1. Find all eigenvalues and eigenvectors of the matrix

$$A = \begin{bmatrix} -8 & 4 \\ 48 & -12 \end{bmatrix}.$$

Solution:

Calculate $A - \lambda I_2$:

$$A - \lambda I_2 = \begin{bmatrix} -8 & 4 \\ 48 & -12 \end{bmatrix} - \lambda \begin{bmatrix} 1.0 & 0.0 \\ 0.0 & 1.0 \end{bmatrix} = \begin{bmatrix} -8 - 1.0\lambda & 4.0 \\ 48.0 & -12 - 1.0\lambda \end{bmatrix}.$$

Then, calculate $\det(A - \lambda I_2)$.

$$\det(A - \lambda I_2) = -192.0 + (-12 - 1.0\lambda)(-8 - 1.0\lambda)$$

Now, we solve $\det(A - \lambda I_2) = 0$.

The matrix A has the eigenvalues $\lambda_1 = -24$ and $\lambda_2 = 4$.