Toxicity Bounds for Dynamic Liquidation Incentives

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Abstract

We derive a slippage-aware toxicity condition for on-chain liquidations executed via a constant-product automated market maker (CP-AMM). For a fixed (constant) liquidation incentive i, the familiar toxicity frontier $\nu < 1/(1+i)$ tightens to $\nu < 1/(1+i)\lambda$ for a liquidity penalty factor, λ, that we derive for both the CP-AMM and a generalised form. Using a dynamic health-linked liquidation incentive $i(h) = i(1-h)$, we obtain a state-dependent bound and, at the liquidation boundary, a liquidity depth-only condition $v < 1/\lambda$. This reconciles dynamic incentives with the impact of the CP-AMM price and clarifies when dynamic liquidation incentives reduce versus exacerbate spiral risk.

1 Introduction

Liquidations in on-chain lending repay debt by selling collateral, typically with a bonus paid to liquidators. When sales route through AMMs, execution moves prices: the sale lowers the pool price and the borrower's remaining collateral is re-marked. If the loss on the remainder exceeds the debt reduction, health falls after the step; repeating such steps creates a *toxic liquidation spiral*. Protocols prefer *partial* liquidations to limit slippage, yet in the toxic regime each partial step raises LTV and drives the position toward *full* liquidation or bad debt.

We consider a borrower with collateral value c (measured in units of the debt asset) and debt q. Let

$$\ell := \frac{q}{c} \quad \text{(LTV)}, \qquad v \in (0,1) \quad \text{(protocol LLTV parameter)}.$$

for health $h := v\,\frac{c}{q} = v/\ell$. Upon a small liquidation repaying da units of debt, the liquidator is entitled to a bonus $i \geq 0$ (possibly state dependent); a fraction $(1+i)\,da$ of collateral value is seized.[1]

2 Slippage-aware toxicity

2.1 CP-AMM price impact model

We consider liquidations that are routed through a CP-AMM with reserves (x, y) for (collateral,debt), price $P = y/x$, and invariant $xy = k$. The local price impact of the CP-AMM is

$$d(\ln P) \;=\; d(\ln y - \ln x) \;=\; -2\,\frac{dx}{x} \tag{1}$$

The sale $(1+i)\,da$ of *value* in collateral implies $dx = \frac{(1+i)\,da}{P}$, and hence $d(\ln P) = -\frac{2(1+i)}{y}\,da$.

[1] For definitions of h and the bonus schedule, and the criterion under which liquidation can reduce health even in fixed-bonus systems [2].

Let collateral $c = sP$, with s the remaining units and P the price. The infinitesimal change in collateral value os $dc = P\,ds + s\,dP$ to first order. The seizure contributes $P\,ds = -(1+i)\,da$, and the price move re-marks the remainder by $s\,dP = c\,d(\ln P)$. Therefore remaining collateral is remarked by

$$c\,d(\ln P) = -\frac{2c(1+i)}{y}\,da,$$

and direct seizure removes $(1+i)\,da$, so

$$dc = -(1+i)\,da \;-\; \frac{2c(1+i)}{y}\,da = -(1+i)\Big(1+\frac{2c}{y}\Big)da, \qquad dq = -da.$$

Differentiating $h = vc/q$ yields

$$dh \;=\; \frac{v}{q}\left[dc - \frac{c}{q}\,dq\right] \;=\; \frac{v}{q}\left[-(1+i)\left(1+\frac{2c}{y}\right) + \frac{c}{q}\right]da$$

A liquidation is *toxic* (reduces health) iff $dh < 0$, i.e.

$$\frac{c}{q}\quad(1+i)\left(1+\frac{2c}{y}\right) \quad\Longleftrightarrow\quad \ell \quad \frac{1}{(1+i)\,\lambda}, \qquad \lambda := 1 + 2\,\frac{c}{y}. \tag{2}$$

In the infinite-liquidity limit $y \to \infty$ (so $\lambda \to 1$), this reduces to the constant incentive frontier $\ell < 1/(1+i)$ [1].

2.2 Linear price impact model

Warmuz, Chaudhary and Pinna [1] propose a linear slippage model to capture execution costs in decentralised liquidations:

$$s(x) = \gamma + \frac{\sigma}{L}\,x,$$

where $s(x)$ is the relative price discount on trade size x, γ is the spread, σ is the slippage parameter, and L is a liquidity scale. The execution price is $1 - s(x)$ relative to the oracle.

Linearising around small trades yields an effective per-unit price impact

$$\phi = \frac{\sigma}{L(1-\gamma)}$$

This is directly analogous to Kyle's λ in market microstructure theory [3], which measures the permanent price impact per unit of order flow. The slippage penalty factor now becomes

$$\lambda = 1 + \phi c$$

3 Removing toxicity

We choose a linear incentive function linked to health, increasing as health falls,

$$i \to i(h) \;=\; i\,\big(1-h\big) \;=\; i\Big(1 - \frac{v}{\ell}\Big),$$

capped at the protocol maximum i. Substituting $i(h)$ into (2) gives

$$\ell \;>\; \frac{1 + i\,v\,\lambda}{(1+i)\,\lambda}. \tag{3}$$

Boundary condition (model-agnostic). At the LLTV boundary we have $\ell = v$ (equivalently $h = 1$). Since the linear function satisfies $i(h) = i(1-h) = 0$ at $h = 1$, substituting $\ell = v$ into (3) removes any dependence on i and yields a depth-only criterion:

$$\boxed{v \;\le\; \frac{1}{\lambda}}. \tag{4}$$

This statement is *model-agnostic*: it holds for any monotone impact summarised by a penalty factor λ. For a CP-AMM, $\lambda = 1 + 2c/y$; for the linear (Kyle) model, $\lambda = 1 + \phi c$. Writing the result in terms of λ avoids unnecessary specialisation and makes clear that greater depth (smaller λ) relaxes the admissible LLTV v.

4 Discussion and limitations

Equation (4) isolates CP-AMM *depth* as the critical determinant of safety at the LLTV boundary: greater depth (larger y) lowers λ and raises the allowable v. The derivation is local (infinitesimal step, CP-AMM); integrating over large sales or routing across venues is straightforward in principle but model-specific. Nevertheless, the local condition precisely characterises when a liquidation step is health-improving versus health-worsening and reconciles dynamic incentives with price impact.

References

[1] J. Warmuz, A. Chaudhary, and D. Pinna, "Toxic Liquidation Spirals," *arXiv preprint arXiv:2212.07306*, 2022. [Online]. Available: `https://arxiv.org/abs/2212.07306`

[2] M. A. Bentley, "Dutch Liquidation Analysis," Euler Labs, Technical Note, 2024.

[3] A. S. Kyle, "Continuous Auctions and Insider Trading," *Econometrica*, vol. 53, no. 6, pp. 1315–1335, Nov. 1985. [Online]. Available: `https://www.jstor.org/stable/1913210`