SSY285 - Linear Control System Design

Linear state estimation and control of DC-motor with flywheel

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Assignment - 3

Group 11 - Mechanical

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1 PROBLEM STATEMENT

The DC motor with flywheel which was the focus of previous assignments, is considered for this assignment as well. Refer figure-1.

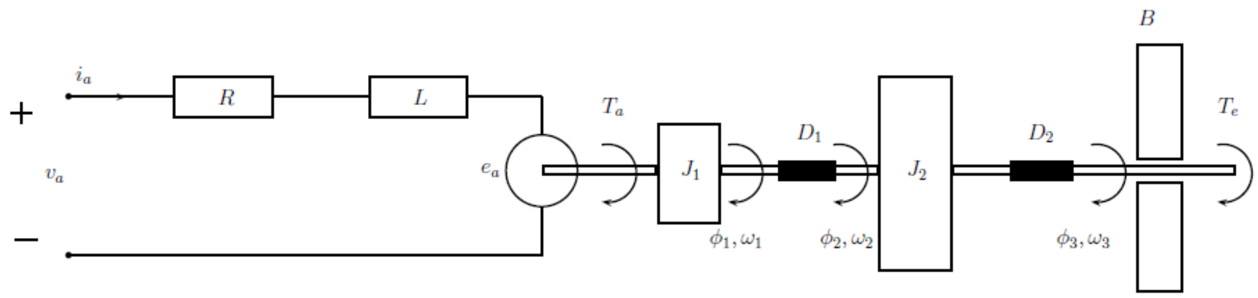

Figure 1: DC motor with flywheel

Starting point of this assignment is the sampled data linear state equation obtained in assignment M2, in questions d and e. The angle and angular velocity of the flywheel are both assumed to be measured in this assignment.

2 QUESTIONS

2.1 Question a: Covariance matrix for disturbance vector w

The Gaussian distribution (refer Figure-2) depicts the 68-95-99.7% rule. As given in the question, the voltage and torque disturbances are both normally distributed and accounts for 99.7% of the realisation as bound. Considering a mean value of zero and upper bounds of 0.3 V for the input voltage and 0.1 Nm for the external torque, the standard deviations for 99.7% distribution can be calculated as -

$$\sigma_1 = \frac{0.3}{3} = 0.1 \tag{1}$$

$$\sigma_2 = \frac{0.1}{3} = 0.033 \tag{2}$$

The covariance matrix is calculated as shown below -

$$Q = \begin{bmatrix} E[(x_1-\mu_1)((x_1-\mu_1)] & E[(x_1-\mu_1)((x_2-\mu_2)] \\ E[(x_2-\mu_2)((x_1-\mu_1)] & E[(x_2-\mu_2)((x_2-\mu_2)] \end{bmatrix} = \begin{bmatrix} E(x_1)^2 & 0 \\ 0 & E(x_2)^2 \end{bmatrix} \tag{3}$$

$$E(x^2) = V(x) + E(x^2) \tag{4}$$

Using the relation given by equation 4 and with mean value as zero, E(x) = V(x) = σ^2. Correspondingly, E(x$_1$)$^2 = \sigma_1^2$ and E(x$_2$)$^2 = \sigma_2^2$.

Further, the covariance matrix for the disturbance vector w having the above components is given as-.

$$Q = \begin{bmatrix} 0.01 & 0 \\ 0 & 0.0011 \end{bmatrix} \tag{5}$$

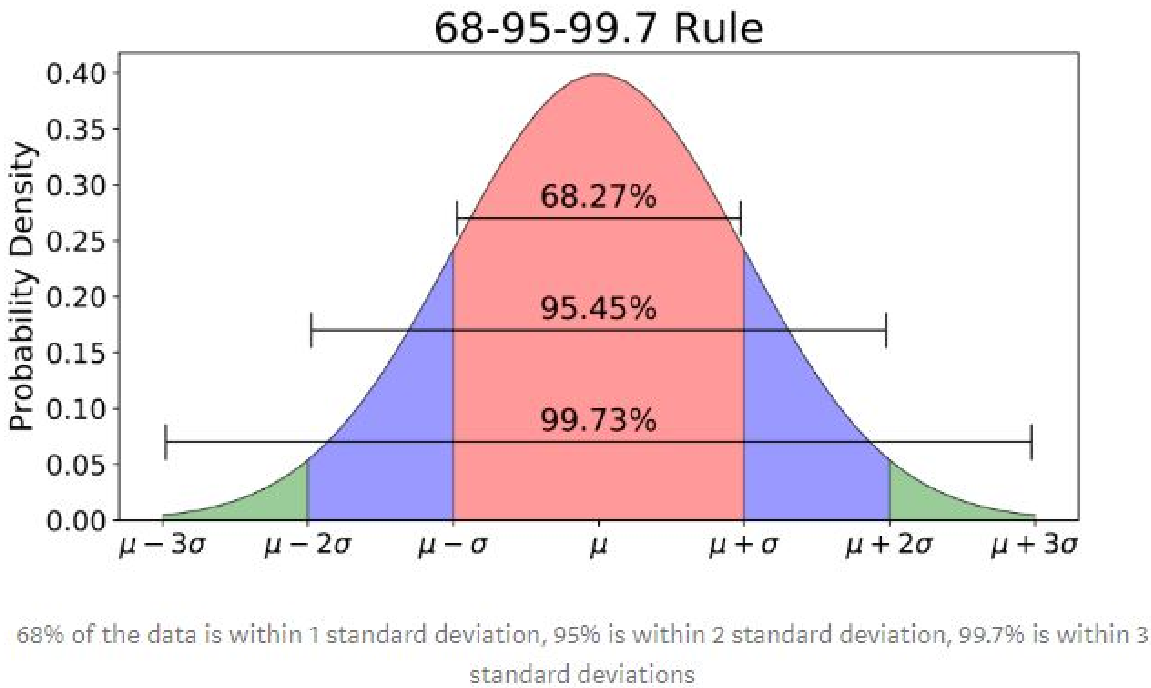

Figure 2: Gaussian distribution

In addition, matrix N is defined as the Noise matrix which is equal to B, since w is included into the input multiplied with B matrix as shown below.

$$\dot{x} = Ax + Bu + Nw \tag{6}$$

$$\dot{x} = Ax + B(u + w) = Ax + Bu + Bw \tag{7}$$

Since equation (6) is equal to equation (7), one would obtain the following-

$$B = N \tag{8}$$

2.2 Question b: Covariance matrix for the measurement disturbance vector n

This question can be solved in a similar method as the previous one. Here, measurement disturbances n_1 and n_2 are added to the output and these are assumed to be discrete time, zero mean uncorrelated white noises with values 0.02 and 0.01 radian per second respectively.

The resulting covarince matrix is given as -

$$Q = 1 * 10^{-4} \begin{bmatrix} 0.4444 & 0 \\ 0 & 0.1111 \end{bmatrix} \tag{9}$$

2.3 Question c: Observer gain matrix (L), Covariance matrix of the state estimation error (P) and observer eigenvalues

This question deals with the calculation of Observer gain matrix (L) and Covariance matrix of the state estimation error (P).

These were calculated by using "kalman" function in MATLAB and the results are as presented below:

$$L = \begin{bmatrix} 0.0005 & 0.0009 \\ 0.0005 & 0.0018 \\ 0.0003 & 0.0113 \\ -0.0001 & 0.6970 \\ -0.0292 & 1.6755 \end{bmatrix} \tag{10}$$

$$P = \begin{bmatrix} 0 & 0 & 0 & 0.0001 & 0.0001 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.0001 & 0.0011 & 0.0031 \\ 0.0001 & 0 & 0.0011 & 0.3429 & 0.0248 \\ 0.0001 & 0 & 0.0031 & 0.0248 & 0.1096 \end{bmatrix} \tag{11}$$

The observer eigen values can be calculated by forming the matrix "Ob" given by equation 9 and calculating the eigen values of the new matrix.

$$Ob = A_{dd} - L * Cdd \tag{12}$$

The resulting observer eigen values are given in Table - 1.

Table 1: Observer eigenvalues

λ_1	-0.6883 + 0.0000i
λ_2	0.9995 + 0.0000i
λ_3	0.1438 + 0.5209i
λ_4	0.1438 - 0.5209i
λ_5	-0.0001 + 0.0000i

2.4 Question d: Designing a discrete time Linear Quadratic Gaussian controller

The objective in this task is to design a discrete time Linear Quadratic Gaussian controller and simulate the closed-loop answer to a step $r_{\omega 2}$ from 10 to 100. Measured states are used as input to the Kalman filter to reconstruct the states, with Kalman filter gain calculated from Task C.

The new states are used to calculate the LQR(Linear Quadratic Regulator) Gain.An integral state is added to find out the difference between the reference value and the measured value.

The reconstructed matrices for A and B used to calculate the LQR gain are given below-

$$A_{lq} = \begin{bmatrix} 0.4010 & 0.5964 & 0.0026 & 0.0004 & 0.0002 & 0 \\ 0.2076 & 0.7749 & 0.0175 & 0.0001 & 0.0009 & 0 \\ 0.0585 & 0.5686 & 0.3729 & 0 & 0.0004 & 0 \\ -782.8328 & 773.7475 & 9.0853 & -0.0489 & 0.5964 & 0 \\ 342.5299 & -370.9589 & 28.4290 & 0.1491 & 0.7749 & 0 \\ 0 & 0 & 0 & 0 & -1 & 1 \end{bmatrix} \tag{13}$$

$$B_{lq} = \begin{bmatrix} 0.0032 & 0.0003 \\ 0.0002 & 0.0032 \\ 0 & 0.3168 \\ 4.4993 & 1.3088 \\ 0.5851 & 8.7634 \\ 0 & 0 \end{bmatrix} \tag{14}$$

The simulink model is designed using the example model provided in the lectures (LQG 2by2 workspace) as reference and modifications are made to the model to suit this task. These modifications include addition of noise, gains and integral states to the system. The schematics of the final simulink model for task D can be seen in figure 3.

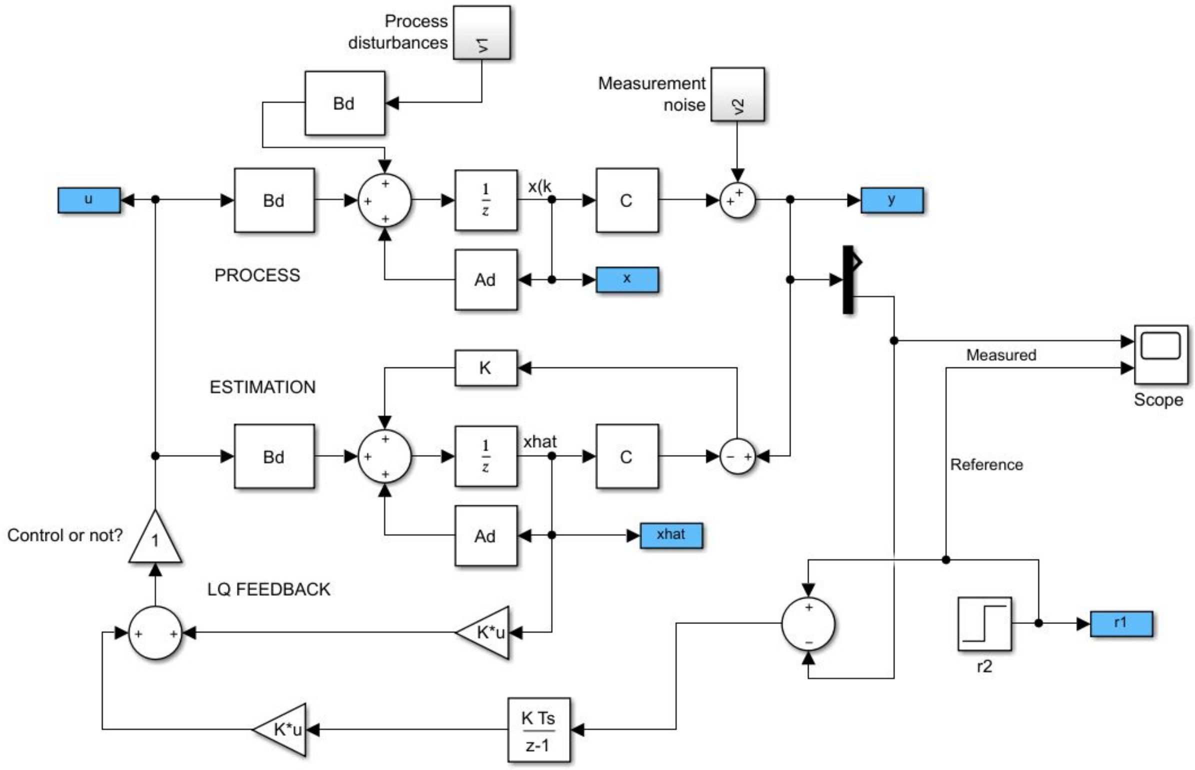

Figure 3: Simulink model for Task D

The weighting matrices Q_x and Q_u used are given below-

$$Q_x = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{15}$$

$$Q_u = \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix} \tag{16}$$

Scope block is used to understand the behaviour of the designed controller and the model is simulated as per the instructions in task D with the step jumping from an initial value of 10 to 100. The resulting plot is shown in figure 4.

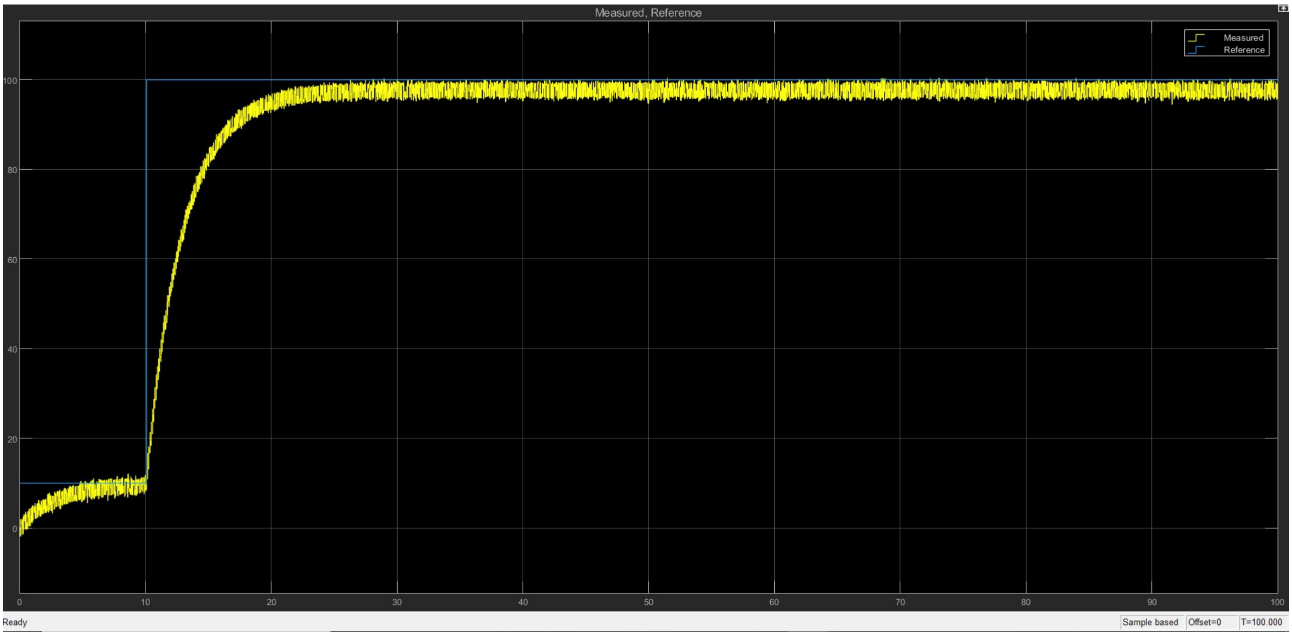

Figure 4: Plot obtained from Simulink model (Scope)

3 REFERENCES

[1] SSY285 - Linear Control System Design Assignment-3 handout
[2] SSY285 – Linear Control System Design – Lecture slides
[3] Control Theory - Multivariable and Nonlinear Methods by Torkel Glad and Lennart Ljung

4 APPENDIX

Matlab Code

```
%% SSY285 Linear Control System Design: Assignment - 3
%% Group 11 - Fikri Farhan Witjaksono,Chintalapudi Adhitya Reddy and Manikanta Venkatesh

clc
clear all
close all

%% Assignment 2 values

R = 1;
Ke = 0.1;
Kt = 0.1;
J1 = 1e-5;
J2 = 4e-5;
Bf = 2e-3;
D1 = 20;
D2 = 2;

AP = [ 0 0 0 1 0 ;
       0 0 0 0  1;
       0 (D2/Bf) (-D2/Bf) 0 0 ;
       (-D1/J1) (D1/J1) 0 (-(Kt*Ke)/(R*J1)) 0;
       (D1/J2) ((-D1-D2)/J2) (D2/J2) 0 0];

B1= [0 0;
     0 0;
     0 (1/Bf);
    (Kt/(R*J1)) 0 ;
       0 0];

C1 = [0 1 0 0 0;
      0 0 0 0 1];

D1 = zeros(2);

S = ss(AP,B1,C1,D1);
p = c2d(S,0.001);
Add=p.A;
Bdd=p.B;
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Cdd=p.C;
Ddd=p.D;

%% Assignment 3
%% Task (a)
mu_1=0;
sigma_1=0.3/3;
sigma_2=0.1*1/3;

a1 = sigma_1^2;
a2 = sigma_2^2;
R1 = [a1 0;
      0 a2];

N = Bdd; % x_dot1 = Ax + Bu + Nw,
         % x_dot2 = Ax + Bu + Bw,
         % Hence B = N

%% Task (b)
mu_2=0;
sigma_3=0.02/3;
sigma_4=0.01/3;

a3 = sigma_3^2;
a4 = sigma_4^2;
R2 = [a3 0;
      0 a4];


%% Task (c)
h = 0.001;
Qn = R1;
Rn = R2;
Nn = zeros(2);

G_kalman = ss(Add,[Bdd Bdd],Cdd,[Ddd Ddd],h);
[KEST,K,P] = kalman(G_kalman,Qn,Rn,Nn)

% Observer eigenvalue
Ob = Add-(K*Cdd) ;
Obsv_eig = eig(Ob)

%% Task (d)

x = [1 1 1 1 1 1];
Qx = diag(x);

u = [1 1];
Qu = 5*diag(u);

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A_lq = [Add [0;0;0;0;0]; 0,0,0,0,-1,1];

B_lq = [Bdd;[0 ,0]];

C_lq = [Cdd, [0;0]];

% LQ solution
[L_LQ,S,P] = dlqr(A_lq,B_lq,Qx,Qu);
L_P = L_LQ(:,1:5);
L_I = L_LQ(:,6);

sim ('Group_11_Assignment_3_Task_D_Simulink_model');
```