Universidade do Minho MIEInf

Departamento de Matemática e Aplicações

Cálculo

Formulário 1 — 2017'18

Algumas propriedades das funções trigonométricas

1. $\forall x \in \mathbb{R} \quad \operatorname{sen}^2 x + \cos^2 x = 1$

2. $\forall x \in \mathbb{R} \setminus \{\frac{\pi}{2} + k\pi : k \in \mathbb{Z}\} \quad 1 + \operatorname{tg}^2 x = \sec^2 x$

3. $\forall x \in \mathbb{R} \setminus \{k\pi : k \in \mathbb{Z}\} \quad 1 + \operatorname{cotg}^2 x = \operatorname{cosec}^2 x$

4. $\forall x \in \mathbb{R} \quad \operatorname{sen}(-x) = -\operatorname{sen} x$ (sen é ímpar)

5. $\forall x \in \mathbb{R} \quad \cos(-x) = \cos x$ (cos é par)

6. $\forall x \in \mathbb{R} \quad \cos(\frac{\pi}{2} - x) = \operatorname{sen} x$ e $\operatorname{sen}(\frac{\pi}{2} - x) = \cos x$

7. $\forall x \in \mathbb{R} \quad \operatorname{sen}(x + 2\pi) = \operatorname{sen} x$ (sen tem período 2π)

8. $\forall x \in \mathbb{R} \quad \cos(x + 2\pi) = \cos x$ (cos tem período 2π)

9. $\forall x,\ y \in \mathbb{R} \quad \operatorname{sen}(x + y) = \operatorname{sen} x \cos y + \operatorname{sen} y \cos x$

10. $\forall x,\ y \in \mathbb{R} \quad \cos(x + y) = \cos x \cos y - \operatorname{sen} y \operatorname{sen} x$

11. $\forall x,\ y \in \mathbb{R} \quad \cos x - \cos y = -2 \operatorname{sen} \frac{x-y}{2} \operatorname{sen} \frac{x+y}{2}$

12. $\forall x,\ y \in \mathbb{R} \quad \operatorname{sen} x - \operatorname{sen} y = 2 \operatorname{sen} \frac{x-y}{2} \cos \frac{x+y}{2}$

Recorde-se que

x	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
$\operatorname{sen} x$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
$\cos x$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0

Algumas propriedades das funções hiperbólicas

$$\begin{array}{rccc} \mathrm{sh}: & \mathbb{R} & \longrightarrow & \mathbb{R} \\ & x & \longmapsto & \dfrac{e^x - e^{-x}}{2} \end{array} \qquad \begin{array}{rccc} \mathrm{ch}: & \mathbb{R} & \longrightarrow & \mathbb{R} \\ & x & \longmapsto & \dfrac{e^x + e^{-x}}{2} \end{array}$$

1. $\forall\, x \in \mathbb{R} \qquad \mathrm{ch}^2 x - \mathrm{sh}^2 x = 1$

2. $\forall\, x \in \mathbb{R} \qquad \mathrm{th}^2 x + \mathrm{sech}^2 x = 1$

3. $\forall\, x \in \mathbb{R} \setminus \{0\} \qquad \coth^2 x - \mathrm{cosech}^2 x = 1$

4. $\forall\, x \in \mathbb{R} \qquad \mathrm{sh}(-x) = -\,\mathrm{sh}\, x$ (a função sh é ímpar)

5. $\forall\, x \in \mathbb{R} \qquad \mathrm{ch}(-x) = \mathrm{ch}\, x$ (a função ch é par)

6. $\forall\, x,\, y \in \mathbb{R} \qquad \mathrm{sh}(x+y) = \mathrm{sh}\, x\ \mathrm{ch}\, y + \mathrm{sh}\, y\ \mathrm{ch}\, x$

7. $\forall\, x,\, y \in \mathbb{R} \qquad \mathrm{ch}(x+y) = \mathrm{ch}\, x\ \mathrm{ch}\, y + \mathrm{sh}\, y\ \mathrm{sh}\, x$

8. $\forall\, n \in \mathbb{N} \quad \forall\, x \in \mathbb{R} \qquad (\mathrm{ch}\, x + \mathrm{sh}\, x)^n = \mathrm{ch}(nx) + \mathrm{sh}(nx)$