Example 9.7.3 *Determine the moment of inertia about X-X axis. All dimensions are in mm.*

VTU : Feb.-04, Dec.-11, Marks 10

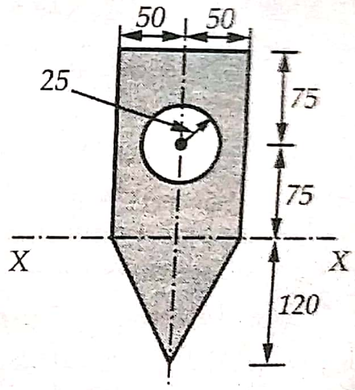

Fig. 9.7.3

Solution : Formulae for moment of inertia about base can be used for rectangle and triangle. Moment of inertia of circle about X-X axis can be obtained using parallel axis theorem.

$$\therefore I_{XX} = \frac{100 \times 150^3}{3} + \frac{100 \times 120^3}{12} - \left[\frac{\pi \times 25^4}{4} + (\pi \times 25^2) \times 75^2\right]$$

$$\therefore \quad \boxed{I_{XX} = 1.155 \times 10^8 \ mm^4}$$

Scanned with CamScanner

Example 9.7.6 *Determine the moment of inertia of the area shown in Fig. 9.7.6 about the axes AB and PQ.* **VTU : Aug.-05, Marks 16**

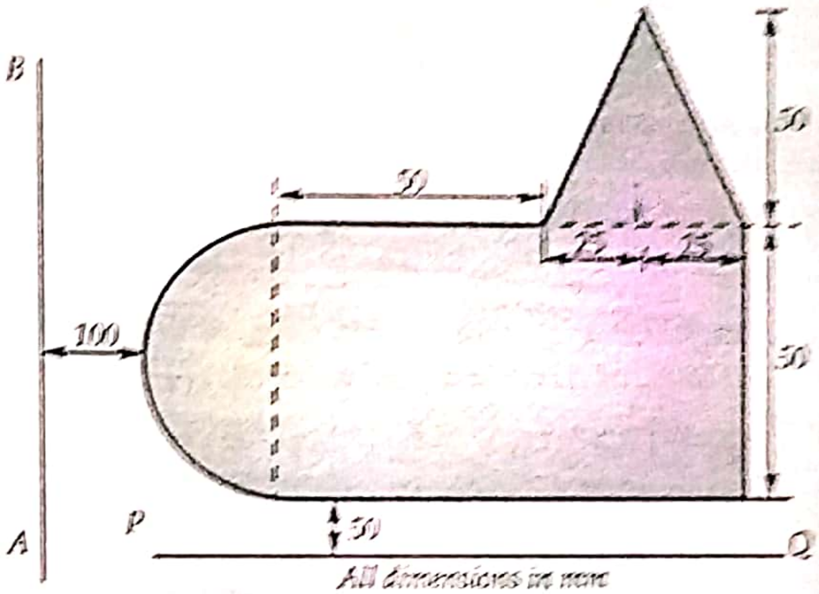

Fig. 9.7.6

Solution : Divide the given area into semicircle – 1, rectangle – 2 and triangle – 3. Let the distances of their centroids from lines PQ and AB be r_{PQ} and r_{AB} respectively. The calculations are tabulated as follows :

Component No.	Component area A (mm^2)	I_G I_x (mm^4)	I_y (mm^4)	r_{PQ} (mm)	r_{AB} (mm)
1.	$\frac{\pi \times 25^2}{2}$	$\frac{\pi \times 25^4}{8}$	0.11×25^4	$50 + 25 = 75$	$125 - \frac{4 \times 25}{3\pi} = 114.39$
2.	100×50	$\frac{100 \times 50^3}{12}$	$\frac{50 \times 100^3}{12}$	75	$125 + 50 = 175$
3.	$\frac{1}{2} \times 50 \times 50$	$\frac{50 \times 50^3}{36}$	$2\left(\frac{50 \times 25^3}{12}\right)$	$100 + \frac{50}{3} = 116.67$	200

$$I_{PQ} = \sum \left(I_X + A r_{PQ}^2\right)$$

$$I_{PQ} = \left[\frac{\pi \times 25^4}{8} + \left(\frac{\pi \times 25^2}{2}\right) \times 75^2\right] + \left[\frac{100 \times 50^3}{12} + (100 \times 50) \times 75^2\right]$$

$$+ \left[\frac{50 \times 50^3}{36} + \left(\frac{1}{2} \times 50 \times 50\right) \times 116.67^2\right]$$

$$\therefore \quad \boxed{I_{PQ} = 52.03 \times 10^6 \ mm^4}$$

$$I_{AB} = \sum \left(I_Y + Ar_{AB}^2\right)$$

$$\therefore \quad I_{AB} = \left[0.11 \times 25^4 + \left(\frac{\pi \times 25^2}{2}\right) \times 114.39^2\right] + \left[\frac{50 \times 100^3}{12} + (100 \times 50) \times 175^2\right]$$

$$+ \left[2\left(\frac{50 \times 25^3}{12}\right) + \left(\frac{1}{2} \times 50 \times 50\right) \times 200^2\right]$$

$$\therefore \quad \boxed{I_{AB} = 2.203 \times 10^8 \ mm^4}$$

Scanned with CamScanner

Example 9.7.9 *Determine the moment of inertia of the shaded area about the axis A-A.*

VTU : Aug.-06, Marks 10

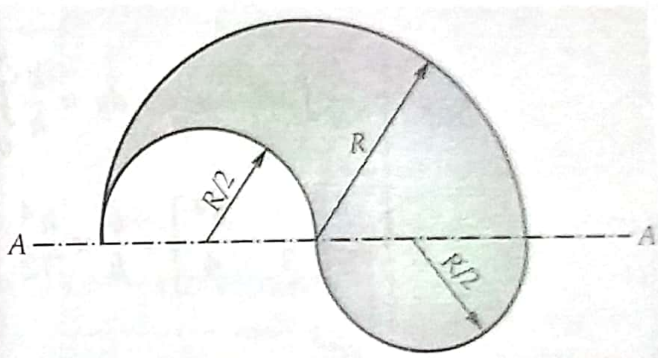

Fig. 9.7.9

Solution : The M.I. of semicircle about diameter is $\frac{\pi r^4}{8}$

$$\therefore \quad I_{AA} = \frac{\pi R^4}{8} + \frac{\pi \times \left(\frac{R}{2}\right)^4}{8} - \frac{\pi \times \left(\frac{R}{2}\right)^4}{8}$$

$$\therefore \quad \boxed{I_{AA} = \frac{\pi R^4}{8}}$$

... enclosed by a right angled triangle

Example 9.7.12 *The cross section of the prestressed concrete beam is as shown in Fig. 9.7.12. Calculate the moment of inertia of this section about the centroidal axes parallel to the top edge and perpendicular to the plane of cross section. Also determine the radius of gyration.*

VTU : Aug.-07, Marks 14

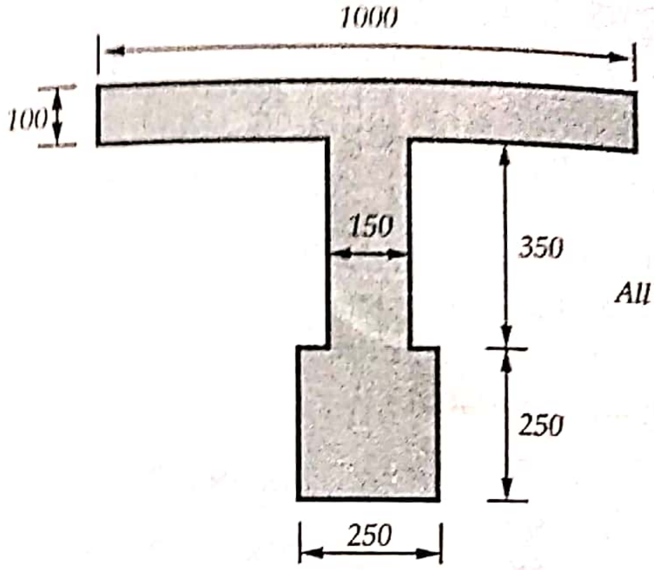

Fig. 9.7.12

Solution : The given area is symmetric about a vertical line passing through the centre. That vertical line is the centroidal Y axis. Divide the area into three rectangles as shown in Fig. 9.7.12 (a). Take base of the given figure as origin.

The calculations are tabulated as follows :

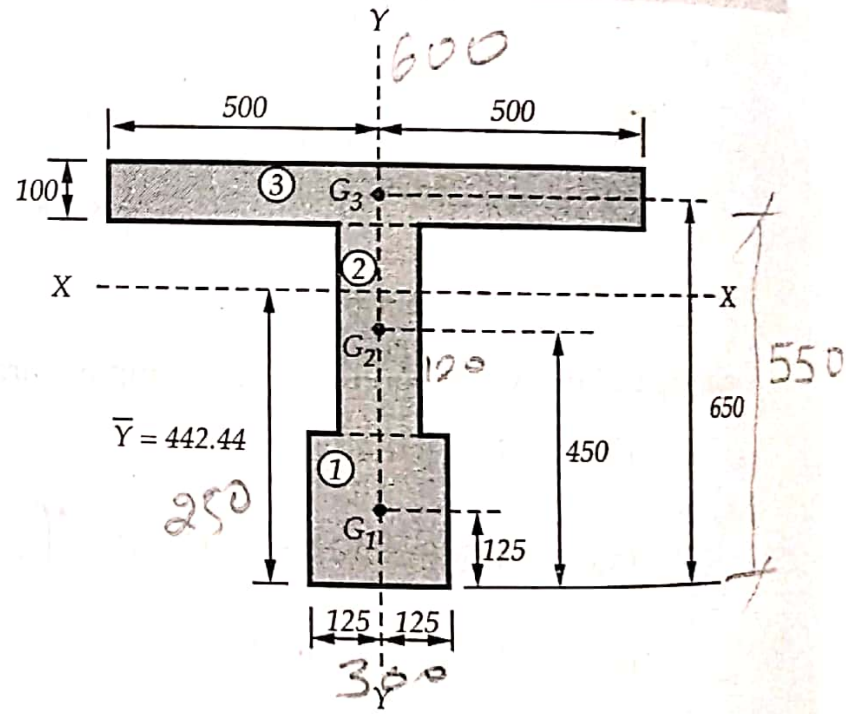

Fig. 9.7.12 (a)

Component No.	Component area A (mm^2)	y (mm)	I_G		$r_x = \overline{Y} - y$
			I_x	I_y	
1.	250×250	125	$\dfrac{250\times250^3}{12}$	$\dfrac{250\times250^2}{12}$	317.44
2.	150×350	425	$\dfrac{150\times350^3}{12}$	$\dfrac{350\times150^3}{12}$	17.44
3.	1000×100	650	$\dfrac{1000\times100^3}{12}$	$\dfrac{100\times1000^3}{12}$	– 207.56

$$\sum A = 250\times250 + 150\times350 + 1000\times100$$

$$\sum A = 215000\ mm^2$$

$$\therefore \quad \overline{Y} = \frac{A_1y_1 + A_2y_2 + A_3y_3}{\sum A}$$

$$= \frac{(250\times250)(125) + (150\times350)(425) + (1000\times100)(650)}{215000}$$

$$\therefore \quad \overline{Y} = 442.44\ mm.$$

$$I_{XX} = \sum (I_X + A\,r_x^2)$$

$$\therefore \quad I_{XX} = \left[\frac{250\times250^3}{12} + (250\times250)\times317.44^2\right] + \left[\frac{150\times350^3}{12} + (150\times350)\times17.44^2\right]$$

$$+\left[\frac{1000\times100^3}{12} + (1000\times100)\times207.56^2\right]$$

$$\therefore \quad \boxed{I_{XX} = 1.1567\times10^{10}\ mm^4}$$

The centroidal axis perpendicular to the plane of the cross section is the Z-Z axis. The M.I. about Z-Z axis can be obtained using,

$$I_{ZZ} = I_{XX} + I_{YY}$$

$$I_{YY} = \frac{250\times250^3}{12} + \frac{350\times150^3}{12} + \frac{100\times1000^3}{12}$$

$$\therefore \quad I_{YY} = 8.7573\times10^9\ mm^4$$

$$I_{ZZ} = 1.1567\times10^{10} + 8.7573\times10^9$$

$$\therefore \quad \boxed{I_{ZZ} = 2.03243\times10^{10}\ mm^4}$$

The radius of gyration for X-X axis is

$$K_{XX} = \sqrt{\frac{I_{XX}}{\sum A}} = \sqrt{\frac{1.1567\times10^{10}}{215000}}$$

$$\boxed{K_{XX} = 231.95\ mm}$$

The radius of gyration for Z-Z axis is

$$k_{ZZ} = \sqrt{\frac{I_{ZZ}}{\sum A}} = \sqrt{\frac{2.03243 \times 10^{10}}{215000}}$$

$$\boxed{k_{ZZ} = 307.46 \text{ mm}}$$

Example 9.7.13 Determine the second moment of the area about the horizontal centroidal axis as shown in Fig. 9.7.13. Also find radius of gyration.

VTU : Feb.-08, Marks 14

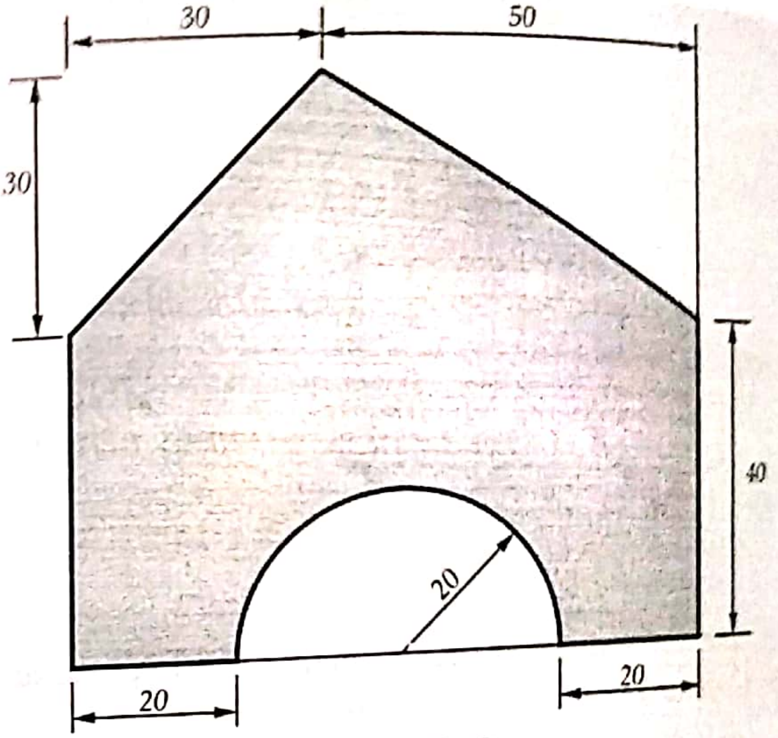

Fig. 9.7.13

Solution : Divide the given area into a rectangle – 1, triangle – 2 from which a semicircle – 3 is to be removed. To find position of horizontal centroidal axis, we find $\bar{Y}$ with respect to the base of the given figure. The calculations are tabulated as follows :

Component No.	Component area A (mm^2)	y (mm)	I_x (mm^4)	$r_x = \bar{Y} - y$ (mm)
1.	80×40	20	$\frac{80 \times 40^3}{12}$	11.46
2.	$\frac{1}{2} \times 80 \times 30$	50	$\frac{80 \times 30^3}{36}$	– 18.54
3.	$-\frac{\pi \times 20^2}{2}$	$\frac{4 \times 20}{3\pi}$	0.11×20^4	22.97

$$\sum A = 3771.68 \text{ mm}^4$$

$$\bar{Y} = \frac{A_1 y_1 + A_2 y_2 + A_3 y_3}{\sum A}$$

$$= \frac{(80 \times 40)(20) + \left(\frac{1}{2} \times 80 \times 30\right)(50) - \left(\frac{\pi \times 20^2}{2}\right)\left(\frac{4 \times 20}{3\pi}\right)}{3771.68}$$

$$\bar{Y} = 31.46 \text{ mm}$$

$$I_{XX} = \sum \left(I_X + A\,r_x^2\right)$$

$$I_{XX} = \left[\frac{80\times40^3}{12} + (80\times40)\times11.46^2\right] + \left[\frac{80\times30^3}{36} + \left(\frac{1}{2}\times 80\times30\right)\times18.54^2\right]$$

$$-\left[0.11\times20^4 + \left(\frac{\pi\times 20^2}{2}\right)\times 22.97^2\right]$$

$$\therefore \boxed{I_{XX} = 9.703\times10^5 \;\; mm^4}$$

$$K_{XX} = \sqrt{\frac{I_{XX}}{\sum A}} = \sqrt{\frac{9.703\times10^5}{3771.68}}$$

$$\therefore \boxed{K_{XX} = 16.04 \text{ mm}}$$

Example 9.7.14 *Find the moment of inertia of the section shown in Fig. 9.7.14 about horizontal centroidal axis and also find the radius of gyration about the same axis.*

VTU : Aug.-08, June-13, Marks 10

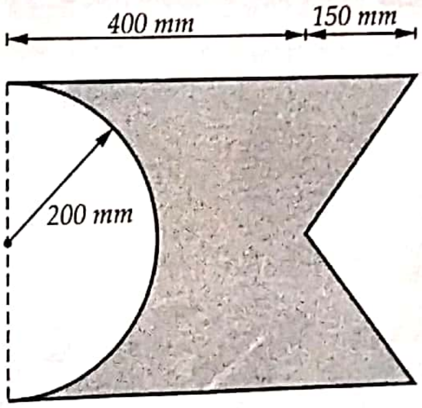

Fig. 9.7.14

Solution : The horizontal centroidal axis passes through the centre. The area can be divided into rectangle from which a semicircle and an isosceles triangle have to be removed. All the three areas have centroids on the horizontal centroidal axis.

$$I_{XX} = \frac{550\times400^3}{12} - \frac{\pi\times200^4}{8} - 2\left(\frac{150\times200^3}{12}\right)$$

$$\boxed{I_{XX} = 2.105\times10^9 \;\; mm^4}$$

$$\sum A = 550\times 400 - \frac{\pi\times200^2}{2} - \frac{1}{2}\times 400\times150$$

$$\sum A = 127168.15 \;\; mm^2$$

$$K_{XX} = \sqrt{\frac{I_{XX}}{\sum A}} = \sqrt{\frac{2.105 \times 10^{9}}{127168.15}}$$

$$\therefore \quad \boxed{K_{XX} = 128.66 \text{ mm}}$$

ample 9.7.17 Determine the second moment of area about horizontal centroidal axis for shaded area shown in Fig. 9.7.17. Also find the radius of gyration about the same axis. Take $R_1 = 50$ mm and $R_2 = 20$ mm. **VTU : Aug.-11, Marks 10**

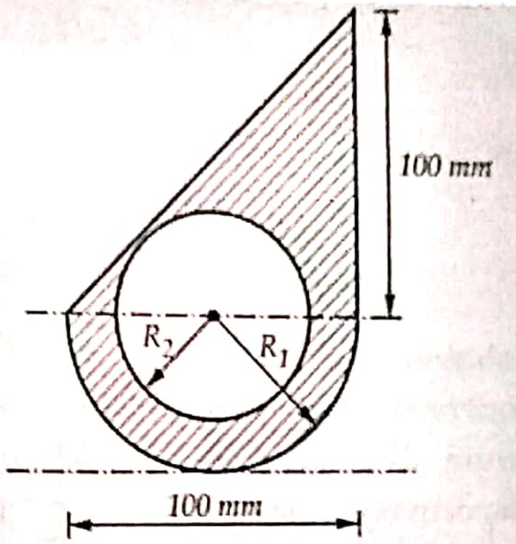

Fig. 9.7.17

olution : Divide the given area into a triangle – 1 d a semicircle – 2 from which the circle – 3 is to be moved. Take base of the given area as reference for lculation of $\bar{Y}$. The calculations are tabulated as llows :

Component No.	Component area A (mm²)	y (mm)	I_X (mm⁴)	$Y_x = \bar{Y} - y$ (mm)
1.	$\frac{1}{2} \times 100 \times 100$	$50 + \frac{100}{3}$	$\frac{100 \times 100^3}{36}$	– 22.47
2.	$\frac{\pi \times 50^2}{2}$	$50 - \frac{4 \times 50}{3\pi}$	0.11×50^4	32.085
3.	$-\pi \times 20^2$	50	$\frac{\pi \times 20^4}{4}$	10.864

$$\sum A = 7670.35 \text{ mm}^2$$

$$\sum Ay = 466851.02 \text{ mm}^3$$

$$\bar{Y} = \frac{\sum Ay}{\sum A} = 60.864 \text{ mm}$$

$$I_{XX} = \sum [I_X + A r_x^2]$$

$$= \left[\frac{100 \times 100^3}{36} + \frac{1}{2} \times 100 \times 100 \times (-22.47)^2\right] + \left[0.11 \times 50^4 + \frac{\pi \times 50^2}{2} \times (32.085)^2\right]$$

$$- \left[\frac{\pi \times 20^4}{4} + \pi \times 20^2 \times (10.864)^2\right]$$

$$\therefore \quad \boxed{I_{XX} = 9.758 \times 10^6 \ mm^4}$$

Example 9.7.18 *Determine the moment of inertia of the symmetrical I-section shown in Fig. 9.7.18 about its centroidal X-X and Y-Y axis.*

VTU : Dec.-13, Marks 10

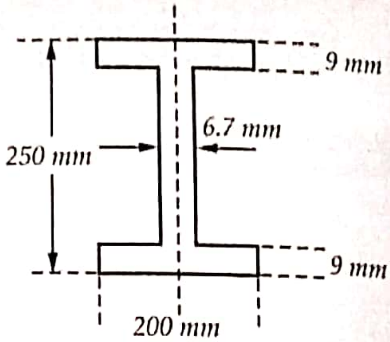

Fig. 9.7.18

Solution : As the given area is symmetric about horizontal and vertical lines pressing through the centre, these lines are the centroidal X-X and Y-Y axes respectively as shown in Fig. 9.7.18 (a).

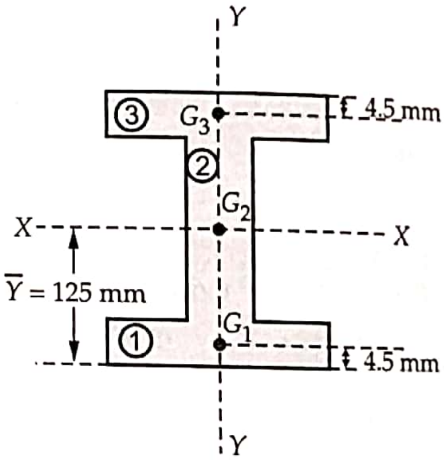

Fig. 9.7.18 (a)

To find I_{XX}, parallel axes theorem is required.

$$I_{XX} = \Sigma\left[I_x + A\left(\overline{Y} - y\right)^2\right]$$

$$A_1 = 200 \times 9 \ mm^2, \ y_1 = 4.5 \ mm$$

$$A_2 = 232 \times 6.7 \ mm^2, \ y_2 = 125 \ mm$$

$$A_3 = 200 \times 9 \ mm^2, \ y_3 = 245.5 \ mm$$

$$\overline{Y} = 125 \ mm$$

$$\therefore \quad I_{XX} = \left[\frac{200 \times 9^3}{12} + 200 \times 9 \times (125 - 4.5)^2\right] + \left[\frac{6.7 \times 232^3}{12} + 0\right] + \left[\frac{200 \times 9^3}{12} + 200 \times 9 \times (125 - 245.5)^2\right]$$

$$\therefore \quad \boxed{I_{xx} = 59.27 \times 10^6 \ mm^4}$$

$$I_{YY} = \Sigma I_Y = \frac{9 \times 200^3}{12} + \frac{232 \times 6.7^3}{12} + \frac{9 \times 200^3}{12}$$

$$\therefore \quad \boxed{I_{YY} = 12 \times 10^6 \ mm^4}$$