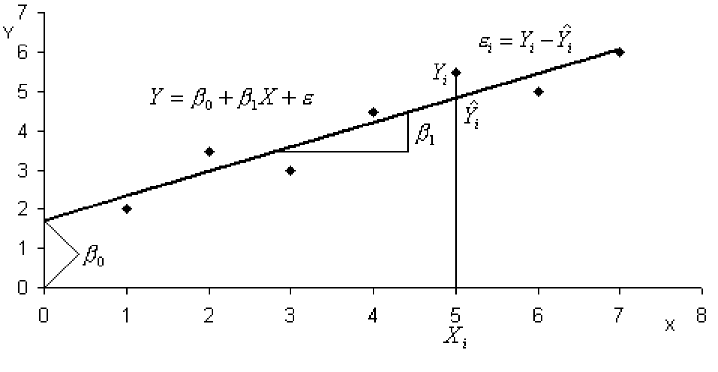
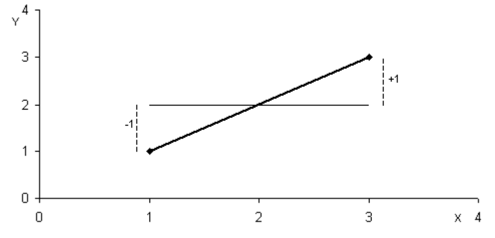
REGRESSÃO E CORRELAÇÃO

AJUSTE DE UMA RETA

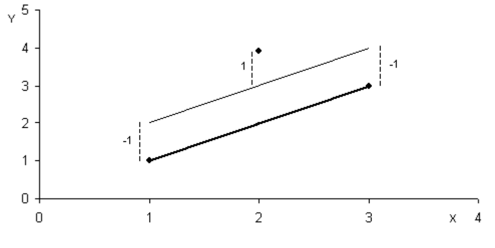


MINIMIZAÇÃO DOS DESVIOS



$$\sum\left(Y_i - \hat{Y}_i\right)$$

MINIMIZAÇÃO DOS DESVIOS ABSOLUTOS



$$\sum\left|Y_i - \hat{Y}_i\right|$$

EXEMPLO 1

- Considere o seguinte conjunto de pontos

X	Y
1	1
2	1
3	2
4	2
5	4

RETAS DE AJUSTE

R1	$Y=-0.1+0.7X$
R2	$Y= 0.5+0.5X$
R3	$Y=-0.7+0.9X$

RETAS

R1	R2	R3
0.6	1	0.2
1.3	1.5	1.1
2	2	2
2.7	2.5	2.9
3.4	3	3.8

DESVIOS

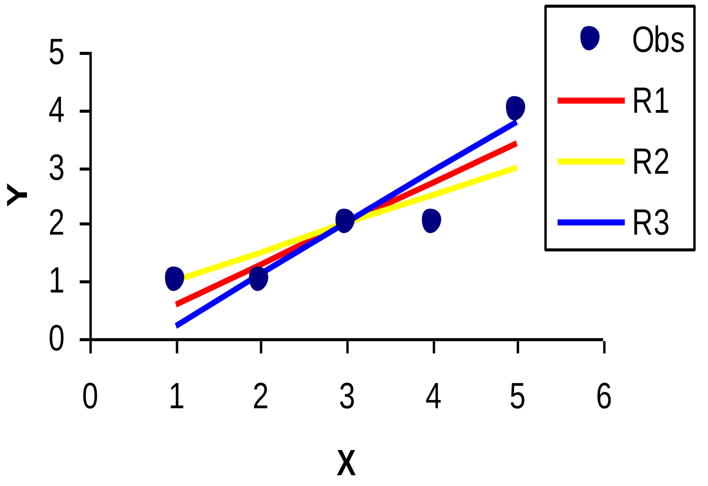
Desv1	Desv2	Desv3
0.4	0	0.8
-0.3	-0.5	-0.1
0	0	0
-0.7	-0.5	-0.9
0.6	1	0.2
0	0	0

DESVIOS ABSOLUTOS

|Desv1|	|Desv2|	|Desv3|
0.4	0	0.8
0.3	0.5	0.1
0	0	0
0.7	0.5	0.9
0.6	1	0.2
2	2	2

QUADRADO DOS DESVIOS

$(Desv1)^2$	$(Desv2)^2$	$(Desv3)^2$
0.16	0	0.64
0.09	0.25	0.01
0	0	0
0.49	0.25	0.81
0.36	1	0.04
1.10	1.50	1.50

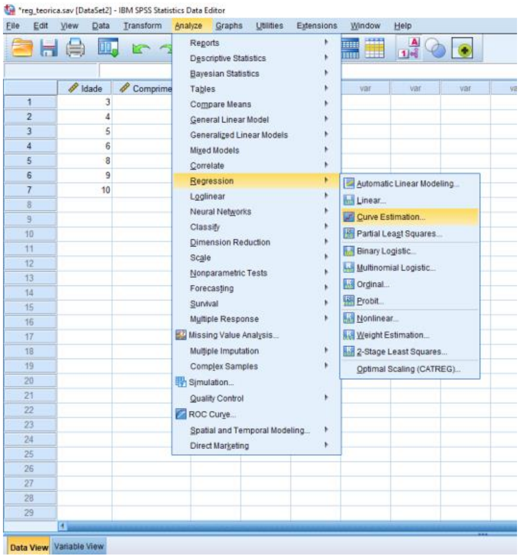


EXEMPLO 2

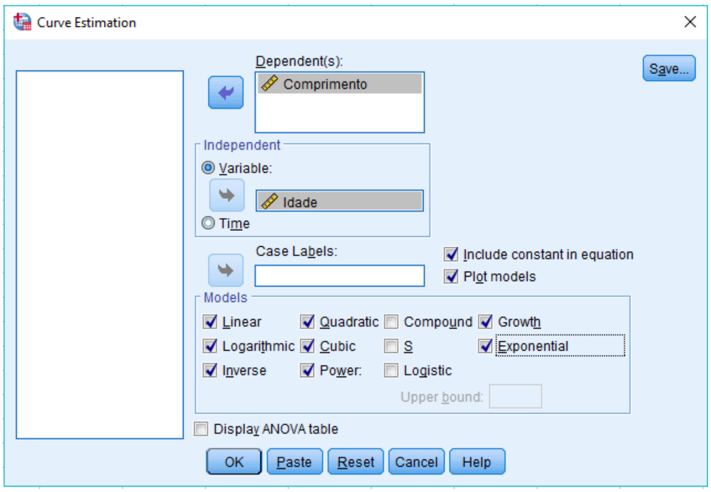
Comprimento alar (cm) em função da idade (dias) para andorinhas

Dias	Comp.
3	1,4
4	1,5
5	2,1
6	2,4
8	3,1
9	3,2
10	3,3

EXEMPLO 2 (SPSS)



EXEMPLO 2



Models (curve estimation algorithms)

Previous Next

CURVEFIT allows the user to specify a model with or without a constant term designated by β_0. If this constant term is excluded, simply set it zero or one depending upon whether it appears in an additive or multiplicative manner in the models listed below.

(1) Linear	$E(Y_t) = \beta_0 + \beta_1 t$
(2) Logarithmic	$E(Y_t) = \beta_0 + \beta_1 \ln(t)$
(3) Inverse	$E(Y_t) = \beta_0 + \beta_1 / t$
(4) Quadratic	$E(Y_t) = \beta_0 + \beta_1 t + \beta_2 t^2$
(5) Cubic	$E(Y_t) = \beta_0 + \beta_1 t + \beta_2 t^2 + \beta_3 t^3$
(6) Compound	$E(Y_t) = \beta_0 \beta_1^t$
(7) Power	$E(Y_t) = \beta_0 t^{\beta_1}$
(8) S	$E(Y_t) = \exp(\beta_0 + \beta_1 / t)$
(9) Growth	$E(Y_t) = \exp(\beta_0 + \beta_1 t)$
(10) Exponential	$E(Y_t) = \beta_0 e^{\beta_1 t}$
(11) Logistic	$E(Y_t) = \left(\frac{1}{u} + \beta_0 \beta_1^t\right)^{-1}$

EXEMPLO 2

*Output1 [Document1] - IBM SPSS Statistics Viewer

File Edit View Data Transform Insert Format Analyze Graphs Utilities Extensions Window Help

Output
- Log
- Curve Fit
  - Title
  - Notes
  - Active Dataset
  - Model Description
  - Case Processing
  - Variable Processi
  - Model Summary a
  - Curvefit for Compr

Curve Fit

[DataSet2] C:\Users\ACB\OneDrive\Aulas2017_18\reg_teorica.sav

Model Description

Model Name		MOD_1
Dependent Variable	1	Comprimento
Equation	1	Linear
	2	Logarithmic
	3	Inverse
	4	Quadratic
	5	Cubic
	6	Power[a]
	7	Growth[a]
	8	Exponential[a]
Independent Variable		Idade
Constant		Included
Variable Whose Values Label Observations in Plots		Unspecified
Tolerance for Entering Terms in Equations		0,0001

a. The model requires all non-missing values to be positive.

- Output
  - Log
  - Curve Fit
    - Title
    - Notes
    - Active Dataset
    - Model Description
    - Case Processing
    - Variable Processi
    - Model Summary a
    - Curvefit for Compr

Variable Processing Summary

		Variables	
		Dependent Comprimento	Independent Idade
Number of Positive Values		7	7
Number of Zeros		0	0
Number of Negative Values		0	0
Number of Missing Values	User-Missing	0	0
	System-Missing	0	0

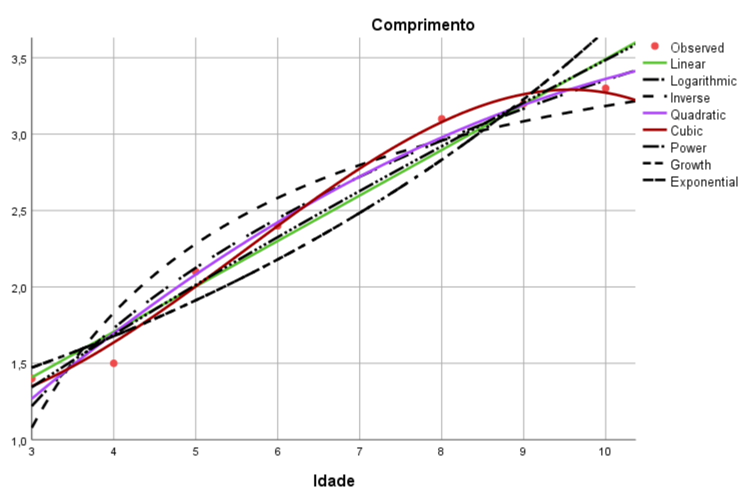
Model Summary and Parameter Estimates

Dependent Variable: Comprimento

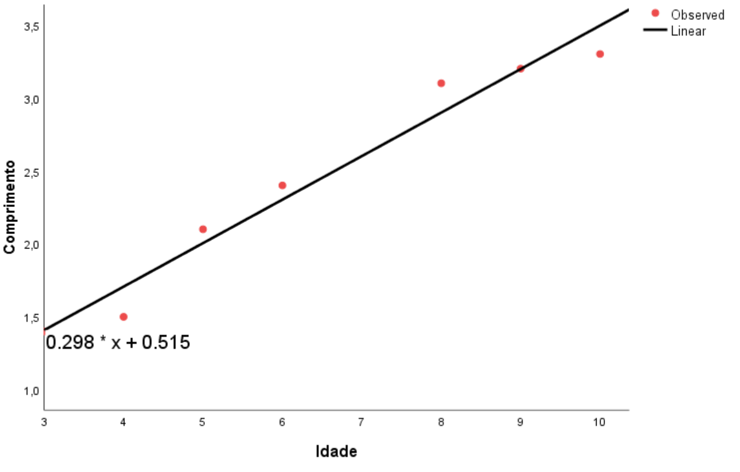
Equation	Model Summary: R Square	F	df1	df2	Sig.	Parameter Estimates: Constant	b1	b2	b3
Linear	$\boxed{0,964}$	132,174	1	5	0,000	0,515	0,298		
Logarithmic	0,971	165,753	1	5	0,000	-0,727	1,772		
Inverse	0,915	53,833	1	5	0,001	4,087	-9,026		
Quadratic	$\boxed{0,980}$	99,685	2	4	0,000	-0,274	0,579	-0,021	
Cubic	$\boxed{0,991}$	106,896	3	3	0,002	1,471	-0,387	0,141	-0,008
Power	0,968	149,638	1	5	0,000	0,563	0,792		
Growth	0,931	67,190	1	5	0,000	-0,006	0,131		
Exponential	0,931	67,190	1	5	0,000	0,994	0,131		

The independent variable is Idade.

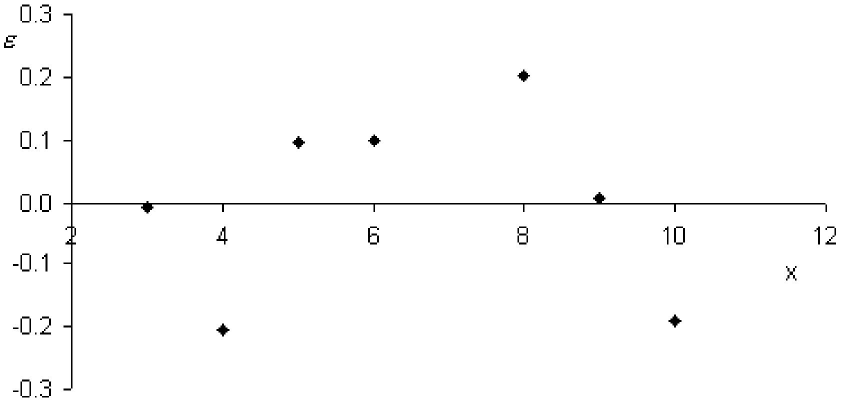
EXEMPLO 2



RECTA DE MÍNIMOS QUADRADOS



RESÍDUOS



Estimadores

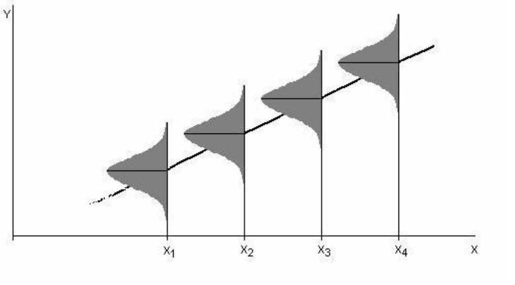
$$Y_i = \beta_0 + \beta_1.\left(X_i - \overline{X}\right) + \varepsilon_i \qquad \text{i}=1,\ldots,\text{n}$$

β_0 $\quad \boxed{\hat{\beta}_0 = \frac{1}{n}\sum_i Y_i = \overline{Y}}$

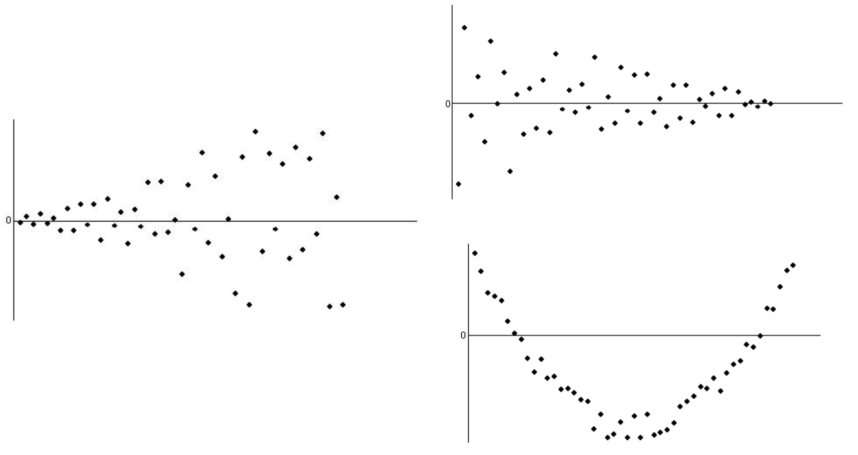
β_1 $\quad \boxed{\hat{\beta}_1 = \frac{\sum_i \left(X_i - \overline{X}\right).\left(Y_i - \overline{Y}\right)}{\sum_i \left(X_i - \overline{X}\right)^2} = \frac{s_{XY}}{s_{xx}}}$

σ^2 $\quad \boxed{s^2 = \frac{1}{n-2}\sum_i \hat{e}_i^2 = \frac{1}{n-2}\sum_i \left\{Y_i - \left[\hat{\beta}_0 + \hat{\beta}_1.\left(X_i - \overline{X}\right)\right]\right\}^2}$

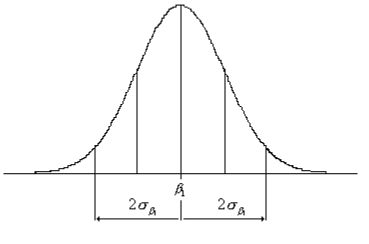
DISTRIBUIÇÃO DOS ERROS



RESÍDUOS



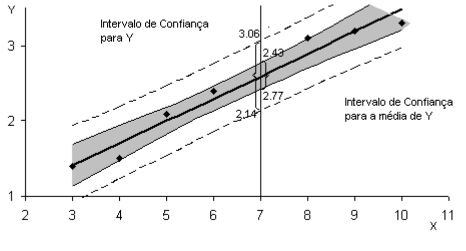
DISTRIBUIÇÃO DO DECLIVE



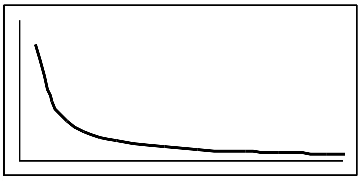
IC e Testes de hipóteses

	IC	TH
β_0	$\hat{\beta}_0 \pm t_{n-2,\left(\alpha/2\right)} \cdot \frac{s}{\sqrt{n}}$	$H_0 : \beta_0 = b_0$ $H_1 : \beta_0 \neq b_0, \beta_0 > b_0$ ou $\beta_0 < b_0$ $ET = \frac{\hat{\beta}_0 - b_0}{s/\sqrt{n}}$ H_0 verdadeira $\Rightarrow ET \sim t_{n-2}$
β_0'	$\left(\hat{\beta}_0 - \bar{X}.\hat{\beta}_1\right) \pm t_{n-2,\left(\alpha/2\right)}.s.\sqrt{\frac{1}{n} + \frac{\bar{X}^2}{s_{XX}}}$	$H_0 : \beta_0' = b_0'$ $H_1 : \beta_0' \neq b_0', \beta_0' > b_0'$ ou $\beta_0' < b_0'$ $ET = \frac{\left(\hat{\beta}_0 - \bar{X}.\hat{\beta}_1\right) - b_0'}{s.\sqrt{\frac{1}{n} + \frac{\bar{X}^2}{s_{XX}}}}$ H_0 verdadeira $\Rightarrow ET \sim t_{n-2}$
β_1	$\hat{\beta}_1 \pm t_{n-2,\left(\alpha/2\right)} \cdot \frac{s}{\sqrt{s_{XX}}}$	$H_0 : \beta_1 = b_{10}$ $H_1 : \beta_1 \neq b_{10}, \beta_1 > b_{10}$ ou $\beta_1 < b_{10}$ $ET = \frac{\hat{\beta}_1 - b_{10}}{\frac{s}{\sum_i \left(x_i - \bar{x}\right)^2}}$ H_0 verdadeira $\Rightarrow ET \sim t_{n-2}$

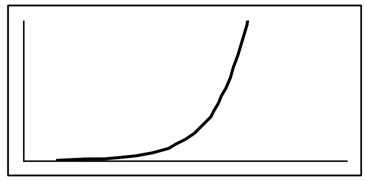
INTERVALO DE CONFIANÇA



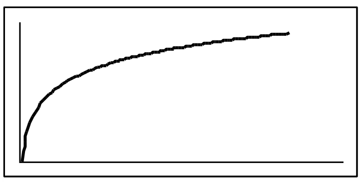
REGRESSÃO NÃO LINEAR



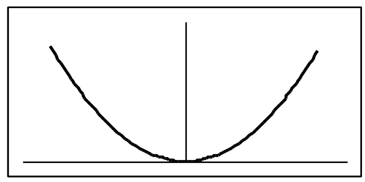
$\hat{Y} = \beta_0 + \beta_1 \frac{1}{X}$



$\hat{Y} = \beta_0 + \beta_1 e^X$



$\hat{Y} = \beta_0 + \beta_1 \ln X$



$\hat{Y} = \beta_0 + \beta_1 X^2$

REGRESSÃO NÃO LINEAR

Modelo	**Transformação**
• $Y_i = \alpha' + \frac{\beta}{X_i} + e_i$	$U_i = \frac{1}{X_i}$ $Y_i = \alpha' + \beta . U_i + e_i$
• $Y_i = e^{\alpha' + \beta . X_i + e_i}$	$Z_i = \ln Y_i$ $Z_i = \alpha' + \beta . X_i + e_i$
• $Y_i = e^{\alpha' + \frac{\beta}{X_i} + e_i}$ com $\alpha' > 0, \beta < 0$	$U_i = \frac{1}{X_i}$ $Z_i = \ln Y_i$ $Z_i = \alpha' + \beta . U_i + e_i$

REGRESSÃO LINEAR E MÚLTIPLA

Um modelo de regressão linear múltipla descreve uma relação entre várias variáveis quantitativas **independentes**, X_1, X_2, ..., X_j, e uma variável quantitativa **dependente**, Y, nos termos seguintes:

$$Y_i = \beta_0 + \beta_1.\left(X_{1i} - \overline{X}_1\right) + \beta_2.\left(X_{2i} - \overline{X}_2\right) + \ldots + \beta_j.\left(X_{ji} - \overline{X}_j\right) + \varepsilon_i \qquad i = 1,\ldots,n \quad j = 1,\ldots,J$$

onde:

- $(X_{1i}, X_{2i}, \ldots, X_{Ji}, Y_i)$ i-ésima observação das variáveis $X_{1i}, X_{2i}, \ldots, X_{Ji}$ e Y.
- $\overline{X}_j$ média aritmética das observações X_{ji}
- $\beta_0, \beta_1, \beta_2, \ldots, \beta_J$ parâmetros fixos da relação linear entre $X_{1i}, X_{2i}, \ldots, X_{Ji}$ e Y
- ε_i erro aleatório associado ao valor observado Y_i

RESÍDUOS

Pressupostos para ε_i

- Têm valor esperado nulo e variância constante, σ^2;
- São mutuamente independentes;
- São normalmente distribuídos.

$\varepsilon_i \sim IN(0, \sigma^2)$

Se estas hipóteses se verificarem então: $Y_i \sim IN(\mu_{Y_i}, \sigma^2)$

ESTIMADORES MÍNIMOS QUADRADOS

$$\hat{\beta}_0 = \frac{1}{n}\sum_i Y_i = \overline{Y}$$

$$\begin{cases} \hat{\beta}_1.S_{X_1X_1} + \hat{\beta}_2.S_{X_1X_2} + \ldots + \hat{\beta}_J.S_{X_1X_J} = S_{X_1Y} \\ \hat{\beta}_1.S_{X_2X_1} + \hat{\beta}_2.S_{X_2X_2} + \ldots + \hat{\beta}_J.S_{X_2X_J} = S_{X_2Y} \\ (\ldots) \\ \hat{\beta}_1.S_{X_JX_1} + \hat{\beta}_2.S_{X_JX_2} + \ldots + \hat{\beta}_J.S_{X_JX_J} = S_{X_JY} \end{cases}$$

$$S_{X_{j_1}X_{j_2}} = \sum_n \left(X_{j_1i} - \overline{X}_{j_1}\right)\left(X_{j_2i} - \overline{X}_{j_2}\right)$$

$$S_{X_jY} = \sum_n \left(X_{ji} - \overline{X}_j\right)\left(Y_i - \overline{Y}\right)$$

$$s^2 = \frac{1}{n-J-1}\sum_i \hat{\varepsilon}_i^2 =$$
$$= \frac{1}{n-J-1}\sum_i \left\{Y_i - \left[\hat{\beta}_0 + \hat{\beta}_1.\left(X_{1i} - \overline{X}_1\right) + \hat{\beta}_2.\left(X_{2i} - \overline{X}_2\right) + \ldots + \hat{\beta}_j.\left(X_{Ji} - \overline{X}_J\right)\right]\right\}^2$$

EXEMPLO 3

Determine a relação existente entre o calor envolvido no endurecimento, representado pela variável Y e os pesos de duas substâncias X_1 e X_2, tendo em consideração os seguintes valores obtidos numa experiência:

Y	78.5	74.3	104.3	87.6	95.6	109.2	102.7	72.5	93.1	115.9
X1	7	1	11	11	7	11	3	1	2	21
X2	26	29	59	31	52	55	71	31	54	47

COEFICIENTE DE CORRELAÇÃO

Coeficiente de correlação de Pearson

$$R = \frac{\sum (X_i - \bar{X})(Y_i - \bar{Y})}{\sqrt{\sum (X_i - \bar{X})^2 \sum (Y_i - \bar{Y})^2}} = \frac{s_{XY}}{\sqrt{s_{xx}} \cdot \sqrt{s_{YY}}}$$

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TESTES DE ASSOCIAÇÃO

Unilateral à direita	Unilateral à esquerda	Bilateral
$H_0 : \rho = 0$	$H_0 : \rho = 0$	$H_0 : \rho = 0$
$H_1 : \rho > 0$	$H_1 : \rho < 0$	$H_1 : \rho \neq 0$

Estatística de teste $t = \frac{r.\sqrt{n-2}}{\sqrt{1-r^2}}$

Região de Rejeição:

$t > t_{n-2,(\alpha)}$	$t < -t_{n-2,(\alpha)}$	$\lvert t \rvert > t_{n-2,(\alpha/2)}$

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EXEMPLO

- Índice de Desenvolvimento de Griffiths
  - avaliações motora e intelectual para 9 crianças com a idade de 4 anos

Motor	Intelectual
84	77
73	85
101	105
74	86
88	108
100	116
86	96
95	100
82	100

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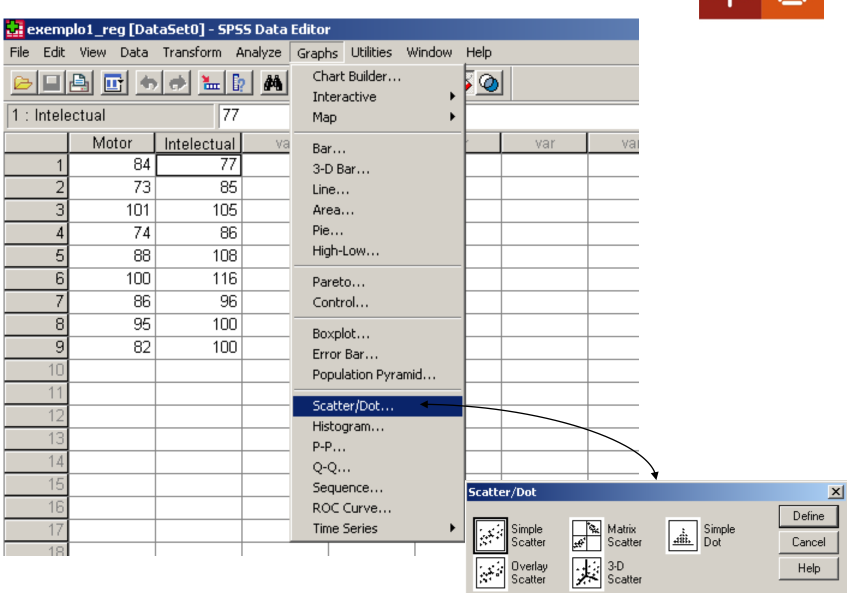

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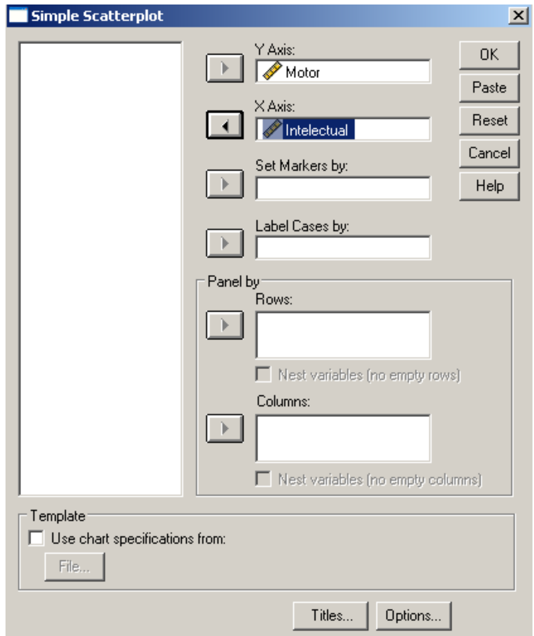
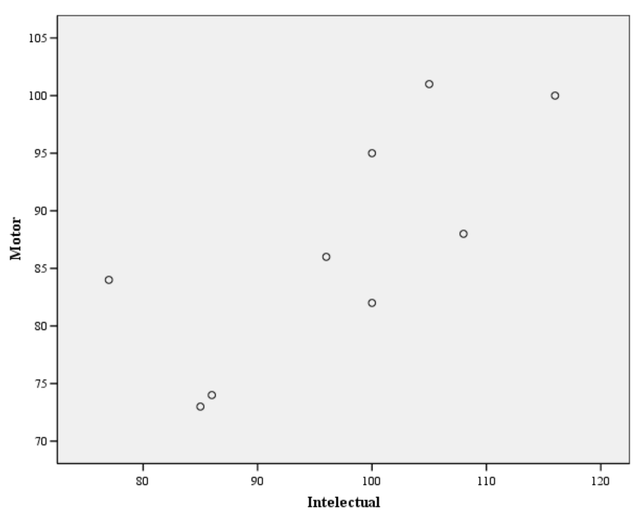
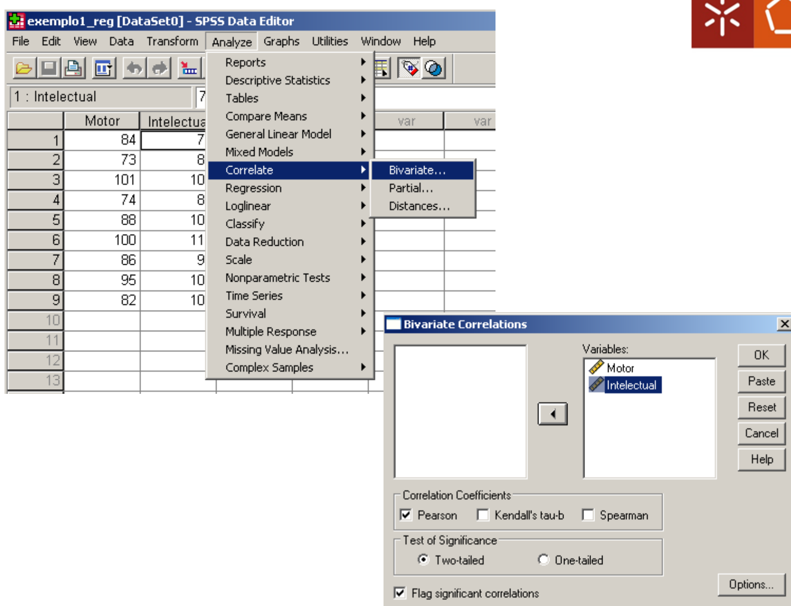
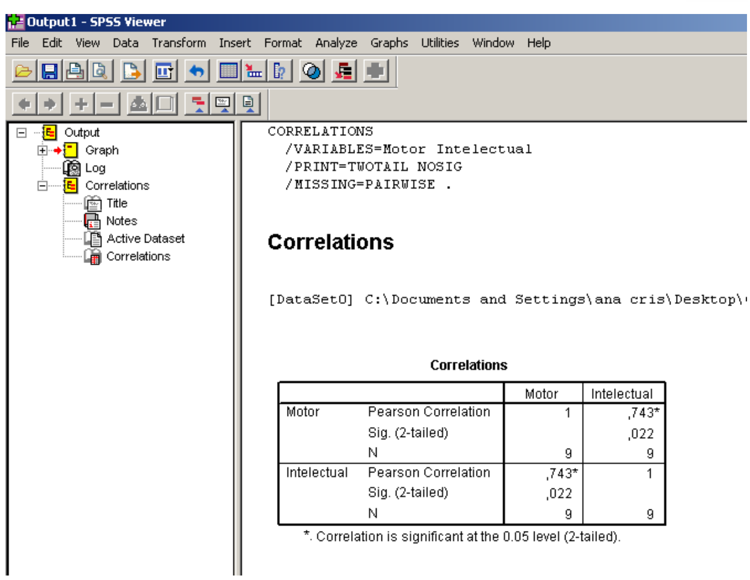


DIAGRAMA DE DISPERSÃO





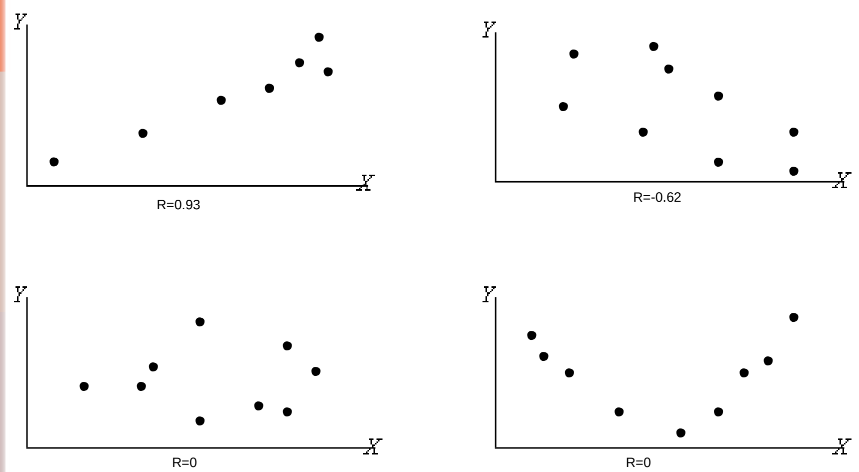


Correlations

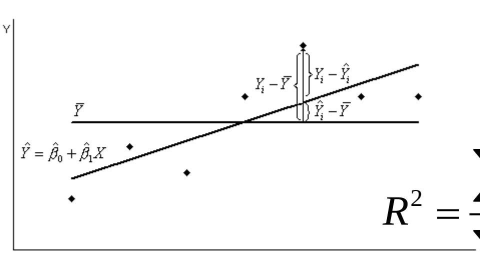
		Motor	Intelectual
Motor	Pearson Correlation	1	,743*
	Sig. (2-tailed)		,022
	N	9	9
Intelectual	Pearson Correlation	,743*	1
	Sig. (2-tailed)	,022	
	N	9	9

*. Correlation is significant at the 0.05 level (2-tailed).

CORRELAÇÃO



COEFICIENTE DE DETERMINAÇÃO



$$R^2 = \frac{\sum\left(\hat{Y}_i - \bar{Y}\right)^2}{\sum\left(Y_i - \bar{Y}\right)^2}$$

Coeficiente de determinação (r^2), representa a proporção da variação de Y que é explicada pela regressão

$$r^2 = \frac{\hat{\beta}_1^2 . s_{XX}}{s_{YY}} = \frac{\hat{\beta}_1^2 . \sum_i \left(X_i - \bar{X}\right)^2}{\sum_i \left(Y_i - \bar{Y}\right)^2} = \frac{\text{variação de } Y \text{ explicada pela regressão}}{\text{variação total de } Y}$$