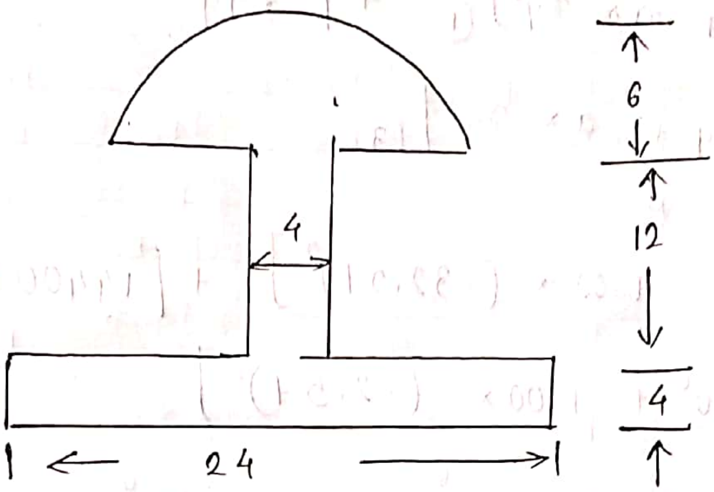
3. Find the polar radius of gyration for the area shown in the figure. All dimensions are in mm. (P1)



In the above numerical reference axes are not given. So the first step would be to choose the reference axes. You can choose left most corner as the reference axes as shown in figure or reference axis can be chosen as shown in fig. 2.

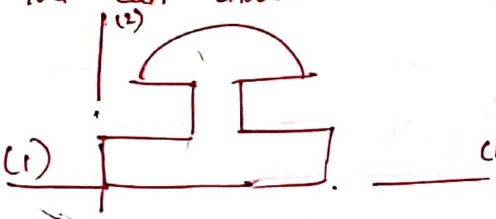
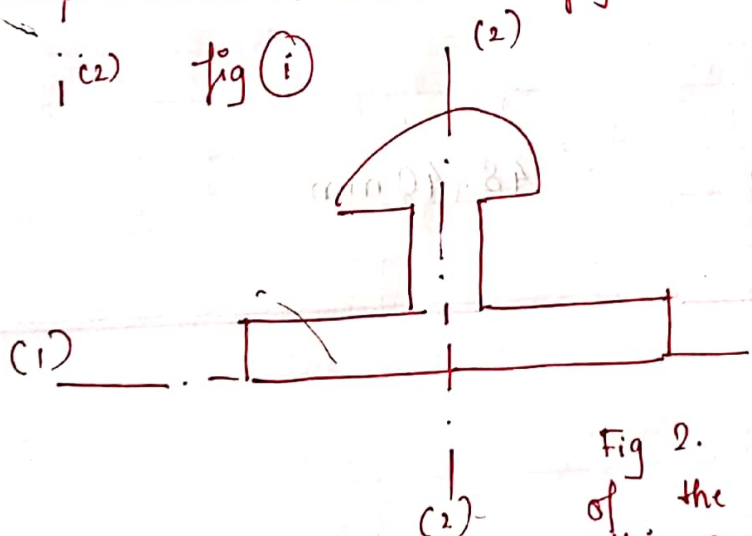


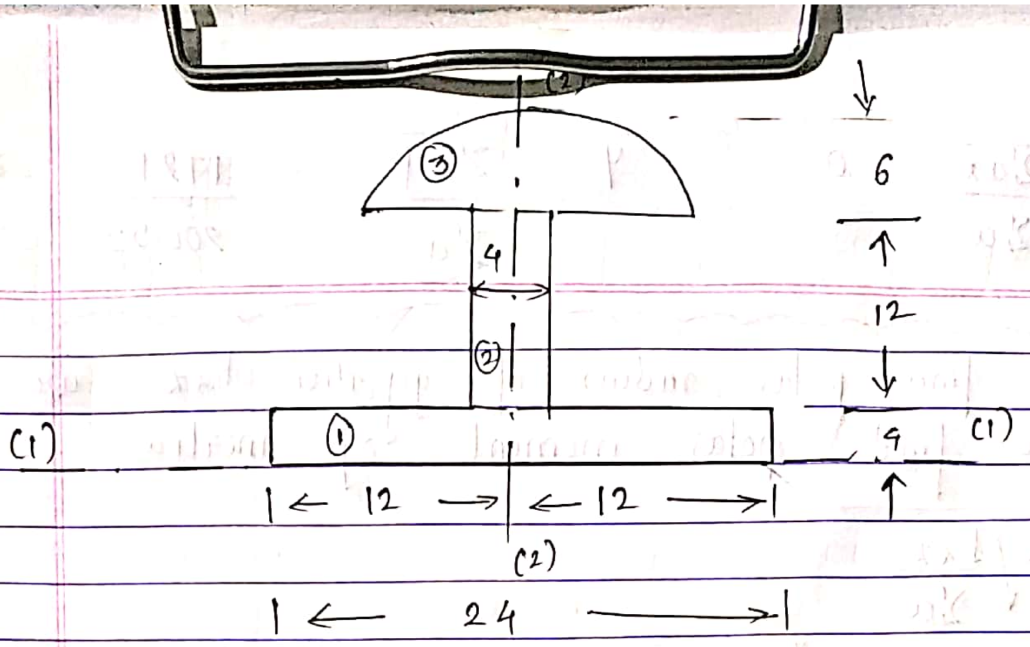
fig (i)



From fig. 2 it can be inferred that (2) – (2) axis co-incides with the symmetrical vertical axis & hence $\bar{X}$ for the (1) whole section is zero.

Fig 2. Since $\bar{X}$ is the distance of the centroid from (2) – (2) axis in this case as symmetrical axis coincide with (2) – (2) axes & centroid lies on symmetrical axis $\bar{X} = 0$. & $\bar{x} = 0$

if $\bar{X} = 0$ $r_y = \bar{X} - \bar{x}$ will also be zero



Component	'a'	$\bar{x}$	$\bar{y}$	$a\bar{x}$	$a\bar{y}$	I_{gx}	I_{gy}	$r_x = \bar{Y} - \bar{y}$	$r_y = \bar{X} - \bar{x}$
1	$24 \times 4 = 96$	0	$\frac{d}{2} = \frac{4}{2} = 2$	0	192	$\frac{bd^3}{12} = \frac{24 \times 4^3}{12} = 128$	$\frac{db^3}{12} = \frac{4 \times 24^3}{12} = 4608$	$8.58 - 2 = 6.58$	0
2	$4 \times 12 = 48$	0	$4 + \frac{d}{2}$, $4 + \frac{12}{2} = 4 + 6 = 10$	0	480	$\frac{bd^3}{12} = \frac{4 \times 12^3}{12} = 576$	$\frac{db^3}{12} = \frac{12 \times 4^3}{12} = 64$	$8.58 - 10 = -1.42$	0
3	$\frac{\pi r^2}{2} = \frac{\pi \times 6^2}{2} = 56.55$	0	$= 16 + \frac{4r}{3\pi}$, 18.55	0	1049	$0.11 r^4 = 0.11(6)^4 = 142.56$	$\frac{\pi r^4}{8} = 508.94$	$8.58 - 18.55 = -9.96$	0
	$\Sigma a = 200.55$			$\Sigma a\bar{x} = 0$	$\Sigma a\bar{y} = 1721$				

$\bar{X} = \dfrac{\Sigma a x}{\Sigma a} = 0$ $\quad$ $\bar{Y} = \dfrac{\Sigma a y}{\Sigma a} = \dfrac{1721}{200.55} = 8.58 \text{ mm}$

NOTE: To find polar radius of gyration K_{zz} we find to find polar moment of inertia

as $K_{zz} = \sqrt{\dfrac{I_{zz}}{\Sigma a}}$

& $I_{zz} = I_{xx} + I_{yy}$

$$I_{xx} = \Sigma\left[I_{gx} + a \times r_x^2\right]$$

$$= \left[I_{gx} + a \times r_x^2\right]_{1} + \left[I_{gx} + a \times r_x^2\right]_{2} + \left[I_{gx} + a \times r_x^2\right]_{3}$$

$$= \left[128 + 96 \times 6.58^2\right] + \left[576 + 48 \times (-1.42)^2\right]$$
$$+ \left[142.56 + 56.55 \times (-9.96)^2\right]$$

$$= 10716.84 \text{ mm}^4$$

$$I_{yy} = \Sigma\left[I_{gy} + a \times r_y^2\right] \quad \because r_y = 0$$

$$= \Sigma\left[I_{gy}\right] = \left[I_{gy}\right]_{1} + \left[I_{gy}\right]_{2} + \left[I_{gy}\right]_{3}$$

$$= 4608 + 64 + 508.94 = 5180.94 \text{ mm}^4$$

$$I_{zz} = I_{xx} + I_{yy} = 10716.84 + 5180.94 = 15897.78 \text{ mm}^4$$

$$K_{zz} = \sqrt{\dfrac{I_{zz}}{\Sigma a}} = \sqrt{\dfrac{15897.78}{200.55}} = \underline{\underline{8.903 \text{ mm}}}$$