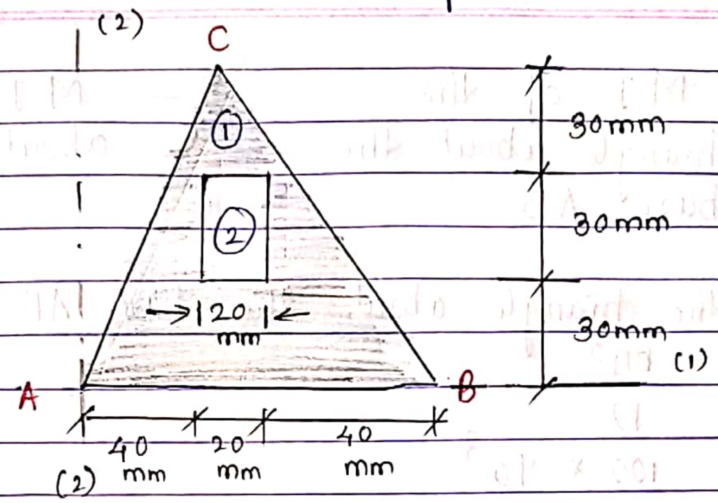
4. Determine the moment of inertia and radii of gyration of the shaded area shown in the figure about the base AB and the centroidal axis parallel to AB.



Soln:

(*) The above numerical is different from the previous numericals. In this numerical M.I. about the base AB is asked which is also the base of the triangle ABC. Hence M.I. of Δ ABC about the base formula is to be used.

i.e M.I of a triangle about the base $I_{AB} = \frac{bh^3}{12}$

and not $\frac{bh^3}{36}$

(*) For the rectangle M.I about the centroidal axis is to be determined and Parallel Axes Theorem is to applied only for rectangular component.

(*) The centroidal axis parallel to AB is the horizontal centroidal axis i.e X-X axis. Therefore I_{xx} is to be determined.

$$I_{xx} = \Sigma\left[I_g + a \times r_x^2\right] \quad \& \quad r_x = \bar{Y} - \bar{y}$$

$\therefore \bar{y}$ is to be determined.

Note: As the rectangle is unshaded portion it is to taken with negative sign

(19) P2

The M.I. of the whole section about the base AB is given by

I_{AB} = M.I. of the triangle about the base AB − M.I. of the rectangle about the base AB

M.I. of the triangle about the base AB is

$$= \frac{bh^3}{12}$$

$$= \frac{100 \times 90^3}{12}$$

$$= 6.075 \times 10^6 \text{ mm}^4.$$

To find

M.I. of the rectangle about the base AB parallel axes theorem is to be used as the component is away from the base AB.

M.I of rectangle about the base AB

= [M.I of rectangle about centroidal axis (X - axis) + Area x d^2]   [i.e $I_{AB} = I_g + A \times d^2$] using parallel axis

$$= \frac{bd^3}{12} + \text{area} \times [\text{distance between the centroid of rectangle \& the base AB}]^2$$

$$= \frac{20 \times 30^3}{12} + (20 \times 30)\left(30 + \frac{d}{2}\right)^2$$

$$= 45000 + (20 \times 30)\left(30 + \frac{30}{2}\right)^2$$

$$= 45000 + 600 \times 45^2 = 1.26 \times 10^6 \text{ mm}^4$$

M.I of the whole section about AB

= M.I of triangle about AB − M.I of rectangle about AB

$= 6.075 \times 10^{6} - 1.26 \times 10^{6}$

$I_{AB} = 4.815 \times 10^{6}\ mm^4$

Radius of Gyration about AB is given by

$K_{AB} = \sqrt{\dfrac{I_{AB}}{\Sigma a}}$

$= \sqrt{\dfrac{4.815 \times 10^{6}}{3900}}$

$K_{AB} = 35.137\ mm$

Σa = Area of triangle − Area of rectangle

$= \frac{1}{2} \times 100 \times 90 - (20 \times 30)$

$= 3900\ mm^2$

To determine the M.I. about the centroidal axis parallel to the base AB i.e I_{xx} by using Parallel Axes Theorem.

$I_{AB} = I_{xx} + \Sigma a \times \bar{y}^2$

In the above equation

$I_{AB} = 4.815 \times 10^{6}\ mm^4$

$\Sigma a = 3900$

$\bar{y}$ = distance of centroid of whole section about from the base AB

$\bar{y} = \dfrac{\Sigma ay}{\Sigma a} = \dfrac{a_1 y_1 - a_2 y_2}{\Sigma a}$

a_1 = area of triangle $= \frac{1}{2} \times 100 \times 90 = 4500\ mm^2$

$y_1 = \dfrac{h}{3} = \dfrac{90}{3} = 30\ mm$

a_2 = area of rectangle

$= 20 \times 30 = 600 \ mm^2$

y_2 = distance between the centroid of triangle and the base AB.

$$= 30 + \frac{d}{2} = 30 + \frac{30}{2} = 45 \ mm.$$

$$\bar{y} = \frac{4500 \times 30 - 600 \times 45}{3900}$$

$$\bar{y} = 27.692 \ mm.$$

$$I_{AB} = I_{xx} + \Sigma a \times \bar{y}^2$$

$$I_{xx} = I_{AB} - \Sigma a \times \bar{y}^2$$

$$= 4.815 \times 10^6 - 3900 \times (27.692)^2$$

$$I_{xx} = 1.824 \times 10^6 \ mm^4$$

Radius of gyration about X-X axis

$$K_{xx} = \sqrt{\frac{I_{xx}}{\Sigma a}} = \sqrt{\frac{1.824 \times 10^6}{3900}}$$

$$K_{xx} = 21.63 \ mm$$