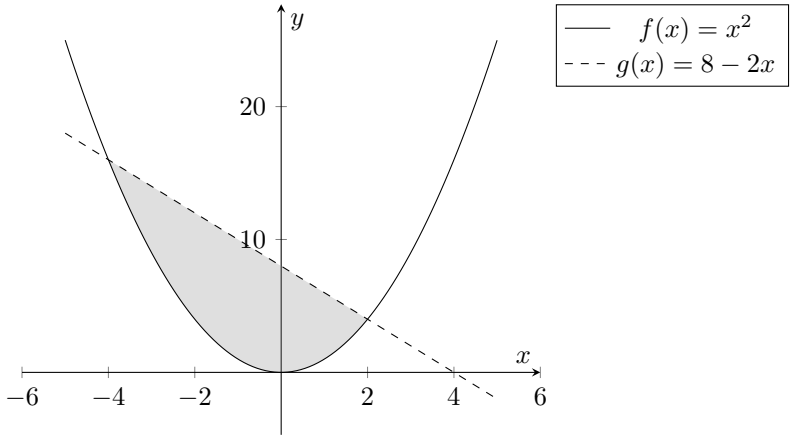
1. (k points) How big is the parabolic segment between the parabola $f(x) = x^2$ and the line $g(x) = 8 - 2x$?

   Sketch a graph to visualize the desired area.

Solution: The functions intersect at $P_1(-4, 16)^T$ and at $P_2(2, 4)^T$. Thus, the area is

$$A = \int_{-4}^{2} g(x) - f(x)\,\mathrm{d}x = \int_{-4}^{2} 8 - 2x - x^2\,\mathrm{d}x = \left[8 - 2x - x^2\right]_{-4}^{2} = -16 + 12x - x^3$$



2. (k points) Given the function

$$f(x) = -7x^2 + 4x^3$$

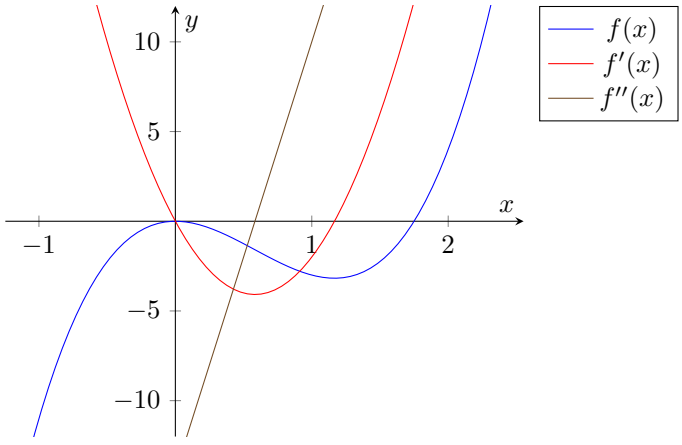
   (a) Sketch f, f' and f'' in one coordinate system.

   (b) Identify all of the minimum and maximum points and find its inflection points.

Solution:

(a) First, calculate the derivatives

$$\begin{aligned} f(x) &= -7x^2 + 4x^3 \\ f'(x) &= -14x + 12x^2 \\ f''(x) &= -14 + 24x \\ f'''(x) &= 24 \end{aligned}$$



(b) The function f has zeros at $x_1 = \frac{7}{4}$ and at $x_2 = 0$. The function f' has zeros at $x_3 = \frac{7}{6}$ and at $x_4 = 0$. The function f has a minimum at $(\frac{7}{6}, 14.0)$ because $f''(x_3) > 0$ and a maximum at $(0, -14)$ because $f''(x_4) < 0$.