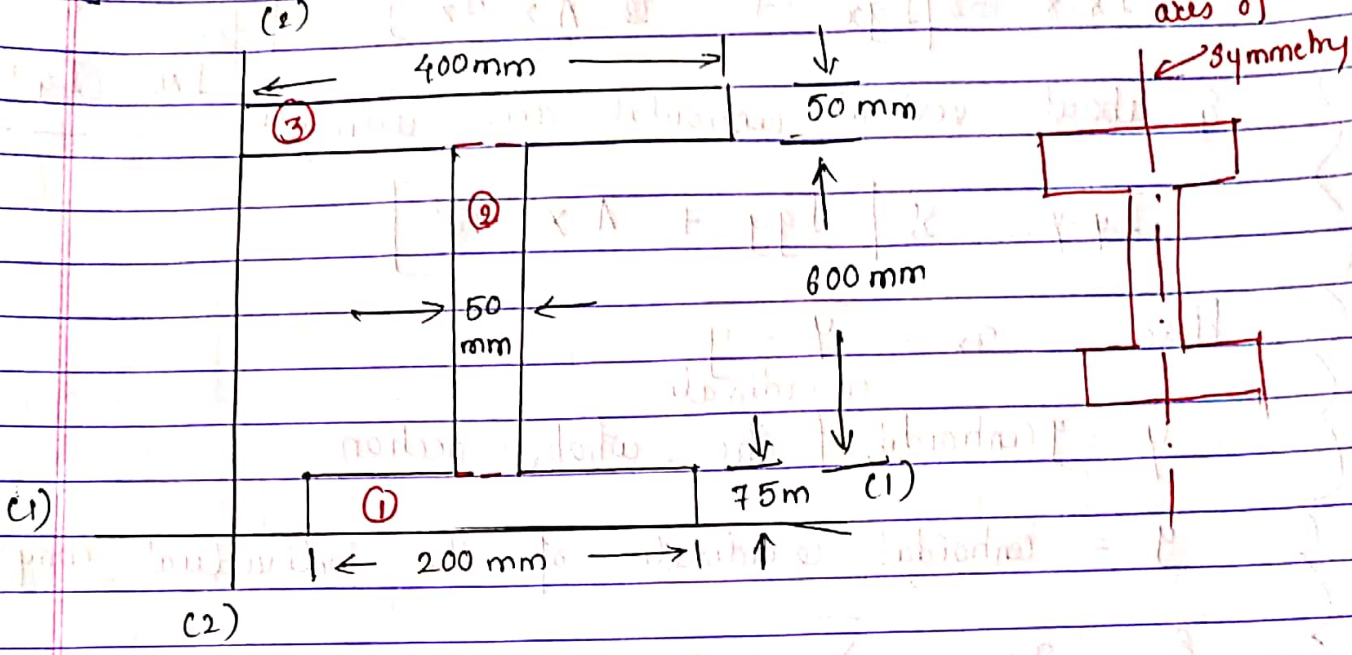
(P1)

Numericals on Moment of Inertia. (M.I.)

1. Find the M.I along the horizontal axis passing through the centroid of the section.



Procedure to find the M.I of the composite sections.

Note: Centroid is to be determined first as M.I. about centroidal axis is asked.

Steps 1: Select the axes of reference. Refer (1)-(1) & (2)-(2) in the above figure.

Step 2: Determine the co-ordinates of the centroid for the given figure using

$$\bar{X} = \frac{\Sigma a x}{\Sigma a} \quad \& \quad \bar{Y} = \frac{\Sigma a y}{\Sigma a}.$$

Step 3: Determine M.I. of each components of the section using the relevant formulae.

In the above figure all '3' components are rectangles use

$$I_{gx} = \frac{bd^3}{12} \quad \& \quad I_{gy} = \frac{db^3}{12}$$

Contd......

Step 4: Use Parallel axes theorem to determine the M.I about the horizontal centroidal axis.

∵ The formula for $I_{AB} = I_G + A \times d^2$

$$I_{x-x} = \Sigma \left[I_{gx} + A \times r_x^2 \right]$$

& about vertical centroidal axis using

$$I_{y-y} = \Sigma \left[I_{gy} + A \times r_y^2 \right]$$

Here. $r_x = \overline{Y} - \bar{y}$

$\overline{Y}$ = y-centroidal co-ordinate of the whole section

$\bar{y}$ = centroidal co-ordinate of the individual component

& $r_y = \overline{X} - \bar{x}$

$\overline{X}$ = x - centroidal co-ordinate of the whole section

$\bar{x}$ = x- centroidal co-ordinate of the individual components.

Soln: Tabulation

Comp.	area 'a'	$\bar{x}$	$\bar{y}$	$a\bar{x}$	$a\bar{y}$	I_{gx}	I_{gy}	$r_x = \overline{Y} - \bar{y}$	$r_y = \overline{X} - \bar{x}$

P.T.O

(P3)

Component	Area 'a' mm^2	$\bar{x}$ distance of centroid from (2)-(2) 'mm'	$\bar{y}$ distance of centroid from (1)-(1) mm	$a\bar{x}$ mm^3	$a\bar{y}$ mm^3	I_{gx} M.I of component about its centroidal X-axis mm^4	I_{gy} M.I of component about its centroidal Y-axis mm^4	r_x $\bar{Y} - \bar{y}$	r_y $= \bar{X} - \bar{x}$
(1)	$200 \times 75 = 15000$	$\frac{400}{2} = 200$ ∵ the section is symmetrical about the vertical axis so $\bar{x}$ will be same for all components	$\frac{d}{2} = \frac{75}{2} = 37.5$	3×10^6	5625×10^3	$\frac{bd^3}{12} = \frac{200 \times 75^3}{12} = 7.031 \times 10^6$	$\frac{db^3}{12} = \frac{75 \times 200^3}{12} = 50 \times 10^6$	$397.08 - 37.5 = 359.58$	$200 - 200 = 0$
(2)	$50 \times 600 = 30 \times 10^3$	$\frac{400}{2} = 200$	$75 + \frac{d}{2} = 75 + \frac{600}{2} = 375$	6×10^6	11.25×10^6	$\frac{bd^3}{12} = \frac{50 \times 600^3}{12} = 9 \times 10^8$	$\frac{db^3}{12} = \frac{600 \times 50^3}{12} = 6.25 \times 10^6$	$397.08 - 375 = 22.08$	$= 200 - 200 = 0$
(3)	$400 \times 50 = 20000$	$\frac{400}{2} = 200$	$675 + d/2 = 700$ $d = 50$	4×10^6	14×10^6	$bd^3/12 = \frac{400 (50)^3}{12}$	$db^3/12 = \frac{50 (400)^3}{12}$		$= 200 - 200 = 0$

b is the dimension parallel to X-axis or horizontal dimension

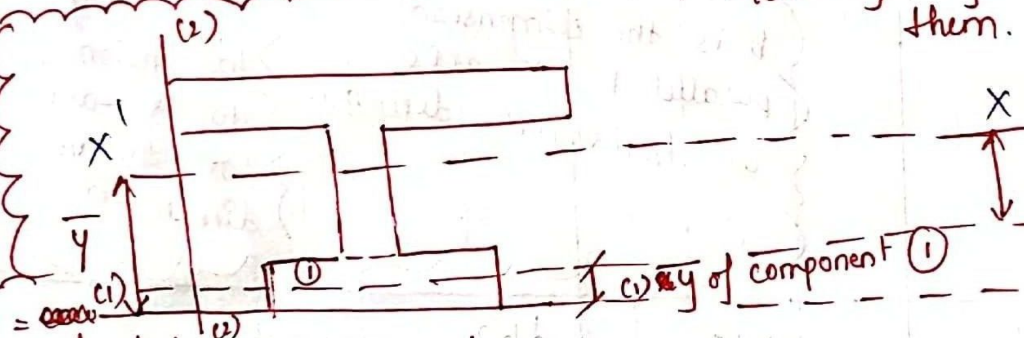
'd' is the dimension ⊥lar to X-axis or the vertical dimension

Comp.	'a'	$\bar{x}$	$\bar{y}$	$a\bar{x}$	$a\bar{y}$	I_{gx}	I_{gy}	$r_x = \bar{Y} - \bar{y}$	$r_y = \bar{X} - \bar{x}$
						4.17×10^6	2.67×10^8	$397.08 - 700 = -302.92$	
	$\Sigma a = 65000$			$\Sigma a\bar{x} = 13 \times 10^6$	$\Sigma a\bar{y} = 25.81 \times 10^6$				

$$\bar{X} = \frac{\Sigma a\bar{x}}{\Sigma a} = \frac{13 \times 10^6}{65000} = 200 \text{ mm}$$

$$\bar{Y} = \frac{\Sigma a\bar{y}}{\Sigma a} = \frac{25.81 \times 10^6}{65000} = 397.08 \text{ mm.}$$

Meaning of r_x & r_y and how to compute them.

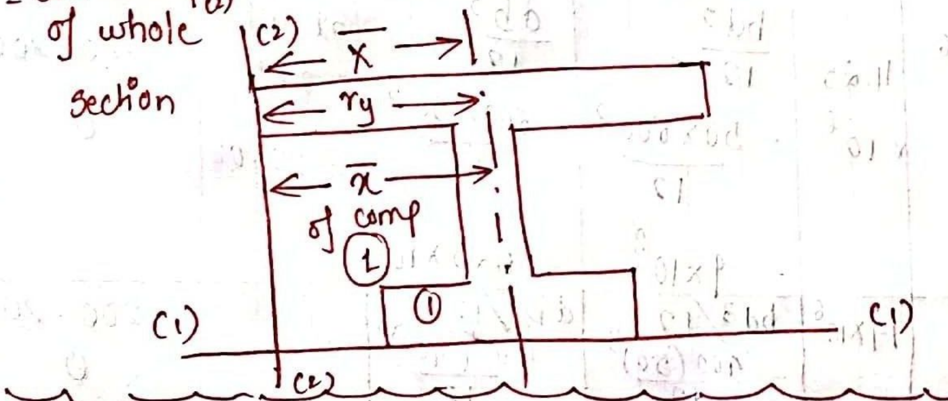


$r_x = \bar{Y} - \bar{y}$

$= 397.08 - 37.5$

$= 359.08$

Here two parallel axes are $X - X'$ & $(1) - (1)$



$r_y = \bar{X} - \bar{x}$

$= 200 - 200$

$= 0$

$\because$ $\bar{X}$ & $\bar{x}$ for each component is same

$r_y = 0$

To compute the M.I of the whole section use Parallel Axes Theorem.

M.I of the section about centroidal X-axis

$$I_{xx} = \Sigma \left[I_{gx} + a \times r_x^2 \right]_i$$

{If there are 3-components use eqn thrice & take summation}

where i = number of component.

$$I_{xx} = \left[I_{gx} + a \times r_x^2 \right]_1 + \left[I_{gx} + a \times r_x^2 \right]_2 + \left[I_{gx} + a \times r_x^2 \right]_3$$

$$I_{xx} = \left[7.031 \times 10^6 + 15000 \times (359.58)^2 \right] + \left[9 \times 10^8 + 30 \times 10^3 \times (22.08)^2 \right] + \left[4.17 \times 10^6 + 20000 \times (-302.92)^2 \right]$$

$$= 1.95 \times 10^9 \ mm^4$$

Similarly I_{yy}

$$I_{yy} = \Sigma \left[I_{gy} + a \times r_y^2 \right]_i \quad \because r_y = 0 \text{ for each component}$$

$$I_{yy} = \left[I_{gy} \right]_1 + \left[I_{gy} \right]_2 + \left[I_{gy} \right]_3$$

$$= 50 \times 10^6 + 6.25 \times 10^6 + 2.67 \times 10^8$$

$$= 3.23 \times 10^8 \ mm^4$$

{Note: I_{y-y} is not asked in the numerical however the above step can be used to compute I_{yy} if M.I about vertical centroidal axis is asked.}