

# Vehicle Dynamics

## Assignment 3 –Vertical Dynamics



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Group 25:

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## **INTRODUCTION**

In Automotive Engineering, Vehicle can be defined as a system within which many components or subsystem interact. The interaction between the whole vehicle and the environment which resulted in the motions could be classified as Vehicle Dynamics. Moreover, in the Vehicle Dynamics study, it is common to divide the vehicle motion based on its direction which include Longitudinal dynamics, Lateral Dynamics and Vertical Dynamics

In Vertical Vehicle Dynamics aspect, it mainly deals with two major types of disturbance, Road surface irregularities and vehicle created disturbances. These two type of disturbance is mainly affecting the vehicle vertical dynamic structure which is related to 3 different output response from the stationary oscillating and transient disturbance input (Vibrations of human, Stress in vehicle structure and Compression of tyre). Moreover, these three different measures are related to Human perception of vibrations (Ride Comfort function), Material Fatigue (Fatigue Life function) and Contact between tyres and road (Road Grip). However, in this assignment, we are focusing on the excitation from the road surface in particular the response which is affecting the ride comfort function. The passenger car in this assignment is modelled with 2-dof quarter car model.

### **Abstract**

This report was made to fulfill the requirements of the Vehicle Dynamics Course. The investigation was done by Fikri Farhan Witjaksono and Marcos Mora Rueda in order to understand the difference between natural frequency obtained through observation of the 2-DOF model compared to through mathematical equation and to propose an optimal wheel suspension stiffness and a damping for a passenger vehicle under road surface excitation. Another purpose is to propose a strategy which will make the initial unacceptable driver vibration exposure to become acceptable by the means of vehicle velocity tuning.

The first main finding was that the observed natural frequencies for front/rear wheel hop and bounce mode corresponds to different values if we compare with the mathematical equation. The second finding was generally the higher the spring stiffness, the higher the value of the rms for both acceleration and tyre force. Moreover, the objective of tyre force requires a higher damping than the acceleration in order to be optimal, because we need the tyre to be in contact with the road all the time so we need higher damping to avoid the vibrations of the wheel. The recommendations for third task was that we should use the vehicle velocity of 103 km/h, 50 km/h and 8.5 km/h while driving in the smooth, rough and very rough condition respectively. This combination will give the best ride comfort for the whole 80 km trip.

## TASK 1: Quarter Car Model Transfer Functions in Matrix Form

### TASK 1.1: Model and derive equations

Since it is 2 dynamics, we have to consider both elasticity between road and wheel (unsprung mass) and also between the wheel and the sprung mass. Also, since it is a quarter model we have to consider different masses for front and rear:

$$\text{Unsprung mass front} = \text{total unsprung mass}/4$$

$$\text{Unsprung mass rear} = \text{total unsprung mass}/4$$

$$\text{Sprung mass rear} = 0.4 \cdot \text{total sprung mass}/2$$

$$\text{Sprung mass front} = 0.6 \cdot \text{total sprung mass}/2$$

This is the free body diagram for the 2-DOF quarter car model:

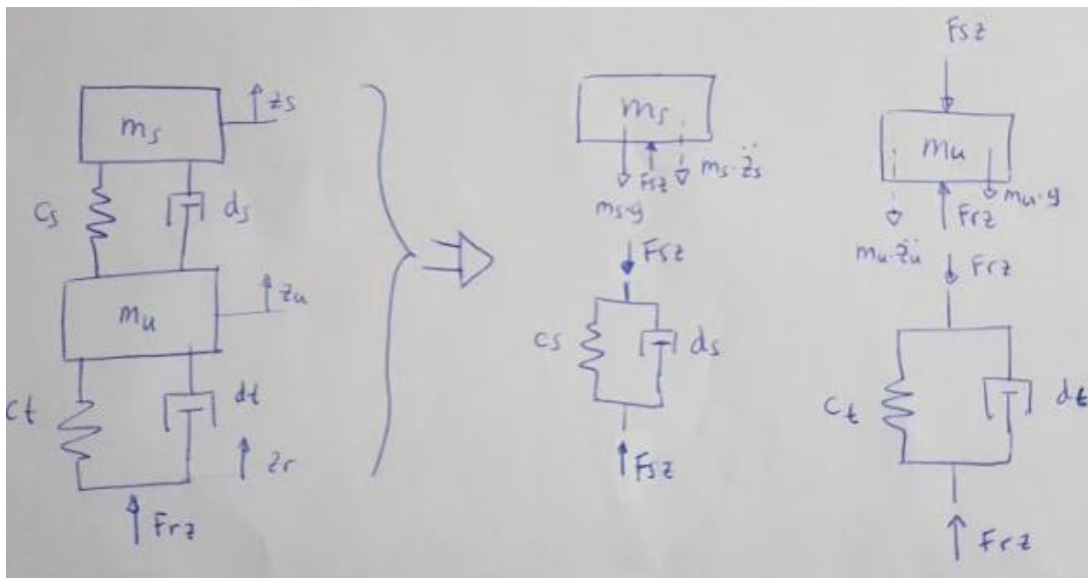


Figure 1: Free body diagram

From the FBD we obtain the following equations:

Equilibrium:

$$m_s \cdot \ddot{Z}_s + m_s \cdot g = F_{sz} \quad (1)$$

$$m_u \cdot \ddot{Z}_u + F_{sz} + m_u \cdot g = F_{rz} \quad (2)$$

Constitution:

$$F_{sz} = c_s(z_u - z_s) + d_s \cdot (\dot{z}_u - \dot{z}_s) + m_s \cdot g \quad (3)$$

$$F_{rz} = c_t \cdot (z_r - z_u) + d_t \cdot (\dot{z}_r - \dot{z}_u) + (m_s + m_u) \cdot g \quad (4)$$

Excitation:

$$z_r = z_r(t) \quad (5)$$

If we introduce equation (3) in (1) and (2), and also we introduce (4) in (2) we obtain the following two equations:

$$m_s \cdot \ddot{z}_s + m_s \cdot g = c_s(z_u - z_s) + d_s \cdot (\dot{z}_u - \dot{z}_s) + m_s \cdot g \quad (6)$$

$$\begin{aligned} m_u \cdot \ddot{z}_u + c_s(z_u - z_s) + d_s \cdot (\dot{z}_u - \dot{z}_s) + m_s \cdot g + m_u \cdot g = \\ = c_t \cdot (z_r - z_u) + d_t \cdot (\dot{z}_r - \dot{z}_u) + (m_s + m_u) \cdot g \quad (7) \end{aligned}$$

Where :

$m_u$  = *Unsprung mass of the quarter model*

$m_s$  = *Sprung mass of the quarter model*

If we write it in the following matrix form:

$$\dot{x} = A \cdot x + B \cdot u \quad (8)$$

We have:

$$\dot{x} = \begin{bmatrix} \dot{z}_s \\ \dot{z}_u \\ \ddot{z}_s \\ \ddot{z}_u \end{bmatrix} \quad x = \begin{bmatrix} z_s \\ z_u \\ \dot{z}_s \\ \dot{z}_u \end{bmatrix} \quad u = \begin{bmatrix} z_r \\ \dot{z}_r \end{bmatrix}$$

$$A = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ \frac{-c_s}{m_s} & \frac{c_s}{m_s} & \frac{-d_s}{m_s} & \frac{d_s}{m_s} \\ \frac{c_s}{m_u} & \frac{-c_t - c_s}{m_u} & \frac{d_s}{m_u} & \frac{-d_s - d_t}{m_u} \end{bmatrix} \quad B = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ \frac{c_t}{m_u} & \frac{d_t}{m_u} \end{bmatrix}$$

### **TASK 1.2: Determine transfer function matrix $H(\omega)$**

$$\dot{x} = A \cdot x + B \cdot u \quad (9)$$

$$y = C \cdot x + D \cdot u \quad (10)$$

From these two equations we will derive using Fourier transform, in order to obtain the transfer function:

$$j\omega F(x) = A \cdot F(x) + B \cdot F(u) \quad (11)$$

$$F(y) = C \cdot F(x) + D \cdot F(u) \quad (12)$$

From equation 11 we obtain:

$$F(x) = (j\omega \cdot I_n - A)^{-1} \cdot B \cdot F(u) \quad (13)$$

Then if we introduce (13) in (12) we obtain:

$$F(y) = C \cdot (j\omega \cdot I_n - A)^{-1} \cdot B \cdot F(u) + D \cdot F(u)$$

And finally:

$$\frac{F(y)}{F(u)} = C \cdot (j\omega \cdot I_n - A)^{-1} \cdot B + D \quad (14)$$

### **TASK 1.3: Plot transfer functions $H(\omega)$**

For this task we will be working with one front wheel and later with one rear wheel so, the index f refers to front and the index r refers to rear. We will write the matrices required for both, the front and the rear during this task.

1. In the first case we want road displacement ( $Z_r$ ) as an input and ride comfort (acceleration of sprung mass,  $\ddot{Z}_s$ ) as an output.

Since we are not considering  $\dot{Z}_r$  the matrix B will be for all the three cases below:

$$B_f = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \frac{c_t}{m_{uf}} \end{bmatrix} \quad B_r = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \frac{c_t}{m_{ur}} \end{bmatrix}$$



Since we want  $\ddot{Z}_s$  as an output the matrices C and D can be obtained from equation (8)

$$C_f = [-c_f/m_{sf} \quad c_f/m_{sf} \quad -d_f/m_{sf} \quad d_f/m_{sf}]$$

$$C_r = [-c_r/m_{sr} \quad c_r/m_{sr} \quad -d_r/m_{sr} \quad d_r/m_{sr}]$$

$$D_f = [0]$$

$$D_r = [0]$$

The by using  $y = C \cdot x + D \cdot u$  we can obtain the output required.

**2.**In the second case we want the road displacement ( $Z_r$ ) as an input and suspension travel  $Z_u - Z_s$  as an output. So we will use the matrix C in order to establish that output:

$$C_f = [-1 \quad 1 \quad 0 \quad 0]$$

$$C_r = [-1 \quad 1 \quad 0 \quad 0]$$

And the, we do not need the matrix D:

$$D_f = [0]$$

$$D_r = [0]$$

**3.**In the third case we want the road grip as an output using the road displacement as an input. Road grip:  $(Z_r - Z_u) \cdot C_t$

Then, the matrix D cannot be 0 in this case:

$$C_f = [0 \quad -C_t \quad 0 \quad 0]$$

$$C_r = [0 \quad -C_t \quad 0 \quad 0]$$

$$D_f = [C_t]$$

$$D_r = [C_t]$$

Finally, we can write the transfer function using eq 14. And plot it for different frequencies:

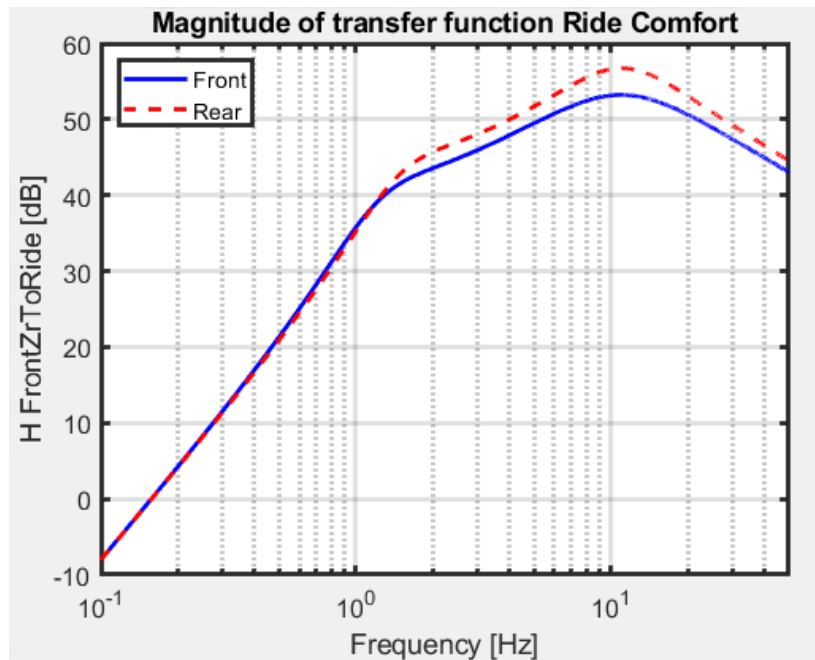


Figure 2: Magnitude of transfer function Ride comfort

We see that for frequencies like 10Hz the magnitude of the transfer function will become very high. Hence, the ride comfort will be worse.

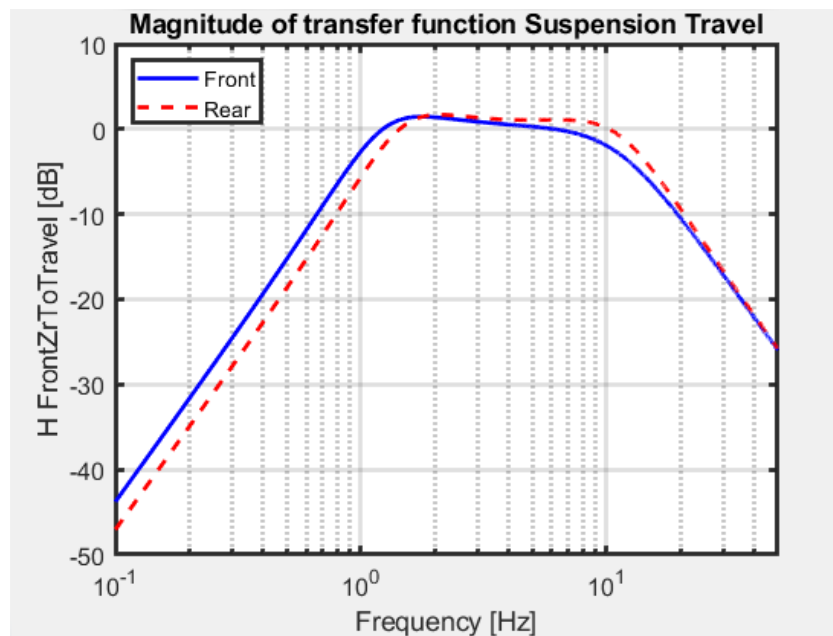


Figure 3: Magnitude of transfer function Suspension travel

We see that the magnitude of transfer function of suspension travel becomes higher for medium frequencies than for small or high frequencies.

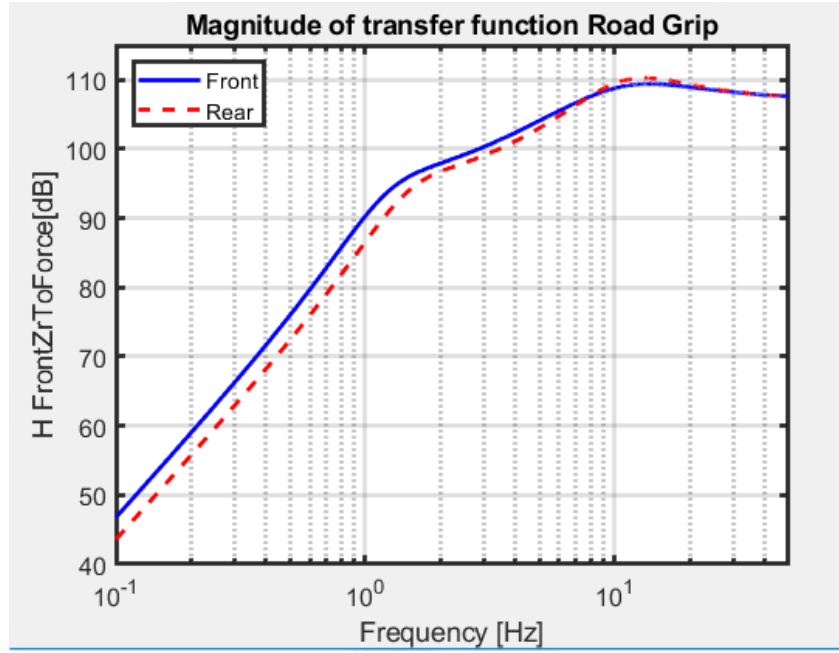


Figure 4: Magnitude of transfer function Road grip

We see that the magnitude of transfer function of road grip becomes higher and higher while increasing the frequency. This means that we have high variations of the tyre force which means poor road grip.

#### **TASK 1.4: Identify natural frequencies**

If we calculate them using the equation 5.51 from compendium:

$$\omega_{Bounce_f} = \sqrt{\frac{1 / (\frac{1}{c_f} + \frac{1}{c_t})}{m_{sf}}} = 7.3522 \frac{rad}{s} \quad (15)$$

$$\omega_{WheelHop_f} = \sqrt{\frac{c_f + c_t}{m_{uf}}} = 75.0927 \frac{rad}{s} \quad (16)$$

$$\omega_{Bounce_r} = \sqrt{\frac{1 / (\frac{1}{c_r} + \frac{1}{c_t})}{m_{sr}}} = 8.8874 \frac{rad}{s} \quad (17)$$

$$\omega_{WheelHop_r} = \sqrt{\frac{c_r + c_t}{m_{ur}}} = 74.9631 \frac{rad}{s} \quad (18)$$

Now, we will convert those frequencies into Hz and we will draw a line in the plot in order to compare the frequencies calculated with the plots (the vertical lines correspond to the frequencies calculated using the equations from compendium:



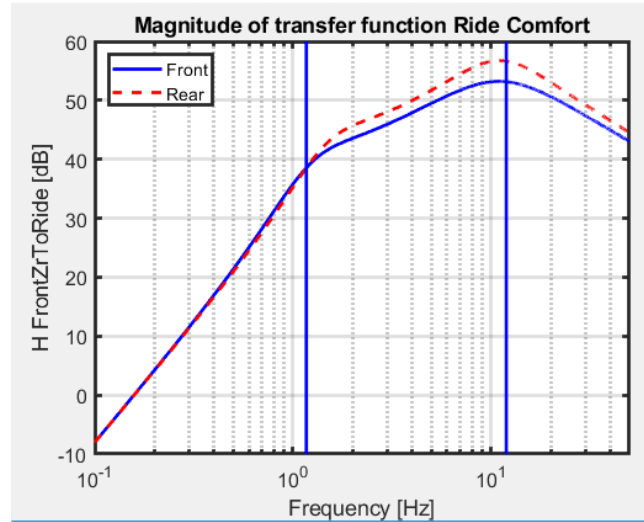


Figure 5: Natural frequencies front wheel

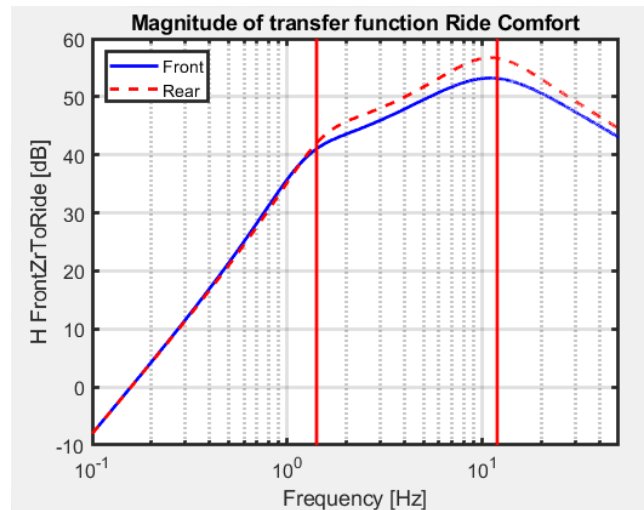


Figure 6: Natural frequencies rear wheel

If we look for the peaks in the plots we could give an approximate value for the natural frequencies in order to compare with those obtained using the equation from compendium. For the front wheel we see that the bounce frequency of the equations is a bit lower than the one of the plots, and the wheel hop frequency of the equation is higher. For the case of the rear wheel we see that both frequencies of the equations are a bit higher than the natural frequencies of the plot. So, see that there is a difference between the values obtained from the plot, and those obtained from the equations. But we need to take into account that the equations used come from an un-damped model ( $ds = 0$ ) and our model we have  $ds \neq 0$ . The model used to obtain the equations is 1-DOF and our model is 2-DOF. In order to obtain those equations, different assumptions are required. To get the equation for the natural frequency bounce we neglect the unsprung mass so that the springs are series connected. And to get the equation of the wheel hop natural frequency it is assumed that the position of the body is fixed so that the springs are parallel connected.



Table 1. Parameters for task 1

Quantity	Notation	Unit	Value
Front Wheel Suspension Stiffness	$c_f$	N/m	30800
Rear Wheel Suspension Stiffness	$c_r$	N/m	29900
Front Wheel Suspension Damping	$d_f$	Ns/m	4500
Rear Wheel Suspension Damping	$d_r$	Ns/m	3500
Tire Stiffness	$ct$	N/m	230000
Sprung Mass Front	$m_{uf}$	kg	502.5
Sprung Mass Rear	$m_{sf}$	kg	335
Unsprung Mass Front	$m_{uf}$	kg	46.25
Unsprung Mass Rear	$m_{ur}$	kg	46.25
Tire damping	$dt$	N/(m/s)	0
Wheelbase	$L$	m	2.675
Distance front axle to CoG	$l_f$	m	1.07

Table 2. Variables for task 1

<u>Quantity</u>	<u>Notation</u>	<u>Unit</u>
Speed of Sprung Mass	$\dot{Z}_s$	m/s
Speed of Unsprung Mass	$\dot{Z}_u$	m/s
Acceleration of Sprung Mass	$\ddot{Z}_s$	m/s <sup>2</sup>
Acceleration of Unsprung Mass	$\ddot{Z}_u$	m/s <sup>2</sup>
Displacement of Sprung Mass	$Z_s$	m
Displacement of Unsprung Mass	$Z_u$	m
Road Vertical Displacement	$Z_r$	m
Road grip	$(Z_r - Z_u) \cdot C_t$	N
Suspension Travel	$Z_u - Z_s$	m

## **TASK 2: Study of the suspension stiffness and damping**

First of all, we have to calculate the road spectrum for one front wheel of a vehicle moving at constant speed using the following equation:

$$G_{zz}(\omega) = v_x^{W-1} \cdot \Phi_0 \cdot \omega^{-W} \quad (19)$$

Where:

$$v_x = 80 \text{ km/h}$$

$$\Phi_0: \text{Road severity} = 10 \cdot 10^{-6} [\text{m}^2/(\text{rad/m})]$$

$$W: \text{Waviness} = 2.5$$

$$\omega: \text{Angular Frequency Vectors} [\text{rad/s}]$$

### **TASK 2.1: Plot response spectrum and calculate rms values**

Here we will evaluate the power spectral density and the rms values of the acceleration of the sprung mass and the tyre force so as to measure the ride comfort and the road grip. The matrices used for the calculation of the transfer function and the expression of the transfer function are the same than in task 1.3.

Once we have calculated the transfer function we can calculate the power spectral density using the following equation:

$$G(\omega) = |H(\omega)|^2 \cdot G_{zz}(\omega) \quad (20)$$

$$rms = \sqrt{\sum_i^N G(\omega_i) \cdot 2 \cdot \pi \cdot 0.01} \quad (21)$$

We can also calculate the rms value of each plot using the equation (21).

Finally, we can plot the PSD values:

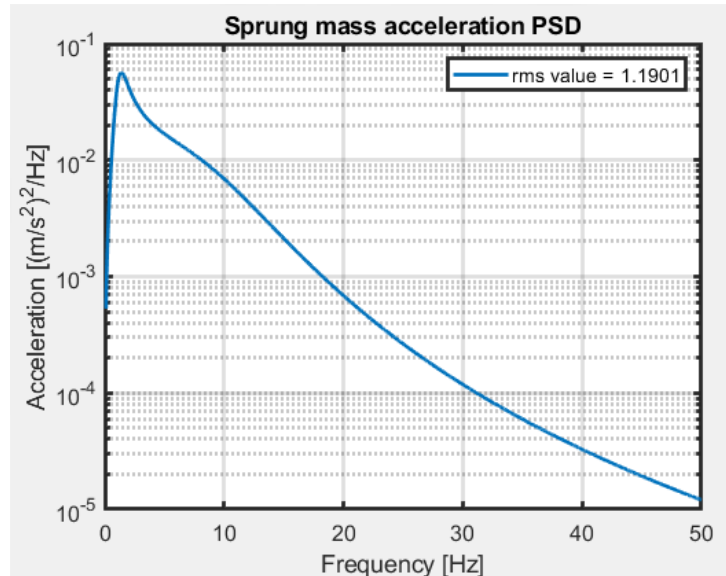


Figure 7: Sprung mass acceleration PSD

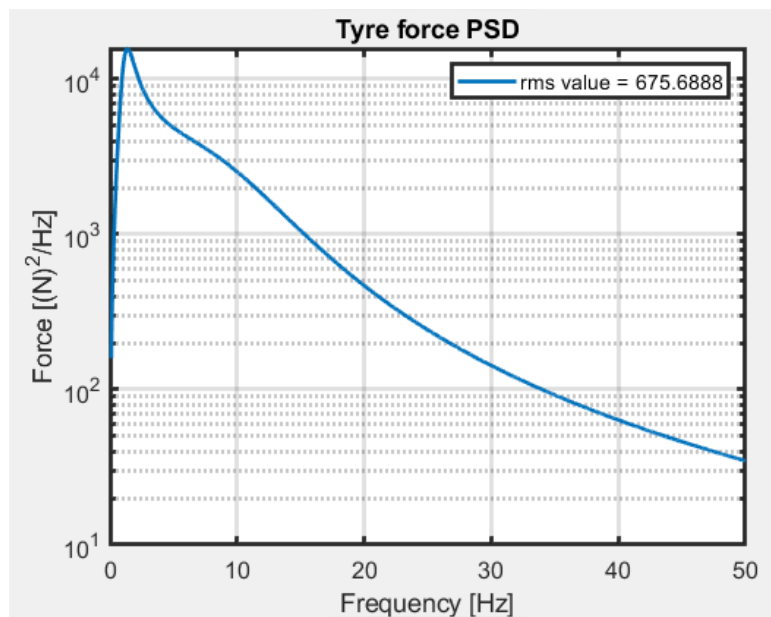


Figure 8: Tyre force PSD

In these two plots, we have the PSD for the tyre force and the sprung mass acceleration, as we see, for low values of the frequency, around 1 Hz, we have high values for PSD. It is also important to take into account that the range of (1Hz to 11Hz) is the range in which the humans have the highest sensitivity.

If the frequency is increased the PSD of tyre force and sprung mass acceleration decreases after we have reached the natural frequency since we are in a system of 2 DOF, the amplitude will be damped. The rms value for the sprung mass acceleration is 1.19 m/s<sup>2</sup> and the rms value for the tyre force is 675.68N.

## **TASK 2.2: Balance Ride comfort and Tyre force/road grip**

In this task we will vary the damping in order to calculate the values of rms of sprung mass acceleration and road grip for different stiffness values, in order to evaluate the ride comfort and the road grip. We have to take into account that high levels of sprung mass acceleration variation mean poor ride comfort, and high levels of the tyre force variation mean poor road grip.

Once we have calculated the new matrices and the PSD and rms values using different loops in Matlab, we are obtaining these two plots:

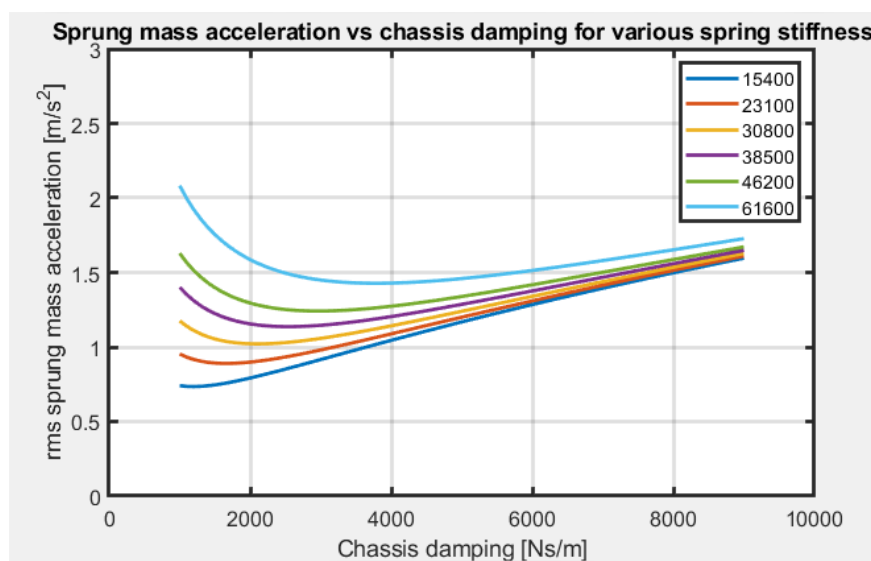


Figure 9. Sprung Mass Acceleration vs Chassis damping with various spring stiffness

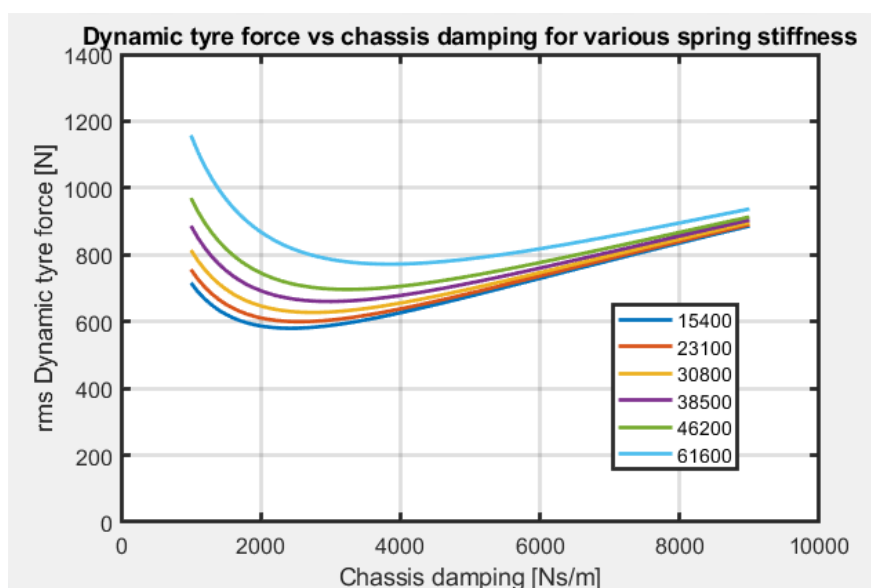


Figure 10. Dynamics tyre force vs Chassis damping with various spring stiffness

As we see from the plots, the higher the spring stiffness, the higher the value of the rms for both acceleration and tyre force. From these two plots, we can obtain the optimal values of stiffness and damping in order to improve the ride comfort and the road grip. In general, the lower the value of the rms the better the ride comfort and the road grip. So, we will store those optimal values and plot them in the following plot below in figure 11.

Table 3: Optimal values of damping for rms values of sprung mass acceleration.

Spring stiffness [N/m]	Optimal damping [Ns/m]	Rms sprung mass acceleration [m/s <sup>2</sup> ]
15400	1200	0.7344
23100	1700	0.8899
30800	2100	1.0209
38500	2500	1.1367
46200	3000	1.2414
61600	3800	1.4277

Table 4: Optimal values of damping for rms values of dynamic tyre force.

Spring stiffness [N/m]	Optimal damping [Ns/m]	Rms dynamic tyre force [N]
15400	2400	580.0754
23100	2600	599.7598
30800	2700	627.4290
38500	3000	660.3032
46200	3200	696.3388
61600	3800	772.0802

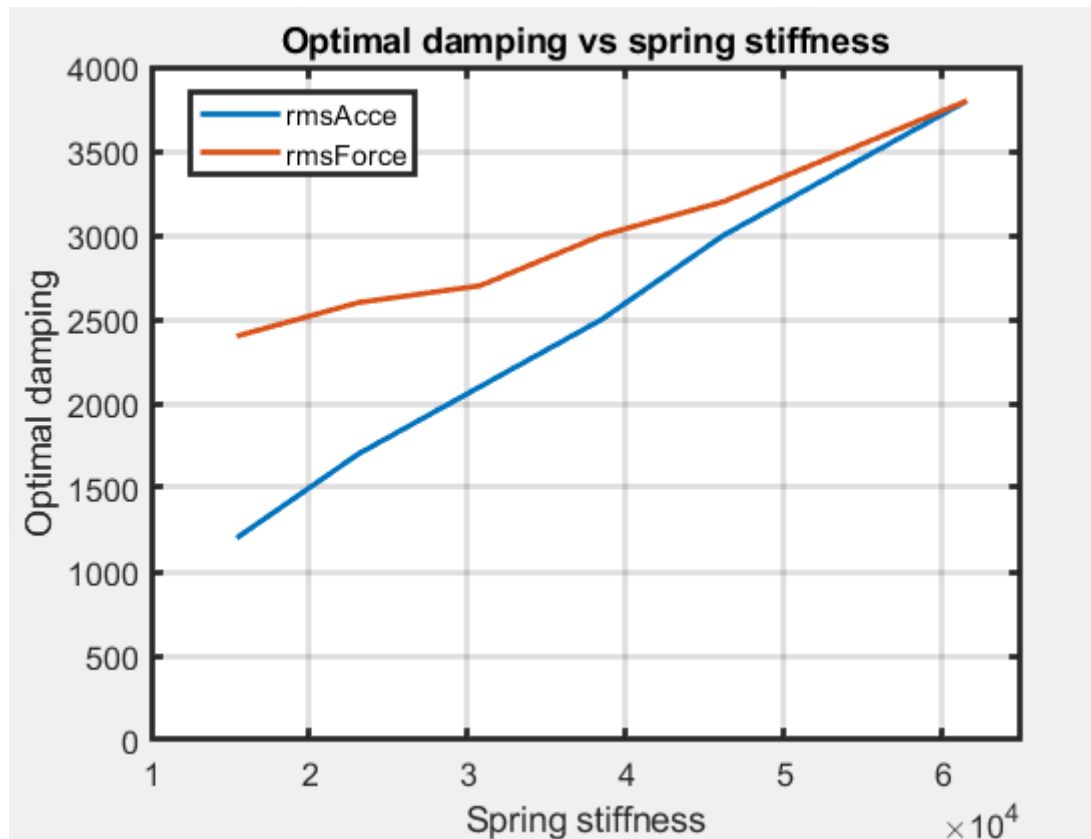


Figure 11. Optimal damping plot using various different Spring Stiffness

As we see, the objective of tyre force requires a higher damping than the acceleration in order to be optimal, because we need the tyre to be in contact with the road all the time so we need higher damping to avoid the vibrations of the wheel. In this case, we have a combined effect because as we see, to increase comfort we need lower damping in order to reduce the acceleration of the sprung mass because with the low damping, the lower mass accelerates more so that it does not transfer acceleration to the upper mass(sprung mass), whereas we need higher damping to improve the road grip.

Table 5: Variables used for task 2

<b>Quantity</b>	<b>Notation</b>	<b>Unit</b>
Acceleration of Sprung Mass	$\ddot{Z}_s$	$\text{m/s}^2$
Displacement of Unsprung Mass	$Z_u$	m
Road Vertical Displacement	$Z_r$	m
Tyre force/ Road grip	$\Delta F_{rz} = (Z_r - Z_u) \cdot C_t$	N
Suspension Stiffness	$C_s$	N/m
PSD of Sprung Mass Acceleration	$G(\omega)\ddot{Z}_s$	$(\text{m/s}^2)^2/\text{Hz}$
Suspension Damping	$d_s$	Ns/m
PSD of Tyre Force	$G(\omega)\Delta F_{rz}$	$\text{N}^2/\text{Hz}$
RMS Tyre Force	$rms_{\Delta F_{rz}}$	N
RMS of Sprung Mass Acceleration	$rms_{\ddot{Z}_s}$	$\text{m/s}^2$

Table 6. Newly Added Parameters for Task 2

<b>Quantity</b>	<b>Notation</b>	<b>Unit</b>
Waviness in different road condition	$w$	[1]
Road Severity	$\Phi_0$	$\text{m}^2 / \text{Rad m}$
Distance travelled in different road condition	$d_{\text{roadcondition}}$	m
Frequency Bandwidth	$\Delta\omega$	Rad/s



### **TASK 3: Ride comfort and the use of ISO-2631.**

In this task, we are tasked to improve the ride comfort of a passenger car which are used for commercial purpose. The car has to drive between City A which is the initial position of the messenger company and B is the customer location. Between A and B, the distance that has to be travelled by the driver is 80 km long. Moreover, the quality of the road travelled in the 80 km distance varied and could be categorized into 3 different levels. These levels are shown below in the Table below.

Table 7: Distribution of the road quality and its characteristics

Quality of Road	Percentage of the total road [%]	$\varphi_0$ (RoadSeverity)	w (Waviness)
Good	70	1E-6	3
Bad	17.5	10E-6	2.5
Very Bad	12.5	100E-6	2

The initial condition was that the driver could drive between A and B in five times per day. However, he has been experiencing bad comfort level of the car. And our task is to improve that condition.

#### **TASK 3.1: Calculation of daily whole-body exposure values**

In order to calculate the daily whole-body exposure values, we make certain assumption that are based on the Directive EEC 2002/44/EC rules for whole-body vibration. These assumptions are shown below.

Table 8: Assumptions used in this task

Driving Speed [km/h]	Vibration exposure Limit [ $\text{m/s}^2$ ]
110	1.15

#### **Method**

The method to calculate the whole body exposure value could be divided into 3 steps, These steps are :

- Calculate the Sprung Mass Acceleration Response Spectra.
- Calculate the ISO filtered Sprung Mass Acceleration.
- Calculate the time averaged vibration exposure value.

Before getting the sprung mass acceleration spectra required in step 1, we have to calculate the Road Spectrum for each road condition. It is done by using this formula below:

$$G_{Zr}(\omega) = v_x^{W-1} \cdot \Phi_0 \cdot \omega^{-W} \quad (22)$$

where the values of waviness and Road Severity are following values specified in Table 7 above.

Secondly, by using the result from the road spectrum calculation, we calculate the transfer function for front wheelZr to Ride. The method is the same as the one that has been used in Task 2. The equation used for the transfer function is eq (14) :

$$\frac{F(y)}{F(u)} = C \cdot (j\omega \cdot I_n - A)^{-1} \cdot B + D \quad (14)$$

where A,B,C,D are matrices that are used as a simplification of the Car Equation of Motions.

Finally, The Sprung Mass Acceleration Power Spectral Density could be calculated by using the equation below:

$$G(\omega) = |H(\omega)|^2 \cdot G_{Zr}(\omega) \quad (23)$$

For the 2nd Step, we need to use the ISO filter which will calculate a filtered acceleration response spectrum and calculate the RMS value of it.

Since human are sensitive to acceleration, the purpose of the creation and usage of ISO 2631 in the automotive industry is to define methods of calculating the whole-body vibration which are related to :

- Human health and comfort
- The probability of vibration perception.
- The incidence of motion sickness.

The Weighted RMS Acceleration ( $a_w(\text{RMS})$ ) could be calculated by finding the weighted MS Acceleration ( $a_w(\text{MS})$ ) using the function given.

For the 3rd step, after obtaining the value of ISO filtered Sprung Mass Acceleration for each road condition, we could calculate the time averaged whole-body vibration exposure by using the equation stated below :

$$a_{w,av} = \sqrt{\frac{(\sum_i a_{wi}^2 \cdot T_i)}{(\sum_i T_i)}} \quad \left[ \frac{m}{s^2} \right] \quad (24)$$

The result from this equation will be the value that shall be compared to the limit that is defined by Directive EEC 2002/44/EC. Ideally, the value shall be lower than 1.15 ( $m/(s^2)$ ).

## **Results**

By inputting vehicle velocity as 110 km/h for all three road conditions, The results of the calculation done in 1.1.1 are shown in the table below:

Table 9: Results of Task 3.1

$a_{w,av}$ (Rule) [m/s <sup>2</sup> ]	$a_{w,av}$ (Case) [m/s <sup>2</sup> ]	Trips (Rule)	Trips (Case)
1.15	1.5492	5	5.5

By observing the table above, we could conclude that the calculated value well exceeds the specified value, thus the vehicle will not pass the requirement that is stipulated in Directive EEC 2002/44/EC and should not be used for driving by human since it will cause uncomfortable driving. Thus, the driving velocity for each different road condition should be adjusted.

## **TASK 3.2: Modification of vehicle velocity**

### **Method**

In this task, we modify the vehicle speed individually for three different road types so that the vibration exposure limit is not exceeded on each road type, comparing the values of weighted rms acceleration of each road type with the limit. The requirements are that the speed selected should not exceed 110 km/h. Moreover, the weighted rms acceleration value shall not exceed 1.15 m/s<sup>2</sup> on each road type. After considering all the requirements above, the additional objective is to get as many times of return trip between the customer place (B) and the company location (A) as well as having the number of trips in integer (ex: 2 is better than 2.5)

### **Results**

By testing different values of velocity for each different road conditions, we come up with these proposed velocity as shown in Table 10. In our first proposed values, we try to keep the weighted rms acceleration for very rough road at 1.1199 (m/s<sup>2</sup>) by fixing the velocity at 8.5 (km/h) in order to keep the value for very rough road under 1.15 (m/s<sup>2</sup>) as well as keeping the maximum integer number of trips that we could have (2 trips).

Table 10. The Two set of proposed value of vehicle velocity Modification

First Proposed Values			
Road condition	Vehicle Velocity [km/h]	RMS weighted acceleration of each section[m/s2]	Number of Trips
Smooth	92.5	0.3313	2
Rough	64.17	0.8175	
Very rough	8.5	1.1199	
Second Proposed Values			
Road condition	Vehicle Velocity [km/h]	RMS weighted acceleration of each section [m/s2]	Number of Trips
Smooth	103.03	0.369	2
Rough	50	0.678	
Very rough	8.5	1.1199	

For the first case, the time weighted acceleration value is 0.9186 m/s<sup>2</sup> and for the second case is 0.91601 m/s<sup>2</sup>, as we see both of them are far from the limit of 1,15m/s<sup>2</sup> but this was not the only requirement, the reason of that the value of the time weighted acceleration is far from the limit is that the weighted acceleration of each road condition should be also lower than 1.15m/s<sup>2</sup>. And the very rough section limits the value.

Since we fixed the value for very rough road at 8.5 km/h, the values that could be tuned further are the velocity for smooth and rough condition. In the first proposed value, the velocity for smooth road is set on 92.5 km/h level and it corresponds to rms weighted acceleration of 0.3313 m/s<sup>2</sup>. However, for the rough road condition, it corresponds to of 0.8175 m/s<sup>2</sup> which is a bit higher than for smooth conditions. That is why we try to propose the second proposed set of vehicle velocity despite the fact that most of the whole trips will be spent on the smooth road condition (56 km vs 14 km).

To balance the driving comfort for the rough road condition we try to adjust the smooth and rough road vehicle velocity by increasing/decreasing it to 103.03 km/h and 50 km/h, which corresponds to rms weighted acceleration of 0.369 m/s<sup>2</sup> for the smooth conditions and 0.678 m/s<sup>2</sup> for rough conditions. This increase of 0.03 m/s<sup>2</sup> comparing the first proposed value is not significant compare to the decrease in rms weighted acceleration of 0.15 m/s<sup>2</sup> for rough conditions. The benefit of using the second set of velocity values is that the driving comfort for the 14 km of rough road conditions will improve significantly sacrificing a bit the driving comfort in the smooth road condition. So, our recommended speeds are those of “Second Proposed Values”.

**Appendix:**

Both students have contributed in the same way.

**References:**

Compendium Vehicle Dynamics MMF062.

Mechanical vibration and shock-Evaluation of human exposure to whole body vibration-Part 1: General requirements.