

Dec 2024

Cryptography & Pacific War

Felix Wu

Pingxiang Middle School

Table of Contents

3	Battle of Midway	11	Cryptanalysis
6	Timeline of the Jap Codes	13	Asymmetric-key Cipher
7	Cryptography	18	Q & A
9	Substitution Cipher	19	References





Richard Halsey Best

The only pilot who has sunk two aircraft carriers (赤城 / 飛龍) in a single day.



Navy Cross: August 7, 1942

Service: United States Navy

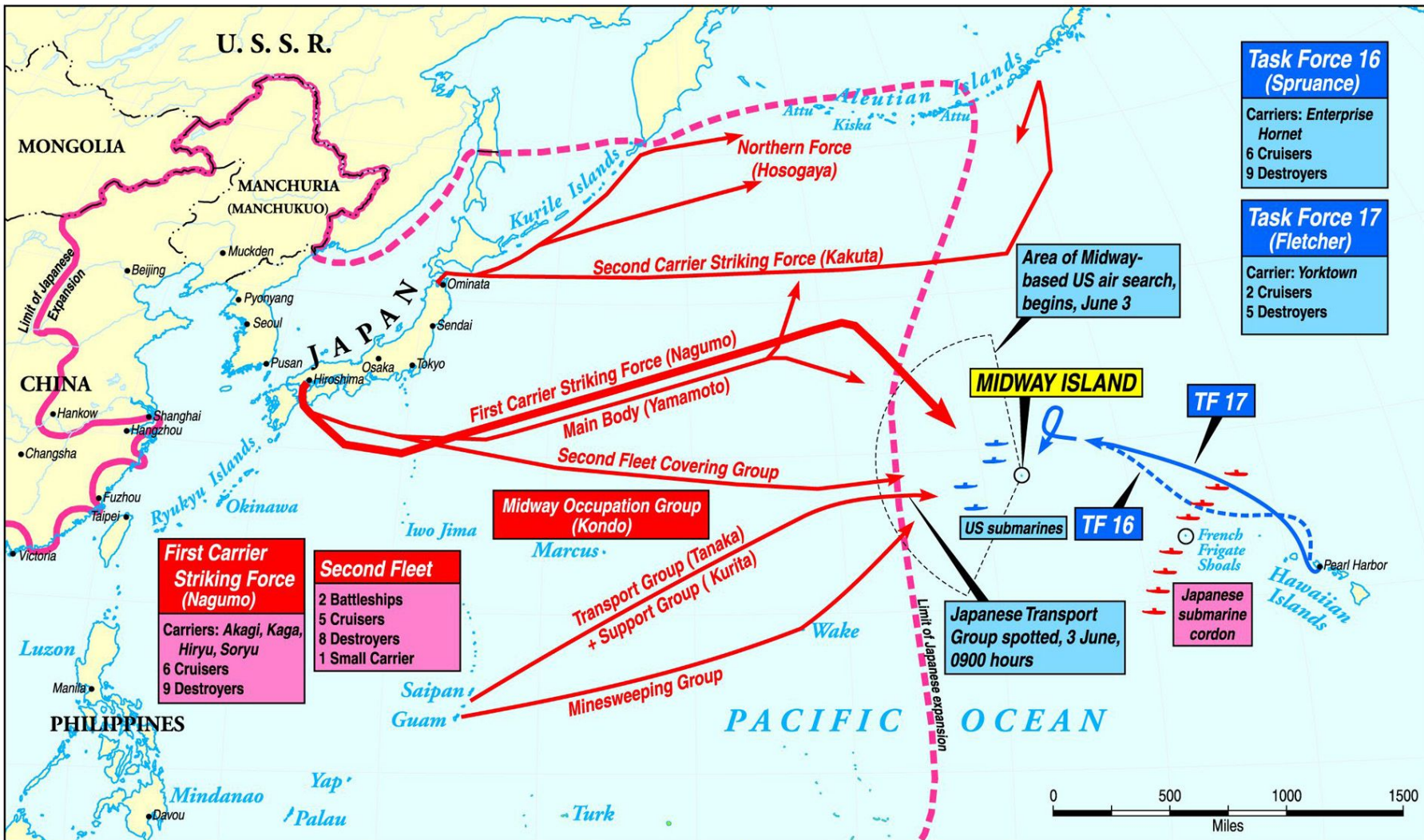
Rank: Lieutenant Commander

Battalion: Bombing Squadron 6 (VB-6)

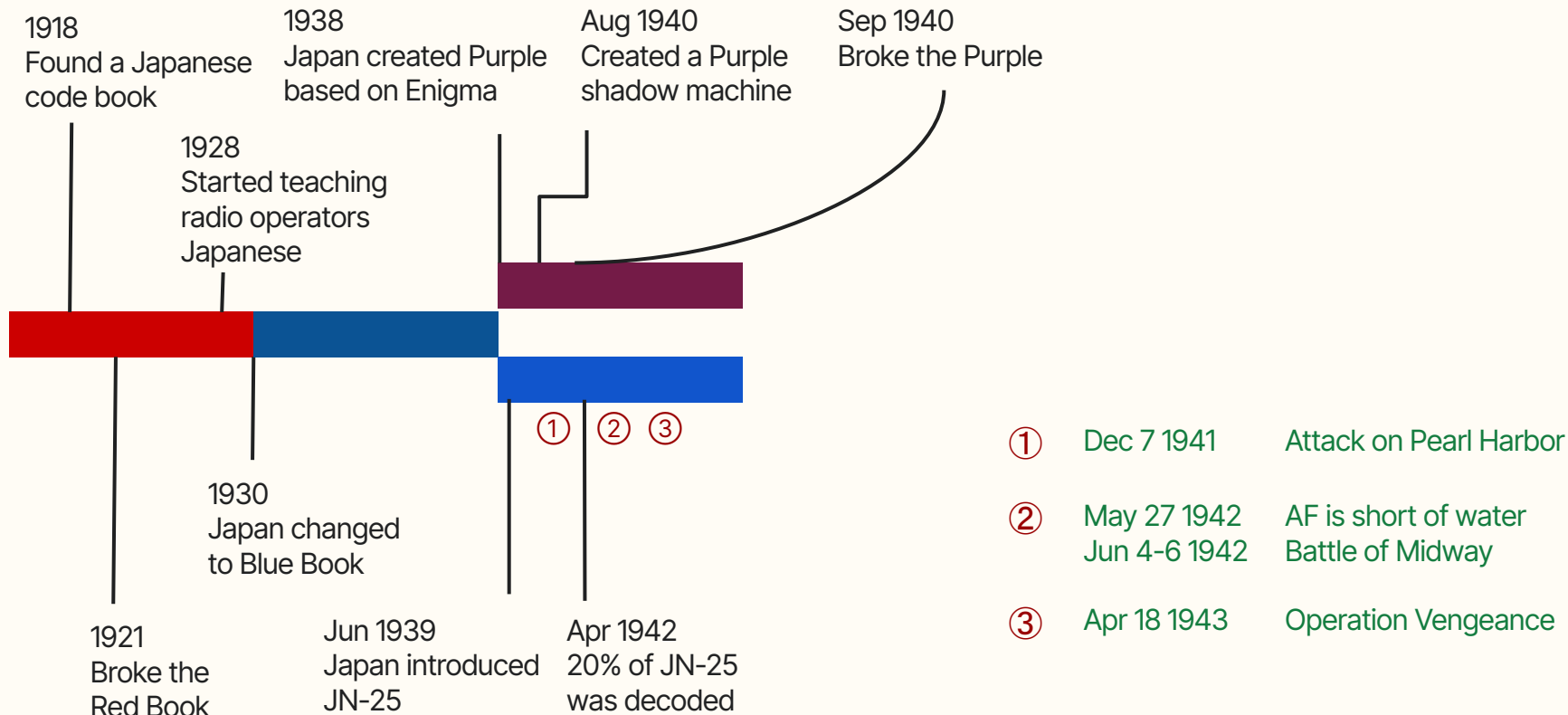
Division: U.S.S. Enterprise (CV-6)

Action Date: June 4 – 6, 1942

Q: Can you name any other famous heroes in the Pacific War?



Why did the US Navy Ambush there?



What is Cryptography?

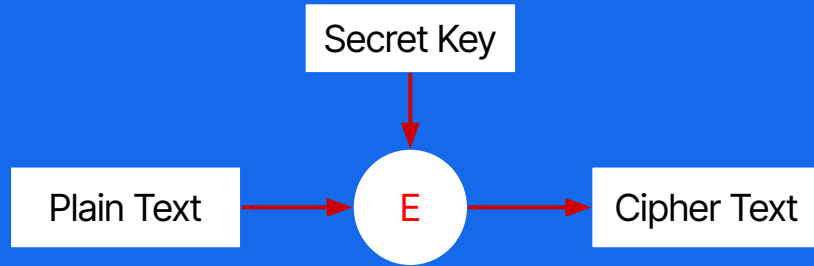
Cryptography is the process of hiding or coding information so that only the person a message was intended for can read it. It's part of Secure Communication.

Decryption

Encryption



Encryption & Decryption

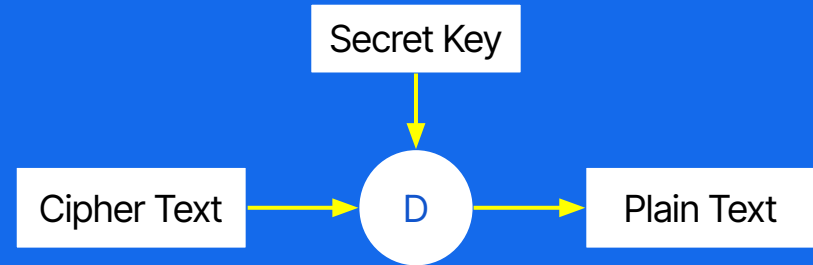


Kerckhoffs's Principle

A cryptosystem should be secure, even if everything about the system except the key is public knowledge.

Shannon's Maxim

The enemy knows the system.



Caesar cipher

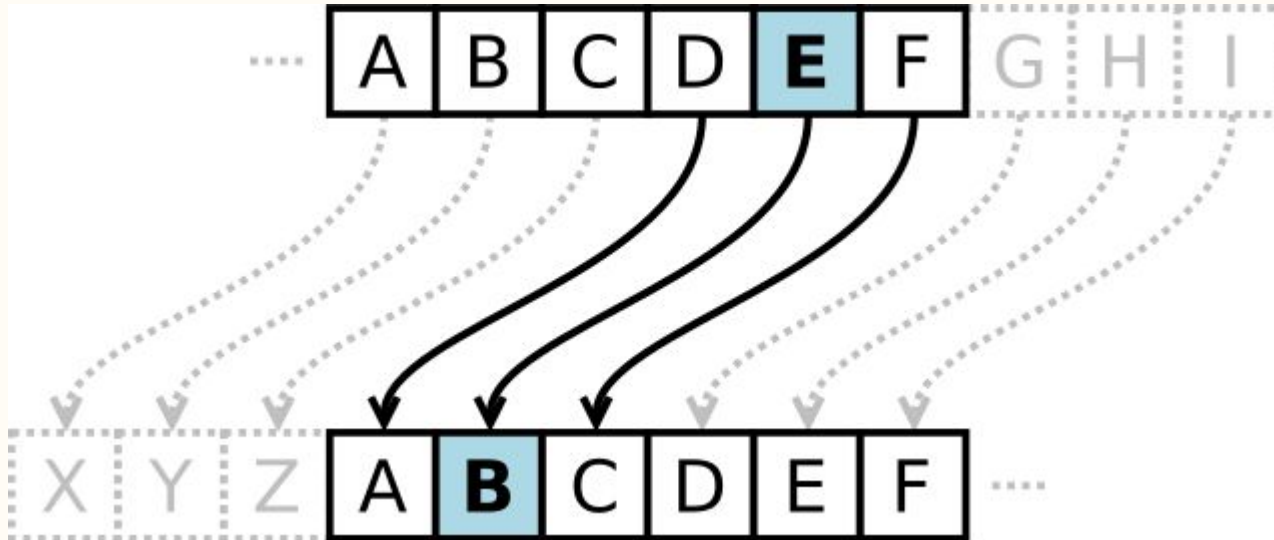
- Developed in ancient Rome
- Named after Julius Caesar

$$E_n(x) = (x + n) \bmod 26$$

$$D_n(x) = (x - n) \bmod 26$$



Secret Key



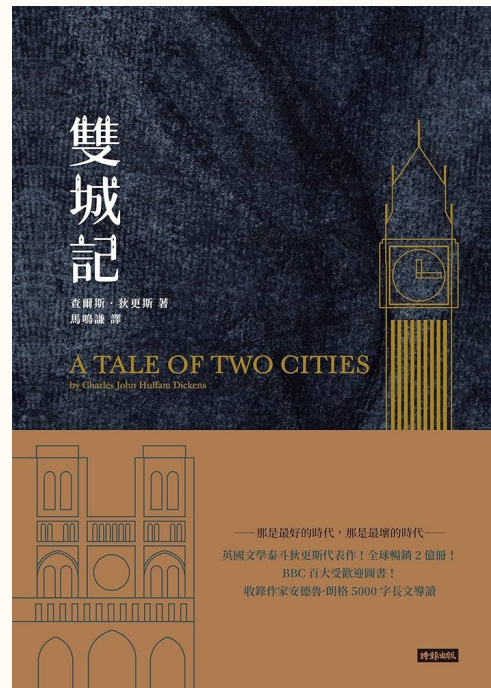
Mixed Alphabet cipher

Example:

It was the best of times, it was the worst of times.

>>>

Er vpq rda xaqr kb reiaq, er vpq rda vkoqr kb reiaq.



Plaintext

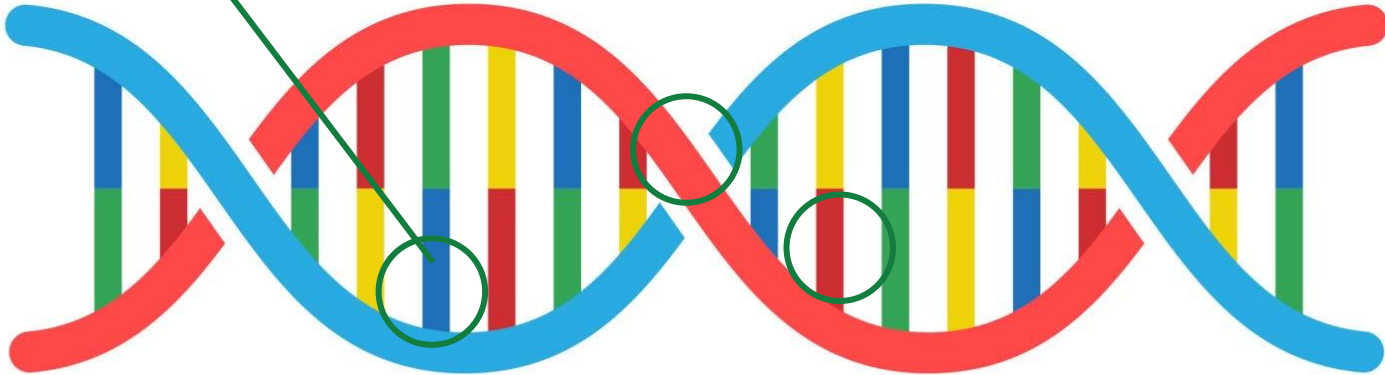
Cypher

A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
P	X	M	S	A	B	C	D	E	F	G	H	I	J	K	L	N	O	Q	R	T	U	V	W	Y	Z

Secret Key

What is Cryptanalysis?

The process of analyzing information systems in order to understand hidden aspects of the systems. It's used to gain access to the contents of encrypted messages, even if the key is unknown.



Brute Force Attack

Caesar cipher

It can be easily broken even in a ciphertext-only scenario. Since there are only a limited number of possible shifts, an attacker can try decrypting with all possible keys and find out the one which generates the meaningful plaintext.

Total possible keys: 25.

Frequency Analysis

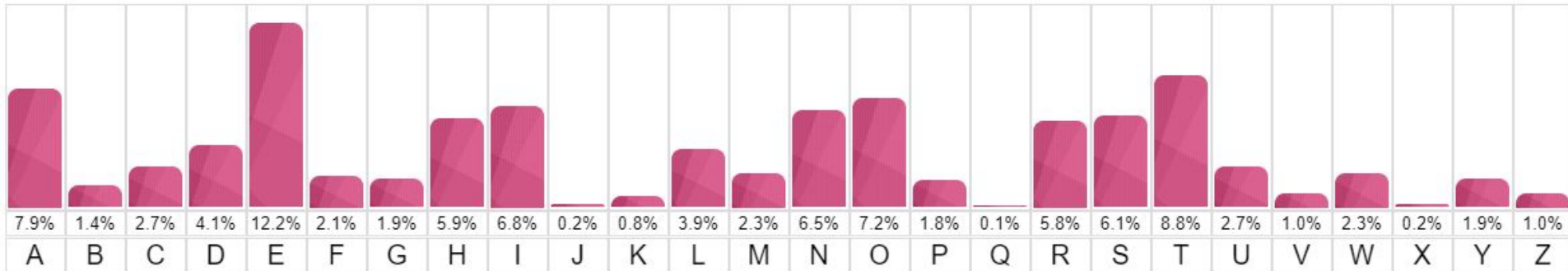
Mixed Alphabet cipher

Q: How many possible keys does it support?

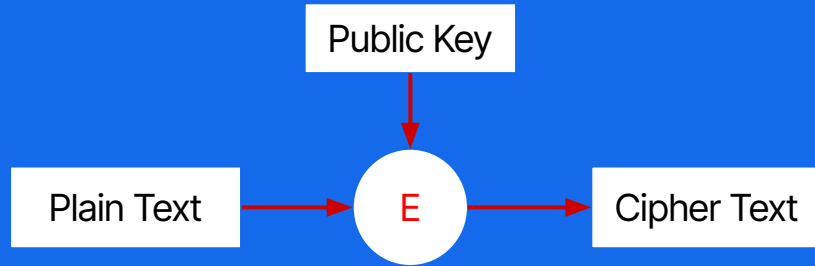
Analyzing the frequency distribution of the ciphertext allows formation of partial words, which can be tentatively filled in, progressively expanding the solution.

Known-plaintext Attack

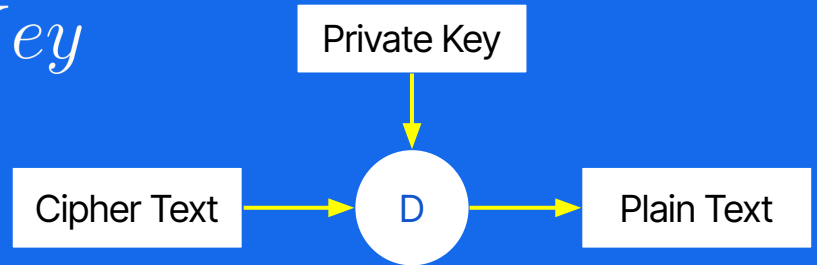
AF is short of water.



Encryption & Decryption



Public Key \neq *Private Key*



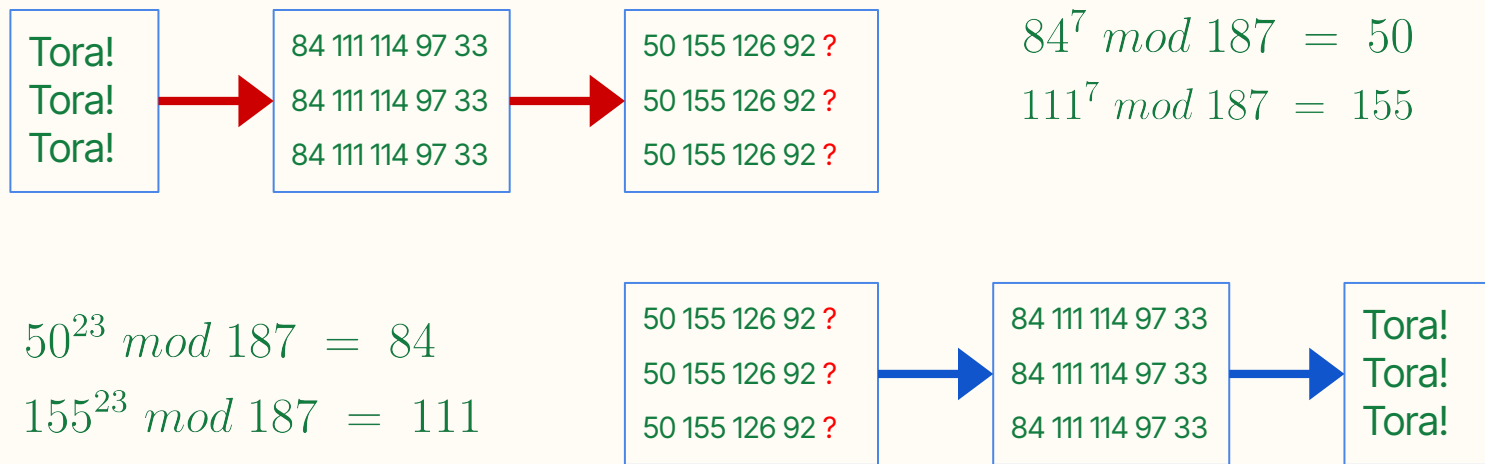
RSA: Example

Public Key: ($n = 187$, $e = 7$)

Private Key: ($n = 187$, $d = 23$)

$$c = p^e \bmod n$$

$$p = c^d \bmod n$$



Q: What is the last cypher number?

RSA: the Magic Numbers

- 1) Choose two large prime numbers p and q randomly.
- 2) Let $n = pq$.
- 3) Let $\varphi = (p - 1)(q - 1)$.
- 4) Choose a large number $e \in [2, \varphi - 1]$ that is coprime to φ .
- 5) Compute $d \in [2, \varphi - 1]$ such that

$$e \cdot d = 1 \pmod{\varphi}$$

There is a unique such d . Furthermore, d must be coprime to φ .

- 6) Announce to the whole world the pair (e, n) , which is the public key.
- 7) Keep d secret, and the private key is (d, n) .

RSA: the Truth behind Magic

Definition

Given an integer $p > 0$, define \mathbb{Z}_p as the set $\{0, 1, \dots, p - 1\}$.

Fermat's Little Theorem

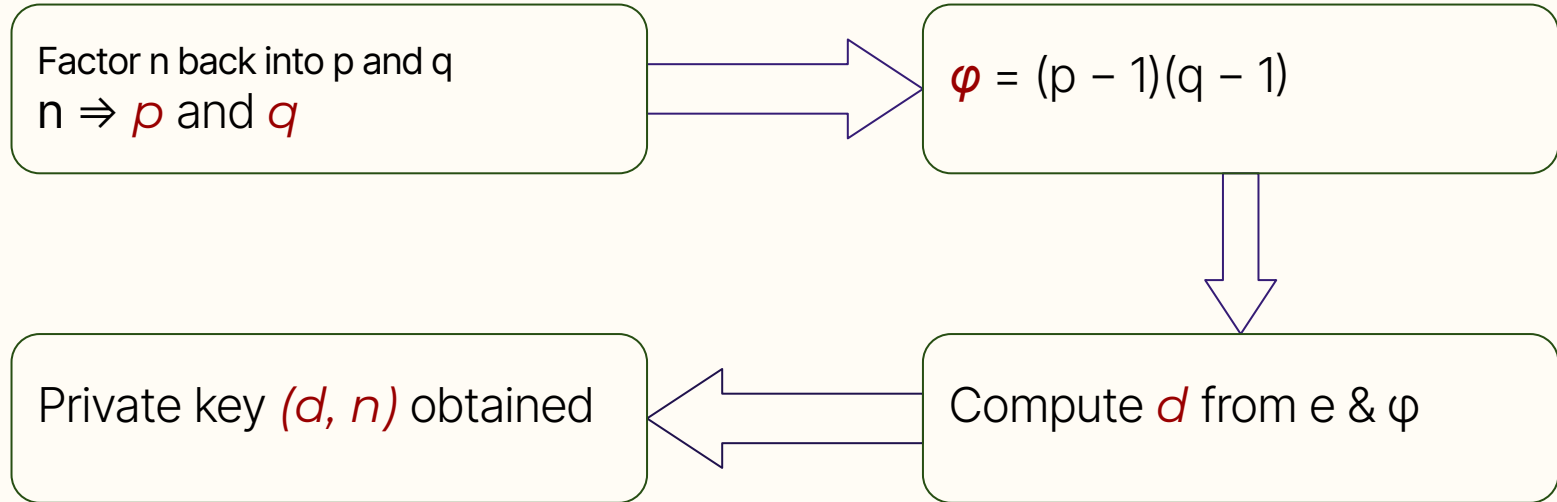
If p is a prime number, for any non-zero $a \in \mathbb{Z}_p$, it holds that

$$a^{p-1} = 1 \mod p.$$

Chinese Remainder Theorem

Let p and q be two co-prime integers. If $x = a \mod p$ and $x = a \mod q$, then $x = a \mod pq$.

RSA: How to Break?



Assumption

Factoring large integers is computationally impossible.



Questions?

References

1. [Battle of Midway](#) (Wikipedia)
2. [Purple](#) (Grand Valley State University)
3. [Genevieve Grotjan Feinstein](#) (Wikipedia)
4. [Marian Rejewski](#) (Wikipedia)
5. [Type B Cipher Machine](#) (Wikipedia)
6. [Operation Vengeance](#) (The National WWII Museum)
7. [Frequency Analysis](#) (101 Computing)
8. [RSA Cyptosystem](#) & [Correctness of RSA](#) (Prof Yufei Tao, CUHK)

Available on GitHub

