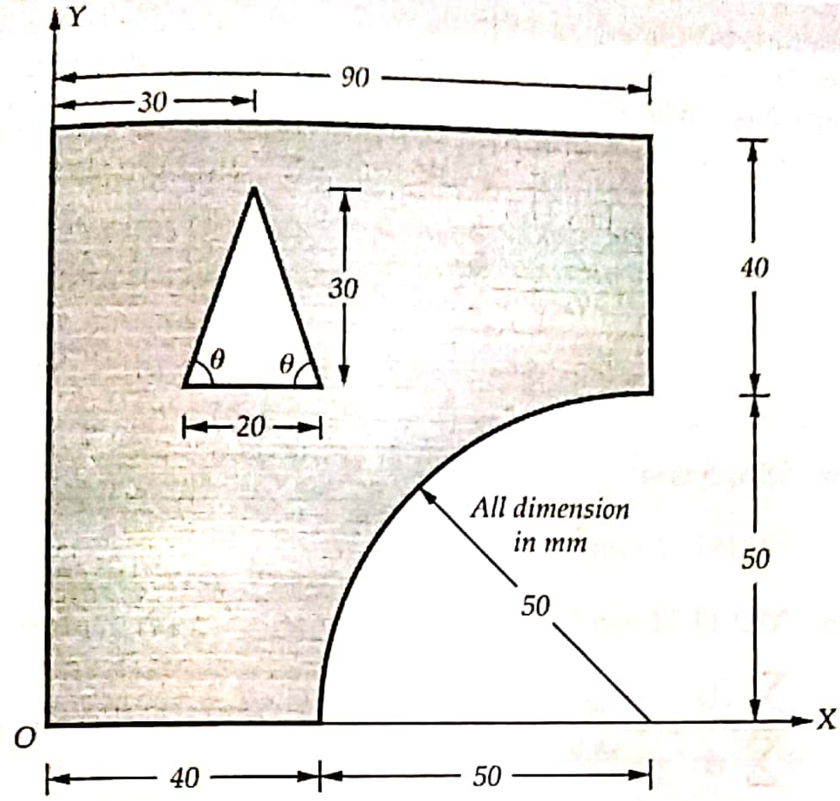

Fig. 8.3.15

Solution : The shaded area can be obtained by removing the quarter circle and the isosceles triangle from the square as shown in Fig. 8.3.15 (a).

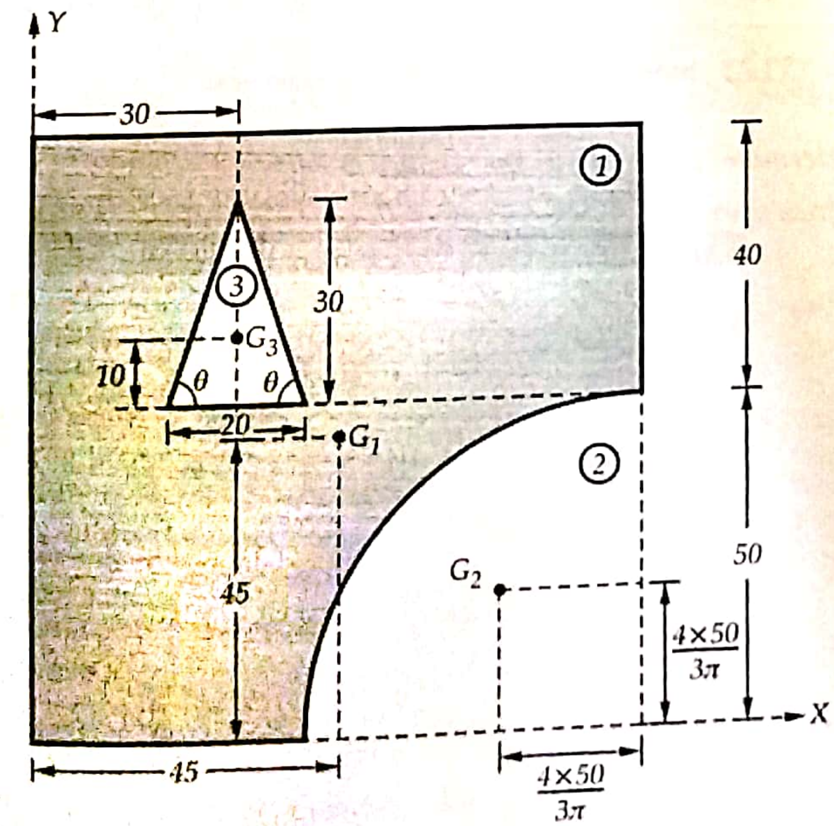

Fig. 8.3.15 (a)

The calculations are tabulated as follows :

Component No.	Component area A (mm^2)	x (mm)	y (mm)
1.	(90) (90)	45	45
2.	$\frac{-\pi \times 50^2}{4}$	$90 - \frac{4 \times 50}{3\pi}$	$\frac{4 \times 50}{3\pi}$
3.	$-\frac{1}{2}$ (20) (30)	30	50+10

$$\sum A = 5836.5 \; mm^2$$

$$\sum Ax = 220452.08 \; mm^3$$

$$\sum Ay = 304833.33 \; mm^3$$

$$\overline{X} = \frac{\sum Ax}{\sum A} = \frac{220452.08}{5836.5}$$

$$\therefore \quad \boxed{\overline{X} = 37.77 \quad mm}$$

$$\overline{Y} = \frac{\sum Ay}{\sum A} = \frac{304833.33}{5836.5}$$

$$\therefore \quad \boxed{\overline{Y} = 52.23 \quad mm}$$

Example 8.3.16 *Determine centroid of the shaded area with reference apex.*

VTU : Aug.-06, Marks 10

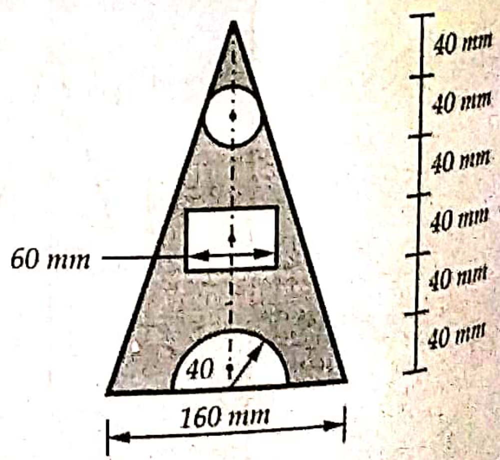

Fig. 8.3.16

Solution : The shaded area can be obtained by removing the circle, rectangle and semi-circle from the triangle. As the area is symmetric about a vertical line passing through the centre, centroid lies on this line of symmetry. We take base of the triangle as reference for writing Y-co-ordinates of centroid.

For triangle, $A_1 = \frac{1}{2}(160)(240)\ mm^2$; $y_1 = \frac{1}{3}(240) = 80\ mm$

For semicircle, $A_2 = -\frac{\pi \times 40^2}{2}\ mm^2$; $y_2 = \frac{4 \times 40}{3\pi}\ mm$

For rectangle, $A_3 = -60 \times 40\ mm^2$; $y_3 = 100\ mm$

For circle, $A_4 = -\pi \times 20^2\ mm^2$; $y_4 = 180\ mm$

$$\bar{Y} = \frac{A_1 y_1 + A_2 y_2 + A_3 y_3 + A_4 y_4}{A_1 + A_2 + A_3 + A_4}$$

$$= \frac{\frac{1}{2}(160)(240)(80) - \frac{\pi \times 40^2}{2} \times \frac{4 \times 40}{3\pi} - 60 \times 40 \times 100 - \pi \times 20^2 \times 180}{\frac{1}{2}(160)(240) - \frac{\pi \times 40^2}{2} - 60 \times 40 - \pi \times 20^2}$$

$\therefore \bar{Y} = 82.02\ mm$ from the base of the triangle.

The distance from apex $= 240 - 82.02 =$ **157.98 mm**

Example 8.3.18 *Determine the position of the centroid of the plane shown in Fig. 8.3.18 with respect to the base.*

VTU : Aug.-07, Marks 13

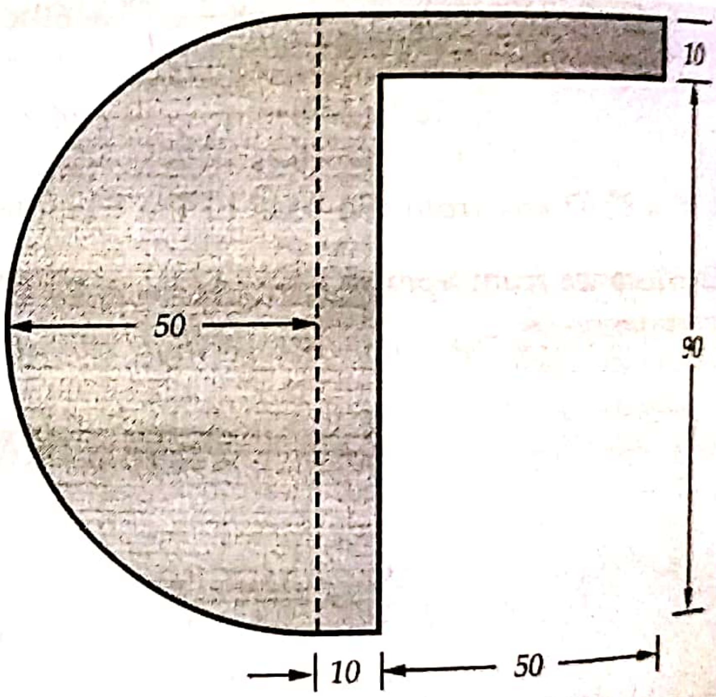

Fig. 8.3.18

Solution : Choose X and Y axis as shown in Fig. 8.3.18 (a) and divide the given area into a semicircle and two rectangles.

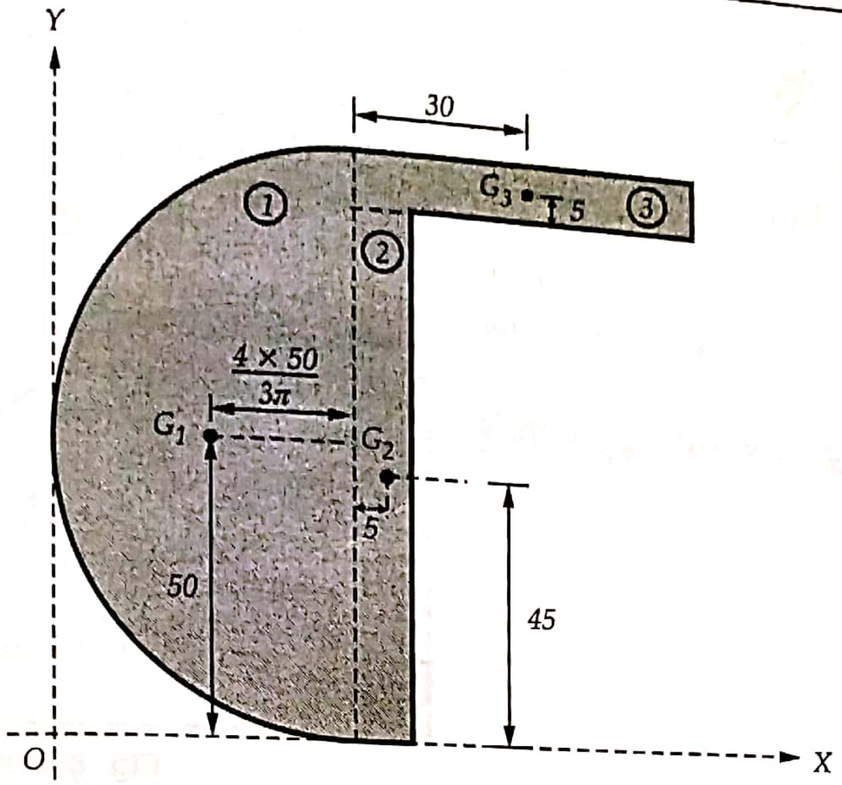

Fig. 8.3.18 (a)

The calculations are tabulated as follows :

Component No.	Component area A	x	y
1.	$\frac{\pi \times 50^2}{2}$	$50 - \frac{4 \times 50}{3\pi}$	50
2.	(10) (90)	55	45
3.	(60) (10)	80	95

$$\sum A = 5426.99$$

$$\sum Ax = 210516.2$$

$$\sum Ay = 293849.54$$

$$\bar{X} = \frac{\sum Ax}{\sum A} = \frac{210516.2}{5426.99}$$

$$\therefore \quad \boxed{\bar{X} = 38.79}$$

$$\bar{Y} = \frac{\sum Ay}{\sum A} = \frac{293849.54}{5426.99}$$

$$\therefore \quad \boxed{\bar{Y} = 54.15}$$

Example 8.3.19 *Determine the position of the centroid for the shaded area with respect to the axes as shown in Fig. 8.3.19.* **VTU : Feb.-08, Marks 10**

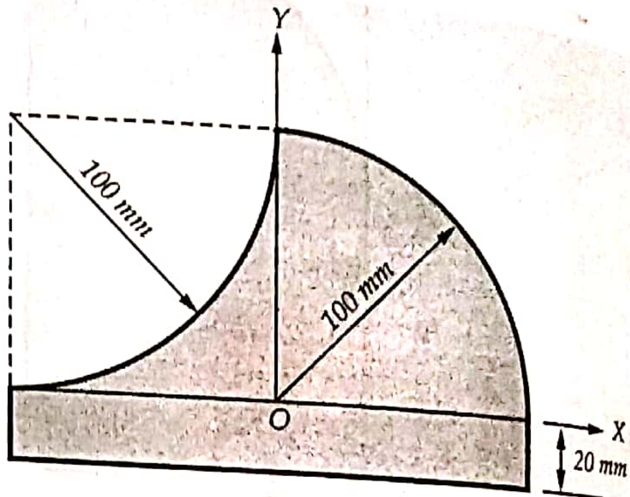

Fig. 8.3.19

Solution : The shaded region can be obtained by adding areas of square, rectangle, quartercircle and then subtracting a quarter circle as shown in Fig. 8.3.19 (a).

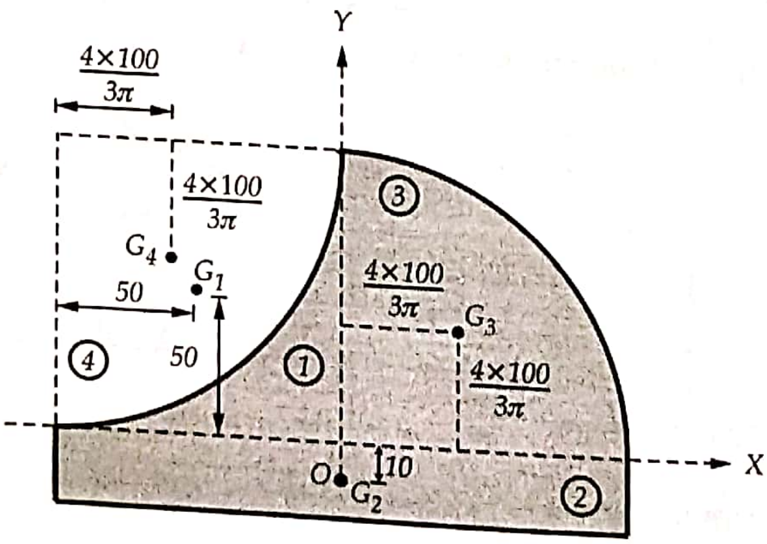

Fig. 8.3.19 (a)

The calculations are tabulated as follows :

Component No.	Component area A (mm^2)	x (mm)	y (mm)
1.	(100) (100)	– 50	50
2.	(200) (20)	0	– 10

3.	$\frac{\pi \times 100^2}{4}$	$\frac{4 \times 100}{3\pi}$	$\frac{4 \times 100}{3\pi}$
4.	$\frac{-\pi \times 100^2}{4}$	$-\left(100 - \frac{4 \times 100}{3\pi}\right)$	$100 - \frac{4 \times 100}{3\pi}$

$$\sum A = 14000 \ mm^2$$

$$\sum Ax = 285398.16 \ mm^3$$

$$\sum Ay = 341268.5 \ mm^3$$

$$\overline{X} = \frac{\sum Ax}{\sum A} = \frac{285398.16}{14000}$$

$$\therefore \quad \boxed{\overline{X} = 20.39 \ mm}$$

$$\overline{Y} = \frac{\sum Ay}{\sum A} = \frac{341268.5}{14000}$$

$$\therefore \quad \boxed{\overline{Y} = 24.38 \ mm}$$