Matriculation number: 10009012
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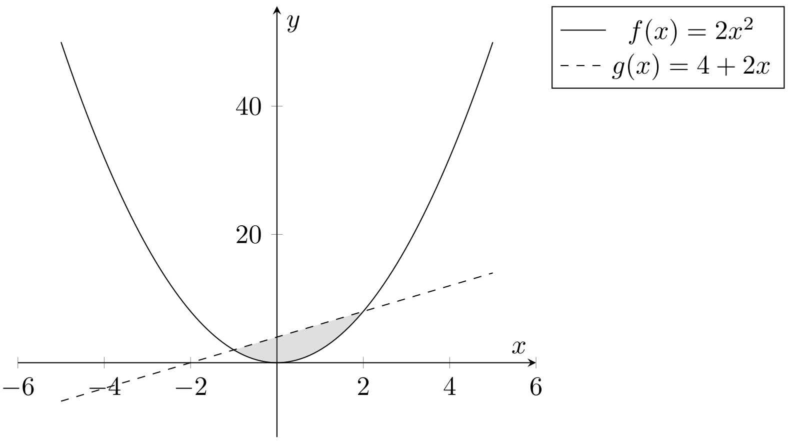
1. (k points) How big is the parabolic segment between the parabola $f(x) = 2x^2$ and the line $g(x) = 4 + 2x$?

   Sketch a graph to visualize the desired area.

Solution: The functions intersect at $P_1(-1, 2)^T$ and at $P_2(2, 8)^T$. Thus, the area is

$$A = \int_{-1}^{2} g(x) - f(x)\,\mathrm{d}x = \int_{-1}^{2} 4 + 2x - 2x^2\,\mathrm{d}x = \left[\frac{1}{3}x\big(12 + 3x - 2x^2\big)\right]_{-1}^{2} = 2\big(-4 + 3x^2 - x^3\big)$$



2. (k points) Given the function

$$f(x) = 5x^2 - 6x^3$$

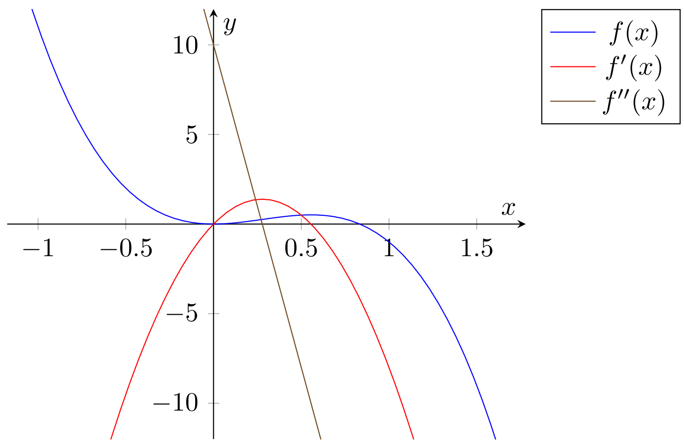
   (a) Sketch f, f' and f'' in one coordinate system.

   (b) Identify all of the minimum and maximum points and find its inflection points.

Solution:

(a) First, calculate the derivatives

$$\begin{aligned} f(x) &= 5x^2 - 6x^3 \\ f'(x) &= 10x - 18x^2 \\ f''(x) &= 10 - 36x \\ f'''(x) &= -36 \end{aligned}$$



(b) The function f has zeros at $x_1 = \frac{5}{6}$ and at $x_2 = 0$. The function f' has zeros at $x_3 = \frac{5}{9}$ and at $x_4 = 0$. The function f has a maximum at $(\frac{5}{9}, 0.51440329218107)$ because $f''(x_3) < 0$ and a minimum at $(0, 0)$ because $f''(x_4) > 0$.

3. Find all eigenvalues and eigenvectors of the matrix

$$A = \begin{bmatrix} 6 & -3 \\ -36 & 9 \end{bmatrix}.$$

Solution:

Calculate $A - \lambda I_2$:

$$A - \lambda I_2 = \begin{bmatrix} 6 & -3 \\ -36 & 9 \end{bmatrix} - \lambda \begin{bmatrix} 1.0 & 0.0 \\ 0.0 & 1.0 \end{bmatrix} = \begin{bmatrix} 6 - 1.0\lambda & -3.0 \\ -36.0 & 9 - 1.0\lambda \end{bmatrix}.$$

Then, calculate $\det(A - \lambda I_2)$.

$$\det(A - \lambda I_2) = -108.0 + (6 - 1.0\lambda)(9 - 1.0\lambda)$$

Now, we solve $\det(A - \lambda I_2) = 0$.

The matrix A has the eigenvalues $\lambda_1 = -3$ and $\lambda_2 = 18$.