Micro - Homework 5

Question 1. Compare the equations of the Kalman Filter's stationary solution with the ones for Dynamic Programming. Do you see the duality?

The stationary solution of Kalman Filter will basically converges to solution of filtering algebraic Riccati equation (assuming v_t is observable)

(1) $L = PC^T[CPC^T + R]^{-1} \longrightarrow$ for $k \in \{0,1,\dots,N-1\}$, $L_k = -AP_kC^T(R + CP_kC^T)^{-1}$

(2) $P_{k+1} = AP_kA^T - AP_kC^T[CP_kC^T + R]^{-1}CP_kA^T + Q$, $:= (A + L_kC)P_k(A + L_kC)^T + Q + L_kRL_k^T$, for $P_0 = Q_f =: P(0)$

The stationary solution of Dynamic Programming solution to the LQ problem could be expressed as below: (3) $K(k) = -(R + B^TP(k+1)B)^{-1}B^TP(k+1)A$

$$= (R + B^TP_kB)^{-1}B^TP_kA, \quad P(N) = P_f$$

(4) $P_{k-1} = Q + A^TP_kA - A^TP_kB(R + B^TP_kB)^{-1}B^TP_kA$

The Kalman Filter state estimator is

(5) $$\hat{x}(k+1) = A\hat{x}(k) + L(k)(C\hat{x}(k) - y(k))$$

Whereas the LQR state observer/estimator is

(6) $$\hat{x}^+ = A\hat{x} + Bu + L(y - C\hat{x})$$

It is clear that there is a duality between LQ and Kalman Filter solution if we compare eq. (1) to (3) and (2) to (4), Hence, we could see that

$$\xi(k+1) = A^T\xi(k) + C^Tu(k),$$ where $\xi(k)$ is state vector of duality, $u(k)$ is control input vector

Question 2. What are the benefits of a moving horizon estimator in comparison with the Kalman Filter?

1. Moving Horizon Estimator (MHE) smoothes the previous estimates hence resulting in higher accuracy.

2. MHE outperforms the Kalman Filter in the presence of large measurement noise.

3. Kalman Filter (e.g. EKF $\longrightarrow$ Extended Kalman Filter) and other types of Kalman filters performs worst when using a short horizon due to large initial state errors, Hence MHE performs better for short horizon condition.