

1. (k points) Given the function

$$f(x) = -6x^2 - x^3$$

- (a) Sketch  $f$ ,  $f'$  and  $f''$  in one coordinate system.  
(b) Identify all of the minimum and maximum points and find its inflection points.

**Solution:**

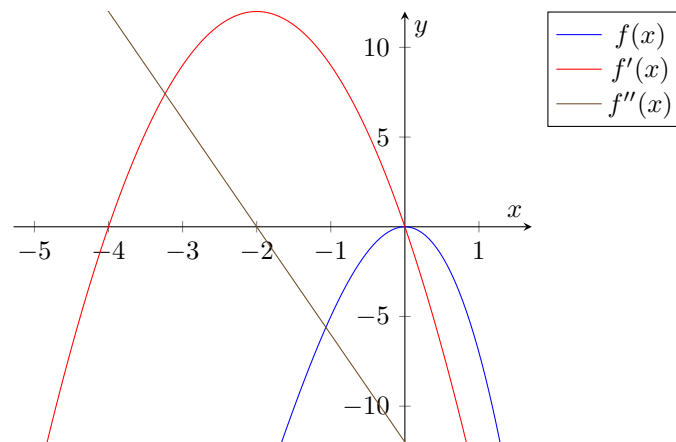
- (a) First, calculate the derivatives

$$f(x) = -6x^2 - x^3$$

$$f'(x) = -12x - 3x^2$$

$$f''(x) = -12 - 6x$$

$$f'''(x) = -6$$



- (b) The function  $f$  has zeros at  $x_1 = -6$  and at  $x_2 = 0$ . The function  $f'$  has zeros at  $x_3 = -4$  and at  $x_4 = 0$ . The function  $f$  has a minimum at  $(-4, -32)$  because  $f''(x_3) > 0$  and a maximum at  $(0, 0)$  because  $f''(x_4) < 0$ .