

1. (k points) Given the function

$$f(x) = -6x^2 + 7x^3$$

- (a) Sketch f , f' and f'' in one coordinate system.
(b) Identify all of the minimum and maximum points and find its inflection points.

Solution:

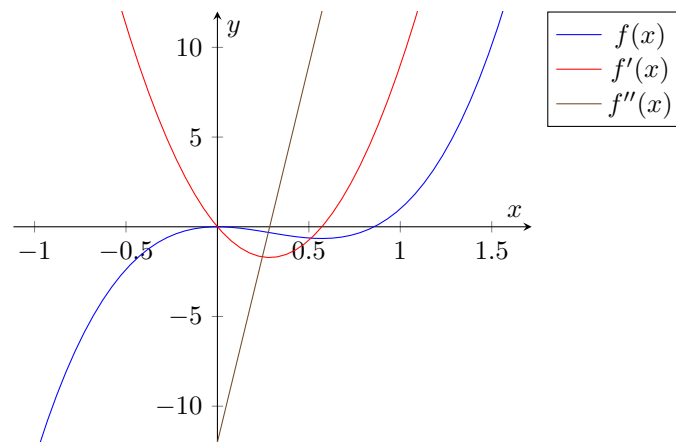
- (a) First, calculate the derivatives

$$f(x) = -6x^2 + 7x^3$$

$$f'(x) = -12x + 21x^2$$

$$f''(x) = -12 + 42x$$

$$f'''(x) = 42$$



- (b) The function f has zeros at $x_1 = \frac{6}{7}$ and at $x_2 = 0$. The function f' has zeros at $x_3 = \frac{4}{7}$ and at $x_4 = 0$. The function f has a minimum at $(\frac{4}{7}, -0.653061224489796)$ because $f''(x_3) > 0$ and a maximum at $(0, 0)$ because $f''(x_4) < 0$.