CHALMERS UNIVERSITY OF TECHNOLOGY

Applied Signal Processing

Project 2 – Adaptive Noise Cancellation

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Empirical Section :

Task 1

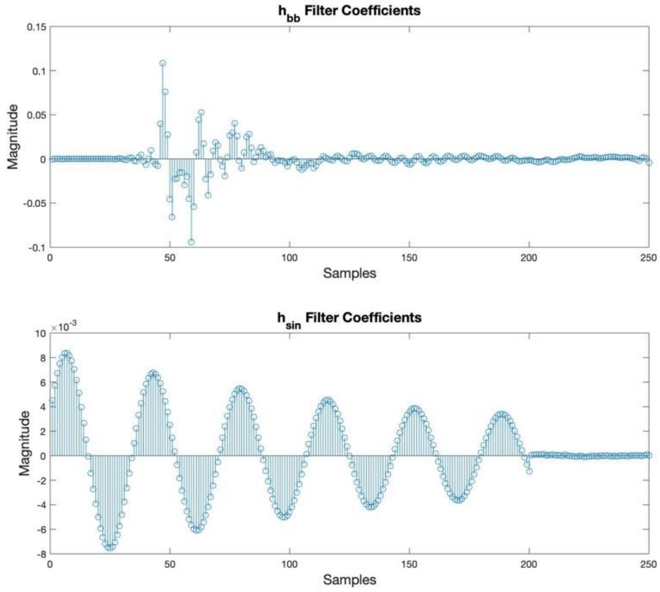


Figure 1 Converged filter coefficients for broadband and sinusoidal noise

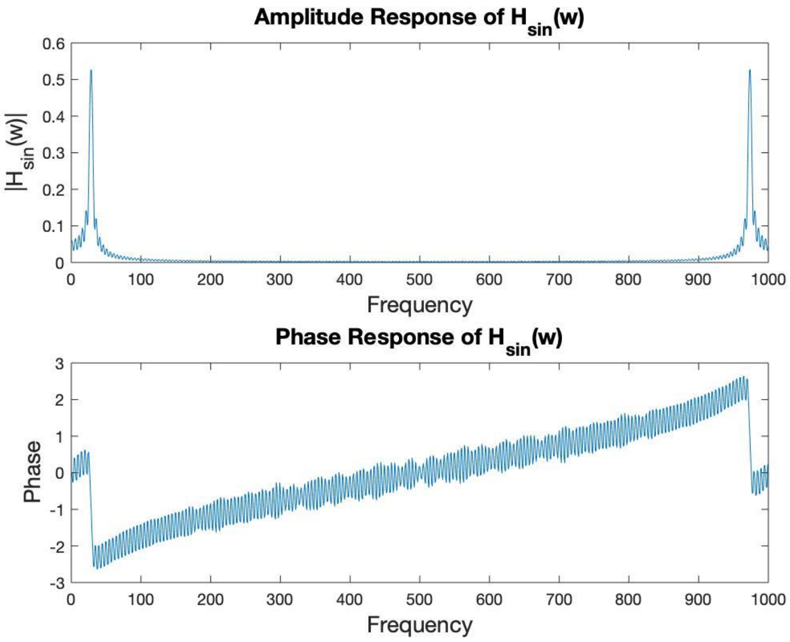


Figure 2 Magnitude and Phase response of LMS filter for sinusoidal noise

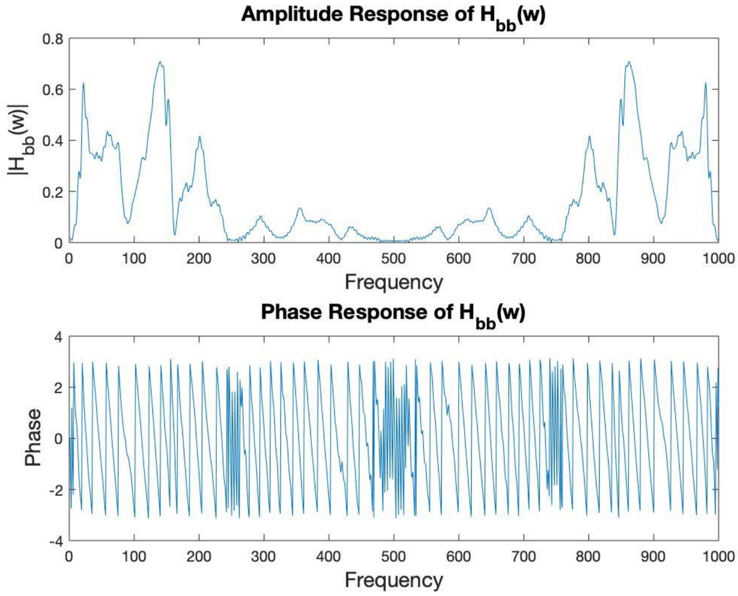


Figure 3 Magnitude and Phase response of LMS filter for broadband noise

Task 2

The LMS filter is being trained for Broadband noise and after convergence the filter coefficients are stopped from updating.

a) Disturbance is introduced by placing a large book between the speaker and the DSP-kit, at this point we noticed some noise. This happens due to the change in channel of the noise source and this change in coefficients is not being updated to the algorithm (i.e. our estimated channel for noise does not correspond to our real channel anymore). Thus, LMS filter was unable to cancel out the noise.

b) While raising/lowering the volume (amplitude) the LMS filter still cancels out the noise as expected, because in this case we did not change the channel of our noise (corresponding to subsection a) and thus our estimated channel holds good for filtering the noise w.r.t the real channel. This implies that filter works irrespective of the amplitude of the signal.

c) As expected, the LMS filter successfully removes the disturbance signal even after changing the position of the cell phone. This happens since we did not change the channel of our noise source (corresponding to subsection a), which implies filter works irrespective of source of music.

Task 3

From Fig 1, the maximum length is taken as 250 and further decreasing in length, the magnitude of filter coefficients at each sample remains unchanged. In this process the filter coefficients need not be reset for the error to converge as it is updated continuously and returns the same coefficients even after resetting. It was observed that the filter does not work equally for all number of coefficients. For a length of 250 to 100 it works as expected but as the length decreases further the performance reduces. If the filter is shorter than 45, it gives very poor performance. Thus, the critical length is 45. It can be observed from Fig 1 that the magnitudes of first 45 filter coefficients are very close to 0 (due to time delay between transmitting and receiving).

Task 4

From Fig 1, when decreasing the length from 100 to 10 the shape remains unchanged while magnitude is scaled significantly. When increasing the number of elements from 10 to 100, it can be observed that the first 10 coefficients have the same magnitude and the rest takes up the initial value as 0. It is not required to reset the coefficients after changing the length, as after some time the coefficients will converge. It is observed that the filter works well for all lengths above 10.

Task 5

Case 1: LMS filter trained for sinusoidal noise and tested for broadband noise

Result: The filter fails to effectively cancel the broadband noise. This is because the filter is trained to attenuate signals with frequencies in the neighbourhood of 440 Hz (due to background noise), corresponding to the training noise frequency. Since the broadband noise is a signal with frequencies spread in the range [0 2000] Hz, all the frequencies other than 440 Hz are not attenuated.

Case 2: LMS filter trained for broadband noise and tested for sinusoidal noise

Result: The filter attenuates the sinusoidal disturbance to a reasonably good extent. There is a clear difference observed when the filter is enabled and disabled. This is due to the fact that while training for broadband noise cancellation, we also train the filter to attenuate signals at 440 Hz. However, how well the filter attenuates the sinusoidal noise depends on the amplitude of the broadband noise signal at 440 Hz.

Task 6

Clipping is a form of distortion that limits a signal once it exceeds a threshold.

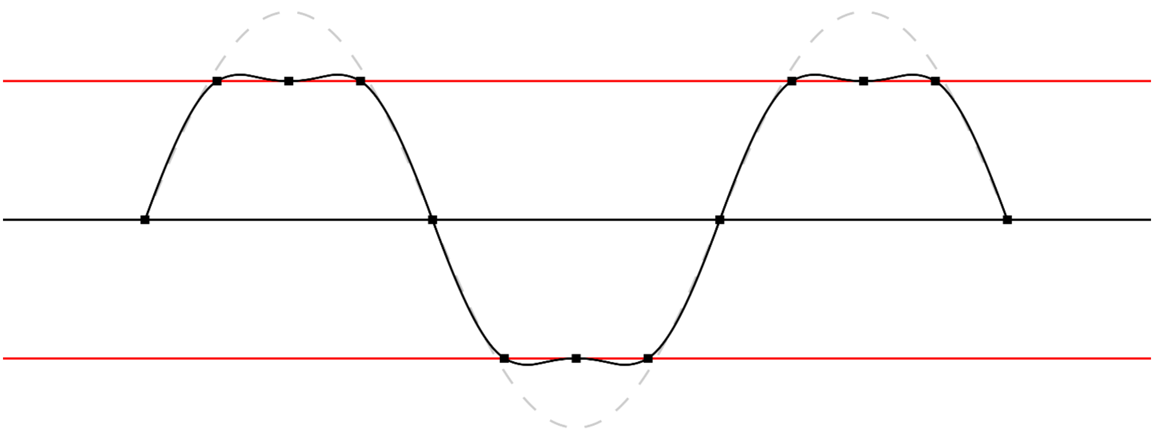


Figure 4. Clipping Mechanis[1]

According to the above mentioned figure, it can be found that:

$$f(x+y) \neq f(x) + f(y)$$

Therefore clipping is a nonlinear process. Since LMS method is based on linear channel assumption, as it is mentioned in the question, for nonlinear channel our mathematical tools won't work anymore. Since, increase in volume for microphone to get saturated cause clipping in channel and makes channel nonlinear. Therefore, hBB and hBB(sat) doesn't look like to each other as it can be seen in the following figures. Hence, hbb (sat) would not converge.

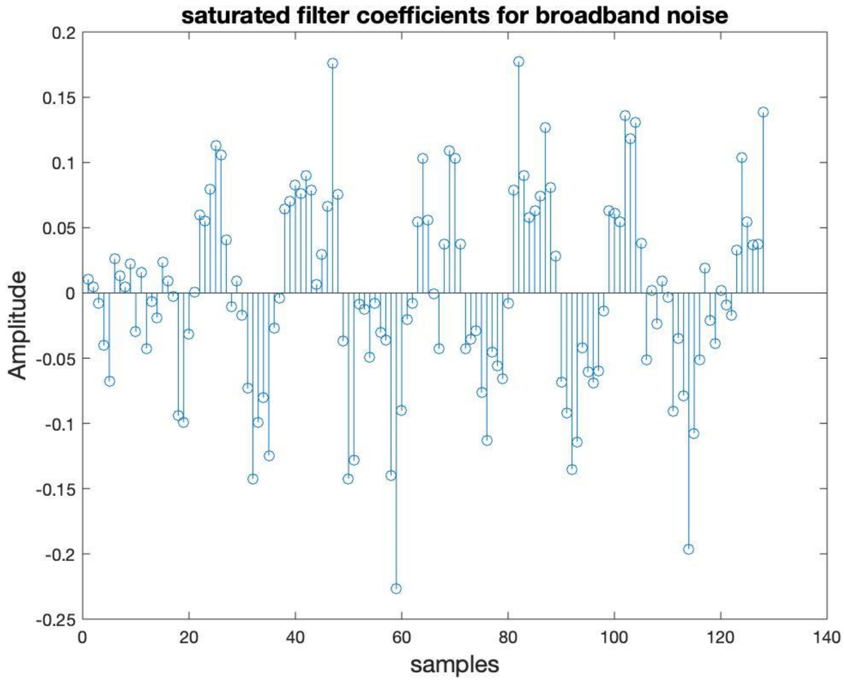


Figure 5 Filter coefficients for saturated channel

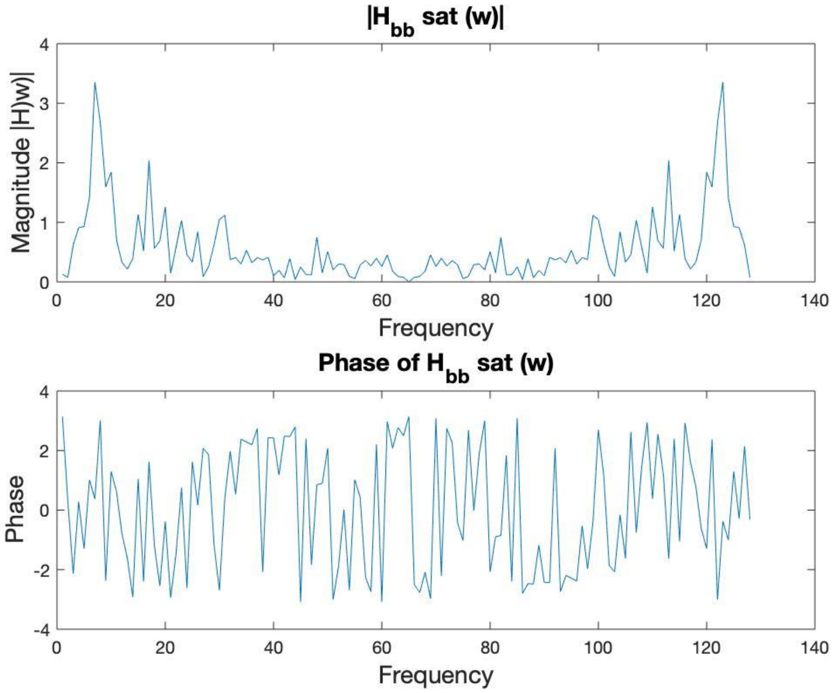


Figure 6: Magnitude and Phase response of LMS filter for saturated channek

Analytical Section:

Task 1

a) The convergence of the LMS filter is inversely proportional to the step size. The LMS iterative filtering equation is given by

$$\hbar(n) = \hbar(n-1) + 2\mu y(n)(x(n) - \hbar^T(n-1)y(n))$$

As seen for the equation, smaller the step size μ, the $\hbar(n)$ will be closer to $\hbar(n-1)$, given the error.

b) A large enough step size will cause the channel estimate to diverge because of the nature of LMS algorithm. Given the quadratic formulation of the LMS convergence problem, a large enough step size will *'push'* the estimate to the other side of the symmetric axis, away from the global minima. This will obviously result in a large error, in response to which the channel estimate is pushed to the former end even higher, resulting in an even higher error.

The second summation term in the equation is a representative of the gradient, which increases rapidly for a product of high μ and a high error $(x(n) - \hbar^T(n-1)y(n))$.

c) The choice of the step length does affect the filter performance even after it has converged. This was tested using the DSP-kit. The filter was allowed to converge and then the step size was increased from $\approx 10^{-3}\ to\ \approx 10^{-1}$. The filter diverged and a loud cracking sound was heard.

This can be explained due to the fact that, the objective function of the LMS filter tries to minimize the error variance, to zero. However, in reality, the filter never really reaches that state due to background noise and thus the filter never really *'converges to zero error'.* Therefore, changing the step size even after a long time will cause the filter to diverge.

Task 2

The difference between sinusoidal and broadband disturbance source could be observed if we compare between Appendix A.2. The shape will differ significantly due to the existence of minimum length which is required to cancel the sinusoidal disturbance which is 10 elements. In addition, when the filter length is modified to 100 elements, the filter still works equally well in canceling noise. Hence since the filter works well with 10 elements as well as 100 elements or theoretically in any number of filter length, we could conclude that the solution to the LMS objective is not unique for the sinusoidal signal depending on the filter length.

On the other hand, for the broadband disturbance, if we increase the filter length above the minimum length as shown in Appendix A.2, it will stop canceling the noise. However, if keep increasing the length, the filter will readapt itself to the initial filter impulse response. Hence, since the filter works not well above the practical minimum length, the solution to the LMS objective will be unique to the certain filter length.

Task 3

In the case that we shift the disturbance for the filter which is trained for filtering sinusoidal disturbance without resetting the filter coefficient ($H_{sin\rightarrow BB}$), the filter **will adapt** to the sudden change in the disturbance type *by matching its magnitude and phase to the newly introduced frequencies outside* $f_0 = 440\ kHz$.

For sinusoidal disturbance, the spectrum would be delta function. Outside the sinusoidal disturbance frequency, the delta function is zero which will imply that the multiplication between Y and $H - \widehat{H}$ in the frequency domain as shown by equation (2) above, would be automatically zero hence e(n) does not need to be zero to get zero E .

On the other hand, At the sinusoidal disturbance frequency, the delta function will be nonzero and the term Y will be nonzero as well. Hence, in order to achieve the minimization criteria the term $H - \widehat{H}$ *will need to be zero only at the value of 440 Hz*.

Similarly, for the broadband disturbance, *at the whole range* of the disturbance [0,fs] = [0,16·10^3]Hz, $H - \widehat{H}$ *will need to be zero* ($H = \widehat{H}$) to cancel the noise properly.

For the case where we shift the disturbance for the filter which is trained for filtering broadband disturbance, the filter **will not adapt** to the change due to the fact that *frequency of the sinusoidal disturbance is included in the frequency of the broadband disturbance*. Moreover, the channel estimate ($\widehat{H}$) should cross with each other at the sinusoidal frequency of 440 Hz and 440 Hz would be called **the crossover frequency**.

By comparison in Appendix A.2, due to the reason explained above, $H_{BB\to sin}$ would look theoretically the same with H_{BB} with minor difference practically, as seen in the observation. On the other hand, $H_{BB\to sin}$ will be totally different to H_{sin} theoretically and practically in the observation except at the single crossover frequency (fs = 440 Hz).

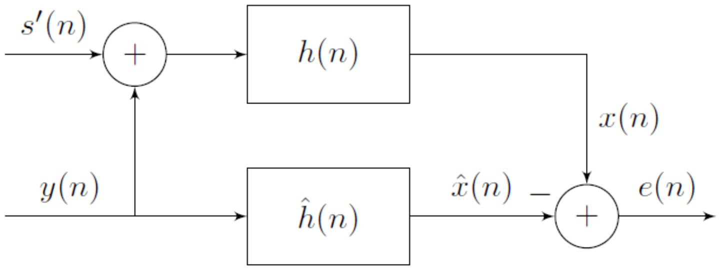
In addition, $H_{sin\to BB}$ would not look the same with H_{sin}. $H_{sin\to BB}$ would look similar to H_{BB} due to the reason explained above.

Task 4

The magnitude and phase of the H_{sin} and H_{BB} for f_0= 440 Hz is shown below in Appendix A.2. Initially before plotting, we assume that they will be *similar practically* for both magnitude and phase due to the fact that the optimization will take place in order to get zero difference between estimation and real. Moreover, we assume that the estimation *for H_{sin} to be the most accurate* of them both since H_{sin} use only single fixed frequency whereas H_{BB} noise source is from an upsampled gaussian process, sampled at a large range of frequency, hence introducing inaccuracies.

After plotting the magnitude and phase, we found out that the magnitude plot is *somewhat similar* whereas the phase angle plot is *entirely different* from each other. The difference in magnitude from the corresponding sample generally **averaged at 4.76%** and **ranged from 1.64 – 23.02%.** On the other hand, the maximum difference in phase angle between both reached about **5.8 radians** due to stark difference between both H_{BB} and H_{sin} in corresponding samples. This shows that our initial expectation is only met by the magnitude plot.
Moreover, both magnitude and phase are totally different in other frequencies since sinusoidal disturbance filter (H_{sin}) is trained only to filter at one specific frequency of 440 Hz whereas broadband disturbance filter (H_{BB}) is trained to filter the whole range of broadband frequency.

Task 5



According to the above figure for sinusoidal disturbance we expect:

$$E = \underline{H}Y - HY = 0$$

Since H is a complex valued function:

$$H = Re + j\,Im$$

And estimated channel is :

$$\underline{H(k)} = \sum_{n=0}^{N} \quad h(n)\, e^{j\frac{2\pi nk}{N}} = \sum_{n=0}^{N} \quad h(n)\, e^{j\frac{w}{ws}}$$

For N=1 have:

$$\underline{H} = h0e^{j\,0} = Re1$$

For N=2 have:

$$\underline{H} = h0e^{j\frac{w}{ws}0} + h1e^{j\frac{w}{ws}1} = Re2 + j\,Im2$$

Since for all frequencies in order to satisfy the first equation, the following equation should be satisfied $H = \underline{H}$.

$$Re + j\,Im = Re2 + j\,Im2$$

Therefore, the minimal theoretical length which could satisfy the conditions 2

Task 6

According to equation LMS algorithm for filter update state we have:

$$\underline{h}(n) = \underline{h}(n-1) + 2y(n)\mu e(n)$$

$\underline{h}(n)$ is the summation of previous filter coefficient and $2y(n)\mu e(n)$. The noise y(n), is sinusoidal and other parameters in the term second are scalar. The second term turn out to be a sinusoidal function. Therefore, $\underline{h}(n)$ or hsin would be sinusoidal which is the summation previous filter coefficient (sinusoidal) and $2y(n)\mu e(n)$.

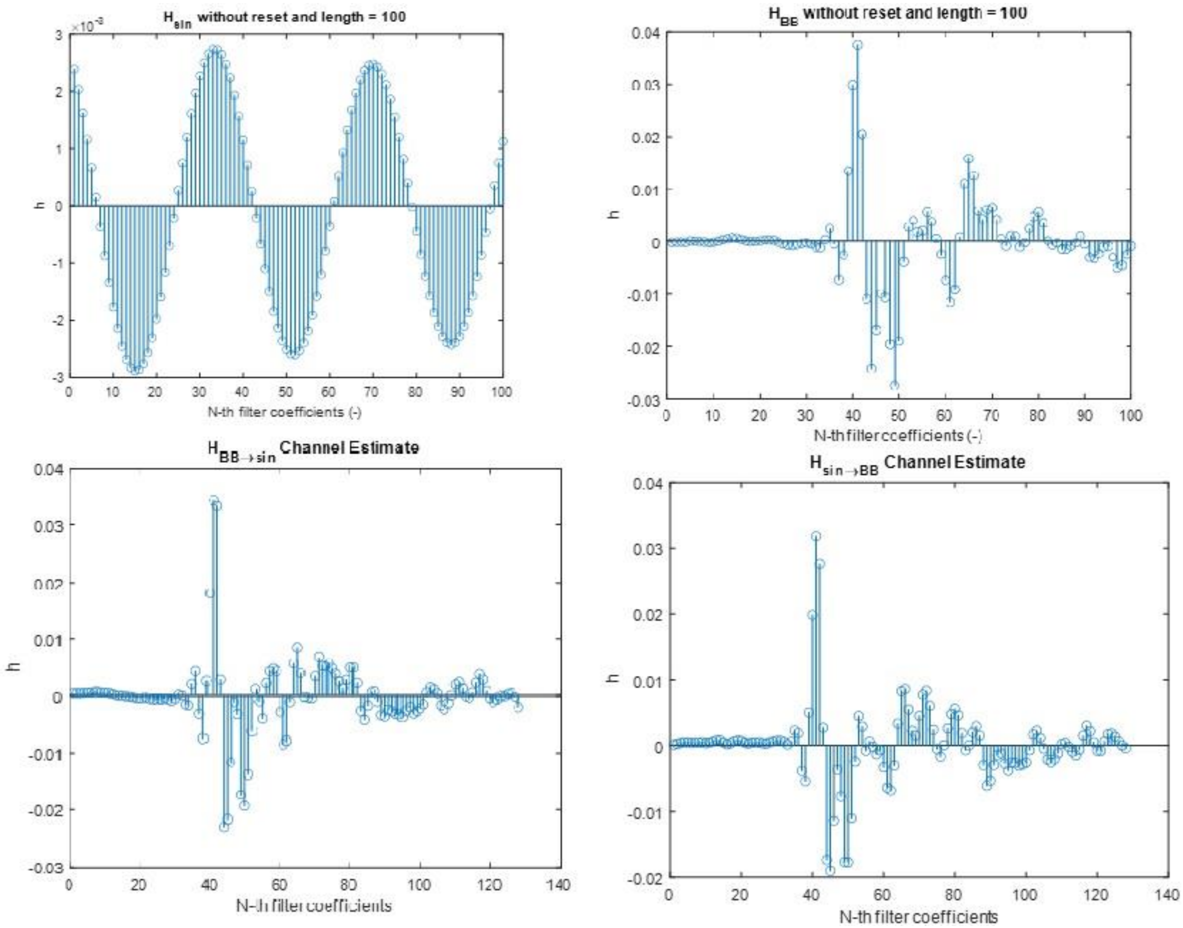
References

[1] https://en.wikipedia.org/wiki/Clipping_(signal_processing)

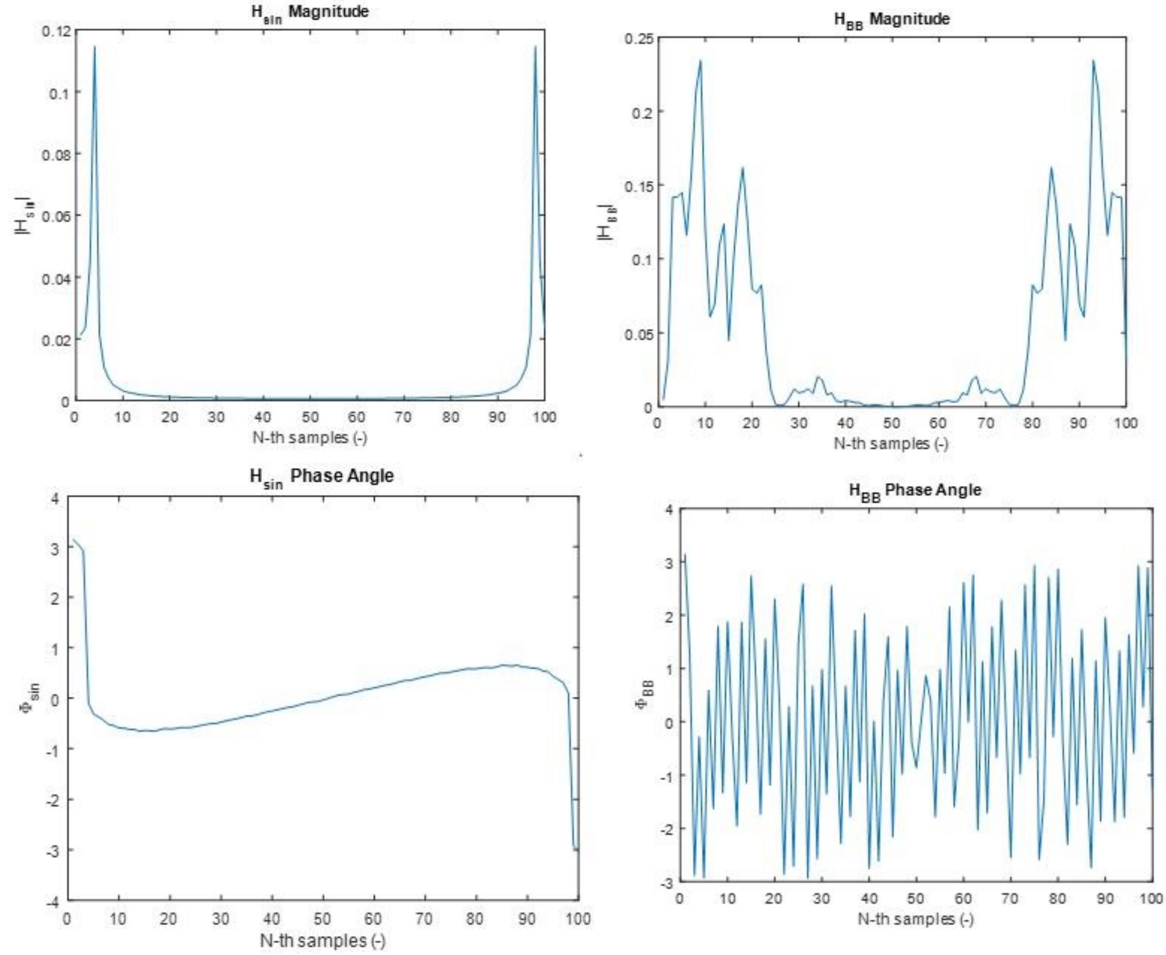
Appendices

```
int i;
int j;
for(i=0;i<block_size;i++){
float *y_book = &lms_state[i];
arm_dot_prod_f32(lms_coeffs , y_book , lms_taps, &xhat[i]);
e[i] = x[i] - xhat[i];
for(j=0;j<lms_taps;j++){
lms_coeffs[j] += (2 * lms_mu * y_book[j] * e[i]) ;
    }
}
```

Appendix A.1. Noise cancelling LMS Algorithm



Appendix A.2. Channel Estimations Plots



Appendix A.3 Magnitude and Phase Angle Plots (in Radians)