Determining centroid of the given section

Formulas to remember

- The coordinates of the centroid $(\bar{x}, \bar{y})$ of a composite area are given by
  $\bar{x} = \frac{\sum a_i x_i}{\sum a_i} \quad \bar{y} = \frac{\sum a_i y_i}{\sum a_i}$

Important points to remember and understand

- The coordinates of the centroid $(\bar{x}, \bar{y})$ depend on the reference axes system chosen
- The centroid of rectangular or square area lies at the centre of these figures
- A basic geometric area (or shape or section) means all those shapes whose location of centroids are known from the readily available formulas. e.g. triangle, square, rectangle, circle, semicircle, sector of a circle, parabola, etc.
- A composite area (or shape or section) means an area made up of a combination of two or more basic geometric shapes.

Problem statement Q1: Locate the centroid of the given T-section (Fig.1) or Determine the coordinates of the centroid for the composite section in Fig. 1

(Use the colour code for easy reference)

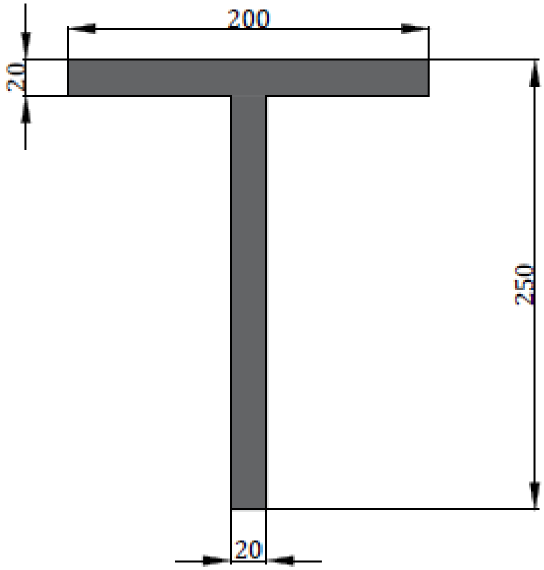

Fig. 1: T-section (all dimensions in mm)

- Given composite area (section) whose centroid is to be located
- Observe that all the necessary dimensions are given

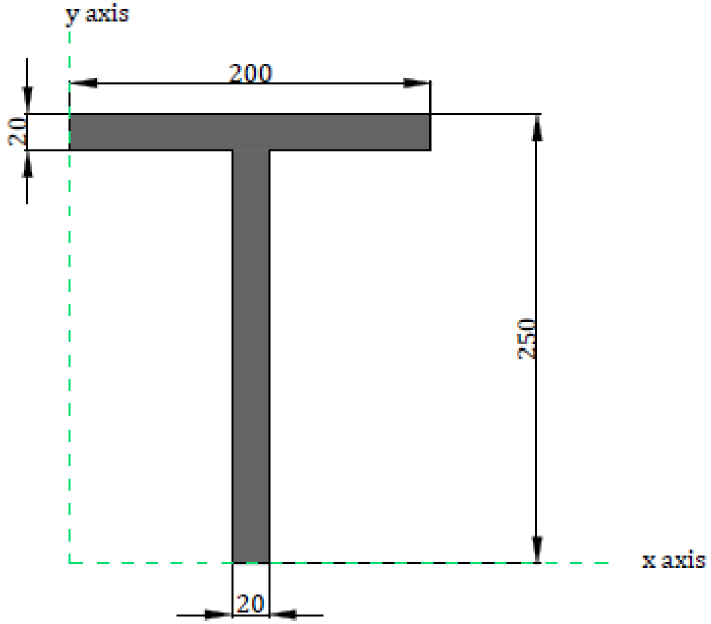

Fig. 2: T-section with reference axes

Step 1: Selecting reference axes system

- Choose a reference axis system such that all the necessary distances from the axes are known or can be determined
- There could be multiple options of an axes system for the same problem
- One must pick that option that reduces computation
- In the present problem the dashed green lines (Fig. 2) represent the reference axes system

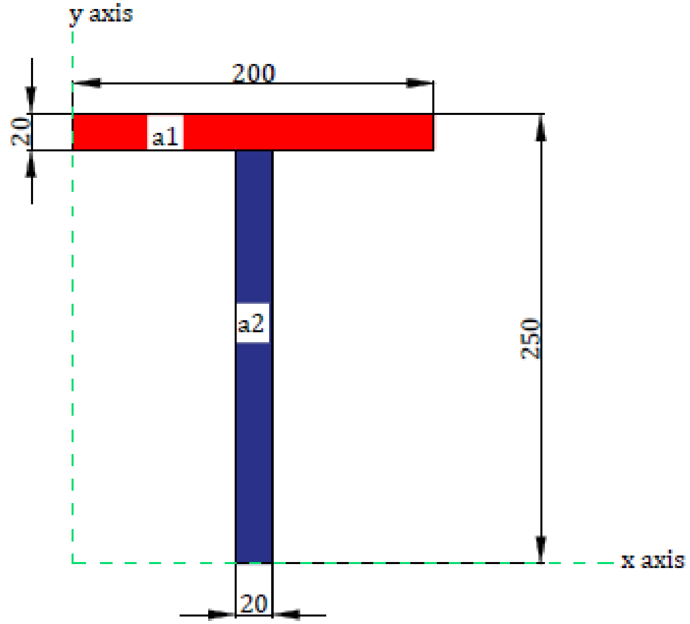

Fig. 3: T-section with basic geometric shapes

Step 2: Divide the composite area

- Divide the composite area (section) into basic geometric shapes (triangle, square, rectangle, circle, semicircle, etc.
- Name each of the basic shapes as a_1, a_2, a_3, ...
- In the present problem the T-section is divided into two rectangular areas a_1 and a_2

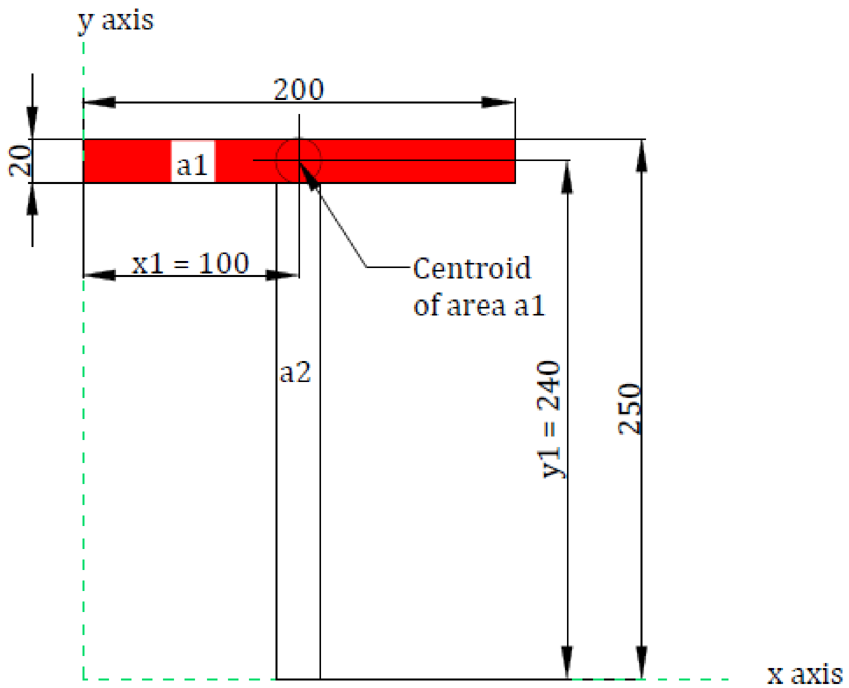

Fig 4: Coordinates (x_1, y_1) of centroid of rectangular area a_1

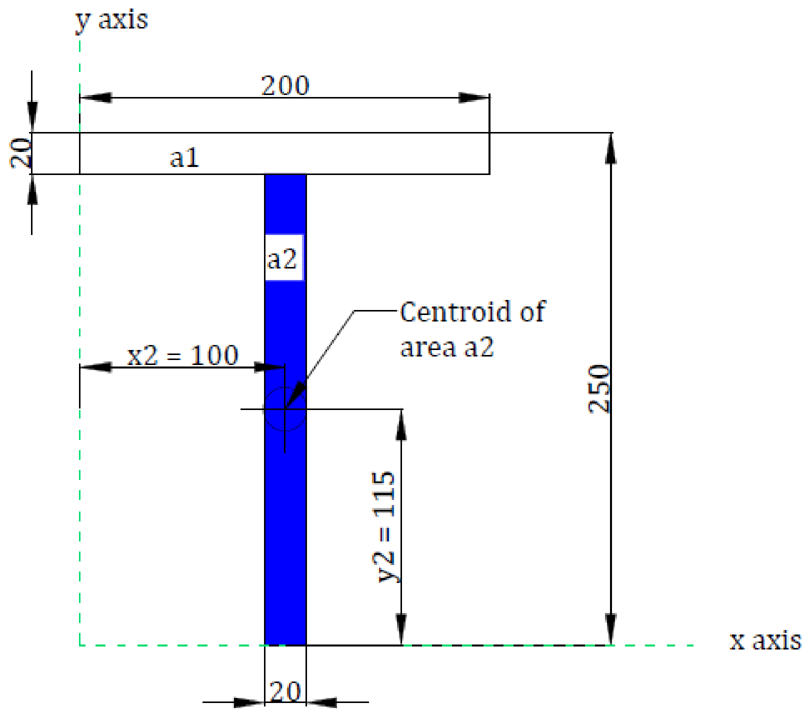

Fig 5: Coordinates (x_2, y_2) of centroid of rectangular area a_2

Step 3: Centroids of each basic geometric shape

- Determining distances to the centroid of each basic shape with respect to the reference axes system chosen
- In the present problem the centroids of rectangular areas a_1 and a_2 are marked (shown in Fig. 4 and 5). It is known that the centroid of a rectangle lies at the geometric centre of the shape.
- From the reference x-axis the centroid of rectangle a_1 is at a distance of y_1 = 240mm and from the reference y-axis the centroid of rectangle a_1 is at a distance of x_1 = 100mm (Fig. 4)
- Therefore, the coordinates of the centroid of area a_1 are (x_1, y_1) = (100, 240) from the reference axes
- Similarly, the coordinates of the centroid of area a_2 are (x_2, y_2) = (100, 115) from the reference axes

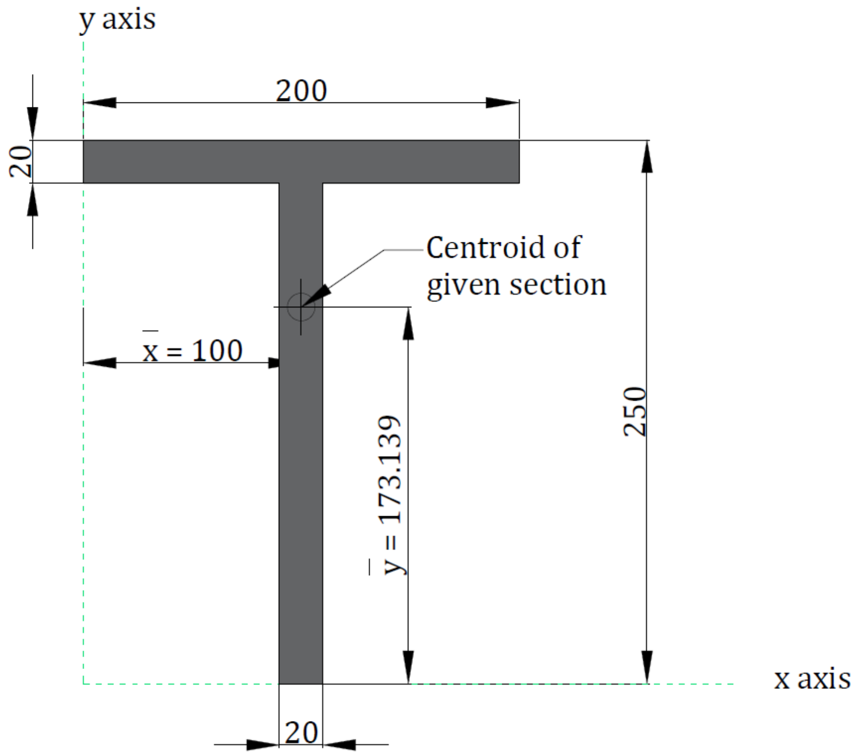

Fig. 6: Coordinates of centroid of the T-section

The calculations shown below can be done in tabular form as follows:

Area, a_i	Dist. from y-axis, x_i	Dist. from x-axis, y_i	$a_i \times x_i$	$a_i \times y_i$
$a_1 = 200\times20=4000$	100	240	4×10^5	9.6×10^5
$a_2 = 230\times20=4600$	100	115	4.6×10^5	5.29×10^5
$\Sigma a_i = 8600$			$\Sigma a_i x_i=8.6\times10^5$	$\Sigma a_i y_i=14.89\times10^5$
$\bar{x} = \frac{(8.6\times10^5)}{8600} = 100$			$\bar{y} = \frac{(14.89\times10^5)}{8600} = 173.139$	

Therefore, the coordinated of the centroid of the given T-section is (100, 173.139) from the coordinate axes chosen

Step 4: Determining the coordinates of the centroid of the given composite

- The coordinates of the centroid of the entire composite from the reference axes can be obtained by using the formulas
$$\bar{x} = \frac{\sum a_i x_i}{\sum a_i} \qquad \bar{y} = \frac{\sum a_i y_i}{\sum a_i}$$
- In the present problem, the composite was divided into two basic shapes of area $a_1 = 200 \times 20 = 4000mm^2$ and $a_2 = 230 \times 20 = 4600mm^2$. $x_1 = 100mm$, $y_1 = 240mm$, $x_2 = 100mm$ and $y_2 = 115mm$. Substituting in the above formula, we get,

$$\bar{x} = \frac{\sum a_i x_i}{\sum a_i} = \frac{a_1 x_1 + a_2 x_2}{a_1 + a_2} = \frac{(4000 \times 100) + (4600 \times 100)}{4000 + 4600} = 100mm \quad \text{(Shown in Fig. 6)}$$

$$\bar{y} = \frac{\sum a_i y_i}{\sum a_i} = \frac{a_1 y_1 + a_2 y_2}{a_1 + a_2} = \frac{(4000 \times 240) + (4600 \times 115)}{4000 + 4600} = 173.139mm \quad \text{(Shown in Fig. 6)}$$