大学物理甲 2 习题解答

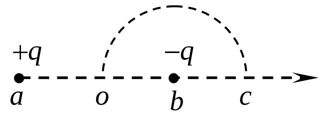
9.24（1）$U_o=\dfrac{1}{4\pi\varepsilon_0}(\dfrac{q_1}{r}+\dfrac{q_2}{r}+\dfrac{q_3}{r}+\dfrac{q_4}{r})=9\times10^9\dfrac{4\times2.0\times10^{-9}}{5.0\times10^{-2}}=1.44\times10^3(\text{V})$

（2）$A=q_0(U_\infty-U_o)=-q_0U_o=-1.44\times10^{-6}(\text{J})$

（3）$\Delta W=W_o-W_\infty=W_o=1.44\times10^{-6}(\text{J})$

9.25（1）$U_o=\dfrac{q}{4\pi\varepsilon_0 l}-\dfrac{q}{4\pi\varepsilon_0 l}=0$　　　$U_c=\dfrac{q}{4\pi\varepsilon_0 3l}-\dfrac{q}{4\pi\varepsilon_0 l}=-\dfrac{q}{6\pi\varepsilon_0 l}$

$A_{oc}=q_0(U_o-U_c)=\dfrac{qq_0}{6\pi\varepsilon_0 l}$



（2）$A_{c\infty}=-q_0(U_c-U_\infty)=\dfrac{qq_0}{6\pi\varepsilon_0 l}$

9.28（a）$E_1=0$　　　$(r<R_1)$

$E_2=\dfrac{Q_1}{4\pi\varepsilon_0 r^2}$　　　$(R_1<r<R_2)$　　　$E_3=\dfrac{Q_1+Q_2}{4\pi\varepsilon_0 r^2}$　　　$(r>R_2)$

$U_1=\int_{0.2}^{0.3}\boldsymbol{E}_2\cdot\mathrm{d}\boldsymbol{r}+\int_{0.3}^{\infty}\boldsymbol{E}_3\cdot\mathrm{d}\boldsymbol{r}=900\,(\text{V})$　　　$U_2=\int_{0.5}^{\infty}\boldsymbol{E}_3\cdot\mathrm{d}\boldsymbol{r}=450\,(\text{V})$

（b）$U_{小}=\dfrac{Q_1}{4\pi\varepsilon_0 r}$　　　$U_{大}=\dfrac{Q_2}{4\pi\varepsilon_0 r}$

$U_1=\dfrac{Q_1}{4\pi\varepsilon_0\times0.2}+\dfrac{Q_2}{4\pi\varepsilon_0\times0.3}=900\,(\text{V})$　　　$U_2=\dfrac{Q_1}{4\pi\varepsilon_0\times0.5}+\dfrac{Q_2}{4\pi\varepsilon_0\times0.5}=450\,(\text{V})$

9.29　$\mathrm{d}U=\dfrac{\mathrm{d}q}{4\pi\varepsilon_0 x}=\dfrac{\lambda\mathrm{d}x}{4\pi\varepsilon_0 x}$

$U_{AB}=U_{CD}=\int_R^{2R}\dfrac{\lambda\mathrm{d}x}{4\pi\varepsilon_0 x}=\dfrac{\lambda}{4\pi\varepsilon_0}\ln2$　　　$U_{BC}=\int_0^{\pi}\dfrac{\lambda R\mathrm{d}\theta}{4\pi\varepsilon_0 R}=\dfrac{\lambda}{4\varepsilon_0}$

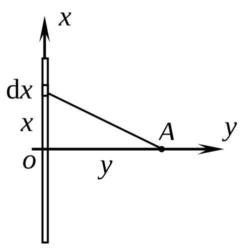
$U_0=U_{AB}+U_{BC}+U_{CD}=\dfrac{\lambda}{4\pi\varepsilon_0}(2\ln2+\pi)$

9.30　$U_+=\dfrac{q}{4\pi\varepsilon_0\sqrt{(x-l/2)^2+R^2}}$　　　$U_-=\dfrac{-q}{4\pi\varepsilon_0\sqrt{(x+l/2)^2+R^2}}$

$U=U_++U_-=\dfrac{q}{4\pi\varepsilon_0}(\dfrac{1}{\sqrt{(x-l/2)^2+R^2}}-\dfrac{1}{\sqrt{(x+l/2)^2+R^2}})$

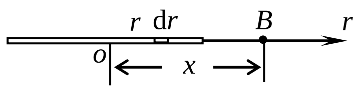
当 $l<<R$ 时，用泰勒级数在 $l=0$ 处展开，略去 l 的二次项，得：　$U=\dfrac{qlx}{4\pi\varepsilon_0(x^2+R^2)^{3/2}}$

9.31（1）$U=\int_{-l}^{l}\dfrac{(Q/2l)\mathrm{d}x}{4\pi\varepsilon_0\sqrt{x^2+y^2}}=\dfrac{Q}{4\pi\varepsilon_0 l}\ln\dfrac{\sqrt{l^2+y^2}+l}{y}$



$E=E_y=-\dfrac{\mathrm{d}U}{\mathrm{d}y}=\dfrac{Q}{4\pi\varepsilon_0 y\sqrt{l^2+y^2}}$

（2）$U=\int_{-l}^{l}\dfrac{Q\mathrm{d}r}{8\pi\varepsilon_0 l(x-r)}=\dfrac{Q}{8\pi\varepsilon_0 l}\ln\dfrac{x+l}{x-l}$

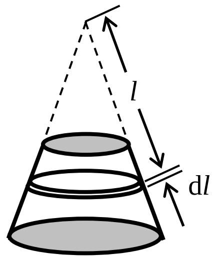


$E=E_x=-\dfrac{\mathrm{d}U}{\mathrm{d}x}=\dfrac{Q}{4\pi\varepsilon_0(x^2-l^2)}$

9.32（a）锥侧长为变量： $\mathrm{d}S = 2\pi r\mathrm{d}l = 2\pi l\sin\theta\mathrm{d}l$

$\mathrm{d}q = \sigma 2\pi l\sin\theta\mathrm{d}l$ $\mathrm{d}U = \dfrac{2\pi l\sigma\sin\theta\mathrm{d}l}{4\pi\varepsilon_0 l}$

$$U = \int_{R_1/\sin\theta}^{R_2/\sin\theta}\frac{2\pi l\sigma\sin\theta\mathrm{d}l}{4\pi\varepsilon_0 l} = \frac{\sigma}{2\varepsilon_0}(R_2 - R_1)$$



（b）锥高为变量： $\mathrm{d}S = 2\pi r\mathrm{d}l = 2\pi x\tan\theta\dfrac{\mathrm{d}x}{\cos\theta}$

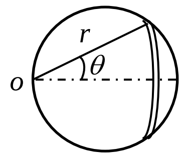
$$U = \int_{R_1/\tan\theta}^{R_2/\tan\theta}\frac{2\pi x\sigma\tan\theta\dfrac{\mathrm{d}x}{\cos\theta}}{4\pi\varepsilon_0\dfrac{x}{\cos\theta}} = \frac{\sigma}{2\varepsilon_0}(R_2 - R_1)$$

（c）半径为变量： $\mathrm{d}S = 2\pi r\mathrm{d}l = 2\pi r\dfrac{\mathrm{d}r}{\sin\theta}$ $U = \displaystyle\int_{R_1}^{R_2}\frac{2\pi r\sigma\dfrac{\mathrm{d}r}{\sin\theta}}{4\pi\varepsilon_0\dfrac{r}{\sin\theta}} = \frac{\sigma}{2\varepsilon_0}(R_2 - R_1)$

9.33 $\sigma = \dfrac{Q}{\pi(b^2 - a^2)}$ $\mathrm{d}q = \sigma\cdot 2\pi r\mathrm{d}r$ $\mathrm{d}U = \dfrac{\mathrm{d}q}{4\pi\varepsilon_0 r} = \dfrac{\sigma}{2\varepsilon_0}\mathrm{d}r$

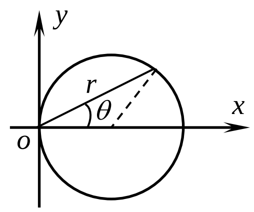
$$U = \int_a^b\frac{\sigma}{2\varepsilon_0}\mathrm{d}r = \frac{\sigma}{2\varepsilon_0}(b - a) = \frac{Q}{2\pi\varepsilon_0(a + b)} = 9\times 10^3(\mathrm{V})$$

9.34（a） $\mathrm{d}q = \sigma\cdot 2\theta r\mathrm{d}r$ $\mathrm{d}U = \dfrac{\mathrm{d}q}{4\pi\varepsilon_0 r} = \dfrac{\sigma\theta}{2\pi\varepsilon_0}\mathrm{d}r$



$r = 2R\cos\theta$ $\mathrm{d}r = -2R\sin\theta\mathrm{d}\theta$

$$U = -(\frac{\sigma R}{\pi\varepsilon_0})\int_{\pi/2}^{0}\theta\sin\theta\mathrm{d}\theta = \frac{\sigma R}{\pi\varepsilon_0}$$



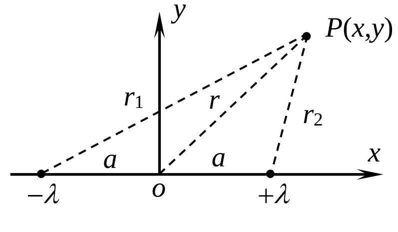
（b） $\mathrm{d}q = \sigma\cdot r\mathrm{d}r\mathrm{d}\theta$ $\mathrm{d}U = \dfrac{\mathrm{d}q}{4\pi\varepsilon_0 r} = \dfrac{\sigma}{4\pi\varepsilon_0}\mathrm{d}r\mathrm{d}\theta$

$$U = \frac{\sigma}{4\pi\varepsilon_0}\int_{-\pi/2}^{\pi/2}(\int_0^{2R\cos\theta}\mathrm{d}r)\mathrm{d}\theta = \frac{\sigma}{4\pi\varepsilon_0}\int_{-\pi/2}^{\pi/2}2R\cos\theta\mathrm{d}\theta = \frac{\sigma R}{\pi\varepsilon_0}$$

9.35 $U_+ = \displaystyle\int_{r_2}^{a}\boldsymbol{E}_+\cdot\mathrm{d}\boldsymbol{r} = \int_{r_2}^{a}\frac{\lambda}{2\pi\varepsilon_0 r}\mathrm{d}r = \frac{\lambda}{2\pi\varepsilon_0}\ln\frac{a}{r_2}$

$$U_- = \int_{r_1}^{a}\boldsymbol{E}_-\cdot\mathrm{d}\boldsymbol{r} = \int_{r_1}^{a}\frac{-\lambda}{2\pi\varepsilon_0 r}\mathrm{d}r = \frac{-\lambda}{2\pi\varepsilon_0}\ln\frac{a}{r_1}$$

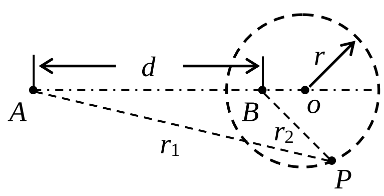
$$U_P = U_+ + U_- = \frac{\lambda}{2\pi\varepsilon_0}\ln\frac{r_1}{r_2} = \frac{\lambda}{4\pi\varepsilon_0}\ln\frac{(x+a)^2 + y^2}{(x-a)^2 + y^2}$$



9.36 $U_P = \dfrac{q_1}{4\pi\varepsilon_0 r_1} - \dfrac{q_2}{4\pi\varepsilon_0 r_2} = 0$ $\dfrac{r_1}{r_2} = \dfrac{q_1}{q_2}$

设 A 点为直角坐标系的原点，P 点的坐标为（x、y、z）：

$$\frac{\sqrt{x^2 + y^2 + z^2}}{\sqrt{(x-d)^2 + y^2 + z^2}} = \frac{q_1}{q_2}$$



$$x^2 - \frac{2q_1^2 d}{q_1^2 - q_2^2}x + y^2 + z^2 = -\frac{q_1^2 d^2}{q_1^2 - q_2^2}$$

$$(x - \frac{q_1^2 d}{q_1^2 - q_2^2})^2 + y^2 + z^2 = \frac{q_1^2 q_2^2 d^2}{(q_1^2 - q_2^2)^2}$$