CHALMERS
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TMA947 - Nonlinear Optimization

Planning of the natural gas transmission

Project Part 2 - Sensitivity Analysis of the Optimal Plan

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1 AMPL Optimization Problem Analysis

The given optimization problem is solved using SNOPT solver. SNOPT is a widely used large-scale optimizer for difficult large-scale nonlinear problems. It is useful to solve problems with linear, quadratic, and smooth nonlinear objectives and constraints in continuous variables.

1. **What is the total cost of the gas supplying the demands in Belgium?**

   To reduce the total cost is the objective of this problem. Thus the total cost of the gas is represented by the variable objective(total_cost) in the output. So, the total cost of gas supplying the demands in Belgium is **1921171.236 $**.

2. **How much gas should be sent from Zomergem to Gent?**

   Since it is given that from Gent to Zomergem **-9.29** m^3 **per day** should be sent, the negative symbol indicates that **9.29** m^3 **per day** gas should be sent from Zomergem to Gent.

3. **How much Algerian gas should be purchased?**

   The Algerian gas is delivered at the Zeebrugge terminal. The amount of gas supplied from this terminal is **8.87 million** m^3 **per day**.

4. **How much gas is being transported from Warnant to Sinsin as a result of the compressor working on this active pipeline?**

   The amount of gas that is being transported from Warnant to Sinsin is **0.222 million** m^3 **per day** irrespective of the arc being active or passive. Therefore, **0** m^3 **per day** is being transported from Warnant to Sinsin as a result of the compressor working on this active pipeline.

2 Optimality Conditions

1. **Is the solution obtained locally optimal? Globally optimal? Explain your answers. Also explain if the approach you adopt to verify optimality works in general cases beyond this exercise**

   Yes, the solution obtained is global optimal. We have concluded this by using Baron solver. We obtained similar results by using BARON and SNOPT solver. BARON is a general nonlinear optimizer capable of solving non-convex optimization problems to global optimality. Special feature of this solver is that it does global optimization for problems that have potentially many locally optimal solutions. This method can be used for all those problems which have linear and nonlinear objectives and constraints including arithmetic, logarithmic, and exponential operators, in continuous and discrete variables. For those problems which are not of the type mentioned above, BARON solver cannot be used for checking if the solution is global optimal. Therefore this approach can be used for all the problems satisfying the above conditions in general, beyond this exercise.

   Another way we can think of is by seeing the supply of gas from each of the terminal (Voeren and Zeebrugge). The supply from each of these terminals is equal to the minimum possible supply. Thus the optimal solution cannot be lower than the present one (Voeren - 20.344 million M^3 per day and Zeebrugge 8.870 million M^3 per day). Therefore the solution obtained is global optimal solution.

2. **In the given AMPL model, the pressure-related decision variables are not exactly the pressures at the nodes. What are these decision variables? Why can we use these variables, and why do we want to use them?**

   In the given optimization problem, square of the pressure is used as the variable. This can be used because the change in the problem constraints is minimal. Moreover, the objective function is not dependent on the pressure but is dependent on only cost and supply. The pressure constraints in the problem are mainly to ensure proper gas flow between nodes but does not actually effect the objective function. Considering the original constraints, square of the pressure is the part of the constraints involving pressure (passive arc equilibrium, active arc equilibrium and fulfill pressure requirements). In the above constraints original constraints have a square term. Thus we can use this reduced form of pressure square as the new variable. This change is made to make the problem easier and simple to remove the non-linearity in the constraints relating pressure.

3. **Describe conceptually how to find a feasible solution without using any given nonlinear programming solver (this is useful in case the solver actually requires you to provide a feasible initial solution).**

   The problem is infeasible using a linear solver because of the ”Passive Equilibrium” constraint, as it is an equality constraint. To find a feasible solution we need linear constraints, so we first need to replace the absolute function used. For this, we could create an algorithm which changes the sign of f^2 by comparing f with 0. It would multiply f^2 by -1 if f is negative or by 1 if f is positive. The constraint is still nonlinear as we have f^2 and c^2 in it. As a remedy to this, the algorithm should be able to create two variables, taking respectively the values of f^2 and c^2. This way, as previously with the use of pi instead of pi^2, we can reformulate a linear constraint. Then, we could change the equality constraint into an inequality constraint by introducing positive slack variables. Finally, we could add a constraint on this slack variable value, depending on other variables of the problem. Using this reformulation, we can get feasible solution for the flow variables.

   To get some feasible solution for the pressure variables, we can formulate a Lagrangian dual function and the pressure is then expressed in terms of the dual variables.

3 Sensitivity Analysis

1. **Assume that Norwegian gas supply is reduced. Specifically, both the minimum and maximum quantities of the supply from Voeren are reduced to 80% of their original values.**

   After the minimum and maximum quantities of the supply from Voeren are reduced to 80% of their original values, the changed minimum and maximum supply values are as mentioned below:

	Minimum Supply	Maximum Supply
Before Change	20.344	22.012
After Change	16.2752	17.6096

   (a) **How does the total amount of Algerian gas purchased change?**

Initially the total supply from Algerian gas (supplied from Zeebrugge) was 8.87 million m^3 per day. After the change of maximum and minimum supply values in Voeren, the total supply of Algerian gas is increased to 11.1534 million m^3 per day.

(b) **How does the total amount of Norwegian gas purchased change?**

Before the change, the total supply of Norwegian gas (supplied from Voeren) was 20.344 million m^3 per day. After the change in the supply values, the amount of gas supplied from this terminal is decreased to 17.6096 million m^3 per day.

(c) **Explain why the above changes occur!**

Initially, since the price is set high for Zeebrugge (Algerian gas), the minimum supply is taken from there and the rest is taken from Voeren (Norwegian gas). But after we change the minimum and maximum supply, the initial supply from Norwegian gas has exceeded the maximum supply. Therefore, after the change, the maximum supply which can be supplied by Norway is taken and the remaining is taken from Algeria.

2. **What and how much can be changed in the data of the problem to make the compressor on the pipeline from Warnant to Sinsin active? How can you tell the compressor is indeed active?**

This part of the project involves changing the data of the optimization and analyze the changes occurred. Different data that may be varied are the pressure's upper bound in Sinsin, the pressure's upper bound in Warnant, the pipe's length and its diameter. The data we chose to analyse here is the upper bound of the pressure in Sinsin. Below is the table denoting the change in the pressure, along with the change in various other variables like flow and pressure at each of these stations. BARON solver is used to solve this optimization problem. We also tabulated the information regarding the constraint that makes a pipe active. That is, to check if the compressor is active or not, we should verify the following constraint:

$$\text{sign}(f_{ij})f_{ij}^2 \geq C_{ij}^2(P_i^2 - P_j^2)$$

where f_{ij} is the supply from town i to town j, C_{ij} is the constant related to the pipe connecting town i and j and P_i is the pressure noted at town i. The pipeline constant for the pipe connecting Warnant and Sinsin is 0.00413627.

Pressure's upper bound in Sinsin	Gas supply from Warnant to Sinsin	Squared pressure measured at Warnant	Squared pressure measured at Sinsin	Constraint satisfied?
63.0	0.222	1733.31	1721.4	Yes
75.0	0.303228	2495.42	3504.79	Yes
85.0	0.328311	2045.06	2211.01	Yes
55.0	0.23671	2040.27	2191.68	Yes
41.0	0.620854	1772.91	1680.37	Yes
40.0	0.835208	1768.65	1600	No
39.0	1.0121	1768.65	1521	No
36.0	1.39821	1768.65	1296	No

Thus, we conclude by saying that the arc, and similarly the compressor, is active only if the pressure's upper bound in Sinsin is greater or equal to 41 bar. By analyzing the table, we can notice that the constraint above holds when the pressure measured in Sinsin is higher than the one measured in Warnant. The reason is that in this case, the result obtained on the right side of the inequality constraint becomes negative, while its left side is always positive. Thus, to keep the compressor active, the pressure in Sinsin must be greater than the pressure in Warnant.

3. **Consider the case of increased demands:**

   (a) **The demands at all demand nodes are doubled, while the supply at each supply node remains the same. Is this optimization model feasible? Explain your answer.**

   This optimization model is infeasible as one of the constraints does not hold. It is the flow balance condition, especially concerning the town of Mons. This is because the demand is doubled but the supply is still the same. Therefore, the demand has an upper bound than the supply, making the model infeasible.

   (b) **The demands at all demand nodes are doubled, and the maximum supplies at all supply nodes are also doubled. Is this optimization model feasible? Explain your answer.**

   By changing these data, we obtained a result with the SNOPT solver but not with BARON. With the BARON solver we obtain an infeasible problem. The reason is that when using SNOPT, an elastic constraint formulation is used to deal with infeasibility. That is why we got solution for the problem while using SNOPT. But in case of BARON solver, if the problem is infeasible, we won't get any solution and a warning comes stating 'infeasible'. Hence this optimization model is **infeasible**.

   To state an analytical reason for this infeasibility, we shall look into the variables. The demands at all demand nodes are doubled, and the maximum supplies at all supply nodes are also doubled. This means that the flow through the pipes is increased because of increased demands. But the values of the maximum pressure are unchanged. In physics there exists a relation between the supply (flow) and the pressure. Thus for the same pipeline (dimensions and properties of pipeline also unchanged), if the flow increases, the pressure also increases. But there would exist a point where this increased pressure exceeds the maximum pressure limit making the problem infeasible. Our observation was verified by doubling the maximum pressures for every node and the problem was feasible. Therefore, we can conclude that just by doubling the demands and supplies makes the optimization problem infeasible.

4. **Consider the marginal reductions of the minimal supplies of Norwegian and Algerian gas. Consider one limit at a time. What are the approximate derivatives of the total cost with respect to changes of the minimal supplies of Norwegian and Algerian gas, evaluated at the original optimal solution? Describe how you obtain these approximate derivatives. In addition, compare the results with the values of the dual variables for the limiting constraints in the optimal solution to the original problem.**

   For this part of the project,

   - We have considered minimal change in minimal supplies of Norwegian and Algerian gas one by one.
   - We calculated the change in the total cost (objective).
   - Then, we calculated the approximate derivatives, i.e., percentage change which is equal to ration of the change in the objective function and the delta change in the minimal supply.

- Finally, we displayed the dual values corresponding of the Demand constraint, corresponding to the supply lower bounds found previously.

Table below shows the marginal reductions of minimal supplies of Norwegian gas (delivered at Voeren terminal) along with the dual values:

Voeren's supply lower bound (in m^3/day)	Objective value (total cost in \$)	Rate change depending on init value	Dual Value
initial value: 20.344	initial objective: 1921171.317	initial rate: 0%	59328
20.244	1915238.453	99.6912%	59328.6
20.144	1909305.59	99.3824%	59328.6
20.044	1903372.727	99.0736%	59328.6
19.944	1897439.864	98.7647%	59328.6
19.894	1894473.432	98.6103%	59328.6
19.893	1894414.103	98.6072%	15555.5
19.889	1894414.103	98.6072%	0
19.000	1894414.103	98.6072%	0
15.000	1894414.103	98.6072%	0
10.000	1894414.103	98.6072%	0

We can observe that as soon as the supply's lower bound of Voeren town is lower or equal to 19.893 m^3/day, the objective always remains the same with a rate of 98.6072%.

Table below shows the marginal reductions of minimal supplies of Algerian gas (delivered at Zeebrugge terminal):

Zeebrugge's supply lower bound (in m^3/day)	Objective value (total cost in \$)	Rate change depending on init value	Dual Value
initial value: 8.870	initial objective: 1921171.317	initial rate: 0%	80517.4
8.770	1913119.574	99.5809%	80517.4
8.370	1883819.704	98.0558%	21188.8
8.000	1875979.849	97.6477%	21188.8
7.000	1854791.052	96.5448%	21188.8
6.770	1849917.628	96.2911%	21188.8
6.755	1849599.796	96.2746%	21188.8
6.752	1849536.23	96.2713%	21188.8
6.751	1849515.041	96.2702%	21027
6.750	1849515.041	96.2702%	0
6.000	1849515.041	96.2702%	0
3.000	1849515.041	96.2702%	0

We can observe that as soon as the supply's lower bound of Zeebrugge town is lower or equal to 6.751 m^3/day, the objective always remains the same with a rate of 96.2702%.

When comparing the dual values to the objective values found by changing the supply lower bounds of each town, we can observe that they correspond. The dual value measures the increase in the objective function's value per unit increase in the variable's value. Actually, the dual values are not equal to zero when the objective value is still changing. It means, when the constraint is binding. But as soon as the rate change is at its minimum, the dual value is equal to zero, meaning that it does not change anymore. Moreover, we can also notice that the dual value's change is corresponding to the change in the Minimum supply (rate of change is similar!!!)