CHALMERS
UNIVERSITY OF TECHNOLOGY

SSY281 - Multi Predictive Control

Beyond linear MPC

Micro Assignment - 13

ID-Number 43

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0.1 Question 1 : Result of constrained optimization problem using RHC if f defined as a nonlinear function

The existing methods are :

1. **Non Linear (NL) MPC**

$$\begin{aligned} \min_{\delta u_k, \delta x_k} \quad & \sum_{i=0}^{N-1} \frac{1}{2} \begin{bmatrix} x_k(i) - x_k^r(i) \\ u_k(i) - u_k^r(i) \end{bmatrix} W_{k,i} \begin{bmatrix} x_k(i) - x_k^r(i) \\ u_k(i) - u_k^r(i) \end{bmatrix} \\ \text{s.t} \quad & \delta x_k(0) = \hat{x_k} \\ & \delta x_k(i+1) = f(x_k(i), u_k(i));\ i = 0, ..., N-1 \\ & h(x_k(i), u_k(i)) \leq 0;\ i = 0, ..., N-1 \end{aligned} \tag{1}$$

The solution to the problem above could be written as following

$$(u_k, r_k) = NLP(\hat{x}_k, u_k^r, x_k^r) \tag{2}$$

It is easy to observe that the solution to NLMPC is in the form of Non-Linear Programming.

2. **Sequential Quadratic Programming (SQP) MPC**
The SQP MPC has the solution in the following form :

$$(u_k, x_k) = SQP(\hat{x}(k), u_k^{guess}, x_k^{guess}, u_k^r, x_k^r) \tag{3}$$

Hence, it is easy to see that the solution is in the form of Sequential Quadratic Programming.

0.2 Question 2 : What if the non-linear model is linearized

Suppose we have the following non-linear function

$$x^+ = f(x, u) \tag{4}$$

with inequality constraints

$$g(x, u) \leq 0 \tag{5}$$

Then, we could basically linearized this problem using 3 different methods as described below

1. **Linear Time Invariant (LTI) MPC**

$$\begin{aligned} \min_{\delta u_k, \delta x_k} \quad & \sum_{i=0}^{N-1} \frac{1}{2} \begin{bmatrix} \delta x_k(i) \\ \delta u_k(i) \end{bmatrix} W \begin{bmatrix} \delta x_k(i) \\ \delta u_k(i) \end{bmatrix} \\ \text{s.t} \quad & \delta x_k(0) = \hat{x_k} - x_k^r(0) \\ & \delta x_k(i+1) = A\delta x_k + B\delta u_k(i); i = 0, ..., N-1 \\ & F\delta u_k(i) + G\delta x_k(i) \leq h; i = 0, ..., N-1 \end{aligned} \tag{6}$$

where the δu_k, δx_k are the deviation variables that needs to be solved. The solution to this problem could be written as below

$$(\delta u_k, \delta x_k) = QP_{MPC}(\hat{x}(k), u_k^r, x_k^r) \tag{7}$$

We could observe that the result of constrained optimization problem is in the form of Quadratic Programming (QP).

2. **Linear Time Varying (LTV) MPC**

$$\begin{array}{ll} \min\limits_{\delta u_k, \delta x_k} & \sum_{i=0}^{N-1} \frac{1}{2} \begin{bmatrix} \delta x_k(i) \\ \delta u_k(i) \end{bmatrix} W_{k,i} \begin{bmatrix} \delta x_k(i) \\ \delta u_k(i) \end{bmatrix} \\ \text{s.t} & \delta x_k(0) = \hat{x_k} - x_k^r(0) \\ & \delta x_k(i+1) = A_{k,i}\delta x_k \ + \ B_{k,i}\delta u_k(i) \ + \ r_k(i); i = 0, ..., N-1 \\ & F_{k,i}\delta u_k(i) \ + \ G_{k,i}\delta x_k(i) \leq h; i = 0, ..., N-1 \end{array} \tag{8}$$

Using this method, we will get the same solution in the form of equation (4) above. Hence, it is a QP form solution as well. The difference is that we use A,B,F,G which is time variant as well as the use of the residual in order to make sure that the reference trajectory satisfies the model equation.

0.3 Question 3 : Difference between using SQP and real-time iteration (RTI) method

In spite of the conceptual similarities between NMPC using SQP and linear MPC, the NMPC is potentially much more computational cost, since a number of QPs have to be solved iteratively instead of just once, and in every iteration the Jacobians have to be computed. This may lead to undesired long sampling periods of the controller or computational delays that become a significant fraction of the sampling period. Moreover there are two significant difference that RTI method has specifically [2] :

1. The linearisation of the system dynamics occurs online and is done at the current state and control prediction rather than on the reference trajectory.
2. the system dynamics are simulated using a numerical integration scheme.

1 REFERENCES

[1] SSY281 - Multi Predictive Control Lecture Notes
[2] S. Gros, M. Zanon, R. Quirynen, A. Bemporad, and M. Diehl. From linear to nonlinear MPC: Bridging the gap via real-time iteration. International Journal of Control,Vol. 93, Issue 1, January 2020.