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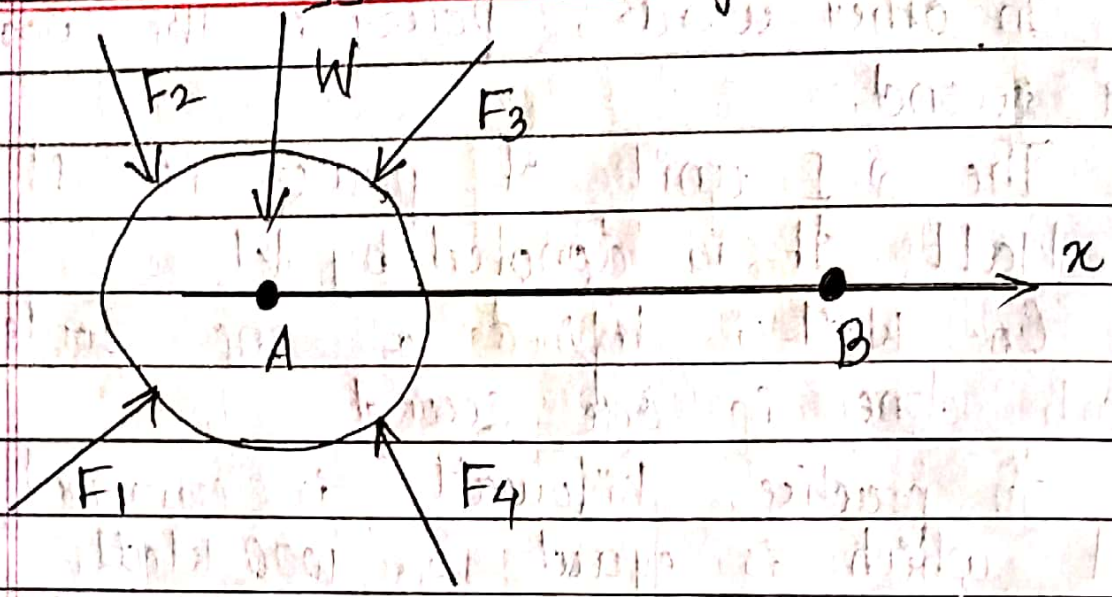
4. Work Energy Equation for Translation :



Consider a body as shown in the figure. subject to a system of forces $F_1, F_2 \ldots$ and moving with an acceleration 'a' in x- direction. Let its initial velocity at A be 'u' and final velocity when it moves a distance AB = S be V. Then the resultant of the system of forces must be in x- direction.

From Newton's second law of motion

$$F = ma \longrightarrow (i)$$

F = Resultant of all forces acting on a body.

m = Mass of the body.

$$WD = FKE - IKE$$

$$F \times S = \frac{1}{2}mv^2 - \frac{1}{2}mu^2$$

a = Acceleration in the direction of the resultant force.

$$a = \frac{v \cdot dv}{ds}$$

Substituting the value of 'a' in eqn (i), we get.

$$F = m \times \frac{v \cdot dv}{ds}$$

or

$$F \cdot ds = m \times v \cdot dv \longrightarrow (ii)$$

But F. ds is the work done by the resultant force F in displacing the body by a small distance ds. The total work done by the resultant force F in displacing the body by a distance S is obtained by integrating the above equation (ii).

Hence integrating equation (ii) on both sides, we get.

$$\int_0^S F \cdot ds = \int_u^v m \times v \times dv$$

where u is the initial velocity when distance is zero and v is the final velocity when distance is S.

$$F \cdot S = m \left[\frac{v^2}{2}\right]_u^v$$

$$= \frac{m}{2}(v^2 - u^2)$$

$$= \frac{mv^2}{2} - \frac{mu^2}{2}$$

or

Work done by resultant force

= change in kinetic energy.

Note:

Acceleration $a = \frac{dv}{dt}$

But $v = \frac{ds}{dt}$

Hence $a = \frac{dv}{dt}$

$= \frac{dv}{ds} \cdot \frac{ds}{dt}$

$= \frac{dv}{ds} \times V$

Or $a = V \frac{dv}{ds}$ this expression is used in work-energy principle.

★ Work Energy Principle Statement:

Work energy principle may be stated as the work done by a system of forces acting on a body during a displacement is equal to change in kinetic energy of the body during the same displacement.