

## Mini Homework Assignment #3

Q.1. Why in the constrained LQ MPC, the control input is calculated online? Why do not we use Dynamic Programming to find an explicit controller?

(a) → Online calculation means that the control input in the future is determined to solve an open-loop optimal control problem cast over a finite size time at each sample time instant.

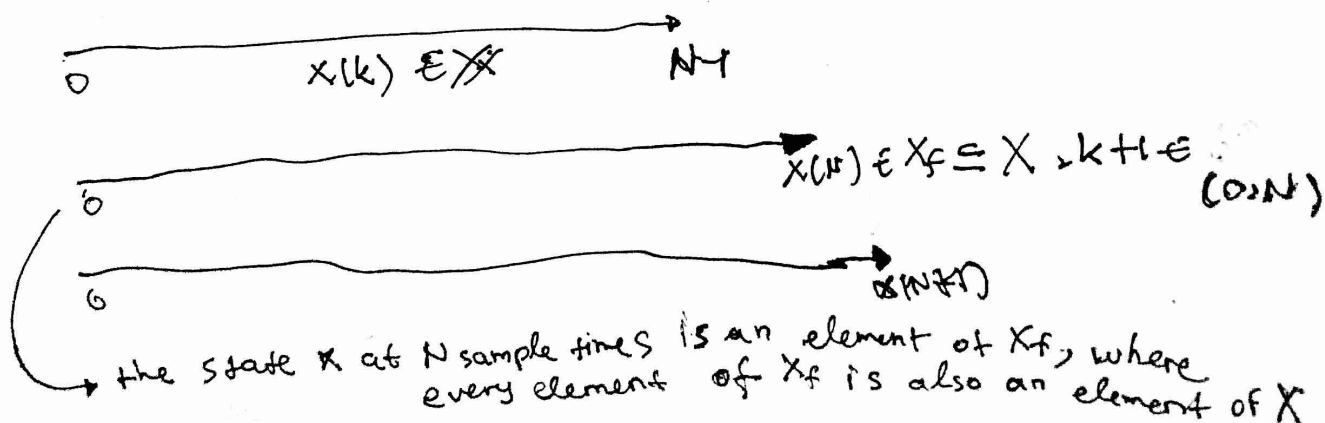
Due to only, it is required to solve the optimization problem in LQ MPC by expressing future states as function of control input at each time iteration inside the constraint.

(b) → It is inefficient due to the fact that the curse-of-dimensionality of Dynamic Programming approach will cause combinatorial explosion in solving the explicit controller at high-dimensional solution spaces.

Q.2. Assume a constrained Linear Quadratic MPC is implemented with  $x(N) \in X_f$  as the terminal constraint. Can you claim that after  $N$  sample times, the system state is contained by  $X_f$ ?

Answer:

$$\begin{aligned} \textcircled{1} \quad & x(k+1) = Ax(k) + Bu(k), \quad x(0) = x \\ & x(k) \in X, \quad u(k) \in U \text{ for all } k \in (0, N-1) \\ & x(N) \in X_f \subseteq X \end{aligned}$$



② By applying the first control input  $u(k) = u^*(0; x)$

③ Let  $k: k+1$

④ At sampling instant  $k = k+1$ , the optimization problem becomes

$$V_{N+1}^*(x) = \min_{u(0:N), x(0:N+1)} \{V_{N+1}(x, u(0:N), x(0:N+1))\}$$

$$V_{N+1}(x, u(0:N), x(0:N+1)) = x^T(N+1) P x(N+1) + \sum_{i=0}^N (x^T(i) Q x(i) + u^T(i) R u(i))$$

⑤ For optimal control and state sequences

$$u^*(0:N; x) = \{u^*(0; x), u^*(1; x), \dots, u^*(N; x)\}$$

$$x^*(0:N+1; x) = \{x^*(0; x), x^*(1; x), \dots, x^*(N+1; x)\}$$

and subject to

$$x(k+1) = A x(k) + B u(k), x(0) = x$$

$$x(k+1) \in X, u(k) \in U \text{ for all } k \in (0, N)$$

Hence  $x(N+1)$  is an element of  $X_f$

(e.g. contained in  $X_f$ ) since  $x(N) \in X_f \subseteq X$