CHALMERS UNIVERSITY OF TECHNOLOGY

Applied Signal Processing

Project 1 – Acoustic Communication System (Part A)

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1a.

- The received symbols after and before equalization would be same since the channel is ideal and the complex channel gain doesn't contain any magnitude scaling or phase shift. This can be seen in Figure 1 and Figure 2.
- Transmitted symbols and received symbols are same because and as the EVM and BER are zero. This fact is because of following reasons:
    1. We don't have any synchronization error. It means that we would not have any phase shift from the received symbols. This will be discussed in 1d.
    2. Signal to noise ratio is infinity. It means that there would be no deviation between transmitted and received signals.
- EVM: 3.01e-16 and BER: 0. BER is calculated from the following equation and since the transmitted symbols are same as received ones therefore, tx would be same as rx. Hence, the numerator would be zero and the final value of BER would be 0. The EVM also approaches zero for the same reasons.

$$BER = 1 - \Sigma(rx == tx)/ N;$$
$$EVM = ||x - r\_eq||/\sqrt{N};$$

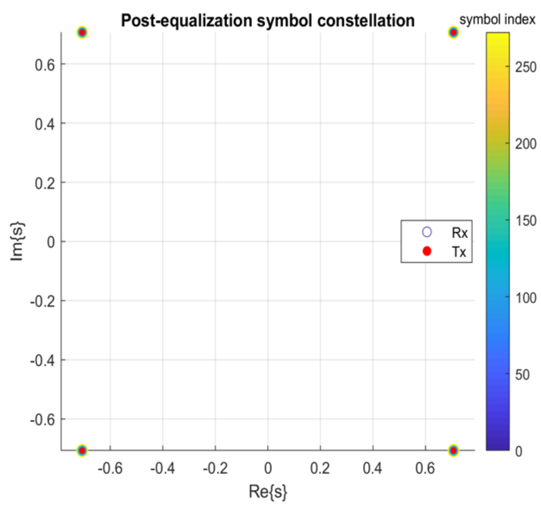

Figure 1

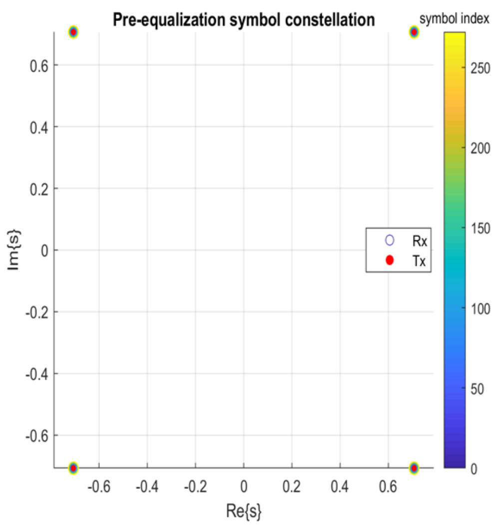

Figure 2

1b)

In this case, since the frequency response of the channel is linear, the symbols will not interfere with one other. Hence, a cyclic prefix would have no effect on the EVM of the system.

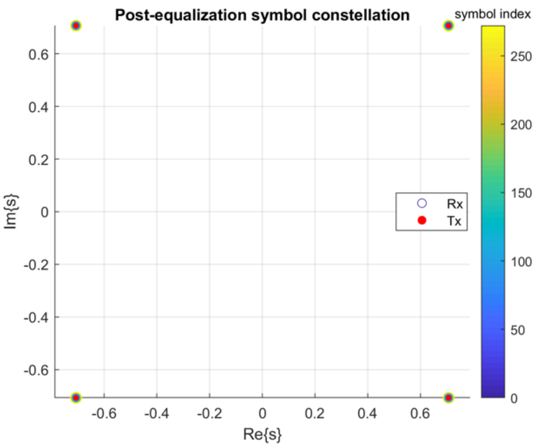

Figure 3

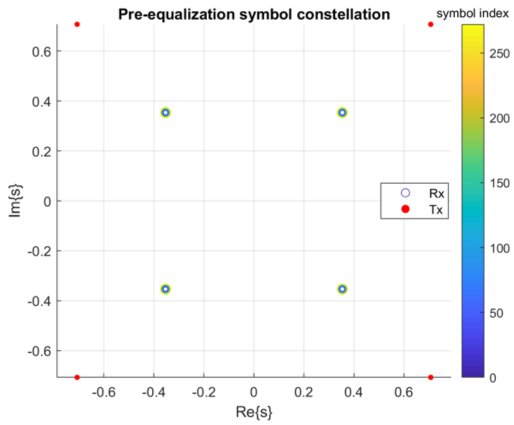

Figure 4

1c)

- In case of h_2, pre equalization constellation diagram would be scaled due to the multiplication of the signal to 0.5 and in case of h_3, pre equalization constellation diagram would rotate due to multiplication of the signal to a complex number ($e^{0.5j}$) as shown in Figure 3,4 and Figure 5,6 respectively. However, since the channel gain is only a constant value, on dividing the pre-equalized data by channel gain we get back the original transmitted information. α in firs case is 0.5 and in the second case is $\cos(0.5) + i\sin(0.5)$. The EVM and BER would be same as the part 1a since no delay or noise is added to the system.

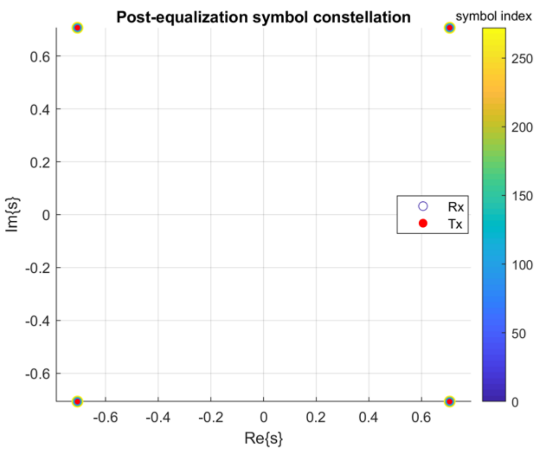

Figure 5

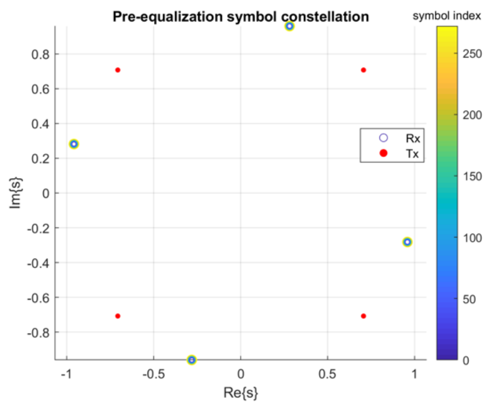

Figure 6

1d)

- A synchronization error in the communication system introduces an additional phase change in the frequency domain, leading to an additional rotation of the symbol in QPSK system. This can be explained as follows:

- The received information through the channel y(n), needs to be converted into symbols r(k), by converting y(n) to the Fourier domain using FFT. A synchronization error of 'D' can be perceived as a phase shift of $e^{-j2\pi kD/N}$, where $2\pi k/N$ is the discrete frequency.

- Therefore, the complete transform of y(n) to r(n) maybe expressed as

$$r(k) = \sum_{n=0}^{N-1} y(n)e^{-j2\pi kn/N}\, e^{-j2\pi kD/N}$$

- The additional phase rotation due to delay, $e^{-j2\pi kD/N}$ is a linear function of the frequency. Since, each symbol is associated with exactly one frequency, the phase rotation affects different symbols differently.

- In QPSK modulation technique, the symbol can be retrieved until the additional phase rotation is within the maximum allowable shift of $\pi/4$ radians (in the same quadrant as the transmitted symbol). For instance, for a synchronization error of D=1, the last symbol for which the OFDM carrier frequency undergoes a phase shift less than the maximum allowed phase shift calculated using

$$2\pi k.1/N \leq \pi/4$$

Yields, k 34 (for N=272). This corresponds to ASCII character number 8.5 (8) in the transmitted text. Hence, only the first 8 letters in the sentence can be recovered. Using a similar argument, at some 'k' further in the spectrum, the phase rotation will be $\geq (3\pi/4 + \pi/4)$ which is found to be 238. This corresponds to ASCII character 59.5 (60). Hence, every character after 60 can be recovered.

Transmitted: 'Alice: Would you tell me, please, which way I ought to go from here?'
Received: **'Alice: W**J_E_E_E_IE_,_5_1>_5_15_3_**om here?**'

- Using a similar argument, the number of ASCII characters that are corrected received can be calculated when synchronization error D = 2.

1e)

snr =30

Transmitted: 'Alice: Would you tell me, please, which way I ought to go from here?'
Received: 'Alice: Would you tell me, please, which way I ought to go from here?'
EVM: 0.0282, BER: 0

snr =5

Transmitted: 'Alice: Would you tell me, please, which way I ought to go from here?'
Received: 'Alice: Wouh& yku tell me, rleaqG, wHich way I ought to go frgm _re?'
EVM: 0.502, BER: 0.0239

On decreasing the SNR from 30 to 5, the magnitude of the noise that is added to the transmitted signal over the channel increases significantly. This allows the symbols to deviate from the original transmitted symbols, leading to a much higher EVM and BER. However, it is interesting to note that for SNR=30, although the EVM is non-zero the BER is zero. This is due to the fact that the EVM is not large enough to push the received symbols to the neighbouring quadrants.

As it can be seen from the Figure 7 and 8 below, the constellation plot for SNR=5 is much more spread out than SNR=30, indicating the presence of excess noise.

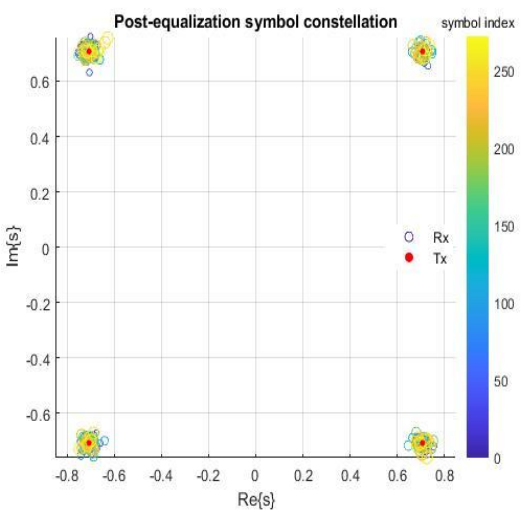

Figure 7

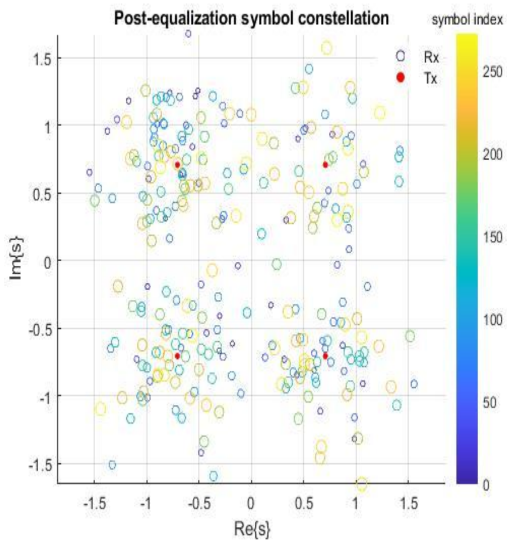

Figure 8

2a)

In part (a), in order to find the correlation between H(k) and the pre-equalization constellation diagram, we could use the equation below

$$r(k)=H(k)\cdot z(k)$$

Let we take H(1) = 5, we will try to find one symbol in the plot that gives a value of $5\cdot 12(\pm 1\pm j)$ as shown in the Figure 9

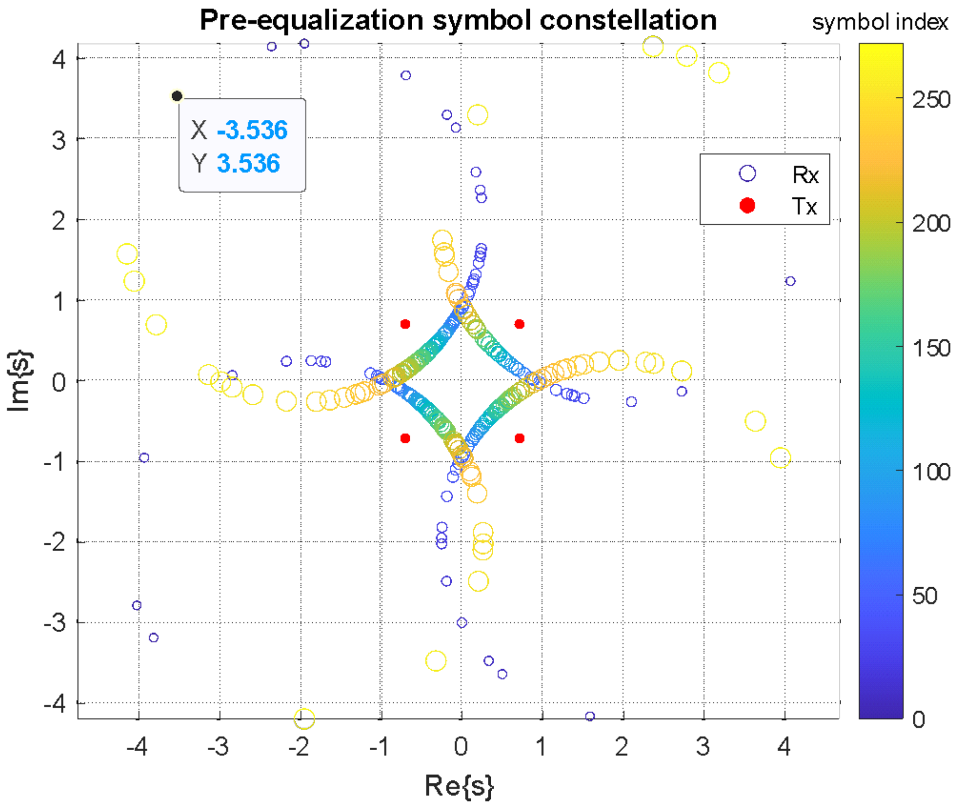

Figure 9

By visual observation of figure 1 plot, we could conclude that the relation between H(k) and constellation diagram is $5\cdot 12(-1+j)$. However, this would depend on the transmitted messages
as well. This is due to the fact that symbs.tx depends on the QPSK coding of the sequence of bits whose input is from the conversion of the strings of the text transmitted into bits. Hence, if we change the transmitted messages length, the relation would ultimately change as well.

2b)

In the part (b), using the Cyclic prefix length (Ncp) set to 59 will output the EVM and BER as $1.11 \cdot 10\text{-}15$ and 0 respectively.

A low pass channel can be expressed as series of Dirac-delta functions as shown in the Figure 10. Therefore, after 59 samples channel would be equivalent to the Dirac-delta function with an amplitude of $\sum_{n=1}^{60} h(n)$ delayed by 59 samples. Hence, prefixing the OFDM package with the last 59 samples will maintain the orthogonality of the OFDM frequencies while starting the convolution of the '*real*' information with a linear filter.

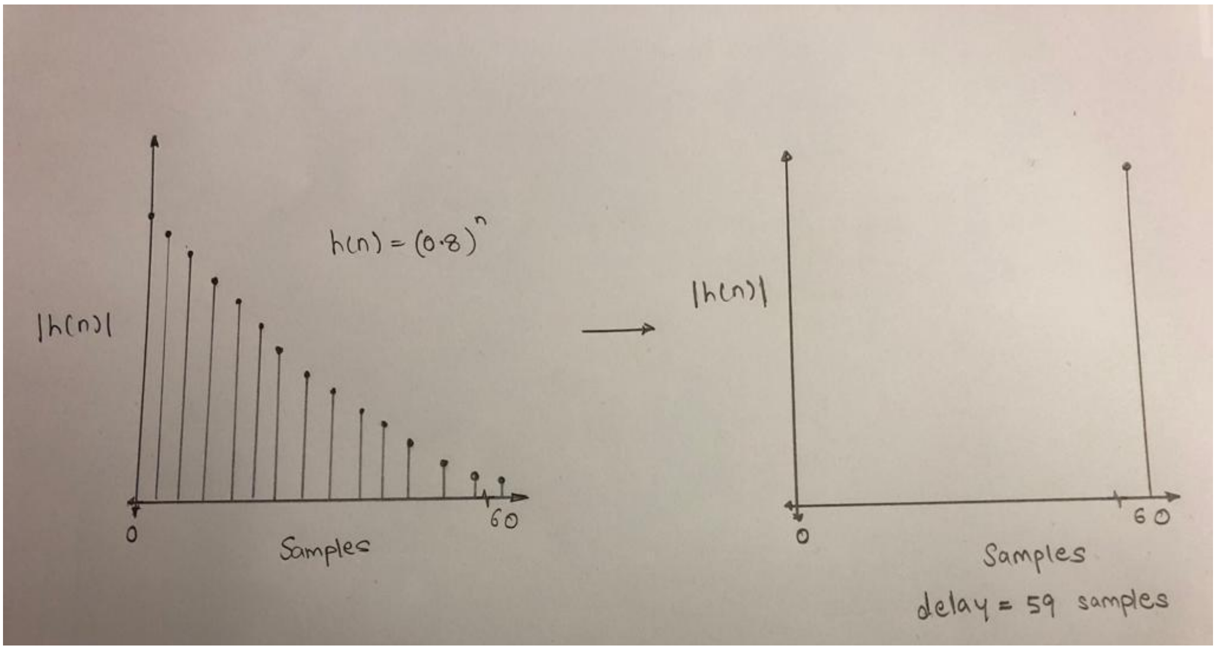

Figure 10

If we experiment with Ncp by *decreasing* it to 0, the Error Vector Magnitude (EVM) will be increased to 0.0469 whereas the Bit Error Rate (BER) will stay unaffected. However, after increasing the Ncp to 59, the EVM and BER stop improving since the received message would same as transmitted message. Therefore, we can say that Ncp=59 is our magic value and increasing Ncp beyond this value would have no effect on EVM. On the other hand, Ncp>59 would just consume more space on the available bandwidth thus providing lower data rates.

2c)

In case of $\mathbf{h}_4 = 0.99^n$ the choice of Ncp would have more importance as compare $\mathbf{h}_4 = 0.8^n$ since the magnitude of the former has a greater impact on the convolution product. Hence, we could conclude that varying N_{cp} using the low pass system of 0.99^n will result in a more sensitive output response of EVM and BER compare to using 0.8^n. However, the required Ncp still remains the same because the length of the impulse response of the channel remains the same. And for Ncp>60, it is shown that it will not change both EVM and BER. In addition, for Ncp < 30, the large value of EVM will increase to the point that it reaches.

EVM and BER variation for $\mathbf{h}_4 = 0.99$n

Ncp	EVM	BER
60	$3.67 \cdot 10^{-15}$	0
50	0.102	0
40	0.285	0
30	0.352	0
20	0.471	0.0129
10	0.628	0.0404
0	0.699	0.0608

3a)

- In the known channel scenario with h (1) = 1, the equalization step includes dividing the received signal with this gain. As a result, the phase shift due to synchronization delay is not taken into account.
- In the unknown channel scenario with synchronization error, the result from the system identification step is shown below.

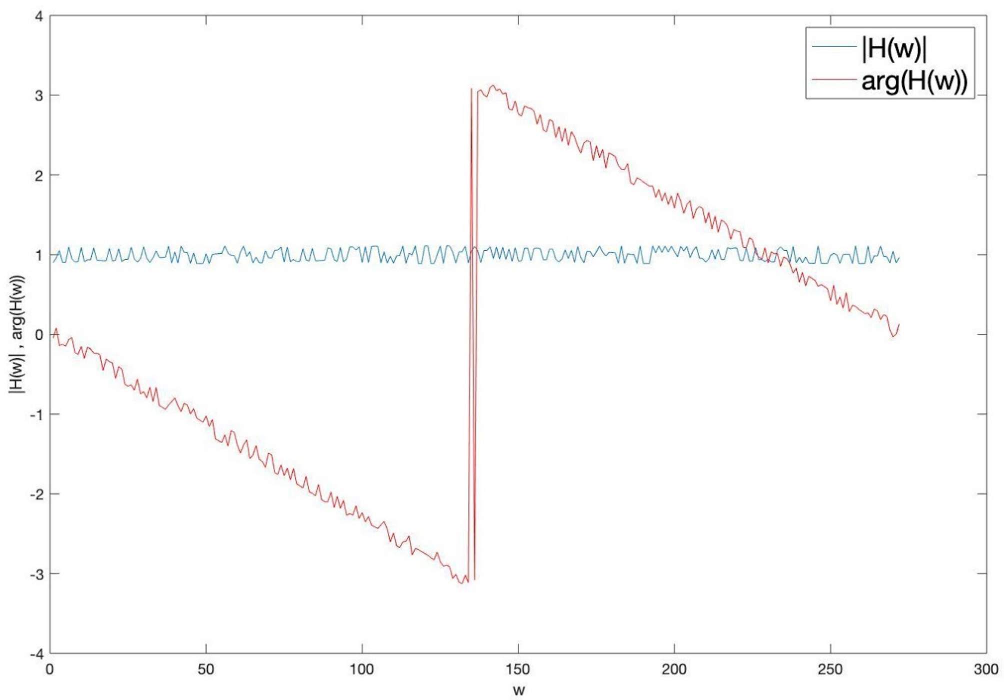

Figure 11

- The system estimates the channel to have a linear phase shift characteristic as seen in Figure 11. This happens as there is no way for the system to know if the phase shift is because of a delay or just a characteristic of channel. Thus, during system identification step, the phase shift due to synchronization error is eliminated and the bit error rate drops down to zero as does the EVM.
  Transmitted: 'Alice: Would you tell me, please, which way I ought to go from here?'

  Received: 'Alice: Would you tell me, please, which way I ought to go from here?'

3b)

- The length of the cyclic prefix for a near zero EVM is till 59, as the number of samples in the filter impulse response is still unchanged.

- The EVM and the BER increase on decreasing the SNR ratio. The unknown channel system is more sensitive to SNR than the known channel system. This is because the unknown channel estimates the noisy complex channel gain as

$$H_{noisy} = \frac{Transmitted\ Pilot\ message}{Noisy\ Received\ Pilot\ message}$$

- The equalization step is then carried out based on this noisy estimate of complex channel gain. Hence, the equalized data has more EVM in this case than in the known channel case where the equalization is carried out with perfectly known channel gain.

3c)

- The impulse response of a multipath channel in the time and frequency domain for h (5) and h' (5) can be represented as shown

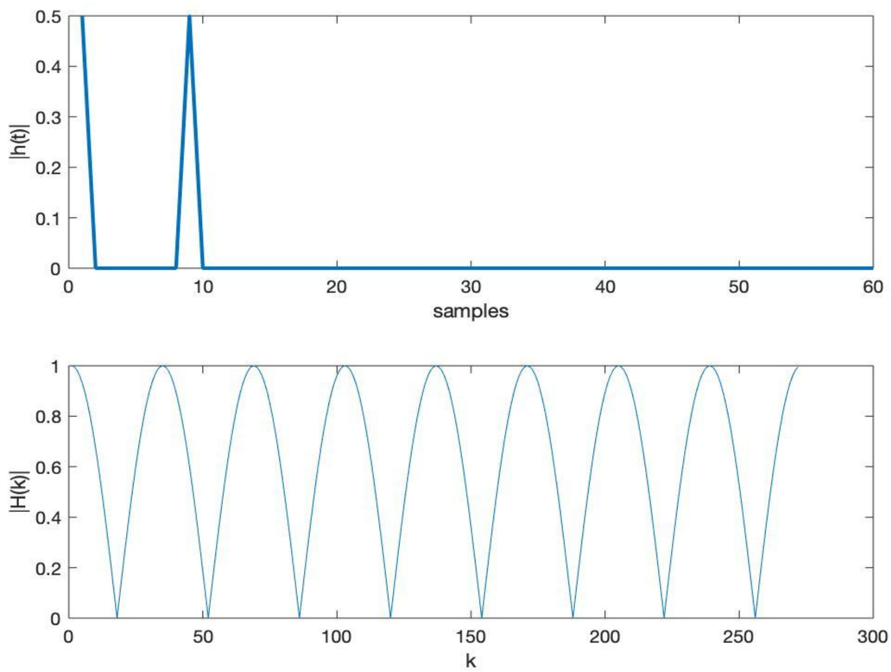

Figure 12

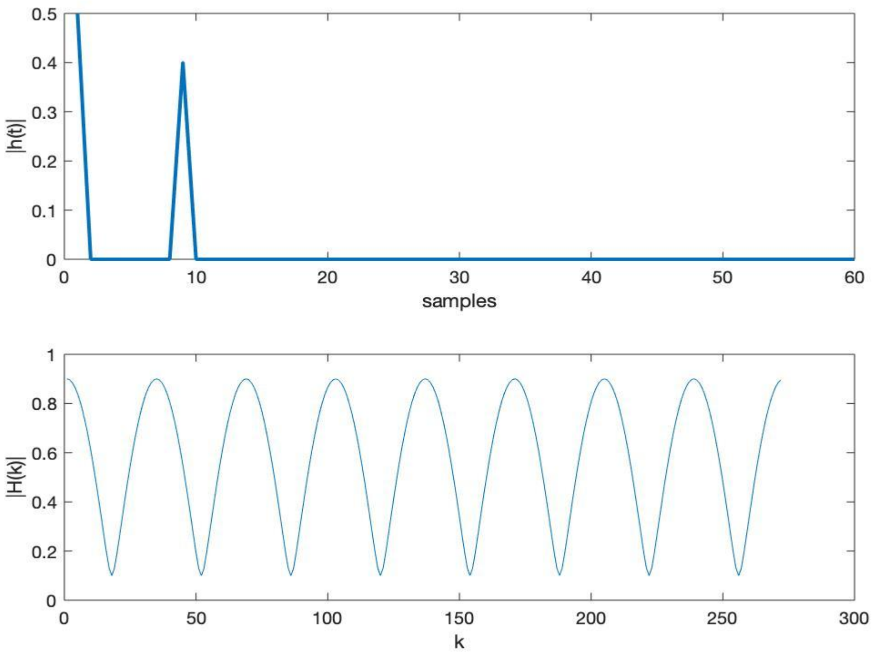

Figure 13

- Case: h (5)
  On observing Figure 12, it can be noticed that the complex sinusoids from the two multipath components (from h (1) and h (9)) interfere constructively and destructively at different points as observed in the frequency domain. On destructive interference, the magnitude of the impulse response drops to zero since the two multipath components have the same magnitude of 0.5. This causes loss of information during the equalization step.

- Case: h' (5)
  On observing Figure 13, the destructive interference of the sinusoids from the two multipath components results in a non-zero [0.5-0.4 = 0.1] frequency response magnitude. Hence, the information in the received symbol can still be retrieved.

- Thus, as along as the amplitudes of the multipath components differ from each other the information in the received signal can be recovered.

3d)

- The cyclic prefix length is set to 8. This value can be considered *'large-enough'* as the maximum delay spread between the impulse responses in the channel corresponds to 8 samples. Thus, inter-symbol interference can be completely avoided by prefixing the last 8 symbols of the OFDM block. In general, for any multipath problem the Ncp can be decided based on the delay corresponding to the last non-zero sample in the impulse response of the channel.
- However, choosing a large enough cyclic prefix doesn't completely eliminate the EVM and BER as the destructive interference between multipath components of equal magnitude as the destructive interference of two components make the transmitted information worthless regardless of cyclic prefix.

3e)

- The following observations can be made when the nature of the channel is unknown:

  - The unknown channel performs worse than the known channel in case the SNR is low. This is due to the fact the unknown channel is estimated based on received signals which are noisy. As shown in Figure 2, the noisy system identification leads to noisier symbol estimation. Thus, the system is not robust to noise.

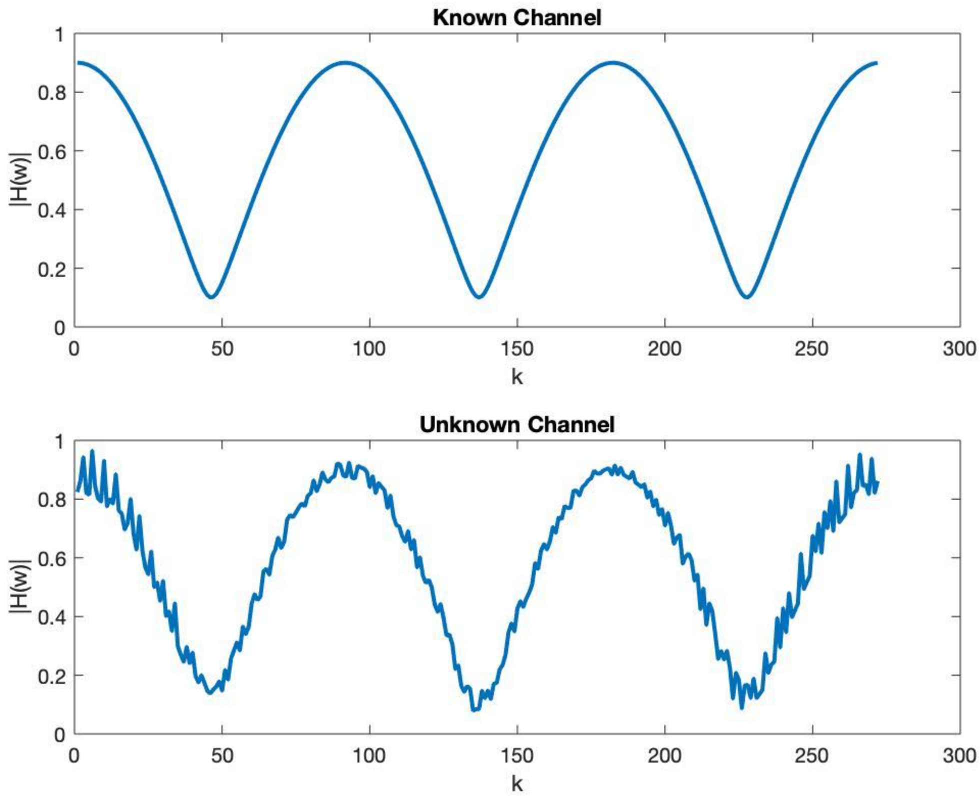

Figure 14

- The system tends to behave differently for positive and negative synchronization delays. In case of large negative synchronization delay (-20 symbols), the system correctly recognizes the phase shifts due to the delay and equalizes them. However, in case of large positive delay (20 symbols), the EVM increases significantly and so does BER. However, the overall performance in the latter case, is still better than the known channel case where it completely fails to account for the delay. Thus, it can be said that the system is robust to small (up to 10 symbols) negative synchronization delay.

- The system identification for unknown channel works well if the channel exhibits linear frequency response resulting in low EVM and BER. However, as in case of low pass and multipath channel models, which exhibit non-linear frequency response, the performance of this communication model drops as the estimation of channel becomes more challenging using the pilot messages.