

1. (k points) Given the function

$$f(x) = x^2 + 6x^3$$

- (a) Sketch  $f$ ,  $f'$  and  $f''$  in one coordinate system.  
(b) Identify all of the minimum and maximum points and find its inflection points.

**Solution:**

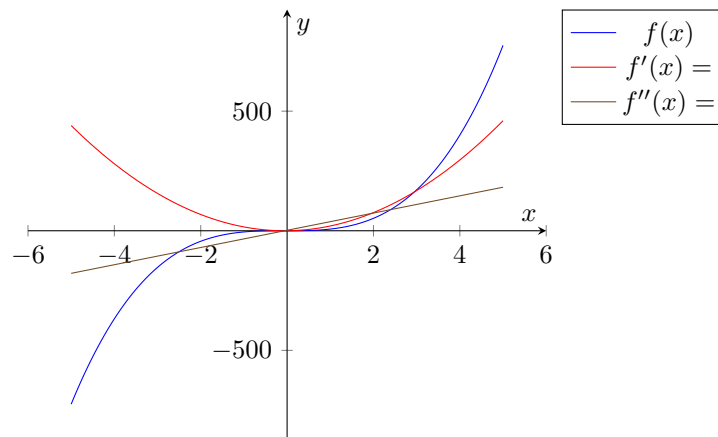
- (a) First, calculate the derivatives

$$f(x) = x^2 + 6x^3$$

$$f'(x) = 2x + 18x^2$$

$$f''(x) = 2 + 36x$$

$$f'''(x) = 36$$



- (b) The function  $f$  has the double zero  $x_0 = 0$  and the single zero  $x_1 = \frac{3}{2}$ .

The first derivative has zeros  $x_2 = 0$  and  $x_3 = 1$ .

Because  $f''(x_2) > 0$  there is a maximum at  $(0,0)$  and because of  $f''(x_3) > 0$  there is a minimum at  $(1,-1)$ .

The second derivative has a zero  $x_4 = \frac{1}{2}$ . Because  $f'''(x_4) \neq 0$  there is an inflection point at  $(\frac{1}{2}, -\frac{1}{2})$ .