



séries de Fourier e edp's

Exercício 1.

- (a) Equação linear de segunda ordem
- (b) Equação linear de terceira ordem
- (c) Equação não linear de segunda ordem
- (d) Equação linear de segunda ordem
- (e) Equação não linear de primeira ordem

Exercício 2.

- (a) Equação elíptica
- (b) Equação parabólica
- (c) Equação hiperbólica
- (d) Equação hiperbólica
- (e)
 - se $x \in \mathbb{R}^+$ e $y \in \mathbb{R}^-$ ou se $x \in \mathbb{R}^-$ e $y \in \mathbb{R}^+$, a equação é hiperbólica
 - se $x = y = 0$, a equação é parabólica
 - se $x \in \mathbb{R}^+$ e $y \in \mathbb{R}^+$ ou se $x \in \mathbb{R}^-$ e $y \in \mathbb{R}^-$, a equação é elíptica

Exercício 5. $u(x, t) = 3 \sin(2\pi x) e^{(1-4\pi^2)t} - 7 \sin(4\pi x) e^{(1-16\pi^2)t}$

Exercício 6.

- (a) $u(y, t) = e^{-3(y+t)} + e^{2(y+t)}$
- (b) $u(y, t) = e^{-4y-5t} + 2e^{-6y-7t} - 14e^{14y+13t}$

Exercício 14.

$$(a) \frac{4}{\pi} \sum_{k=1}^{\infty} \frac{\text{sen}((2k-1)x)}{2k-1}$$

$$(b) \frac{2}{\pi} \sum_{n=1}^{\infty} \frac{\text{sen}(n\pi x)}{n}$$

$$(c) \frac{8}{\pi^3} \sum_{k=1}^{\infty} \frac{\text{sen}((2k-1)\pi x)}{(2k-1)^3}$$

$$(d) -\frac{16}{\pi} \sum_{k=1}^{\infty} \frac{\text{sen}((2k-1)x)}{(2k-3)(2k-1)(2k+1)}$$

Exercício 15.

$$(a) \pi - \frac{8}{\pi} \sum_{k=1}^{\infty} \frac{\cos((2k-1)x)}{(2k-1)^2}$$

$$(b) \frac{1}{2} + \frac{2}{\pi} \sum_{k=1}^{\infty} \frac{(-1)^{k-1} \cos((2k-1)\pi x)}{2k-1}$$

$$(c) -\frac{8}{\pi} \sum_{k=1}^{\infty} \frac{\cos((2k-1)x)}{(2k-3)(2k+1)}$$

$$(d) \pi + \frac{32}{\pi} \sum_{k=1}^{\infty} \frac{\cos((2k-1)x)}{(2k-3)(2k-1)^2(2k+1)}$$

Exercício 16.

$$(a) u(x, t) \sim e^{-3\pi^2 t} \text{sen}(\pi x) + \frac{1}{2} e^{-3(4\pi)^2 t} \text{sen}(4\pi x) - e^{-3(5\pi)^2 t} \text{sen}(5\pi x)$$

$$(b) u(x, t) \sim \frac{4}{\pi^2} \sum_{k=1}^{\infty} \frac{(-1)^{k-1}}{(2k-1)^2} e^{-3((2k-1)\pi)^2 t} \text{sen}((2k-1)\pi x)$$

$$(c) u(x, t) \sim \frac{8}{\pi^3} \sum_{k=1}^{\infty} \frac{1}{(2k-1)^3} e^{-3((2k-1)\pi)^2 t} \text{sen}((2k-1)\pi x)$$

Exercício 17. $u(x, t) \sim -\frac{16}{\pi} \sum_{k=1}^{\infty} \frac{1}{(2k-3)(2k-1)(2k+1)} e^{-5(2k-1)^2 t} \text{sen}((2k-1)x)$

Exercício 19.

(a) $u(x, t) \sim 1 + \frac{1}{2} e^{-8t} \cos(2x) + \frac{1}{3} e^{-18t} \cos(3x) + 5e^{-72t} \cos(6x)$

(b) $u(x, t) \sim \frac{\pi}{4} - \frac{2}{\pi} \sum_{k=1}^{\infty} \frac{1}{(2k-1)^2} e^{-2(4k-2)^2 t} \cos((4k-2)x)$

(c) $u(x, t) \sim \pi + \frac{32}{\pi} \sum_{k=1}^{\infty} \frac{1}{(2k-3)(2k-1)^2(2k+1)} e^{-2(2k-1)^2 t} \cos((2k-1)x)$

Exercício 21.

(a) $u(x, t) \sim \cos(4\pi t) \text{sen}(2\pi x) + \frac{1}{5} \cos(6\pi t) \text{sen}(3\pi x) + \cos(10\pi t) \text{sen}(5\pi x)$

(b) $u(x, t) \sim \frac{1}{2\pi} \text{sen}(2\pi t) \text{sen}(\pi x) + \frac{1}{2\pi} \text{sen}(4\pi t) \text{sen}(2\pi x) + \frac{1}{30\pi} \text{sen}(10\pi t) \text{sen}(5\pi x)$

(c) $u(x, t) \sim \cos(6\pi t) \text{sen}(3\pi x) + \cos(10\pi t) \text{sen}(5\pi x) + \frac{1}{8\pi} \text{sen}(4\pi t) \text{sen}(2\pi x) + \frac{1}{18\pi} \text{sen}(6\pi t) \text{sen}(3\pi x) +$
 $+\frac{2}{5\pi} \text{sen}(10\pi t) \text{sen}(5\pi x)$

(d) $u(x, t) \sim \frac{4}{\pi^2} \sum_{k=1}^{\infty} \frac{(-1)^{k-1}}{(2k-1)^2} \cos(2(2k-1)\pi t) \text{sen}((2k-1)\pi x) + \frac{1}{3\pi} \text{sen}(6\pi t) \text{sen}(3\pi x) +$
 $+\frac{5}{8\pi} \text{sen}(8\pi t) \text{sen}(4\pi x)$

(e) $u(x, t) \sim \cos(2\pi t) \text{sen}(\pi x) - 2 \cos(4\pi t) \text{sen}(2\pi x) + 3 \cos(6\pi t) \text{sen}(3\pi x) +$
 $+\frac{4}{\pi^4} \sum_{k=1}^{\infty} \frac{1}{(2k-1)^4} \text{sen}(2(2k-1)\pi t) \text{sen}((2k-1)\pi x)$

(f) $u(x, t) \sim \frac{8}{\pi^3} \sum_{k=1}^{\infty} \frac{1}{(2k-1)^3} \cos(2(2k-1)\pi t) \text{sen}((2k-1)\pi x) +$
 $+\frac{2}{\pi^3} \sum_{k=1}^{\infty} \frac{(-1)^{k-1}}{(2k-1)^3} \text{sen}(2(2k-1)\pi t) \text{sen}((2k-1)\pi x)$