

Problem P1.3

Prove the following vector identities, all of which are frequently used in this course.

$$\nabla \times (\nabla \times \overline{E}) = \nabla (\nabla \cdot \overline{E}) - \nabla^2 \overline{E}$$

$$\nabla \cdot (\overline{E} \times \overline{H}) = \overline{H} \cdot (\nabla \times \overline{E}) - \overline{E} \cdot (\nabla \times \overline{H})$$

$$\nabla \cdot (\nabla \times \overline{A}) = 0$$

$$\nabla \times (\nabla \phi) = 0$$

Problem P1.4

Consider a wave which has the form of $\cos(kz - \omega t)$, where the phase term is $\phi = kz - \omega t$. Take derivative of the phase ϕ with respect to time t and let $\partial\phi/\partial t = 0$ to calculate the wave velocity. Can we calculate the wave velocity by taking derivative of the phase with respect to z ? Why?

Run the following code in Matlab and you will get a figure showing the propagation of this wave. Trace the line where each point has the same phase as the origin. Then the slope of the line represents the wave velocity. Here we let $k = 1$ first. What will happen if k is increased?

```
k=1;
omega = 2;
[z,t] = meshgrid(0:0.01:10, 0:0.01:10);
wave = cos(k*z - omega*t);
pcolor(t,z,wave);shading flat;colorbar;
xlabel('t');ylabel('z');axis square
```