



For polystyrene foam,

$$\frac{\sigma}{\epsilon\omega} = \frac{4 \times 10^{-6}}{1.03 \times 8.85 \times 10^{-12} \times 2\pi \times 2.5 \times 10^9} = 2.79 \times 10^{-5} \ll 1,$$

we get

$$d_p \approx \frac{2}{\sigma\eta} = \frac{2\sqrt{1.03}}{4 \times 10^{-6} \times 377} = 1346.0(m)$$

(b) For good conductor,

$$\frac{\epsilon\omega}{\sigma} < 0.1$$

thus

$$\omega < 0.1 \times \frac{\sigma}{\epsilon} = 0.1 \times \frac{5 \times 10^{-3}}{10 \times 8.85 \times 10^{-12}} = 5.65(rad/sec),$$

or

$$f_{\max} = \frac{\omega}{2\pi} = 0.899(MHz)$$

(c) Since  $\frac{\sigma}{\epsilon\omega} \gg 1$ , thus

$$d_p \approx \sqrt{\frac{2}{\omega\mu_o\sigma}} = \sqrt{\frac{2}{2\pi \times 10^8 \times 4\pi \times 10^{-7} \times 3.54 \times 10^7}} = 8.46 \times 10^{-6}(m).$$

So the thickness of the aluminum layer is  $5d_p = 4.24 \times 10^{-5}(m)$ . The thickness of ordinary aluminum foil is

$$10^{-3}(inch) = 2.54 \times 10^{-5}(m) < 4.24 \times 10^{-5}(m),$$

which is not thick enough.

(d)

(1) For  $f = 100(Hz)$ ,

$$\frac{\sigma}{\omega\epsilon} = \frac{4}{2\pi \times 100 \times 80 \times 8.85 \times 10^{-12}} = 8.99 \times 10^6,$$

and skin depth

$$d_p \approx \sqrt{\frac{2}{\omega\mu_o\sigma}} = \sqrt{\frac{2}{2\pi \times 100 \times 4\pi \times 10^{-7} \times 4}} = 25.2(m).$$

(2) For  $f = 5(MHz)$ , the loss tangent is

$$\frac{\sigma}{\omega\epsilon} = \frac{4}{2\pi \times 5 \times 10^6 \times 80 \times 8.85 \times 10^{-12}} = 180,$$

and the skin depth is

$$d_p \approx \sqrt{\frac{2}{\omega\mu_o\sigma}} = \sqrt{\frac{2}{2\pi \times 5 \times 10^6 \times 4\pi \times 10^{-7} \times 4}} = 0.11(m).$$

(e) The penetration depth at frequency  $f = 1.0(kHz)$  is

$$d_p \approx \sqrt{\frac{2}{\omega\mu_o\sigma}} = \sqrt{\frac{2}{2\pi \times 1000 \times 4\pi \times 10^{-7} \times 4}} = 7.96(m) = \frac{1}{k_I}.$$

The attenuation term for the radiation power is

$$e^{-2k_I z} = e^{-2 \times \frac{100}{7.96}} = 1.22 \times 10^{-11} = -109.1(dB).$$

### Solution P3.3

(a)

$$\overline{T} = \hat{r} \times \overline{F} = \frac{1}{2} l_b \hat{x} \times (\hat{z} \times \hat{x} I l_z B_0) - \frac{1}{2} l_b \hat{x} \times (-\hat{z} \times \hat{x} I l_z B_0) = \hat{z} I A B_0$$

From  $\overline{M} = -\hat{y} I A$ , and  $\overline{B} = \hat{x} B_0$ , we find that,  $\overline{T} = \overline{M} \times \overline{B} = \hat{z} I A B_0$ .

(b) The magnetic moment,  $\overline{M}$  of the current loop can be found by applying  $\overline{M} = \hat{n} A_0 I_l = \frac{1}{\sqrt{2}}(\hat{x} - \hat{y}) A_0 I_l$ .

To find the magnetic field,  $\overline{H}$ , at the position of the loop due to the straight wire carrying current,  $I_0$ , we use the integral form of Ampere's Law, that is,

$$\oint_C \overline{H} \cdot d\vec{l} = \int_s \overline{J} \cdot d\vec{s} = I_0$$

By symmetry the  $\overline{H}$  field will be of uniform amplitude at a fixed radius and in the  $\hat{\phi}$  direction yielding,

$$\oint_C \overline{H} \cdot d\vec{l} = \int_0^{2\pi} H_\phi d\phi = 2\pi d H_\phi$$

which implies that at the loop's position

$$\overline{B} = \hat{\phi} \frac{I_0 \mu_0}{2\pi d} = \hat{x} \frac{I_0 \mu_0}{2\pi d}$$

where the fact that  $\overline{B} = \mu_0 \overline{H}$  was used. To calculate the torque, we apply,

$$\overline{T} = \overline{M} \times \overline{B} = \hat{z} \frac{A_0 I_l I_0 \mu_0}{2\sqrt{2}\pi d}$$

which means that the current loop will move about the  $z$ -axis in a counter-clockwise direction.