

1. (k points) Given the function

$$f(x) = 2x^2 - 8x^3$$

- (a) Sketch f , f' and f'' in one coordinate system.
(b) Identify all of the minimum and maximum points and find its inflection points.

Solution:

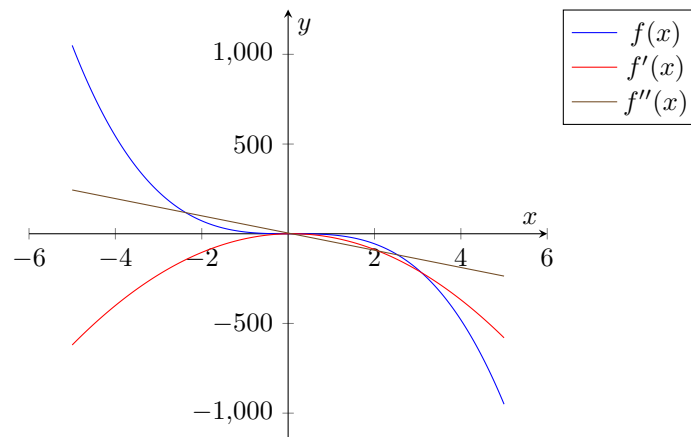
- (a) First, calculate the derivatives

$$f(x) = 2x^2 - 8x^3$$

$$f'(x) = 4x - 24x^2$$

$$f''(x) = 4 - 48x$$

$$f'''(x) = -48$$



- (b) The function has zeros at $x_1 = \frac{1}{4}$ and at $x_2 = 0$. The function f' has zeros at $x_1 = \frac{1}{6}$ and at $x_2 = 0$.

Because $f''(x_2) > 0$ there is a maximum at $(0,0)$ and because of $f''(x_3) > 0$ there is a minimum at $(1,-1)$.

The second derivative has a zero $x_4 = \frac{1}{2}$. Because $f'''(x_4) \neq 0$ there is an inflection point at $(\frac{1}{2}, -\frac{1}{2})$.