<u>Determining centroid of a composite with a portion cut out</u>

<u>Formulas to remember</u>

- The coordinates of the centroid $(\bar{x}, \bar{y})$ of a composite area are given by
  $\bar{x} = \frac{\sum a_i x_i}{\sum a_i} \quad \bar{y} = \frac{\sum a_i y_i}{\sum a_i}$

<u>Important points to remember and understand</u>

- The coordinates of the centroid $(\bar{x}, \bar{y})$ depend on the reference axes system chosen
- The centroid of rectangular or square area lies at the centre of these figures
- The centroid of a quarter circle lies at a distance of (4R/3π) from the radii encompassing it
- When a portion is cut out from a shape, the cut-out area must be deducted from the total area. To do this, the cut-out area is written with a negative sign (see the solved example below)

<u>Problem statement Q7:</u> Locate the centroid of the given area (Fig.1). The area shown is a square of sides 200mm with a quarter of a circle of radius R = 100mm cut out from the square.

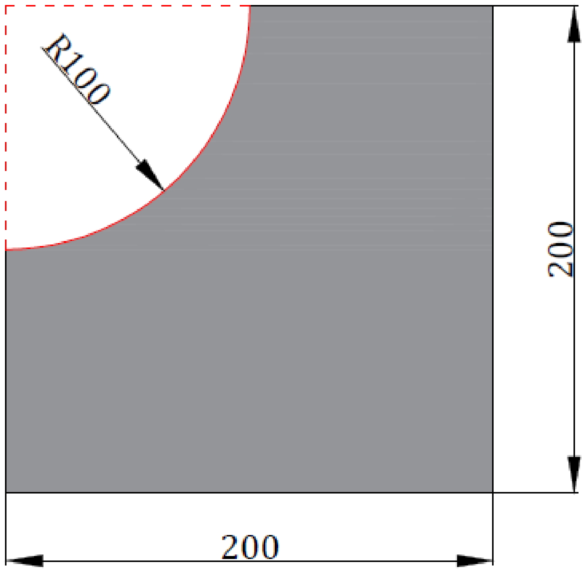

Fig. 1: Given area (all dimensions in mm)

- Given composite area (section) whose centroid is to be located
- Observe that all the necessary dimensions are given
- The area is actually a square. However, a portion (a quarter circle) has been cut-out

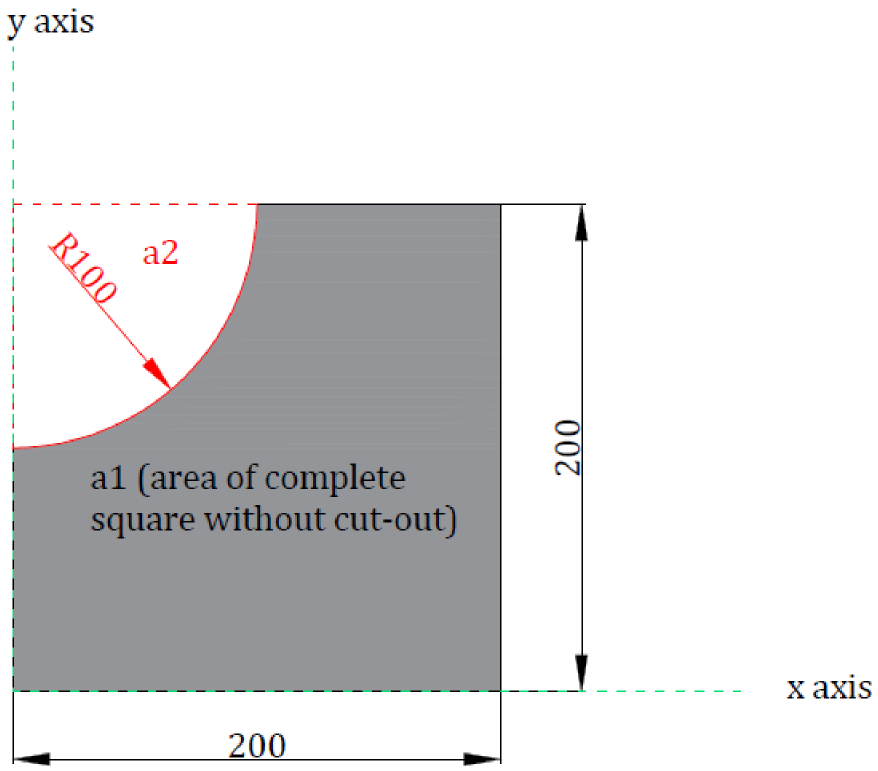

Fig. 2: Given area with reference axes and divided into basic shapes

Step 1 and 2: Selecting reference axes system and dividing the area

- Choose a reference axis system such that all the necessary distances from the axes are known or can be determined
- There could be multiple options of an axes system for the same problem
- One must pick that option that reduces computation
- In the present problem the dashed green lines (Fig. 2) represent the reference axes system

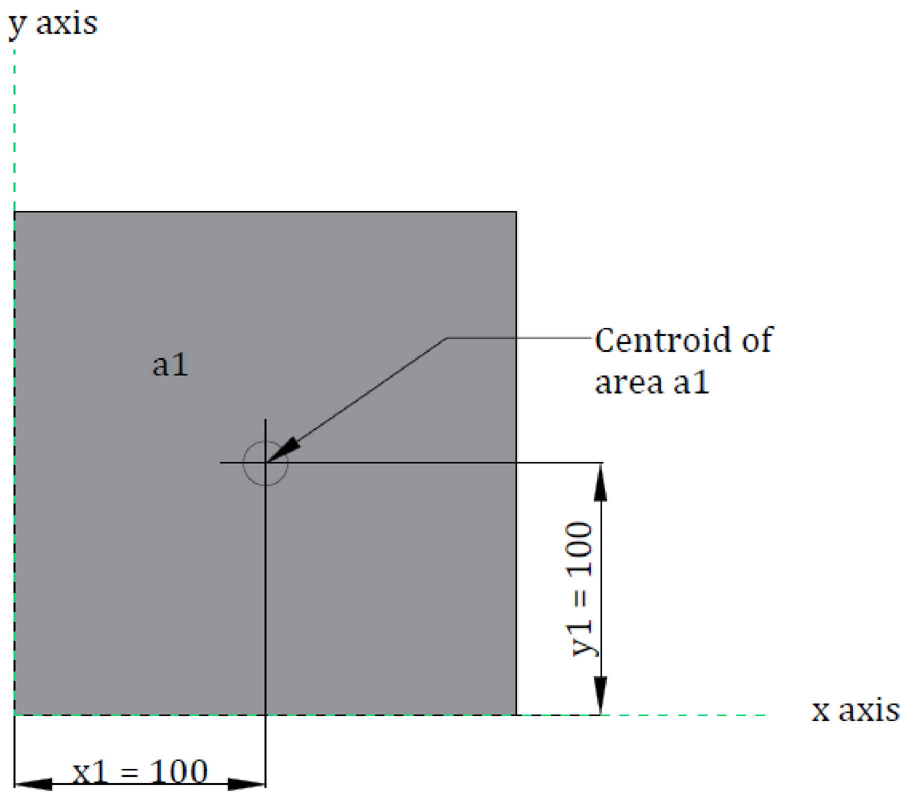

Fig 3: Coordinates (x_1, y_1) of centroid from reference axes of square area a_1 (w/o cut-out)

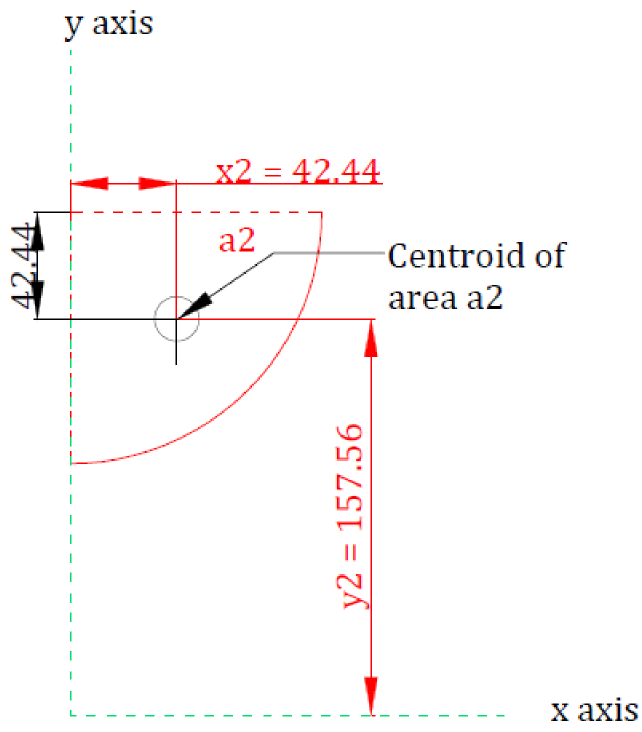

Fig 4: Coordinates (x_2, y_2) of centroid of quarter-circle a_2

Step 3: Centroids of each basic geometric shape

- Determining distances to the centroid of each basic shape with respect to the reference axes system chosen
- In the present problem the centroid of square area a_1 is marked (shown in Fig. 3). It is known that the centroid of a square lies at the geometric centre of the shape.
- From the reference x-axis the centroid of square a_1 is at a distance of y_1 = 100mm and from the reference y-axis the centroid of rectangle a_1 is at a distance of x_1 = 100mm (Fig. 3)
- Therefore, the coordinates of the centroid of area a_1 are (x_1, y_1) = (100, 100) from the reference axes
- Similarly, the coordinates of the centroid of area a_2 are (x_2, y_2) = (42.22, 157.56) from the reference axes

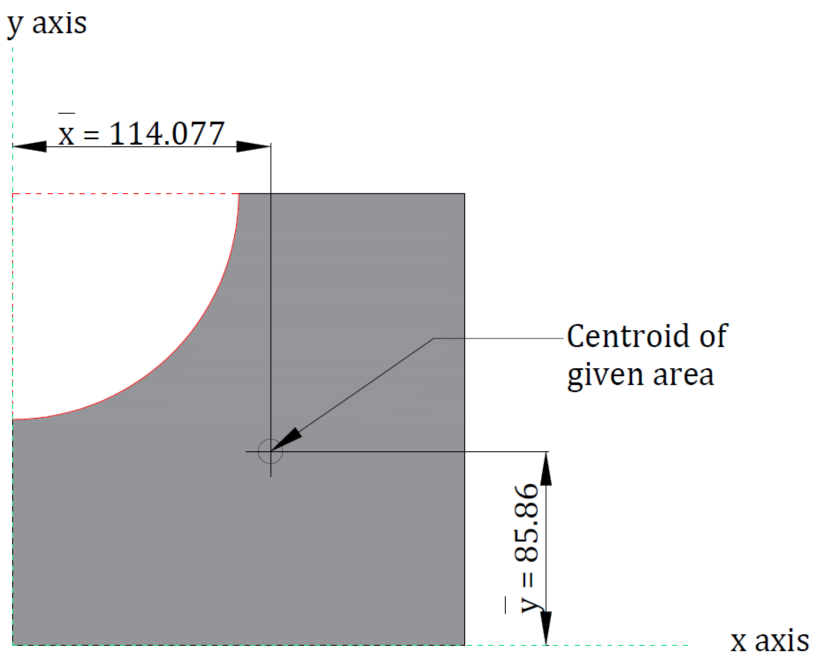

Fig. 5: Coordinates of centroid of the given area

The calculations shown below can be done in tabular form as follows:

Area, a_i	Dist. from y-axis, x_i	Dist. from x-axis, y_i	$a_i \times x_i$	$a_i \times y_i$
$a_1 = 200\times200=40000$	100	100	4×10^6	4×10^6
$a_2 = -(3.142\times100^2)/4= -7855$	42.44	157.56	-3.33×10^5	-1.24×10^6
$\Sigma a_i = 32145$			$\Sigma a_i x_i=3.667\times10^6$	$\Sigma a_i y_i=2.76\times10^6$
$\bar{x} = \frac{(3.667\times10^6)}{32145} = 114.077$			$\bar{y} = \frac{(2.76\times10^6)}{32145} = 85.86$	

Therefore, the coordinated of the centroid of the given T-section is (114.077, 85.86) from the coordinate axes chosen

Step 4: Determining the coordinates of the centroid of the given composite

- The coordinates of the centroid of the entire composite from the reference axes can be obtained by using the formulas
  $\bar{x} = \frac{\sum a_i x_i}{\sum a_i} \quad \bar{y} = \frac{\sum a_i y_i}{\sum a_i}$
- In the present problem, the composite was divided into two basic shapes of area $a_1 = 200 \times 200 = 40000mm^2$ and $a_2 = -(3.142\times100^2)/4=-7855mm^2$. $x_1 = 100mm$, $y_1 = 100mm$, $x_2 = 42.44mm$ and $y_2 = 157.56mm$. Substituting in the above formula, we get,

$$\bar{x} = \frac{\sum a_i x_i}{\sum a_i} = \frac{a_1 x_1 + a_2 x_2}{a_1 + a_2} = \frac{(40000\times100) + (-7855\times42.44)}{40000 + (-7855)} = 114.077mm \quad \text{(Shown in Fig. 5)}$$

$$\bar{y} = \frac{\sum a_i y_i}{\sum a_i} = \frac{a_1 y_1 + a_2 y_2}{a_1 + a_2} = \frac{(40000\times100) + (-7855\times157.56)}{40000 + (-7855)} = 85.86mm \quad \text{(Shown in Fig. 5)}$$