

In the PEPS setting, the parameter index  $\alpha$  naturally factorizes as

$$\alpha \equiv (\mathbf{x}, p, lrdu)$$

where  $\mathbf{x}$  is the site index,  $p$  is the physical index and  $lrdu$  are the virtual indices.

There is a fact:

$$O[\mathbf{x}](s)_{p,lrdu} = 0 \text{ if } p \neq s(\mathbf{x})$$

One thus only keeps non-zero elements in Jacobians  $o[\mathbf{x}](s)_{lrdu} = O[\mathbf{x}](s)_{s(\mathbf{x}),lrdu}$  to improve minSR:

$$\begin{aligned} \sum_{\alpha} O_{s\alpha} O_{\alpha s'}^{\dagger} &= \sum_{\mathbf{x}} \sum_p \sum_{lrdu} O[\mathbf{x}](s)_{p,lrdu} O^{\dagger}[\mathbf{x}](s')_{p,lrdu} \\ &\stackrel{\text{Fact}}{=} \sum_{\mathbf{x}} \sum_p \sum_{lrdu} O[\mathbf{x}](s)_{p,lrdu} O^{\dagger}[\mathbf{x}](s')_{p,lrdu} \mathbf{1}[p = s(\mathbf{x})] \mathbf{1}[p = s'(\mathbf{x})] \\ &= \sum_{\mathbf{x}} \sum_p \sum_{lrdu} \underbrace{O[\mathbf{x}](s)_{s(\mathbf{x}),lrdu}}_{o[\mathbf{x}](s)_{lrdu}} \underbrace{O^{\dagger}[\mathbf{x}](s')_{s'(\mathbf{x}),lrdu}}_{o^{\dagger}[\mathbf{x}](s')_{lrdu}} \mathbf{1}[p = s(\mathbf{x})] \mathbf{1}[p = s'(\mathbf{x})] \\ &= \sum_p \sum_{\mathbf{x}} \sum_{lrdu} \underbrace{o[\mathbf{x}](s)_{lrdu} \mathbf{1}[p = s(\mathbf{x})]}_{\tilde{o}} \underbrace{o^{\dagger}[\mathbf{x}](s')_{lrdu} \mathbf{1}[p = s'(\mathbf{x})]}_{\tilde{o}^{\dagger}} \end{aligned}$$

which is exactly the small-o trick in minSR, with a mere difference that one has no need to store a  $N_s \times N_p$  sample tensor. But maybe in practice one still calculates a  $N_s \times N_p$  sample tensor for better GEMM.

For the small-o trick, one enumerates over  $p$ . One can also consider other enumeration orders.