

A Course in Applied Physics lab

Dr. Chetan V. Kotabage

Department of Physics,
KLS Gogte Institute of Technology, Belagavi, India
E-mail: cvkotabage@git.edu

Contents

1	Newton's rings	2
1.1	Introduction	2
1.2	Why concentric dark and bright rings are observed?	2
1.3	Why the outer rings are thinner?	2
1.4	Newton's rings formula	2
2	Laser diffraction by a grating	3
2.1	Introduction	4
2.2	Diffraction due to single slit	4
2.3	Diffraction due to N slits	5
3	Numerical aperture of an optical fiber	8
3.1	Total internal reflection	8
3.2	Numerical aperture of optical fiber	8
4	Study of I-V characteristics of a photodiode	9
4.1	Working of a photodiode	9
5	Estimation of Fermi energy and a study of variation of resistance with temperature for a metal	9
5.1	Fermi-Dirac distribution function	9
6	Velocity of ultrasonic waves by using Ultrasonic interferometer	10
6.1	Piezoelectric effect and generation of ultrasonic waves	10
6.2	Standing waves	10
6.3	Compressibility	11
7	Rigidity modulus by Torsion pendulum	11
7.1	Derivation of relation between torsional constant and shear modulus	12
7.2	Moment of inertia of the ring	12
8	Resonance of LCR circuit	12
8.1	Resonance in LCR series circuit	13
8.2	Resonance in LCR parallel circuit	13

1. NEWTON'S RINGS

These rings were first observed by Boyle and Hook. Newton did measurement analysis of these rings and Thomas Young gave correct explanation of these rings.

1.1 Introduction

In Newton's rings experiment, a monochromatic source of light is used. In the experimental set up, light from a sodium vapour lamp of wavelength 5893 \AA is directed to a plano-convex lens by a reflecting glass plate, which is inclined at an angle of 45° with the vertical. The curved surface of plano-convex lens is placed in a plane glass plate. Due to this, a thin film of air, which has thickness of few microns, is formed between the plano-convex lens and glass plate.

The light rays directed to the plano-convex lens get reflected from the top and the bottom surface of this thin air film and these rays interfere. Depending on the phase difference between these two rays, concentric dark or bright rings are observed through microscope. If the point of contact between the plano-convex lens and the glass plate is perfect, then a dark spot is observed at the center of the interference pattern.

1.2 Why concentric dark and bright rings are observed?

In case of Newton's rings, instead of a parallel fringe pattern, we observe circular concentric rings. To understand this, consider a circle of radius R drawn on the plan glass plate. Let the center of the circle be at the point of contact between plano-convex lens and the glass plate. On the circumference of this circle, the thickness of the air film is uniform. So, the light rays getting reflected from such air film, will maintain the same phase difference. So, the interference pattern produced by these waves will be either constructive or destructive. Hence, the pattern that we observe is of concentric dark and bright rings. So, in short, the geometry of air film is responsible for the concentric rings.

1.3 Why the outer rings are thinner?

The thickness of the rings decreases gradually away from the center. The expression of fringe width of a wedge shaped film can explain this feature of Newton's rings. The fringe width is given by

$$w = \frac{\lambda}{2\mu\theta} , \quad (1)$$

where w is the fringe width, λ is the wavelength of light, μ is the refractive index of air and θ is the angle of the wedge. The angle θ at a point on the curved surface of plano-convex lens is the angle made by the tangent at that point with the horizontal surface. Thus, this angle goes on increasing from the point of contact towards the edge of the glass plate. Since the fringe width is inversely proportional to this angle, the fringe width decreases and the outer rings become thinner.

1.4 Newton's rings formula

Consider a Newton's n^{th} ring (either bright or dark) of radius r_n . This ring is formed due to air film of thickness t . R is the radius of curvature of plano-convex lens.

The Pythagoras theorem in $\triangle OMN$ gives

$$R^2 = (R - t)^2 + r_n^2 . \quad (2)$$

Hence,

$$2Rt = r_n^2 + t^2 . \quad (3)$$

Since $t \ll r_n$, we get

$$t = \frac{r_n^2}{2R} . \quad (4)$$

For a wedge shaped film, the condition for constructive interference is

$$2\mu t \cos(r + \theta) = (2n - 1)\lambda/2 , \quad (5)$$

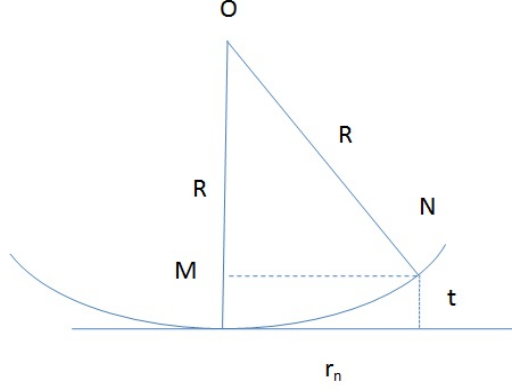


Figure 1. Newton's n^{th} ring of radius r_n .

where μ is refractive index of air, t is the thickness of the film, r is the angle of refraction, θ is the angle of wedge, λ is the wavelength of light, and $n = 1, 2, \dots$. Since, r and θ are small, above equation reduces to

$$t = \frac{(2n-1)\lambda}{4\mu} . \quad (6)$$

Hence from equation (4) and (6), the radius and diameter of the n^{th} bright ring are

$$r_n^2 = \frac{(2n-1)\lambda R}{2\mu} \quad (7)$$

$$D_n^2 = \frac{2(2n-1)\lambda R}{\mu} . \quad (8)$$

For a wedge shaped film, the condition for destructive interference is

$$2\mu t \cos(r + \theta) = n\lambda , \quad (9)$$

where μ is refractive index of air, t is the thickness of the film, r is the angle of refraction, θ is the angle of wedge, λ is the wavelength of light, and $n = 1, 2, \dots$. Since, r and θ are small, above equation reduces to

$$t = \frac{n\lambda}{2\mu} . \quad (10)$$

Hence from equation (4) and (10), the radius and diameter of the n^{th} bright ring are

$$r_n^2 = \frac{n\lambda R}{\mu} \quad (11)$$

$$D_n^2 = \frac{4n\lambda R}{\mu} . \quad (12)$$

2. LASER DIFFRACTION BY A GRATING

The deviation of a light beam from its linear path occurs when the beam comes across an obstacle, which could be opaque or transparent. Due to such obstacle, the energy distribution of the deviated light gets modified. This effect is called as diffraction. Physically, interference and diffraction are almost identical. When few waves meet at a point, it is called as interference and when the number of waves is large, it is called as diffraction.

2.1 Introduction

The diffraction is classified as Fresnel and Fraunhofer diffraction. In this experiment, Fraunhofer diffraction is studied, where the source and the screen are at a large distance from the diffraction grating.

A diffraction grating consists of N number of slits, where each slit of width b , separated by opaque surface of width a . Thus, the distance between two neighbouring slits is $d = a + b$, which is called as grating constant or grating element. Before the analysis of diffraction grating is carried out, the analysis of diffraction due to single slit is required.

2.2 Diffraction due to single slit

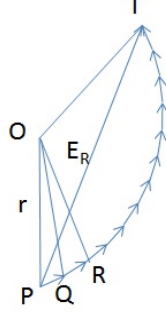


Figure 2. Diffraction due to single slit.

Consider a single slit of width b . The slit can be split into n number of small slits of equal widths. These slits work as coherent sources. Let ϕ be the phase difference between the waves emitted by neighbouring slits. Since each slit is of same width, the magnitude of electric field vector, E , is same. Thus, at a point on the screen, electric field vectors due to these slits add up as shown in figure. For large number of slits, n , addition of vectors, which is an equiangle polygon, forms a circle of radius r . The resultant electric field vector, E_R , is found by vector sum of all these vectors. The magnitude of resultant electric field vector governs the intensity of light observed at a point on the screen. For the brightest spot on the screen, the magnitude of the vector is maximum while for the darkest spot on the screen the magnitude of the vector is zero.

In $\triangle OPQ$, the geometric relation between OP and PQ and in $\triangle OQR$, the geometric relation between OQ and OR is same. So, $\angle POQ = \angle QOR = \phi$.

If a perpendicular is drawn from O on PQ , then

$$\sin \phi/2 = \frac{E/2}{r} . \quad (13)$$

In $\triangle OPR$, $\angle POR = n\phi$. If a perpendicular is drawn from O on PR , then

$$\sin \frac{n\phi}{2} = \frac{E_R/2}{r} . \quad (14)$$

Equations (13) and (14) yield

$$E_R = \frac{E \sin(n\phi/2)}{\sin(\phi/2)} . \quad (15)$$

Since the angle ϕ is small, above equation can be written as

$$E_R = \frac{E \sin(n\phi/2)}{\phi/2} . \quad (16)$$

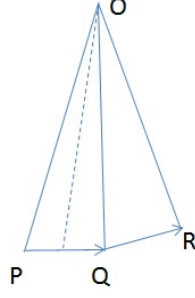


Figure 3. Relation between phase difference and magnitude of electric field.

Multiplication and division by n on RHS of above equation yields,

$$E_R = E_0 \frac{\sin \alpha}{\alpha}, \quad (17)$$

where $\alpha = n\phi/2$ and $E_0 = nE$. Thus, the intensity at any point on the screen is given by

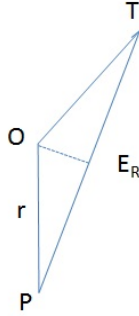


Figure 4. Relation between total phase difference and resultant magnitude of electric field.

$$I = I_0 \left(\frac{\sin \alpha}{\alpha} \right)^2, \quad (18)$$

where I_0 is the intensity of light incident on the slit.

2.3 Diffraction due to N slits

Consider a grating consisting N slits. Let b be the width of each slit and a is the separation between the slits. Here, the above same method is adapted to find the intensity of light at a point on the screen. For N slits, there are N coherent sources. Let the phase difference between the light waves emerging from neighbouring slits be δ . Since each slit is of same width, the magnitude of electric field vector, E' , is same. Thus, at a point on the screen, electric field vectors due to these slits add up as shown in figure. For large number of slits, N , addition of vectors, which is an equiangle polygon, forms a circle of radius r' . The resultant electric field vector, E'_R , is found by vector sum of all these vectors. The magnitude of resultant electric field vector governs the intensity of light observed at a point on the screen. For the brightest spot on the screen, the magnitude of the vector is maximum while for the darkest spot on the screen the magnitude of the vector is zero.

In $\triangle O'P'Q'$, the geometric relation between $O'P'$ and $P'Q'$ and in $\triangle O'Q'R'$, the geometric relation between $O'Q'$ and $O'R'$ is same. So, $\angle P'O'Q' = \angle Q'O'R' = \delta$.

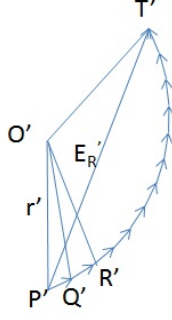


Figure 5. Diffraction due to Diffraction grating.

If a perpendicular is drawn from O' on $P'Q'$, then

$$\sin \delta/2 = \frac{E'/2}{r'} . \quad (19)$$

In $\triangle O'P'R'$, $\angle P'O'R' = n\delta$. If a perpendicular is drawn from O' on $P'R'$, then

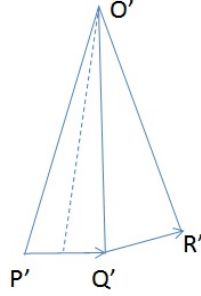


Figure 6. Relation between phase difference and magnitude of electric field.

$$\sin \frac{N\delta}{2} = \frac{E'_R/2}{r'} . \quad (20)$$

Equations (19) and (20) yield

$$E'_R = \frac{E' \sin(N\delta/2)}{\sin(\delta/2)} . \quad (21)$$

If $\beta = \delta/2$, then

$$E'_R = E' \frac{\sin(N\beta)}{\sin \beta} . \quad (22)$$

Thus, the intensity at point on the screen is given by

$$I' = I \frac{\sin^2(N\beta)}{\sin^2 \beta} , \quad (23)$$

where I is the intensity of light at a given point due to single slit. Thus, equations (18) and (23) yield the intensity at a point on the screen due to all N slits as

$$I' = I_0 \frac{\sin^2 \alpha}{\alpha^2} \frac{\sin^2(N\beta)}{\sin^2 \beta} . \quad (24)$$

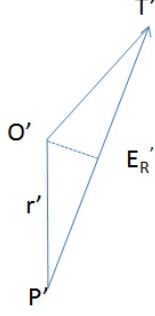


Figure 7. Relation between total phase difference and resultant magnitude of electric field.

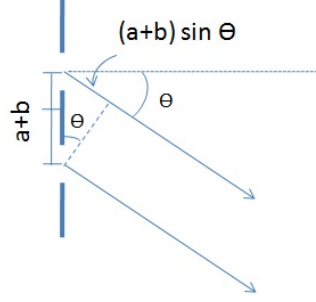


Figure 8. Optical path difference between rays emerging from two neighbouring slits.

Since in the experiment we do not measure the phase δ but angle θ , the relation between them is required. To this end, consider light emerging from two neighbouring slits as shown in figure 8.

From the geometry, the path difference between two rays emerging from two slits is $(a + b) \sin \theta$. Hence, this path difference is related to phase difference as

$$\delta = \frac{2\pi}{\lambda} (a + b) \sin \theta . \quad (25)$$

Since $\beta = \delta/2$, above equation yields,

$$\beta = \frac{\pi}{\lambda} (a + b) \sin \theta . \quad (26)$$

From equation (23), the maximum intensity is obtained when $\beta = n\pi$, $n = 0, \pm 1, \pm 2, \dots$. In this limit,

$$\lim_{\beta \rightarrow n\pi} \frac{\sin(N\beta)}{\sin \beta} = \lim_{\beta \rightarrow n\pi} \frac{N \cos(N\beta)}{\cos \beta} = \pm N . \quad (27)$$

Thus, as $\beta \rightarrow n\pi$,

$$I' = I_0 \frac{\sin^2 \alpha}{\alpha^2} N^2 . \quad (28)$$

Hence, as the number of slits of the grating increases, intensity of these maxima increase. Thus, $\beta = n\pi$ yields equation (26) as,

$$n\lambda = (a + b) \sin \theta . \quad (29)$$

The above equation gives the condition for constructive interference. In the experiment, this equation is utilized to figure out the wavelength of light emitted by the laser.

The intensity would be minimum, when $I' = 0$, i.e., $\sin N\beta = 0$. This condition implies, $\beta = m\pi/N$, where $m = 0, \pm 1, \pm 2, \dots$. This condition yields equation (26) as

$$m\lambda = N(a + b) \sin \theta . \quad (30)$$

The above equation gives the condition for destructive interference. But $m \neq nN$ because at these values the condition of constructive interference is satisfied.

3. NUMERICAL APERTURE OF AN OPTICAL FIBER

The light gathering capacity of any optical instrument is quantified by numerical aperture. An optical fiber is a device through which light can be transmitted over a long distance with a little loss. The working principle of optical fiber is total internal reflection. The light from a source is coupled to optical fiber such that it undergoes multiple total internal reflections and can propagate over a long distance.

3.1 Total internal reflection

Snell's law states that

$$n_1 \sin \theta_i = n_2 \sin \theta_r, \quad (31)$$

where light moves from material of refractive index n_1 to n_2 . θ_i and θ_r are the angle of incidence and angle of refraction respectively.

When light moves from a denser medium to a rarer medium, i.e., $n_1 > n_2$, the refracted ray turns away from the normal, i.e., $\theta_r > \theta_i$. If the angle of incidence goes on increasing, then at a certain angle, called as critical angle, the angle of refraction is 90° . If the angle of incidence is greater than critical angle, then light ray undergoes reflection. This is known as total internal reflection.

The critical angle is given by

$$\theta_c = \sin^{-1} \left(\frac{n_2}{n_1} \right). \quad (32)$$

3.2 Numerical aperture of optical fiber

Optical fiber is made up of core and cladding. The core has higher refractive index than cladding. The cladding is covered by plastic jacket, which protects the core and cladding.

Consider a ray of light entering the core of a fiber from air. The refractive indices of air, core and cladding are $n_{\text{air}} = 1$, n_1 and n_2 respectively. At the air-core boundary, Snell's law yields

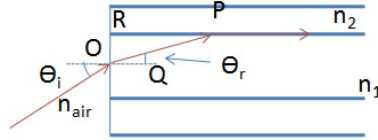


Figure 9. Numerical aperture of optical fiber.

$$\sin \theta_i = n_1 \sin \theta_r. \quad (33)$$

From the geometry of the figure, $\angle RPO = \angle POQ$. Thus, at core-cladding boundary, the angle of incidence is $90 - \theta_r$. So, if the angle of refraction at core-cladding boundary is 90° , then Snell's law yields

$$n_1 \sin(90 - \theta_r) = n_2. \quad (34)$$

Thus,

$$\cos \theta_r = \frac{n_2}{n_1}. \quad (35)$$

Equations (33) and (35) yield

$$\sin \theta_i = n_1 \sqrt{1 - \cos^2 \theta_r} = n_1 \sqrt{1 - \frac{n_2^2}{n_1^2}} = \sqrt{n_1^2 - n_2^2}. \quad (36)$$

The angle for which above condition is satisfied is called as angle of acceptance, i.e., $\theta_i = \theta_a$. The sine of this angle defines numerical aperture of the fiber.

4. STUDY OF I-V CHARACTERISTICS OF A PHOTODIODE

A photodiode is p-n junction, which is utilized to detect light. The detection mechanism in a photodiode is exactly opposite to that of LED. Under forward bias condition in LED, electrons and holes recombine to generate light. A photodiode is operated under reverse bias condition and utilized to detect light.

4.1 Working of a photodiode

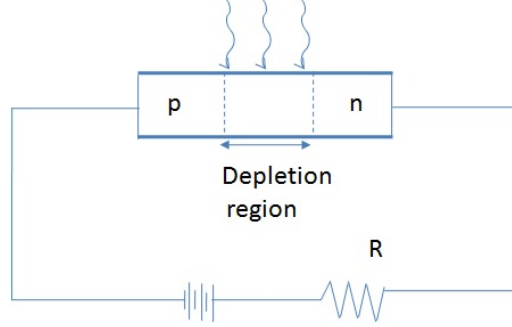


Figure 10. Detection of light by a photodiode.

When a photon of energy $h\nu > E_g$ is absorbed by depletion region of a reverse biased photodiode, electron hole pair generation takes in the region. The electron and holes are swept to n side and p side respectively by the applied electric field. It results in the current in the circuit. The number of electron hole pairs generated is directly proportional to the number of photons absorbed. Hence, the current in the circuit is proportional to the intensity of light absorbed by the diode.

5. ESTIMATION OF FERMI ENERGY AND A STUDY OF VARIATION OF RESISTANCE WITH TEMPERATURE FOR A METAL

The most simple model that describes conducting electrons in a metal considers no interaction between electrons. Introduction of quantum mechanical ideas into this model gives rise to a concept of Fermi energy of a metal. In metals, valence electrons are loosely bound to the nucleus. Due to this, valence electrons can easily get dissociated from the parent atom leaving behind a positive ion. The model assumes that electrons move in a region of zero potential energy instead of encountering periodic Coulombic potential due to positive ions. The Coulombic repulsion between electrons is also discarded. Under such conditions, electrons can be treated as particles confined in a three dimensional infinite potential well. The highest energy level occupied by electrons when all lower energy levels are filled at absolute zero is known as Fermi energy.

5.1 Fermi-Dirac distribution function

The probability of occupancy of electrons in a given energy level is given by Fermi-Dirac distribution function. The Fermi-distribution function for a level of energy E at temperature T is defined as

$$F(T) = \frac{1}{e^{(E-E_F)/kT} + 1} , \quad (37)$$

where E_F is Fermi energy. At $T = 0$ K and $E > E_F$, i.e., the level is above Fermi level, then

$$F(0K) = \frac{1}{e^\infty + 1} = 0 . \quad (38)$$

It means all the energy levels above Fermi energy have zero probability of occupancy, i.e., these levels are empty.

At $T = 0$ K and $E < E_F$, i.e., the level is below Fermi level, then

$$F(0K) = \frac{1}{e^{-\infty} + 1} = 1 . \quad (39)$$

It means all the energy levels above Fermi energy have probability of occupancy as one, i.e., these levels are completely filled.

Hence, another definition of Fermi energy is that the energy level up to which all energy levels are completely filled by electrons at absolute zero.

6. VELOCITY OF ULTRASONIC WAVES BY USING ULTRASONIC INTERFEROMETER

Ultrasonic waves are sound waves, which are longitudinal waves. The sound waves of frequency above 20 kHz are known as ultrasonic waves.

6.1 Piezoelectric effect and generation of ultrasonic waves

A certain class of crystals, known as anisotropic crystals, have a property that when a pressure is applied across a pair of opposite faces of a crystal, equal and opposite charge appears on the other two faces of the crystal as shown in the diagram. This is known as piezoelectric effect. The inverse of this effect is also possible, i.e., when

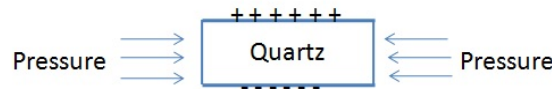


Figure 11. Piezoelectric effect.

potential difference is applied across a pair of opposite faces, dimensions of the crystal change along the other two faces of the crystal. This principle is the basis of generation of ultrasonic waves.

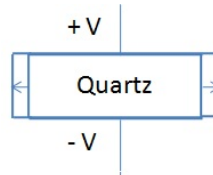


Figure 12. Inverse piezoelectric effect.

If the voltage across the faces keeps changing with frequency f , then the other pair of opposite faces contracts and expands with frequency f . This leads to generation of longitudinal waves of frequency f in the surrounding medium. If the frequency of vibration is greater than 20 kHz, then ultrasonic waves are generated in the surrounding medium.

6.2 Standing waves

The phenomenon of reflection and interference gives rise to standing waves. Consider a wave travelling on a string, where one end of the string is attached to a wall. The wave will be reflected from the wall and it will be out of phase. This wave interferes constructively with the incoming wave and results in a standing wave, where the wave pattern appears to be stationary. For such a wave, amplitude varies on the string but the pattern remains the same. The points of maximum and minimum amplitude of the stationary wave are called as antinodes and nodes respectively.

Since ultrasonic waves are longitudinal waves, these can be described either in terms of displacement of particles or pressure. For stationary waves of ultrasonic waves, a displacement node is a pressure antinode and a displacement antinode is a pressure node.

In the given experimental set up, ultrasonic waves, which are generated at the bottom of a water column by a quartz crystal, propagate up the column and are reflected by a metal reflector. Hence, the transmitted and

reflected wave form a standing wave pattern. As the reflector moves up the water column, nodes and antinodes appear alternately at the ends of the water column. The positions of pressure antinode or displacement node is found by the maximum current. The minimum current refers to pressure node or displacement antinode. In the experiment, the positions of pressure antinode are recorded.

Since the distance between two consecutive nodes or antinodes is $\lambda/2$, the wavelength of ultrasonic waves is determined by measurement of distance between two consecutive pressure antinodes.

6.3 Compressibility

Compressibility is defined as inverse of bulk modulus. If a fluid has compressibility β and density ρ , then at equilibrium, Hook's law states that

$$P = \frac{1}{\beta} \frac{\partial \eta}{\partial x} , \quad (40)$$

where P is local pressure, i.e., stress and η is local displacement. The strain is quantified by $\partial \eta / \partial x$. The local acceleration is given by

$$\frac{\partial^2 \eta}{\partial t^2} = \frac{1}{\rho} \frac{\partial P}{\partial x} . \quad (41)$$

Equations (40) and (41) yield

$$\frac{\partial^2 \eta}{\partial x^2} = \rho \beta \frac{\partial^2 \eta}{\partial t^2} . \quad (42)$$

The wave equation is given as

$$\frac{\partial^2 \eta}{\partial x^2} = \frac{1}{v^2} \frac{\partial^2 \eta}{\partial t^2} . \quad (43)$$

Hence, equations (42) and (43) yield

$$\beta = \frac{1}{\rho v^2} . \quad (44)$$

7. RIGIDITY MODULUS BY TORSION PENDULUM

A torsion pendulum is made up of an extended body suspended by a wire. The extended body, which is a disk here, is twisted through a small angle θ and released. The wire exerts a torque on the disk and the disk, which has moment of inertia I_D , undergoes simple harmonic motion. The torque is given by

$$\tau = -\kappa \theta , \quad (45)$$

where κ is torsion constant. But the torque is related to angular acceleration as

$$\tau = I_D \frac{d^2 \theta}{dt^2} . \quad (46)$$

Thus, equations (45) and (46) yield period of oscillation as

$$T = 2\pi \sqrt{\frac{I_D}{\kappa}} \quad (47)$$

7.1 Derivation of relation between torsional constant and shear modulus

Consider a wire of length L and radius r . Consider a small radius element dr' at a distance r' from the center. The area of this small element is $dA = 2\pi r' dr'$.

Torque $d\tau$ produced in the wire when a force of dF is applied to twist the wire is $d\tau = -r' dF$. The negative sign indicates restoring torque. Thus, shear stress is

$$\text{Shear stress} = \frac{dF}{2\pi r' dr'} . \quad (48)$$

If the wire is twisted through a small angle θ , then shear strain is

$$\text{Shear strain} = \frac{r'\theta}{L} . \quad (49)$$

Thus, shear modulus is

$$\eta = \frac{\text{Shear stress}}{\text{Shear strain}} = \frac{L dF}{2\pi r'^2 dr' \theta} . \quad (50)$$

Multiplying numerator and denominator by r' , the torque is written as

$$d\tau = -\frac{2\pi r'^3 dr' \theta \eta}{L} . \quad (51)$$

Integration of above equation yields

$$\tau = -\frac{2\pi\theta\eta}{L} \int_0^r r'^3 dr' = -\frac{\pi r^4 \eta}{2L} \theta . \quad (52)$$

The comparison of above equation with (45) yields

$$\kappa = \frac{\pi r^4 \eta}{2L} . \quad (53)$$

The comparison of above equation with (47) yields

$$\eta = \frac{8\pi I_D L}{r^4 T^2} . \quad (54)$$

7.2 Moment of inertia of the ring

The shear modulus of wire is independent (i.e. constant) whether a disk or disk+ring are suspended by the wire. Let T_1 be the period of oscillation for disk and T_2 be the period for oscillation of disk+ring. Thus, eq. (54) gives

$$\eta = \frac{8\pi I_D L}{r^4 T_1^2} = \frac{8\pi (I_D + I_R) L}{r^4 T_2^2} . \quad (55)$$

Rearrangement of terms gives moment of inertia of ring as

$$I_R = I_D \left(\frac{T_2^2}{T_1^2} - 1 \right) \quad (56)$$

8. RESONANCE OF LCR CIRCUIT

If a closed loop has a current i flowing through the wire, then it has some magnetic flux Φ_B passing through it. If the loop has N turns, then inductance in the circuit is given by

$$L = \frac{N\Phi_B}{i} . \quad (57)$$

Any pair of conductors that are separated by insulator forms a capacitor. The capacitance of a capacitor is defined as

$$C = \frac{Q}{V} , \quad (58)$$

where Q is the equal and opposite charge on the conductors and V is the potential difference between those two.

8.1 Resonance in LCR series circuit

The resistance offered by inductor and capacitor in ac circuit is called as inductive reactance, X_L and capacitive reactance, X_C respectively. These are given by

$$X_L = \omega L \quad (59)$$

$$X_C = \frac{1}{\omega C} \quad (60)$$

For dc current $\omega \rightarrow 0$. Hence $X_L = 0$ and $X_C = \infty$. It means an inductor offers no resistance to dc current while a capacitor does. For ac current $\omega \rightarrow \infty$. Hence $X_L = \infty$ and $X_C = 0$. It means an inductor offers resistance to ac current while a capacitor does not.

The resonance in series circuit is given by

$$X_C = X_L \quad (61)$$

Thus, the resonant frequency is given by

$$f = \frac{1}{2\pi} \frac{1}{\sqrt{LC}} \quad (62)$$

The impedance of LCR series circuit is given by

$$Z = \sqrt{R^2 + (X_L - X_C)^2} \quad (63)$$

Since inductive reactance and capacitive reactance are same at resonance frequency, the impedance is equal to R . At resonance frequency power in the circuit is maximum, i.e.,

$$P_m = \frac{V_m^2}{R} \quad (64)$$

Bandwidth is a range of frequencies over which the power in the circuit is at least half of the maximum. Thus,

$$\frac{P_m}{2} = \frac{1}{R} \left(\frac{V_m}{\sqrt{2}} \right)^2 \quad (65)$$

So, due to this, f_1 and f_2 are calculated where voltage is $0.707V_m$. Quality factor quantifies the spread of the resonance curve as it is inversely proportional to bandwidth.

8.2 Resonance in LCR parallel circuit

The resonance in parallel circuit is given by

$$X_C = X_L \quad (66)$$

Thus, the resonant frequency is given by

$$f = \frac{1}{2\pi} \frac{1}{\sqrt{LC}} \quad (67)$$

The impedance of LCR parallel circuit is given by

$$\frac{1}{Z} = \sqrt{1/R^2 + (1/X_C - 1/X_L)^2} \quad (68)$$

Since inductive reactance and capacitive reactance are same at resonance frequency, the impedance is equal to R . At resonance frequency power in the circuit is minimum, i.e.,

$$P_{min} = \frac{V_{min}^2}{R} \quad (69)$$

Bandwidth is a range of frequencies over which the power in the circuit is not more than double of the minimum power in the circuit. Thus,

$$2P_{min} = \frac{1}{R} (\sqrt{2}V_{min})^2 \quad (70)$$

So, due to this, f_1 and f_2 are calculated where voltage is $V_{min}/0.707$.