Methods in Artificial Intelligence Homework 4

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1 Candy Company Problem

1.1 Problem statement

We have candies with two flavors:

- 70% are strawberry, and 30% are anchovy
- 80% of strawberry candies are round, while 90% of anchovy candies become square.
- 80% of strawberry candies are given red wrappers, and 90% of anchovy candies are given brown wrappers.
- The candies are sold in identical black boxes, preventing customers from knowing any details about the candies before purchase.

1.2 Correct Representation of Dependencies

To correctly represent *P(Flavor, Wrapper, Shape)*, a network must capture the dependencies between variables described in the problem.

- **Network (i):** Assumes that Wrapper and Shape are conditionally independent given Flavor. This is incorrect as having candy of a certain shape greatly increases it's chances of being of a certain wrapper. Therefore Netwrok (i) can't represent our problem
- **Network (ii):** This network infers that wrapper influences shape. This is correct as these are both based on flavor, and knowing the wrapper of a candy, let's us know which shape is most likely.
- **Network (iii):** Assumes Flavor directly causes both Wrapper and Shape, which is correct.

We can therefore conclude that **Networks (ii) and (iii) correctly represent our problem**.

1.3 Best Representation of the Problem

The ideal Bayesian network should correctly model dependencies while making the probability calculations simple. As we know Network (i) incorrectly represents the model, we only look at (ii) and (iii):

- **Network (ii)** contains a cyclar dependency as it is fully connected. Therefore it is harder to work with.
- **Network (iii)** also represents the correct dependencies but it is not fully connected. Wrapper and Shape are conditionally independent given Flavor, requiring only: $P(Wrapper|Flavor)$ and $P(Shape|Flavor)$.

We can therefore conclude that **Network (iii) is the best choice** due to its simpler probability calculations.

1.4 Independence of Wrapper and Shape in Network (i)

Network (i) asserts that Wrapper and Shape are independent given Flavor because both Wrapper and Shape have direct edges from Flavor but not between each other. This means that once Flavor is known, knowing the Wrapper provides no additional information about the Shape, and vice versa. Mathematically, this implies:

$$P(Wrapper, Shape|Flavor) = P(Wrapper|Flavor) \times P(Shape|Flavor)$$

Thus, **Network (i) explicitly asserts that Wrapper and Shape are conditionally independent given Flavor**.

1.5 Probability of Red Wrapper

We calculate the total probability of a candy having a red wrapper by summing the chances of both flavors having a red wrapping.

$$P(Red) = P(Red|Strawberry) \times P(Strawberry) + P(Red|Anchovy) \times P(Anchovy)$$

Substituting the given values:

$$P(Red) = 0.8 \times 0.7 + 0.1 \times 0.3$$

$$P(Red) = 0.59 = 59\%$$

The probability that a randomly chosen candy has a red wrapper is **0.59**.

1.6 Probability that Flavor is Strawberry Given Red Wrapping

To solve this, we can use Bayes' theorem. We can therefore use:

$$P(Strawberry|Red) = \frac{P(Red|Strawberry)P(Strawberry)}{P(Red)}$$

We can substitute to get:

$$P(Strawberry|Red) = \frac{(0.8 \times 0.7)}{0.59}$$

$$P(Strawberry|Red) = \frac{0.56}{0.59} \approx 0.9492$$

Thus, the probability that a candy is strawberry given that it has a red wrapper is **0.949**.

1.7 Expected Value of an Unopened Candy Box

The expected value $E[V]$ of an unopened candy box is given by the probability of each candy multiplied by its value:

$$E[V] = P(\text{Strawberry}) \cdot s + P(\text{Anchovy}) \cdot a$$

Substituting the given probabilities:

$$E[V] = (0.7 \cdot s) + (0.3 \cdot a)$$

Thus, the expected value of an unopened candy box is:

$$\mathbf{0.7s + 0.3a}$$

2 Exponential Utility and Risk Aversion

2.1 Certainty vs Lottery

We have the utility function:

$$U(x) = -e^{-x/R}$$

where $R = 500$. Mary has two choices:

- **Option 1:** Receive 500$
- **Option 2:** Participate in a lottery with:
  - 60% probability of winning 5000$.
  - 40% probability of winning nothing.

Option 1: Guaranteed 500

Since the outcome is guaranteed, the utility is:

$$U(500) = -e^{-500/500} = -e^{-1} \approx -0.3679$$

Option 2: Lottery

Here, we calculate the utility in two parts, based on the probability of winning the lottery:

$$EU_{\text{lottery}} = 0.6 \cdot U(5000) + 0.4 \cdot U(0)$$

Calculating for winning the lottery:

$$U(5000) = -e^{-5000/500} = -e^{-10} \approx -4.54 \times 10^{-5}$$

Calculating for loosing the lottery

$$U(0) = -e^{0} = -1$$

Substituting the values:

$$EU_{\text{lottery}} = 0.6 \cdot U(5000) + 0.4 \cdot U(0)$$
$$EU_{\text{lottery}} = 0.6 \cdot (-4.54 \times 10^{-5}) + 0.4 \cdot (-1)$$
$$EU_{\text{lottery}} = -2.72 \times 10^{-5} - 0.4$$
$$EU_{\text{lottery}} \approx -0.40003$$

Comparing utilities

We see that the two options give the following utilities:

$$U(500) \approx -0.3679, \quad EU_{\text{lottery}} \approx -0.40003$$

Since $U(500) > EU_{\text{lottery}}$, **Mary would choose the certain \$500 over the lottery**, given that she acts rationally.

2.2 Calculating R for Indifference

We have the two choices:

- **Option 1:** Receive 100$
- **Option 2:** Participate in a lottery with:
  - 50% probability of winning 500$.
  - 50% probability of winning nothing.

To find the risk tolerance R that makes an individual indifferent between the two choices, we solve for R in:

$$U(100) = EU_{lottery}$$

$$U(100) = 0.5 \cdot U(500) + 0.5 \cdot U(0)$$

Expanding each term using the utility function $U(x) = -e^{-x/R}$:

$$-e^{-100/R} = 0.5 \cdot (-e^{-500/R}) + 0.5 \cdot (-e^{0/R})$$

$$-e^{-100/R} = 0.5 \cdot (-e^{-500/R}) + 0.5 \cdot (-1)$$

$$-e^{-100/R} = -0.5e^{-500/R} - 0.5$$

As everything is negative, we can multiply by -1 to get:

$$e^{-100/R} = 0.5e^{-500/R} + 0.5$$

$$e^{-100/R} - 0.5 = 0.5e^{-500/R}$$

$$2e^{-100/R} - 1 = e^{-500/R}$$

This can be solved numerically, but I chose to solve this trough graphs. I therefore moved everything to the right:

$$e^{-500/R} - 2e^{-100/R} + 1 = 0$$

We can rewrite this to

$$f(R) = e^{-500/R} - 2e^{-100/R} + 1 = 0$$

Plotting this with geogebra, shown in Figure 1 and Figure 2. We get the following solution:

$$R = 152.37956$$

Approximating to 3 decimals, we get:

$$R \approx 152.380$$

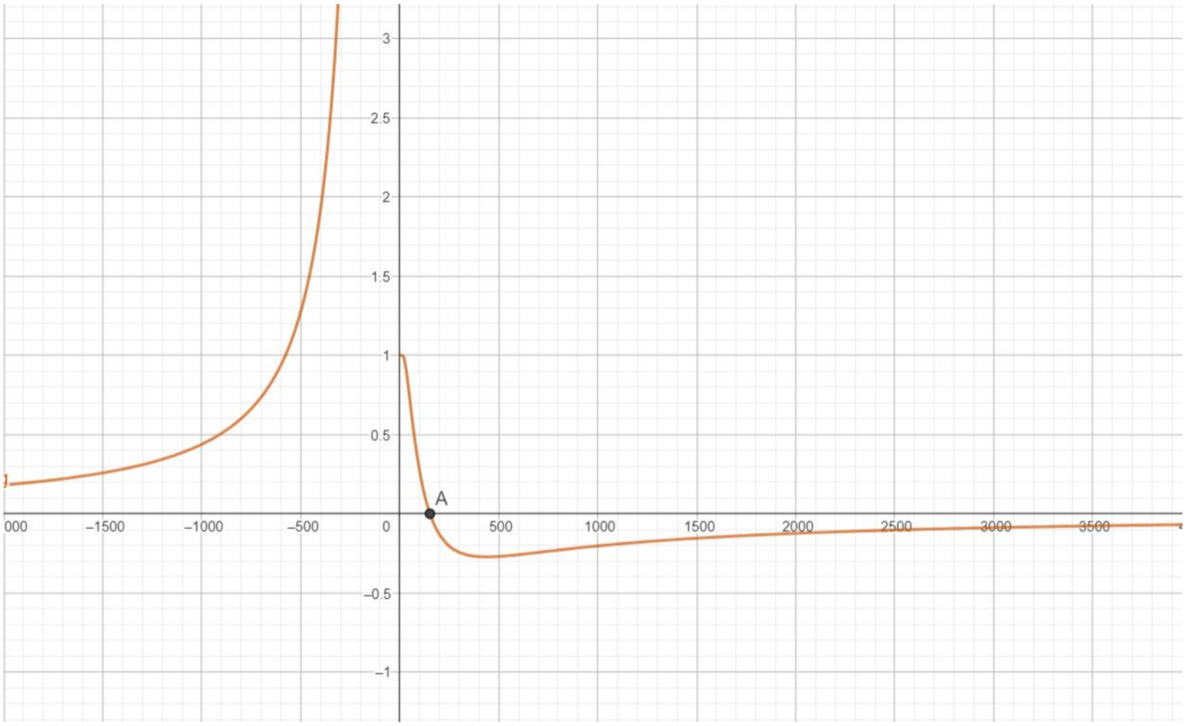

Figure 1: The graph of the function I need to solve. We see that when $R \to \inf$
, we go towards a solution. There also exists negative solutions. But as R has to be a positive constant, neither of these solutions are valid.

$$g(x) = e^{-\frac{500}{x}} - 2\,e^{-\frac{100}{x}} + 1$$

$$A = \mathrm{Skjæring}(g, \mathrm{xAkse}, (152.37956,\ 0))$$

$$= (152.37956,\ 0)$$

Figure 2: The numerical values of the graph and corresponding solution in geogebra. The only positive constant value for $R = 152.37956$