CHALMERS

UNIVERSITY OF TECHNOLOGY

SSY281 - Multi Predictive Control

Stability

Micro Assignment - 9

ID-Number 43

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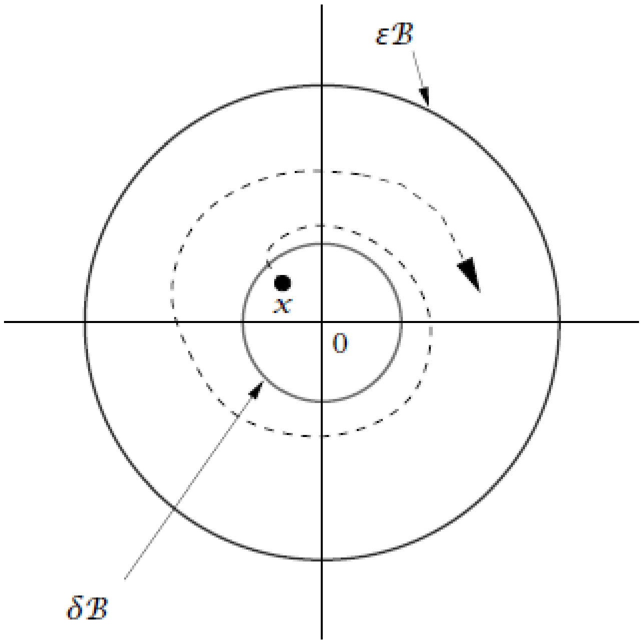

Figure 1: Stability of the origin, **B** denotes the unit ball [1]

0.1 Question 1: Definition of local stability

Assume we have an autonomous system

$$x^+ = f(x) \tag{1}$$

and assume that $f(\cdot)$ is locally bounded, and that the set $\mathbf{A}$ is closed and positive invariant which implies that it is locally stable. For $x^+ = f(x)$ if, for all $\epsilon > 0$, there exists a $\delta > 0$ such that $|x|_{\mathbf{A}} < \delta$ implies $|\phi(i;x)|_{\mathbf{A}} < \epsilon$ for all i$\in I_{\geq 0}$. Stability at the origin, is valid if there exist continuity of the map $x \to (x, \phi(1;x), \phi(2:x), ...)$, $\mathbb{R} \to \ell_\infty$ so that $\|x\| \to 0$ as $x \to 0$. The figure above showed the definition of the local stability at the origin.

0.2 Question 4: List the tuning parameter in a RH controller

List of tuning parameters that affect closed loop stability:

1. **Control Weighting** :
   Increasing the weights R(i) (in the cost function) on the control moves relative to the weights Q(i) on the tracking errors has the effect of reducing the control activity while increasing this too much will reduce the control activity to zero. With a stable plant, the result would be a stable closed loop by increasing R(i) moderately. The drawback would be it will be slow response to disturbance. In unstable plant, increasing R(i) too much will result in an unstable feedback loop.

2. **The Optimization Algorithm**:
   For example, using interior point methods over quadratic programming might give advantage as they have polynomial complexity bounds as well as better performance for large problems.

3. **Lyapunov's Function assurance of the cost function** $(J_N(\mathbf{x}(k))$
   To assure that the cost function is a Lyapunov's Function, we could use :

   (a) Terminal state constraint $\mathbf{x}(k+N) = 0$
   (b) Terminal constraint set $\mathbf{x}(k+N) \in X_T$
   (c) A Terminal cost $(F(x(k+N))$

4. **Specify the prediction horizon N for which closed-loop stability is ensured (e.g. choose N as large as possible)**

5. **Using "Artificial" Lyapunov function (e.g. Piecewise Affine or Piecewise Quadratic Lyapunov)**

6. **Disturbance models and Observers tuning (e.g. Observer dynamics should be faster than closed-loop settling time**

0.3 Question 5: Provide an intuitive explanation

1. **Usage of Terminal set as a control invariant set** is due to the fact that solving a unconstrained optimal control problem with terminal state constraint requires an infinite number of iterations of the optimization algorithm due to the fact that it requires the exact solution, at every instant, of an optimal control problem. It is also done in order for the following to hold

$$\min_{u \in U} \{V_f(f(x,u)) + l(x,u) | f(x,u) \in X_f\} \leq V_f(x), \forall x \in X_f \tag{2}$$

2. **Usage of Terminal cost as an optimal cost-to-go**
   The terminal cost can be seen as an approximation cost beyond the prediction horizon (from k=N+1 to k =∞) so that infinite time horizon can be emulated which will result in closed-loop stability guarantee.

1 REFERENCES

[1] Model Predictive Control: Theory, Computation, and Design page 697 by James B. Rawlings,David Q. Mayne and Moritz M. Diehl