

Problem P5.3

In the Čerenkov radiation, the total energy radiated out of a cylinder of path ℓ and radius ρ is given by

$$S_\rho = \frac{\mu q^2 \ell}{4\pi} \int_0^\infty d\omega \omega \left(1 - \frac{1}{n^2 \beta^2}\right).$$

So the energy lost per unit length per unit frequency band is

$$\frac{d^2 S_\rho}{d\ell d\omega} = \frac{\mu q^2}{4\pi} \omega \left(1 - \frac{1}{n^2 \beta^2}\right).$$

- (a) By $E_{\text{photon}} = \hbar\omega$ and $d\omega/d\lambda = 2\pi c/\lambda^2$, show that the number of photons radiated on unit path at wavelength λ is

$$\frac{d^2 N}{d\ell d\lambda} = \frac{q^2 c}{2\lambda^2 \hbar} \mu \left(1 - \frac{1}{n^2 \beta^2}\right)$$

and show the frequently used formula $\frac{dN}{d\ell} \propto \frac{d\lambda}{\lambda^2} \sin^2 \theta$, which gives the dependence of N on λ and θ .

- (b) Gas Čerenkov detector is widely used in high energy particle experiments. The refractive index of the gas n is typically 1.002. What will be the angle for the Čerenkov radiation in case of $\beta = 1$?
- (c) Most energy is radiated by the waves in the band 350 nm \sim 550 nm. How many photons can you get on unit path? In order to get 100 photons for the detector, how long is the path (ℓ)? This is the size the detector should be. Note that the parameters are as follows: $\hbar = 6.63 \times 10^{-34} / 2\pi \text{ J} \cdot \text{s/rad}$, $q = 1.6 \times 10^{-19} \text{ C}$, $\beta = 1$.