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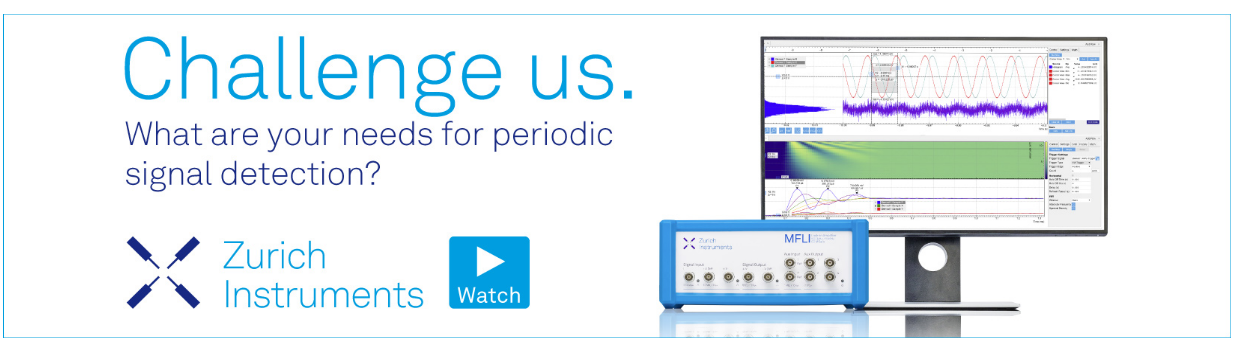

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Application of Graph Theory Concept for Traffic Light Control at Crossroad

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Abstract. Graph theory can be applied to solving systems of traffic lights at crossroads. By modeling the system of traffic flows into compatible graph, 2 vertices are represented as the flow connected by an edge if and only if the flow at the crossroads can be moved simultaneously without causing crashes. Influenced by the volume of traffic flows and the weights of the traffic flow, thus to be created a mathematical model in the form of the total time of all flows function by establishing required conditions, such as minimizing running time of each flow. The solution of this model is in the form of maximum total running time is not unique, since it is influenced by volume that is per hour based on the rush-hour at each crossroad.

INTRODUCTION

Traffic management is important in order to avoid collisions, consequently we need a system to regulate the flow of traffic. One of the systems that regulate the flow of traffic is traffic lights. As the city equipment, traffic lights play the most important role on the impact of traffic. If the traffic arrangements are not optimal, then it will not only affect traffic order, but also lead to an accident.

The traffic lights are lights that control the flow of traffic and attach at a crossroads, pedestrian crossings (zebra crossing), and another traffic flow places (UU no. 22/2009 tentang Lalu Lintas dan Angkutan Jalan : Alat Pemberi Isyarat Lalulintas or APILL). These lights indicate when the vehicles have to continue or stop driving alternately from different directions. The setting of traffic lights at the crossroads is intended as vehicle's movement regulation in each group of movements, so that vehicles will not interfere the existing flow. However, in reality shows that there are more solid volume in the specific crossroads rather than congested crossroads. This situation happens because the traffic control system is less effective and efficient.

To represent whether or not the flow of traffic be effective, we can view from its direction. If there are two different directions that can move jointly and efficiently, i.e. with total time flows (when light is green) are maximum, then it can be transformed into one of the branches of mathematics: graph theory. In this graph, the dots show the objects that will be set, and the sides indicate the compatible object. Particular traffic flows can be called compatible if two flows will not result in an accident caused by vehicles moving on multiple flows simultaneously (Hosseini & Orooji, 2009). By using compatible graph, optimal waiting time at the crossroads can be determined.

In this article, the authors make the traffic control systems work effectively and efficiently. The system is not synchronized with other traffic controller, but with cascading vehicle volume data from the main or other direction. Although there are changes in the volume of arrival vehicles from various directions, with this system, the expected duration of the maximum flow of vehicles can be made especially for the main direction, in this case the author mentions optimal.

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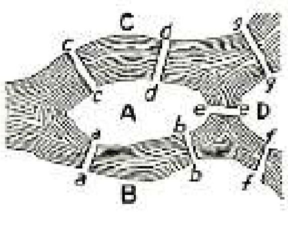
GRAPH THEORY AND ITS APPLICATION ON TRAFFIC LIGHT

History of Graph Theory

Graph theory emerged from the Konigsberg bridge problem in 1736 by Swiss mathematician, L Euler. Konigsberg is a small town located in the European continent. In the city, there is a big river and there are two deltas (small island).



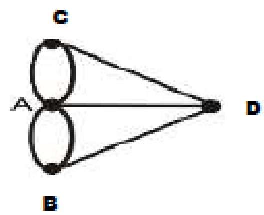


FIGURE 1. (a) Konigsberg Bridges, (b) Graph that represents Konigsberg Bridges.

Konigsberg bridge problem is whether it is possible to cross the seven bridges exactly in once, and come back to the original position.

Graph Theory

Graph theory has a very wide range of applications in everyday life and in various disciplines such as Computer Science, Engineering Science, even in Social Sciences.
Graph $G = (V(G), E(G))$ is a set of ordered pairs of $V(G)$ and $E(G)$ where:
$V(G)$ = a non empty finite set of objects called points (*vertices set)*
E (G) = a finite set (can be empty) so that each element of $E(G)$ is an unsorted pair of elements in $V(G)$ (*edge set*) $\{e_1,e_2,\ldots,e_n\}$ called edges.

a. Complete Graph
   An edge of the graph that connects appoint with itself is called a *loop*. Meanwhile, if there is more than one side connecting two vertices (u and v) in a graph the graph is said to have multiple edges. A simple graph is a graph having no loops and no multiple edges. While a graph which has multiple edges but does not have a loop called a multi graph (*multiple graph)*. A complete graph with n vertices denoted by K_n, is a simple undirected graph with n edges in which every pair of distinct vertices is connected by a unique edge. A graph having no edges is called an empty graph (null graph).
b. Section Graph
   Let$G$=($V(G)$, $E(G)$) be a graph. Graph H=($V(H)$, $E(H)$) is called a section graph G, written as $H\subseteq G$, if $V(H) \subseteq V(G)$ and $E(H)\subseteq E(G)$. If $H\subseteq G$ and $V(H)=V(G)$, then H is called the graph of the range section G, moreover if H is a complete piece of graph G, then H refers to a clique on G.

Compatible Graph

Definition 1

The compatible graph of traffic flows are defined as the traffic flow which is expressed by an edge, two edges of the traffic flow are connected when the traffic flow does not cause a crashes if it flows together.

An example of a traffic flow system at a crossroads and its graphical representation can be seen as follows

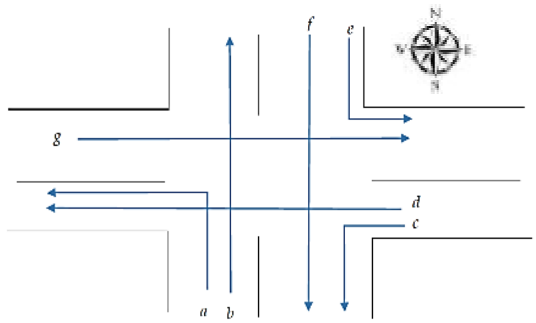


Figure 2 Examples of traffic flows along with the directions

Description :

- For the turned left flow at the current index (*a*), it must follow the light so that the current flow (edges) intersects with the current flow (*d*)
- For the turned left flow at the current index (*c*) and (*e*), it does not follow the flow of light so that flows (side) do not intersect with the flows (*f*) and (*g*)

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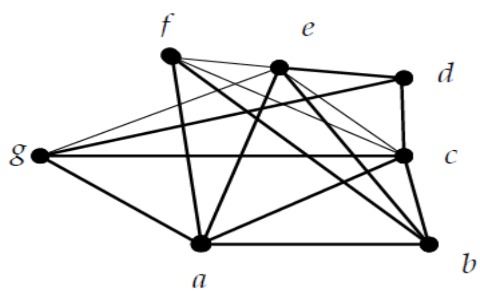

Figure 3 Examples of compatible graph of Figure 2.

In a graph form, we can model it as follows :

1. Observe many currents and directions of flows.
2. Labeling the first flow and giving dots mark (vertices).
3. Provide initial assumptions, such as whether or not to turn left to follow the light, whether or not flow must stream with the current left turn who has the same end vertices (on Figure 2 flow *a* and *d* may not run concurrently, but the flow *c* and *f* with *e* and *g* may run concurrently.
4. Representing each flow with a vertice on the graph.
5. Connect two vertices on the graph by an edge if and only if the two flows are represented by the compatible two vertices (not occurs the collision when running simultaneously, such as flow *a* compatible with flow *b, c, f,* and *g*.

Operation Research

Operations research requires the LINDO program which has acronym for *Linear Interactive Discrete Optimizer* This program will be operated to solve linear optimization problems. The program was developed by LINDO Corp. Mathematically, optimal solution of a case related to the linear programming optimal solution of another linear programming cases.

In the optimization of traffic lights, the objective function is "Total time flow" which is the flows in a given time period. Hence, in this case we are concerned to maximize total time flows. If the function of the objective is "Total time flow" which is flows in a period of specific time, then in this case we are concerned to minimize the total waiting time.

Crossroads

Crossroads are inseparable part of systems. Crossroad can be defined as the place one road crosses another/an crossroads of two or more roads, including roads and roadside facilities for traffic movement.

The purpose of the crossroads making is to reduce the potential conflicts among drivers (including pedestrians) and to provide a great comfort and to ease movement of vehicles.

Characteristics of Traffic Flow

The characteristics of traffic flow are necessary to learn in order to analyze traffic flow. To be able to represent the traffic characteristics, there are three (3) main parameters that should be known where those parameters are mathematically interconnected, with each other. Those are as follows :

1. Volume of traffic flows is the number of vehicles that pass a certain point in a certain roads within a certain time unit, usually expressed in passenger car units per hour for short (smp / hour)
2. Density of traffic flows is the number of vehicles that are within a certain unit of road length usually expressed in units of vehicles per km or abbreviated to (veh / km)
3. Density of traffic flows is a distance which can be reached within a certain time unit, usually expressed in units (km / hour)

Characteristics of traffic flow that will be used in this article is the volume.

Identification of Crossroad Performance

Identification of crossroad performance is the crossroad of performance values / criteria of a junction that is used to optimize the performance of crossroad among others.

1. Saturated traffic flow

   Saturated traffic flow is the maximum number of the units used are vehicles / hour. The saturation vehicles can be the base of the crossroad per hour. flow is not the same at the crossroads, because there are some things that affect the saturated flow rate as follows.

   a) Climbs and an instance at each crossroad
   b) Traffic of compositions
   c) Whether there is a traffic that will be turned to the right which ran into the traffic coming from the opposite direction
   d) Curve radius

2. Fase

   Fase is part of the cycle of traffic lights, provided for certain combinations of traffic movements. This phase will be partitioned into several parts sub set of flows, so that we know where the currents are moving and where the currents are stopped. For example, from the observations in the field to the junction at Darmo Crossroads– The main street of Dr.Soetomo.Obtained phase of each crossroads is divided into 3 phases.

Illustration of Cross Roads

The discussion begins with one illustration of the flow of traffic at the crossroads city of Surabaya, precisely on Darmo Crossroads– The main street of Dr.Soetomo.

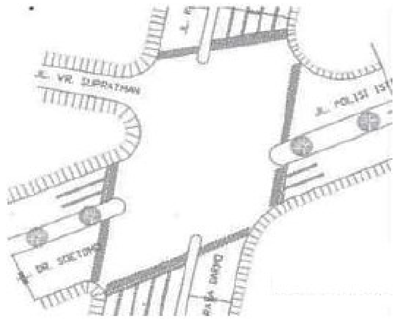

Figure 4 Darmo Crossroads– The main street of Dr.Soetomo (source : Dishub Surabaya)

Graph Model of Traffic Problem at the Crossroads

As we can seen there are 8 flows which are then labeled by *a, b, c, d, e, f, g,* and *h*. Direction of traffic flows can be seen at the following figure 5.

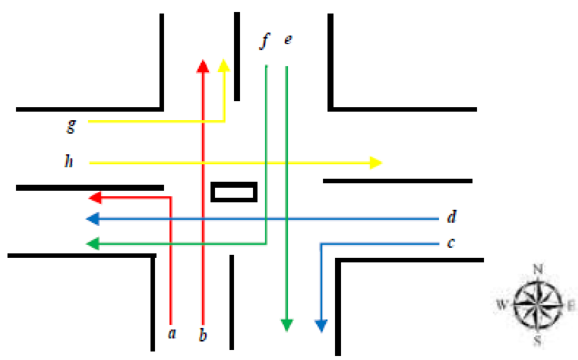

Figure 5 Darmo Crossroads– The main street of Dr.Soetomo. and the direction of movements

The flows are compatible which can be seen in the following

1. A flow *a* is compatible with the flows *b, c, e, g, h*
2. A flow *b* is compatible with the flows *a, c, e*
3. A flow *c* is compatible with the flows *a, b, d, e, f, g, h*
4. A flow *d* is compatible with the flows *c, g, h*
5. A flow *e* is compatible with the flows *a, b, c, f, g*
6. A flow *f* is compatible with the flows *c, e, g*
7. A flow *g* is compatible with the flows *a, c, d, e, f, h*
8. A flow *h* is compatible with the flows *a, c, d, g.*

In observation of the crossroads forms are assumptions, including:

- The flow turn left (*c*) does not follow the light, meaning that the flow can move at any time by the waiting time 0 (zero).
- The flow of The main street Darmo that turn left from the north (*e*) does not relate directly to the junction for the left turn lane there before the crossroads.
- For other flow turn left (*a* and *g*) the movement of currents follow the light.
- There is only one flow turn right (*f*).

Compatible Graph Model of Traffic Problem at the Crossroads

There are 8 currents flowing in the crossroads, among whom four straight flows that are *b, d, e,* and *h*. In currents that turn left there are three flow that *a, c, g* and then there is 1 which flow right turn *f.* Two vertices on the graph are connected by an edge if and only if the two flows are corresponding to the two points are compatible (can be flow together without causing a crashes). For example, flow *a* and flow *b* can flow together without any collision that point that represents the flows *a* and the point which represents the flows *b* connected by an edge, as well as the flows *a* and *b* flowing, thus vertices that represent the flows *a, b,* and flows *c*are interconnected directly. Thus all three vertices are formed a clique (complete section graph)

Alternative Solution Waiting Time on Crossroad a Possible

The purpose of the control is to organize the traffic lights at the crossroads of the traffic system is to avoid collisions and total waiting time is minimal. Each current given an interval to flowing, at a time when current is flowing then no current is not flowing. The following description of the differences of alternative solutions that may be waiting time of each flow.

TABLE 1 Description of differences from each current that stop (Red light) and flowing (green light)

Alternative	**Information Sub Arus**	**Waiting Time (red light) Total**	**Flowing time (green light) Total**	**Cycle Total**
I (The flow before partition)	Homogeneous flow	840 second	120 second	960 second
II (The flow after partition)	Flows *A={a,b,c}* *B ={c,d,g,h}* *C={f,g,c}*	400 second	560 second	960 second
III	Flows *A={a,b,c}* *B ={f,e,c}* *C={c,d,g,h}*	440 second	520 second	960 second
IV	Flows *A={a,b,c,e}* *B ={e,f,g,c}* *C={c,d,g,h}*	480 second	480 second	960 second

If the setting up a way to do that is compatible flow can be flowing simultaneously (compatible flow can move together) then this will greatly reduce the total waiting time. Compatible with existing graph regard, it is not possible to partition the set of flows to less than three sub sets of compatible flow.

The above situation is some illustrations which use the assumption that ignores the number of vehicles passing in each of the current and the time intervals are created equally. There is another illustration, which would otherwise ignore the number of vehicles waiting time given different weights.

The Setting of Traffic System at a Crossroads by Watching Thickness Current is Flowing

In the previous sections each sub-interval has the same length and weight all the same flow. Previous currents solid by flowing time longer than the flow relatively quiet, such as currents flowing "straight" relatively more dense than the current turn left or right, so that the currents that flow straight should be given time span flow longer than the current turn, say the volume ratio and flows straight turn is with a straight flow and is the turn flows. In the graph model are compatible 4 the current that flowing straight, namely *b, d, e, h* and 4 the remaining flow *a, c, f, g* flowing turn.

Note that minimizing the waiting time results in a slowing down of the total time in the current flows, for example if the total time flow is symbolized by *M*, then it is supposed that the green time on each lane is not less than 15 seconds.

When *b* is in the interval x_1, *d* is in the interval x_3, *e* is in the interval $x_1$and x_2, *h* is in the interval x_3, *a* is in the interval x_1, *c* is in the interval x_1, x_2, and x_3, *f* is in the interval x_2, and *g* is in the interval $x_2$and x_3. So that the mathematical model of the above problems is to maximize value *M,* when

$$M = \alpha_1(x_1 + x_3 + x_1 + x_2 + x_3) + \alpha_2(x_1 + x_1 + x_2 + x_3 + x_2 + x_2 + x_3) \Leftrightarrow$$
$$M = \alpha_1(2x_1 + x_2 + 2x_3) + \alpha_2(2x_1 + 3x_2 + 2x_3)$$

If required any current flowing in a minimal 15 seconds, then the above problems can be modeled as follows :
Max value *M,* with value *M* is

$$M = \alpha_1(2x_1 + x_2 + 2x_3) + \alpha_2(2x_1 + 3x_2 + 2x_3)$$

with the provision of $x_1 + x_2 + x_3 = 120;$

$$x_1 \geq 15 \;;$$
$$x_1 + x_2 \geq 15 \;;$$
$$x_2 \geq 15;$$
$$x_3 \geq 15;$$
$$x_1 + x_2 + x_3 \geq 15$$

Results of Data Collection

Time Data

Based on data from the time at the cross roads of Darmo Crossroads – The main street of Dr. Soetomo.obtained results from Survey Kinerja Lalu Lintas Dishub Kota Surabaya 2015 Tahap 2 are listed in Table 2

TABLE 2 Description of Differences from Each current that Stop (Red light) and Flowing (Green light)

Alternative	Time		
	Red Light	**Green Light**	**Siklus Total**
Raya Darmo southern (*a,b*)	70	130	200
Raya Darmo northern (*e*)	58	142	200
Raya Darmo northern turn right (*e*)	143	57	200
Raya Polisi Istimewa (*d)*	150	50	200
Jalan Raya Dr. Soetomo (*g, h)*	150	50	200
TOTAL	**571**	**429**	

Volume Data

Here is the dataof the number of vehicles passing through each crossroad on Darmo Crossroads – The main street of Dr. Soetomo is served in Table 3

Table 3. Actual Vehicle Volume at Each Crossroad

No	Flows Indeks	Actual Traffic Flow (07.00-08.00)
1	*a*	127
2	*b*	2943
3	*c*	503
4	*d*	2865
5	*e*	3387
6	*f*	2445
7	*g*	331
8	*h*	2277
	TOTAL	14878

AnalysisData

The details of each flow are as follows:

- Straight flows (North-South) → 6.330 ↔ 42,55 % v 0,43 as α_1
- Straight flows (West-East) → 5.142 ↔ 34,56 % v 0,35 as α_2
- Left flows → 3.406 ↔ 22,89 % v 0,23 as α_3,

if add up the the percentages are 100%.

Furthermore, from the presentation it will be used to provide constant values in $\alpha_1 : \alpha_2 : \alpha_3$.

Maximize value *M,* with values *M*

$$M = 1{,}32x_1 + 1{,}12x_2 + 1{,}19x_3$$

If required according to data on any current flowing minimal 50 seconds total flows (green light)429 seconds, then the above problems can be modeled as follows

Max value *M,* with *M*

$$M = 1{,}32x_1 + 1{,}12x_2 + 1{,}19x_3$$

with the provision of

$$\begin{aligned} x_1 + x_2 + x_3 &= 429; \\ x_1 &\geq 50\ ; \\ x_1 + x_2 &\geq 50; \\ x_2 &\geq 50; \\ x_2 + x_3 &\geq 50; \\ x_3 &\geq 50; \\ x_1 + x_2 + x_3 &\geq 50; \end{aligned}$$

By using the help of LINDO software, we can get the value x_1 =329, x_2 = 50, and x_3= 50 with the objective function value is 549,78 in due time second then rounding into ≈ 550.

Comparison of actual time and time calculations can be seen in Table 4.

Table4. Comparison of actual time (data) and Saturated time (simulation)

Initial Observation	Actual Time Green light (second)	Saturated Time Green light (second)
The mainstreet of Darmo southern (*a, b)*	130	167
The main street of Darmonorthern (*e)*	142	182
The main street of Darmo northernturn right (*f)*	57	73
The main street of Polisi Istimewa (*d)*	50	64
The main street of Dr. Soetomo (*g, h)*	50	64
TOTAL	**429**	**550**

In addition to determining the time of saturated (simulation), we can also calculate the predicted number of passing vehicles. This is because the time duration of the green the number of passing vehicles. Description for comparison of the actual time (data) and the number of vehicles and the time saturated (simulation) and the number of vehicles (prediction), can be seen in Table 5

Tabel5. Comparison of Actual Time (Data) and Number of Vehicles and then Time Calculation (Simulation) and Number of Vehicles (Prediction)

Flows	Actual			Saturated		
	Green light (second)	Red Light (second)	Number of vehicles/ second (data)	Green light (second)	Red Light (second)	Number of vehicles/ second (prediction)
a	130	70	5	167	55	6
b			106			36
c			28			31
d	50	150	40	64	118	51
e	142	58	134	182	46	172
f	57	143	39	73	113	50
g	50	150	5	64	118	6
h			32			41
Total	429	571	389	550	450	493

Green light time on calculations appear longer than the actual time on the green and red on a calculation time also looks shorter than actual green light time. By the time the green light on the calculation also affects the amount of (prediction) of passing vehicles. The amount of the predictions that can be flashed even seen more than the number of vehicles on actual data. So this thesis qualifies effective that the same the current that flows in a collision and also not efficient which time the maximum flow and minimum stop time and consideration can be made to be applied at the crossroad of Darmo Crossroads – The main street of Dr. Soetomo.

Application of the concept of graph theory in this paper in the same way can be adopted to the other way, which is certainly the time to flow and the number of different vehicles.

CONCLUSIONS

Based on the discussion that has been described in previous chapters, the following conclusions can be taken.

1. Graph theory can be applied to the problem of traffic system settings at a crossroads in the following way
    - Modeling system traffic flow at crossroads into the compatible graph
    - Dividing the set vertices of the compatible graph into several sections complete graph such that many parts as possible or the number of dots in each graph section complete as much as possible

- Consider the weight of the current that flows to the flow grouping based on the volume flow
- Make a mathematical model in the form of the function total time all the current flowing in the time intervals of flows grouping
- Establish the conditions required e.g. minimum running time of each flow
- Resolving mathematical model in point d.

2. The calculation of the optimal cycle states traffic light cycle at different times at crossroads and when the green light in all directions. But in fact, the traffic light settings are very complicated and no single, which involves a variety of factors, and cannot adopt a suitable model to solve all problems.

Suggestions

In a subsequent study we are planning to do the following things

1. The method to generate the graph model are compatible from the research that can be applied to different traffic flow, using a graph definition compatible (some currents can move together without a crashed) also accompanied the initial assumptions to simplify the modeling of such neglect things less directly related to the flow of traffic, such as road conditions, weather, the form of roads, wide roads, and also the type of passing vehicles.
2. Need to be a improvement compatible graph model of the traffic system at the crossroad of Darmo Crossroads – The main street of Dr. Soetomo or crossroad with simulated design computer program in solving pattern of traffic control by the real conditions every minute or every hour, so when the current at an crossroads that is very dense then automatically flow at the crossroads very crowded time duration flows (time green light) longer compared to other crossroads that are not so crowded

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