}{g} \times (v^2 - u^2) + \frac{1}{2} \times \frac{W_2}{g} \times (v^2 - u^2)$$

$$(-F_1 + P - F_2 - W_2 \sin\theta) \times S$$

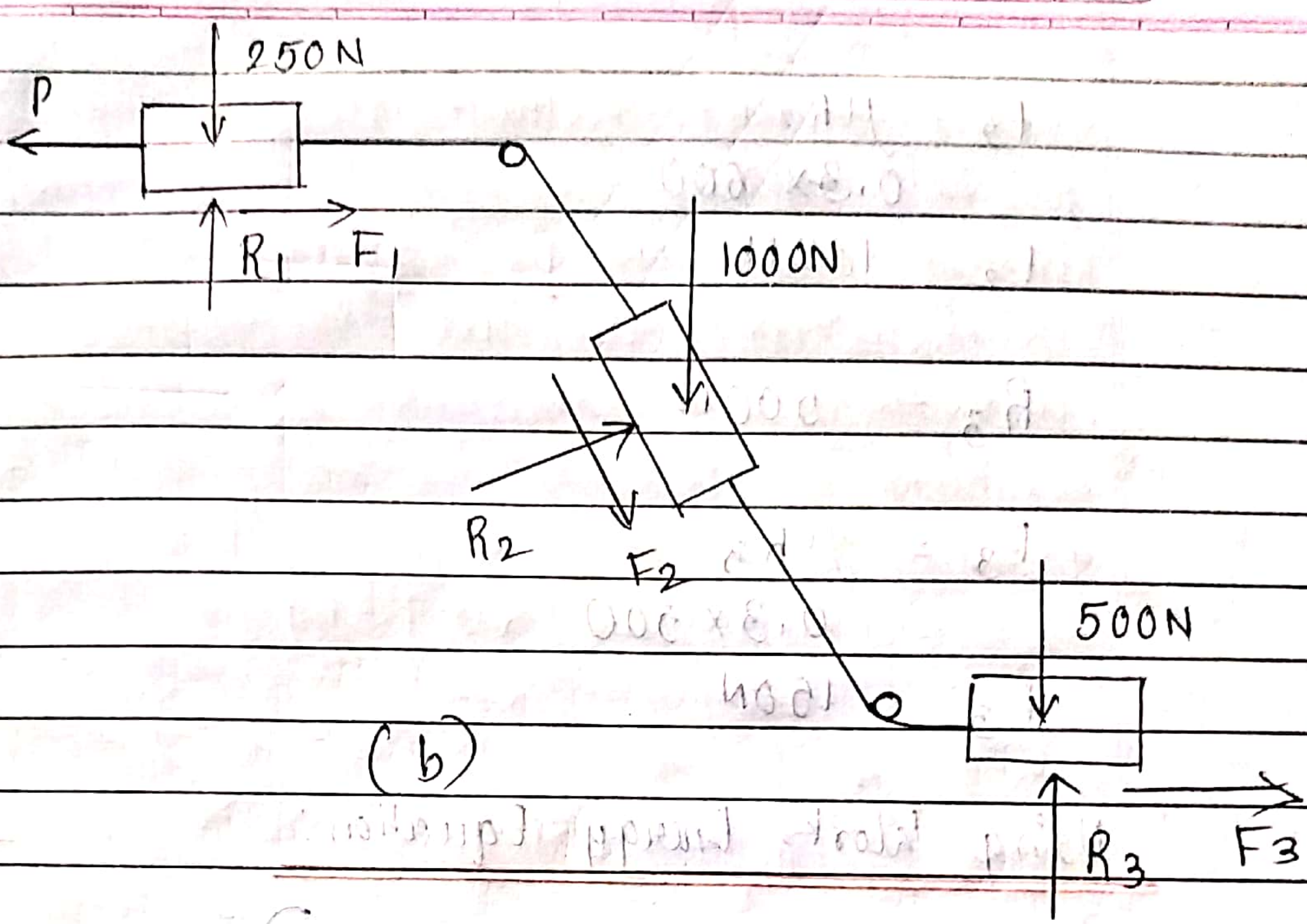
$$= \frac{W_1 + W_2}{2g} (v^2 - u^2)$$

Numericals on connected bodies :

1. Determine the constant force P that will give the system of bodies shown in figure (a), a velocity of 3 m/s after moving 4.5 m from rest. Coefficient of friction between the blocks and the plank is 0.3. Pulleys are smooth.



(a)



(b)

The system of forces acting on connected bodies is shown in figure. b.

$$R_1 = 250\,N$$

$$F_1 = \mu R_1 = 0.3 \times 250$$

$$F_1 = 75\,N$$

$$R_2 = 1000 \sin(90^\circ - \theta)$$

$$= 1000 \cos\theta$$

$$= 1000 \times \frac{3}{5}$$

$$R_2 = 600\,N$$

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$F_2 = \mu R_2$
$= 0.3 \times 600$
$F_2 = 180N$

$R_3 = 500N$

$F_3 = \mu R_3$
$= 0.3 \times 500$
$F_3 = 150N$

Using Work Energy Equation:

$$\left(P - F_1 - F_2 - 1000 \sin\theta - F_3\right) \times 5 = \frac{W_1 + W_2 + W_3}{2g}\left(v^2 - u^2\right)$$

$$\left(P - 75 - 180 - 1000 \times 0.8 - 150\right) \times 4.5 = \frac{250 + 1000 + 500}{2 \times 9.81}\left(3^2 - 0\right)$$

$P = 1383.39N.$

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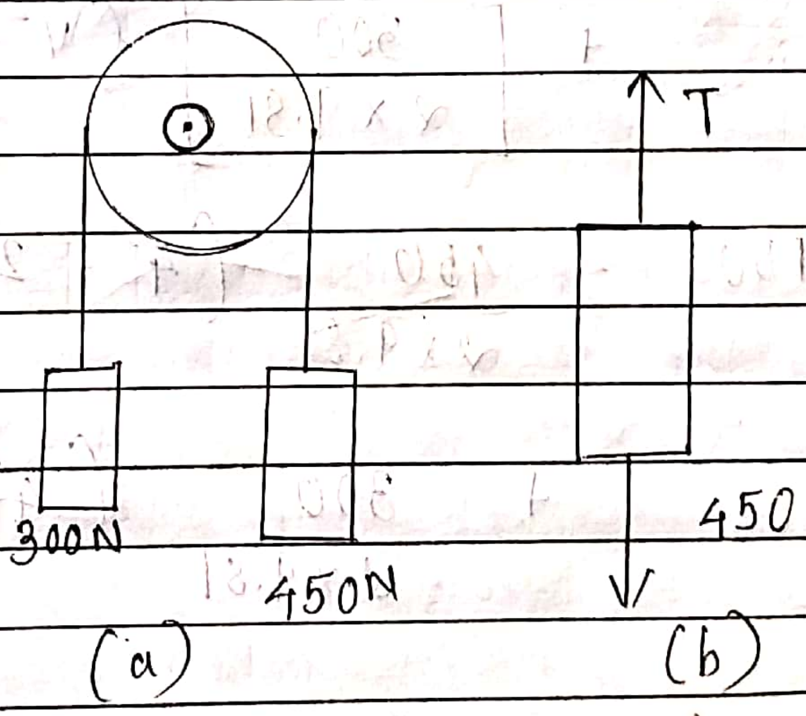
2) Two bodies weighing 300 N and 450 N are hung to the ends of a rope passing over an ideal pulley as shown in figure (a). How much distance the blocks will move in increasing the velocity of system from 2 m/s to 4 m/s? How much is the tension in the string? Use work energy method.

Soln:



450 N block moves down and 300 N block moves up. The arrangement is such that both bodies will be having same velocity and both will move by the same distance.

Let 's' be the distance moved.

Writing work energy equation for the system, we get;

$$450s - 300s = \left[\frac{450}{2 \times 9.81}\right](v^2 - u^2)$$

$$+ \left[\frac{300}{2 \times 9.81}\right](v^2 - u^2)$$

$$150s = \frac{450}{2 \times 9.81}(4^2 - 2^2)$$

$$+ \frac{300}{2 \times 9.81}(4^2 - 2^2)$$

$$s = 3.058 \text{ m.}$$

Let 'T' be the tension in the string.
Consider work energy equation for any one body,

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say 450 N body as shown in figure b.

$$450 s - T s = \left[\frac{450}{2 \times 9.81}\right] (4^2 - 2^2)$$

$$(450 - T) \times 3.058 = \frac{450}{2 \times 9.81} \times 12$$

$$T = 360 N.$$