SSY281 - Multi Predictive Control

Alternative formulations of MPC

Micro Assignment - 12

ID-Number 43

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0.1 Question 1: What is the parameter of parametric program used to solve explicit MPC [1]

The parameter is the current state x, which has the form

$$V^o(x) = \min_u \quad \{V(x,u) | u \in U(x)\} \tag{1}$$

Compared to conventional optimization problem, The solution to the parametric program approach is a function $x \to u^o(x)$ which is generally a set valued whereas in the conventional one, it is either a point or a set. In addition, the domain of the $V^0(\cdot)$ and $u^0(\cdot)$ is in the set X which could be defined as.

$$X = \{x \mid \exists\, u \,\text{such that}\, (x,u) \in Z\} = \{x \mid U(x) \neq \emptyset\} \tag{2}$$

The parametric constraint $u \in U(x)$ can be expressed in $(x,u) \in Z$ as below

$$U(x) = \{u \mid (x,u) \in Z\} \tag{3}$$

0.2 Question 2: How does the control law look like in explicit MPC

The control law which is optimal and affine on each of the partition regions (R_{x_i}) could be calculated or expressed with the offline-procedure below (Algorithm (1)) which could also be used in the case of finding a solution to multi-parametric (mp) Quadratic Programming problem [2]:
Steps

1. Let $X \subseteq \mathbb{R}^{\mathrm{n}}$ be the set of parameters (states);
   procedure partition Y
   (a) let $x_0 \in Y$ and ϵ the solution to the LP problem;
   (b) if $\epsilon \leq 0$ then exit; (no full dimensional CR is in Y);
   (c) For $x = x_0$, compute the optimal solution (z_0, λ_0) of the QP;
   (d) Determine the set of active constraints when $z = z_0$, $x = x_0$, and build $\tilde{G}$,$\tilde{W}$,$\tilde{S}$;
   (e) If r=rank($\tilde{G}$) is less than the number m of rows of $\tilde{G}$, take a subset of r linearly independent rows, and redefine $\tilde{G}, \tilde{W}, \tilde{S}$ accordingly;
   (f) Determine $\tilde{\lambda}(x)$,$z(x)$;
   (g) Characterize the Complex Polyhedral Region (CR_i);
   (h) Define and partition the rest of the region;
   (i) For each new sub-region R_i, **partition($\mathbf{R}_i$)**;
   end procedure
2. execute **partition(X)**;
3. for all regions where z(x) is the same and whose union is a convex set, compute such a union as described by Algorithm (2);
4. end.

The algorithm solves the mp-QP problem by partitioning the given parameter set X into N_r convex polyhedral regions. For the characterization of the MPC controller, in **step 3**, the union of regions is computed where the first N_u components of the solution z(x) are the same,
by using the Algorithm (2) as defined below for checking convexity of the union,
and for generating an H-representation of the union [3]:

1. Construct $env(P,Q)$ by removing non-valid constraints,
   let $\tilde{A}x \leq \tilde{\alpha}$, $\tilde{B}x \leq \tilde{\beta}$ be the set of removed constraints
   let $env(P,Q)$ =$\{x : Cx \leq \gamma\}$ the resulting envelope;

2. Remove from env(P,Q) possible duplicates (B_j, βj)=(σA_i, $\sigma \alpha_i$), $\sigma > 0$

3. **for** each pair $\tilde{A}_i x \leq \alpha_i$, $\tilde{B}_j x \leq \beta_j$ **do**

4. Determine ϵ^* by solving the linear program
   $\epsilon^* = max_{x,\epsilon}\, \epsilon$
   subject to $\tilde{A}_i x \geq \alpha_i + \epsilon$, $\tilde{B}_j x \geq \beta_j + \epsilon$, $\tilde{C}x \leq \gamma$;
   /*$\epsilon^* = -\infty$ if the LP is infeasible, $\epsilon^* = \infty$ if the LP is unbounded.

5. if $\epsilon^* > 0$, **stop**;**return** nonconvex;

6. **endfor**;

7. return env(P,Q). /*$P \cup Q$ is convex.*/

The algorithm presented above arise from the Theorem below which represents a result for convexity recognition of the union of two H-polyhedra.

Theorem 1. $P \cup Q$ is *convex* $\leftrightarrow P \cup Q = env(P,Q)$

Therefore, through the offline-procedure/algorithm above, we could find the control law which is continuous and piecewise-affine defined as below

$$f(x) = F^i x + g^i \;\text{ if }\; H^i x \leq k^i, i = 1, ..., N_{mpc} \tag{4}$$

where the polyhedral sets $\{H^i \leq k^i\}$, $i = 1, ..., N_{mpc} \leq N_r$ are a partition of a given set of states X.

0.3 Question 3: Online phase feature in an explicit MPC controller

The online operation in Explicit MPC consist of :

1. Identifying the region to which x belongs for the measured state x

2. Look up the controller parameters for the found region

3. Compute the next control Signal

4. Repeat step 1

1 REFERENCES

[1] SSY281 - Multi Predictive Control Lecture Notes
[2] Bemporad, Alberto; Morari, Manfred; Dua, Vivek; Pistikopoulos, Efstratios N. (2002). ”The explicit linear quadratic regulator for constrained systems”. Automatica. 38 (1): 3–20.
[3]Bemporad, Fukuda, and Torrisi (2001). ”Convexity recognition of the union of polyhedra”. Computational Geometry. 18 (2001) 141–154.