CHALMERS UNIVERSITY OF TECHNOLOGY

Applied Signal Processing

Project 2 – Acoustic Communication System (Part B)

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Question 1

Based on visual observation of Figure 1, *the bandwidth of the transmitted audio signa*l will be **1.22 kHz** and *the transmission band where frequency lies* is between **0.738 kHz** until **1.958 kHz**. These numbers come from the calculation of only one non-zero signal. We could only consider one signal since it is a symmetric so that we can move the second non-zero signal from Right Half Plane (RHP) to Left Half Plane (LHP) which will produce identical signal in LHP. The output that we want to consider is a result of interpolation as well as modulation, but before it is transmitted through the channel. Moreover, the result of modulation is then processed by discarding the imaginary part of the signal since it will be transmitted through the channel which only works with the real valued signal. The result of this the one that is plotted in Figure 1 here.

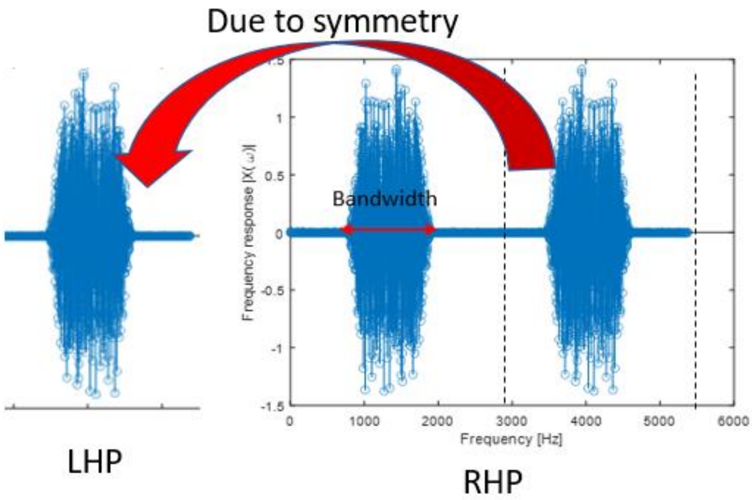


Figure 1. Frequency Response of the Transmitted Signal in Frequency Domain

Question 2

In part A, prefixing the OFDM package with the last 59 samples will maintain the orthogonality of the OFDM frequencies while starting the convolution of the '*real*' information with a linear filter. Therefore, it will not result in EVM as the cyclic prefix will guard the accuracy of the output signal without the need to pass through a physical channel.

One of the reason that the EVM is becoming a nonzero value is due to non-ideal process of the low pass filtering in the interpolation process which eliminates some amount of information which belongs to the high frequency element (*aliasing*). These eliminated signals will mostly contain ripple. On the other hand, the upsampling process in the interpolation and decimation will result in ideal process whose output signal does not result in the loss of information. The general process of interpolation and decimation is shown in Appendix A.1 and A.2.

Another possible suspected cause is the transmission over the ideal physical channel will result in some small deviation in the EVM. However, these deviation will be negligible since both with or without the channel will still output nonzero EVM ($> 10^{-15}$) and no improvement to the state of EVM as shown in the Table 1 below. Therefore, we conclude that this *will be a negligible additional cause* for the deviation of EVM.

Table 1. EVM value variation

With simulated physical channel	Without simulated physical channel (h=1)
0.00375	$4.86 \cdot 10^{-14}$

Modulation is essentially a necessary process in order to transmit the signal through the physical channel which is located at higher frequency than the signal.

Modulation process is receiving input signal of the result of interpolation process which creates distortion to the original information due to non-ideal process. However, modulation *will not add further distortion* to the signal just by shifting the frequency of non-zero signal's middle point to center modulation frequency ($\frac{fcm}{fs} = \frac{1}{4}$), hence modulation *will not result in the loss of information*. Similar effect is given by the demodulation process where it *will not result in the loss of information.* Modulation process could be denoted as shown in the equation below where $e^{j\omega_M t}$ is the additional shifting term to the original signal $X(\omega)$.

$$X(\omega_M) = e^{j\omega_M t} \cdot X(\omega)$$

Moreover, demodulation process will not be resulting in perfectly back-shifted signal to the baseband signal since it is shifting back a distorted signal. As a result, the distorted signal that we received will be the main reason to the nonzero EVM in addition to the distortion in decimation process, which is in similar effect with interpolation, before it is going through equalization and QPSK-to-bit conversion. The comparison between the output of interpolation process **before Modulation** and **after Demodulation** process is shown below in Figure 2 and 3 respectively.

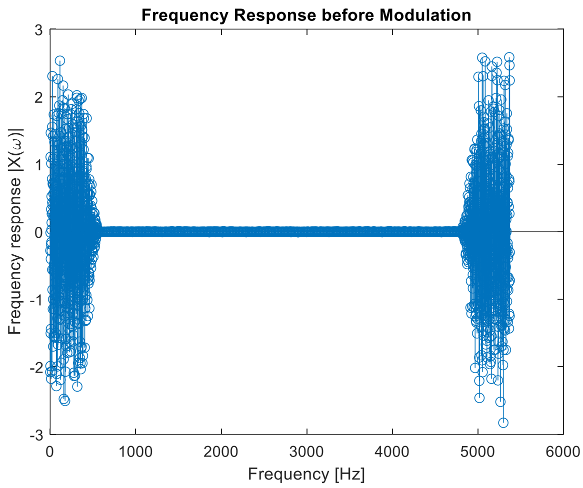


Figure 2. Frequency Response of the signal before Modulation in Frequency Domain (FFT plot)

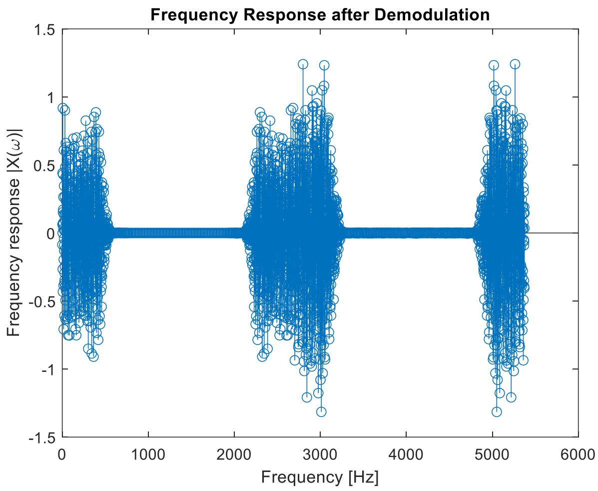


Figure 3. Frequency Response of the signal after Demodulation (FFT plot)

Question 3

During interpolation we up-sample the baseband signal and use low-pass filter to it. Up-sampling means to increase the size of samples from $z(n)$ to $z_u(n)$ by a factor of R and low-pass filter removes the R-1 images of z_u. Hence this stage has significant contribution to H. At the Decimation stage we apply the same low-pass filter again and down-sample. Here we apply the low-pass filter to avoid aliasing and then reduce the size of samples by the same factor R which in turn reduces the magnitude of the signal. It is to be noted that convolution of signal with filter causes distortions, at the stages like interpolation and decimation where low pass filter is convoluted with the signal hence contributes to the H.

Transmission over a physical channel requires only the real parts of the signal, however by excluding the imaginary part no loss of information occurs. Since our channel is air medium disturbance occurs, and if not taken into account then equalization step would not be required. For the above reasons it is necessary to consider this step.

Modulation shifts the absolute position of the signal to the passband with the center modulation frequency f_{cm} whereas demodulation shifts the modulated signal to bring it back to the base-band, which is achieved by modulation with negative centre modulation frequency f_{cm}. Hence the effect of modulation is completely removed by this process and does not affect the estimation of H.

Question 4

By up sampling we transform the information and place them around another w0 frequency which is suitable for channel. Since the channel is real value the signal need to be real value as well. But when we modulate the signal it contains the real and imaginary part. Therefore, it won't be possible to transmit the real and imaginary part. If we take only the real parts what will happen is we keep the half of original data and add the half of complex conjugate of the data. The following figure illustrates the process described above. Now, since the signal is real value it is possible to transmit it from the channel.

$$Re = \frac{1}{2}(X + \bar{X})$$

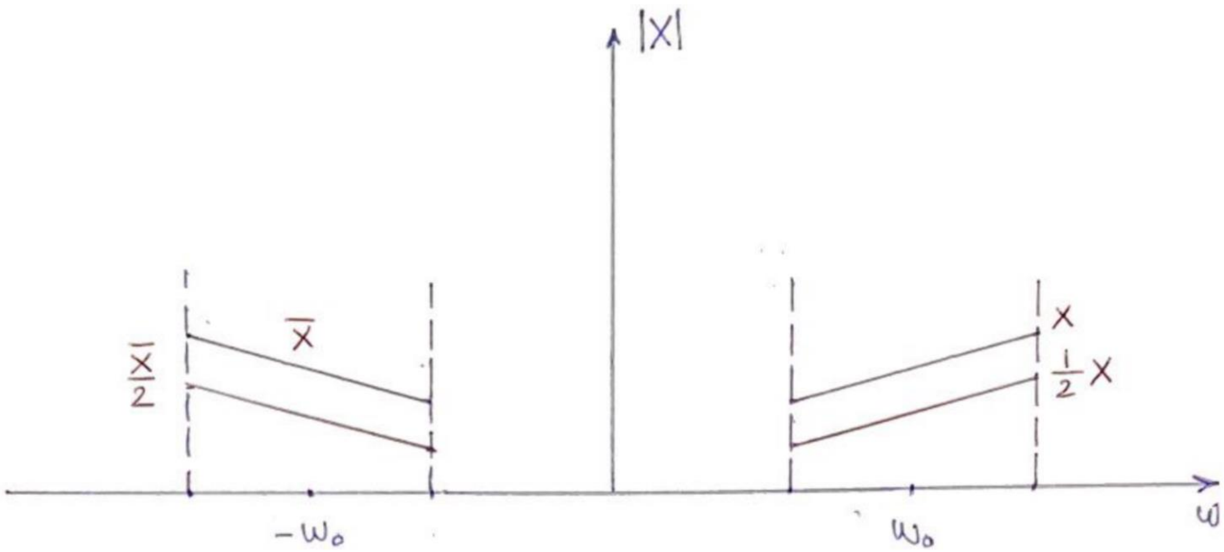


Figure 4.

If we take the Imaginary part, it can be calculated by:

$$IM = \frac{1}{2J}(X - \bar{X})$$

In a same way what will happen is we keep the half of original data and add the half of complex conjugate of the data. Therefore, in the magnitude plot it will look the same. Which means that, there is nothing magic in the real part because it contains the same information as the imaginary part. We can also say this is the effect modulation. Since our initial signal (before modulation) contains both imaginary and real part. And both of the part are important and we need them all otherwise we lose information. But since we increase the sample rate, we would have same information, and when we modulate we shift information and the real and imaginary part become redundant, and they contain same information.

Question 5

Interpolation Stage:

- **Passband ripple**: The passband ripple of the interpolation low pass filter, in the frequency domain, can be estimated during equalization process. The passband ripple in a sense, becomes the property of the channel when transmitted, and hence does not affect the equalization accuracy as such.
- **Stopband attenuation**: The stopband attenuation of the low pass filter is quite an important property, as low stopband attenuation may corrupt the original signal with multiple images of the signal at higher frequencies produced as a result of upsampling.
- **Transition band width:** The transition band width of the low pass filter becomes more important as the upsampling factor increases. As the factor increases, more images of the original signal is created in the frequency spectrum, as the spectrum ‘squeezes’ together. Thus, for a highly upsampled signal, the transition band width becomes very important to avoid aliasing from signal images at higher frequencies.
- **Phase linearity:** The phase change of the signal (linear or non- linear) due to the low pass filter can be estimated during the equalization step. It can be considered to become a property of the channel which is accounted during the equalization step. Hence, the phase linearity is not such an important property.

Decimation Stage:

- **Passband ripple:** The passband ripple of the decimation low pass filter, can be estimated during the equalization process and hence is not such an important property.
- **Stopband attenuation:** The stopband attenuation of the lowpass filter here is still important, but not as critical as in the interpolation stage as most of the signal images between the base frequency and upsampled sampling frequency would be filtered after interpolation stage.
- **Transition band width:** The transition band width depends on the spacing between the spectral images of the signal in the frequency domain and the downsampling factor. Thus, for lower downsampling rates, if the spectral images are well separated during interpolation stage, the transition band width is not so critical.
- **Phase linearity:** The phase change of the signal (linear or non- linear) due to the low pass filter can be estimated during the equalization step. It can be considered to become a property of the channel which is accounted during the equalization step. Hence, the phase linearity is not such an important property.

Question 7

The Magnitude Equivalency Proof

Simplifying the more computationally demanding method of estimating the unknown channel's amplitude, we get the equation below assuming that T,H and R be the Fourier transforms of the transmitted signal, channel and received signal respectively

$$|\widehat{H}(k)|_{demanding} = \left|\frac{R}{T}\right| = |R| \cdot \left|\frac{1}{T}\right| \quad (1)$$

Comparing it to the less computationally demanding method of estimating the channel's amplitude that is being used on the test in the DSP-kit, we get the equation below since magnitude of conjugate T ($|\overline{T}|$) equal to magnitude of T and due to its commutative property

$$\left|\widehat{H}(k)\right|_{non-demanding} = \left|\overline{T} \cdot R\right| = |R| \cdot |T| \quad (2)$$

By assuming that equation (1) = equation (2) which is the point that we ultimately want to prove we get the equation

$$|R| \cdot \left|\frac{1}{T}\right| = |R| \cdot |T| \quad (3)$$

Since R is the same for both Left Hand Side (LHS) and Right Hand Side (RHS) equation, we could simplify |R| to get the equation

$$\left|\frac{1}{T}\right| = |T| \quad (4)$$

Assuming that |T| has the magnitude of 1 due to the distance from the origin of the 4 symbols in the unit circle of the constellation plot ($|T_1|$, $|T_2|$, $|T_3|$, $|T_4|$) , which is 1, as well as the location of each symbol which is $\sqrt{\frac{1}{2}}(\pm 1 \pm j)$ as shown in Figure 4 below, we get

$$\left|\frac{1}{1}\right| = |1| \quad (4)$$

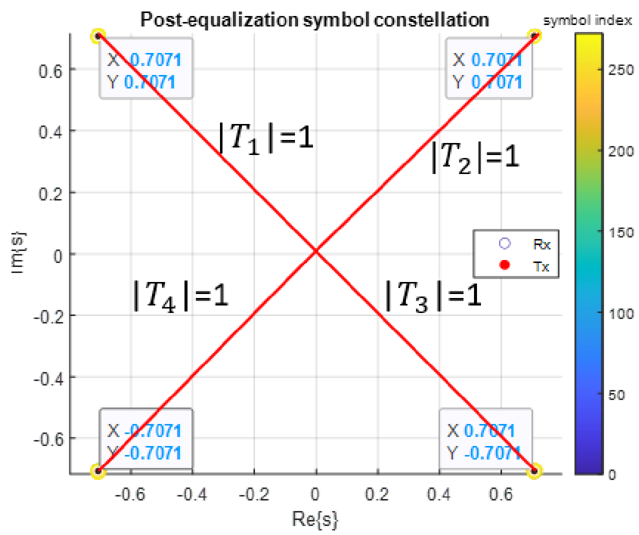


Figure 4. Constellation Plot of the unit circle

Equation (4) proved the equivalency of using the less computationally demanding way of channel estimation

The Phase Equivalency Proof

Using the more computationally demanding way of channel estimation, we get the equation below

$$\widehat{H}(k)_{demanding} = \frac{R}{T} = \frac{|R| \cdot e^{j\theta_1}}{|T| \cdot e^{j\theta_2}} \qquad (5)$$

Calculating for the Phase of the channel ($\angle H_{demanding}$) we will get the following equation

$$\angle H_{demanding} = \theta_1 - \theta_2 \qquad (6)$$

whereas for the computationally less demanding way of channel estimation we get the equation below

$$\widehat{H}(k)_{non-demanding} = \overline{T} \cdot R = |T| \cdot e^{j(-\theta_2)} \cdot |R| \cdot e^{j\theta_1} \qquad (7)$$

Calculating for the Phase of the channel ($\angle H_{non-demanding}$) we will get the following equation

$$\angle H_{non-demanding} = -\theta_2 + \theta_1 \qquad (8)$$

Hence, from equation (6) and (8), we have prove that

$\angle H_{demanding} = \angle H_{non-demanding}$

Question 8.

Consider a pilot signal X(n) as an input signal. By passing from the channel the received signal would be:

$$|H(w0)| = X(n)e^{j<H(w0)}$$

Therefore, the channel would be Fourier transform of the received signal divided Fourier transform of transformed signal.

$$H(w) = \frac{FFT(R_{recieved})}{FFT(R_{transmitted})}$$

$$H(w) = \frac{|H(w0)|e^{j<H(w0)}\sum_{n=0}^{N-1} R(n)e^{-j2\pi kn/N}}{\sum_{n=0}^{N-1} R(n)e^{-j2\pi kn/N}} = |H(w0)|e^{j<H(w0)}$$

And the equalized can be calculated as following:

$$Req = \frac{|H(w0)|e^{j<H(w0)}\sum_{n=0}^{N-1} R(n)e^{-j2\pi kn/N}}{|H(w0)|e^{j<H(w0)}} = \sum_{n=0}^{N-1} R(n)e^{-j2\pi kn/N}$$

Therefore, as it can be seen both the phase and the magnitude of the channel would be cancel out, and it correctly equalize the phase of Req.

Question 9

The task was examined with two cases:

Case a) With high amplitude signal and a constant background noise level:

- The constellation plot and EVM were found as shown in the figure:

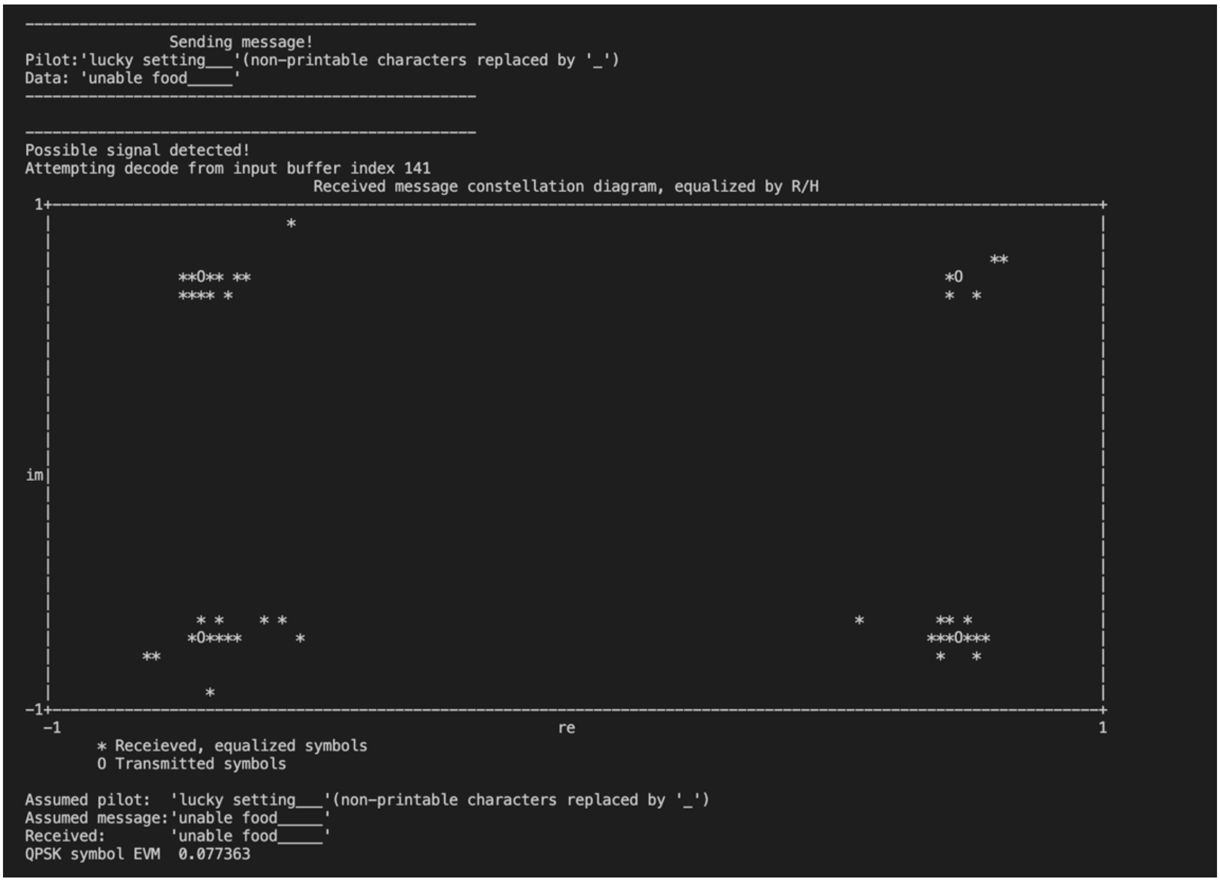


Figure 6. Constellation plot for high amplitude signal

- The EVM was found to be 0.07763. All the transmitted symbols were recovered perfectly as seen in the figure above. This is because, the high amplitude of the transmitted signal leads to a high Signal to Noise Ratio which is good during the equalization process. Thus, with a sufficiently high amplitude, the transmitted message can be perfectly recovered in the presence of noise.

Case b) With low amplitude signal and a constant background noise level:

- The constellation plot obtained for Case b is as shown below

Figure 7. Constellation plot for low amplitude signal

The EVM for this case was found to be 0.761, which is almost 10 times higher than the previous case. This is because, when the amplitude of the signal from the speaker is reduced, the Signal to Noise Ratio of the transmitted signal becomes small, and hence the received signal cannot be recovered properly as observed.

Question 10

From the constellation diagram it is clear that moving the speaker towards the DSP kit after initiating the transmission causes a rotation (phase shift) of received symbols in the anticlockwise direction and while moving the speaker away from the DSP kit causes a rotation (phase shift) of received symbols in the clockwise direction.

This happens due to the transmission of pilot and data at different channels (i.e. different distances between the speaker and the DSP kit as we change the position of the speaker).

Relation between the frequency and time domain can be given as

a rotation (phase shift) in frequency domain can be related to a delay(shift) in time domain which is represented as $X(w)\, e^{jwt_o} = x(t - t_o)$.

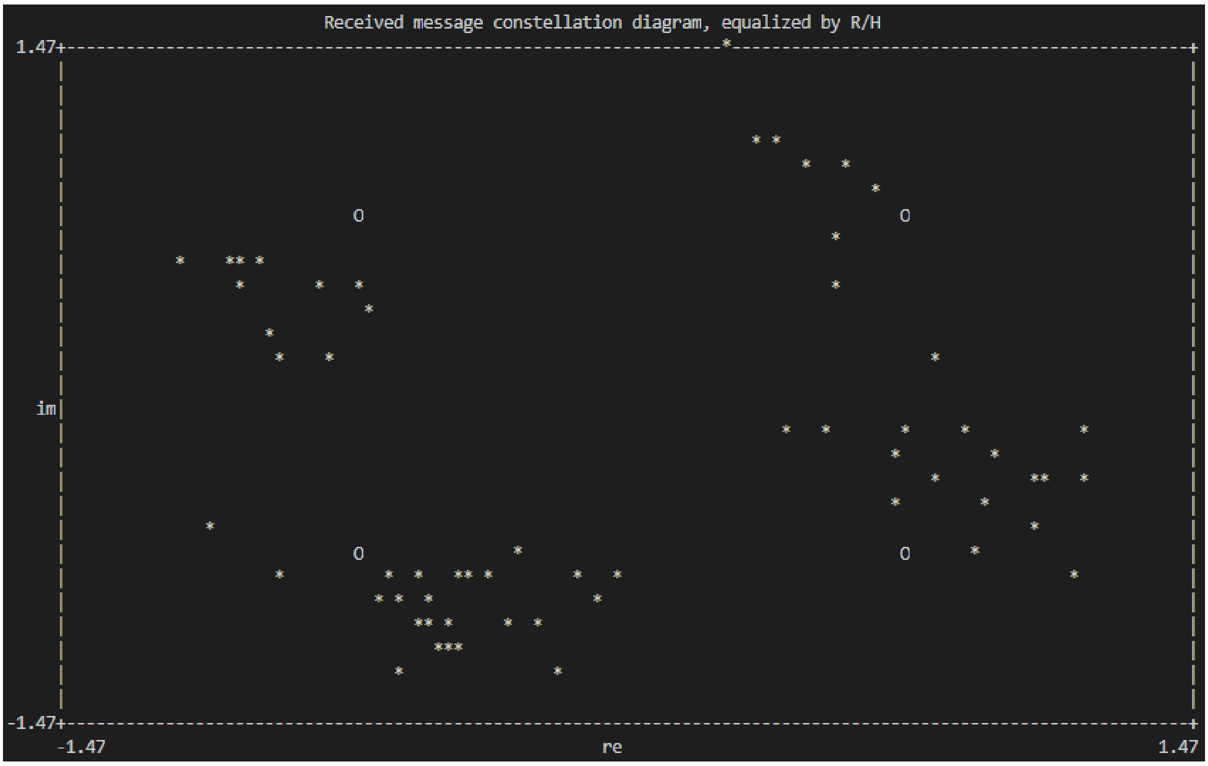


Figure 8 representing the constellation plot of speaker moving towards the DSP kit

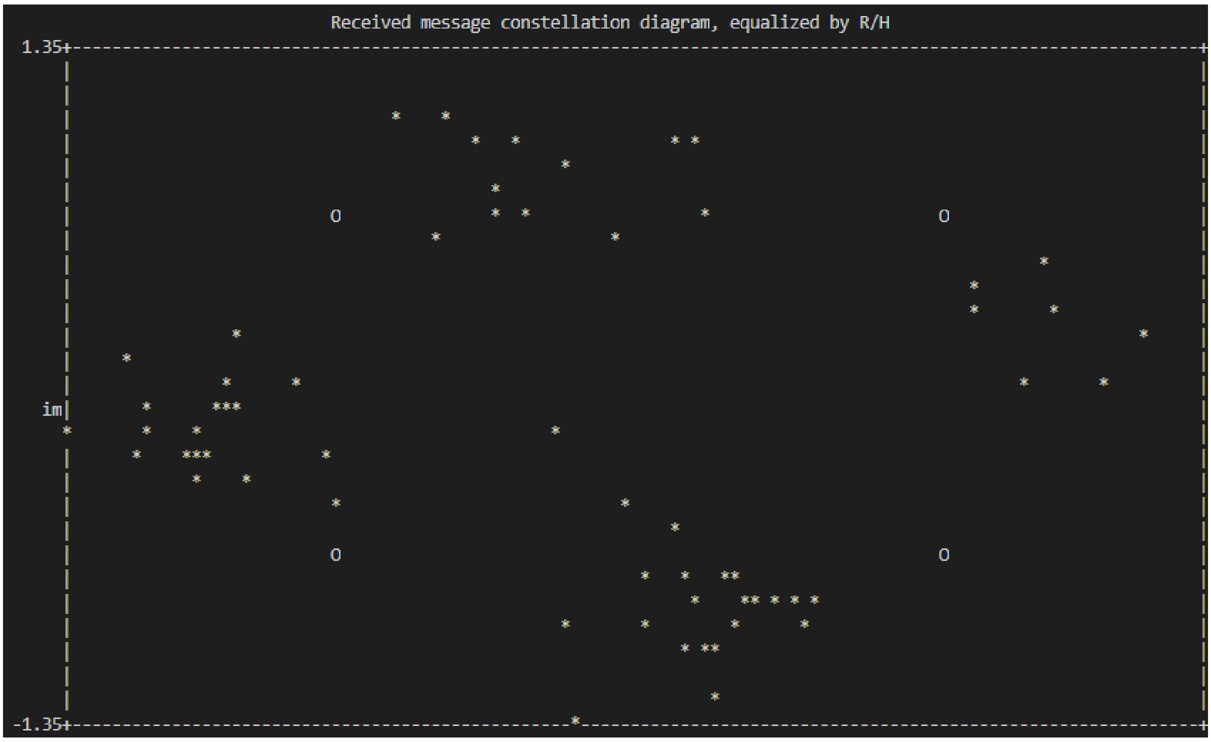


Figure 9 representing the constellation plot of speaker moving away from the DSP kit

To not have any bit errors the phase shift must not exceed 45 degrees (pi/4 rad). Based on the constellation diagram assume a phase shift of about 20 degrees (~ 0.35 rad).

Calculation of relative velocity between the speaker and the DSP kit.

Starting from known parameters Velocity of sound in air is 343 m/s, Symbols were transmitted in a band between 3 and 5 kHz, thus assume an average of 4khz, hence w_{avg} = 8000pi in rad/s and sampling frequency f_s is 16khz.

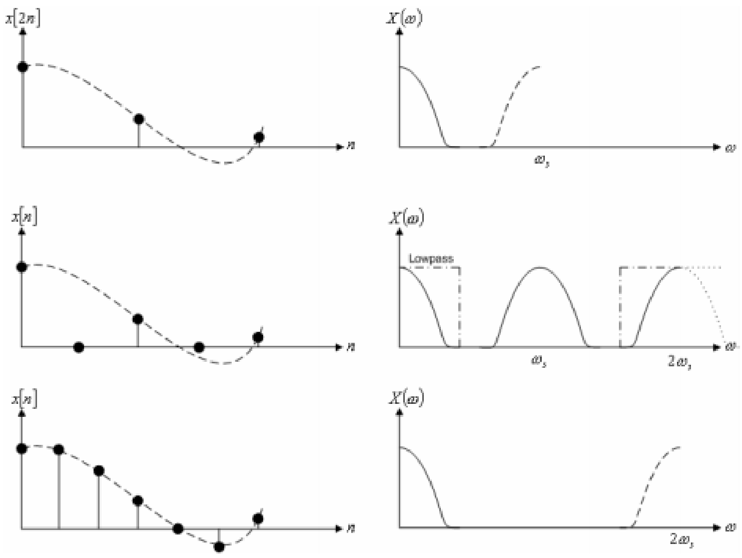
We can calculate the distance w.r.t time delay which is represented by delta_d = velocity of sound * t_o where t_o is the average time delay given by t_o = phase shift (in rad) / w_{avg}. Thus delta_d = velocity of sound * t_o = 343 * (0.35 / 8000pi).

Time difference between first pilot sample and first data sample is given by delta_t = $N_{samples}$ /f_s = bits/f_s = 0.048 s.

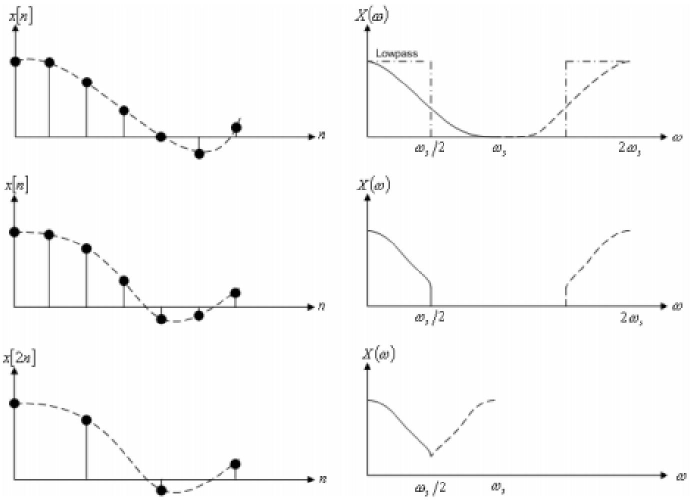
And finally, the relative velocity (between the speaker and the DSP kit) is given by V= delta_d / delta_t = ~10 cm/s.

For maximum velocity a phase shift of about pi/4 (i.e. 0.785) rad and w = 10000pi rad/s are to be assumed. Thus, by repeating the same procedure we get the maximum relative velocity V_max of ~ 18cm/s.

Appendix A



Appendix A.1 Ideal Interpolation Process by Upsampling factor 2 [1]



Appendix A.2 Ideal Decimation Process by Upsampling factor 2 [2]

Reference
[1] Engelbert, Anna & Hallqvist, Carl (2008). Digital filters Computable efficient recursive filters.

Accessed from : http://publications.lib.chalmers.se/records/fulltext/74710.pdf (2019/12/03)