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TMA947 - Nonlinear Optimization

Planning of the natural gas transmission

Project Part 1 - Modelling

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1 PROBLEM STATEMENT

The aim of this article is to model a nonlinear programming model for minimizing the total cost of supplying the demand of gas distributed over different nodes at some minimal guaranteed pressure in Belgium. In this article we consider a situation where the gas merchant and transmission functions are integrated in a single company referred to as the gas company operating a transmission network. The company must decide on the quantities of gas to buy from several sources in order to satisfy the demand distributed over different nodes at some minimal guaranteed pressure with the aim to minimize the total supply cost of a gas transmission. The map of the Belgium gas network is shown in figure 1.

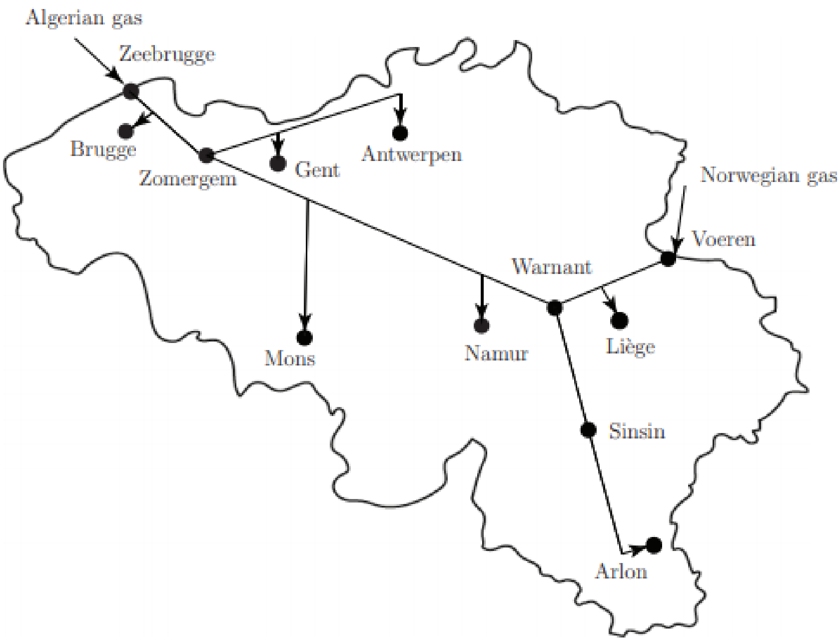

Figure 1: Belgium gas network

2 PROBLEM FORMULATION

2.1 Sets

The network of a gas company consists of several supply nodes where the gas is injected into the system, several demand nodes where the gas flows out of the system, and other intermediate nodes where the gas is rerouted. Pipelines are represented by arcs linking the nodes. The network is defined as the pair (N, A).

$$N \longrightarrow \text{Set of all nodes where each node denotes a city in the network}$$

$$N_s \longrightarrow \text{Set of supply nodes in the network}$$

$$N_i \longrightarrow \text{Set of intermediary nodes in the network}$$

$$N_d \longrightarrow \text{Set of demand nodes in the network}$$

$$A \subseteq N \times N \longrightarrow \text{Set of arcs connecting the nodes}$$

There are two types of arcs: passive arcs correspond to pipelines, and active arcs correspond to pipelines with a compressor.

$$A_a \longrightarrow \text{Set of Active arcs}$$
$$A_p \longrightarrow \text{Set of Passive arcs}$$

2.2 Parameters

The total demand of each Belgian province considered has been assigned to the main town of the province. This demand is given as follows:

$$d_i \longrightarrow \text{ Daily demand}(m^3/\text{day}) \text{ at node i, where i} \in N_d$$

A gas contract specifies an average daily quantity to be taken by the transmission company from the producer.Depending on the flexibility of the contracts, the transmission company has the possibility of lifting a quantity ranging between a lower and an upper bounds on the contracted quantity.

$$Gmax_i \longrightarrow \text{Maximum contracted quantity of a gas at node(city) i, where i} \in N$$
$$Gmin_i \longrightarrow \text{Minimum contracted quantity of a gas at node(city) i, where i} \in N$$

The pressure at the nodes is bounded between a minimum and maximum values.

$$Pmax_i \longrightarrow \text{Maximum gas pressure at node(city) i, where i} \in N$$
$$Pmin_i \longrightarrow \text{Minimum gas pressure at node(city) i, where i} \in N$$

The purchase price of the gas delivered is measured in dollars per million British thermal units [$/MBTU].

$$c_i \longrightarrow \text{Purchase price[\$/MBTU] at node i, where i} \in N_s$$
$$cs_i \longrightarrow \text{Converted purchase price[\$/m}^3\text{] at node i, where i} \in N_s$$

Pipelines are represented by arcs linking the nodes. the parameters describing these pipelines are as listed below.

$$L_{ij} \longrightarrow \text{Length of pipeline[km] between nodes i and j, where (i,j)} \in A$$

$$D_{ij} \longrightarrow \text{Interior diameter of the pipeline[mm] between nodes i and j, where (i,j)} \in A$$
$$C_{ij} \longrightarrow \text{Constant that depends on parameters of pipeline between nodes i and j, where (i,j)} \in A$$
$$T \longrightarrow \text{Gas temperature (K)}$$
$$\varepsilon \longrightarrow \text{Absolute pipe roughness (mm)}$$
$$\delta \longrightarrow \text{Density of gas relative to air}$$
$$z \longrightarrow \text{ gas compressibility factor}$$

2.3 Variables

The variable that we are interested to know about in the discussed problem is the flow of gas from node i to node j. This is formulated as:

$$f_{ij} \longrightarrow \text{flow of gas between nodes i and j, where (i,j)} \in A$$

There are two additional variables which are associated with each node i of the network.At each node the gas pressure is measured and the net supply of gas at each node. This supply could be negative, positive or zero depending upon the type of node.

$$P_i \longrightarrow \text{ Gas pressure[bar] measured at node i, where i} \in N$$
$$S_i \longrightarrow \text{ Net gas supply}(m^3\text{/day) in node i, where i} \in N$$

Mathematically, S_i could be expressed as below

$$S_i = \begin{cases} s_i & \text{A positive } s_i \text{ corresponds to a supply of gas at node i, where i} \in N_s \\ 0 & \text{for i } \in N_i \\ -s_i & \text{A negative } s_i \text{ implies a gas demand at node i, where i} \in N_d \end{cases}$$

2.4 Objective Function

The objective function of the gas company is to minimize the total cost of the supplies.The optimization problem could be understood as network flow optimization. This problem could specifically be formulated as below :

$$\text{minimize} \quad Z = \sum_{i \in N_s} \; cs_i S_i$$

2.5 Constraints

At a supply node i, the gas inflow S_i must remain within the limitations specified in the contracts.

$$\text{Gmin}_i \leq S_i \leq \text{Gmax}_i, \; i \in N_s$$

At a demand node, the gas outflow $-S_i$ must be greater than or equal to d_i, the demand at this node i.

$$-\text{S}_i \leq \text{d}_i, \; i \in N_d$$

The gas transmission company cannot take gas at a pressure higher than the one ensured by the supplier at the entry point. Conversely, at each exit point, the demand must be satisfied at a minimal pressure guaranteed to the industrial user or to the local distribution company.

$$\text{Pmin}_i \leq P_i \leq \text{Pmax}_i, \; i \in N$$

The flow conservation ensuring the gas balance at each node has to be satisfied.

$$\sum_{(i,j) \in A} f_{ij} = \sum_{(j,i) \in A} f_{ji} + S_i, \; i \in N$$

Now, consider the constraints on the arcs. We distinguish between the passive and active arcs. For a passive arc, the relation between the flow f_{ij} in the arc (i, j) and the pressures P_i and P_j is of the following form. f_{ij} are unrestricted in sign.If $f_{ij} < 0$, the flow $-f_{ij}$ goes from node j to node i.

$$\text{sign}(f_{ij})f_{ij}^2 = C_{ij}^2(P_i^2 - P_j^2), \;\; \forall(i,j) \in A_p$$

For an active arc corresponding to a pipeline with a compressor, which increases the pressure along the pipe, the relation between the flow and the corresponding pressures is

$$\text{sign}(f_{ij})f_{ij}^2 \geq C_{ij}^2(P_i^2 - P_j^2), \;\; \forall(i,j) \in A_a$$

There is an upper bound on the pressure at the exit of the compressor for each active arc.

$$P_j \leq Pmax_j, \;\; \forall(i,j) \in A_a$$

In practice, there is also a cost associated with the pressure increase. We do not consider this cost here. For active arcs, the direction of the flow is fixed.

$$f_{ij} \geq 0, \;\; \forall(i,j) \in A_a$$

In this problem, the constant C_{ij} could be expressed as the following

$$C_{ij}^2 = 96.074830(10^{-15})\frac{D_{ij}^2}{\lambda_{ij} z T L_{ij} \delta}$$

where,

$$\frac{1}{\lambda_{ij}} = [2\log_{10}(\frac{3.7D_{ij}}{\epsilon})]^2$$

2.6 Complete Formulation of the problem

The optimization problem could be understood as network flow optimization. This problem could specifically be formulated as below :

$$\begin{aligned}
\min \quad & \textstyle\sum_{i\in N_s} \; cs_i S_i \\
\text{s.t} \quad & \text{sign}(f_{ij})f_{ij}^2 = C_{ij}^2(P_i^2 - P_j^2), \;\; \forall(i,j) \in A_p \\
& \text{sign}(f_{ij})f_{ij}^2 \geq C_{ij}^2(P_i^2 - P_j^2), \;\; \forall(i,j) \in A_a \\
& \textstyle\sum_{(i,j)\in A} f_{ij} - \sum_{(j,i)\in A} f_{ji} = S_i, \; i \in \mathcal{N} \\
& f_{ij} \geq 0, \;\; \forall(i,j) \in A_a \\
& P_j \leq Pmax_j, \;\; \forall(i,j) \in A_a \\
& -S_i \leq d_i, \;\; i \in N_d \\
& \text{Gmin}_i \leq S_i \leq \text{Gmax}_i, \; i \in N_s, N_i \\
& \text{Pmin}_i \leq P_i \leq \text{Pmin}_i, \; i \in N
\end{aligned}$$