Problem 1

(1)

Solution. In order to derive the identity for time-harmonic field, first we need to write E and H in complex form,

$$\mathbf{E}(\mathbf{r},t) = Re[\mathbf{E}(\mathbf{r})e^{j\omega t}] = \frac{1}{2}[\mathbf{E}(\mathbf{r})e^{j\omega t} + \mathbf{E}^*(\mathbf{r})e^{-j\omega t}]$$
$$\mathbf{H}(\mathbf{r},t) = Re[\mathbf{H}(\mathbf{r})e^{j\omega t}] = \frac{1}{2}[\mathbf{H}(\mathbf{r})e^{j\omega t} + \mathbf{H}^*(\mathbf{r})e^{-j\omega t}]$$

then,

$$\begin{aligned}
&\mathbf{S}(\mathbf{r},t) = \mathbf{E}(\mathbf{r},t) \times \mathbf{H}(\mathbf{r},t)\\
&= \frac{1}{4}[\mathbf{E}(\mathbf{r}) \times \mathbf{H}(\mathbf{r})^* + \mathbf{E}^*(\mathbf{r}) \times \mathbf{H}(\mathbf{r}) + \mathbf{E}(\mathbf{r}) \times \mathbf{H}(\mathbf{r})e^{2j\omega t} + \mathbf{E}^*(\mathbf{r}) \times \mathbf{H}^*(\mathbf{r})e^{-2j\omega t}]\\
&= \frac{1}{2}Re[\mathbf{E}(\mathbf{r}) \times \mathbf{H}^*(\mathbf{r}) + \mathbf{E}(\mathbf{r}) \times \mathbf{H}(\mathbf{r})e^{2j\omega t}]
\end{aligned}$$

The average value of Poynting vector in a period is,

$$\begin{aligned}
&\mathbf{S}^{av}(\mathbf{r},t) = \frac{1}{T}\int_0^T \mathbf{S}(\mathbf{r},\mathbf{t})\,dt\\
&= \frac{1}{T}\int_0^T \frac{1}{2}Re[\mathbf{E}(\mathbf{r}) \times \mathbf{H}^*(\mathbf{r}) + \mathbf{E}(\mathbf{r}) \times \mathbf{H}(\mathbf{r})e^{2j\omega t}]\,dt\\
&= \frac{1}{2}Re[\mathbf{E}(\mathbf{r}) \times \mathbf{H}^*(\mathbf{r})]
\end{aligned}$$

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(2)

Solution. Total emitted power from a dipole is,

$$W_T = \int_{S_6} \frac{1}{2}Re[\mathbf{E}(\mathbf{r}) \times \mathbf{H}^*(\mathbf{r})] \cdot\, dS$$

Dissipation power in metal cathode is,

$$W_M = -\int_{S_1+S_2} \frac{1}{2}Re[\mathbf{E}(\mathbf{r}) \times \mathbf{H}^*(\mathbf{r})] \cdot\, dS$$

Dissipation power in ITO layer is,

$$W_I = -\int_{S_3+S_4} \frac{1}{2}Re[\mathbf{E}(\mathbf{r}) \times \mathbf{H}^*(\mathbf{r})] \cdot\, dS$$

Dissipation power in glass substrate is,

$$W_G = -\int_{S_4+S_5} \frac{1}{2}Re[\mathbf{E}(\mathbf{r}) \times \mathbf{H}^*(\mathbf{r})] \cdot\, dS$$

Emission power in air is,

$$W_E = \int_{S_{upper}} \frac{1}{2}Re[\mathbf{E}(\mathbf{r}) \times \mathbf{H}^*(\mathbf{r})] \cdot\, dS$$

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In LED, emission power is equal to total power minus power dissipation inside the LED, so the power dissipation in organic layer is,

$$W_O = -(W_T + W_M + W_I + W_G - W_E)$$

Problem 2

Solution. Use three dimension delta function in the form,

$$\delta^3(\mathbf{r}-\mathbf{r}') = -\frac{1}{4\pi}\nabla^2\frac{1}{|\mathbf{r}-\mathbf{r}'|}$$

Use the vector identities

$$\begin{aligned}
&\nabla^2\mathbf{a} = \nabla(\nabla\cdot\mathbf{a}) - \nabla\times(\nabla\times\mathbf{a})\\
&\nabla\cdot(\psi\mathbf{a}) = \psi\nabla\cdot\mathbf{a} + \mathbf{a}\cdot\nabla\psi\\
&\nabla\times(\psi\mathbf{a}) = \psi\nabla\times\mathbf{a} + \nabla\psi\times\mathbf{a}
\end{aligned}$$

Then the vector $\mathbf{F(r)}$ can be written as,

$$\begin{aligned}
&\mathbf{F}(\mathbf{r}) = \int_V \mathbf{F}(\mathbf{r}')\delta^3(\mathbf{r}-\mathbf{r}')\,dV'\\
&= \int_V \mathbf{F}(\mathbf{r}')(-\frac{1}{4\pi}\nabla^2\frac{1}{|\mathbf{r}-\mathbf{r}'|})\,dV'\\
&= -\frac{1}{4\pi}\int_V \nabla^2\frac{\mathbf{F}(\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|}\,dV'\\
&= -\frac{1}{4\pi}[\nabla(\nabla\cdot\int_{\mathbf{V}}\frac{\mathbf{F}(\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|}\,\mathbf{dV'}) - \nabla\times(\nabla\times\int_{\mathbf{V}}\frac{\mathbf{F}(\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|}\,\mathbf{dV'})]
\end{aligned}$$

Inside, $\mathbf{F(r')}$ is a vector (invariant respect to $\mathbf{r}$), so,

$$\begin{aligned}
&\nabla\cdot\frac{\mathbf{F}(\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|} = \frac{1}{|\mathbf{r}-\mathbf{r}'|}\nabla\cdot\mathbf{F}(\mathbf{r}') + \mathbf{F}(\mathbf{r}')\cdot\nabla\frac{1}{|\mathbf{r}-\mathbf{r}'|}\\
&= \mathbf{F}(\mathbf{r}')\cdot\nabla\frac{1}{|\mathbf{r}-\mathbf{r}'|}\\
&\nabla\times\frac{\mathbf{F}(\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|} = \frac{1}{|\mathbf{r}-\mathbf{r}'|}\nabla\times\mathbf{F}(\mathbf{r}') + \nabla\frac{1}{|\mathbf{r}-\mathbf{r}'|}\times\mathbf{F}(\mathbf{r}')\\
&= \nabla\frac{1}{|\mathbf{r}-\mathbf{r}'|}\times\mathbf{F}(\mathbf{r}')
\end{aligned}$$

Use the curl with respect to $\mathbf{r'}$

$$\nabla\frac{1}{|\mathbf{r}-\mathbf{r}'|} = -\nabla'\frac{1}{|\mathbf{r}-\mathbf{r}'|}$$

Therefore,

$$
\begin{aligned}
\mathbf{F}(\mathbf{r}) &= -\frac{1}{4\pi}[\nabla(\nabla\cdot\int_{\mathbf{V}}\frac{\mathbf{F}(\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|}\,\mathbf{dV}') - \nabla\times(\nabla\times\int_{\mathbf{V}}\frac{\mathbf{F}(\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|}\,\mathbf{dV}')] \\
&= -\frac{1}{4\pi}[\nabla(\int_{\mathbf{V}}\mathbf{F}(\mathbf{r}')\cdot\nabla\frac{\mathbf{1}}{|\mathbf{r}-\mathbf{r}'|}\,\mathbf{dV}') - \nabla\times(\int_{\mathbf{V}}\nabla\frac{\mathbf{1}}{|\mathbf{r}-\mathbf{r}'|}\times\mathbf{F}(\mathbf{r}')\,\mathbf{dV}')] \\
&= -\frac{1}{4\pi}[-\nabla(\int_{\mathbf{V}}\mathbf{F}(\mathbf{r}')\cdot\nabla'\frac{\mathbf{1}}{|\mathbf{r}-\mathbf{r}'|}\,\mathbf{dV}') - \nabla\times(\int_{\mathbf{V}}\mathbf{F}(\mathbf{r}')\times\nabla'\frac{\mathbf{1}}{|\mathbf{r}-\mathbf{r}'|}\,\mathbf{dV}')]
\end{aligned}
$$

Then using the vector identities

$$
\begin{aligned}
\mathbf{a}\cdot\nabla\psi &= \nabla\cdot(\psi\mathbf{a}) - \psi\nabla\cdot\mathbf{a} \\
\mathbf{a}\times\nabla\psi &= -\nabla\times(\psi\mathbf{a}) + \psi\nabla\times\mathbf{a}
\end{aligned}
$$

We get,

$$
\begin{aligned}
&\mathbf{F}(\mathbf{r}')\cdot\nabla'\frac{1}{|\mathbf{r}-\mathbf{r}'|} = \nabla'\cdot(\frac{\mathbf{F}(\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|}) - \frac{1}{|\mathbf{r}-\mathbf{r}'|}\nabla'\cdot\mathbf{F}(\mathbf{r}') \\
&\mathbf{F}(\mathbf{r}')\times\nabla'\frac{1}{|\mathbf{r}-\mathbf{r}'|} = -\nabla'\times(\frac{\mathbf{F}(\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|}) + \frac{1}{|\mathbf{r}-\mathbf{r}'|}\nabla'\times\mathbf{F}(\mathbf{r}') \\
&\mathbf{F}(\mathbf{r}) = -\frac{1}{4\pi}[-\nabla(\int_{\mathbf{V}}\mathbf{F}(\mathbf{r}')\cdot\nabla'\frac{\mathbf{1}}{|\mathbf{r}-\mathbf{r}'|}\,\mathbf{dV}') + \nabla\times(\int_{\mathbf{V}}\mathbf{F}(\mathbf{r}')\times\nabla'\frac{\mathbf{1}}{|\mathbf{r}-\mathbf{r}'|}\,\mathbf{dV}')] \\
&= -\frac{1}{4\pi}[-\nabla(\int_{\mathbf{V}}[\nabla'\cdot(\frac{\mathbf{F}(\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|}) - \frac{\mathbf{1}}{|\mathbf{r}-\mathbf{r}'|}\nabla'\cdot\mathbf{F}(\mathbf{r}')]\,\mathbf{dV}') \\
&-\nabla\times(\int_{\mathbf{V}}[-\nabla'\times(\frac{\mathbf{F}(\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|}) + \frac{\mathbf{1}}{|\mathbf{r}-\mathbf{r}'|}\nabla'\times\mathbf{F}(\mathbf{r}')]\,\mathbf{dV}')]
\end{aligned}
$$

Use divergence theorem,

$$
\begin{aligned}
&\mathbf{F}(\mathbf{r}) = -\frac{1}{4\pi}[-\nabla(\int_{\mathbf{S}}\mathbf{n}'\cdot(\frac{\mathbf{F}(\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|})\,\mathbf{dS}' - \int_{\mathbf{V}}\frac{\mathbf{1}}{|\mathbf{r}-\mathbf{r}'|}\nabla'\cdot\mathbf{F}(\mathbf{r}')\,\mathbf{dV}') \\
&-\nabla\times(\int_{\mathbf{S}}-\mathbf{n}'\times(\frac{\mathbf{F}(\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|})\,\mathbf{dS}' + \int_{\mathbf{V}}\frac{\mathbf{1}}{|\mathbf{r}-\mathbf{r}'|}\nabla'\times\mathbf{F}(\mathbf{r}')\,\mathbf{dV}')] \\
&= -\nabla(-\frac{1}{4\pi}\int_{\mathbf{S}}\mathbf{n}'\cdot(\frac{\mathbf{F}(\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|})\,\mathbf{dS}' + \frac{\mathbf{1}}{4\pi}\int_{\mathbf{V}}\frac{\mathbf{1}}{|\mathbf{r}-\mathbf{r}'|}\nabla'\cdot\mathbf{F}(\mathbf{r}')\,\mathbf{dV}') \\
&+\nabla\times(-\frac{\mathbf{1}}{\mathbf{4}\pi}\int_{\mathbf{S}}\mathbf{n}'\times(\frac{\mathbf{F}(\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|})\,\mathbf{dS}' + \frac{\mathbf{1}}{\mathbf{4}\pi}\int_{\mathbf{V}}\frac{\mathbf{1}}{|\mathbf{r}-\mathbf{r}'|}\nabla'\times\mathbf{F}(\mathbf{r}')\,\mathbf{dV}')
\end{aligned}
$$

So,

$$
\begin{aligned}
\psi(\mathbf{r}) &= -\frac{1}{4\pi}\int_{\mathbf{S}}\mathbf{n}'\cdot(\frac{\mathbf{F}(\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|})\,\mathbf{dS}' + \frac{\mathbf{1}}{4\pi}\int_{\mathbf{V}}\frac{\mathbf{1}}{|\mathbf{r}-\mathbf{r}'|}\nabla'\cdot\mathbf{F}(\mathbf{r}')\,\mathbf{dV}' \\
\mathbf{A}(\mathbf{r}) &= -\frac{\mathbf{1}}{\mathbf{4}\pi}\int_{\mathbf{S}}\mathbf{n}'\times(\frac{\mathbf{F}(\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|})\,\mathbf{dS}' + \frac{\mathbf{1}}{\mathbf{4}\pi}\int_{\mathbf{V}}\frac{\mathbf{1}}{|\mathbf{r}-\mathbf{r}'|}\nabla'\times\mathbf{F}(\mathbf{r}')\,\mathbf{dV}'
\end{aligned}
$$

When $r \to \infty$, F(r') vanishes while the curl and divergence of it still remain. Therefore,

$$\psi(\mathbf{r}) = \frac{1}{4\pi}\int_V \frac{1}{|\mathbf{r}-\mathbf{r}'|}\nabla'\cdot\mathbf{F}(\mathbf{r}')\,dV'$$
$$\mathbf{A}(\mathbf{r}) = \frac{1}{4\pi}\int_V \frac{1}{|\mathbf{r}-\mathbf{r}'|}\nabla'\times\mathbf{F}(\mathbf{r}')\,dV'$$
$$\mathbf{F} = -\nabla\psi + \nabla\times\mathbf{A}$$

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Problem 3

(1)

Solution. Assume that A (SI unit: m^2) is a small surface centred at a given point M and orthogonal to the motion of the charges at M. If I_A(SI unit: A) is the electric current flowing through A, then electric current density j at M is given by the limit

$$J = \lim_{A \to 0} \frac{I_A}{A} = \frac{I_A}{dA}$$

Then

$$I = \frac{dQ}{dt} = \frac{\rho v \cdot dt \cdot dA}{dt} = \rho v \cdot dA$$

$$\mathbf{J} = \rho \mathbf{v}$$

In the farfield expression of EM fields at Page 24 or wave equation, the source of the field is j$\omega\mathbf{J}$, which is $\frac{\partial \mathbf{J}}{\partial t}$ in time domain. j$\omega\mathbf{J}$ will be equal to zero, since no radiation, if electron is not accelerated ($\frac{d\mathbf{v}}{dt} = 0$).

Then, for the mechanical rotation of an object, the direction of v is changing and there will be centripetal acceleration $\mathbf{a} = v^2/R$ ■

Problem 4

Solution. Consider the original problem in the following photography:

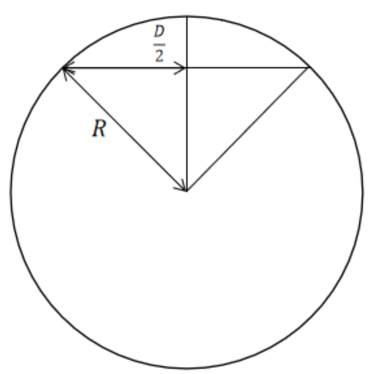

Figure 1: farfield Approximation

According to that phase delay should be less than $\pi/8$, we get inequation

$$\frac{R - \sqrt{R^2 - \left(\frac{D}{2}\right)^2}}{\lambda} \cdot 2\pi < \frac{\pi}{8}$$

Then we can derive that

$$R > \frac{2D^2}{\lambda}$$

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Problem 5

Solution. Far field expression of a dipole is

$$\mathbf{E}(\mathbf{k}) = -j\omega\mu_0 \frac{e^{-jkr}}{4\pi r} \int_v (\mathbf{a}_\theta \mathbf{a}_\theta + \mathbf{a}_\phi \mathbf{a}_\phi) \cdot \mathbf{J}(\mathbf{r}') exp(jk\mathbf{r}' \cdot \mathbf{a_R}) d\tau'$$

We can obtain the source distribution with mirror principle, as shown in figure 2 .The two dipoles orient toward z and -z direction.The current density can be written in spherical coordinate

$$\mathbf{a}_\theta \mathbf{a}_\theta + \mathbf{a}_\phi \mathbf{a}_\phi = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\mathbf{J_d} = \begin{bmatrix} J_d \cdot sin(\theta) \\ J_d \cdot cos(\theta) \\ 0 \end{bmatrix}$$

With superposition principle, the inner part of the integral can be written as

$$\begin{aligned} &(\mathbf{a}_\theta \mathbf{a}_\theta + \mathbf{a}_\phi \mathbf{a}_\phi) \cdot (\mathbf{J_d} \cdot exp(jkr' \cdot a_R) - \mathbf{J_d} \cdot exp(-jkr' \cdot a_R)) \\ &= 2jJ_d cos(\theta) sin(kdcos(\theta)) \end{aligned}$$

When d=$\lambda/4$, the expression is

$$2jJ_d cos(\theta) sin(\frac{\pi}{2} cos(\theta))$$

So the radiation is unidirectional. ■

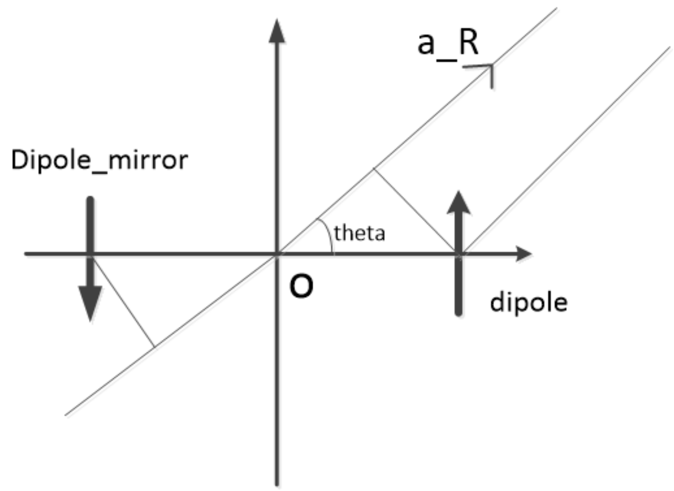

Figure 2: source distribution

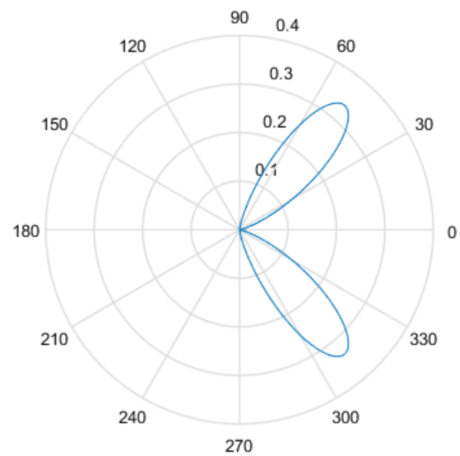

Figure 3: farfield pattern when d=lambda/2

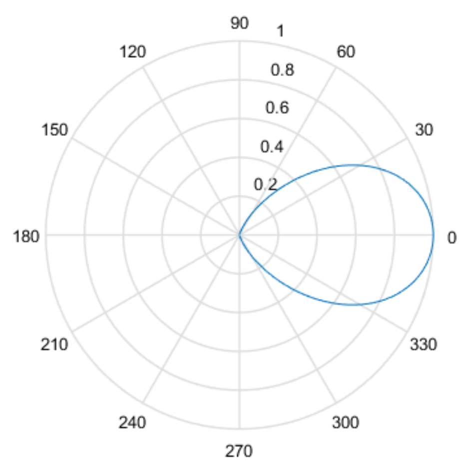

Figure 4: farfield pattern when d=lambda/4