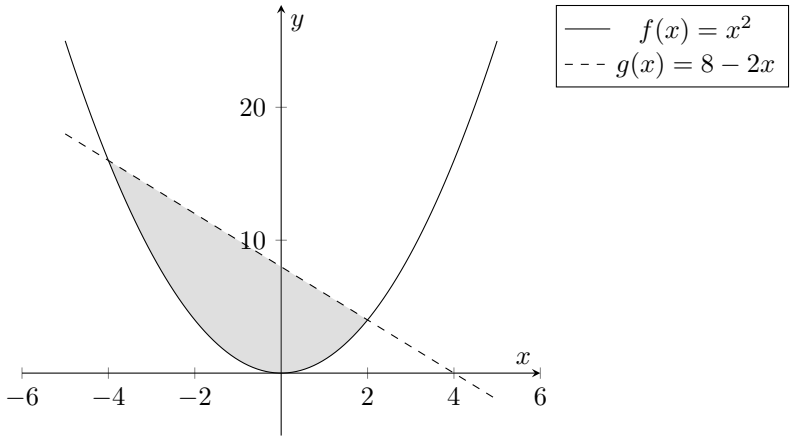
1. (k points) How big is the parabolic segment between the parabola $f(x) = x^2$ and the line $g(x) = 8 - 2x$?

   Sketch a graph to visualize the desired area.

**Solution:** The functions intersect at $P_1(-4, 16)^T$ and at $P_2(2, 4)^T$. Thus, the area is

$$A = \int_{-4}^{2} g(x) - f(x)\,\mathrm{d}x = \int_{-4}^{2} 8 - 2x - x^2\,\mathrm{d}x = \left[\frac{1}{3}x\left(24 - 3x - x^2\right)\right]_{-4}^{2} = -16 + 12x - x^3$$




2. (k points) Given the function

$$f(x) = -x^2 + 10x^3$$

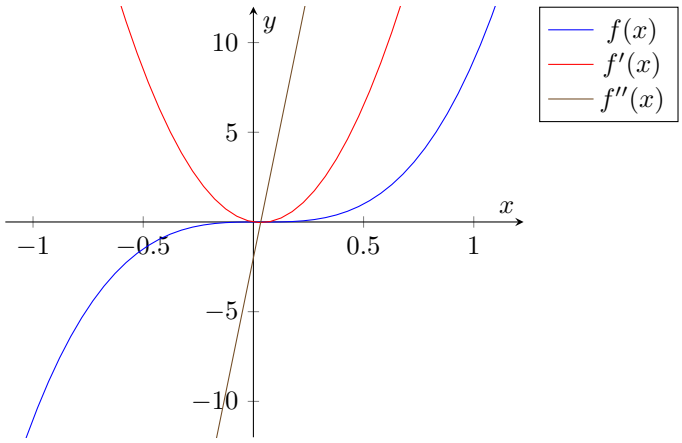
   (a) Sketch $f, f'$ and $f''$ in one coordinate system.

   (b) Identify all of the minimum and maximum points and find its inflection points.

**Solution:**

(a) First, calculate the derivatives

$$\begin{aligned} f(x) &= -x^2 + 10x^3 \\ f'(x) &= -2x + 30x^2 \\ f''(x) &= -2 + 60x \\ f'''(x) &= 60 \end{aligned}$$



(b) The function $f$ has zeros at $x_1 = \frac{1}{10}$ and at $x_2 = 0$. The function $f'$ has zeros at $x_3 = \frac{1}{15}$ and at $x_4 = 0$. The function $f$ has a minimum at $(\frac{1}{15}, 2.0)$ because $f''(x_3) > 0$ and a maximum at $(0, -2)$ because $f''(x_4) < 0$.