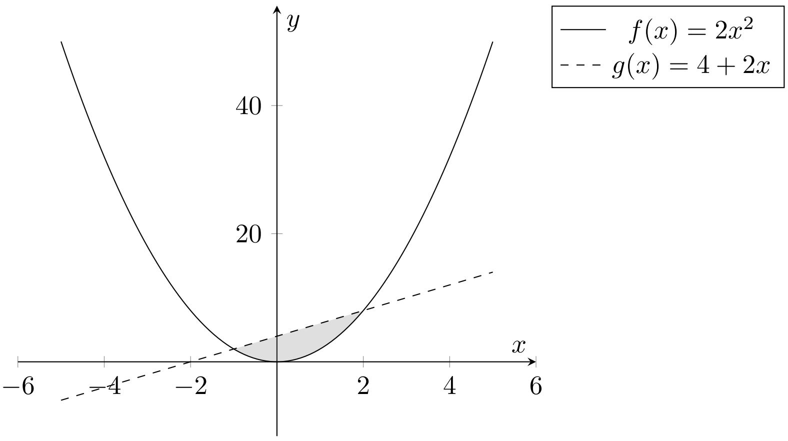
1. (k points) How big is the parabolic segment between the parabola $f(x) = 2x^2$ and the line $g(x) = 4 + 2x$?

   Sketch a graph to visualize the desired area.

Solution: The functions intersect at $P_1(-1, 2)^T$ and at $P_2(2, 8)^T$. Thus, the area is

$$A = \int_{-1}^{2} g(x) - f(x)\,\mathrm{d}x = \int_{-1}^{2} 4 + 2x - 2x^2\,\mathrm{d}x = \left[4 + 2x - 2x^2\right]_{-1}^{2} = 2\left(-4 + 3x^2 - x^3\right)$$



2. (k points) Given the function

$$f(x) = -9x^2 - 7x^3$$

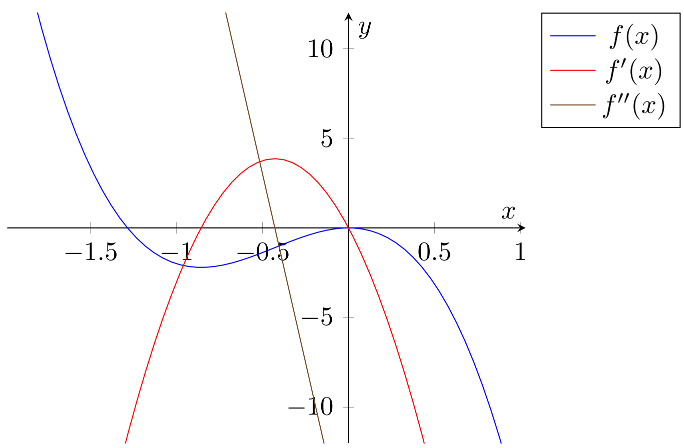
   (a) Sketch f, f' and f'' in one coordinate system.

   (b) Identify all of the minimum and maximum points and find its inflection points.

Solution:

(a) First, calculate the derivatives

$$\begin{aligned} f(x) &= -9x^2 - 7x^3 \\ f'(x) &= -18x - 21x^2 \\ f''(x) &= -18 - 42x \\ f'''(x) &= -42 \end{aligned}$$



(b) The function f has zeros at $x_1 = \frac{-9}{7}$ and at $x_2 = 0$. The function f' has zeros at $x_3 = \frac{-6}{7}$ and at $x_4 = 0$. The function f has a minimum at $(\frac{-6}{7}, 18.0)$ because $f''(x_3) > 0$ and a maximum at $(0, -18)$ because $f''(x_4) < 0$.