

# Toxicity Bounds for Dynamic Liquidation Incentives

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October 8, 2025

## Abstract

We derive a slippage-aware toxicity condition for on-chain liquidations executed via a constant-product automated market maker (CP-AMM). For a fixed (constant) liquidation incentive  $i$ , the familiar toxicity frontier  $\nu < 1/(1+i)$  tightens to  $\nu < 1/(1+i)\lambda$  for a liquidity penalty factor,  $\lambda$ , that we derive for both the CP-AMM and a generalised form. Using a dynamic health-linked liquidation incentive  $i(h) = i(1-h)$ , we obtain a state-dependent bound and, at the liquidation boundary, a liquidity depth-only condition  $v < 1/\lambda$ . This reconciles dynamic incentives with the impact of the CP-AMM price and clarifies when dynamic liquidation incentives reduce versus exacerbate spiral risk.

## 1 Introduction

Liquidations in on-chain lending repay debt by selling collateral, typically with a bonus paid to liquidators. When sales route through AMMs, execution moves prices: the sale lowers the pool price and the borrower's remaining collateral is re-marked. If the loss on the remainder exceeds the debt reduction, health falls after the step; repeating such steps creates a *toxic liquidation spiral*. Protocols prefer *partial* liquidations to limit slippage, yet in the toxic regime each partial step increases LTV and drives the position toward *full* liquidation or bad debt.

We consider a borrower with collateral value  $c$  (measured in units of the debt asset) and debt  $q$ . Let

$$\ell := \frac{q}{c} \quad (\text{LTV}), \quad v \in (0, 1) \quad (\text{protocol LLTV parameter}).$$

for health  $h := v \frac{c}{q} = v/\ell$ . Upon a small liquidation repaying  $da$  units of debt, the liquidator is entitled to a bonus  $i \geq 0$  (possibly state dependent); a fraction  $(1+i)da$  of collateral value is seized.<sup>1</sup>

## 2 Slippage-aware toxicity

### 2.1 CP-AMM price impact model

We consider liquidations that are routed through a CP-AMM with reserves  $(x, y)$  for (collateral, debt), price  $P = y/x$ , and invariant  $xy = k$ . The local price impact of the CP-AMM is

$$d(\ln P) = d(\ln y - \ln x) = -2 \frac{dx}{x} \tag{1}$$

The sale  $(1+i)da$  of *value* in collateral implies  $dx = \frac{(1+i)da}{P}$ , and hence  $d(\ln P) = -\frac{2(1+i)}{y} da$ .

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<sup>1</sup>For definitions of  $h$  and the bonus schedule, and the criterion under which liquidation can reduce health even in fixed-bonus systems [2].

Let collateral  $c = sP$ , with  $s$  the remaining units and  $P$  the price. The infinitesimal change in the collateral value is  $dc = P ds + s dP$  to the first order, neglecting  $dsdP$ . The seizure contributes  $P ds = -(1+i) da$ , and the price move re-marks the remainder by  $s dP = c d(\ln P)$ . Therefore, the remaining collateral is re-marked by

$$c d(\ln P) = -\frac{2c(1+i)}{y} da,$$

and direct seizure removes  $(1+i) da$ , so

$$dc = -(1+i) da - \frac{2c(1+i)}{y} da = -(1+i) \left(1 + \frac{2c}{y}\right) da, \quad dq = -da.$$

Differentiating  $h = vc/q$  yields

$$dh = \frac{v}{q} \left[ dc - \frac{c}{q} dq \right] = \frac{v}{q} \left[ -(1+i) \left(1 + \frac{2c}{y}\right) + \frac{c}{q} \right] da$$

A liquidation is *toxic* (reduces health) iff  $dh < 0$ , i.e.

$$\frac{c}{q} (1+i) \left(1 + \frac{2c}{y}\right) \quad \text{equivalently} \quad \ell \frac{1}{(1+i)\lambda}, \quad \lambda := 1 + 2 \frac{c}{y}. \quad (2)$$

In the infinite-liquidity limit  $y \rightarrow \infty$  (so  $\lambda \rightarrow 1$ ), this reduces to the constant incentive frontier  $\ell < 1/(1+i)$  [1].

## 2.2 Linear price impact model

Warmuz, Chaudhary and Pinna [1] propose a linear slippage model to capture execution costs in decentralised liquidations:

$$s(x) = \gamma + \frac{\sigma}{L} x,$$

where  $s(x)$  is the relative price discount on trade size  $x$ ,  $\gamma$  is the spread,  $\sigma$  is the slippage parameter, and  $L$  is a liquidity scale. The execution price is  $1 - s(x)$  relative to the oracle.

Linearising around small trades yields an effective per-unit price impact

$$\phi = \frac{\sigma}{L(1-\gamma)}$$

This is directly analogous to Kyle's  $\lambda$  in market microstructure theory [3], which measures the permanent price impact per unit of order flow. The slippage penalty factor now becomes

$$\lambda = 1 + \phi c$$

## 3 Removing toxicity

We choose a linear incentive function linked to health, increasing as health falls,

$$i \rightarrow i(h) = i(1-h) = i\left(1 - \frac{v}{\ell}\right),$$

capped at the protocol maximum  $i$ . Substituting  $i(h)$  into (2) gives

$$\ell > \frac{1 + i v \lambda}{(1+i)\lambda}. \quad (3)$$

**Boundary condition (model-agnostic).** At the LLTV boundary we have  $\ell = v$  (equivalently  $h = 1$ ). Since the linear function satisfies  $i(h) = i(1 - h) = 0$  at  $h = 1$ , substituting  $\ell = v$  into (3) removes any dependence on  $i$  and yields a depth-only criterion:

$$v \leq \frac{1}{\lambda} \quad (4)$$

This statement is *model-agnostic*: it holds for any monotone impact summarised by a penalty factor  $\lambda$ . For a CP-AMM,  $\lambda = 1 + 2c/y$ ; for the linear (Kyle) model,  $\lambda = 1 + \phi c$ . Writing the result in terms of  $\lambda$  avoids unnecessary specialisation and makes clear that a greater depth (smaller  $\lambda$ ) relaxes the admissible LLTV  $v$ .

## 4 Discussion and limitations

Equation (4) isolates CP-AMM *depth* as the critical determinant of safety at the LLTV boundary: greater depth (larger  $y$ ) lowers  $\lambda$  and raises the allowable  $v$ . The derivation is local (infinitesimal step, CP-AMM); integrating over large sales or routing across venues is straightforward in principle but model-specific. Nevertheless, the local condition precisely characterises when a liquidation step is health-improving versus health-worsening and reconciles dynamic incentives with price impact.

## References

- [1] J. Warmuz, A. Chaudhary, and D. Pinna, “Toxic Liquidation Spirals,” *arXiv preprint arXiv:2212.07306*, 2022. [Online]. Available: <https://arxiv.org/abs/2212.07306>
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- [3] A. S. Kyle, “Continuous Auctions and Insider Trading,” *Econometrica*, vol. 53, no. 6, pp. 1315–1335, Nov. 1985. [Online]. Available: <https://www.jstor.org/stable/1913210>