SSY098 - Image Analysis

Triangulation

Lab - 4

Group-Number 49

Fikri Farhan Witjaksono   fikrif@student.chalmers.se

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0.1 Part 4.1 : Using RANSAC

0.1.1 Exercise 4.1.

First, Assume the Standard Deviation of the Gaussian Noise is equal to 3 σ which is a standard way of assumption. Suppose we have

$$u = \begin{bmatrix} x \\ y \end{bmatrix} \tag{1}$$

We could add one at the 3rd row of image points of each camera a and b to obtain 3-vector representation

$$u_a = \begin{bmatrix} x_a \\ y_a \\ 1 \end{bmatrix},\ u_b = \begin{bmatrix} x_b \\ y_b \\ 1 \end{bmatrix} \tag{2}$$

Then, by constructing the quadratic system using the affine transformation form

$$\begin{bmatrix} \tilde{x_j} \\ \tilde{y_j} \end{bmatrix} = A \begin{bmatrix} x_j \\ y_j \end{bmatrix} + t \tag{3}$$

And solving this could be done in the form of

$$M\theta = b \tag{4}$$

will yield 5 unknowns. Hence, taking only the first three solution of θ will give the desired coordinate of the corresponding 3D point that we want. Checking with the test case output at a random iteration gives a similar result with slight deviation as shown below

$$U_{true} = \begin{bmatrix} 0.8690 \\ 0.2743 \\ 0.4363 \end{bmatrix},\ U = \begin{bmatrix} 0.8604 \\ 0.2846 \\ 0.1921 \end{bmatrix} \tag{5}$$

0.1.2 Exercise 4.2.

This could be done by checking whether the depth (λ) is positive or negative. Then, the result would be array of boolean values of length 2 since we have 2 camera matrices.

0.1.3 Exercise 4.3.

The reprojection errors could be calculated by the equation below

$$r_i(\theta) = \left\| \begin{bmatrix} x_i \\ \hat{x}_i \end{bmatrix} - \begin{bmatrix} y_i \\ \hat{y}_i \end{bmatrix} \right\| \tag{6}$$

where i corresponds to each different camera image points in x and y-coordinate.

0.1.4 Exercise 4.4

This step could be done by using the RANSAC algorithm to find the model with the best number of inliers (support model). The number of inlier is defined as a measurement with positive depth ($\lambda \in \mathbb{R}^+$) and the reprojection error ($r(\theta)$) less than the threshold. In this case, the threshold is defined as 5 pixels. The termination criterion (η) is defined as 90 percent meaning the iteration will stop if

$$(1 - \epsilon^n)^{k_{max}} = \eta \tag{7}$$

0.1.5 Exercise 4.5

From the method described in 4.4 as well as using a separate script **triangulatesequence** as well as predefined function **clearforplot**, the result of the plot, assuming that we collect all triangulated with at least two inliers is obtained as following. Note that there is a considerable between using all examples and only 1000 examples.

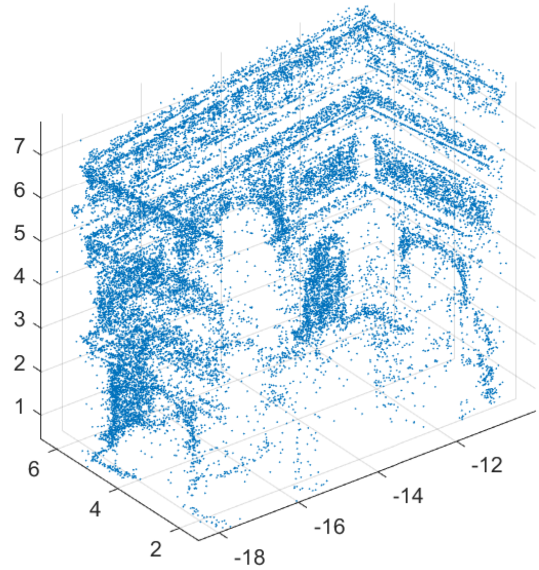

Figure 1: Full 3D reconstruction of Arc d'Triomphe

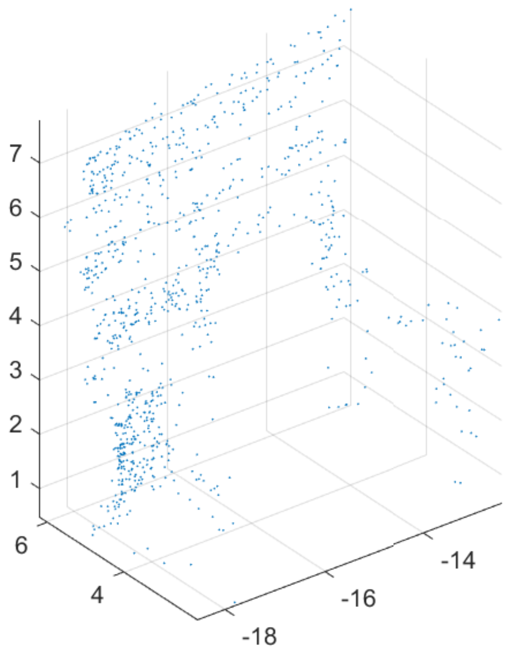

Figure 2: 3D reconstruction with 1000 examples

0.2 Part 4.2. Least Square Triangulation

0.2.1 Exercise 4.6

0.2.2 Exercise 4.7

The stacked vectors of reprojection residuals can be expressed mathematically as below

$$\bar{r}(\theta) = \begin{bmatrix} r_{1,x}(\theta) \\ r_{1,y}(\theta) \\ r_{2,x}(\theta) \\ r_{2,y}(\theta) \\ \vdots \\ r_{n,x}(\theta) \\ r_{n,y}(\theta) \end{bmatrix} \tag{8}$$

Here, the size would be (2N x 1) for N is the number of cameras used.

0.2.3 Exercise 4.8-4.9

The jacobian of equation (8) above could be expressed mathematically as below

$$J = \begin{bmatrix} \frac{\partial r_{1,x}}{\partial \theta_1} & \frac{\partial r_{1,x}}{\partial \theta_2} & \frac{\partial r_{1,x}}{\partial \theta_3} \\ \frac{\partial r_{1,y}}{\partial \theta_1} & \frac{\partial r_{1,y}}{\partial \theta_2} & \frac{\partial r_{1,y}}{\partial \theta_3} \\ \frac{\partial r_{2,x}}{\partial \theta_1} & \frac{\partial r_{2,x}}{\partial \theta_2} & \frac{\partial r_{2,x}}{\partial \theta_3} \\ \vdots & \vdots & \vdots \end{bmatrix} \tag{9}$$

The size of equation (9) would be (2N x 3) where N is number of cameras. For our case where we have N = 2, the equation could be simplified to

$$J = \begin{bmatrix} \frac{\partial r_{1,x}}{\partial \theta_1} & \frac{\partial r_{1,x}}{\partial \theta_2} & \frac{\partial r_{1,x}}{\partial \theta_3} \\ \frac{\partial r_{1,y}}{\partial \theta_1} & \frac{\partial r_{1,y}}{\partial \theta_2} & \frac{\partial r_{1,y}}{\partial \theta_3} \\ \frac{\partial r_{2,x}}{\partial \theta_1} & \frac{\partial r_{2,x}}{\partial \theta_2} & \frac{\partial r_{2,x}}{\partial \theta_3} \\ \frac{\partial r_{2,y}}{\partial \theta_1} & \frac{\partial r_{2,y}}{\partial \theta_2} & \frac{\partial r_{2,y}}{\partial \theta_3} \end{bmatrix} = \begin{bmatrix} \frac{a_1^T}{\lambda_1} & - & \frac{(a_1^T \hat{U})c_1^T}{\lambda_1^2} \\ \frac{b_1^T}{\lambda_1} & - & \frac{(b_1^T \hat{U})c_1^T}{\lambda_1^2} \\ \frac{a_2^T}{\lambda_2} & - & \frac{(a_2^T \hat{U})c_2^T}{\lambda_2^2} \\ \frac{b_2^T}{\lambda_2} & - & \frac{(b_2^T \hat{U})c_2^T}{\lambda_2^2} \end{bmatrix} \tag{10}$$

0.2.4 Exercise 4.10

To refine the triangulation, we could iterate using the Gauss-Newton iteration method defined as below

$$\theta^{(k+1)} = \theta^{(k)} - \underbrace{(J^T J)^{-1} J^T \bar{r}}_{\text{at iteration k}} \tag{11}$$

where $\theta^(k)$ is the parameter estimate U, J and $\bar{r}$ is defined in equation (9) and (8) respectively. Moreover the sum of squared residuals after each Gauss Newton step always decreases at each iteration. Hence, we could conclude that the value of equation 11 below converges to zero and the minimization method (e.g. Gauss-Newton) works.

$$\sum_i \|r_i\|^2 \to 0 \tag{12}$$

0.2.5 Exercise 4.11

The result of applying the function **refinetriangulation** is shown as follows

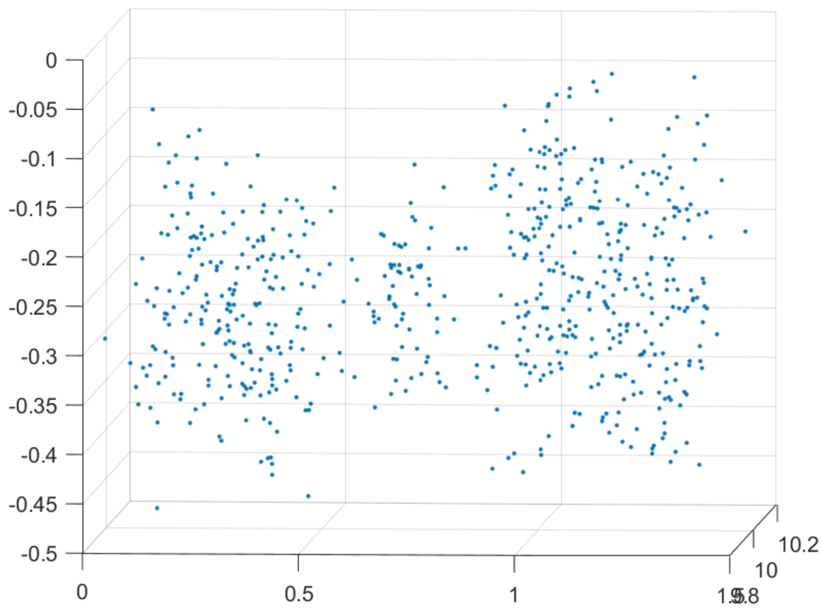

Figure 3: Scatter plot of Uhat **Before refinement**

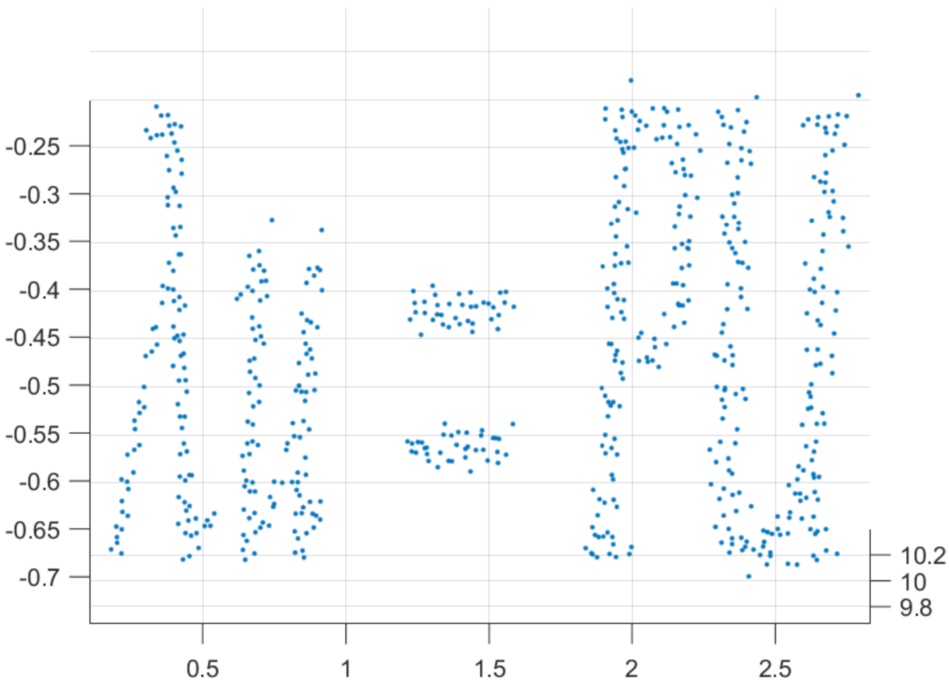

Figure 4: Scatter plot of Uhat **After refinement**

0.2.6 Exercise 4.14

We could find the essential matrix through the equation (13) and (14) below

$$0 = \hat{x_2}^T [t]_x R \hat{x_1} \tag{13}$$

$$0 = x_2^T E \hat{x}_1$$

where $[t]_x$ is the matrix representation of the cross product with t and R is defined as follows

$$R = R_2 R_1^{-1} \tag{14}$$

$[t]_x$ could be further decomposed as

$$[t] = t_2 - Rt_1 \tag{15}$$

$$[t]_x = \begin{bmatrix} 0 & -t_3 & t_2 \\ t_3 & 0 & -t_1 \\ -t_2 & t_1 & 0 \end{bmatrix} \tag{16}$$

where [t] in equation (5) is a matrix of size (3 x 1). Hence we could simplify the Essential matrix (denoted as E) to be as follows

$$E = R[t]_x \tag{17}$$

where E is 3x3 matrix as follows

$$E = \begin{bmatrix} 2.568 & -0.243 & -0.375 \\ -1.446 & 0.658 & -2.898 \\ 1.863 & 0.089 & -1.859 \end{bmatrix} \tag{18}$$