

Matriculation number: 10003456

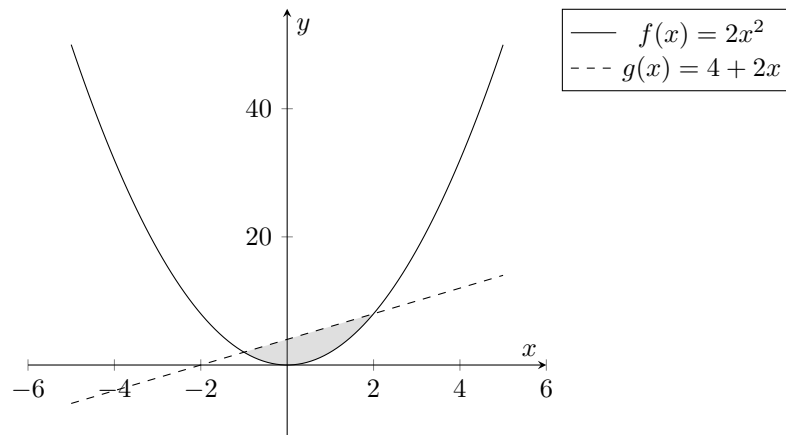
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1. (k points) How big is the parabolic segment between the parabola  $f(x) = 2x^2$  and the line  $g(x) = 4 + 2x$ ?

Sketch a graph to visualize the desired area.

**Solution:** The functions intersect at  $P_1(-1, 2)^T$  and at  $P_2(2, 8)^T$ . Thus, the area is

$$A = \int_{-1}^2 g(x) - f(x) \, dx = \int_{-1}^2 4 + 2x - 2x^2 \, dx = \left[ \frac{1}{3}x(12 + 3x - 2x^2) \right]_{-1}^2 = 2(-4 + 3x^2 - x^3)$$



2. (k points) Given the function

$$f(x) = 4x^2 - 6x^3$$

- (a) Sketch  $f, f'$  and  $f''$  in one coordinate system.  
(b) Identify all of the minimum and maximum points and find its inflection points.

**Solution:**

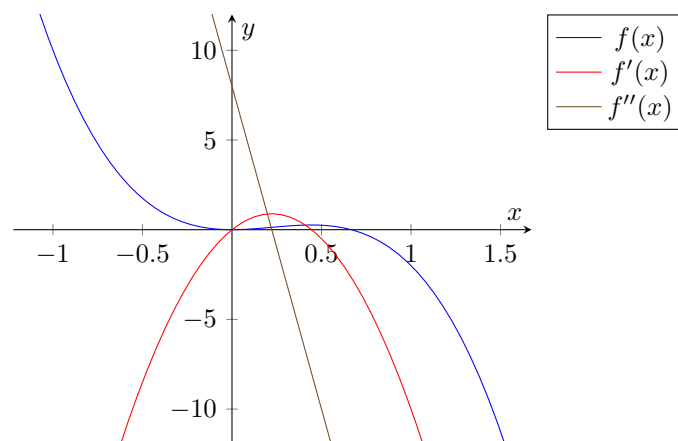
- (a) First, calculate the derivatives

$$f(x) = 4x^2 - 6x^3$$

$$f'(x) = 8x - 18x^2$$

$$f''(x) = 8 - 36x$$

$$f'''(x) = -36$$



(b) The function  $f$  has zeros at  $x_1 = \frac{2}{3}$  and at  $x_2 = 0$ . The function  $f'$  has zeros at  $x_3 = \frac{4}{9}$  and at  $x_4 = 0$ . The function  $f$  has a maximum at  $(\frac{4}{9}, 0.263374485596708)$  because  $f''(x_3) < 0$  and a minimum at  $(0, 0)$  because  $f''(x_4) > 0$ .

3. Find all eigenvalues and eigenvectors of the matrix

$$A = \begin{bmatrix} -12 & 15 \\ 15 & -12 \end{bmatrix}.$$

**Solution:**

Calculate  $A - \lambda I_2$ :

$$A - \lambda I_2 = \begin{bmatrix} -12 & 15 \\ 15 & -12 \end{bmatrix} - \lambda \begin{bmatrix} 1.0 & 0.0 \\ 0.0 & 1.0 \end{bmatrix} = \begin{bmatrix} -12 - 1.0\lambda & 15.0 \\ 15.0 & -12 - 1.0\lambda \end{bmatrix}.$$

Then, calculate  $\det(A - \lambda I_2)$ .

$$\det(A - \lambda I_2) = -225.0 + (-12 - 1.0\lambda)^2$$

Now, we solve  $\det(A - \lambda I_2) = 0$ .

The matrix  $A$  has the eigenvalues  $\lambda_1 = -27$  and  $\lambda_2 = 3$ .