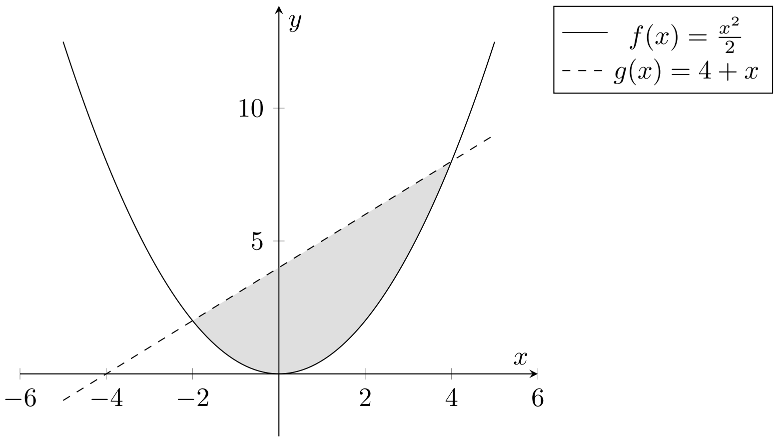
1. (k points) How big is the parabolic segment between the parabola $f(x) = \frac{x^2}{2}$ and the line $g(x) = 4 + x$?

   Sketch a graph to visualize the desired area.

Solution: The functions intersect at $P_1(-2, 2)^T$ and at $P_2(4, 8)^T$. Thus, the area is

$$A = \int_{-2}^{4} g(x) - f(x)\,\mathrm{d}x = \int_{-2}^{4} 4 + x + \left(\frac{-1}{2}\right) x^2\,\mathrm{d}x = \left[\frac{1}{6}x\left(24 + 3x - x^2\right)\right]_{-2}^{4} = -16{+}3x^2 - 0.5x^3$$



2. (k points) Given the function

$$f(x) = 5x^2 + 7x^3$$

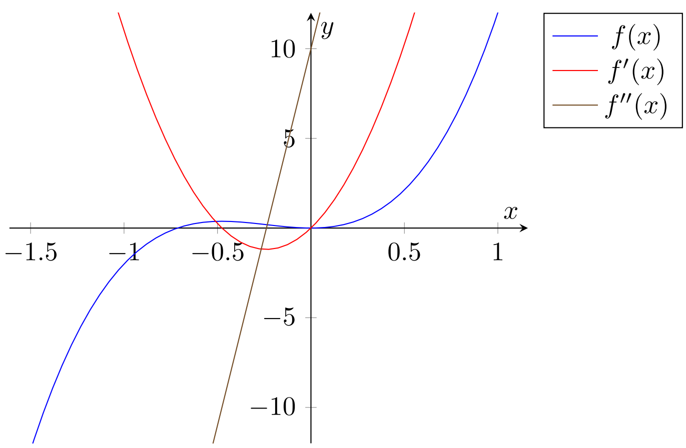
   (a) Sketch f, f' and f'' in one coordinate system.

   (b) Identify all of the minimum and maximum points and find its inflection points.

Solution:

(a) First, calculate the derivatives

$$\begin{aligned} f(x) &= 5x^2 + 7x^3 \\ f'(x) &= 10x + 21x^2 \\ f''(x) &= 10 + 42x \\ f'''(x) &= 42 \end{aligned}$$



(b) The function f has zeros at $x_1 = \frac{-5}{7}$ and at $x_2 = 0$. The function f' has zeros at $x_3 = \frac{-10}{21}$ and at $x_4 = 0$. The function f has a maximum at $(\frac{-10}{21}, -10.0)$ because $f''(x_3) < 0$ and a minimum at $(0, 10)$ because $f''(x_4) > 0$.