

Matriculation number: 10003456

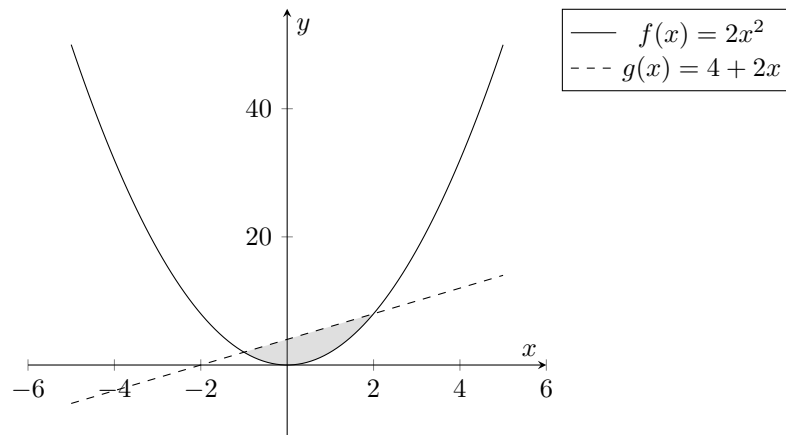
Student name: Joe Schmoe

1. (k points) How big is the parabolic segment between the parabola $f(x) = 2x^2$ and the line $g(x) = 4 + 2x$?

Sketch a graph to visualize the desired area.

Solution: The functions intersect at $P_1(-1, 2)^T$ and at $P_2(2, 8)^T$. Thus, the area is

$$A = \int_{-1}^2 g(x) - f(x) \, dx = \int_{-1}^2 4 + 2x - 2x^2 \, dx = \left[\frac{1}{3}x(12 + 3x - 2x^2) \right]_{-1}^2 = 2(-4 + 3x^2 - x^3)$$



2. (k points) Given the function

$$f(x) = 4x^2 - 6x^3$$

- (a) Sketch f, f' and f'' in one coordinate system.
(b) Identify all of the minimum and maximum points and find its inflection points.

Solution:

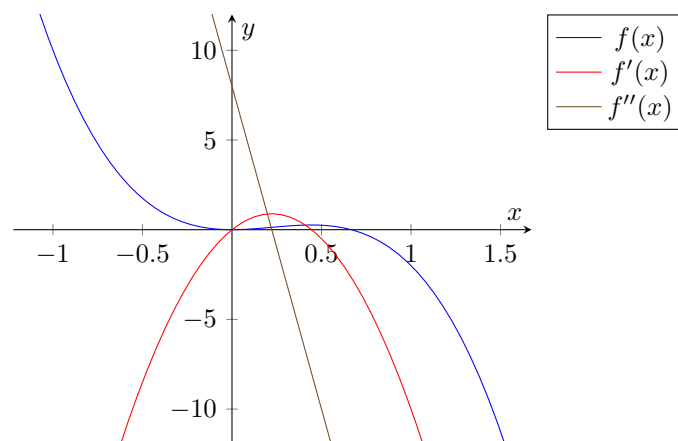
- (a) First, calculate the derivatives

$$f(x) = 4x^2 - 6x^3$$

$$f'(x) = 8x - 18x^2$$

$$f''(x) = 8 - 36x$$

$$f'''(x) = -36$$



(b) The function f has zeros at $x_1 = \frac{2}{3}$ and at $x_2 = 0$. The function f' has zeros at $x_3 = \frac{4}{9}$ and at $x_4 = 0$. The function f has a maximum at $(\frac{4}{9}, 0.263374485596708)$ because $f''(x_3) < 0$ and a minimum at $(0, 0)$ because $f''(x_4) > 0$.

3. Find all eigenvalues and eigenvectors of the matrix

$$A = \begin{bmatrix} -12 & 15 \\ 15 & -12 \end{bmatrix}.$$

Solution:

Calculate $A - \lambda I_2$:

$$A - \lambda I_2 = \begin{bmatrix} -12 & 15 \\ 15 & -12 \end{bmatrix} - \lambda \begin{bmatrix} 1.0 & 0.0 \\ 0.0 & 1.0 \end{bmatrix} = \begin{bmatrix} -12 - 1.0\lambda & 15.0 \\ 15.0 & -12 - 1.0\lambda \end{bmatrix}.$$

Then, calculate $\det(A - \lambda I_2)$.

$$\det(A - \lambda I_2) = -225.0 + (-12 - 1.0\lambda)^2$$

Now, we solve $\det(A - \lambda I_2) = 0$.

The matrix A has the eigenvalues $\lambda_1 = -27$ and $\lambda_2 = 3$.