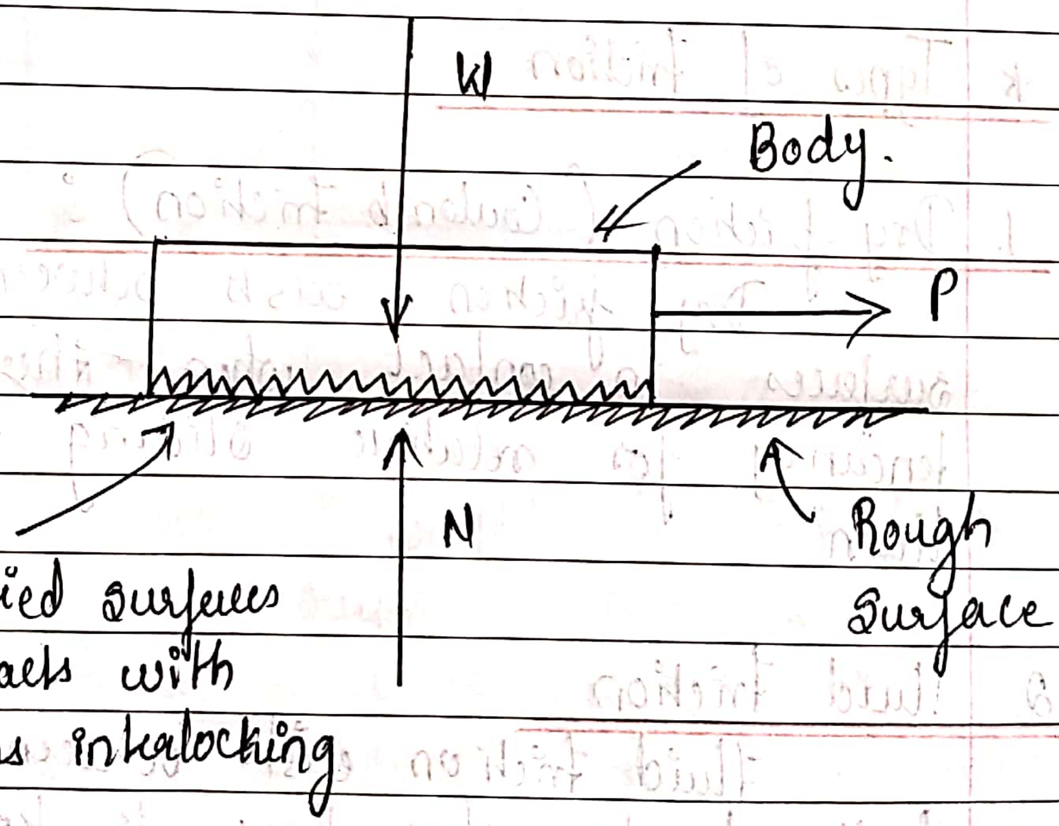
Friction

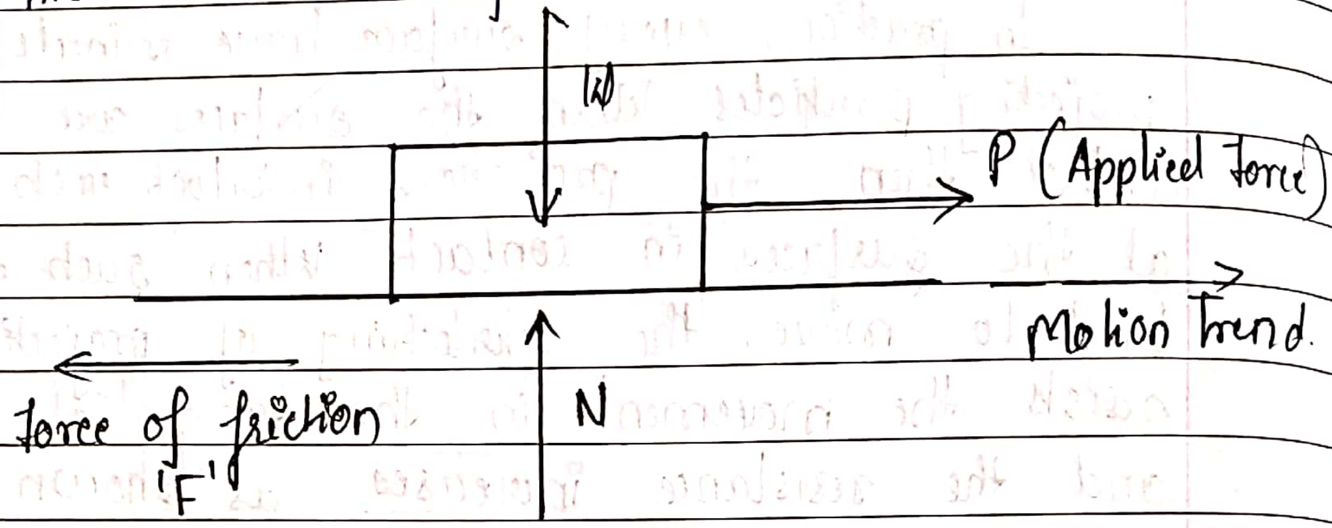
* Introduction :

In practice, every surface have minute projecting particles. When the surfaces are in contact then, the projections interlock each other at the surfaces in contact. When such surfaces tend to move, the interlocking of projections resists the movement in the tangential direction and the resistance increases as shown.



* Definition of Friction :

When one body tends to move in contact over other body, a resistance to its movement is set-up. This resistance to movement is called friction or force of friction.



* Types of friction

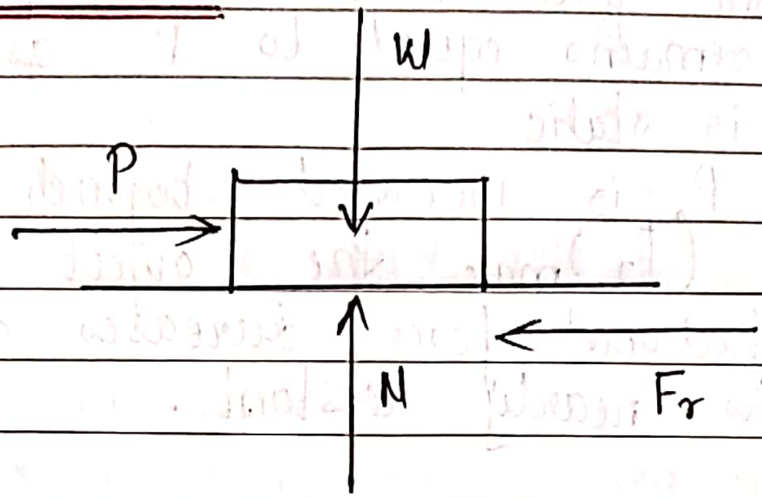
1. Dry friction (Coulomb friction):

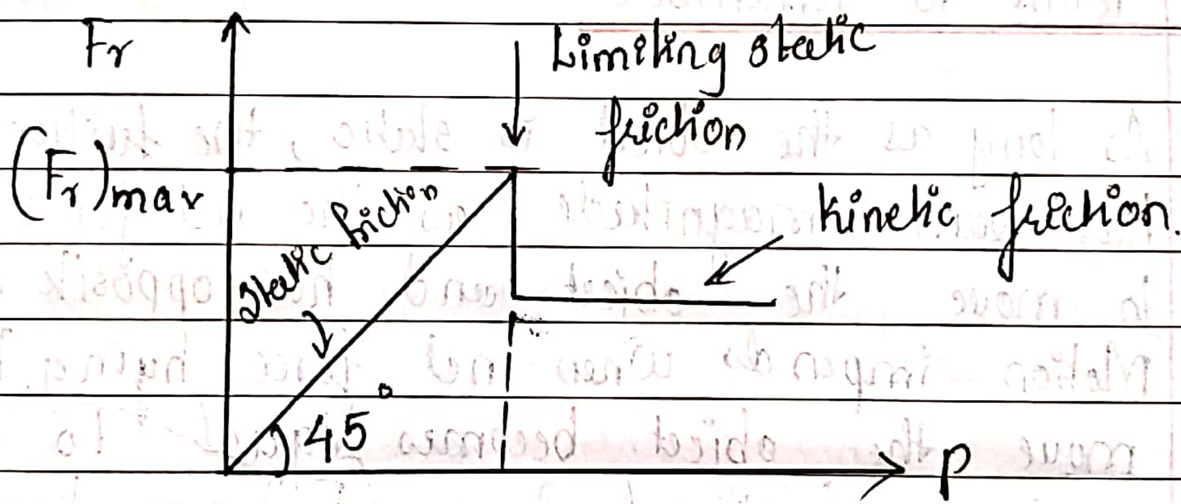
Dry friction exists between the two surfaces in contact when there is tendency for relative sliding motion between them.

2. Fluid friction

Fluid friction exists between two layers of fluid when there is tendency for relative sliding motion between them.

* Dry friction :





i. Suppose a block of weight 'W' is kept on a rough horizontal surface and a horizontal force 'p' is applied to it as shown.

ii. When force 'P' is small, the block does not move. This is because of the frictional force which balances P.

iii. The friction force is largely due to interlocking of irregularities in the two surfaces in contact.

iv. If the force P is increased, F_r increases and remains equal to P as long as object is static.
v. When P is increased beyond a certain value $(F_r)_{max}$ the object starts moving, the frictional force decreases and then remains nearly constant.

* Points to remember:

i. As long as the object is static, the frictional force has same magnitude as the net force trying to move the object and has opposite direction.
ii. Motion impends when net force trying to move the object becomes equal to the maximum frictional force $(F_r)_{max}$ known as the limiting static friction force.
iii. When object starts moving, the frictional force is constant, independent of the net applied force.
iv. $(F_r)_{max} = \mu_s N$ → static friction

$(F_K)_f = \mu_K . N$ → Kinetic friction

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* Laws of Static friction

i. Under static conditions, the friction force opposes tendency for relative motion between the two surfaces in contact and acts tangential to the surfaces.

ii. The limiting static friction force, which is the maximum value of the friction force, is directly proportional to the normal reaction between the two surfaces in contact i.e.

$$(F_r)_{max} \propto N.$$

$$(F_r)_{max} = \mu_s . N.$$

iii. Limiting force of static friction is independent of the area of the two surfaces in contact.

iv. Limiting force of static friction depends on the nature and material of the two surfaces in contact.

* Laws of Kinetic Friction

i. The force of kinetic friction opposes the relative motion between two surfaces in contact.

ii. The force of kinetic friction is directly proportional to the normal reaction between the two surfaces in contact.

i.e $F_K \propto N$

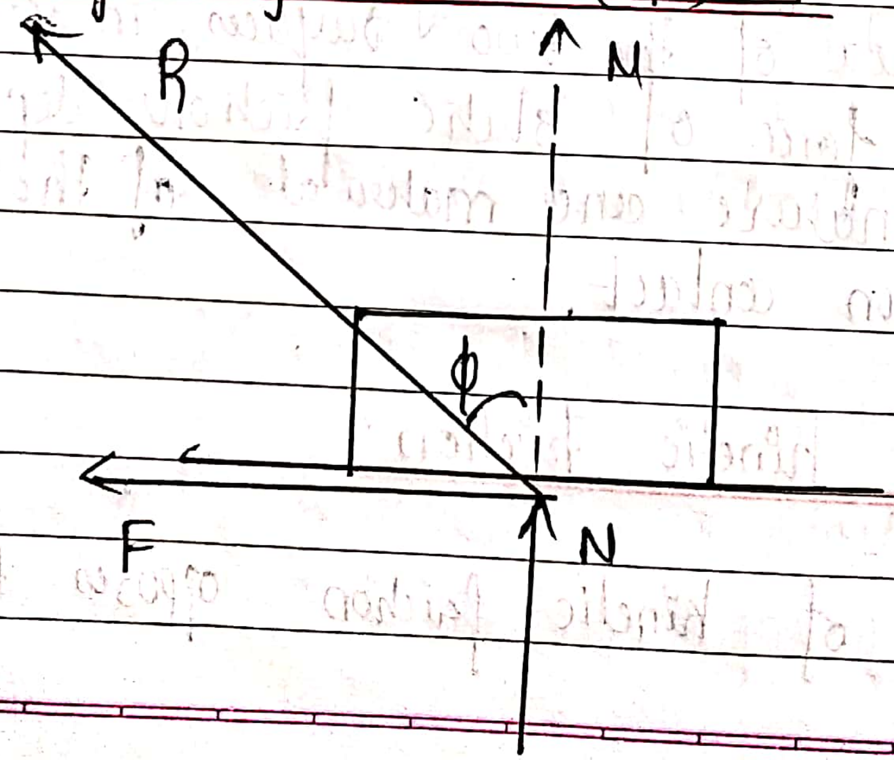
$F_K = \mu_k . N$

iii. Force of kinetic friction is independent of the area of the two surfaces in contact.

iv. Force of kinetic friction depends on the nature and material of the two surfaces in contact.

v. Force of kinetic friction is independent of velocity for small velocities.

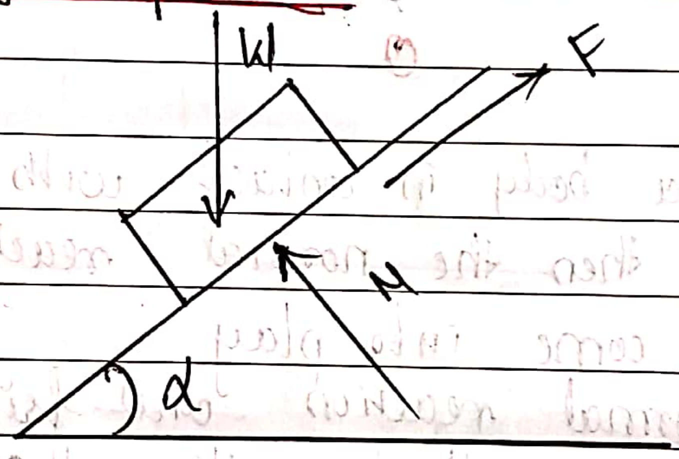
* Angle of friction (ϕ).



The angle which the resultant reaction R due to the normal reaction N and friction F makes with the normal to the surface is called Angle of friction (ϕ).

$$\tan\phi = \frac{F}{N} = \mu.$$

* Angle of Repose



If a body is placed on an inclined plane, then the angle at which the body is just on the point or verge of sliding down is called Angle of Repose.

Angle of repose (α) = Angle of Limiting friction (ϕ).

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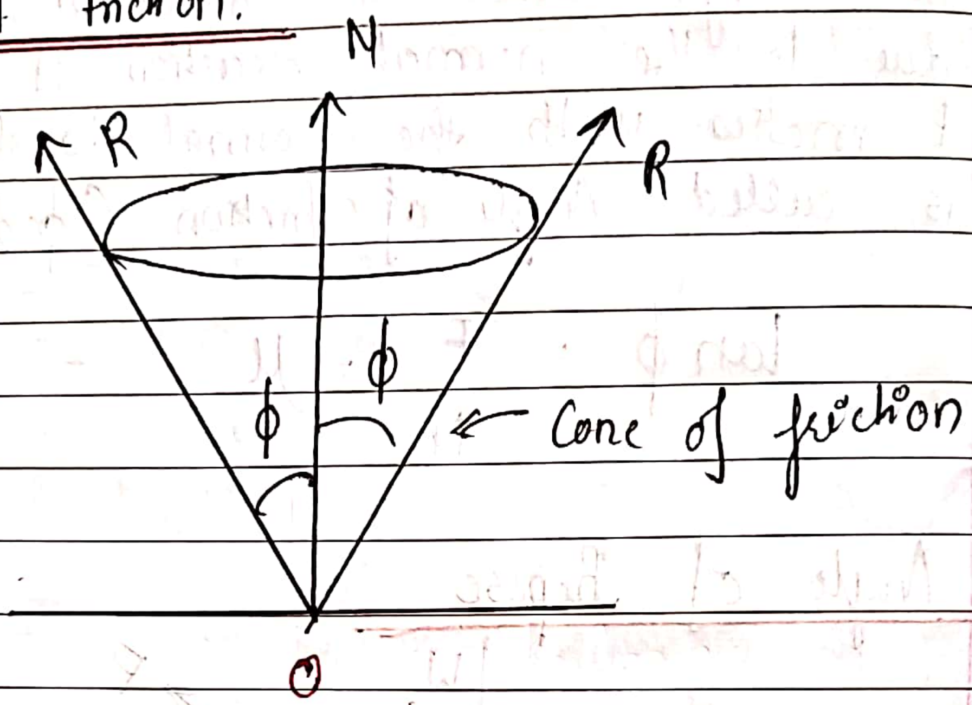
* Cone of friction.



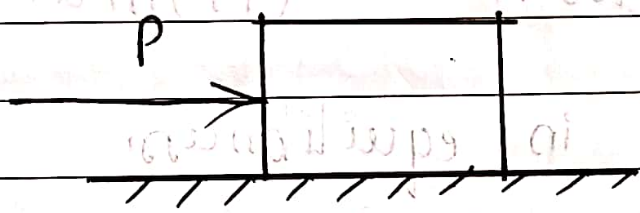
Whenever a body in contact with other tend to move, then the normal reaction ON and friction come into play.
The normal reaction and friction can be replaced by resultant reaction OR. When this resultant reaction OR making angle ϕ is revolved around point 'O' will form a right circular cone called cone of friction.

* Numericals :

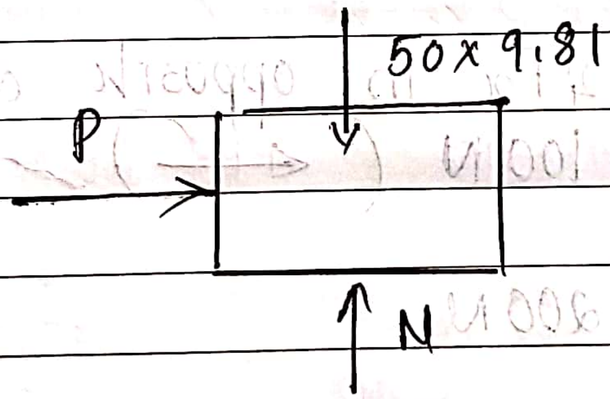
1. Determine whether the 50 kg block shown in figure is in equilibrium and find the magnitude and direction of the frictional force when

(a) $P = 100N$ (b) $P = 200N$

Take $\mu_s = 0.3$ and $\mu_k = 0.2$



Soln:



The FBD of the block is as shown in the figure excluding the frictional force

$$\sum F_y = 0$$

$$N - 50 \times 9.81 = 0$$

$$N = 490.5 N$$

$(F_r)_{max} = \mu_s . N$

$= 0.3 \times 490.5$

$(F_r)_{max} = 147.15 N.$

$\sum F_x = P \; (\longrightarrow)$

a) For $P = 100 N.$

$\sum F_x = 100 N < (F_r)_{max}.$

$\therefore$ Block is in equilibrium

$F_r = \sum F_x$ in opposite direction

$\therefore F_r = 100 N \; (\longleftarrow).$

b) For $P = 200 N$

$\sum F_x = 200 N$

$\sum F_x > (F_r)_{max}$

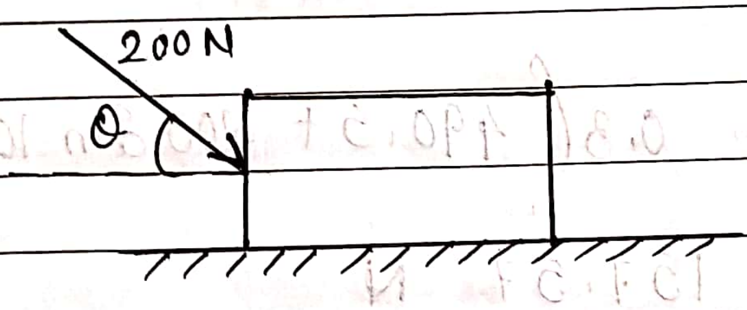
$\therefore$ Object moves towards right

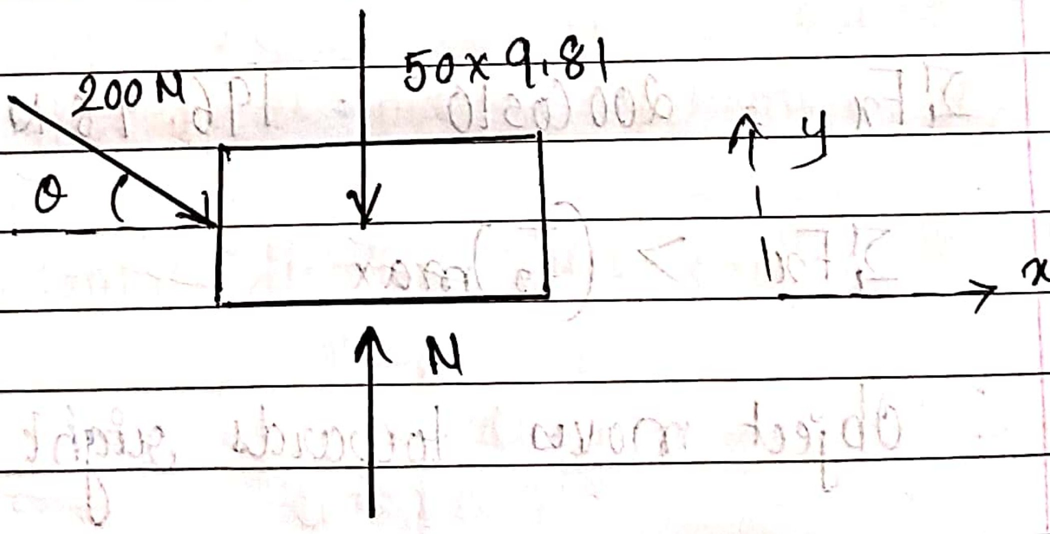
$F_r = \mu_k . N = 0.2 \times 490.5$

$F_r = 98.1 N \ (\leftarrow)$

2. Determine whether the 50 kg block shown in figure is in equilibrium and find the magnitude and direction of the frictional force when
a) $\theta = 10°$ and b) $\theta = 40°$, Take $\mu_s = 0.3$ and $\mu_k = 0.2$

Soln :





The F.B.D of block is as shown in the figure without the frictional force

$$\sum F_y = 0$$

$$N - 50 \times 9.81 - 200 \sin\theta = 0$$

$$N = (490.5 + 200 \sin\theta)$$

$$(F_r)_{max} = \mu_s . N.$$

$$= 0.3\,(490.5 + 200 \sin\theta)$$

a) For $\theta = 10^\circ$

$$(F_r)_{max} = 0.3\,(490.5 + 200 \sin 10^\circ)$$

$$= 157.57 \text{ N}$$

$$\sum F_x = 200 \cos 10^\circ = 196.96 \text{ N}$$

$$\sum F_x > (F_r)_{max}$$

$\therefore$ Object moves towards right

$$F_r = \mu_k . N$$

$$= 0.2\,(490.5 + 200 \sin 10^\circ)$$

$$F_r = 105.05 \text{ N}$$

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b) For $\theta = 40^\circ$

$(F_r)_{max} = 0.3\,(490.5 + 200 \sin 40^\circ)$
$= 185.72\,N$

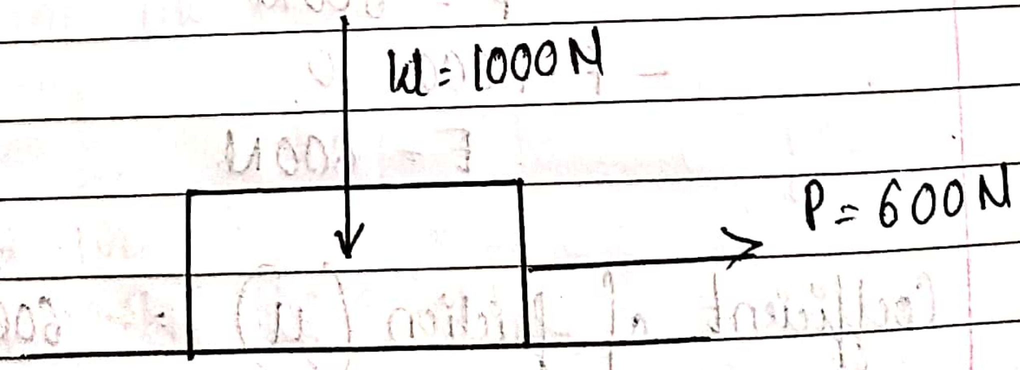
$\Sigma F_x = 200 \sin 40^\circ$
$= 153.21\,N \;(\longrightarrow)$

$\Sigma F_x < (F_r)_{max}$ $\therefore$ Object doesn't move.

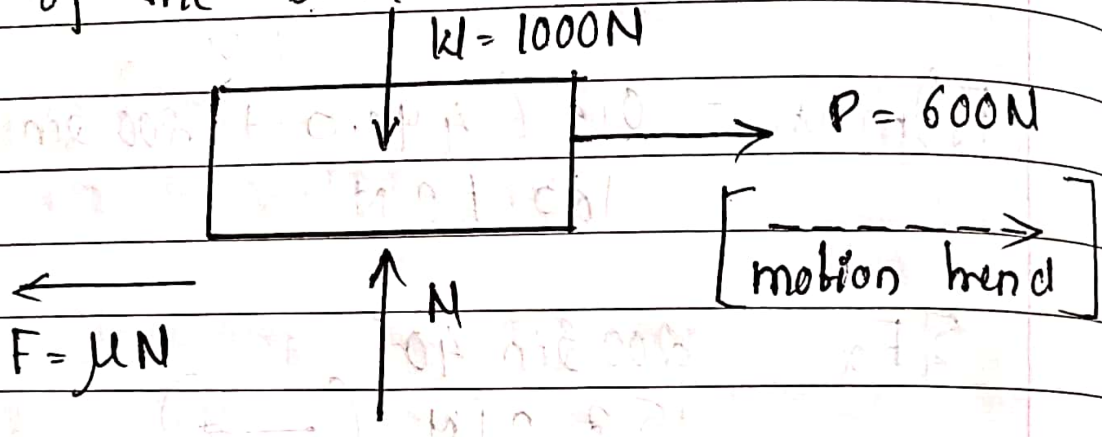
$F_r = \Sigma F_x$
$F_r = 153.21\,N \;(\longleftarrow)$

3. A body of weight 1000 N is placed on a rough horizontal plane. Determine the coefficient of friction due to a force of 600 N in horizontal direction just causes the body to slide.

Soln:



F.B.D of the block.



$F = \mu N$

$\mu = \dfrac{F}{N}$

$\Sigma F_y = 0 \qquad W - N = 0$

$1000 - N = 0$

$N = 1000N$

$\Sigma F_x = 0$

$-F + P = 0$

$P = 600N$

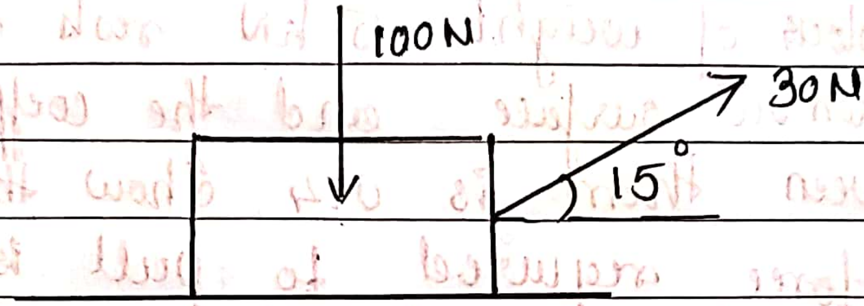
$-F + 600 = 0$

$F = 600N$

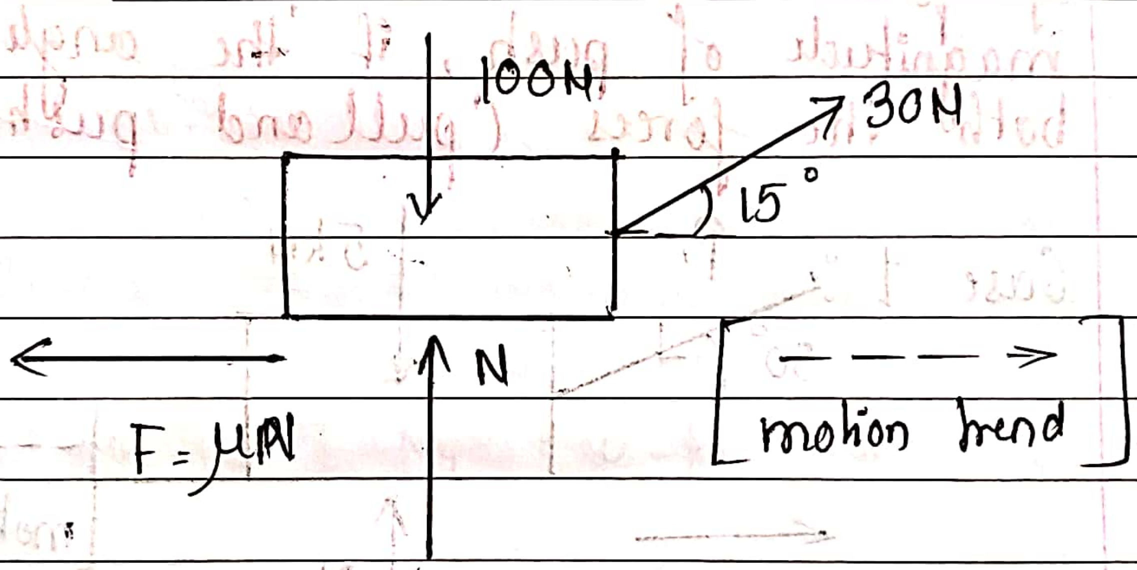
Coefficient of friction $(\mu) = \dfrac{600}{1000}$

$\mu = 0.6$.

4. A body weighing 100 N is placed on a rough horizontal plane and is pulled by a force of 30 N inclined at 15° with horizontal. Find the coefficient of friction.



n:



F = μN or $\mu = \frac{F}{N}$

F.B.D of the block

$\sum F_y = 0$

$30 \sin 15 + N - 100 = 0$

$N = 92.24 N$

$\sum F_x = 0$

$-F + 30 \cos 15^{\circ} = 0$

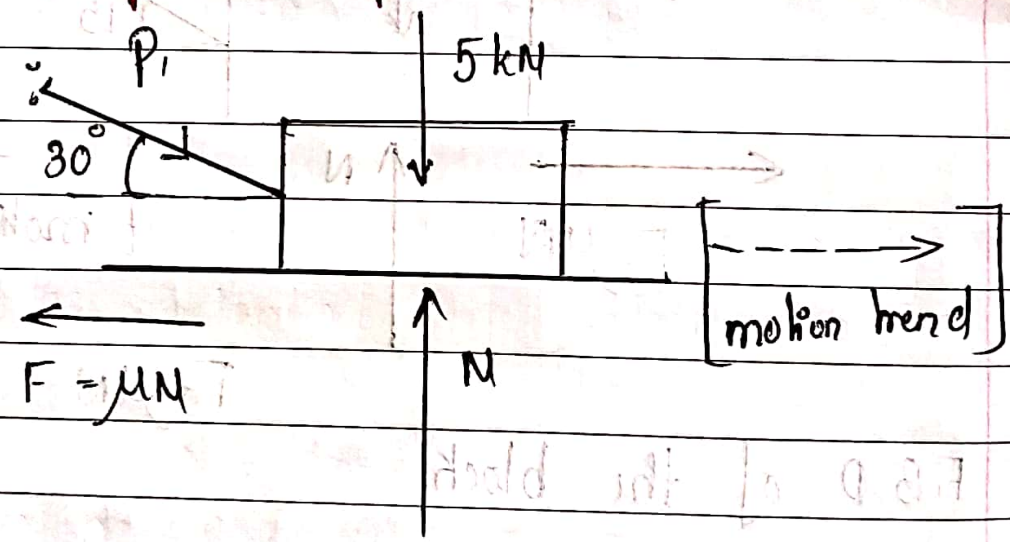
$F = 28.98 N$

$$\mu = \frac{28.98}{92.24}$$

$$\mu = 0.314$$

5. A block of weight 5 kN rests on a rough horizontal surface and the coefficient of friction between them is 0.4. Show that the magnitude of force required to pull is less than the magnitude of push, if the angle made by both the forces (pull and push) is 30°.

Soln: Case 1 :-



$\sum F_y = 0$

$$-5 - P_1 \sin 30^\circ + N = 0$$

$$N = P_1 \sin 30^\circ + 5$$

$\sum F_x = 0$

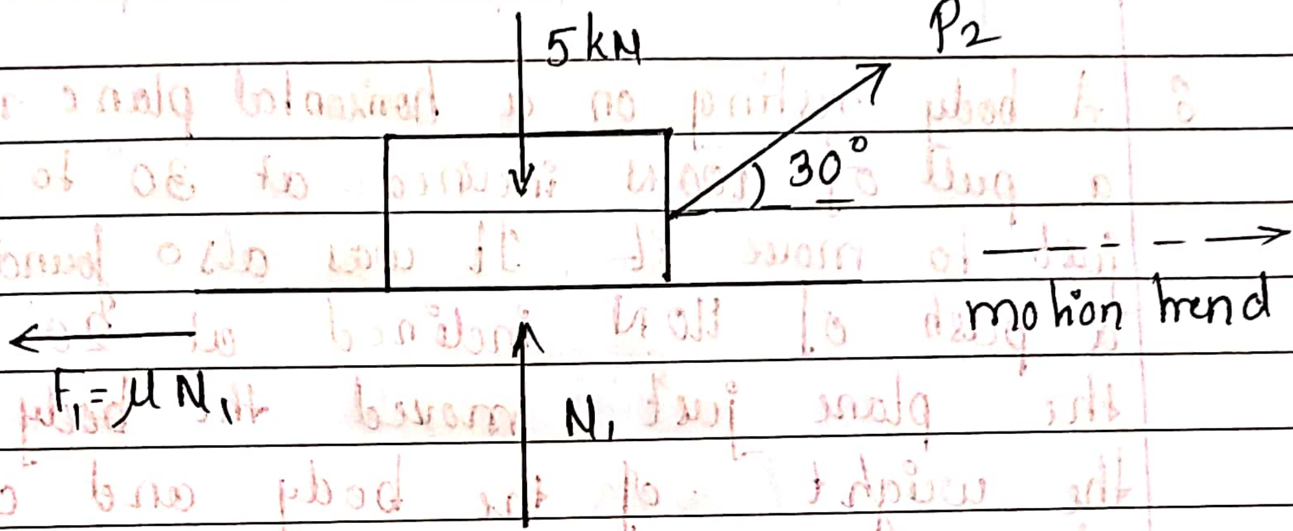
$$P_1 \cos 30^\circ - F = 0$$

$$P_1 \cos 30^\circ - \mu N = 0$$

$$P_1 \cos 30^\circ - 0.4\,(P_1 \sin 30^\circ + 5) = 0$$

$$P_1 = 3.12 \text{ kN.}$$

Case 2 :



$\sum F_y = 0$

$$N_1 - 5 + P_2 \sin 30^\circ = 0$$

$$N_1 = 5 - P_2 \sin 30^\circ$$

$\sum F_x = 0$

$$-F_1 + P_2 \cos 30^\circ = 0$$

$$P_2 \cos 30^\circ = \mu N_1$$

$$P_2 \cos 30^\circ = 0.4\,(5 - P_2 \sin 30^\circ)$$

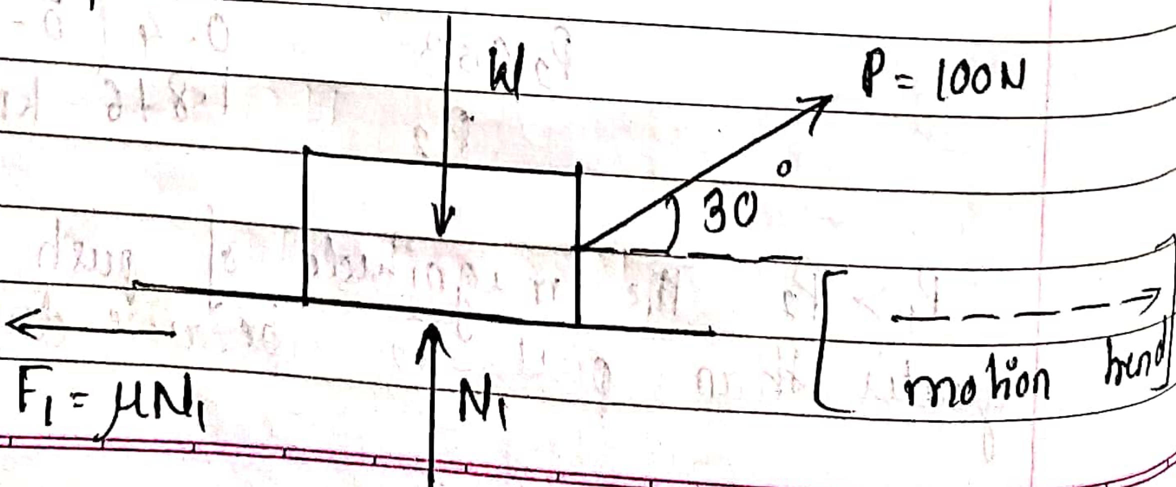
$$P_2 = 1.876 \text{ kN.}$$

$P_1 > P_2$. The magnitude of push P_1 is greater than pull P_2 because during push the vertical component of push acts downwards when resolved vertically, adding to the weight of the body. which inturn increases the reaction and thereby increases the friction. Therefore pull is preferred over push.

6 A body resting on a horizontal plane required a pull of 100 N inclined at 30° to horizontal just to move it. It was also found that a push of 110 N inclined at 20° to the plane just moved the body. Determine the weight of the body and coefficient of friction.

Soln: Let 'W' be the weight of the body and 'μ' be the coefficient of friction.

Case 1: When a pull of 100 N inclined at 30° to plane.



Consider F.B.D of the body.

$\sum F_y = 0$

$$-W + N_1 + 100 \sin 30^\circ = 0$$

$\sum F_x = 0$

$$-F_1 + 100 \cos 30^\circ = 0$$

$$-\mu N_1 + 100 \cos 30^\circ = 0$$

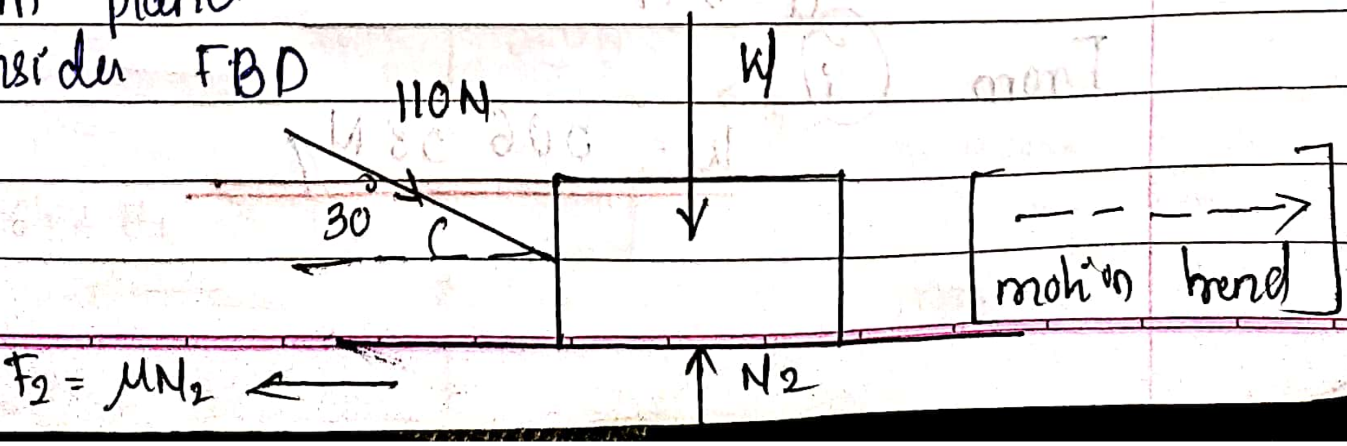
$$N_1 = \frac{86.6}{\mu}$$

$$-W + \frac{86.6}{\mu} + 100 \sin 30^\circ = 0$$

$$W - \frac{86.6}{\mu} = 50 \longrightarrow (i)$$

Case 2: When a push of 110 N inclined at 20° with plane.

Consider FBD



$\sum F_y = 0$

$-W - 110\sin 30^\circ + N_2 = 0$

$\sum F_x = 0$

$-F_2 + 110\cos 30^\circ = 0$

$-\mu N_2 + 110\cos 30^\circ = 0$

$$N_2 = \frac{103.37}{\mu}$$

$$-W - 110\sin 30^\circ + \frac{103.37}{\mu} = 0$$

$$-W + \frac{103.37}{\mu} = 37.62 \longrightarrow (ii)$$

Solving (i) and (ii)

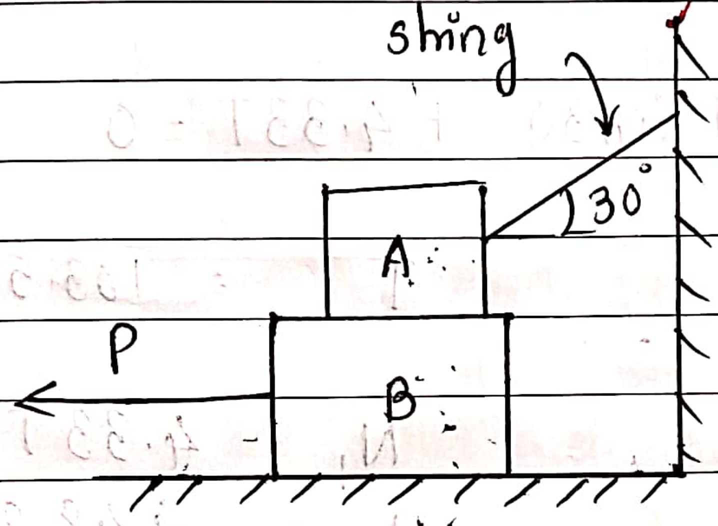
$\mu = 0.19$

From (i)

$W = 506.38\,N$

7. Find the force P just required to slide the block B in the arrangement shown on figure. Find also the tension in the string.

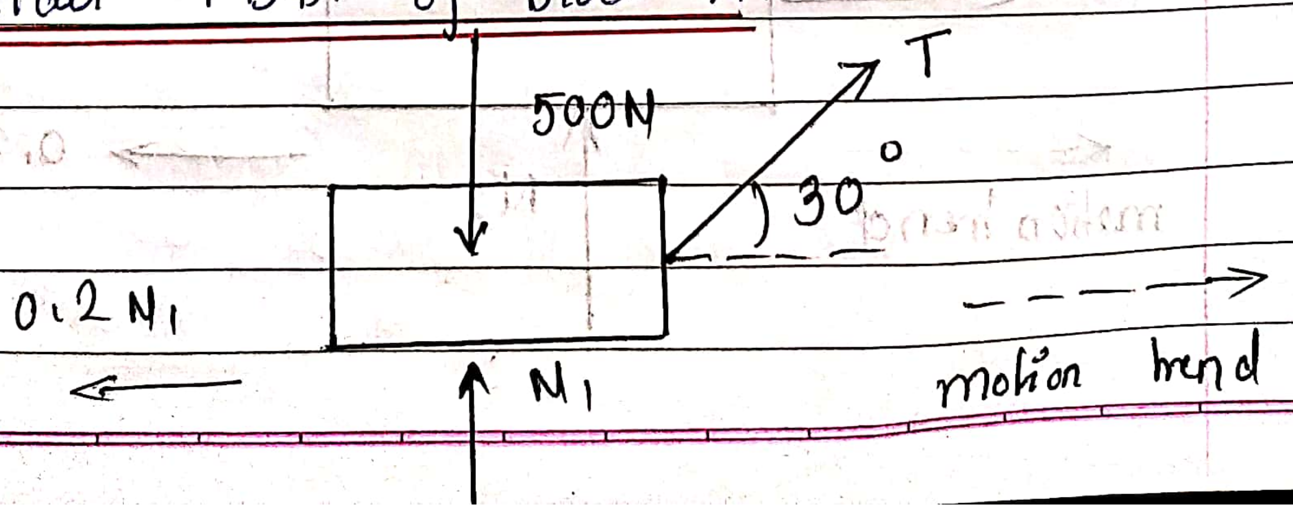
Take, Weight of block A = 500N
Weight of block B = 1000N
μ = 0.2



Soln: Block B tends to move left due to P and Block A tends to move towards right w.r.t Block B.

Let T be the tension in string.

Consider F.B.D. of block A:



$\Sigma F_y = 0$

$-500 + T\sin 30^\circ + N_1 = 0$

$\Sigma F_x = 0$

$T\cos 30^\circ - 0.2 N_1 = 0$

$N_1 = 4.33T \rightarrow (i)$

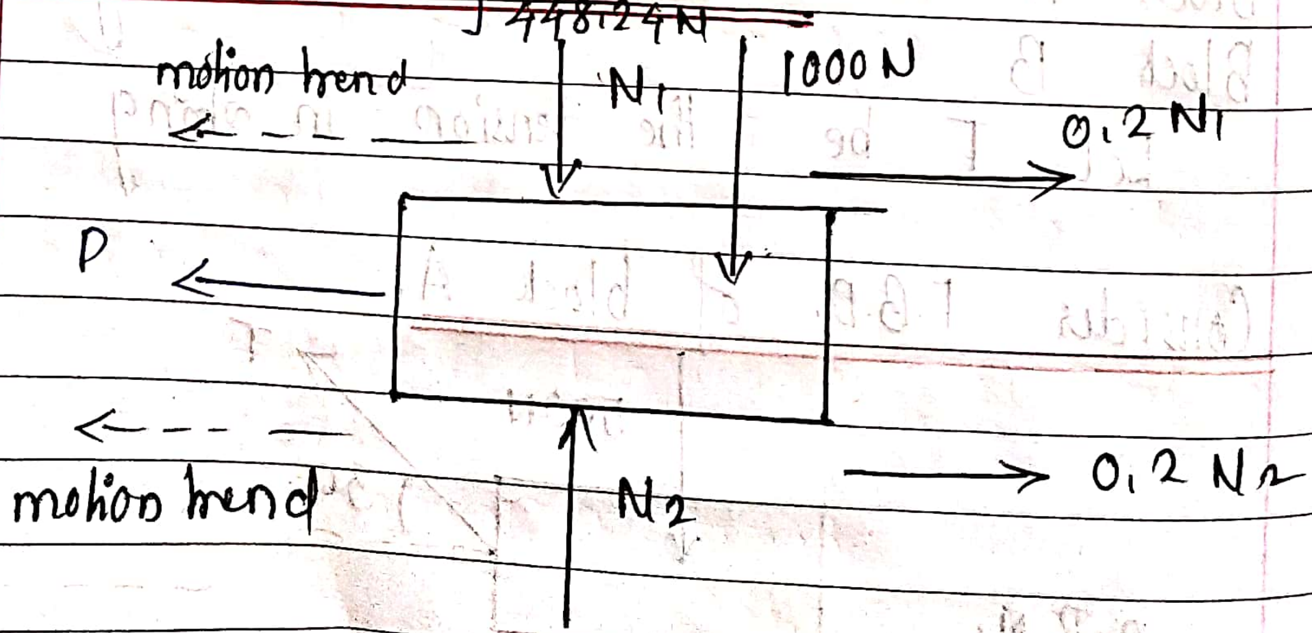
$-500 + T\sin 30^\circ + 4.33T = 0$

$T = 103.52 \text{ N}.$

$N_1 = 4.33T$

$\underline{N_1 = 448.24 \text{ N}}$

Consider FBD of Block B



$\Sigma F_y = 0$

$-448.24 - 1000 + N_2 = 0$

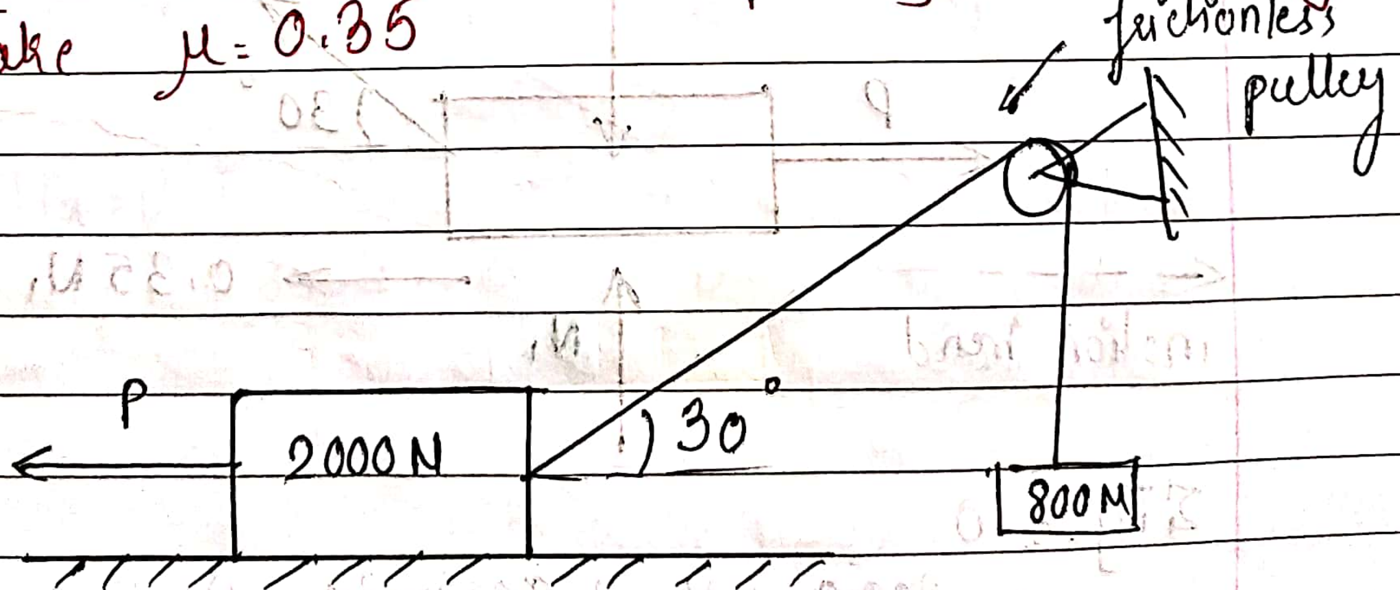
$N_2 = 1448.24 N$

$\Sigma F_x = 0$

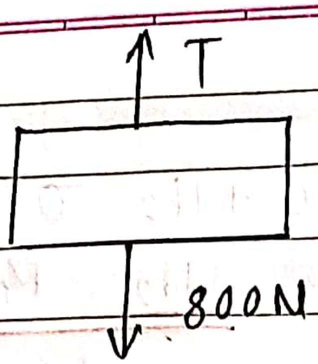
$-P + 0.2N_1 + 0.2N_2 = 0$

$P = 379.30 N$

8. A block A weighing 2000 N is attached to one end of the chord which passes round a frictionless pulley as shown, carries a load of 800 N at other end. Find the value of P
(i) If the motion is impending towards left.
(ii) If the motion is impending towards right.
Take $\mu = 0.35$



Soln:



Let T be the tension in string.

Consider F.B.D of 800N load.

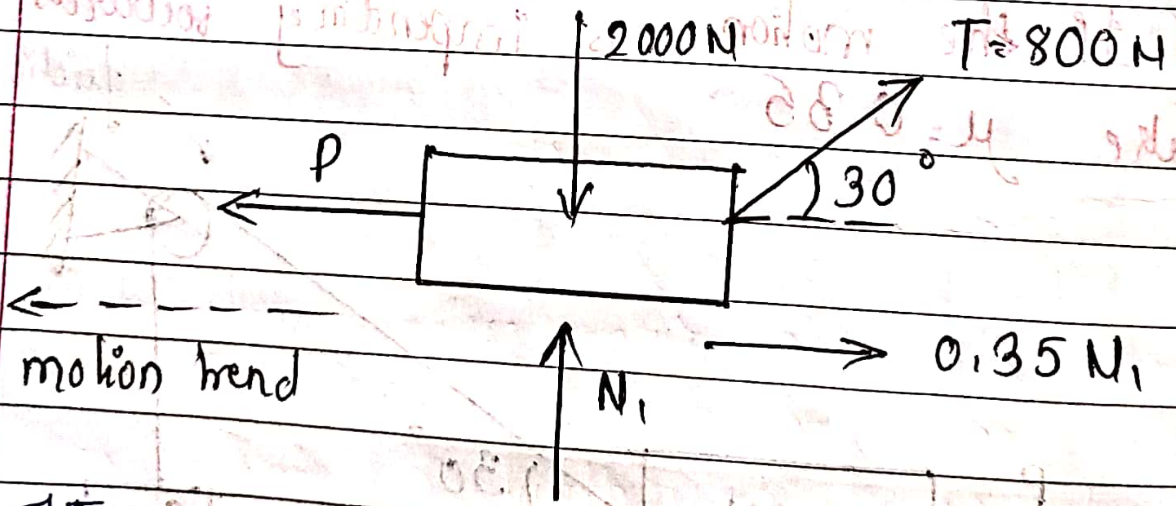
$\sum F_y = 0$

$T - 800 = 0$

$\underline{T = 800N}$

Case (i): If the motion is impending towards left

Consider F.B.D of 2000 N.



$\sum F_y = 0$

$-2000 + N_1 + 800 \sin 30° = 0$

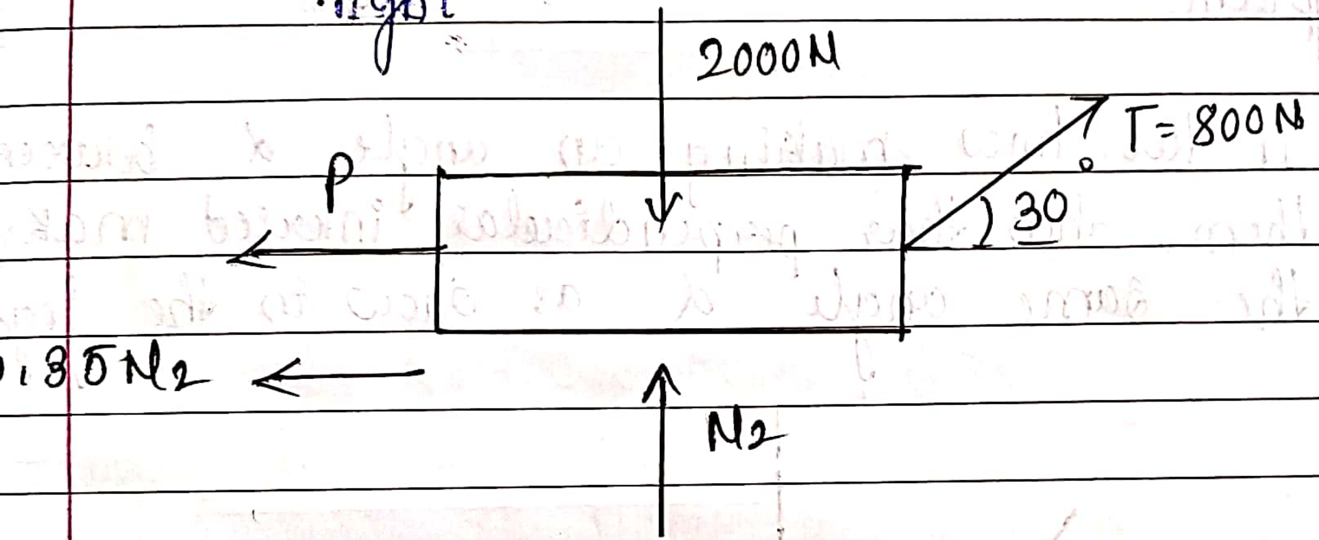
$N_1 = 1600N$

$\Sigma F_x = 0$

$-P + 800\cos 30^\circ + 0.35 \times 1600 = 0$

$P = 1252.82\,N.$

Case (ii). If the motion is impending towards right



$\Sigma F_y = 0$

$-2000 + N_2 + 800\sin 30^\circ = 0$

$N_2 = 1600N$

$\Sigma F_x = 0$

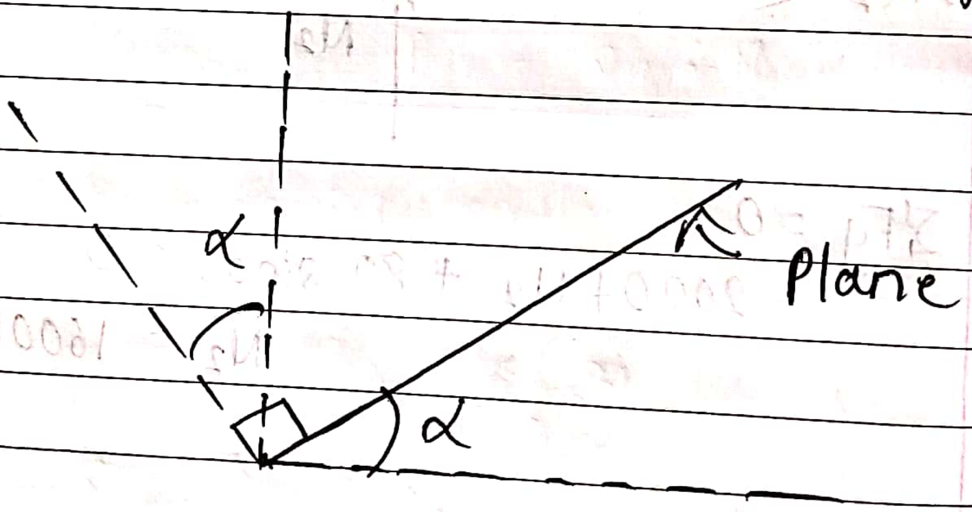
$-P + 800\cos 30^\circ - 0.35 \times 1600 = 0$

$P = 132.82\,N.$

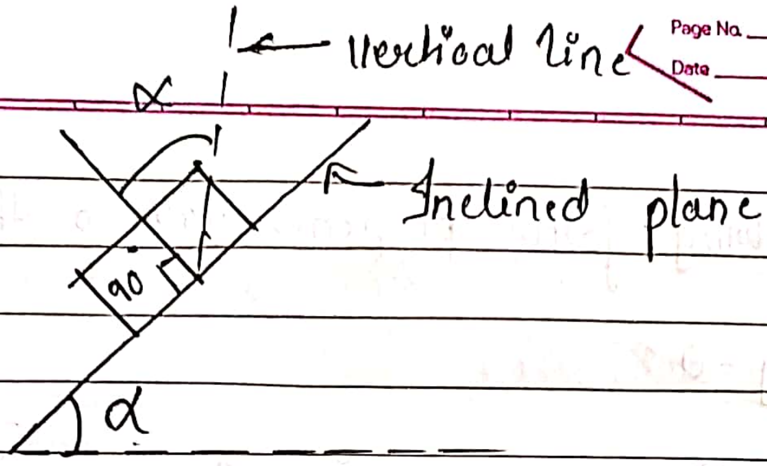
* Problems on Blocks Placed on Inclined Planes

i. In inclined planes it is convenient to resolve the forces along or parallel to the inclined plane and perpendicular to the inclined plane.

ii. If two lines making an angle α between them, then their perpendicular intersect make the same angle α as show in the figure

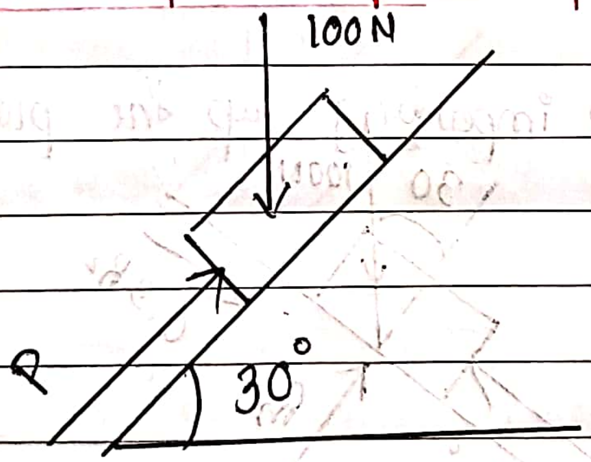


iii. If a plane makes angle α with horizontal then the perpendicular to the plane will make same angle α with vertical as shown in the figure.

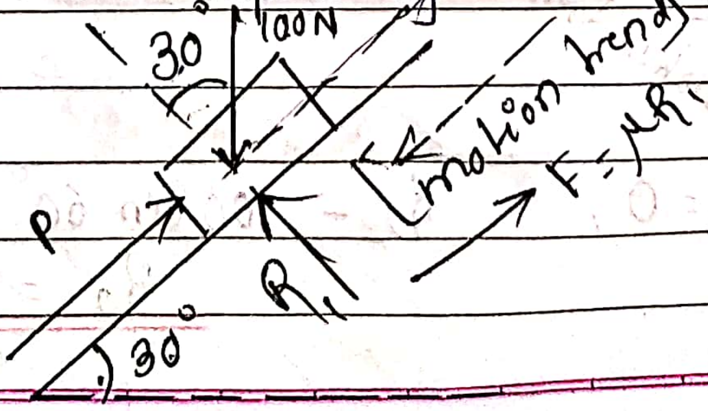


* **Numericals:**

(1) Find the value of 'P' so that the body will not impend down the plane. Also find the value of P for the body to impend down the plane. Also find the value of P for the body to impend up the plane. Take $\mu = 0.3$



Soln: Case i: Motion impending down the plane.



Resolving forces perpendicular to the plane.

$\sum F_y = 0$

$R_1 - 100\cos 60^\circ = 0$

$R_1 = 86.60\,N.$

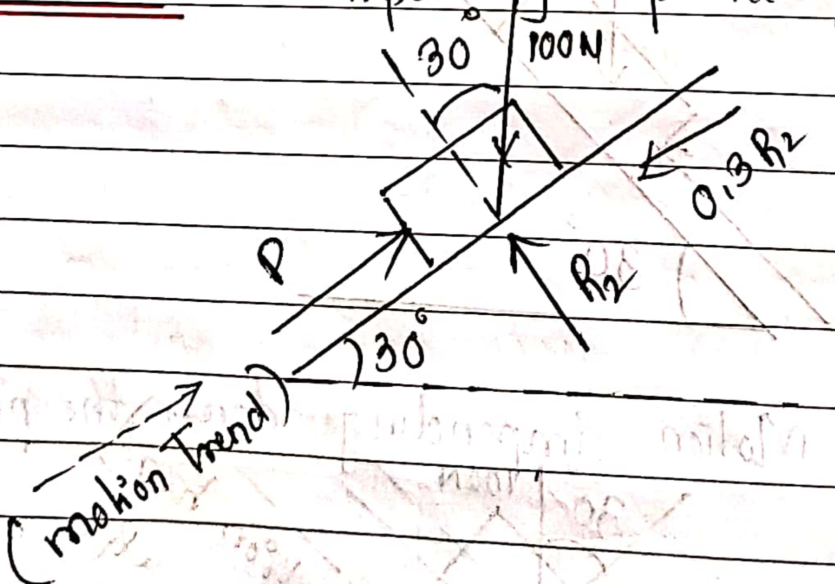
Resolving forces parallel to the plane

$\sum F_x = 0$

$P + 0.3R_1 - 100\cos 60^\circ = 0$

$P = 24.2\,N.$

Case ii. Motion impending up the plane.



$\sum F_y = 0, \quad R_2 - 100\sin 60^\circ = 0$

$R_2 = 86.6\,N$

$\Sigma F_x = 0$,

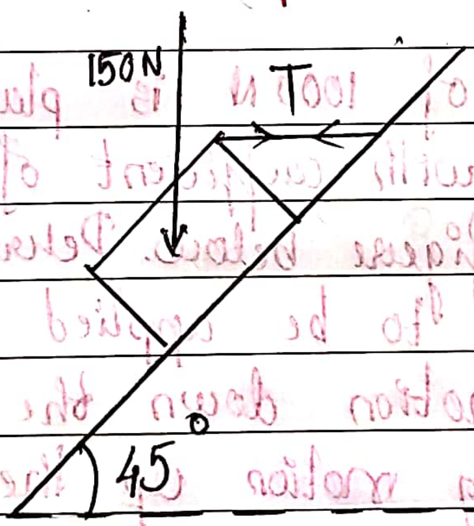
$P - 0.3 R_2 - 100 \cos 60^\circ = 0$

$P = 75.98 N$.

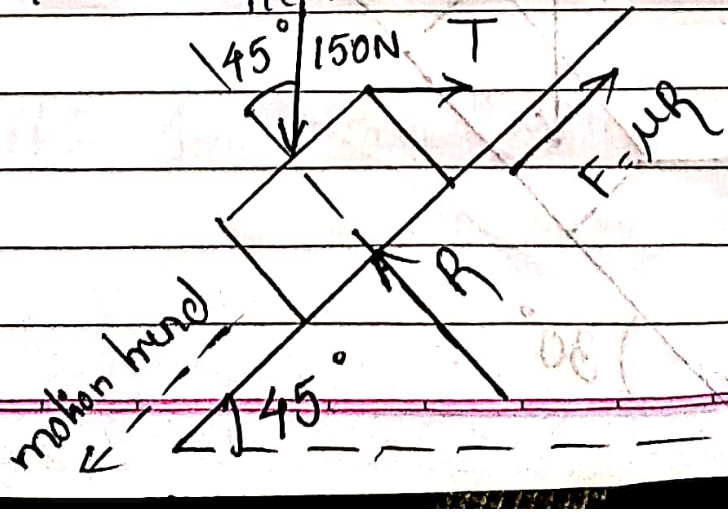
(2) A block is resting on a rough inclined plane as shown in figure below. The block is tied up by a horizontal string which has a tension of 50 N. Find :

(a) The friction force on the block and the normal reaction in the inclined plane.

(b) the coefficient of friction between contact faces.



Solⁿ: Let 'T' be the tension in string, given $T = 50N$.



$\sum_i F_y = 0$

$R - 150 \sin 45^\circ - 50 \sin 45^\circ = 0$

$R = 141.42\,N.$

$\sum_i F_x = 0$

$F - 150 \cos 45^\circ + 50 \cos 45^\circ = 0$

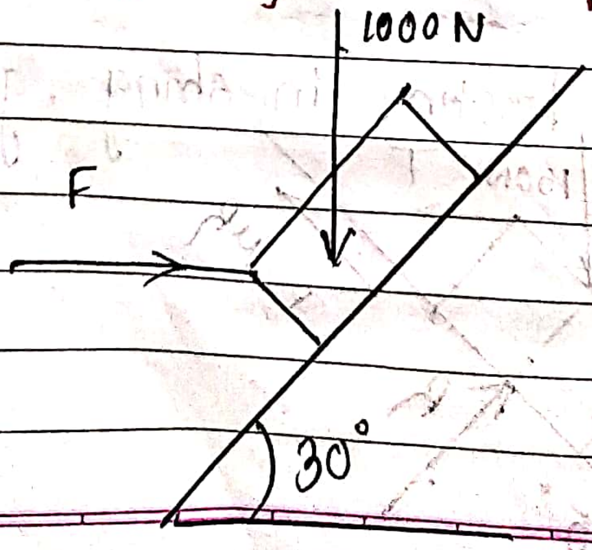
But $F = \mu R = \mu 141.42.$

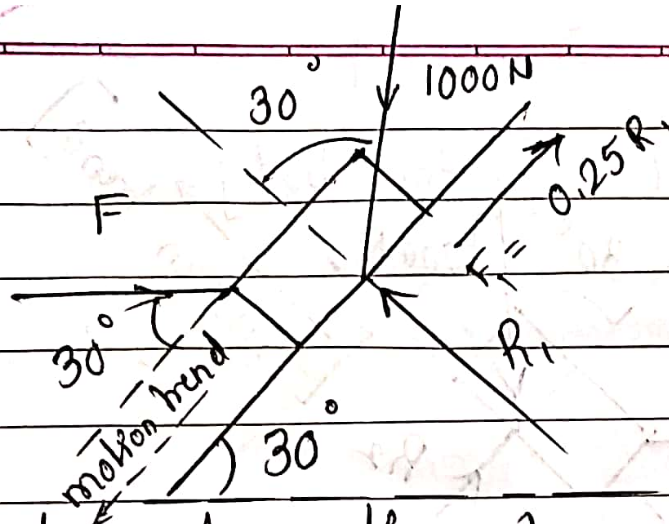
$\mu = 0.50.$

(3) A small block of 1000 N is placed on a 30° inclined plane with coefficient of friction of 0.25 as shown in figure below. Determine the horizontal force to be applied for

(i) impending motion down the plane.

(ii) the impending motion up the plane.





For motion down the plane.

Case i : Let F be the force applied for impending motion down the plane.

Consider FBD of the body

$\sum F_y = 0$

$$R_1 - 1000 \sin 60^\circ - F \sin 30^\circ = 0$$

$$R_1 = F \sin 30^\circ + 1000 \sin 60^\circ \longrightarrow (i)$$

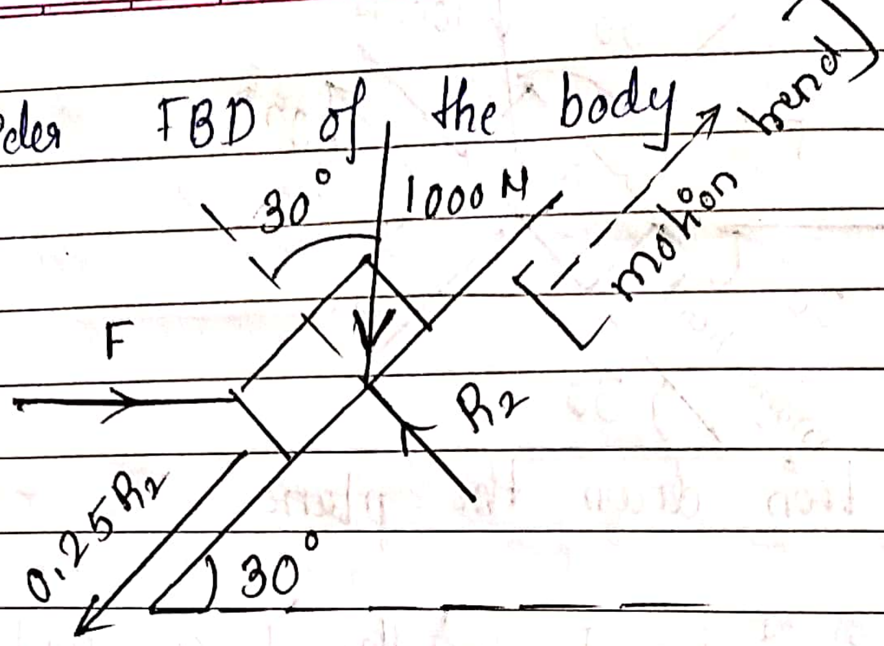
$\sum F_x = 0$

$$0.25 R_1 - 1000 \cos 60^\circ + F \cos 30^\circ = 0$$

$$F = 286.36\ N.$$

Case ii : Impending motion up the plane

Consider FBD of the body



$\Sigma F_y = 0$

$$R_2 - 1000\sin 60^\circ - F\sin 30^\circ = 0$$

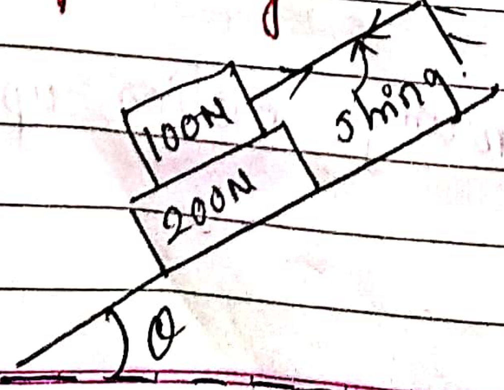
$$R_2 = F\sin 30^\circ - 1000\sin 60^\circ$$

$\Sigma F_x = 0$

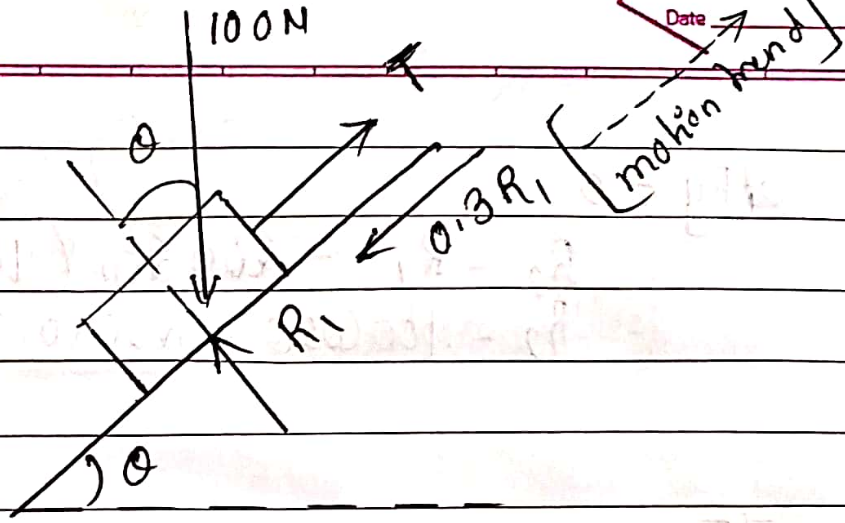
$$F\cos 30^\circ - 0.25R_2 - 1000\cos 60^\circ = 0$$

$$F = 968.24N$$

(4) What should be the value of angle 'θ' so that motion of the block 200N impends down the plane take $\mu = 0.3$ for all contact surface. Figure shown below.



Soln:



Let 'T' be the tension in the string.
Consider F.B.D of 100 N block.

$\sum F_y = 0$

$$R_1 - 100 \sin(90° - \theta) = 0$$

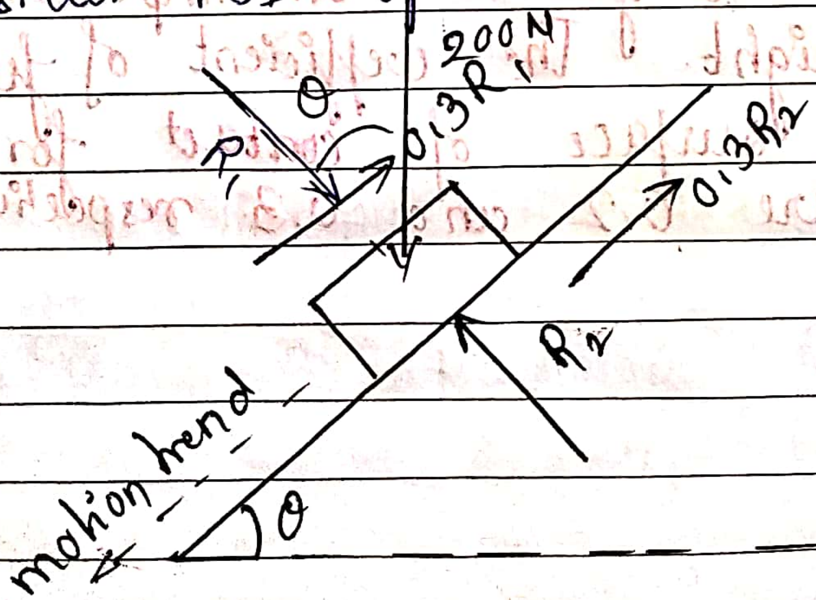
$$R_1 = 100 \cos\theta \longrightarrow (i)$$

$\sum F_x = 0$

$$T - 100 \cos(90° - \theta) - 0.3(100\cos\theta) = 0$$

$$T = 100 \sin\theta + 30\cos\theta \longrightarrow (ii)$$

Consider FBD of 200 N block



$\sum F_y = 0$

$$R_2 - R_1 - 200 \sin(90^\circ - \theta) = 0$$
$$R_2 - 100\cos\theta - 200\cos\theta = 0$$
$$R_2 = 300\cos\theta$$

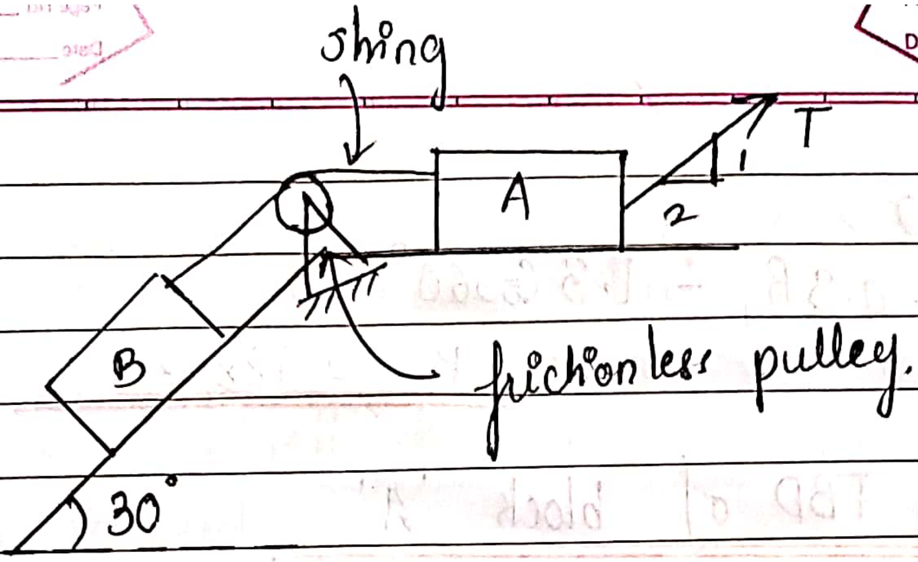
$\sum F_x = 0$

$$0.3R_1 + 0.3R_2 - 200\cos(90^\circ - \theta) = 0$$

$$\theta = 30.93^\circ$$

$$T = 77.14N$$

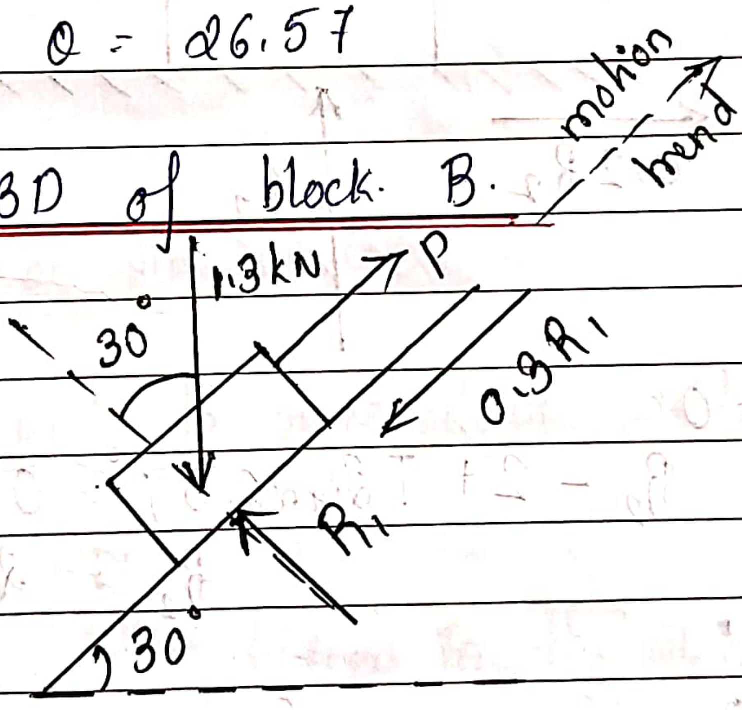
(5) Two blocks A and B weighing 3 kN and 1.3 kN respectively are connected up by a string over a frictionless pulley as shown in figure below. Find the minimum value of force T to generate an impending motion to the right. The coefficient of friction for the surface of contact for block A and B are 0.2 and 0.3 respectively.



Let 'θ' be the angle made by 'T' with horizontal

$$\tan\theta = 1/2$$
$$\theta = 26.57$$

Consider FBD of block B.



Let 'P' be the pull or tension in string.

μ for Block B = 0.3

$\Sigma F_y = 0$ $\quad R_1 - 1.3 \sin 60^\circ = 0$

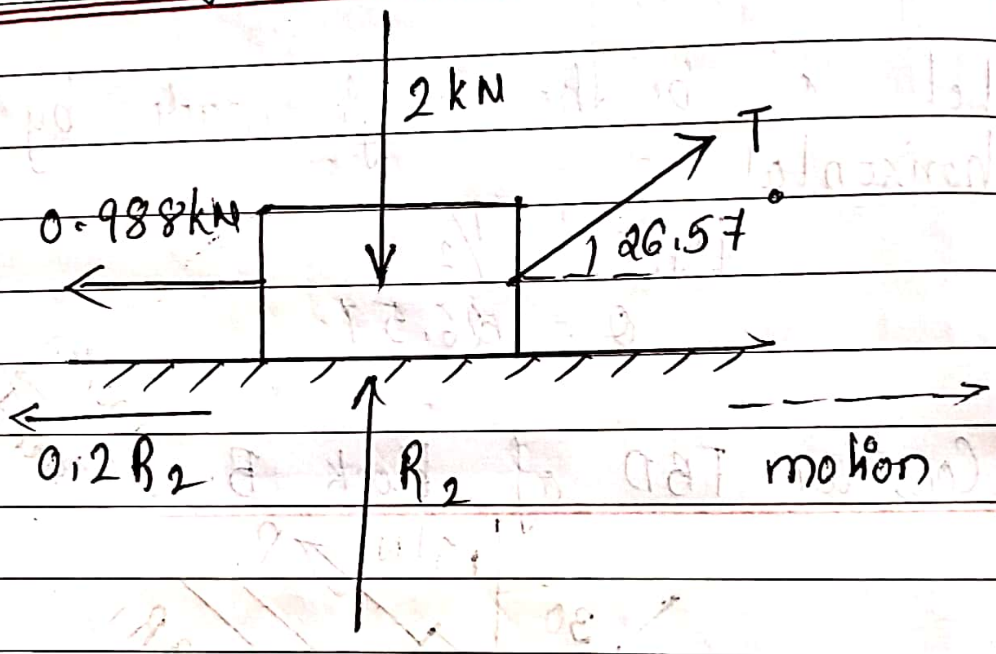
$R_1 = 1.1261$ kN

$\Sigma F_x = 0$

$P - 0.3R_1 - 1.3\cos 60^\circ = 0$

$\underline{P = 0.988 \text{ kN}}$

Consider FBD of block 'A':



$\Sigma F_y = 0$

$R_2 - 2 + T\sin 26.57^\circ = 0$

$R_2 = 2 - T\sin 26.57^\circ$

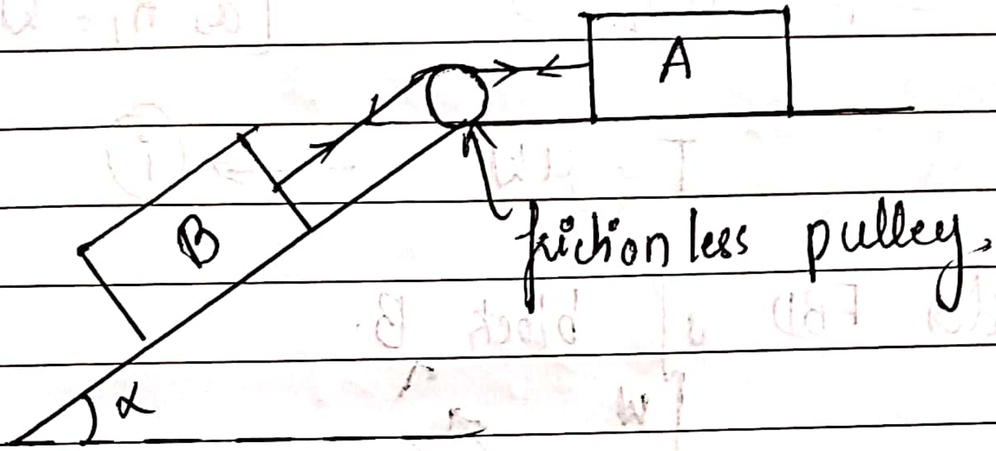
$\Sigma F_x = 0$

$T\cos 26.57^\circ - 0.988 - 0.2R_2 = 0$

$\underline{T = 1.41 \text{ kN.}}$

6) Two blocks A and B weighing W_1 and W_2 are connected as shown in the figure.

If $\mu_1 = \mu_2$ and if μ is the coefficient of friction for all contact surfaces. Find the angle of inclination of inclined plane α at which the motion of the system will impend.



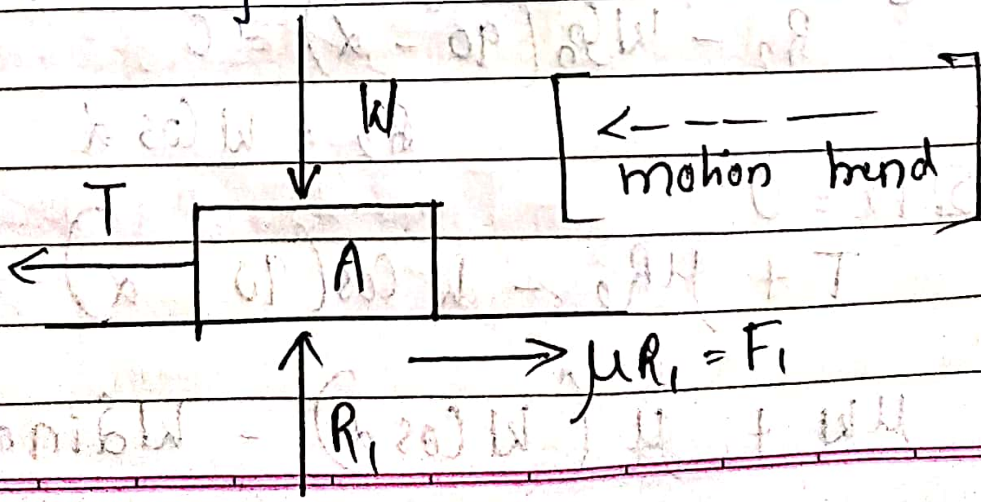
Soln: Let 'α' be the angle.

Block B tend to slide down and block A moves towards left.

Given $\mu_1 = \mu_2 = \mu$.

Let 'T' be the tension in the string.

Consider FBD of block A.



$\sum F_y = 0$

$R_1 - W = 0$

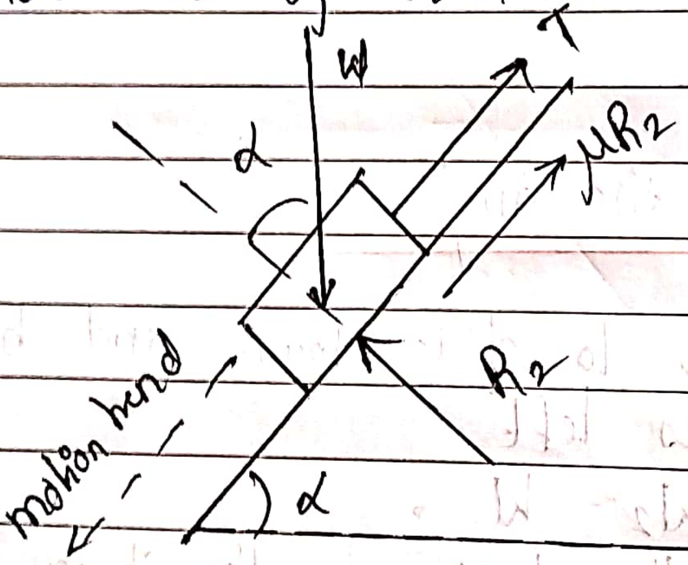
$R_1 = W.$

$\sum F_x = 0$

$-T + \mu R_1 = 0$ $\boxed{\text{as } R_1 = W}$

$T = \mu W \longrightarrow (i)$

Consider FBD of block B.



$\sum F_y = 0$

$R_2 - W \sin(90° - \alpha) = 0$

$R_2 = W \cos \alpha$

$\sum F_x = 0$

$T + \mu R_2 - W \cos(90° - \alpha) = 0$

$\mu W + \mu (W \cos \alpha) - W \sin \alpha = 0$

$$\mu W(1+\cos\alpha) = W\sin\alpha$$

$$\sin\alpha = \mu(1+\cos\alpha)$$

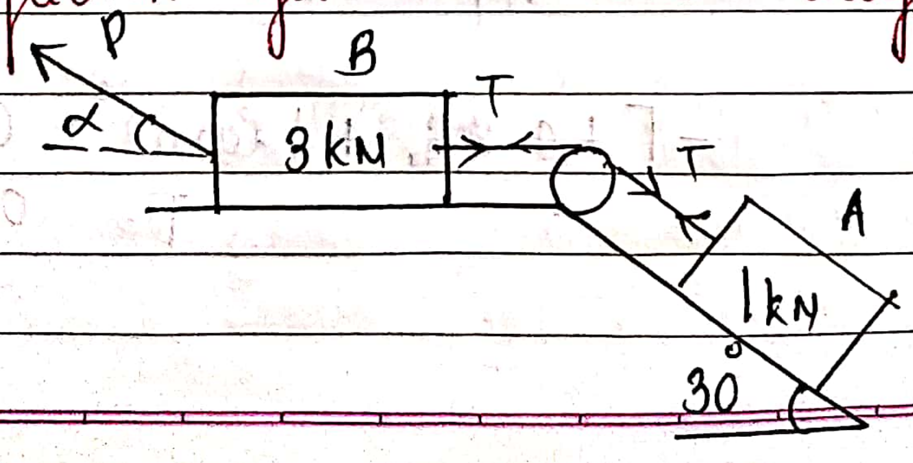
$$\mu = \frac{\sin\alpha}{1+\cos\alpha}$$

$$= \frac{2\sin\alpha/2\ \cos\alpha/2}{2\cos^2\left(\frac{\alpha}{2}\right)}$$

$$\tan\frac{\alpha}{2} = \mu$$

$$\alpha = 2\tan^{-1}\mu$$

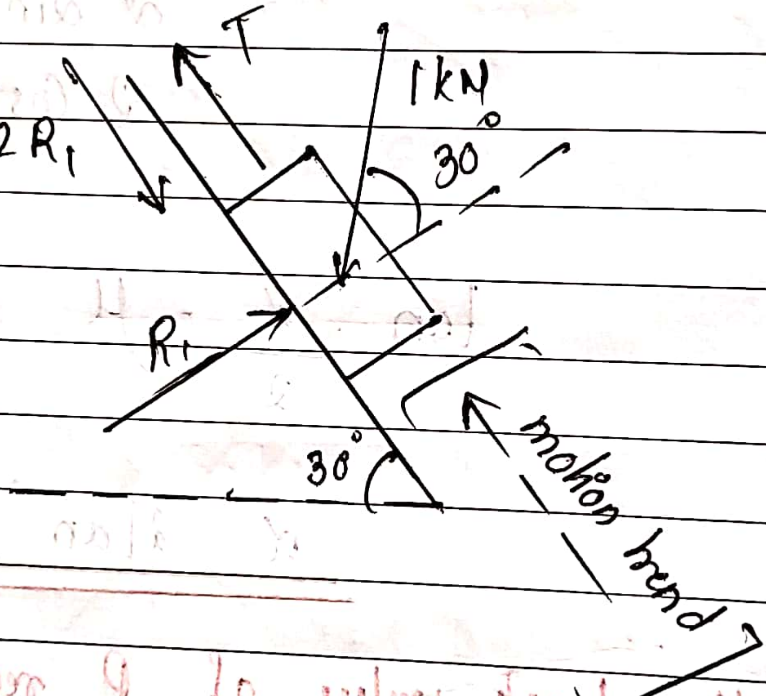
7) Find the least value of P required to cause the system of block shown in figure to have impending motion to the left. The coefficient of friction for all contact surfaces is 0.2



Soln: Let 'T' be the tension in the string.

Under the action of force 'P', the block B moves to left and A tends to move up the plane.

Consider FBD of block A.



$\sum F_y = 0$

$R_1 - 1 \sin 60° = 0$

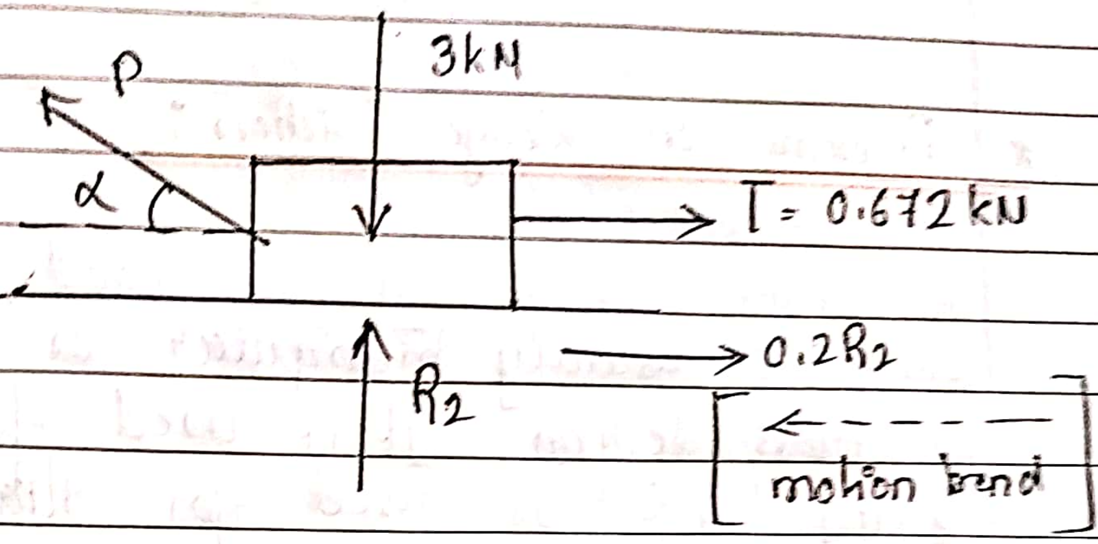
$R_1 = 0.86$ kN

$\sum F_x = 0$

$-T + 0.2 R_1 + 1 \cos 60° = 0$

$T = 0.672$ kN

Consider FBD of block B



$\Sigma F_y = 0$

$$R_2 - 3 + P\sin\alpha = 0$$

$$R_2 = 3 - P\sin\alpha$$

$\Sigma F_x = 0$

$$0.672 + 0.2R_2 - P\cos\alpha = 0$$

$$0.672 + 0.2(3 - P\sin\alpha) - P\cos\alpha = 0$$

for least value of P, $\frac{dP}{d\alpha} = 0$

$$0.2P\cos\alpha - P\sin\alpha = 0$$

$$\tan\alpha = 0.2$$

$$\alpha = 11.3$$

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$P = 1.247$ kN.

* Problems on Wedge Friction :

A wedge is a piece of wood or metal which is usually triangular or trapezoidal in cross section. It is used for lifting heavy loads or used for slight adjustment in the position of body.

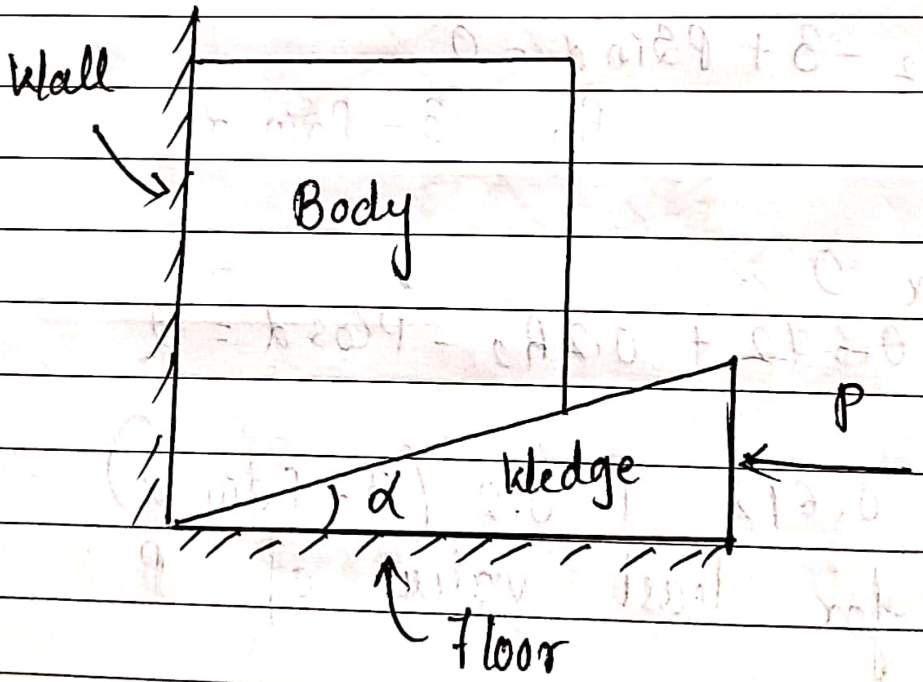
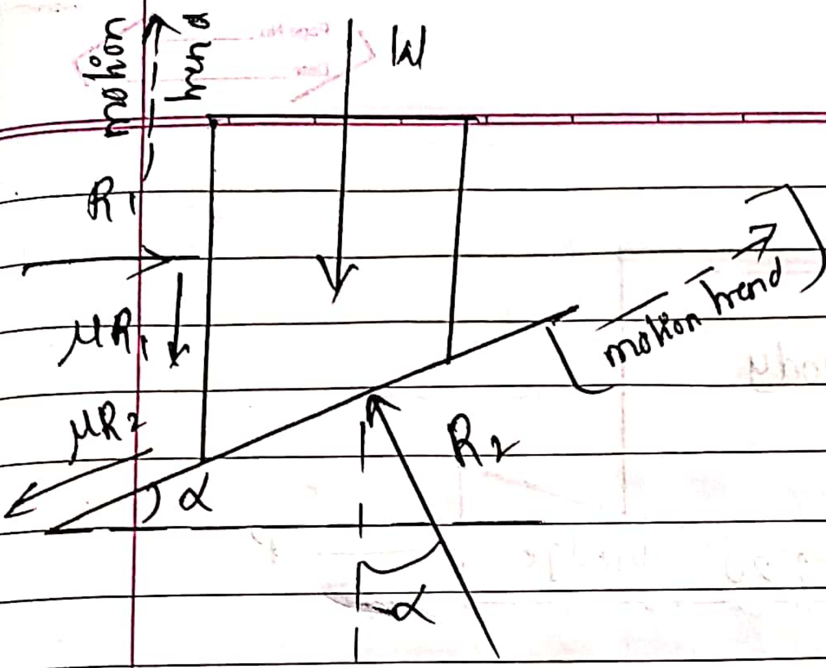
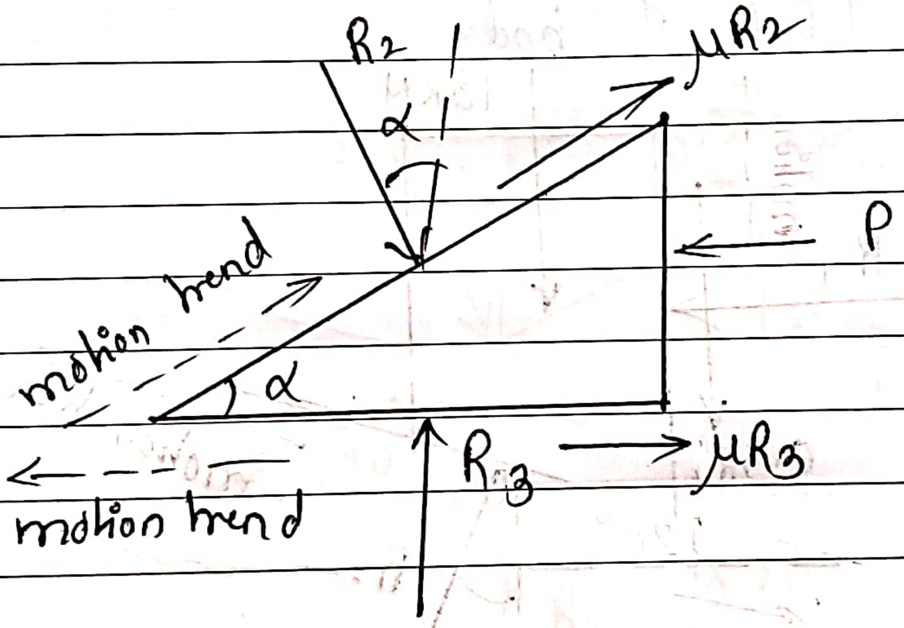


Figure shows a body being lifted by force P applied to the wedge. Let us consider F.B.D of body and wedge separately as below.





(1) A block weighing 10kN is to be raised by means of 20° wedge as shown below. Find the horizontal force which will just raise the block if coefficient of friction for all surfaces of contact is 0.3.

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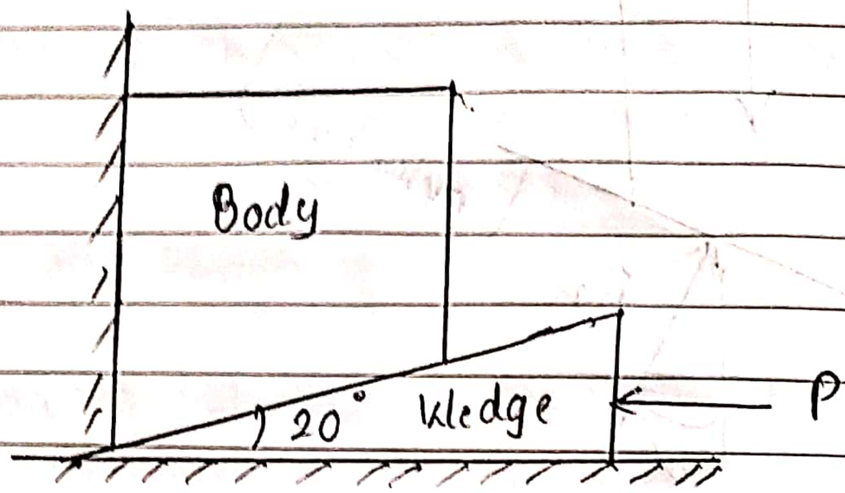
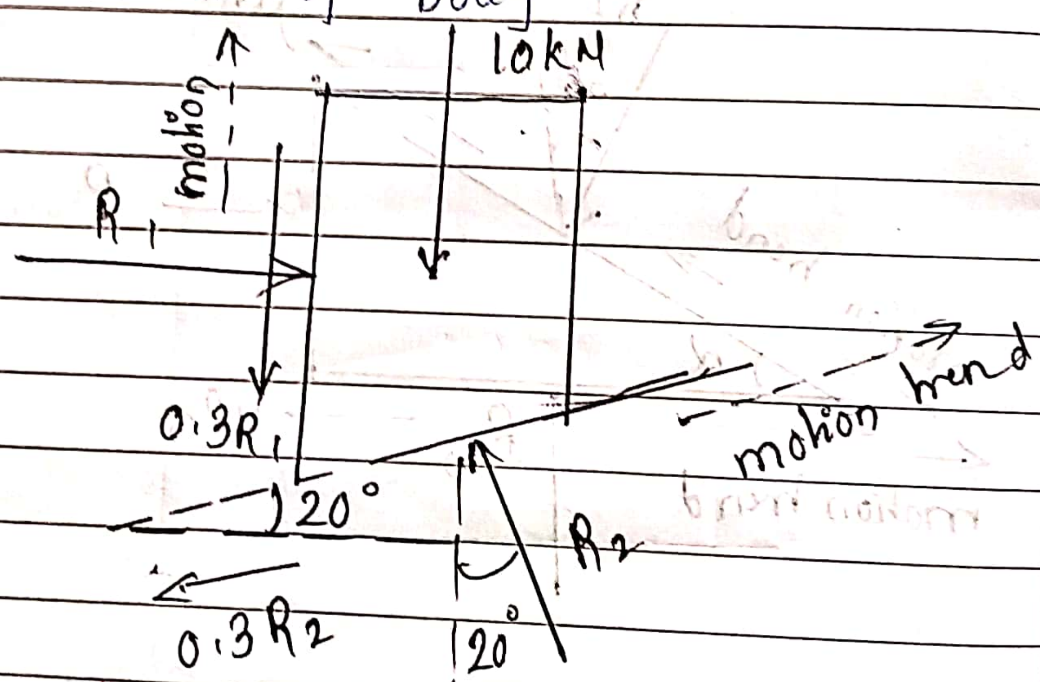


Consider F.B.D of body



$\Sigma F_y = 0$

$-10 - 0.3R_1 + R_2 \sin 70° - 0.3R_2 \sin 20 = 0$

$-0.3R_1 + 0.84R_2 = 10$

$\sum F_x = 0$

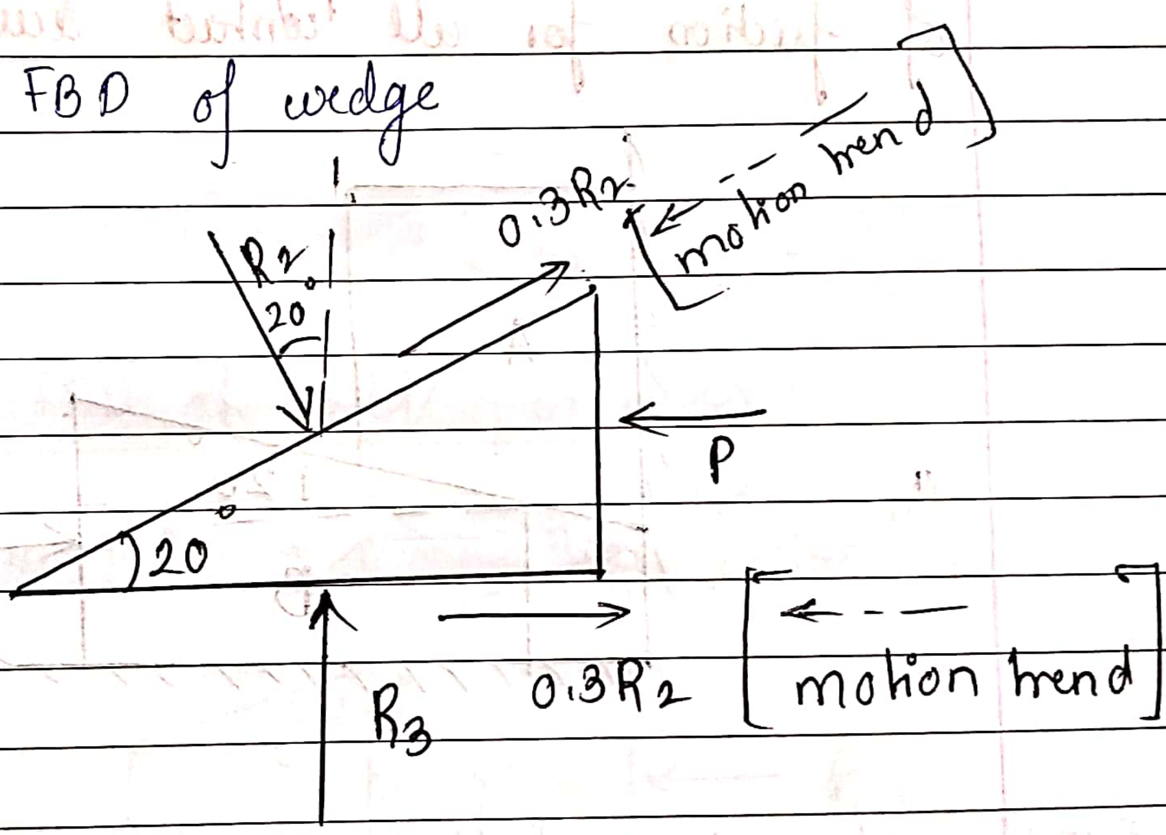
$R_1 - R_2 \cos 70^\circ - 0.3 R_2 \cos 20^\circ = 0$

$R_1 = 0.62 R_2$

$R_2 = 15.29 \text{ kN}$

$R_1 = 9.48 \text{ kN}$

Consider FBD of wedge



$\sum F_y = 0$

$R_3 - R_2 \sin 70^\circ + 0.3 R_2 \sin 20^\circ = 0$

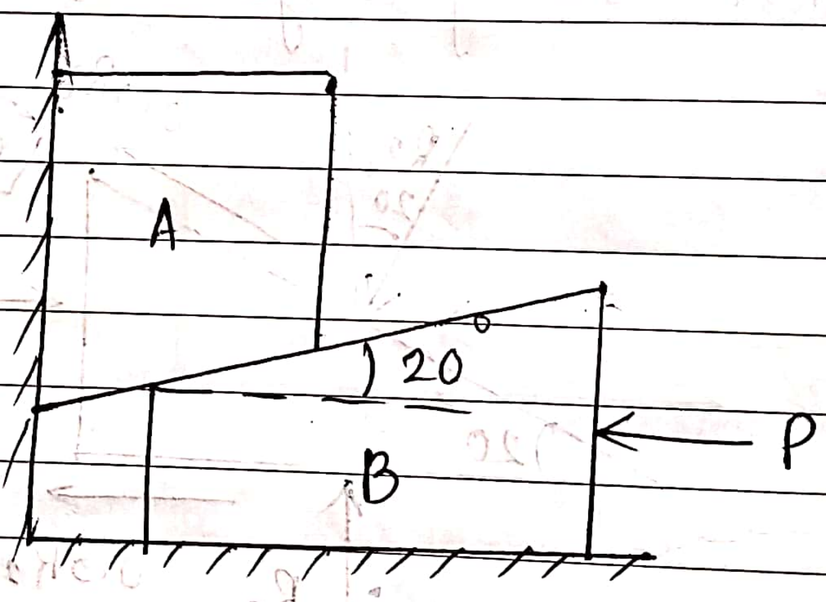
$R_3 = 12.80 \text{ kN}$

$\sum F_x = 0$

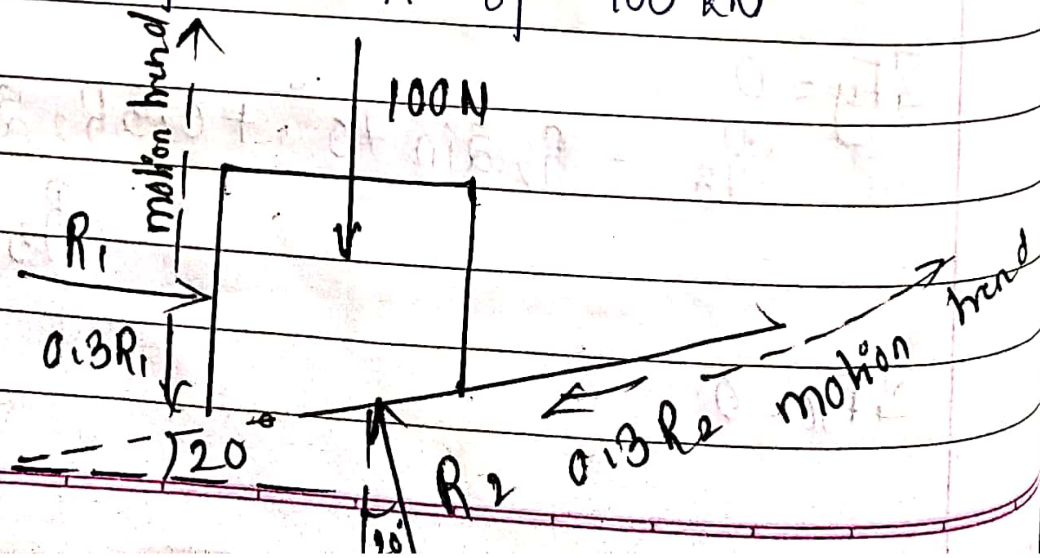
$$-P + 0.3R_3 + 0.3R_2 \cos 20^\circ + R_2 \cos 70^\circ = 0$$

$$P = 13.34 \text{ kN}$$

(2) Determine the force P that must be applied to the 20 kN block B to lift the 100 kN block A shown in figure below. The coefficient of friction for all contact surface is $\mu = 0.3$



Soln; Consider FBD of block A of 100 kN



$$\sum F_y = 0$$

$$-100 - 0.3R_1 + R_2 \sin 70^\circ - 0.3R_2 \sin 20^\circ = 0$$

$$-0.3R_1 + 0.84R_2 = 100$$

$$\sum F_x = 0$$

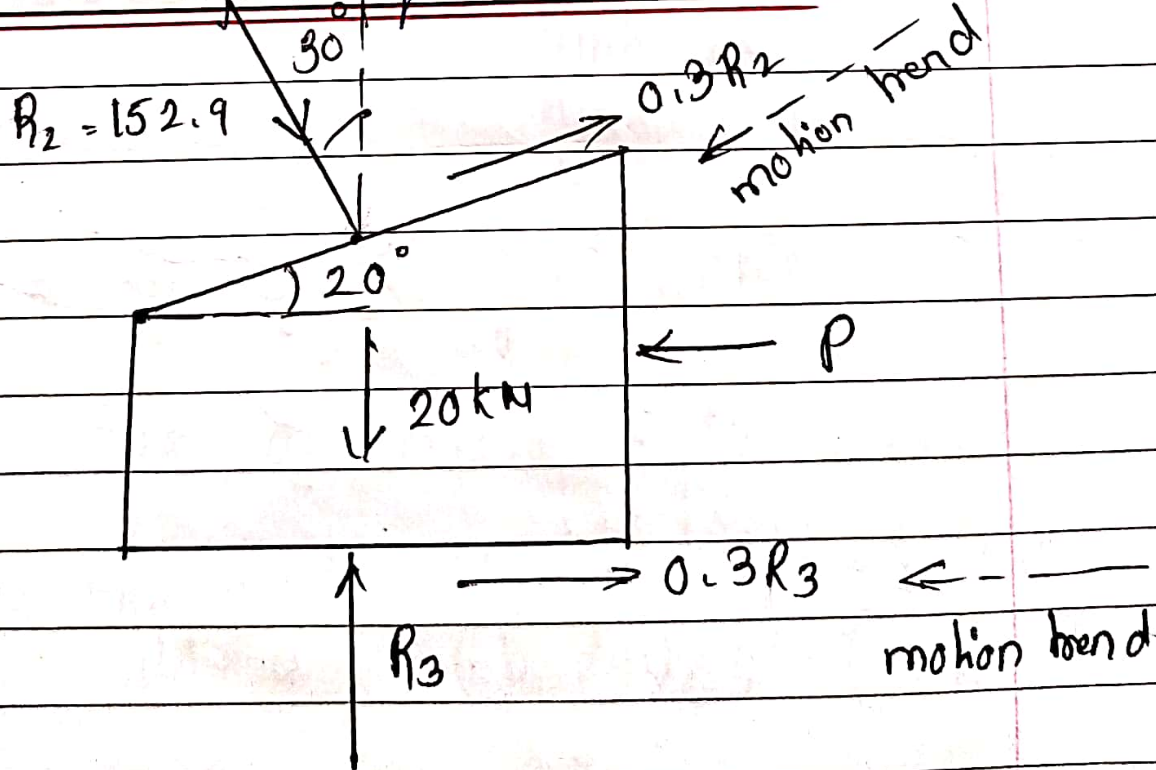
$$R_1 - R_2 \cos 70^\circ - 0.3R_2 \cos 20^\circ = 0$$

$$R_1 - 0.62R_2 = 0$$

$$R_1 = 0.62R_2$$

$$\underline{R_2 = 152.9\ kN.}$$

Consider FBD of Wedge or Block B.



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$\sum_i F_y = 0$

$R_3 - R_2 \sin 70^\circ + 0.3 R_2 \sin 20^\circ - 20 = 0$

$R_3 = 148\,kN.$

$\sum_i F_x = 0$

$-P + R_2 \cos 70^\circ + 0.3 R_2 \cos 20^\circ + 0.3 R_3 = 0$

$P = 139.8\,kN.$