

1. Find all eigenvalues and eigenvectors of the matrix

$$A = \begin{bmatrix} -6 & 3 \\ 36 & -9 \end{bmatrix}.$$

Solution:

Calculate $A - \lambda I_2$:

$$A - \lambda I_2 = \begin{bmatrix} -6 & 3 \\ 36 & -9 \end{bmatrix} - \lambda \begin{bmatrix} 1.0 & 0.0 \\ 0.0 & 1.0 \end{bmatrix} = \begin{bmatrix} -6 - 1.0\lambda & 3.0 \\ 36.0 & -9 - 1.0\lambda \end{bmatrix}.$$

Then, calculate $\det(A - \lambda I_2)$.

$$\det(A - \lambda I_2) = -108.0 + (-9 - 1.0\lambda)(-6 - 1.0\lambda)$$

Now, we solve $\det(A - \lambda I_2) = 0$.

The matrix A has the eigenvalues $\lambda_1 = -18$ and $\lambda_2 = 3$.