

Solution P1.3

$$\begin{aligned}
\nabla \times (\nabla \times \overline{E}) &= \nabla \times \begin{bmatrix} \hat{x} & \hat{y} & \hat{z} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ E_x & E_y & E_z \end{bmatrix} \\
&= \begin{bmatrix} \hat{x} & \hat{y} & \hat{z} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ \left(\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z}\right) & \left(\frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x}\right) & \left(\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y}\right) \end{bmatrix} \\
&= \left[\frac{\partial}{\partial x} \left(\frac{\partial E_y}{\partial y} + \frac{\partial E_x}{\partial x} + \frac{\partial E_z}{\partial z} \right) - \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \right) E_x \right] \hat{x} \\
&\quad + \left[\frac{\partial}{\partial y} \left(\frac{\partial E_y}{\partial y} + \frac{\partial E_x}{\partial x} + \frac{\partial E_z}{\partial z} \right) - \left(\frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \right) E_y \right] \hat{y} \\
&\quad + \left[\frac{\partial}{\partial z} \left(\frac{\partial E_y}{\partial y} + \frac{\partial E_x}{\partial x} + \frac{\partial E_z}{\partial z} \right) - \left(\frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \right) E_z \right] \hat{z} \\
&= \nabla (\nabla \cdot \overline{E}) - \nabla^2 \overline{E}
\end{aligned}$$

$$\begin{aligned}
\nabla \cdot (\overline{E} \times \overline{H}) &= \nabla \cdot \begin{bmatrix} \hat{x} & \hat{y} & \hat{z} \\ E_x & E_y & E_z \\ H_x & H_y & H_z \end{bmatrix} \\
&= \frac{\partial}{\partial x} (E_y H_z - E_z H_y) + \frac{\partial}{\partial y} (E_z H_x - E_x H_z) + \frac{\partial}{\partial z} (E_x H_y - E_y H_x) \\
&= H_x \left(\frac{\partial}{\partial y} E_z - \frac{\partial}{\partial z} E_y \right) + H_z \left(\frac{\partial}{\partial x} E_y - \frac{\partial}{\partial y} E_x \right) + H_y \left(\frac{\partial}{\partial z} E_x - \frac{\partial}{\partial x} E_z \right) \\
&\quad - E_x \left(\frac{\partial}{\partial y} H_z - \frac{\partial}{\partial z} H_y \right) - E_y \left(\frac{\partial}{\partial z} H_x - \frac{\partial}{\partial x} H_z \right) - E_z \left(\frac{\partial}{\partial x} H_y - \frac{\partial}{\partial y} H_x \right) \\
&= \overline{H} \cdot (\nabla \times \overline{E}) - \overline{E} \cdot (\nabla \times \overline{H})
\end{aligned}$$

$$\begin{aligned}
\nabla \cdot (\nabla \times \overline{A}) &= \nabla \cdot \begin{bmatrix} \hat{x} & \hat{y} & \hat{z} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ A_x & A_y & A_z \end{bmatrix} \\
&= \frac{\partial}{\partial x} \left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z} \right) + \frac{\partial}{\partial y} \left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x} \right) + \frac{\partial}{\partial z} \left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y} \right) \\
&= 0
\end{aligned}$$

$$\begin{aligned}
\nabla \times (\nabla \phi) &= \begin{bmatrix} \hat{x} & \hat{y} & \hat{z} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ \partial\phi/\partial x & \partial\phi/\partial y & \partial\phi/\partial z \end{bmatrix} \\
&= \left(\frac{\partial^2 \phi}{\partial y \partial z} - \frac{\partial^2 \phi}{\partial y \partial z} \right) \hat{x} + \left(\frac{\partial^2 \phi}{\partial x \partial z} - \frac{\partial^2 \phi}{\partial x \partial z} \right) \hat{y} + \left(\frac{\partial^2 \phi}{\partial x \partial y} - \frac{\partial^2 \phi}{\partial x \partial y} \right) \hat{z} \\
&= 0
\end{aligned}$$

Solution P1.4

The phase ϕ is a function of both space and time. To get the wave velocity, one needs to fix a phase and trace this constant phase in space and time. In other words, $\Delta\phi = 0$ in both space and time coordinates. Therefore, we can take derivative with respect to time or space to get the wave velocity.

If k is increased, the slope tends to be flat, meaning the wave velocity decreased. In addition, we can see that the wave becomes denser in space.