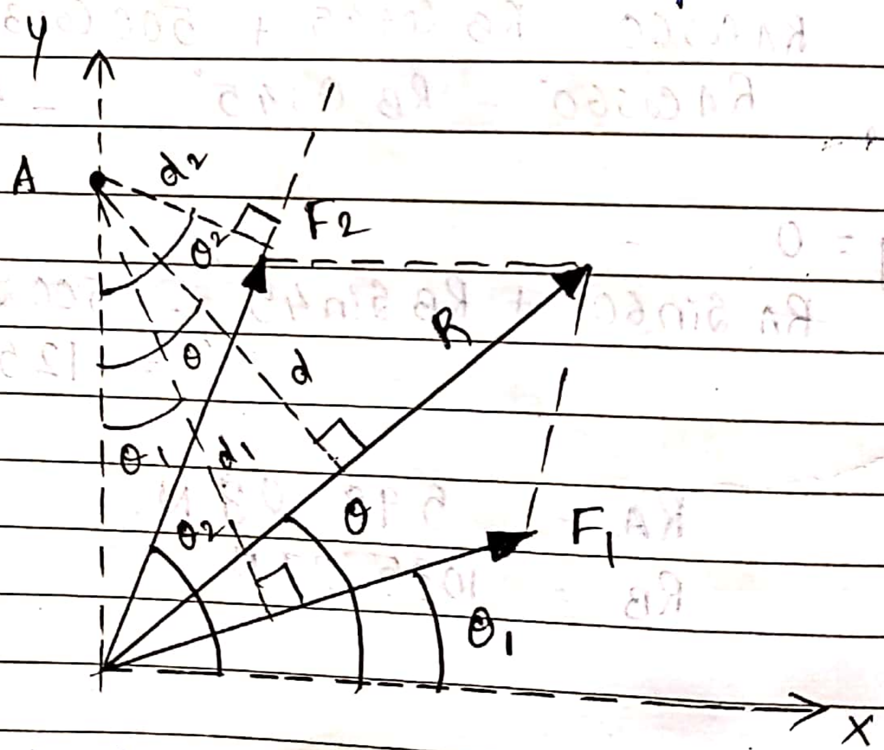
Unit III

Composition of Coplanar Non-concurrent Force Systems :

Varignon's Theorem of Moments :

Statement :

The algebraic sum of moments due to all forces acting on an object about any point is equal to the moment of their resultant about the same point.



Consider two forces F_1 and F_2 making angles θ_1 and θ_2 respectively with x-axis and concurrent at the origin as shown in figure

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Let their resultant R make angle θ with x-axis. The perpendicular distances of F_1, F_2 and R from point A are d_1, d_2 and d respectively. The distances d_1, d_2 and d can be expressed in terms of OA as follows:

$$\cos\theta_1 = \frac{d_1}{OA}$$

or $$d_1 = OA \cdot \cos\theta_1$$

$$d_2 = OA \cos\theta_2$$

$$d = OA \cos\theta$$

The moment of R about A is

$$M_R = R \times d$$
$$= R \times (OA \cos\theta)$$

$$M_R = OA\,(R\cos\theta)$$

The x-component of resultant is

$$R_x = R\cos\theta$$
$$M_R = OA \cdot R_x \longrightarrow \text{(i)}$$

The moment of F_1 about A is

$$M_1 = F_1 \times d_1$$

$$= F_1 (OA \cos\theta_1)$$

$$M_1 = OA (F_1 \cos\theta_1)$$

But $F_1 \cos\theta_1 = F_{1x}$ i.e x-component F_1

$$M_1 = OA\, F_{1x} \longrightarrow \text{(ii)}$$

The moment of F_2 about A is

$$M_2 = F_2 \cdot d_2$$

$$M_2 = F_2 (OA \cos\theta_2)$$

But $F_2 \cos\theta_2 = F_{2x}$ i.e x-component of

$$M_2 = OA\, F_{2x} \longrightarrow \text{(iii)}$$

Adding equations (ii) and (iii)

$$M_1 + M_2 = OA\, F_{1x} + OA\, F_{2x}$$

$$M_1 + M_2 = OA (F_{1x} + F_{2x})$$

But $F_{1x} + F_{2x} = R_x$ is the x- component of resultant.

$$M_1 + M_2 = OA \cdot R_x$$

From (i)

$$M_1 + M_2 = M_R$$

Thus the algebraic sum of moments due to F_1 and F_2 about A is equal to the moment of their resultant about A.

(*) Numericals:

1. For the non-concurrent coplanar system shown in figure below. Determine the magnitude, direction and position of resultant force with reference to 'A'.