



From Ampere's law, we find, since  $\overline{\omega}_c = qB_0/m$ ,

$$\nabla \times$$

$$\mathbf{H} = -i\omega\epsilon_0\overline{\mathbf{E}} + \overline{\mathbf{J}} = i\omega \left\{ i\epsilon_g \times \overline{\mathbf{E}} + \epsilon\overline{\mathbf{E}} + \frac{\omega_p^2\omega_c^2\epsilon_0}{\omega^2(\omega^2 - \omega_c^2)} \cdot \overline{\mathbf{E}} \right\} = -i\omega\overline{\epsilon} \cdot \overline{\mathbf{E}}$$

where  $\epsilon_g$  and  $\epsilon$  are as defined. Writing  $\overline{\epsilon}$  in the matrix form, we find

$$\epsilon_z = \epsilon_0 \left[ 1 - \frac{\omega_p^2/\omega^2}{1 - \omega_c^2/\omega^2} + \frac{\omega_p^2\omega_c^2/\omega^4}{1 - \omega_c^2/\omega^2} \right] = \epsilon_0 \left[ 1 - \frac{\omega_p^2}{\omega^2} \right]$$

Carrying on the inverse of  $\overline{\epsilon}$ , we find for  $\overline{\kappa} = \overline{\epsilon}^{-1}$

$$\begin{aligned} \kappa &= \frac{\epsilon}{\epsilon^2 - \epsilon_g^2} = \frac{1}{\epsilon_0} \left[ \frac{1 - \omega_p^2/\omega^2 - \omega_c^2/\omega^2}{(1 - \omega_p^2/\omega^2)^2 - \omega_c^2/\omega^2} \right] \\ \kappa_g &= \frac{-\epsilon_g}{\epsilon^2 - \epsilon_g^2} = \frac{1}{\epsilon_0} \left[ \frac{\omega_c\omega_p^2/\omega^3}{(1 - \omega_p^2/\omega^2)^2 - \omega_c^2/\omega^2} \right] \\ \kappa_z &= \frac{1}{\epsilon_z} = \frac{1}{\epsilon_0} \left[ \frac{1}{1 - \omega_p^2/\omega^2} \right] \end{aligned}$$

It is easily shown that  $\overline{\epsilon} \cdot \overline{\kappa} = \overline{\mathbf{I}}$ .

In the case of an infinitely strong magnetic field,  $\omega_c \rightarrow \infty$  and we have

$$\begin{aligned} \epsilon &= \epsilon_0 & \kappa &= \frac{1}{\epsilon_0} \\ \epsilon_g &= 0 & \kappa_g &= 0 \\ \epsilon_z &= \epsilon_0 \left( 1 - \frac{\omega_p^2}{\omega^2} \right) & \kappa_z &= \frac{1}{\epsilon_0} \left[ \frac{1}{1 - \omega_p^2/\omega^2} \right] \end{aligned}$$

and the medium becomes a uniaxial plasma.

#### Solution P2.4

$$\overline{\mathbf{E}} = \hat{x}e^{ikz} = \frac{1}{2} [(\hat{x} + i\hat{y}) + (\hat{x} - i\hat{y})] e^{ikz}$$