1. (k points) Given the function

$$f(x) = -6x^2 + 6x^3$$

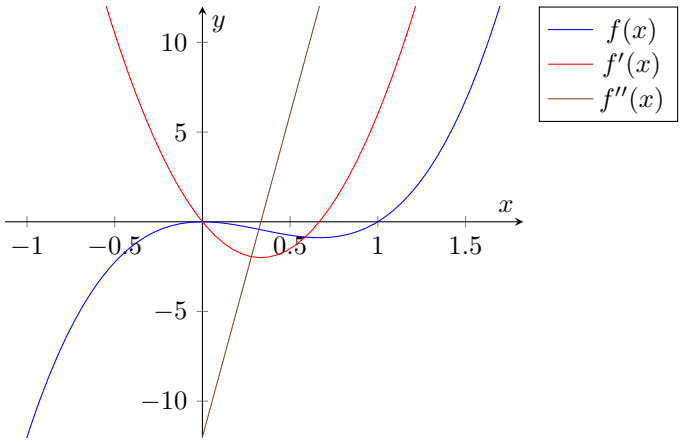
(a) Sketch f, f' and f'' in one coordinate system.

(b) Identify all of the minimum and maximum points and find its inflection points.

Solution:

(a) First, calculate the derivatives

$$\begin{aligned} f(x) &= -6x^2 + 6x^3 \\ f'(x) &= -12x + 18x^2 \\ f''(x) &= -12 + 36x \\ f'''(x) &= 36 \end{aligned}$$



(b) The function f has zeros at $x_1 = 1$ and at $x_2 = 0$. The function f' has zeros at $x_3 = \frac{2}{3}$ and at $x_4 = 0$. The function f has a minimum at $(\frac{2}{3}, -0.888888888888889)$ because $f''(x_3) > 0$ and a maximum at $(0, 0)$ because $f''(x_4) < 0$.