



Fórmulas trigonométricas úteis

1. $\sin^2 x + \cos^2 x = 1$.
2. $\operatorname{tg} x = \frac{\sin x}{\cos x}$, $\operatorname{cotg} x = \frac{\cos x}{\sin x}$.
3. $\sec x = \frac{1}{\cos x}$, $\operatorname{cosec} x = \frac{1}{\sin x}$.
4. $\sin 2x = 2 \sin x \cos x$.
5. $\cos 2x = \cos^2 x - \sin^2 x$.
6. $\cos^2 x = \frac{1 + \cos 2x}{2}$, $\sin^2 x = \frac{1 - \cos 2x}{2}$.

	$\pi/6$	$\pi/4$	$\pi/3$
sen	$1/2$	$\sqrt{2}/2$	$\sqrt{3}/2$
cos	$\sqrt{3}/2$	$\sqrt{2}/2$	$1/2$

	0	$\pi/2$	π	$3\pi/2$
sen	0	1	0	-1
cos	1	0	-1	0

Fórmulas hiperbólicas úteis

$$\begin{aligned} \operatorname{sh} : \mathbb{R} &\longrightarrow \mathbb{R} \\ x &\longmapsto \frac{e^x - e^{-x}}{2} \end{aligned}$$

$$\begin{aligned} \operatorname{ch} : \mathbb{R} &\longrightarrow \mathbb{R} \\ x &\longmapsto \frac{e^x + e^{-x}}{2} \end{aligned}$$

1. $\operatorname{ch}^2 x - \operatorname{sh}^2 x = 1$.
2. $\operatorname{th} x = \frac{\operatorname{sh} x}{\operatorname{ch} x}$, $\operatorname{coth} x = \frac{\operatorname{ch} x}{\operatorname{sh} x}$.
3. $\operatorname{sech} x = \frac{1}{\operatorname{ch} x}$, $\operatorname{cosech} x = \frac{1}{\operatorname{sh} x}$.
4. $\operatorname{sh} 2x = 2 \operatorname{sh} x \operatorname{ch} x$.
5. $\operatorname{ch} 2x = \operatorname{ch}^2 x + \operatorname{sh}^2 x$.
6. $\operatorname{ch}^2 x = \frac{\operatorname{ch} 2x + 1}{2}$, $\operatorname{sh}^2 x = \frac{\operatorname{ch} 2x - 1}{2}$.



Algumas propriedades das funções trigonométricas

1. $\forall a \in \mathbb{R} \quad \sin^2 a + \cos^2 a = 1$
2. $\forall a \in \mathbb{R} \setminus \{\frac{\pi}{2} + k\pi : k \in \mathbb{Z}\} \quad 1 + \operatorname{tg}^2 a = \sec^2 a$
3. $\forall a \in \mathbb{R} \setminus \{k\pi : k \in \mathbb{Z}\} \quad 1 + \operatorname{cotg}^2 a = \operatorname{cosec}^2 a$
4. $\forall a \in \mathbb{R} \quad \sin(-a) = -\sin a \quad (\sin \text{ é ímpar})$
5. $\forall a \in \mathbb{R} \quad \cos(-a) = \cos a \quad (\cos \text{ é par})$
6. $\forall a \in \mathbb{R} \quad \cos(\frac{\pi}{2} - a) = \sin a \quad \text{e} \quad \sin(\frac{\pi}{2} - a) = \cos a$
7. $\forall a \in \mathbb{R} \quad \sin(a + 2\pi) = \sin a \quad (\sin \text{ tem período } 2\pi)$
8. $\forall a \in \mathbb{R} \quad \cos(a + 2\pi) = \cos a \quad (\cos \text{ tem período } 2\pi)$
9. $\forall a, b \in \mathbb{R} \quad \sin(a + b) = \sin a \cos b + \sin b \cos a$
10. $\forall a, b \in \mathbb{R} \quad \cos(a + b) = \cos a \cos b - \sin b \sin a$
11. $\forall a, b \in \mathbb{R} \quad \cos a - \cos b = -2 \sin \frac{a-b}{2} \sin \frac{a+b}{2}$
12. $\forall a, b \in \mathbb{R} \quad \sin a - \sin b = 2 \sin \frac{a-b}{2} \cos \frac{a+b}{2}$

Algumas propriedades das funções hiperbólicas

$$\begin{aligned} \operatorname{sh} : \mathbb{R} &\longrightarrow \mathbb{R} \\ x &\longmapsto \frac{e^x - e^{-x}}{2} \end{aligned}$$

$$\begin{aligned} \operatorname{ch} : \mathbb{R} &\longrightarrow \mathbb{R} \\ x &\longmapsto \frac{e^x + e^{-x}}{2} \end{aligned}$$

1. $\forall a \in \mathbb{R} \quad \operatorname{ch}^2 a - \operatorname{sh}^2 a = 1$
2. $\forall a \in \mathbb{R} \quad \operatorname{th}^2 a + \operatorname{sech}^2 a = 1$
3. $\forall a \in \mathbb{R} \setminus \{0\} \quad \operatorname{coth}^2 a - \operatorname{cosech}^2 a = 1$
4. $\forall a \in \mathbb{R} \quad \operatorname{sh}(-a) = -\operatorname{sh} a \quad (\text{a função sh é ímpar})$
5. $\forall a \in \mathbb{R} \quad \operatorname{ch}(-a) = \operatorname{ch} a \quad (\text{a função ch é par})$
6. $\forall a, b \in \mathbb{R} \quad \operatorname{sh}(a + b) = \operatorname{sh} a \operatorname{ch} b + \operatorname{sh} b \operatorname{ch} a$
7. $\forall a, b \in \mathbb{R} \quad \operatorname{ch}(a + b) = \operatorname{ch} a \operatorname{ch} b + \operatorname{sh} b \operatorname{sh} a$
8. $\forall n \in \mathbb{N} \quad \forall a \in \mathbb{R} \quad (\operatorname{ch} a + \operatorname{sh} a)^n = \operatorname{ch}(na) + \operatorname{sh}(na)$