$$f(x_1, x_2) = x_1^2 + 5x_1 + x_2^3 - 3x_2^2$$

Cond. Otimalidade

$$\nabla f = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \qquad \begin{bmatrix} 2x_1 + 5 \\ 3x_2^2 - 6x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\rightsquigarrow x_1 = -\frac{5}{2} = -2.5$$

$$3x_2^2 - 6x_2 = 0 \Leftrightarrow 3x_2(x_2 - 2) = 0$$

$$\rightsquigarrow 3x_2 = 0 \quad \vee \quad x_2 - 2 = 0$$

$$x_2 = 0 \quad \vee \quad x_2 = 2$$

$\therefore$ 2 pontos estacionários $(-2.5, 0)$, $(-2.5, 2)$

$$\nabla^2 f = \begin{bmatrix} 2 & 0 \\ 0 & 6x_2 - 6 \end{bmatrix}$$

$\begin{bmatrix} -2.5 \\ 0 \end{bmatrix}$:

$$\nabla^2 f\left(\begin{bmatrix} -2.5 \\ 0 \end{bmatrix}\right) = \begin{bmatrix} 2 & 0 \\ 0 & -6 \end{bmatrix}$$

$\det([2]) > 0$

$\det\begin{pmatrix} 2 & 0 \\ 0 & -6 \end{pmatrix} = -12 < 0$

$\hookrightarrow$ Mat. indef.

$\begin{bmatrix} -2.5 \\ 0 \end{bmatrix} \rightarrow$ Ponto SELA

$\begin{bmatrix} -2.5 \\ 2 \end{bmatrix}$:

$$\nabla^2 f\left(\begin{bmatrix} -2.5 \\ 2 \end{bmatrix}\right) = \begin{bmatrix} 2 & 0 \\ 0 & 6 \end{bmatrix}$$

$\det([2]) > 0$

$\det\left(\begin{bmatrix} 2 & 0 \\ 0 & 6 \end{bmatrix}\right) = 12 > 0$

$\hookrightarrow$ Mat. def. Positiva

$\begin{bmatrix} -2.5 \\ 2 \end{bmatrix} \rightarrow$ MINIMIZANTE