1. Consider a rectangular waveguide, how to excite the TE_{10} mode with a metallic small loop? and how to excite the TE_{10} and TE_{20} mode simultaneously with a metallic short wire? Please draw your design configurations and gives the working frequency range of the excitations. (50%, medium)

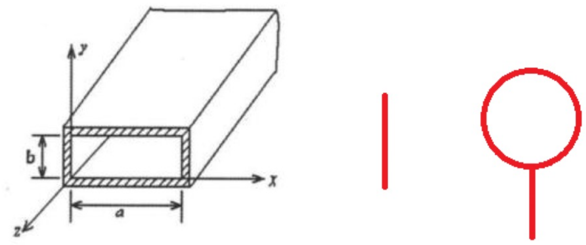
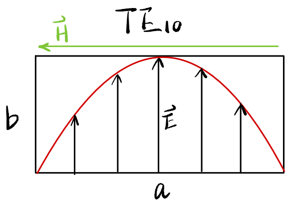
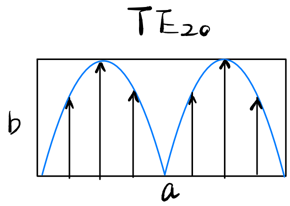


Fig. 1. Rectangular waveguide, short wire and small loop.

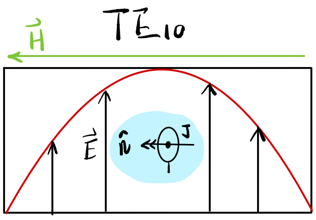
Solution:





(1) Small loop can be regarded as magnetic dipole.

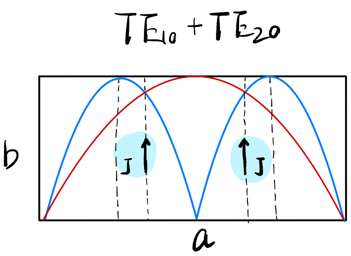
So the normal direction of the loop should be set along the magnetic line to match the magnetic field.



working frequency rage: $TE_{10}: K_1 = \frac{\pi}{a}$ $TE_{20}: K_2 = \frac{2\pi}{a}$

$cK_1 \le \omega < cK_2$, i.e. $\frac{1}{\sqrt{\mu_0\varepsilon_0}} \cdot \frac{\pi}{a} \le \omega_{10} < \frac{1}{\sqrt{\mu_0\varepsilon_0}} \cdot \frac{2\pi}{a}$

(2)



TE_{10}: $K_x = \frac{\pi}{a}$, $k_y = 0$

TE_{20}: $K_x = \frac{2\pi}{a}$, $k_y = 0$

working frequency rage: $\omega \ge \frac{1}{\sqrt{\mu_0\varepsilon_0}} \cdot \frac{2\pi}{a}$

2. For a grating structure, the substrate has a finite thickness, which supports a guided mode with a propagation constant β. For a plane wave incidence, please answer whether the guided mode can be excited or not? How to set up the incident angle of the plane wave by using the phase matching condition? The propagation constant of the guided mode in the grating is real or complex? Please think about these questions and write a correct phase-matching condition. (50%, medium)

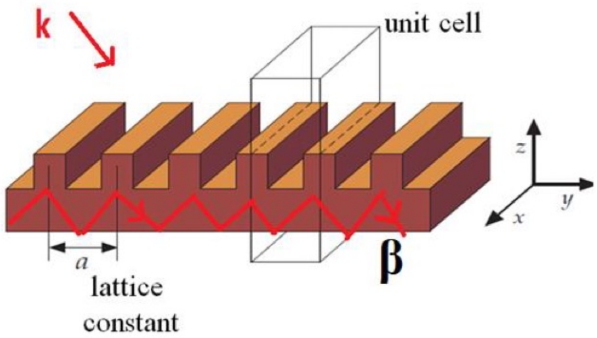
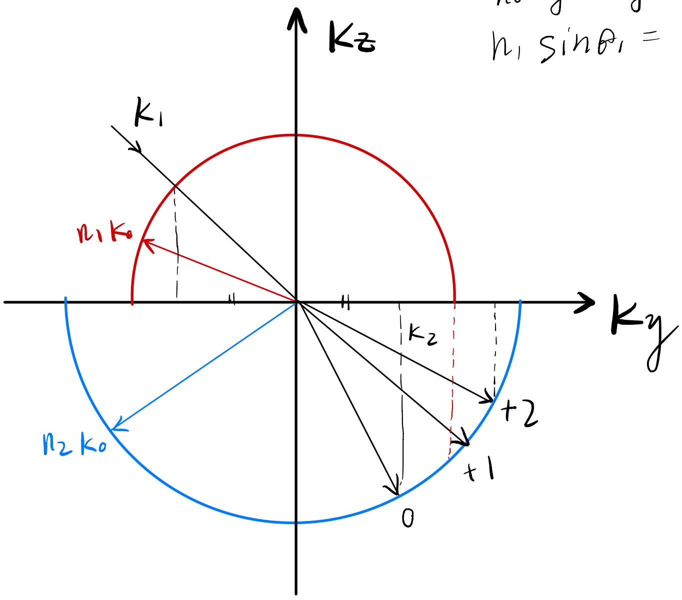


Fig. 2. Guided wave bounded in a grating structure excited by a plane wave from air with the wavevector **k**.

no grating:
$n_1 \sin\theta_1 = n_2 \sin\theta_2$

Solution:



The structure is periodic along y, phase matching condition:

切向 K_ρ 连续 $Re(\pm K_{2y}) = K_{1y} + \frac{2\pi m}{a}$, $m = 0, \pm 1, \pm 2 \cdots$

To excite guided mode along y: $\sqrt{K_{1y}^2 + K_{1z}^2} < K_\rho < \sqrt{K_{2y}^2 + K_{2z}^2}$

$$\begin{cases} K_\rho^2 + K_{1z}^2 = n_1^2 K_0^2 = K_1^2 \\ K_\rho^2 + K_{2z}^2 = n_2^2 K_0^2 = K_2^2 \end{cases} \Rightarrow K_1 < K_\rho < K_2$$

$K_{1z}^2 < 0$ $K_{2z}^2 > 0$

According to the reciprocity theorem
quasi-guided mode 耦入 ⟷ 泄漏
β: complex

Guided mode (along y) can be excited from high order diffraction.