

This is the superposition of two linearly polarized waves.

Solution P2.4

$$\lambda = \frac{1}{A} = \frac{1}{100} = 0.01 \text{ (m)}$$

The wave propagates in the $+\hat{z}$ direction.

At $z = 0$,

$$\overline{E}(z = 0, t) = E_0 [\hat{x} \cos(\omega t) - \hat{y} \sin(\omega t)]$$

This is a left-hand circularly polarized plane wave.

At $t = 0$,

$$E_x = E_0 \cos\left(\frac{2\pi}{\lambda}z\right)$$

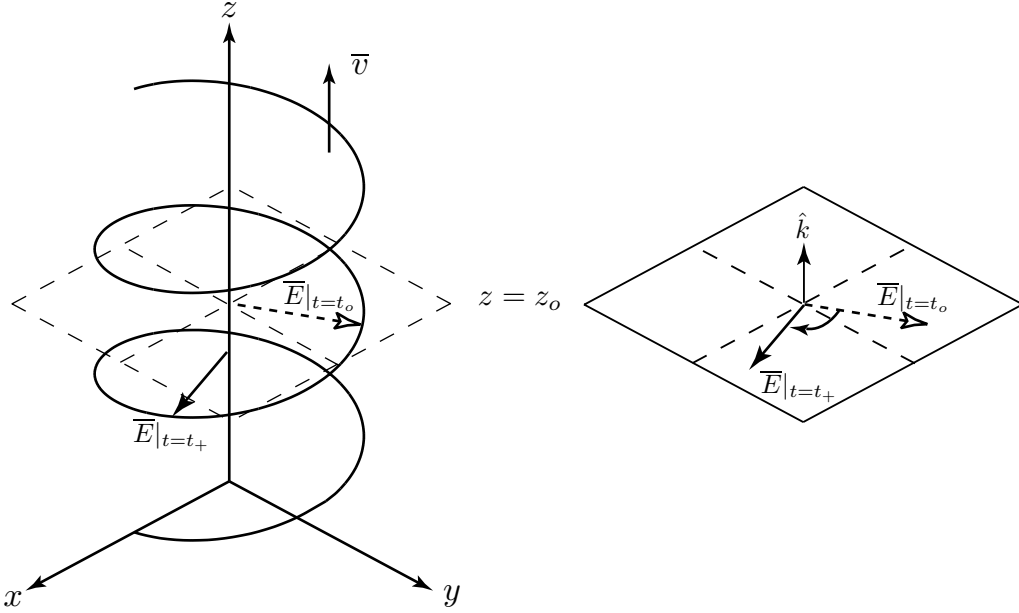
$$E_y = E_0 \sin\left(\frac{2\pi}{\lambda}z\right)$$

The parametric equation of a helix is

$$x = R \cos\left(\frac{2\pi}{p}z\right)$$

$$y = R \sin\left(\frac{2\pi}{p}z\right)$$

where p is the pitch of the helix. Thus the locus of the tip point of the electric field vector measured along the z axis is a right-handed helix with the pitch $p = \lambda$. From the figure below, we see that $\overline{E}|_{t_0} \times \overline{E}|_{t_+}$ is in the direction of $-\hat{k}$, thus the wave is temporally l.h.c.p.



At $\omega t = \pi/4$,

$$E_x = E_0 \cos\left[\frac{2\pi}{\lambda}\left(z - \frac{\lambda}{8}\right)\right]$$

$$E_y = E_0 \sin\left[\frac{2\pi}{\lambda}\left(z - \frac{\lambda}{8}\right)\right]$$

The helical locus advances along $+\hat{z}$ without rotating!

Solution P2.5

- (a) Speed of light $c = 3 \times 10^8 \text{m/sec}$
 Distance between Sun and Earth $D = 150 \times 10^9 \text{m}$
 Travelling time of light from Sun to Earth $T = D/C = 500 \text{sec} = 8.33 \text{min}$

- (b) Power received by the Earth $P_r = S(\text{power density}) \times A(\text{cross section area})$

$$A = \pi r^2 = \pi \times (6.4 \times 10^6)^2 \text{m}^2$$

$$S = 1.5 \text{kW/m}^2$$

$$P_r = 1.93 \times 10^{14} \text{kW}$$

- (c) The total power radiated by the Sun is

$$P_{total} = S(\text{power density at distance } D) \\ \times A_T(\text{Total surface area of sphere with radius } D)$$

$$A_T = 4\pi D^2 = 4\pi \times (150 \times 10^9)^2 \text{m}^2$$

$$S = 1.5 \text{kW/m}^2$$

$$\text{Mass of Sun } m = 2 \times 10^{30} \text{kg}$$

$$\text{Efficiency } E_{eff} = 1\%$$

Total time that Sun can radiate

$$T = \frac{mc^2 E_{eff}}{P_{Total}} = \frac{mc^2 E_{eff}}{S 4\pi D^2} \\ = \frac{(2 \times 10^{30}) \times (3 \times 10^8)^2 \times 0.01}{(1.5 \times 10^3) \times 4\pi \times (150 \times 10^9)^2} = 4.2 \times 10^{18} \text{sec} \approx 1.34 \times 10^{11} \text{years}$$

- (d)

$$\text{Poynting power } S = P(\text{power density per Hz}) \times W(\text{bandwidth}) \\ = 10^{-20} \text{Wm}^{-2} \text{Hz}^{-1} \times 10^9 \text{Hz} = 10^{-11} \text{Wm}^{-2}$$

Since $S = E^2/2\eta$, we have

$$E = \sqrt{2\eta S} = \sqrt{2 \times 120\pi \times 10^{-11}} = 8.68 \times 10^{-5} \frac{\text{volt}}{\text{m}}$$