Study Sheet
Test I
OPER 527

Name _____

Be sure to show all work.

Any problems that require a picture generated by a computer must be generated using R and the code must be below the figure.

You may work with no other people on this test.

1. Consider the following function:

$$f(x) = 10\left(e^{-(x-7)^2} + e^{-0.5(x-3\pi)^6}\right),\ 0 \leq x \leq 15$$

   (a) (5 pts) Is f concave or convex or neither? Please justify your answer other than by looking at a picture.

   (b) (10 pts) What is the maximum and maximizer for f on this domain?

   (c) (10 pts) Let $g(x) = \sin(x) + 2\sin(2x)$ and we want the to impose the constraint $g(x) \geq 0$ what is the feasible region?

   (d) (10 pts) Find the solution to $\max f(x)$ subject to $g(x) \geq 0$. Report the maximum and maximizer.

2. A chemical manufacturer is interested in finding the optimal allocation of reaction columns for six petroleum products, P_1, P_2, P_3, P_4, P_5, P_6 and P_7. He has reaction columns A, B, and Cat his disposal. We know that A can produce any of the chemicals, B and C can produce P_1, P_2, P_3, P_6 and P_7. He needs to determine what to produce on which machines given the following information: The machines can produce each product at the following rates of litres per hour and Demand for the next 24 hours.

Product	A	B	C	Demand
P_1	523	721	639	21,000
P_2	622	615	492	6,300
P_3	624	709	512	6,400
P_4	307	0	0	600
P_5	293	0	0	550
P_6	309	314	304	1,800
P_7	112	217	97	800

   The cost of running each item on each machine per hour is:

Product	A	B	C
P_1	53	71	39
P_2	59	63	42
P_3	64	78	52
P_4	30	0	0
P_5	41	0	0
P_6	72	83	26
P_7	21	26	22

The goal is to minimize the costs of producing the items while meeting demand.

(a) (5pts) Clearly define the variables that will be optimized over (there may be alot of these). Write these by hand.

(b) (5pts) Clearly define the objective function using the variable notation above. Write this by hand.

(c) (5pts) Clearly define the constraints using the variable notation above. Write these by hand.

(d) (10 pts) Define $\mathbf{c}$, $\mathbf{A}$ and $\mathbf{b}$ for the linear program. Write these by hand.

(e) (10 pts) Write the R code that will solve the above linear program.

(f) (5 pts) Write the solution from the linear program if it exists.

(g) (10 pts) Write the solution to the linear program in a manner that a manner that a manager could employ... how about a schedule of what to run when and on what machine.

3. (15 pts) Consider the following set $Q = \{(x, y) \in \mathbf{E}_2 : x^2 + y^2 \leq 1\}$. Show that Q is convex.