Problem 1

Firstly, measure the conductivity σ_n without the applied B field as illustrated in the Figure (1).

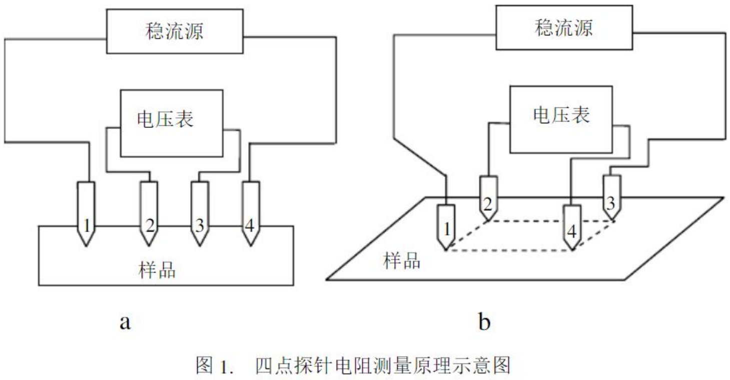

Figure 1: Schematic diagram for measuring the conductivity.

When applied B field, the motion of the electrons is shown in Figure (2), which generates the Hall electric field E_y.

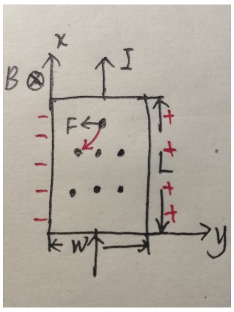

Figure 2: This is the motion of electrons in n type semiconductor with B field.

$$-qE_y - qv_xB = 0 \tag{1}$$

The Hall voltage is known, so the Hall electric filed is

$$E_y = \frac{V_H}{W} = -v_xB \tag{2}$$

where the mean drift velocity is $v_x = -\mu_n E_x$.

As for E_x,

$$E_x = \frac{J}{\sigma_n} = \frac{I}{\sigma_n WD} \tag{3}$$

In conclusion, based on Equation (2) and (3), the mobility of the semiconductor is derived by

$$\mu_n = -\frac{v_x}{E_x} = \frac{V_H}{WBE_x} = \frac{V_H}{WB} \cdot \left(\frac{\sigma_n WD}{I}\right) = \frac{V_H \sigma_n D}{BI} \tag{4}$$

Problem 2

To derterime the carrier concentration in thermal equilibrium, we need to know the density of states (the distribution of the energy states in the energy bands) and the Fermi function (the probability of an energy state being occupied by an electron).

$$g_c(E) = 4\pi V \frac{(2m_n^*)^{3/2}}{h^3}(E - E_c)^{1/2} \tag{5}$$

$$g_v(E) = 4\pi V \frac{(2m_p^*)^{3/2}}{h^3} (E_v - E)^{1/2} \tag{6}$$

$$f(E) = \frac{1}{1 + \exp\left(\frac{E - E_F}{k_0 T}\right)} \tag{7}$$

Assuming a continuous distribution of energy levels in the energy band, we can divide the energy band into infinitly small dE. Among the energy E with $E + dE$, the distribution of electrons in the conduction band can be found by the product $g_c(E) f(E) dE$. And the distribution of holes in the valence band can be found by the product $g_v(E) [1 - f(E)] dE$.

The carrier concentration is derived by integrating over all energies:

- Electron concentration

$$n = \int_{E_c}^{\infty} g_c(E) f(E) dE \tag{8}$$

- Hole concentration

$$p = \int_{-\infty}^{E_v} g_v(E) [1 - f(E)] dE \tag{9}$$

In thermal equilibrium, the electron concentration per unit volume is given by

$$\begin{aligned} n_0 = \frac{n}{V} &= \int_{Ec}^{\infty} \exp\left(-\frac{E - E_F}{k_0 T}\right) 4\pi \frac{(2m_n^*)^{3/2}}{h^3} (E - E_c)^{1/2} dE \\ &= 4\pi \frac{(2m_n^*)^{3/2}}{h^3} (k_0 T)^{3/2} \exp\left(-\frac{E - E_F}{k_0 T}\right) \int_0^{\infty} x^{1/2} e^{-x} dx \\ &= N_c \exp\left(-\frac{E_c - E_F}{k_0 T}\right) \end{aligned} \tag{10}$$

where $x = (E - E_c) / (k_0 T)$, $\int_0^{\infty} x^{1/2} e^{-x} dx = \frac{\sqrt{\pi}}{2}$. The effective density of states for electrons in the conduction band is $N_c = 2 \frac{(2\pi m_n^* k_0 T)^{3/2}}{h^3}$.

Similarly, the hole concentration per unit volume is

$$p_0 = N_v \exp\left(\frac{E_v - E_F}{k_0 T}\right) \tag{11}$$

where the effective density of states for holes in the valence band is $N_v = 2 \frac{(2\pi m_p^* k_0 T)^{3/2}}{h^3}$.

The product of carrier concentration is obtained from

$$n_0 p_0 = N_c N_v \exp\left(-\frac{E_c - E_v}{k_0 T}\right) = N_c N_v \exp\left(-\frac{E_g}{k_0 T}\right) = n_i^2 \tag{12}$$

where n_i is the intrinsic carrier density.

Problem 3

The processes of nonradiative Shockley-Read-Hall (SRH) recombination are illustrated in the Figure (1).

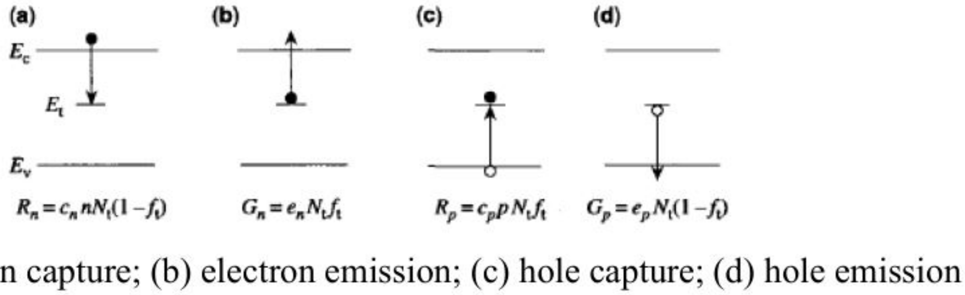

Figure 1: The four processes of SRH recombination

- Electron capture: The recombination rate for electrons is

$$R_n = c_n n N_t (1 - f_t) \tag{1}$$

- Electron emission: The generation rate of the electrons is

$$G_n = e_n N_t f_t \tag{2}$$

- Hole capture: The recombination rate of the holes is

$$R_p = c_p p N_t f_t \tag{3}$$

- Hole emission: The generation rate of the holes is

$$G_p = e_p N_t (1 - f_t) \tag{4}$$

where N_t is the concentration of the traps, $N_t (1 - f_t)$ term represents the concentration of empty traps, e_n and e_p are the emission probability for electrons and holes respectively. And c_n and c_p are the capture coefficient for the electrons and holes, respectively.

At thermal equilibrium, we have derived the expressions:

$$\frac{e_n}{c_n} = n_1, \frac{e_p}{c_p} = p_1, n_1 p_1 = n_i^2 \tag{5}$$

where n_1 and p_1 are the conduction equilibrium electron carrier concentration and the valence equilibrium hole carrier concentration when the Ferimi level is coincident with the recombination center level, respectively.

However, f_t is unknown in non-equilibrium and has to be calculated. Considering the stable condition $R_n - G_n = R_p - G_p$, we have

$$c_n n N_t (1 - f_t) - e_n N_t f_t = c_p p N_t f_t - e_p N_t (1 - f_t) \tag{6}$$

$$f_t = \frac{c_n n N_t + e_p N_t}{e_n N_t + c_p p N_t + c_n n N_t + e_p N_t} = \frac{c_n n + c_p p_1}{c_n (n + n_1) + c_p (p + p_1)} \tag{7}$$

So the SRH recombination rate is derived by

$$U = R_n - G_n = \frac{c_n c_p N_t (np - n_1 p_1)}{c_n (n + n_1) + c_p (p + p_1)} = \frac{np - n_i^2}{\tau_p (n + n_1) + \tau_n (p + p_1)} \tag{8}$$

where $\tau_p = \frac{1}{c_p N_t}$, $\tau_n = \frac{1}{c_n N_t}$.

Problem 4

Solution. Carrier concentration in thermal non-equilibrium of holes is

$$p = N_v \exp(\frac{E_v - E_{F_p}}{k_B T}),$$

where N_v is the effective density of states for valance band, E_v is the valence level and E_{F_p} is the quasi-Fermi level of holes. Then the quasi-Fermi level of holes can be derived as

$$E_{F_p} = E_v - k_B T \ln(\frac{p}{N_v}) = E_0 - \chi - E_g + k_B T \ln N_v - k_B T \ln p,$$

where E_0 is base level, χ is the vacuum level and E_g is the band gap between the conduction and valence levels. According to quasi-Fermi potential

$$\phi_p = \varphi + \frac{k_B T}{q} \log(\frac{p}{n_i})$$

and the defination of electrostatic field

$$\mathbf{E} = -\nabla \varphi,$$

the expression of the hole current can be written as

$$J_p = -q\mu_p p \nabla \phi_p = q\mu_p p \mathbf{E} - qD_p \nabla p,$$

where D_p satisfies the Einstein relation

$$D_p = \mu_p \frac{k_B T}{q}.$$

Then, the hole current can be also expressed as

$$J_p = -\mu_p p \nabla (q\phi_p) = -\mu_p p \nabla E_{F_p} = -\mu_p p \nabla [E_0 - \chi - E_g + k_B T \ln N_v - k_B T \ln p].$$

In order to derive the expression of electrostatic field of holes (i.e., $\mathbf{E}_p$), we need to wirte the hole current equation in the form including $\mathbf{E}$. Therefore,

$$\begin{aligned} J_p &= q\mu_p p[-\nabla(\frac{E_0 - \chi - E_g + k_B T \ln N_v - k_B T \ln p}{q})] \\ &= q\mu_p p[-\nabla(\frac{E_0 - \chi - E_g + k_B T \ln N_v}{q})] - qD_p \nabla p \\ &= q\mu_p p \mathbf{E}_p - qD_p \nabla p \end{aligned}$$

Eventually, we get

$$\begin{aligned} \mathbf{E}_p &= -\nabla \phi_p = -\nabla(\frac{E_0 - \chi - E_g + k_B T \ln N_v}{q}) \\ \phi_p &= \frac{E_0 - \chi - E_g + k_B T \ln N_v}{q} = \varphi + \frac{\chi}{q} + \frac{E_g}{q} - \frac{k_B T}{q} \ln N_v. \end{aligned}$$

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