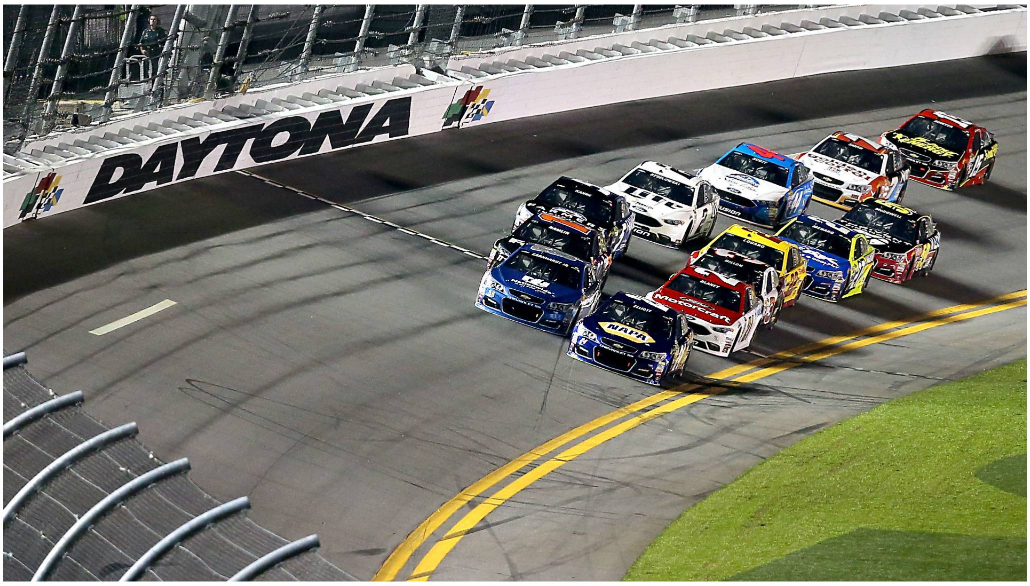
Assignment 2: Lateral Dynamics

Hugo Cécilia-Joseph and Fikri Farhan Witjaksono



Chalmers University of Technology

Vehicle Dynamics

May 9, 2022

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Introduction

In this assignment, we are required to investigate the Steady State and the Stationary Oscillating Cornering at High Speed using the one-track model. We are also investigating the effect of adding load transfer equation and adding the combined tyre slip model to the axle lateral stiffness. Another purpose is to understand the differences in the results of the simulation model from our Simulink model and the CASTER vehicle simulator model.

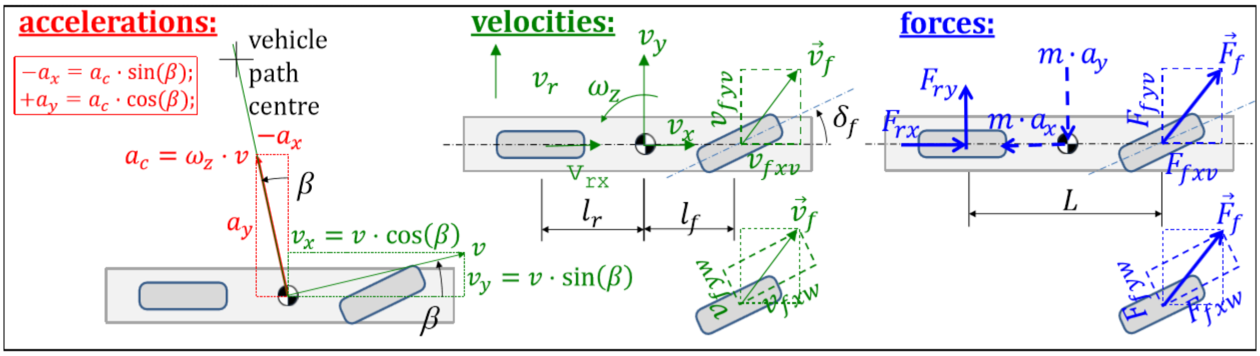
Abstract

The first main finding was that the load case 1, load case 2 and load case will be understeered, oversteered and neutral steered respectively for driving in a curve radius of 200 m. The second finding was that the load case 1 will be the quickest setup with respect to output transient response with changed steering input. It is evident since by observing the lateral acceleration, we could see that the steepest slope of curve between all load cases is the load case 1. Moreover, we recommend the use of the roll stiffness distribution ratio of 0.22 that will results in the desired neutral steering for steering angle of 10 degrees. Although, it should not be put in a production car for several reasons that will be seen in the report. In addition, we also recommend the use of brake force distribution of 0.765 in order to achieve neutral steer in the added combined slip model and brake demand of 4000 N. For another variation of brake demand, we recommend the use of EBD system and ABS in order to vary the brake distribution ratio thus maintaining the vehicle stability.

1 Steady state cornering characteristics

1.1 Steady state cornering equations

To solve this problem, we've used the Figure 4-15 from the compendium which shows accelerations, forces and velocities applied on a one-track model. Due to the fact that we are studying a one-track vehicle model at high speed, all time derivatives are equal to zero. The vehicle is driven on a circle with constant yaw velocity. So $\dot{V}_x = \dot{V}_y = \dot{\omega}_z = 0$; $R = constant\, and\, \beta = constant$.



Assumptions:

1. Path >> Radius so all forces are approximately codirected

2. Small tyre and vehicle side slip; so angle = sin(angle) = tan(angle)

Equilibrum:

$$m.V_x^2 = F_{ry} + F_{fy} \tag{1}$$

$$0 \approx F_{fy}.L_f - F_{ry}.L_r \tag{2}$$

Constitution:

$$F_{fy} = -C_f.S_{fy} \tag{3}$$
$$F_{ry} = -C_r.S_{ry} \tag{4}$$

Compatibility:

$$V_{fy} = V_y + L_f.\omega_z - V_x.\delta \tag{5}$$

$$V_{fx} = V_x \tag{6}$$

$$(5)\,and\,(6) \Longrightarrow S_{fy} = \frac{V_{fy}}{Vrx} \approx \frac{V_y + L_f.\omega_z}{V_x} - \delta_f \tag{7}$$

$$V_{ry} = V_y - L_f.\omega_z - V_x.\delta \tag{8}$$

$$V_{rx} = V_x \tag{9}$$

$$(8)\,and\,(9) \Longrightarrow S_{ry} = \frac{V_{ry}}{Vrx} \approx \frac{V_y - L_f.\omega_z}{V_x} \tag{10}$$

$$\omega_z \approx \frac{V_x^2}{R_p} \tag{11}$$

Eliminate: $\mathrm{F}_{ry}, F_{fy}, S_{fy}, S_{ry}, \omega_z, V_y$

$$\delta_f \approx \frac{L}{R_p} + K_u.\frac{m.V_x^2}{R_p} \tag{12}$$

with $\mathrm{K}_u = \frac{C_r.L_f - C_f.L_f}{C_f.C_r.L}$

1.2 Understeer gradient

An equilibrum helps to determine the normal load force applied on each wheel. Then, these values enable to solve for the understeer gradient Ku for each load case.

Thus,

$$F_{zfl} = F_{zfr} = \frac{m.g}{2.L}.L_r$$

$$F_{zrl} = F_{zrr} = \frac{m.g}{2.L}.L_f$$

Then,

$$C_f = C_0.(F_{zfl} + F_{zfr}) + C_1.(F_{zfl}^2 + F_{zfr}^2)$$

$$C_r = C_0.(F_{zrl} + F_{zrr}) + C_1.(F_{zrl}^2 + F_{zrr}^2)$$

And,

$$K_u = \frac{C_r.L_f - C_f.L_f}{C_f.C_r.L}$$

Given data:

Load case 1: L_f=0.37$L\,with\,L$=2.675$\,m$

Load case 2: L_f=0.63$L\,with\,L$=2.675$\,m$

Load case 3: L_f=0.47$L\,with\,L$=2.675$\,m$

Solve the understeer gradient:

$$\mathbf{K}_u = 1.6.10^{-6}.[0.6996 \quad -0.6996 \quad 0.1593]$$

1.3 Critical and Characterisitc speeds

$V_{x,crit}$ is the speed above which the yaw velocity soars up for a small steer angle. In other terms, $\delta_f \approx 0$

$$\delta_f = \frac{L}{R_p} + K_u.\frac{m.V_{x,crit}^2}{R_p} = 0 \tag{13}$$

$$(13) \implies V_{x,crit} = \sqrt{\frac{-L}{K_u.m}} \tag{14}$$

$V_{x,char}$ is the speed at which the vehicle requires twice as steer angle for a certain path radius as required at low speed. In other terms, $\delta_f = 2.\delta_f$

$$\delta_f = \frac{L}{R_p} + K_u.\frac{m.V_{x,char}^2}{R_p} = 2.\delta_f \tag{15}$$

$$(15) \implies V_{x,crit} = \sqrt{\frac{L}{K_u.m}} \tag{16}$$

Critical speeds:

Load case 1: Not defined

Load case 2: $V_{x,crit}$ = 44.78 m/s

Load case 3: Not defined

Characteristic speeds:

Load case 1: $V_{x,char}$ = 44.78 m/s

Load case 2: Not defined

Load case 3: $V_{x,char}$ = 100.13 m/s

1.4 Steering wheel angle for one certain operating point

We know that $V_x = 100\ km/h$ and a $= 4\ m/s^2$. Thus,

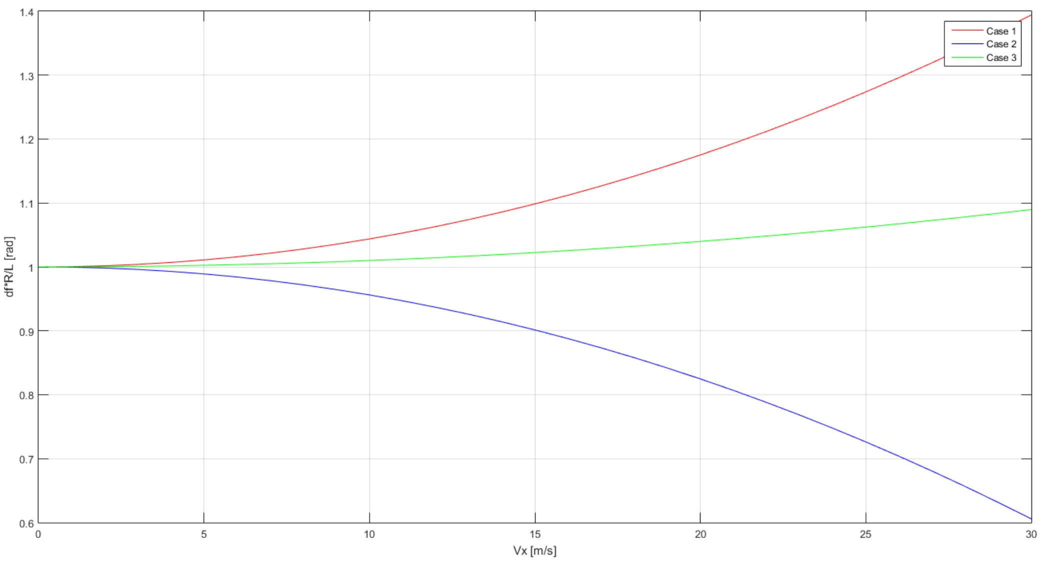
$$a = \frac{V_x^2}{R_p} = 4\ m/s^2 \tag{17}$$

$$(17) \implies R_p = 192.9012m \tag{18}$$

Then, equation (12) gives the required δ_f :

$\delta_f = [16.9035 \quad 8.3626 \quad 13.6053] \quad in\ degrees$

1.5 Steering wheel angle for varying operating point

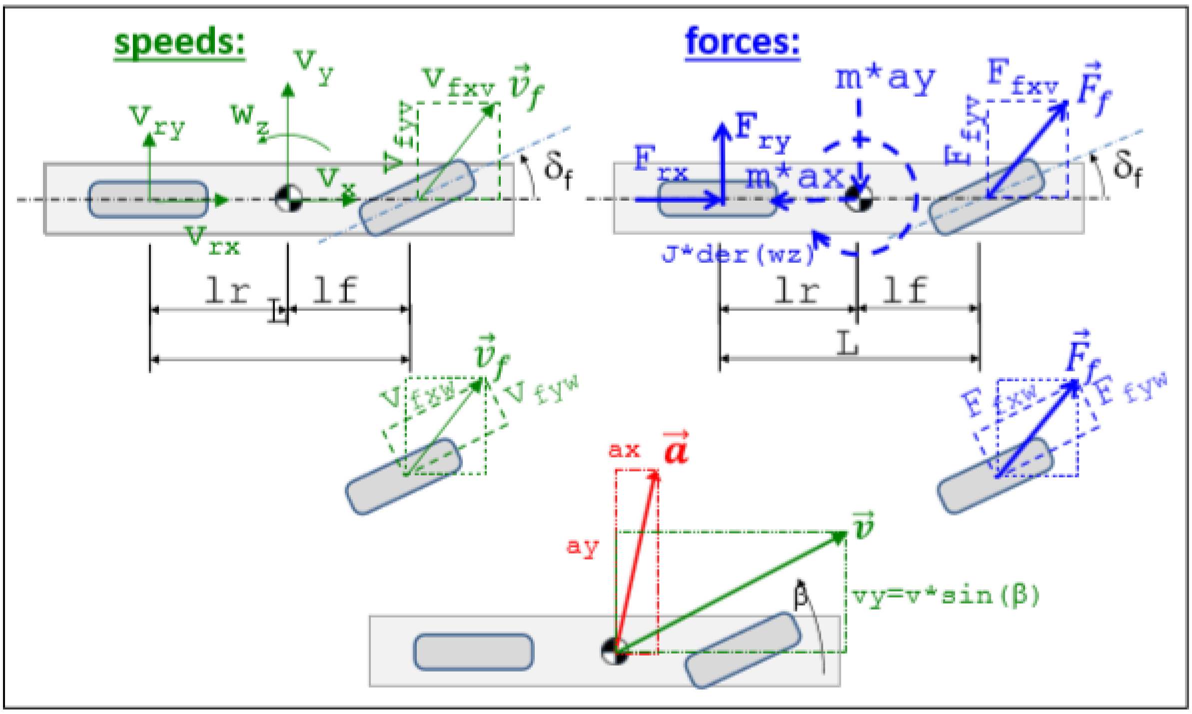


From the picture, we can say that load case 1 is shows an understeered vehicle, case 2 shows an oversteered one and case 3 corresponds to a relatively neutral steered vehicle.

2 Simulation with single track model (without load transfer and without combined tyre slip)

2.1 Implement model

Now we don't assume small steering and slip angles anymore. We ignore aerodynamic drag and rolling resistance.



By rearranging the equilibrium equations in Equation 4.47 in the compendium which is shown below in equation (19), (20), (21):

$$m.(\dot{v_x} - \omega_z.v_y) = F_{fxw}.cos(\delta_f) - F_{fyw}.sin(\delta_f) + F_{rx} \tag{19}$$

$$m.(\dot{v_y} + \omega_z.v_x) = F_{fxw}.sin(\delta_f) + F_{fyw}.cos(\delta_f) + F_{ry} \tag{20}$$

$$J.\dot{\omega_z} = (F_{fxw}.sin(\delta_f) + F_{fyw}.cos(\delta_f)).l_f - F_{ry}.l_r \tag{21}$$

We could get these simplified expression shown in equation (22), (23), (24):

$$\dot{v_x} = \frac{(cos(\delta_f).F_{fxw} - sin(\delta_f).F_{fyw} + (F_{rx} + m.w_z.v_y))}{m} \tag{22}$$

$$\dot{v_y} = \frac{(sin(\delta_f).F_{fxw} + cos(\delta_f).F_{fyw} + (F_{rx} - m.w_z.v_y))}{m} \tag{23}$$

$$\dot{\omega_z} = \frac{((F_{fxw}.sin(\delta_f) + F_{fyw}.cos(\delta_f).l_f) - (F_{ry}.l_r))}{J} \tag{24}$$

The slip characteristic could be derived by the equations below:

$$s_{fy} = \frac{v_{fyw}}{|v_{fyw}|} \tag{25}$$

$$s_{ry} = \frac{v_y - l_r.\omega_z}{|v_x|} \tag{26}$$

v_{fxw} and v_{fyw} could be defined as:

$$v_{fxw} = (v_y + l_f.\omega_z).sin(\delta_f)) + v_x.cos(\delta_f) \tag{27}$$

$$v_{fyw} = (v_y + l_f.\omega_z).cos(\delta_f)) - v_x.sin(\delta_f) \tag{28}$$

Then, by inserting equations (27) and (28) into equations (25) and (26), we get the slip equations :

$$s_{fy} = \delta_f - tan^{-1}(\frac{(v_y + (l_f.w_z))}{v_x}) \tag{29}$$

$$s_{ry} = -tan^{-1}(\frac{(v_y - (l_r.w_z))}{v_x}); \tag{30}$$

In the end, in order to obtain the axle stiffness in front and rear axle, we could use these equations below :

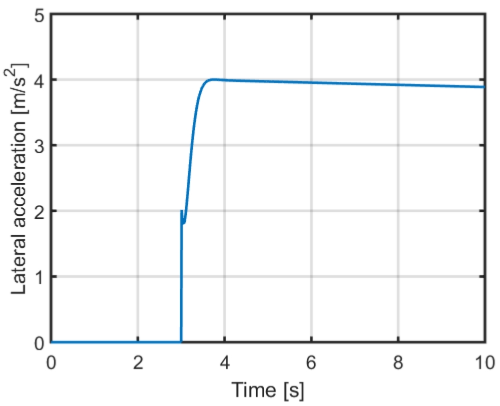
$$C_f = (c_0.F_{zrf} + c_1.(F_{zrf})^2) + (c_0.F_{zlf} + c_1.(F_{zlf})^2) \tag{31}$$

$$C_r = (c_0.F_{zrr} + c_1.(F_{zrr})^2) + (c_0.F_{zlr} + c_1.(F_{zlr})^2) \tag{32}$$

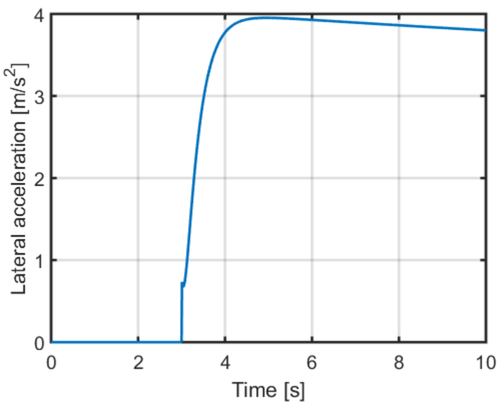
These equations are required in order to simulate the transient single-track model in Simulink.

2.2 Verify model for moderate a_y

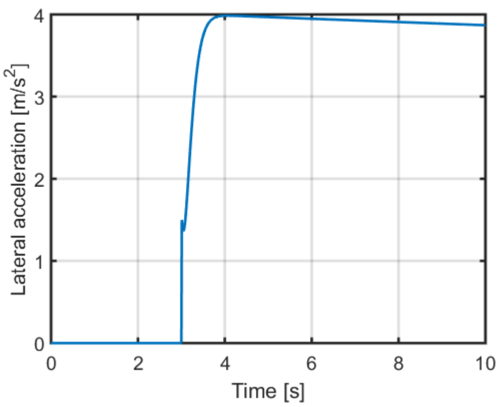
By running the model for the three load cases, we can verify that the 4 m/s^2 is reached for each load case.



(a) Case 1 Lateral Acceleration



(b) Case 2 Lateral Acceleration



(c) Case 3 Lateral Acceleration

2.3 Transient response

By analyzing and comparing the lateral acceleration plots for the different load cases, we've determined that the quickest response occurs with the understeered vehicle. The yaw rate peak is also higher and reached faster. The slip angles are bigger most of the time except for the rear slip angle that is bigger in the neutral steered configuration.

Fig. 2.3.1: Understeered

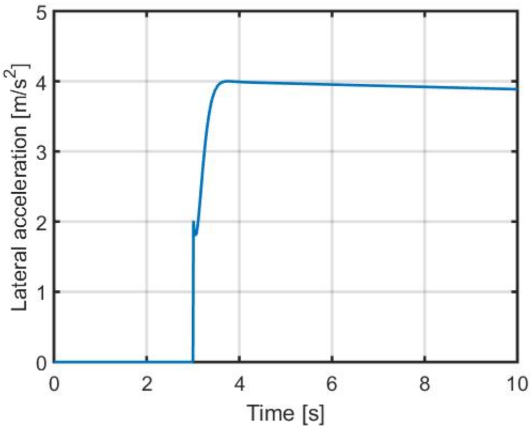


Fig. 2.3.2: Oversteered

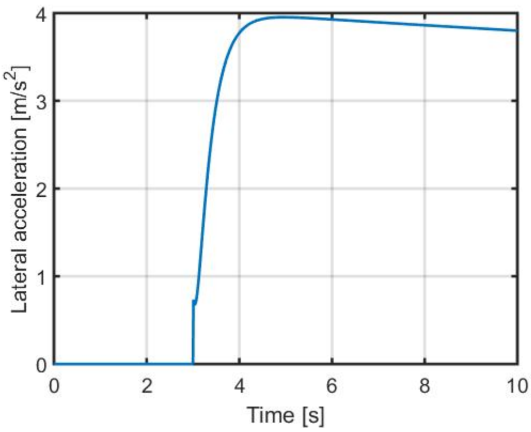


Fig. 2.3.3: Neutral steered

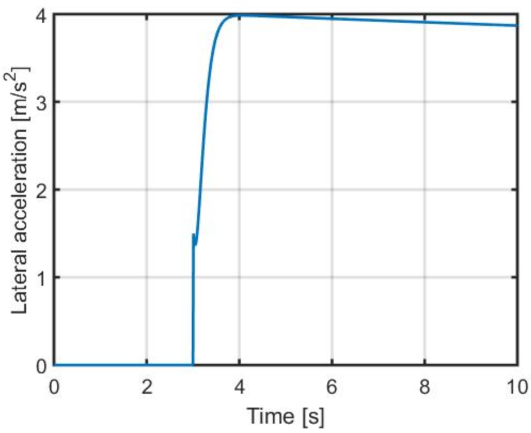


Fig. 2.3.4: Lateral acceleration in each vehicle set-up

Fig. 2.3.5: Understeered

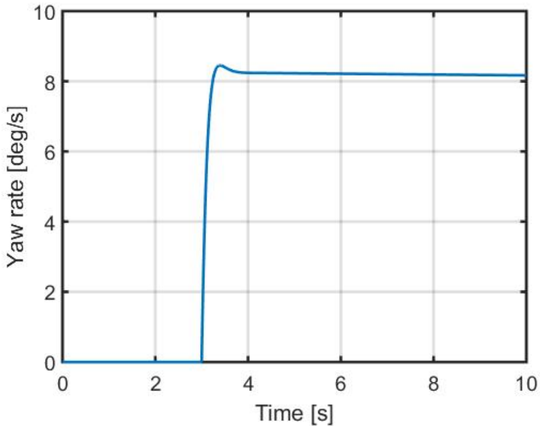


Fig. 2.3.6: Oversteered

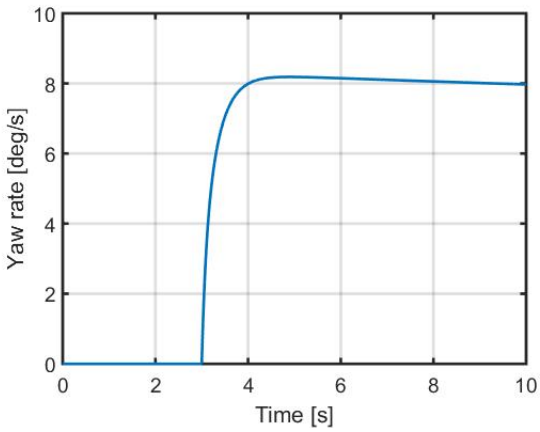


Fig. 2.3.7: Neutral steered

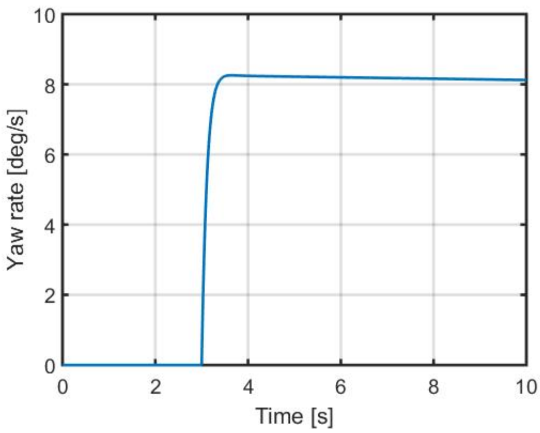


Fig. 2.3.8: Yaw rate in each vehicle set-up

Fig. 2.3.9: Understeered

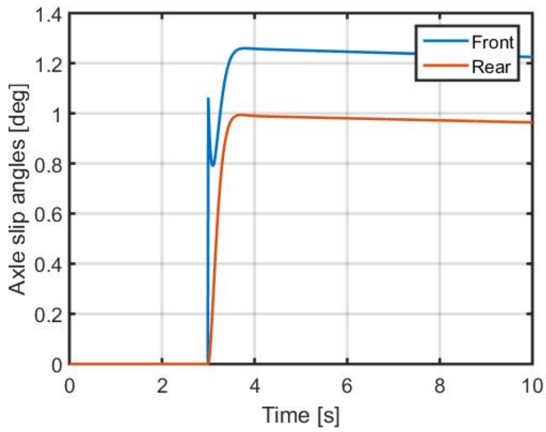


Fig. 2.3.10: Oversteered

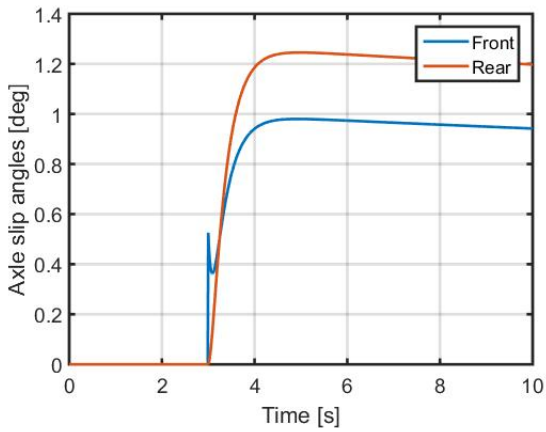


Fig. 2.3.11: Neutral steered

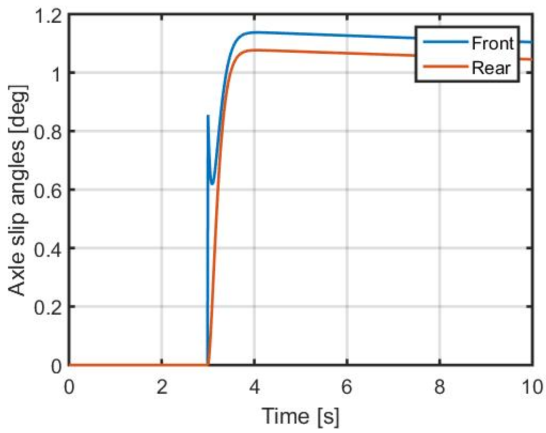
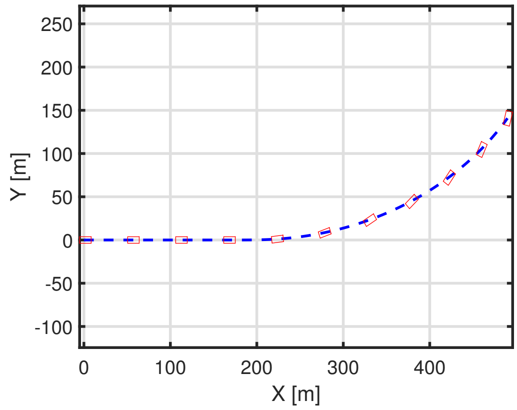
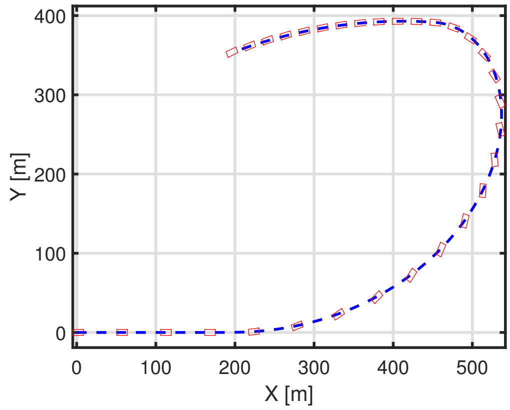


Fig. 2.3.12: Axle slip angles in each vehicle set-up

2.4 Try model at over-critical speed



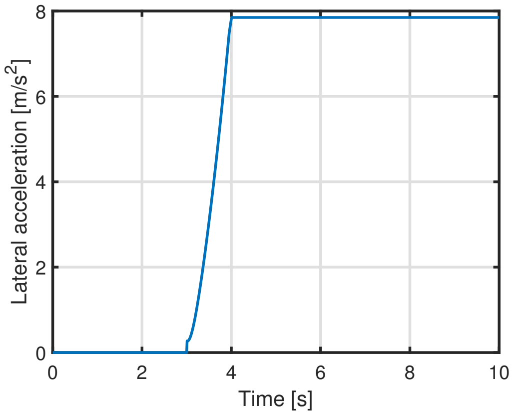
(a) Before changing simulation time



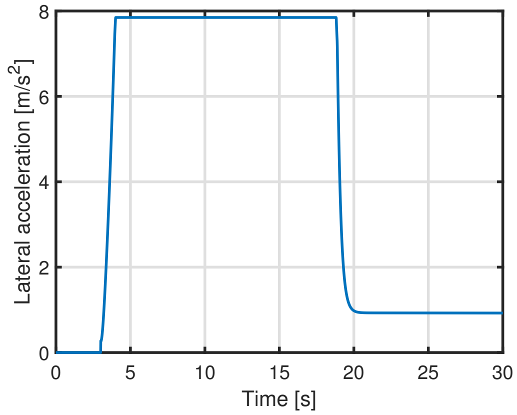
(b) After changing simulation time

Fig. 2.4.1: Effect of the changing the simulation time to curvature radius

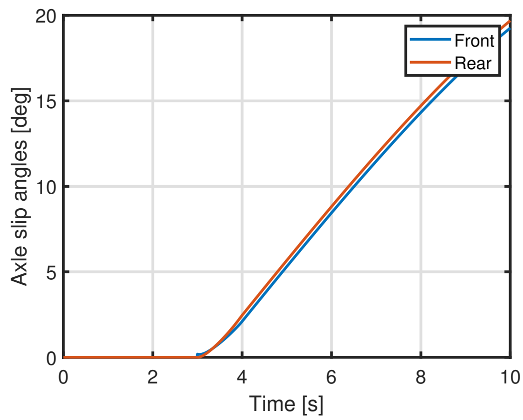
We are above the critical speed so theoretically, we should observe an infinite peak in the yaw velocity gain. The slip angles values should be high too because at this speed the tyres should be saturated during the cornering manoeuvre and the car should thus be unstable.This is not clearly seen for a 10-second simulation but for a 30-second simulation we can see what was expected to happen. Moreover, the trajectory of the car shows that at a certain time, the car starts sliding (drifting) instead of cornering. The tyres are saturated and there's not enough later force to available to make the car turn. As a consequence, the vehicle is forced to change its trajectory. This longer simulation shows that the expectations from task 1 are in line with task 2 in case we're above the critical speed.



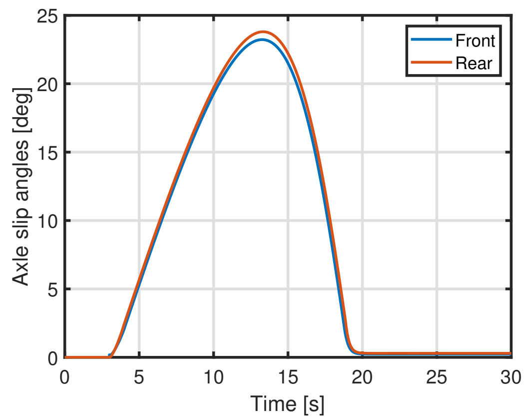
(a) Lateral Acceleration (Before Change)



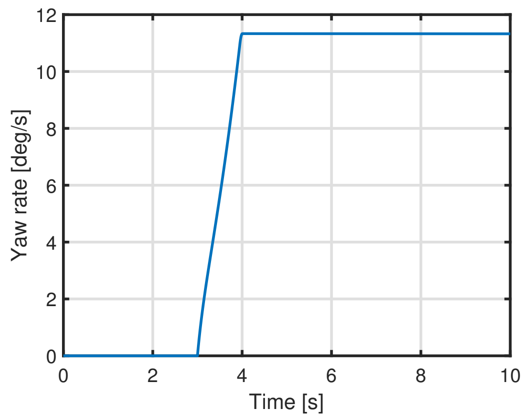
(b) Lateral Acceleration (After Change)



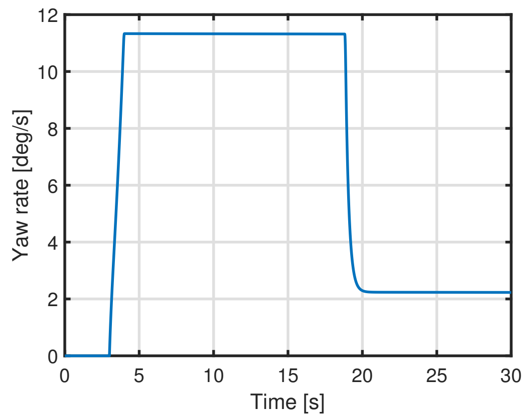
(c) Side Slip Angle (Before change)



(d) Side Slip Angle (After change)



(e) Yaw Rate (Before change)



(f) Yaw Rate (After change)

Fig. 2.4.2: Effect of the changing the simulation time to Lateral Acceleration, Yaw Rate and Side Slip Angle

3 Load transfer

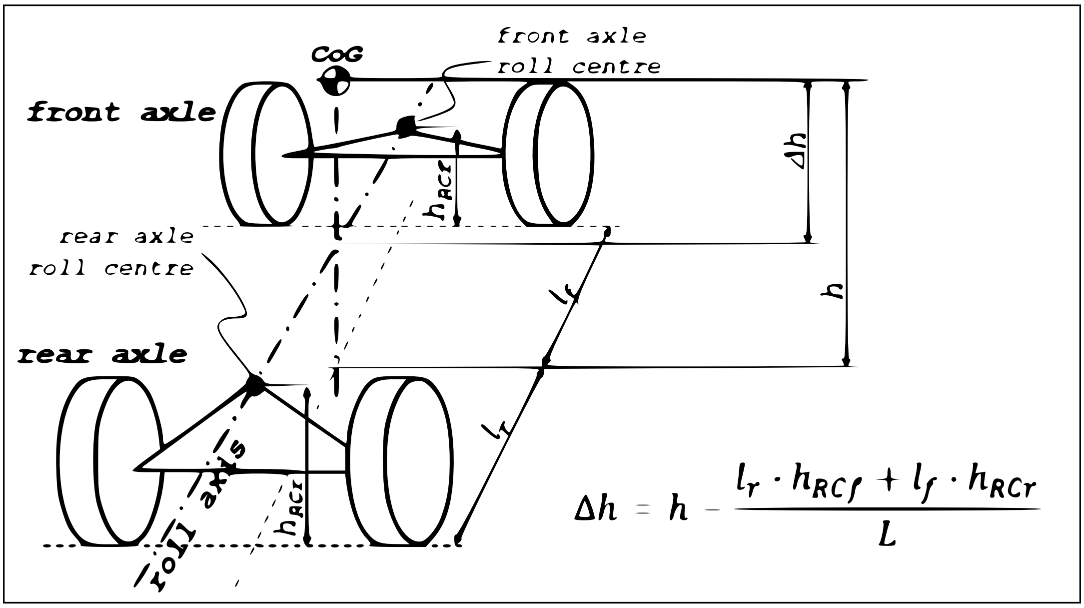


Fig. 3.0.1: The Roll axis for a Two Axle Vehicle

3.1 Load transfer addition to model

The figure above shown the roll axis for a two-axle vehicle and the geometric parameters that are used in the addition of load transfer to the steady-state model of a vehicle. h_{RCf} denotes the roll centre height of the front axle, h_{RCr} denotes the roll centre height of the rear axle and Δh denotes the height of the COG of the vehicle from the ground (h) - the ratio of (distance from the rear axle to CoG (l_r) times roll center height of the front axle + distance from the front axle to CoG (l_f) times roll center height of the front axle) and the wheel base).

According to the compendium, the equation for steady state longitudinal and lateral distribution by neglecting the body forces (Aerodynamic and gravity components) can be defined as :

$$Ff_{lz} = m.(\frac{g.l_r}{2L} - \frac{a_x.h}{2L} - a_y.(\frac{h_{RCf}.l_r}{L.w} + \frac{\Delta h}{w}.\frac{c_{f,roll}}{c_{vehicle,roll}})) \tag{33}$$

$$Ff_{rz} = m.(\frac{g.l_r}{2L} - \frac{a_x.h}{2L} + a_y.(\frac{h_{RCf}.l_r}{L.w} + \frac{\Delta h}{w}.\frac{c_{f,roll}}{c_{vehicle,roll}})) \tag{34}$$

$$Fr_{lz} = m.(\frac{g.l_f}{2L} + \frac{a_x.h}{2L} - a_y.(\frac{h_{RCr}.l_f}{L.w} + \frac{\Delta h}{w}.\frac{c_{r,roll}}{c_{vehicle,roll}})) \tag{35}$$

$$Fr_{lz} = m.(\frac{g.l_f}{2L} + \frac{a_x.h}{2L} + a_y.(\frac{h_{RCr}.l_f}{L.w} + \frac{\Delta h}{w}.\frac{c_{r,roll}}{c_{vehicle,roll}})) \tag{36}$$

By using equation (33)-(36) and separating the longitudinal fraction and lateral fraction of the Steady State Distribution equation, we could obtain the following equations for longitudinal distribution :

$$\Delta F_{flz,Long} = -m.(\frac{a_x.(h)}{2L}); \tag{37}$$

$$\Delta F_{frz,Long} = -m.(\frac{a_x.(h)}{2L}); \tag{38}$$

$$\Delta F_{rlz,Long} = m.(\frac{a_x.(h)}{2L}); \tag{39}$$

$$\Delta F_{rrz,Long} = m.(\frac{a_x.(h)}{2L}); \tag{40}$$

Moreover, we could also find these following equations for lateral distribution :

$$\Delta F_{flz,Lat} = -a_y.(\frac{h_{RCf}.l_r}{L.w} + \frac{\Delta h}{w}.\frac{c_{f,roll}}{c_{vehicle,roll}}) \tag{41}$$

$$\Delta F_{frz,Lat} = +a_y.(\frac{h_{RCf}.l_r}{L.w} + \frac{\Delta h}{w}.\frac{c_{f,roll}}{c_{vehicle,roll}}) \tag{42}$$

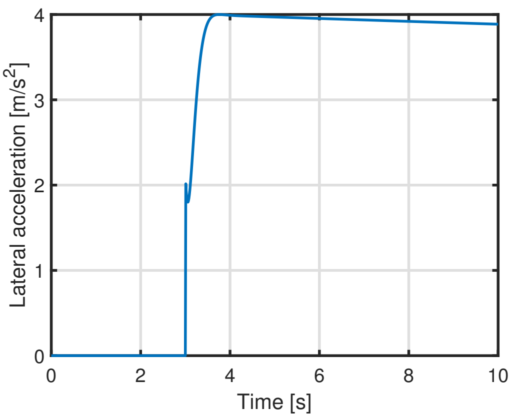
$$\Delta F_{rlz,Lat} = -a_y.(\frac{h_{RCr}.l_f}{L.w} + \frac{\Delta h}{w}.\frac{c_{r,roll}}{c_{vehicle,roll}}) \tag{43}$$

$$\Delta F_{rrz,Lat} = +a_y.(\frac{h_{RCr}.l_f}{L.w} + \frac{\Delta h}{w}.\frac{c_{r,roll}}{c_{vehicle,roll}}) \tag{44}$$

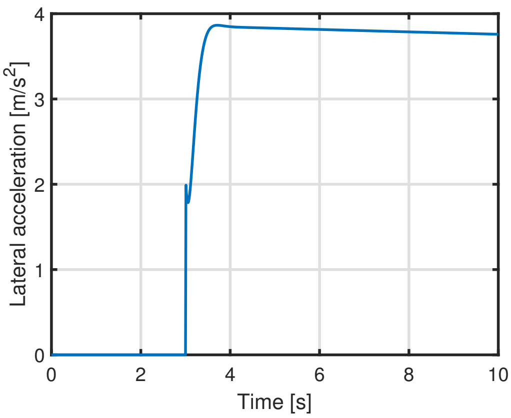
By using these equations, we can see that the models runs correctly.

3.2 Influence from load transfer on steering response

In this task, we used the default roll stiffness distribution of 0.65 and simulate the 3 load cases as shown in Task 1.2 with respective understeer gradient. After that, we plot the lateral acceleration as shown below.

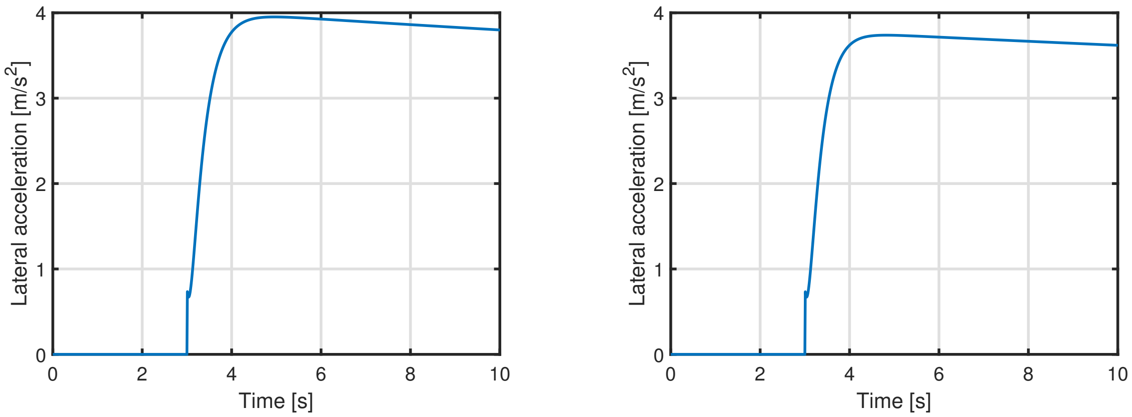


(a) Before Load transfer Addition



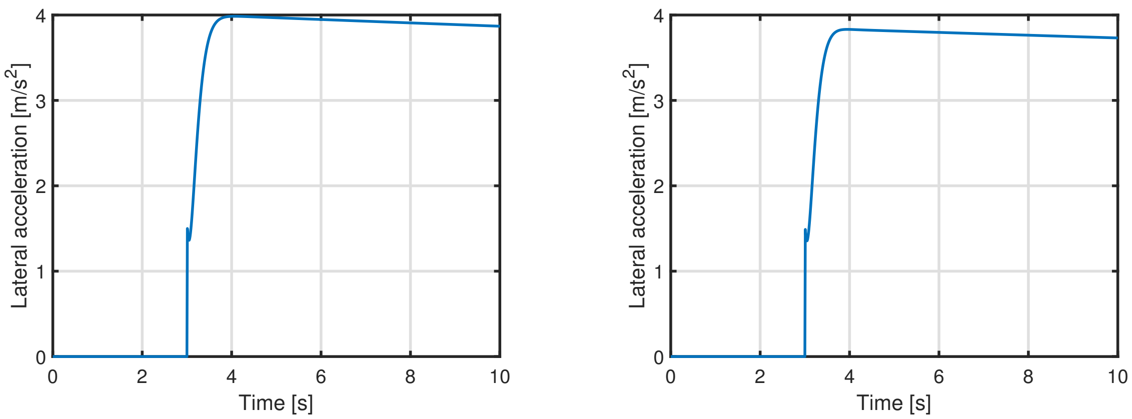
(b) After Load transfer Addition

Fig. 3.2.1: Lateral Acceleration (Load case 1)



(a) Before Load transfer Addition **(b)** After Load transfer Addition

Fig. 3.2.2: Lateral Acceleration (Load case 2)



(a) Before Load transfer Addition **(b)** After Load transfer Addition

Fig. 3.2.3: Lateral Acceleration (Load case 3)

Table 1: Lateral Acceleration Peak Values (Load case 1)

$PeakValue(Before)(m/s^2)$	$PeakValue(After)(m/s^2)$	$ChangeinValue[\%]$
4.0	3.861	3.48

By observing the peak values of lateral acceleration that were displayed above and comparing the percentage in change of the values, we could assume for load case 1 that the lateral acceleration after including the load transfer equation for steady-state condition does not cause much change in the peak value. Only 3.48 % change was observed.

Table 2: Lateral Acceleration Peak Values (Load case 2)

$PeakValue(Before)(m/s^2)$	$PeakValue(After)(m/s^2)$	$ChangeinPercentage$
3.95	3.735	5.44

By observing the peak values of lateral acceleration that were displayed above and comparing the percentage in change of the values, we could assume for load case 1 that the lateral acceleration after including the load transfer equation for steady-state condition does not cause much change in the peak value. Only 5.44 % change was observed.

Table 3: Lateral Acceleration Peak Values (Load case 3)

$PeakValue(Before)(m/s^2)$	$PeakValue(After)(m/s^2)$	$ChangeinPercentage$
3.985	3.83	3.89

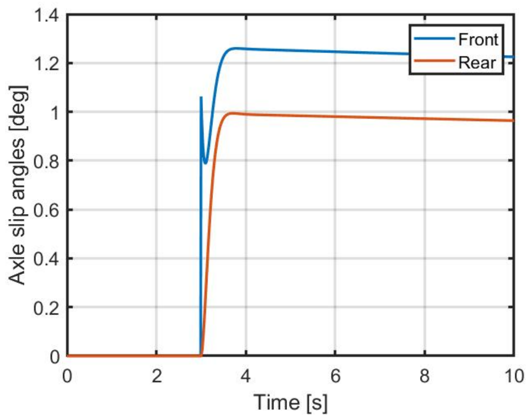
By observing the peak values of lateral acceleration that were displayed above and comparing the percentage in change of the values, we could assume for load case 1 that the lateral acceleration after including the load transfer equation for steady-state condition does not cause much change in the peak value. Only 3.89 % change was observed.

If we compare the two values of lateral acceleration between the one before/after load transfer addition, we can observe that both values are quite similar with only a little variation (3-5%).

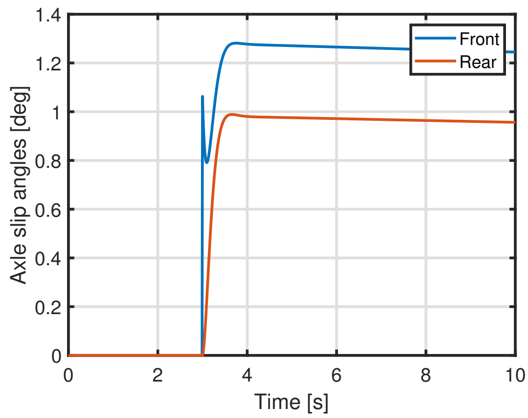
The side slip angle evaluation is correlated to the steering condition. Let's consider the steering angle as a variable during the transient manoeuvre. For instance, a vehicle is understeered if the tyre side slip is larger on front axle than on rear axle, in other terms if $|\alpha_f| > |\alpha_r|$, and it's oversteered if $|\alpha_f| < |\alpha_r|$. Therefore, neutral steering condition would mean that $|\alpha_f| = |\alpha_r|$.

Thus, by finding the difference between the front slip angle and rear slip angle (α_f-α_r), we could define whether the vehicle is in an understeered, an oversteered or a neutral steered condition.

The variation in side slip angle before/after load transfer addition as shown by fig. 3.2.6 could be described as very small. And the transient state and steady state behaviour in the before/after curves are also similar.

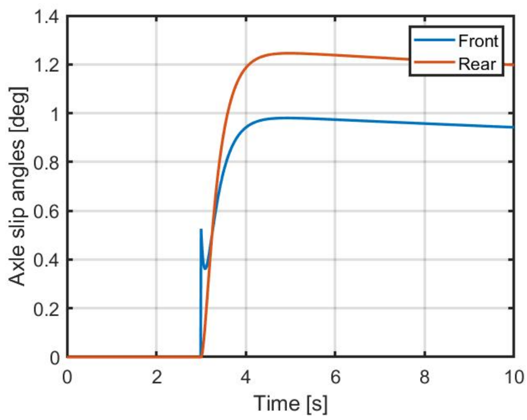


(a) Before Load transfer Addition

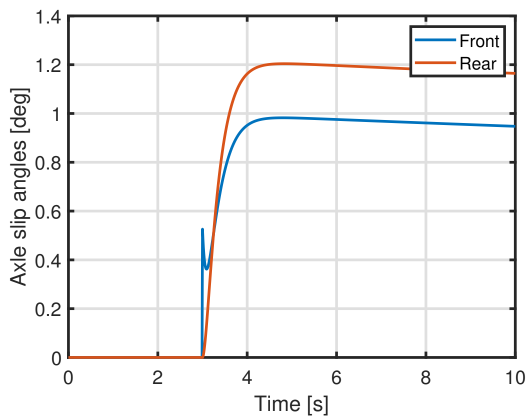


(b) After Load transfer Addition

Fig. 3.2.4: Side Slip Angle (Load case 1)

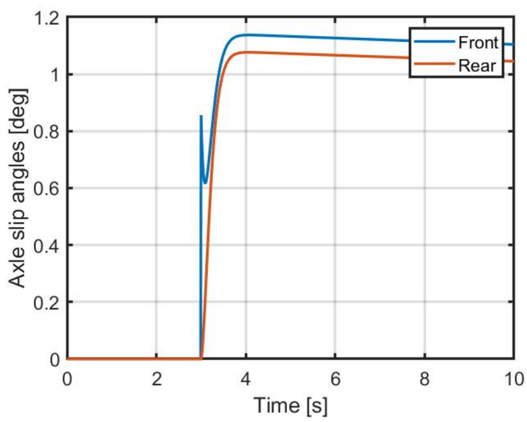


(a) Before Load transfer Addition

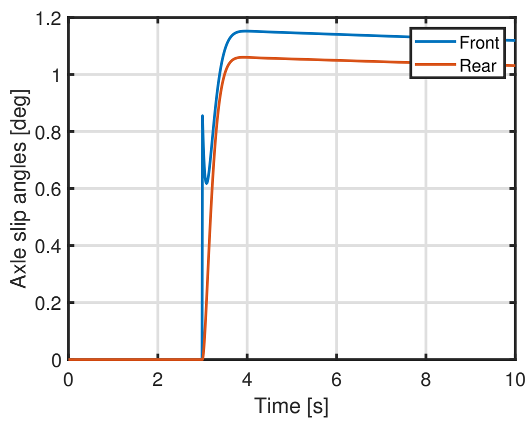


(b) After Load transfer Addition

Fig. 3.2.5: Side Slip Angle (Load case 2)



(a) Before Load transfer Addition



(b) After Load transfer Addition

Fig. 3.2.6: Side Slip Angle (Load case 3)

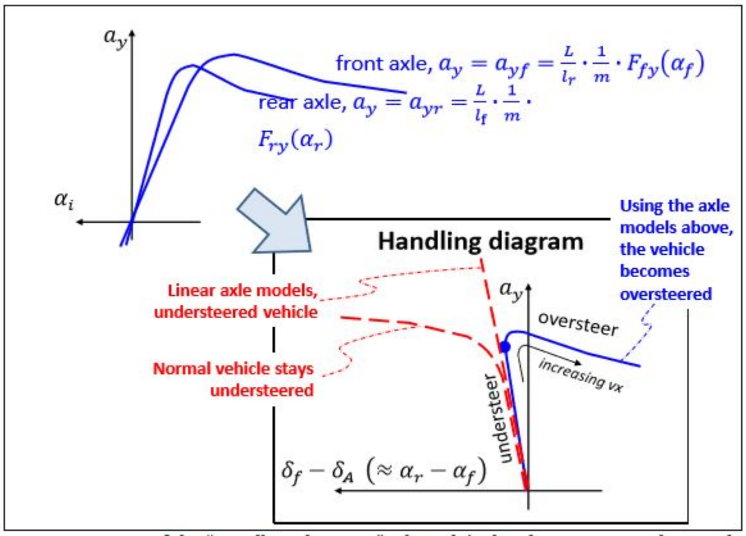


Fig. 3.2.7: Handling Diagram Construction (Side Slip Angle - Lateral Acceleration) [1]

Therefore, according to Fig. 3.2.7, which shows that handling characteristic depends on lateral acceleration response and side slip angle response, we can conclude that the handling characteristic of the linear 2-DOF single track model used in task 2 will roughly output the same handling characteristic as the one which includes the load transfer if it is used in its linear range.

3.3 Propose roll stiffness distribution

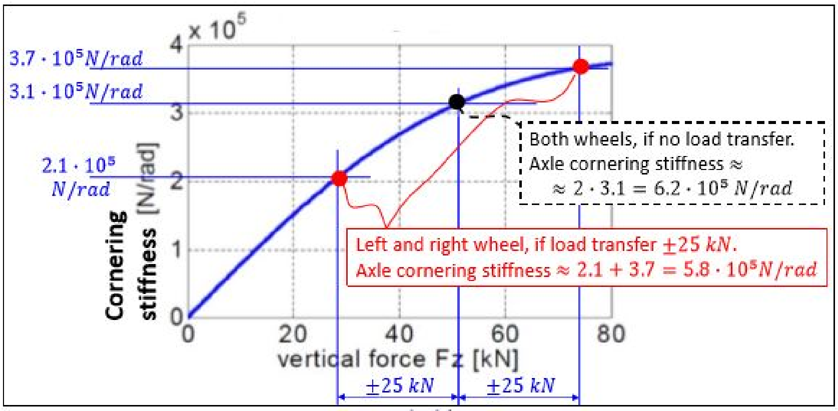
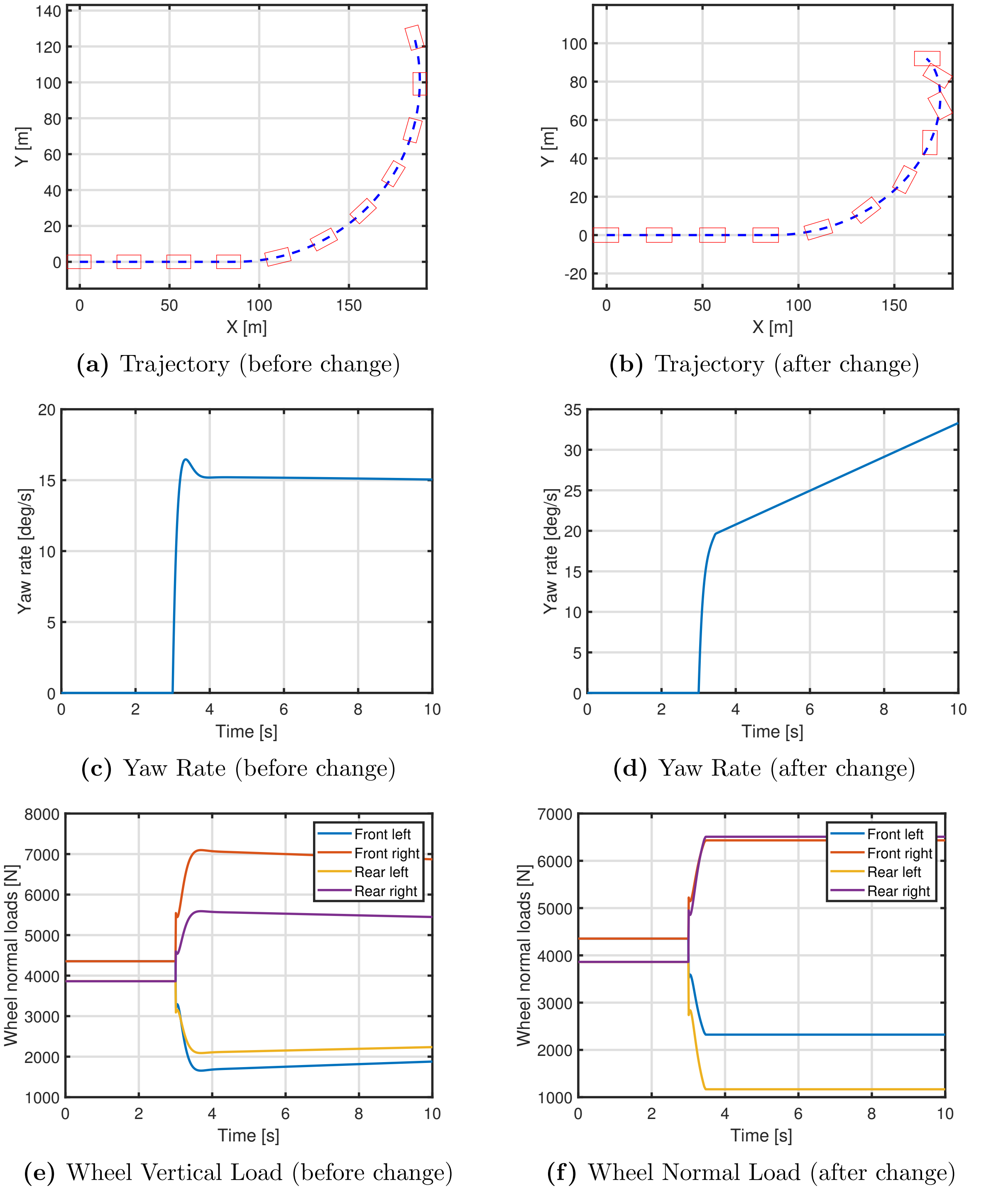


Fig. 3.3.1: Relationship between The axle cornering stiffness with increased load transfer. [1]

In Task 3.3, the roll stiffness distribution is changed from the initial condition of 0.65 into 0.45. We also set the parameter of steering wheel angle of 30 degrees. Our task is to see the effect of changing roll stiffness distribution to the steering response of the vehicle.

As a theoretical background, in the event of cornering, the vertical load is shifted towards the outer wheels. Depending on the roll stiffness of each axle, the axles take differently much of this lateral load transfer. This also influences the yaw balance. The more roll stiff an axle is, the more of the lateral load shift it takes [1].



(a) Trajectory (before change)

(b) Trajectory (after change)

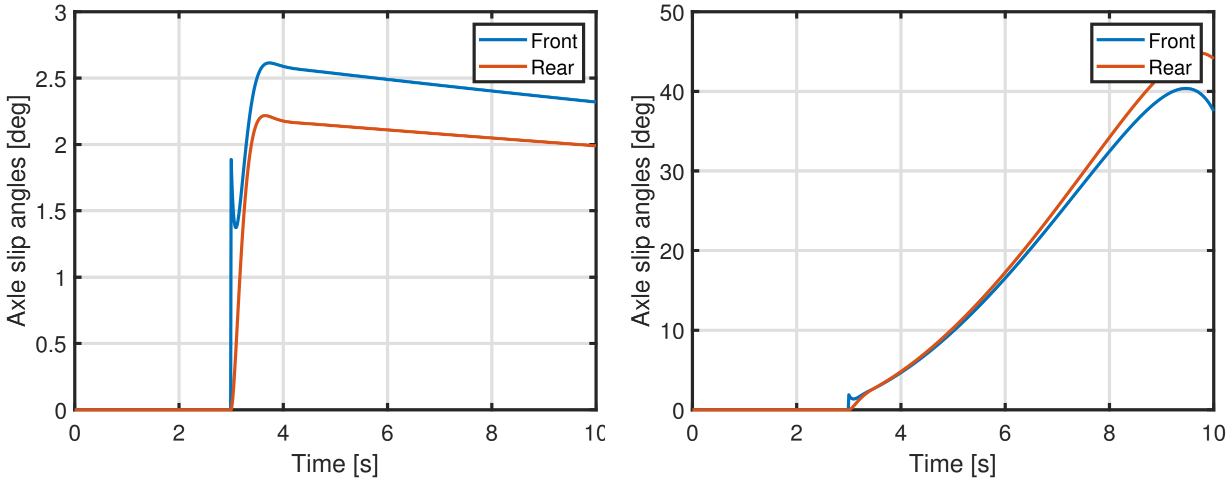
(c) Yaw Rate (before change)

(d) Yaw Rate (after change)

(e) Wheel Vertical Load (before change)

(f) Wheel Normal Load (after change)

Fig. 3.3.2: Trajectory, Yaw Balance and Wheel Normal Load (Fz) with changed roll stiffness distribution



(a) Side Slip Angle (before change) **(b)** Side Slip Angle (after change)

Fig. 3.3.3: Side Slip Angle with changed roll stiffness distribution

In Fig. 3.3.2, the path trajectory before changing the roll stiffness distribution shows a normal cornering condition. Increasing the roll stiffness on rear axle has an implication: it increases the front cornering stiffness as well as it decreases the rear cornering stiffness. As a result, the front axle will tend to become more and more oversteered and it results in yaw rate that keeps increasing linearly as the outer rear wheel normal load is increasing with the same rate with the outer front wheel normal load until it reaches a higher steady state load than the outer front wheel normal load. The side slip in both axles mainly drives the vehicle to drift out the normal cornering trajectory until it spins out as the yaw rate reaches very high level.

Normally, the outer front wheel normal load should be higher than that of the outer rear wheel normal load as it could be seen by Fig.3.3.2 (e). In addition, the inner rear wheel normal load and outer rear wheel normal load experiencing a step decrease to about 1000 N and 2300 N respectively. Comparatively, before changing the roll stiffness distribution, the value for both respective wheel normal load was about 1800 N and 2000 N.

The most stark difference could be observed in the side slip angle values. With the changed roll stiffness distribution, the side slip angle peak value is very high compare to the initial roll stiffness distribution (> 40 degrees vs $(2 - 2.5)$ degrees). Moreover, the side slip angle for front and rear will increase in the same rate until the side slip angle for the rear axle have a higher value. This will happen as the side slip force of the rear axle dominates the front axle and the car will be drifted out the normal cornering trajectory. The illustration of this side slip force and how it works in the tyre is shown in Fig. 3.3.4.

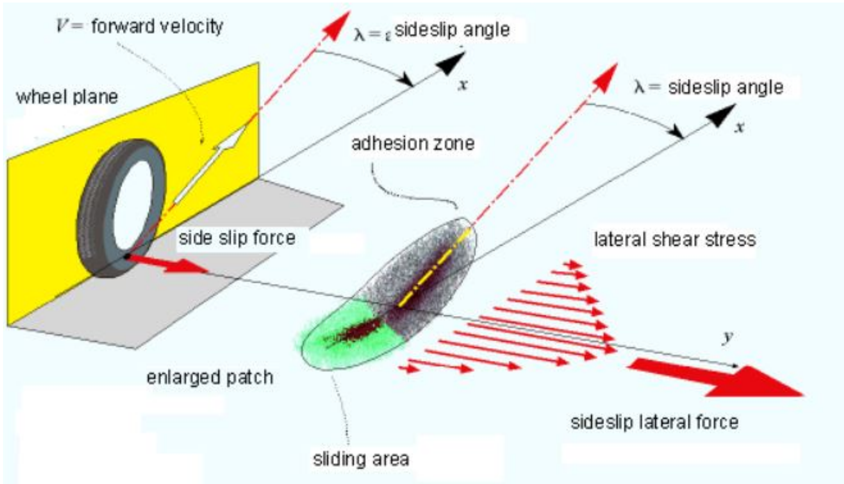


Fig. 3.3.4: Side Slip Force and Angle Illustration Diagram [2]

3.4 Roll stiffness distribution proposition

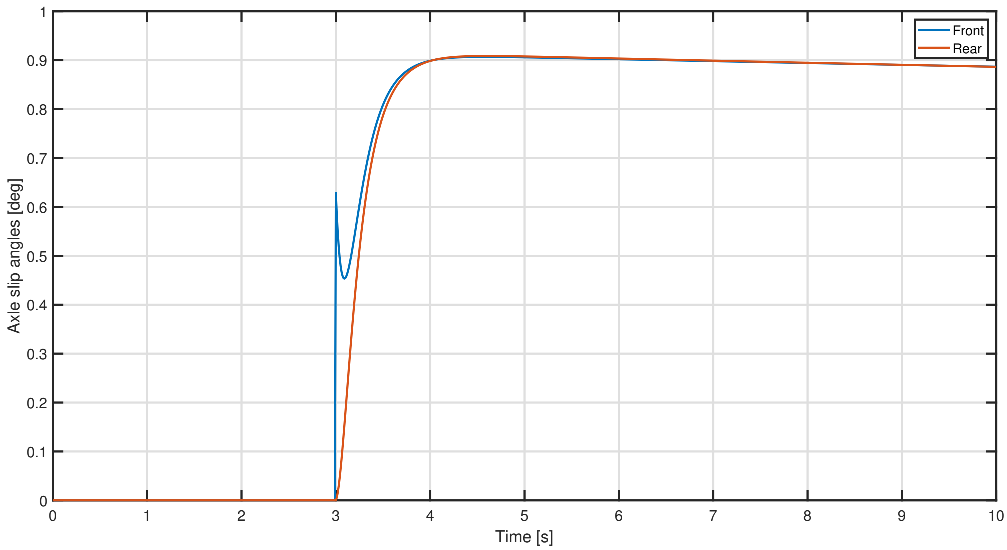


Fig. 3.4.1: The Neutral Steered Slip Angle (tuned roll stiffness)

The tuned roll stiffness distribution for steering wheel angle of 10 degrees and initial longitudinal speed of 100 km/h that is proposed here is 0.22. By using this value, we could obtain almost perfect neutral-steer as the $|\alpha_f| = |\alpha_r|$.

However, this value of roll stiffness distribution is not suitable to be used in a production car even if it has the same load case because we need to also consider a lot of different steering angle situation (e.g. Steady state cornering at low speed, cambering, driving on a straight path etc.) as well as different initial longitudinal speeds and other initial parameters and situations.

In addition, this model which runs in Simulink needs to be verified by means of comprehensive DIL/Driving Simulator, HIL or even Real vehicle test despite being validated in the CASTER driving simulator before we can consider applying it to a production vehicle.

Moreover, the tuned roll stiffness should also depends car configuration (e.g. 4WD/FWD/RWD). Since the model used in this simulation is a one-track model, other factors should be taken into account in the real life. These factors show that we can't use this tuned roll stiffness distribution in a production car :

1. Exclusion of Combined Tyre Slip effect on cornering stiffness,
2. Exclusion of Braked Force Distribution effect
3. It does not simulate the added yaw moment effect due to left/right-asymmetric wheel torque, such as ESC interventions.
4. It does not simulate well the roll motion effect in comparison with the two track models.
5. It does not take into account the deviations from Ackerman geometry within an axle as it does in two-track models.
6. It does not simulate the influence from vertically uneven road.
7. Control functions (e.g. ABS,TC and ESC) are not included in the model.
8. Exclusion of Aerodynamic Drag and Rolling Resistance effect.

Normally, the vehicle dynamic engineers tend to design the front axle roll stiffness to be higher than the rear axle. This is done in order to keep an understeered configuration.

4 Combined tyre slip

4.1 Add combined slip to model

The corrected axle lateral stiffness is shown by the equations below :

$$C_f, corrected = \left(\sqrt{\frac{(\mu.F_z)^2 - (F_x)^2)}{\mu.F_z}}\right).C_f \tag{45}$$

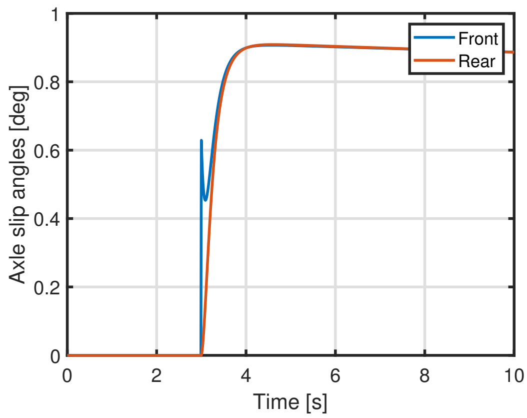
$$C_r, corrected = \left(\sqrt{\frac{(\mu.F_z)^2 - (F_x)^2)}{\mu.F_z}}\right).C_r \tag{46}$$

The performance of the tyre under combined tyre slip is important[5].The neutral steered condition is dependent on the condition of the front slip angle and rear slip angle whether it is roughly equal or not ($|\alpha_f| = |\alpha_r|$). And the lateral side slip itself depends directly on the value of the axle lateral stiffness as shown by Constitution equation (47) and (48) below :

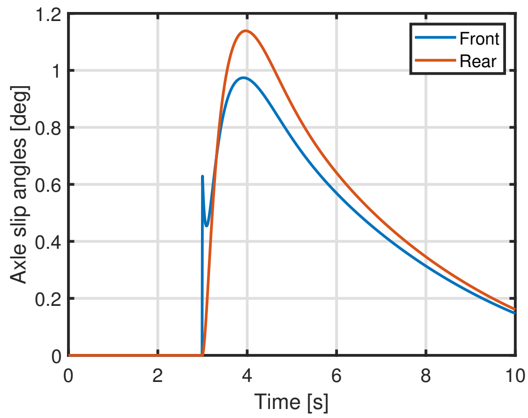
$$F_{fyw} = -C_{fy}.s_{fy}; F_{ry} = -C_{ry}.s_{ry} \tag{47}$$

$$F_{fyw} = -C_{\alpha}.s_{\alpha_f}; F_{ry} = -C_{\alpha}.s_{\alpha_r} \tag{48}$$

By equation (48), changing the axle lateral stiffness will results in changed α_r and α_f as well as the lateral tyre force (F_{fyw}) and (F_{ry}). Then, by correcting the axle lateral stiffness to include the combined tyre slip effect, the effect will be seen in Fig.4.1.1 as the initial response shows similar curve between (a) and (b). However, as it approaches 4 seconds of simulation, instead of levelling out, the slip angle keeps increasing until it peaks out around 0.95 and 1.17 degrees for front and rear slip angle respectively. After the value peaks out, it decreases "exponentially" until it reaches roughly 0.2 degrees for both slip angles. In conclusion, the vehicle handling characteristic will change from neutral steered ($|\alpha_f|$ =$|\alpha_r|$, uncorrected stiffness) to oversteered ($|\alpha_f| < |\alpha_r|$, with corrected stiffness). Moreover, by adding a brake demand of 4000 N and applying alateral force, there will be a step increase in the rear longitudinal axle force demand in the negative direction and as we apply the braking forces when cornering at a constant steer angle, the steering forces (F_y) will decrease "exponentially" as seen in Fig. 4.1.2 (b) because of the tire slip limit in the friction circle (Appendix A.0.1) has been reached.

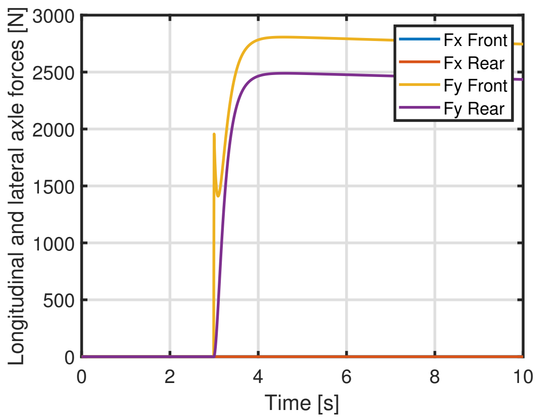


(a) Before Correction

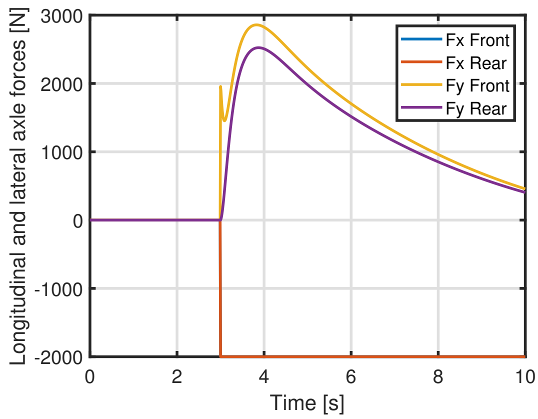


(b) After Correction

Fig. 4.1.1: Effect of the Corrected Axle Lateral Stiffness on Side Slip Angle



(a) Before Correction



(b) After Correction

Fig. 4.1.2: Effect of the Corrected Axle Lateral Stiffness on Lateral Axle Force)

4.2 Influence of brake force distribution

In Task 4,2, we are tasked to modify the default brake distribution of 0.5 into 0.2 and compare the ouput responses between the two. The responses are shown in Fig. 4.2.1 and Fig. 4.2.2 in the next two pages.

By comparing both the responses, we could summed up the comparison below :

- Initially, the car motion follows the same trajectory for the first three seconds, after three seconds the yaw rate in the initial condition spikes to about 8 (deg/s) where on the other hand in the modified condition it in crease to gradually to about 20 (deg/s) before having a plateau and increasing steeply to about 120 (deg/s). This very high value of yaw rate definitely cause the vehicle to spin out compared to when brake distribution ratio isn't increased. After that, the yaw rate is decreasing step by step and reaches a value close to 0 at the 8 seconds mark. This marks the time when the vehicle is completely spun out which is a condition where the front axle is in the place of rear axle and the car finishes the rest 2 seconds trip in a backward motion.

- The side slip response also differs sharply between the unchanged and changed brake distribution force value. In the fig.4.2.1 (a), it can be observed that the rear and front axle slip angles peaked at about (1.15 deg) and 0.8 degrees respectively before decreasing "exponentially" to about 0.2 degrees at around 10 seconds. On the other hand, in the changed brake distribution side slip angle, the side slip angle increases moderately between 4-6 seconds before having a spike and peaks out at around 90 degrees after 7 seconds mark. After that, the value decreases step by step until a value close to 0 for the front side slip angle and around -5 degrees for the rear side slip angle after 8 seconds before levelling out in the (8-10 seconds) .

- The other output responses that we could analyze are the Longitudinal,Lateral Axle Force and the Wheel Normal Loads. These responses are shown in Fig. 4.2.2. In a normal driving, the response should look like the one in (a) and (c) without some unexpected step increase/decrease in wheel normal loads and axle force.

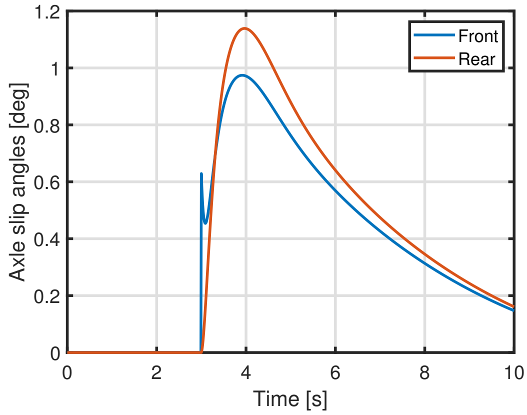
- As stated in the compendium, An ideal brake distribution would be if each axle always as the same fraction of available friction force. Moreover, A vehicle with locked rear wheels tends to be unstable. It makes the vehicle turns around and ends up sliding with the rear first. Hence, it is desirable to have the front wheels locked instead of the rear wheels locked. With most of the brake distribution in the rear axle, as just before the vehicle starts cornering at 3 seconds, most of the brake force is applied in the rear axle and the tire longitudinal friction force limit is exceeded and any additional side slip demand would not be distributed. As a result, the demand is compensated by the lateral force. The tyre lateral friction force limit related to this lateral force is exceeded too and the front tires just turn around which leads the vehicle to completely turn around and will be constant on that point for the rest of 4 seconds since the rear tire will drive the vehicle with maximum propulsion force.

- In addition, by analyzing the wheel normal load curve, the highest wheel normal load is focused on the outward rear wheel instead of outer front wheel in the normal conditions. This condition has caused oversteer and cause rear wheel lock where the tyre could not supply the brake demand anymore. Moreover, if we want to analyze the stability we should look at the equation (b) below that the change in yaw rate/yaw rotation in the curve shall be more than 0. However, that is not the case, so the vehicle loses stability and doesn't keep it's trajectory.

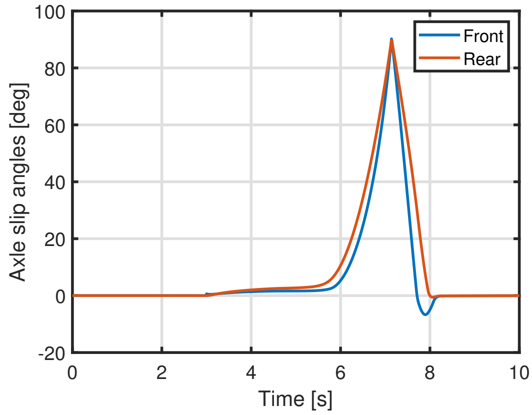
$$\frac{F_{fx}}{\mu_f \cdot F_{fz}} = \frac{F_{rx}}{\mu_r \cdot F_{rz}} \Rightarrow \begin{Bmatrix} Assume \\ \mu_f = \mu_r; \end{Bmatrix} \Rightarrow \frac{F_{fx}}{F_{rx}} = \frac{F_{fz}}{F_{rz}} \qquad \Rightarrow \frac{d\dot{\omega}_z}{d\omega_z} = \frac{m \cdot v_x \cdot l_f}{J} > 0; \Rightarrow Unstable)$$

(a) **(b)**

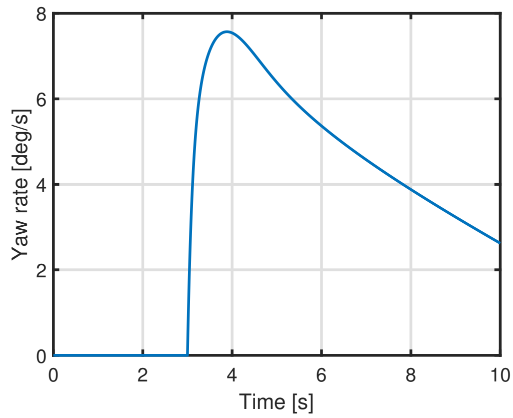
Fig. 4.2.1: Ideal condition of Brake Force Distribution Equation and Unstable Equation in Rear Wheel Lock condition



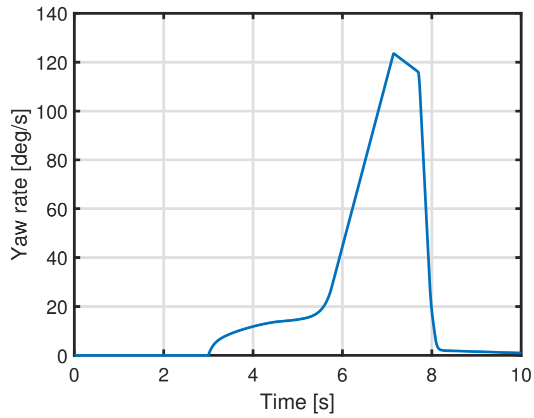
(a) Before Change



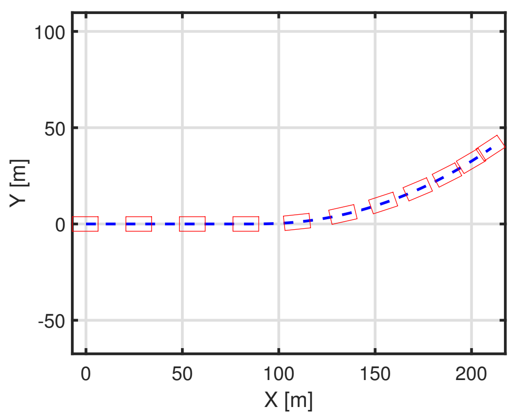
(b) After Change



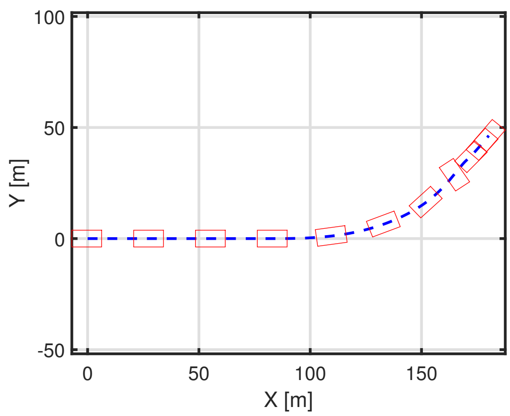
(c) Yaw rate (Before change)



(d) Yaw rate (After change)

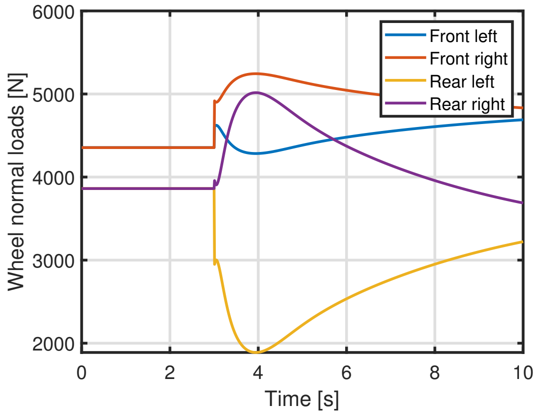


(e) Cornering trajectory (Before change)

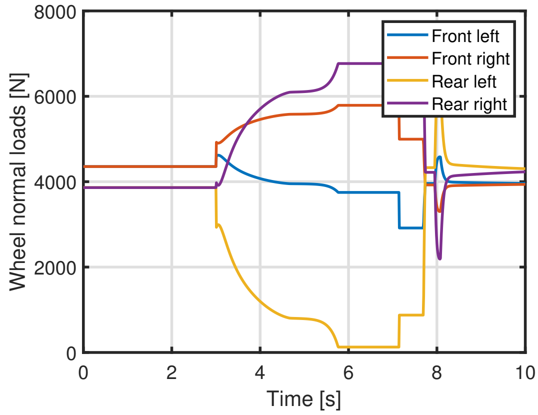


(f) Cornering trajectory (After change)

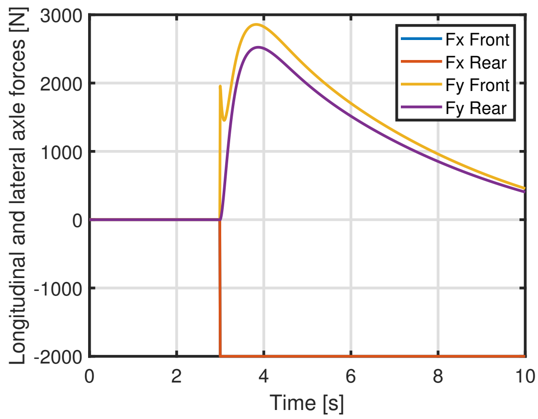
Fig. 4.2.2: Effect of the changing Brake Force Distribution Cornering Trajectory, Yaw Rate and Side Slip Angle



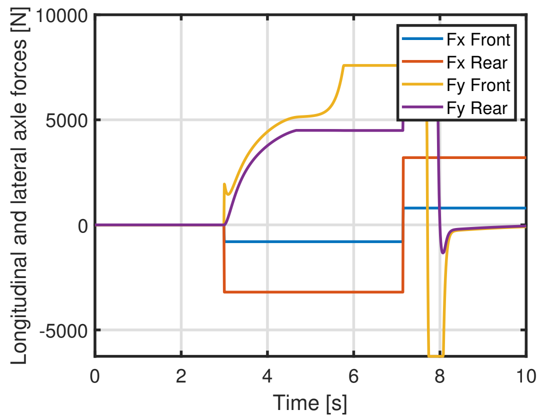
(a) Wheel Normal Loads (Before change)



(b) Wheel Normal Loads (After change)



(c) Axle Forces (Before change)



(d) Axle Forces (After change)

Fig. 4.2.3: Effect of the changing Brake Force Distribution on Wheel Normal Loads as well as Longitudinal and Lateral Axle Forces

4.3 Propose brake force distribution

In this task, we are tasked to find a brake distribution ratio which corresponds to neutral-steering behaviour with the added brake demand of 4000 N and using the proposed value of roll stiffness distribution in Task 3.4. We propose the brake distribution ratio of 0.765 (76.5% front axle and 23.5% rear axle). This value results in roughly equal front slip angle and rear slip angle, thus neutral-steered for brake demand of 4000 N.

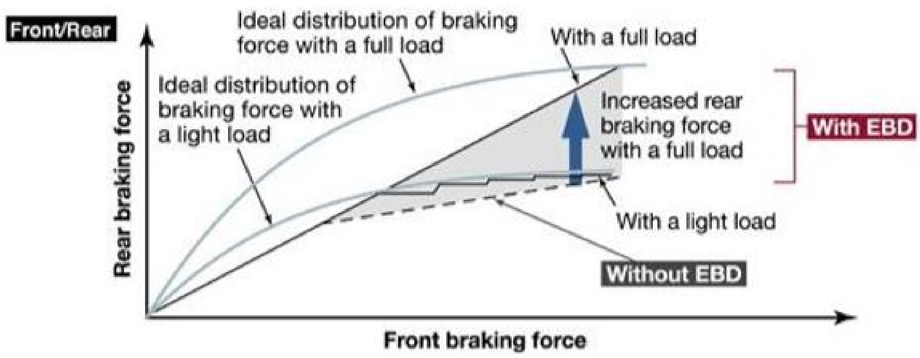
However, as shown by Fig. 4.3.1, when we shift the brake demand in +/- 2000 N steps from 4000 N, we observe a mild change for brake demand of 2000 N and 4000 N and quite considerable change with the brake demand of 8000 N where the vehicle is well understeered. Thus, One possible solution so this is that the brake distribution ratio should variably change as the brake demand is changing and it should be controllable. One way to do this is by introducing Electronic Brake Distribution (EBD) System to the car.

As it is written in compendium, brake distribution design (Proportioning) is highly correlated with the front/rear wheel lock prevention. The wheel lock itself if happens in either front/rear wheel has different implication as shown below [1]:

1. A vehicle with locked front wheels tends to be stable. However, steering ability is lost, so vehicle continues straight, incapable of curving its path. In most driving situations, it is preferred that the front wheels lock first

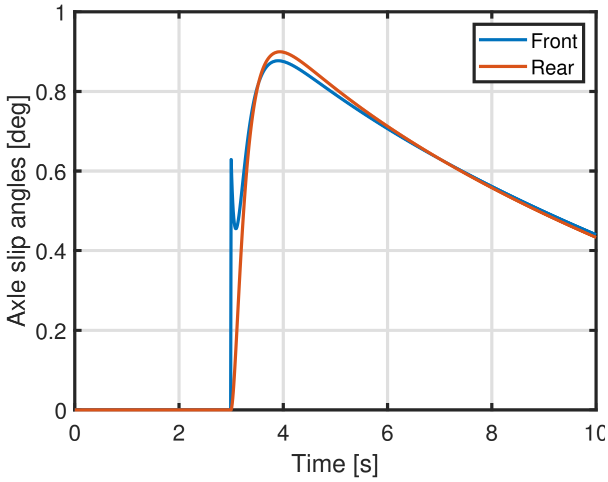
2. A vehicle with locked rear wheels tends to be unstable. It turns around and ends up sliding with the rear wheel in the position of front wheel.

In order to understand the effect of change in brake demand one should take a look at the Fig.4.3.2. To avoid rear axle lock-up, one restricts the brake pressure to the rear axle. By using EBD system, the braking force is more distributed and the proportion between rear braking force and front braking force curve is becoming closer to the ideal distribution of braking force with a light load and full load. Thus, preventing front/rear wheel lock. Moreover, the use of EBD system is usually in parallel with the usage of Anti-Lock Braking System (ABS) as both of them is now a requirement in modern vehicle design.

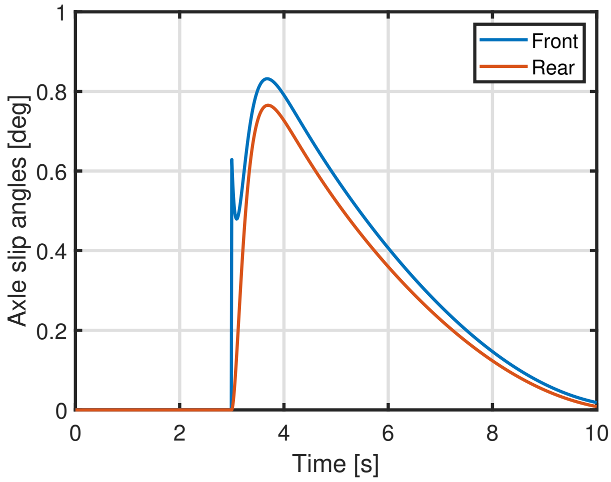


2.JPG

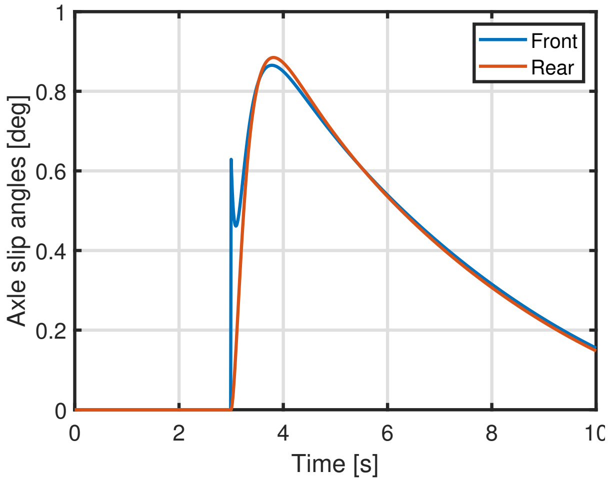
Fig. 4.3.1: Electronic Brake Distribution Working Principle [3]



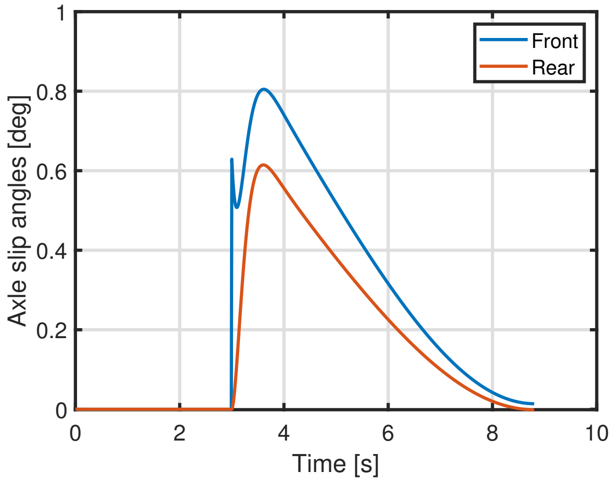
(a) Brake Demand = 2000 N



(b) Brake Demand = 6000 N



(c) Brake Demand = 4000 N



(d) Brake Demand = 8000 N

Fig. 4.3.2: Slip Angles for Proposed Brake Distribution (=0.765)

5 Driving experience in sumlator

5.1 Participate in model validation session

From the first Caster session, we've obtained the following results. Each list contains the data from case 1, 2 and then 3.

$\mathrm{a}_{x,max} = [0 \quad 0 \quad 0] \quad in \quad \mathrm{m/s^2}$

$\mathrm{a}_{x,min} = [-0.0118 \quad -1.1941 \quad -0.5981] \quad in \quad \mathrm{m/s^2}$

$\mathrm{a}_{y,max} = [1.0790 \quad 0 \quad 0.3266] \quad in \quad \mathrm{m/s^2}$

$\mathrm{a}_{y,min} = [0 \quad 0 \quad 0] \quad in \quad \mathrm{m/s^2}$

$\omega_{z,max} = [1.9102 \quad 0 \quad 0.595] \quad in \quad \mathrm{rad/s}$

$\omega_{z,min} = [0 \quad 0 \quad 0] \quad in \quad \mathrm{rad/s}$

$\omega_{\dot{z},max} = [37.6875 \quad 0 \quad 11.4096] \quad in \quad \mathrm{rad/s^2}$

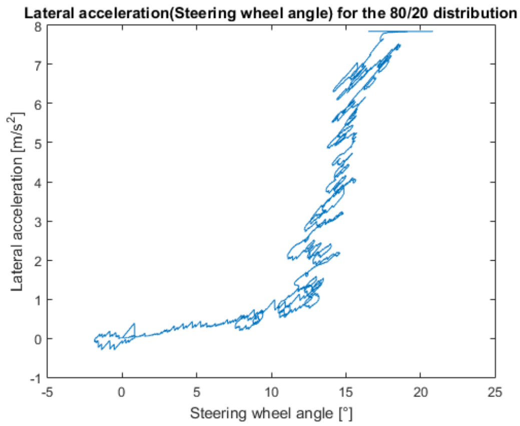
$\omega_{\dot{z},min} = [0 \quad 0 \quad 0] \quad in \quad \mathrm{rad/s^2}$

5.2 Participate in simulator drive session

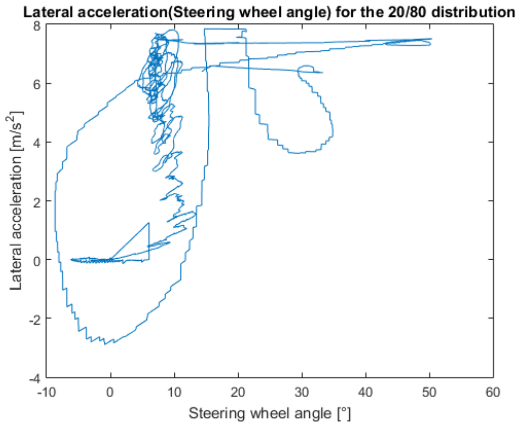
We've taken part in the Caster driving session.

5.3 Documentation in assignment report

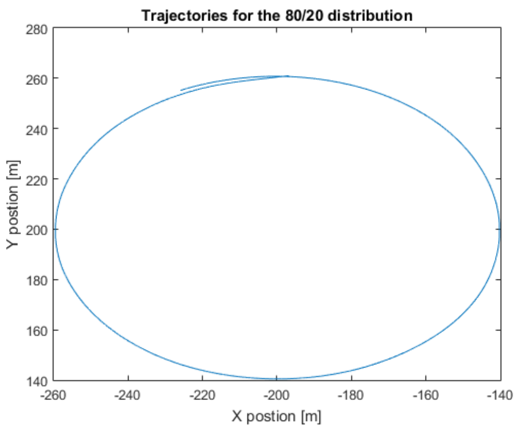
By comparing our model to the caster one we can quote a few differences. The caster vehicle model takes more parameters into account. This model uses vertical dynamics parameters so we feel the surface of the road while driving. It also takes into account the drag and more generally the aerodynamic forces. The roll resistance too. As a result, in the caster model the speed of the car reduces if we release the throttle whereas in our model the speed of the car doesn't decrease which is not realistic. Finally, in our model there's only one gear whereas there are 6 gears in the caster model and we have to select the good one in regard of the regime of the engine.



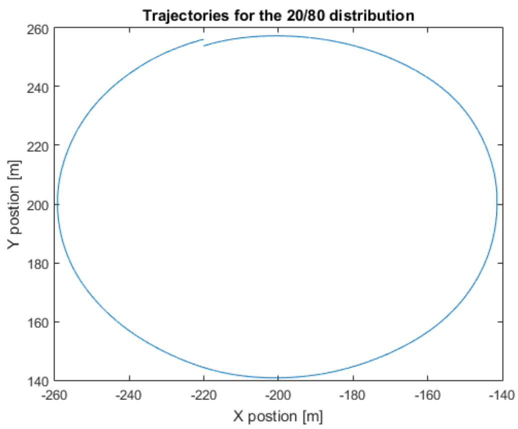
(a) 80/20



(b) 20/80



(c) 80/20



(d) 20/80

For the steady state constant radius state. We tried to follow a line with a smooth acceleration until 80 kph. From picture (a) we can clearly see that we're in the Under-steered configuration because the Steering wheel angle is much higher. On figure (b) there's still some noise but we can notice a part of the curve that is similar to the one on figure (a). This configuration is the oversteered one because we see that the steering wheel angle is smaller. Moreover, the steering wheel angle reduces when the lateral acceleration increases.
The trajectories curves (c) and (d) show that we managed to keep a fine trajectory. However, we've selected the best lap.

Conclusion

his assignment has enabled us to study some fundamental parameters used in analyzing the vehicle cornering performance. We've seen the influence of the load distribution on the vehicle's handling. We were able to create a model for the vehicle and to refine it step by step by adding several parameters. We've also compared the different models to determine the most suitable ones. This assignment was the occasion to learn how to use Simulink and also to discover the Caster Simulator. The driving session was really interesting insofar as it has enabled us to try our models by driving a virtual car. We've feeled the difference between an understeered car and an oversteered car. We've also understood that it's more difficult that it may seem to drive a sport car when we've tried the Porsche Cayman!

References

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`http://www.dynamotion.it/eng/dinamoto/8on-linepapers/Pneumatici/Pneumatii-eng.htm`

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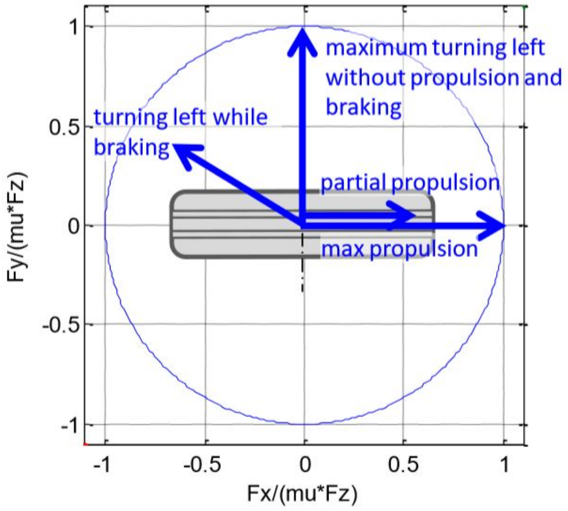
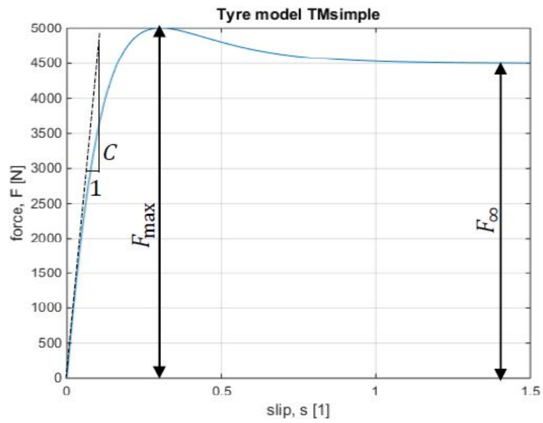
A Tyre Friction Circle and TMsimple Curve



(a) Tyre Friction Circle Diagram and Longitudinal/Lateral Force change in different driving condition



(b) Force-Slip Characteristic Curve using Magic Formula

Fig. A.0.1: Tyre Friction Circle and Force-Slip Characteristic