CHALMERS UNIVERSITY OF TECHNOLOGY

Vehicle Dynamics

(MMF062)

ASSIGNMENT-1

Longitudinal Vehicle Dynamics

Pair Number : 13

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INTRODUCTION

In Automotive Engineering, Vehicle can be defined as a system within which many components or subsystem interact. The interaction between the whole vehicle and the environment which resulted in the motions could be classified as Vehicle Dynamics. Moreover, in the Vehicle Dynamics study, it is common to divide the vehicle motion based on its direction including Longitudinal dynamics, Lateral Dynamics and Vertical Dynamics.

In Longitudinal Vehicle Dynamics, the vehicle tyres are the primary force interface for all motion which includes the disturbances from the environment. Moreover, Aerodynamic loads which results in wind resistance should also be taken into account in the study of longitudinal vehicle dynamics.

In this assignment, we are tasked with a problem of the Longitudinal Dynamics aspect of a vehicle. The objective is to propose a front or rear wheel drive for a two-axle driven passenger vehicle which will be used for a car for the competition. The road condition is wet as well as with an uphill slope of 8 degrees. To propose it, we are required to model:

1. The tire slip based on different road surface conditions

2. The Longitudinal Vehicle Dynamics with Equation of Motion

3. Physical Model, Mathematical Model and Explicit Form Model with/without considering the road gradient

Using these three models, we are able to implement and validate model of the vehicle as well as finding the best vehicle configuration (RWD/FWD) in order to be optimized for a racing track of 100 m long. Moreover, we were also able to investigate the influence of road friction on the vehicle longitudinal dynamics performance based on the models that we have built. In addition, in this report, the lateral vehicle dynamics aspect of the car is neglected in order to answer only the relevant questions to the assignment

METHODS AND RESULTS:

The following analysis of Longitudinal Vehicle Dynamics is basically divided into 3 major components:

a. Making use of a Tire Slip Model on different road conditions (such as Dry Asphalt, Wet Asphalt and Ice) to understand the influence of different parameters on the tire's normalized traction force.
b. Using equations of motion to model and understand the influence of different longitudinal forces acting on the vehicle by splitting it into smaller components.
c. Simulating the equations generated in the previous task and adding it to a simplified vehicle to understand the influence of different parameters such as road gradient, torque distribution and the road conditions, on the vehicle.

TASK 1: TYRE CHARACTERISTICS AT DIFFERENT ROAD CONDITIONS

a. Plot Tyre Model

In order to understand the behavior of a vehicle's tyre in different surface conditions such as on a dry asphalt, wet asphalt and on ice, a tyre model has been provided which includes a dynamic formula for the calculation of normalized longitudinal traction force. This formula accounts for the relation between longitudinal traction force and the wheel slip which results from the compression of contact patch. The relation, also referred to as 'Magic Tire Formula', is given by the following relation:

$$\frac{F_x}{F_z} = D * \sin\big(C * arctan\big(B * s_x - E(B * s_x - arctan(B * s_x))\big)\big)$$

&

$$B = 100 * \frac{\arctan(K)}{(C * D)} \; and \; K = \frac{3\pi}{180}$$

Where,

$\frac{F_x}{F_z}$ is the Longitudinal Traction Force or Normalized Longitudinal Tyre Force

$B, C, D \; and \; E$ are the variables whose values are dependent on different road conditions

This task was carried out by creating a *function* called 'Magic Tyre' containing the formula mentioned above and a plot Traction Force vs Longitudinal Wheel Slip is generated to understand the influence of Wheel Slip on Normalized Longitudinal Traction Force. This function was then referenced multiple times to accommodate the change in values of the variables $\boldsymbol{B, C, D \; and \; E}$ at different road conditions which are given as arguments for the function created above.

RESULTS:

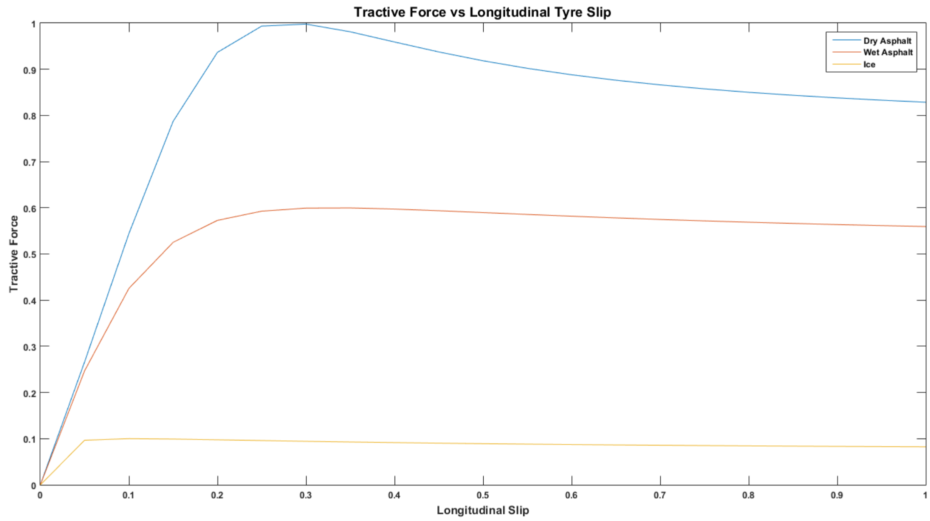

Fig. 3 : Variation of Traction Force with Longitudinal Slip at varying Road Conditions

From this plot, it is clear that the Dry Asphalt road conditions provides the maximum traction force and Icy road conditions account for lesser Traction Forces between the wheels and the road. It is also worth noting that that upto approximately 5% Slip, the Traction Force increases almost linearly for both Dry and Wet Asphalts and the percentage is greater when Dry Asphalt is considered.

The physical meaning of Longitudinal Tyre Slip refers to the relative measure of the speed of the wheel when it is rotating after it comes in contact with the road surface with the speed of the vehicle when it is not loaded or when it is not in contact with the road. It is essentially a measure of the energy loss.

b. Plot Experimental Tyre Behavior

In order to carry out this task, a set of experimental values of Normalized Longitudinal Tyre Force have been provided corresponding to a set of Wheel Slip values. These experimental values are plotted against each other. To understand how each of the different variables, namely $\boldsymbol{C, D \text{ and } E}$ influences the shape of the curve, three nested loops for the three variables are created. Now, the value of two variables are manipulated iteratively keeping one constant. This procedure is carried out for each of the three variables until a set of values for the three variables is found which helps in generating a curve that is almost identical to the experimental plot. Here, the parameter E is allowed to vary between -4 and -0.5.

RESULTS:

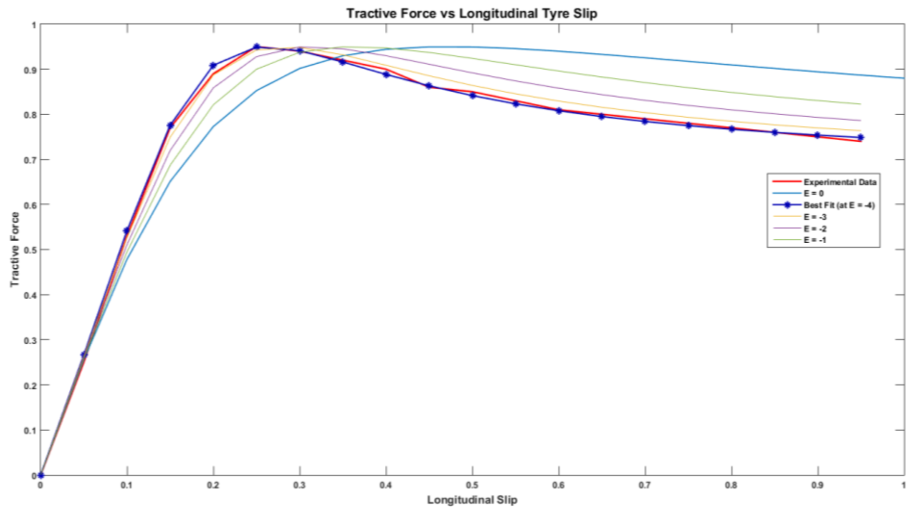

Fig. 4: Experimental Tractive Force vs Longitudinal Slip with varying values of E

The values of $C, D \text{ and } E$ which would generate a curve closer to the curve for Experimental value is listed in the Table below:

C	D	E	K	B
1.50	0.95	- 4	0.05	3.67

By iterating different values of E from 0 through -4, we can understand how the Traction Force reacts for a particular value of slip. The best fit curve is obtained for a value of E = -4. As the value of E keeps on increasing and moves towards a positive value, so does the rate of reduction in the value of traction forces. This means that, for a higher value of E, i.e. for values of E moving from -4 to 0 the rate of decrease in the value of Traction force from maximum (once it reaches the maximum value for a particular value of wheel slip) is very slow. This, in turn, means that for higher values of slip, the tractive forces are higher as compared to tractive force at lower values of E, as we move towards a positive E.

TASK 2: VEHICLE MODEL

a. Physical Model (Including Free Body Diagram)

A free body diagram of the given vehicle model is created. A separate diagram is shown in 2D for each of the 2 wheels, the chassis and the ground. In order to create this, the following assumptions were made in order to simply things along the way.

1. Each Axle i.e. front and rear, have one wheel
2. Translational Inertia is negligible compared to the Rotational Inertia
3. Rolling Resistance and Air Resistance should be taken into account

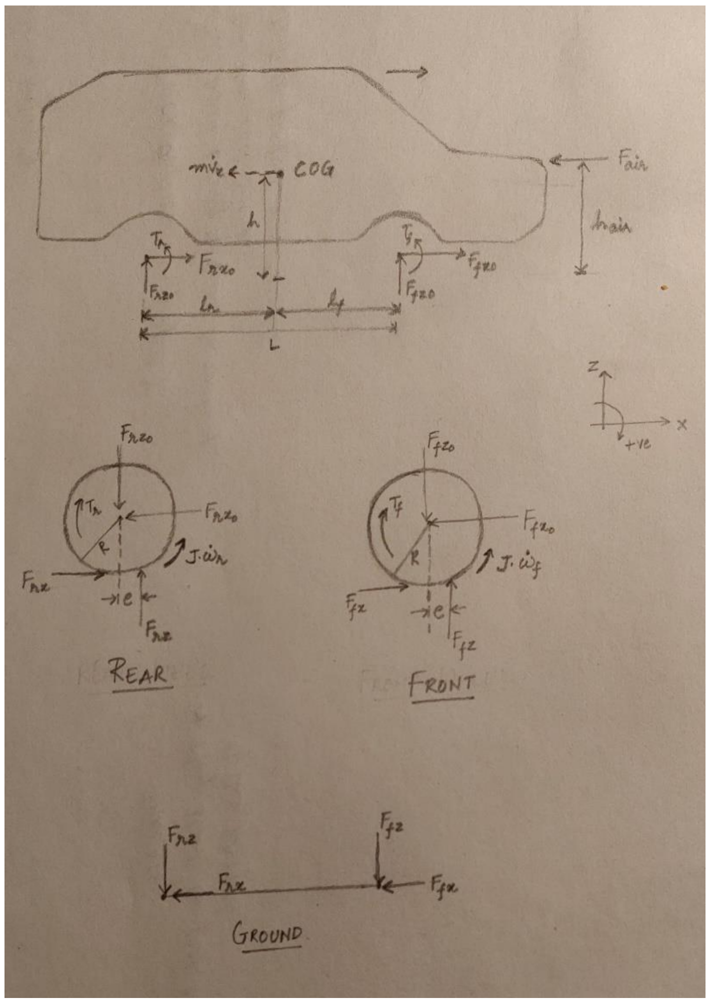

Fig. 1: Free Body Diagram showing the Chassis (Upper), the Wheels (middle), the Road(bottom)

The Figure above shows the Free Body Diagram of the Vehicle Model with the chassis, the 2 wheels and the road shown separately.

The Forces shown in Figure 1 with suffix as **0** are forces between the chassis and the wheels. *These forces essentially cancel out each other* if the wheels are placed on a suspension.

It is assumed that height at which F_{air} acts is equal to the height of the COG of the vehicle.

Here, only the 2 Traction Forces (Front and Rear), 2 Normal Forces (Front and Rear), 2 Angular Accelerations and 1 Linear Acceleration are the quantities that are unknown. So, there are essentially, 7 unknowns and 7 equations. Out of the two equations for Horizontal and Vertical Forces, either of them can be used to calculate the unknowns.

The list of all 7 unknown Variables that are necessary to derive the equations of motion are:

Sl. No.	**Variable**	**Description**	**Units**
1.	F_{fx}	Traction Force on the Front Axle in the Horizontal Direction	N
2.	F_{rx}	Traction Force on the Rear Axle in the Horizontal Direction	N
3.	F_{fz}	Normal Force on the Front Axle in the Vertical Direction	N
4.	F_{rz}	Normal Force on the Rear Axle in the Vertical Direction	N
5.	$\dot{\omega}_r$	Rotational Acceleration of the Rear Wheel	rad/s^2
6.	$\dot{\omega}_f$	Rotational Acceleration of the Front Wheel	rad/s^2
7.	$\dot{v}_x$	Linear Acceleration of the vehicle	m/s^2

And, the Parameters are:

Sl. No.	**Variable**	**Description**	**Units**
1.	F_{air}	Air Drag in the Horizontal Direction	N
2.	T_f	Torque on the Front Axle	N-m
3.	T_r	Torque on the Rear Axle	N-m
4.	m	Mass of the Vehicle	kg
5.	g	Acceleration due to Gravity	m/s^2
6.	L	Wheelbase of the vehicle	m
7.	l_f	Horizontal distance between the Front Wheel and the COG of the vehicle	m
8.	l_r	Horizontal distance between the Rear Wheel and the COG of the vehicle	m
9.	h	Height of the COG from the ground	m
10.	e	Horizontal Offset between the Vertical forces on the wheel and the Hub	

- Horizontal Offset 'e' between the Vertical forces on the wheel and the Hub which is given by the relation:

$$e = RR_{const} \cdot R$$

- RR_{const} is the Rolling Resistance Constant
- R is the Radius of the Wheel

b. Mathematical Model

The Free Body diagram helps in deriving the equations of motion of the complete vehicle model which are represented as follows:

Sum of Horizontal Forces $\sum F_x = 0$

$$F_{rx} + F_{fx} - m\dot{v}_x - F_{air} = 0 \qquad (2)$$

Sum of Vertical Forces $\sum F_z = 0$

$$F_{rx} + F_{fx} - mg = 0 \qquad (3)$$

Sum of Moments about the Front Axle $\sum M_{front} = 0$

$$(F_{rz} \cdot L) - (mg \cdot l_f) - (m\dot{v}_x \cdot h) - (F_{air} \cdot h) = 0 \qquad (4)$$

Sum of Moments about the Rear Axle $\sum M_{rear} = 0$

$$-(F_{fz} \cdot L) + (mg \cdot l_r) - (m\dot{v}_x \cdot h) - (F_{air} \cdot h) = 0 \qquad (5)$$

Sum of Moments about the center of the Front Wheel $\sum M_{front\_wheel} = 0$

$$-T_f + \left(F_{fz} \cdot e\right) + \left(F_{fx} \cdot R\right) + (J \cdot \dot{\omega}_f) = 0 \qquad (6)$$

Sum of Moments about the center of the Rear Wheel $\sum M_{rear\_wheel} = 0$

$$-T_r + (F_{rz} \cdot e) + (F_{rx} \cdot R) + (J \cdot \dot{\omega}_r) = 0 \qquad (7)$$

Sum of Friction Forces at the Front Wheel $\sum F_{\mu,front\_wheel} = 0$

$$F_{fx} - (\mu_f \cdot F_{fz}) = 0 \qquad (8)$$

Sum of Friction Forces at the Rear Wheel $\sum F_{\mu,rear\_wheel} = 0$

$$F_{rx} - (\mu_r \cdot F_{rz}) = 0 \qquad (9)$$

c. Update Models with Road Gradients

The Free Body Diagram for a vehicle driving up a slope with an inclination θ can be represented as follows:

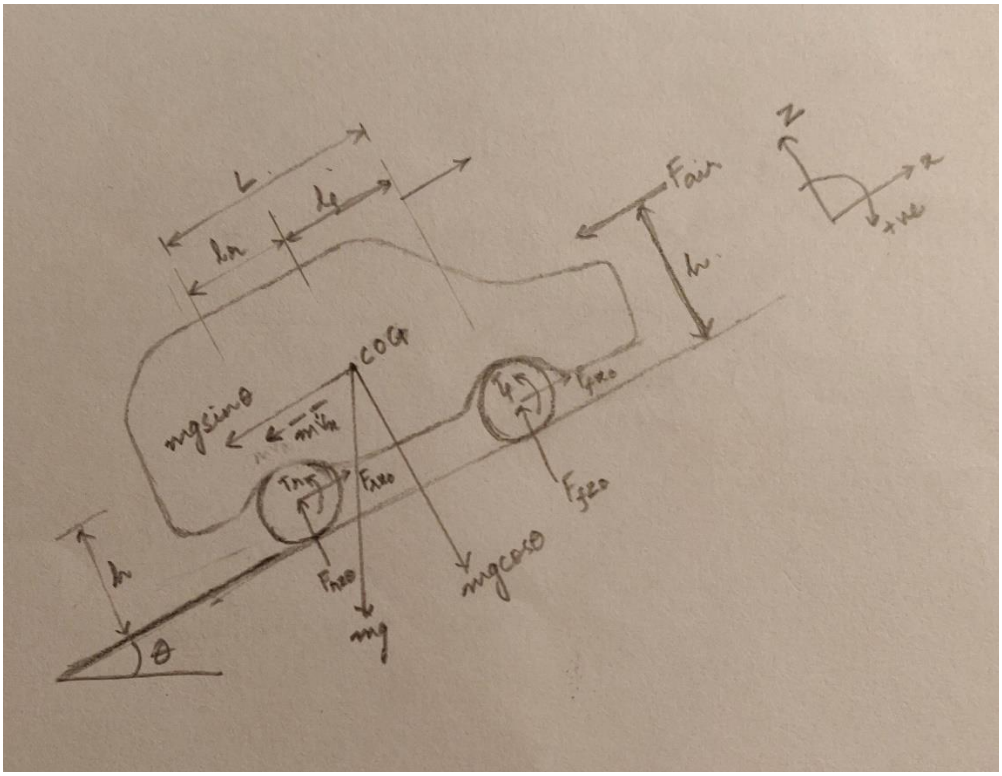

Fig. 2: Vehicle Moving up an inclined slope with gradient 'θ'

When the vehicle is moving upward on an inclined surface, the vertical Force $'mg'$ acting vertically downwards due to the weight of the vehicle can be resolved into 2 components:

1. $mg\ cos\theta$ along the downward direction which perpendicular to the inclined surface
2. $mg\ cos\theta$ along the horizontal direction which is opposite to the motion of the vehicle

With this change in the gradient of the road surface, the Force and Moment components are affected. These equations can be re-written as follows:

Sum of Horizontal Forces $\sum F_x = 0$

$$F_{rx} + F_{fx} - m\dot{v}_x - F_{air} - \boldsymbol{mg\ sin\theta} = 0$$

Sum of Vertical Forces $\sum F_z = 0$

$$F_{rx} + F_{fx} - \boldsymbol{mg\ cos\theta} = 0$$

Sum of Moments about the Front Axle $\sum M_{front} = 0$

$$(F_{rz} \cdot L) - (\boldsymbol{mg\ cos\theta \cdot l_f}) - (m\dot{v}_x \cdot h) - (F_{air} \cdot h) - (\boldsymbol{mg\ sin\theta \cdot h}) = 0$$

Sum of Moments about the Rear Axle $\sum M_{rear} = 0$

$$-(F_{fz} \cdot L) + (\boldsymbol{mg\ cos\theta \cdot l_r}) - (m\dot{v}_x \cdot h) - (F_{air} \cdot h) - (\boldsymbol{mg\ sin\theta \cdot h}) = 0$$

Sum of Moments about the center of the Front Wheel $\sum M_{front\_wheel} = 0$

$$-T_f + (F_{fz} \cdot e) + (F_{fx} \cdot R) + (J \cdot \dot{\omega}_f) = 0$$

Sum of Moments about the center of the Rear Wheel $\sum M_{rear\_wheel} = 0$

$$-T_r + (F_{rz} \cdot e) + (F_{rx} \cdot R) + (J \cdot \dot{\omega}_r) = 0$$

Sum of Friction Forces at the Front Wheel $\sum F_{\mu,front\_wheel} = 0$

$$F_{fx} - (\mu_f \cdot F_{fz}) = 0$$

Sum of Friction Forces at the Rear Wheel $\sum F_{\mu,rear\_wheel} = 0$

$$F_{rx} - (\mu_r \cdot F_{rz}) = 0$$

These modified equations are used along with the equations for moments about the Front and Rear Wheels and the Friction Forces acting at the Front and the Rear Wheels.

d. Solve the Equations

The equations are written, as they've been represented in Task 2c, in MATLAB using the symbolic toolbox for the all variables. The '**syms**' command is used to initialize all the variables that are used in the equations mentioned above. Along with this, the function '**solve**' is used to get the independent expressions for each of the 7 unknowns.

The final simplified expressions for the 7 unknowns are:

$$F_{fx} = -\frac{\left(l_f * mgcos\theta * h * \mu_f * \mu_r - L * l_r * mgcos\theta * \mu_f + l_r * mgcos\theta * h * \mu_f * \mu_r\right)}{(L * (L + (\mu_f - \mu_r) * h))}$$

$$F_{rx} = \frac{\left(L * l_f * mgcos\theta * \mu_r + l_f * mgcos\theta * h * \mu_f * \mu_r + l_r * mgcos\theta * h * \mu_f * \mu_r\right)}{(L * (L + (\mu_f - \mu_r) * h))}$$

$$F_{fz} = -\frac{\left(l_f * mgcos\theta * h * \mu_r - L * l_r * mgcos\theta + l_r * mgcos\theta * h * \mu_r\right)}{(L * (L + (\mu_f - \mu_r) * h))}$$

$$F_{rz} = \frac{\left(L * l_f * mgcos\theta + l_f * mgcos\theta * h * \mu_f + l_r * mgcos\theta * h * \mu_f\right)}{(L * (L + (\mu_f - \mu_r) * h))}$$

$$\dot{\omega_f} = \left(L^2 * T_f + L * T_f * h * \left(\mu_f - \mu_r\right) - L * l_r * mgcos\theta * R * \left(RR_{const} + \mu_f\right) + mgcos\theta * R * RR_{const} * h * \mu_r * \left(l_f + l_r\right) + mgcos\theta * h * \mu_f * \mu_r * R * \left(l_f + l_r\right)\right) / (J * L * (L + \left(\mu_f - \mu_r\right) * h))$$

$$\dot{\omega_r} = \left(L^2 * T_r + L * T_r * h * \left(\mu_f - \mu_r\right) - L * l_f * mgcos\theta * R * (RR_{const} + \mu_r) - mgcos\theta * R * RR_{const} * h * \mu_f * \left(l_f + l_r\right) - mgcos\theta * h * \mu_f * \mu_r * R * \left(l_f + l_r\right)\right) / (J * L * (L + \left(\mu_f - \mu_r\right) * h))$$

$$\dot{v_x} = -\left(F_{air} * L + F_{air} * h * \left(\mu_f - \mu_r\right) + L * mg\, sin\theta - mg\, cos\theta * (l_f * \mu_r + l_r * \mu_f) + mg\, sin\theta * h * \left(\mu_f - \mu_r\right)\right) / (M * (L + \left(\mu_f - \mu_r\right) * h))$$

RESULTS:

The effect of aerodynamic drag force acting on the vehicle can be given by given by the moment equation:

$$\sum M_{air} = F_{air} \cdot (h_{air} - h)$$

If the aerodynamic drag force acts a height different than the height of the COG of the vehicle, it will add to the downward weight transfer and hence, the moment generated due to this will cause the vehicle to squat greater.

As mentioned previously, the aerodynamic drag force in our case is assumed to be acting at the same height where the COG of the vehicle lies. Thus, this moment does not appear in the vertical forces F_{fz} and F_{rz}.

e. Integrate the Equations

The final expressions for all the 7 unknowns listed above are used to analyze the behavior of the vehicle under the influence of certain road condition and road gradient.

In order to do this, as stated, we begin by creating a function called '**TyreSlipModel**' which takes the values of Slip and Road Conditions (Dry, Wet or Ice) as the arguments and then calculates and returns the value of the Longitudinal Tyre Force or the Traction Force for the different road conditions. To create a function 'TyreSlipModel', we use the input argument for Slip and Road Conditions. A **structure** is created to refer to the values of $\boldsymbol{C}, \boldsymbol{D}\ \boldsymbol{and}\ \boldsymbol{E}$ which was mentioned in Task 1a. Based on the input argument for Road Condition and Slip, the 'Magic Tire' formula is used in a loop to iterate through the elements of the structure and calculate the Normalized Longitudinal Tyre Force. This is passed to the function as the output.

After this, another function called ‘**Slip**’ is created to calculate the amount of Wheel spin in the Front and the Rear. Slip of vehicle is given by the relation:

$$Slip = \frac{(R \cdot \omega) - v}{(R \cdot \omega)}$$

Where,

R is the Radius of the Wheel

ω is the Rotational Velocity of the Wheel

v is the Linear Velocity

The function ‘**Slip**’ takes the input as v, ω and R as the inputs in order to calculate the Wheel Slip. This value of wheel slip can then be used in the function ‘**TyreSlipModel**’ to calculate the Normalized Longitudinal Tyre Force.

After selecting the required parameters and choosing the required conditions, the expressions obtained in Task 2d are integrated from time t = 0 to t = 20 seconds in a for loop.

RESULTS:

In order to carry out this task, the following conditions were assumed:

Road Condition: Dry Asphalt; Initial Velocity: 0.1 m/sec; Slope: 5 Degrees

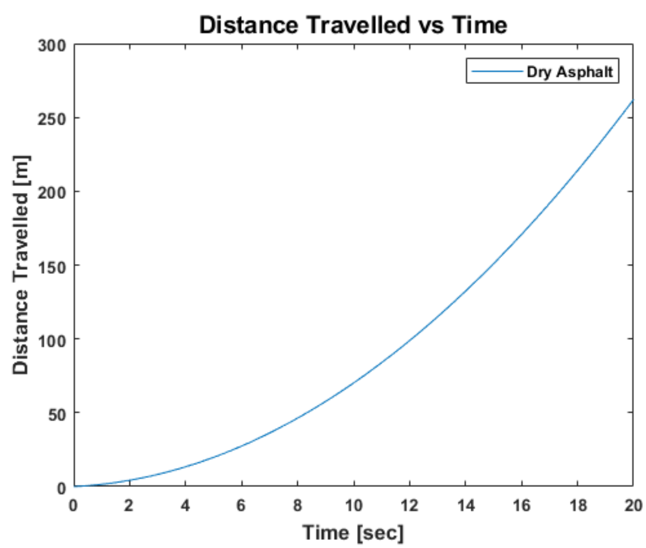

Fig. 4: Distance vs Time

When the Distance vs Time plot obtained in this Task is compared with the results obtained in Task 3, it is quite visible that the amount of time needed to cover a distance of 100m is relatively higher here (around 12 seconds compared to 7 seconds in Task 3). This huge difference may be attributed to the fact that here, the propulsion torques have a constant value throughout.

TASK 3 : SIMULATION OF LONGITUDINAL DYNAMICS

The final expressions obtained in Task 2d for a vehicle moving on an incline, can be used as an input to a simulation file. This simulation helps to understand certain behaviors of the vehicle under the influence of factors such as Road Gradient, Road Conditions and Torque Distribution. The simulation is carried out using a set of MATLAB Scripts and Functions which is already provided.

The results in this Task are to be analyzed from the perspective of a race engineer of a drag race team. So, it is imperative that the time need to travel a certain distance is as less as possible taking into consideration the fact that it is moving under the influence of different road conditions and road gradients.

The vehicle used for this purpose is a Saab 9-3.

a. Implement and Validate Model

As stated above, this task is started by stating the final expressions obtained for the unknowns in Task 2d i.e. expressions for F_{fz}, $F_{rz}, \dot{\omega}_f, \dot{\omega}_r$ and $\dot{v_x}$ are used in a MATLAB script called *Sub_vehicle_dynamics.m*.

These expressions from the above MATLAB Script are fed as inputs to the main Simulation file '*Simulation_main.m*'. Here the values of different parameters such as Road Gradients, Road Conditions and Torque Distribution can be manipulated to obtain the values of time and the speed of the vehicle in order to travel a distance of 100m.

In this case, first, the Road Gradient is kept constant at **2°** and the Road Condition is set to '**Dry**'. And the parameter Torque Distribution is varied from 0 to 1. The value 0 refers to the Rear Wheel Drive and 1 refers to a 'Front Wheel Drive'. The parameter used here is the torque distribution for the front axle. So, a value of 0.8 would mean that 80% of the torque is transmitted to the Front Axle and the remaining 20% to the rear wheel. The lowest time is noted down during variation. This would help us decide if the vehicle would run faster on a RWD or FWD. Also, changing these parameters changes the plots for Normalized Longitudinal Tyre Forces, the Tractive Forces and the Wheel Slip in both the front and the rear end of the wheels.

Now, the road condition is changed to '**Wet**' and the same iterative process is repeated. The same procedure can be repeated by selecting different road gradients.

This would help us understand better how the vehicle would behave under the influence of different conditions so that the vehicle can be setup more efficiently for faster lap times.

RESULTS:

- *Case 1: Road Gradient = 2°, Road Condition – 'Dry Asphalt', Torque Distribution = 0 (RWD) [Fig. 5 & 6]*

Keeping the Road Gradient and Road conditions constant, the Torque distribution is varied starting from 0 which means that the behavior of the vehicle is observed if a RWD configuration is assumed for the current setup. From the graphs (Fig. 5 and 6), it is clear that in order to travel a distance of **100m**, the vehicle takes **7.328 seconds**. Consequently, the vehicle speed and the accelerations are around **86.6 kmph** and **2.5m/s²** respectively. This seems quite reasonable for a passenger car.

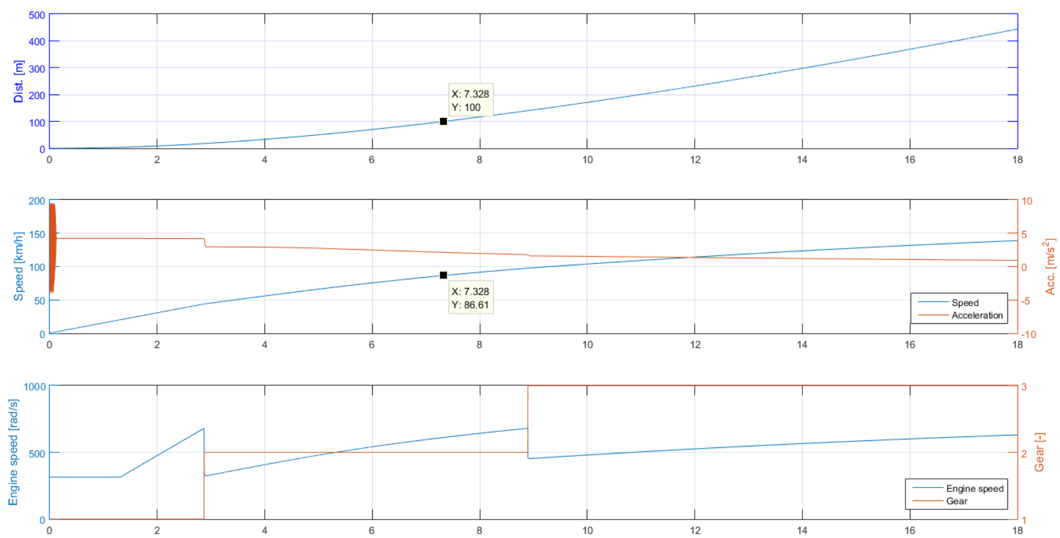

Fig. 5: Displacement, Speed, Acceleration, Engine Speed and Gear vs Time (RWD, Dry Asphalt)

If, for the same conditions, the plots for Normal Forces, the driving Torques and the Slip at the front and the rear is observed, we can see that there seems to be no driving torque at the front axle which is why it never rises above 0 throughout. Since there is no torque on the front, consequently the slip on the front wheels is zero and on the rear wheels its maximum which reduces with time. These factors confirm that the results achieved are quite reasonable.

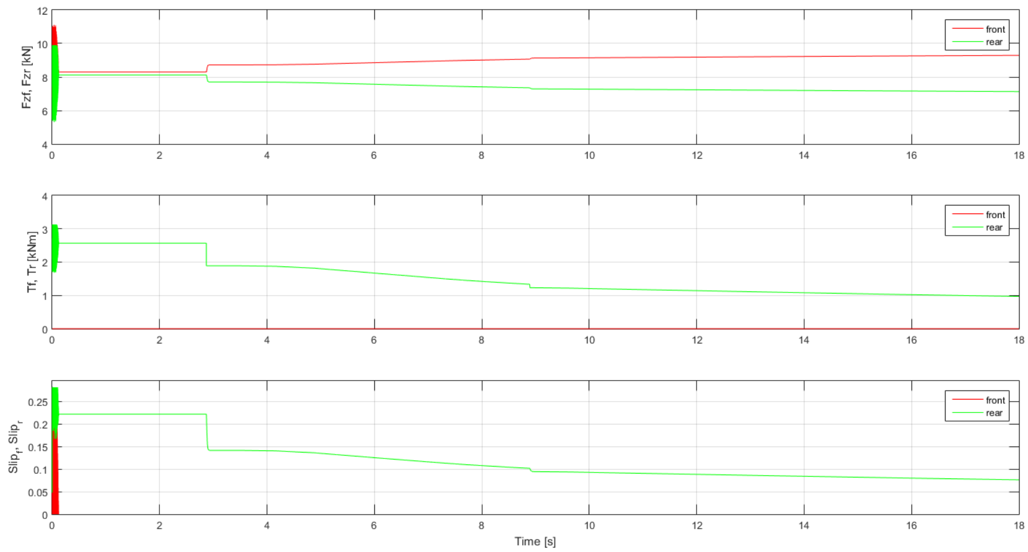

Fig. 6: Normal Forces, Driving Torques, Wheel Slip (at Front & Rear) vs Time (RWD, Dry Asphalt)

- *<u>Case 2</u>: Road Gradient = 2°, Road Condition – 'Dry Asphalt', Torque Distribution = 1 (FWD) [<u>Fig. 7 & 8</u>]*

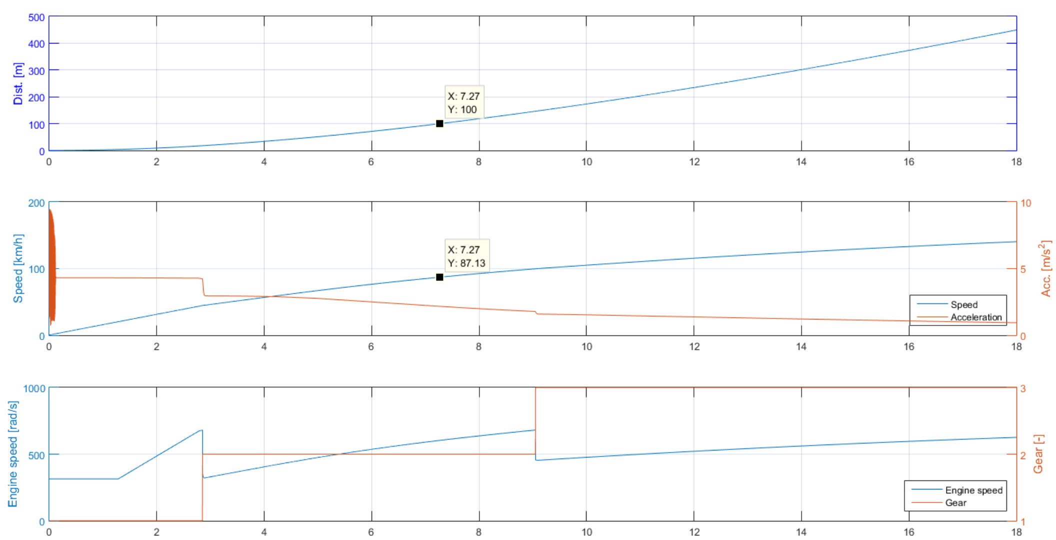

Fig. 7: Displacement, Speed, Acceleration, Engine Speed and Gear vs Time (FWD, Dry Asphalt)

Similarly, the amount of time required for covering a distance of 100m is observed in this case. In this scenario, the vehicle takes **7.27 seconds** to travel a distance of **100m**. The speed achieved during this time period in this scenario is **87.13 kmph**. These seem to be normal for a a normal passenger vehicle with this kind of Torque Distribution.

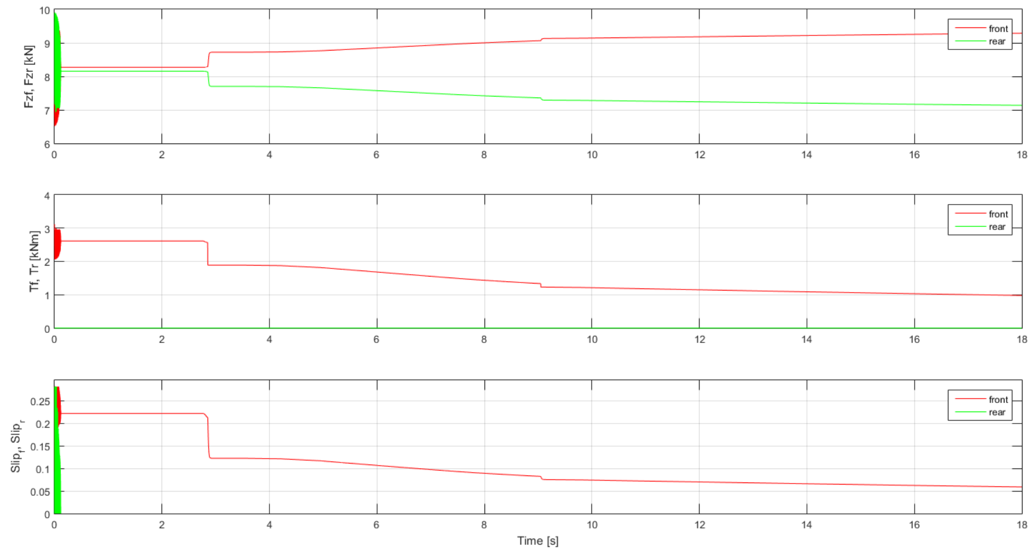

Fig. 8: Normal Forces, Driving Torques, Wheel Slip (at Front & Rear) vs Time (FWD, Dry Asphalt)

In the same way, albeit an inverse of the effect described in Case 1, it can be observed here which seems to justify our results. For e.g.: Since the torque distribution is 100% towards the front wheels, the driving torque curves never rise above 0 for the Rear axles. The same goes for the wheel slip of the rear which never increases.

Comparing these 2 results at the for the Dry Asphalt Road Condition and a Road Gradient of 2 degrees, we can observe that the vehicle speed is comparatively higher for the Front Wheel Drive Condition and therefore, the time taken to reach a distance of 100m is smaller.

b. Select Between FWD and RWD

In this task, the Road Gradient has been changed to **8°** and there is also a possibility of rain which means that the influence of rain on the dynamics of the car has to be taken into consideration while setting it up before the race.

In order to achieve this, the same procedure, as mentioned above in Task 3a, is repeated keeping the parameters Road Gradient as **8°** and switching the Road Conditions between Wet and Dry. Again, the Torque Distribution between the front and the Rear is varied between 0 and 1 in order to make a decision about FWD or RWD.

For this, the same simulation file is used i.e. '*Simulation_main.m*'.

RESULTS:

In this scenario, when the vehicle moves at a higher gradient (i.e. **8°**), in order to get the lowest times, we need to adjust the torque distribution between the front and the rear axles. On iterating through different values of Torque Distribution for a particular Road Condition, we can observe that the lowest times are clocked (for travelling a distance of 100m) when the vehicle is setup in such a way it gets a **Torque Distribution of 0.45** which means that the Front Axles receive 45% of the available Torque and the remaining 55% goes to the Rear Wheel. Thus, it can be inferred that it should be used as a **rear wheel drive (RWD)**.

Configuration	Road Condition	Time (in seconds)	Distance (in meters)
FWD (F-75%, R-25%)	Dry	7.4	100
RWD (F-45%, R-55%)	Dry	7.22	100
FWD (F-75%, R-25%)	Wet	8.48	100
RWD (F-45%, R-55%)	Wet	7.66	100

- *Case 1: Dry Asphalt, RWD – (Front – 45%, Rear – 55%) [Fig. 9 & 10]*

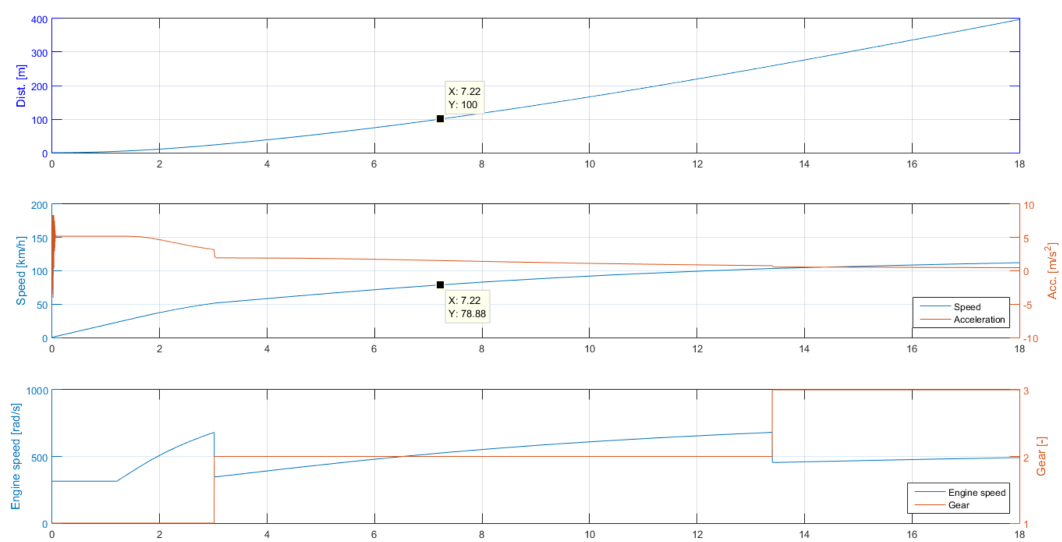

*Fig. 9: Displacement, Speed, Acceleration, Engine Speed and Gear vs Time (**RWD, Dry Asphalt**)*

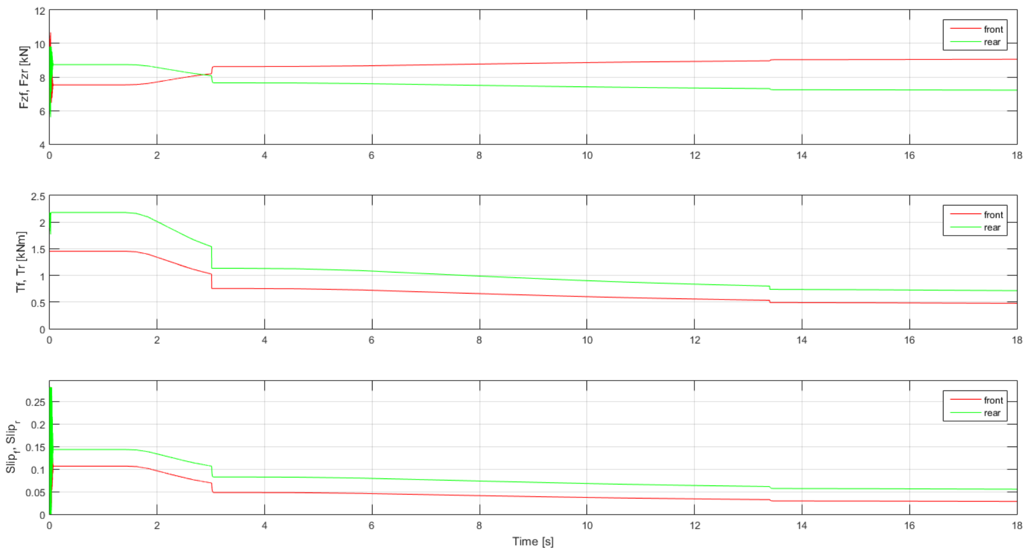

*Fig. 10: Normal Forces, Driving Torques, Wheel Slip (at Front & Rear) vs Time (**RWD, Dry Asphalt**)*

The plots shown above (i.e. Fig. 9 & 10) are the Plots for Dry Asphalt for a Torque Distribution of 45% to the Front and 55% to the Rear (i.e. RWD). ***This particular value of Torque Distribution (i.e. 0.45) is chosen specifically because it gives the lowest times for covering a distance of 100m***.

From Fig. 9, it can be observed that the amount of time required for covering a distance of 100m is **7.22 seconds**. The speed achieved during this time period in this scenario is **78.9 kmph**. The lap times clocked here are the lowest when compared to other values of Torque Distribution (between 0 and 1) for a vehicle moving at a gradient of 8 degrees.

<u>*Case 2*</u>*: Wet Asphalt, RWD – (Front – 45%, Rear – 55%) [<u>Fig. 11 & 12</u>]*

Fig. 11 shows the variation of Displacement, Speed, Acceleration with time for a **Wet Asphalt**. Again, a Torque Distribution of 0.45 is taken into consideration since this value yields the lowest lap times for the given setup. The time achieved here for travelling a distance of **100m** is **7.66 seconds** and the speed attained during this time period is **77.5 kmph**. This is the lowest time that can be achieved by the vehicle for travelling a distance of 100 m on a wet asphalt.

When the plots for the Slip values of the front and rear wheels for a vehicle moving in wet and dry conditions are compared (i.e. Figures 10 and 12 are compared), it can be observed that the slip values on a wet surface (i.e. 0.2) is higher than the values obtained on a dry asphalt (i.e. 0.15). Although, when these values are compared to a vehicle with a Front Wheel Drive (for e.g. a vehicle with a Torque Distribution of 0.75), slip values are higher for both wet and dry surfaces. *This shows that it makes more sense to use a vehicle with a Rear Wheel Drive setup on both Wet and Dry Asphalts. The reason being lower time achieved to reach a distance of 100m*. This can also be proved by the values mentioned in the Table shown above for this Task. The values marked in green show the optimum configuration for the respective torque distribution.

So, in this case, i.e. at a gradient of 8 degrees, **RWD** will be the optimum one for us (for a Torque Distribution of 45% to the Front and 55% to the Rear).

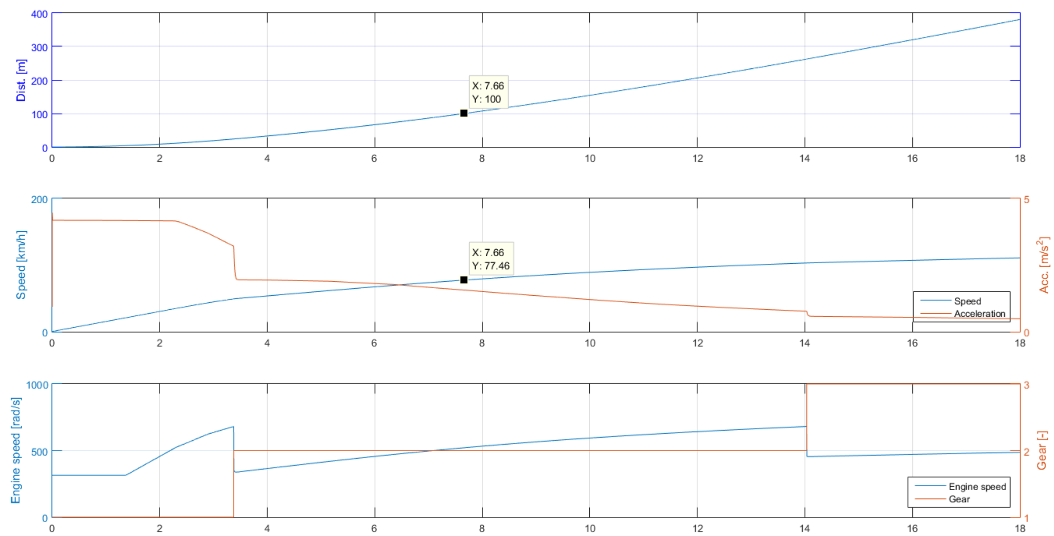

*Fig. 11: Displacement, Speed, Acceleration, Engine Speed and Gear vs Time (**RWD, Wet Asphalt**)*

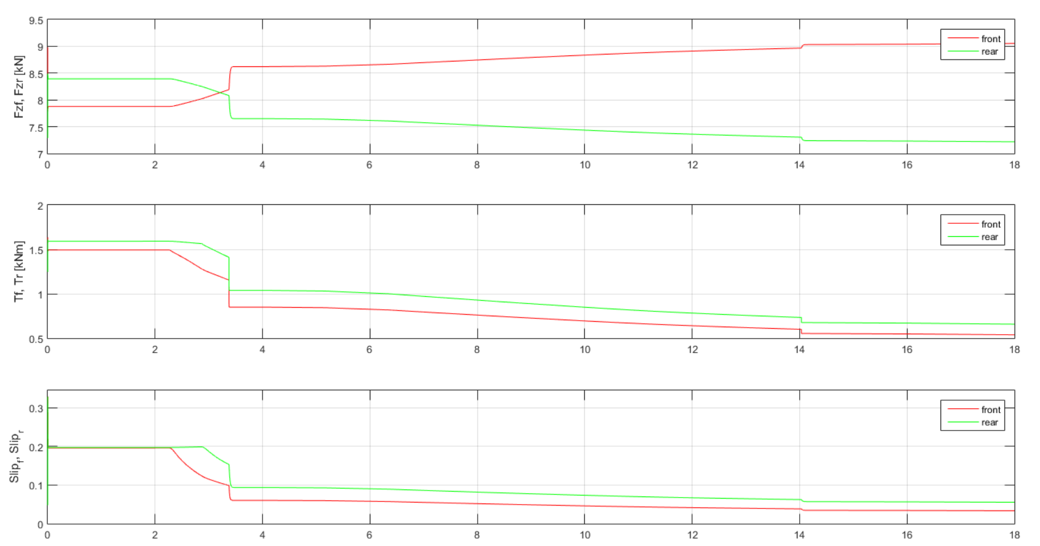

*Fig. 12: Normal Forces, Driving Torques, Wheel Slip (at Front & Rear) vs Time (**RWD, Wet Asphalt**)*