

1. (k points) How big is the parabolic segment between the parabola $f(x) = 2x^2$ and the line $g(x) = 4 + 2x$?

Sketch a graph to visualize the desired area.

Solution: The functions intersect at $P_1(-1, 2)^T$ and at $P_2(2, 8)^T$. Thus, the area is

$$A = \int_{-1}^2 g(x) - f(x) \, dx = \int_{-1}^2 4 + 2x - 2x^2 \, dx = \left[\frac{1}{3}x(12 + 3x - 2x^2) \right]_{-1}^2 = 2(-4 + 3x^2 - x^3)$$

