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## Formulário séries de Fourier e EDPs

- Função periódica de período  $2L$

*Coefficientes de Fourier*

$$a_n = \frac{1}{L} \int_{-L}^L f(x) \cos\left(\frac{n\pi x}{L}\right) dx, \quad n = 0, 1, 2, \dots$$

$$b_n = \frac{1}{L} \int_{-L}^L f(x) \sin\left(\frac{n\pi x}{L}\right) dx, \quad n = 1, 2, \dots$$

- Função definida em  $[0, L]$

- *Coefficientes da série de Fourier de cossenos*

$$a_n = \frac{2}{L} \int_0^L f(x) \cos\left(\frac{n\pi x}{L}\right) dx, \quad n = 0, 1, 2, \dots$$

- *Coefficientes da série de Fourier de senos*

$$b_n = \frac{2}{L} \int_0^L f(x) \sin\left(\frac{n\pi x}{L}\right) dx, \quad n = 1, 2, \dots$$

- Problema da condução do calor

- **Caso 1**

$$\begin{cases} \frac{\partial u}{\partial t} = \sigma \frac{\partial^2 u}{\partial x^2}, & 0 < x < L, t > 0 \\ u(0, t) = u(L, t) = 0, & t \geq 0 \\ u(x, 0) = f(x), & 0 \leq x \leq L \end{cases}$$

*Solução formal*

$$u(x, t) \sim \sum_{n=1}^{\infty} b_n e^{-\sigma \left(\frac{n\pi}{L}\right)^2 t} \sin\left(\frac{n\pi x}{L}\right),$$

onde  $b_n$  são os coeficientes da série de Fourier de senos de  $f$ .

- **Caso 2**

$$\begin{cases} \frac{\partial u}{\partial t} = \sigma \frac{\partial^2 u}{\partial x^2}, & 0 < x < L, t > 0 \\ \frac{\partial u}{\partial x}(0, t) = \frac{\partial u}{\partial x}(L, t) = 0, & t \geq 0 \\ u(x, 0) = f(x), & 0 \leq x \leq L \end{cases}$$

*Solução formal*

$$u(x, t) \sim \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n e^{-\sigma \left(\frac{n\pi}{L}\right)^2 t} \cos\left(\frac{n\pi x}{L}\right),$$

onde  $a_n$  são os coeficientes da série de Fourier de cossenos de  $f$ .



## Formulário séries de Fourier e EDPs

- Problema da corda vibrante

$$\left\{ \begin{array}{l} \frac{\partial^2 u}{\partial t^2} = \alpha^2 \frac{\partial^2 u}{\partial x^2}, \quad 0 < x < L, t > 0 \\ u(0, t) = u(L, t) = 0, \quad t \geq 0 \\ u(x, 0) = f(x), \quad 0 \leq x \leq L \\ \frac{\partial u}{\partial t}(x, 0) = g(x), \quad 0 \leq x \leq L \end{array} \right.$$

*Solução formal*

$$u(x, t) \sim \sum_{n=1}^{\infty} b_n \cos\left(\frac{n\pi\alpha t}{L}\right) \sin\left(\frac{n\pi x}{L}\right) + \sum_{n=1}^{\infty} \frac{L}{n\pi\alpha} c_n \sin\left(\frac{n\pi\alpha t}{L}\right) \sin\left(\frac{n\pi x}{L}\right),$$

onde  $b_n$  e  $c_n$  são os coeficientes das séries de Fourier de senos de  $f$  e  $g$ , respetivamente.

- Alguns integrais úteis

$$\int_0^L \sin\left(\frac{n\pi x}{L}\right) dx = \frac{1 - (-1)^n}{n\pi} L, \quad n = 1, 2, \dots$$

$$\int_0^L \cos\left(\frac{n\pi x}{L}\right) dx = 0, \quad n = 1, 2, \dots$$

$$\int_0^L x \sin\left(\frac{n\pi x}{L}\right) dx = \frac{(-1)^{n+1}}{n\pi} L^2, \quad n = 1, 2, \dots$$

$$\int_0^L x \cos\left(\frac{n\pi x}{L}\right) dx = \frac{-1 + (-1)^n}{n^2\pi^2} L^2, \quad n = 1, 2, \dots$$

$$\int_0^L x^2 \sin\left(\frac{n\pi x}{L}\right) dx = \frac{(-1)^n(2 - n^2\pi^2) - 2}{n^3\pi^3} L^3, \quad n = 1, 2, \dots$$

$$\int_0^L x^2 \cos\left(\frac{n\pi x}{L}\right) dx = \frac{2(-1)^n}{n^2\pi^2} L^3, \quad n = 1, 2, \dots$$