

1. (k points) How big is the parabolic segment between the parabola $f(x) = \frac{x^2}{2}$ and the line $g(x) = 4 + x$?

Sketch a graph to visualize the desired area.

Solution: The functions intersect at $P_1(-2, 2)^T$ and at $P_2(4, 8)^T$. Thus, the area is

$$A = \int_{-2}^4 g(x) - f(x) \, dx = \int_{-2}^4 4 + x + \left(\frac{-1}{2}\right) x^2 \, dx = \left[\frac{1}{6} x (24 + 3x - x^2) \right]_{-2}^4 = -16 + 3x^2 - 0.5x^3$$

