DTM - Deterministic Turing Machine
NTM - Non-deterministic Turing Machine
PT - Probabilistic Turing Machine
PDTM - Deterministic Turing Machine *running in polynomial time*
PNTM - Non-deterministic Turing Machine *running in polynomial time*
PPT - Probabilistic Turing Machine *running in polynomial time*
n - Input length

Time Complexity Classes

$$\text{DTIME}(f) = \{A \in \Sigma^* \mid \exists \text{ DTM } T : T \text{ decides } A \wedge T \text{ runs in } \mathcal{O}(f(n))\ \}$$
$$\text{NTIME}(f) = \{A \in \Sigma^* \mid \exists \text{ NTM } T : T \text{ decides } A \wedge T \text{ runs in } \mathcal{O}(f(n))\ \}$$

$$P = \bigcup_{k \geq 1} \text{DTIME}(n^k) \qquad NP = \bigcup_{k \geq 1} \text{NTIME}(n^k)$$
$$E = \bigcup_{k \geq 1} \text{DTIME}(2^{kn}) \qquad NE = \bigcup_{k \geq 1} \text{NTIME}(2^{kn})$$
$$EXP = \bigcup_{k \geq 1} \text{DTIME}(2^{n^k}) \qquad NEXP = \bigcup_{k \geq 1} \text{NTIME}(2^{n^k})$$

Space Complexity Classes

$$\text{DSPACE}(s) = \{A \in \Sigma^* \mid \exists \text{ DTM } T : T \text{ decides } A \wedge T \text{ uses } \mathcal{O}(s(n)) \text{ space }\}$$
$$\text{NSPACE}(s) = \{A \in \Sigma^* \mid \exists \text{ NTM } T : T \text{ decides } A \wedge T \text{ uses } \mathcal{O}(f(n)) \text{ space }\}$$

$$\begin{aligned} LOGSPACE = L = &\quad \text{DSPACE}(\log n) \\ NLOGSPACE = NL = &\quad \text{NSPACE}(\log n) \\ LINSPACE = &\quad \text{DSPACE}(n) \end{aligned}$$

$$PSPACE = \bigcup_{k\geq 1} \text{DSPACE}(n^k) \quad = \quad NPSPACE = \bigcup_{k\geq 1} \text{NSPACE}(n^k)$$

$$ESPACE = \bigcup_{k\geq 1} \text{DSPACE}(2^{kn}) \quad = \quad NESPACE = \bigcup_{k\geq 1} \text{NSPACE}(2^{kn})$$

$$EXPSPACE = \bigcup_{k\geq 1} \text{DSPACE}(2^{n^k}) \quad = \quad NEXPSPACE = \bigcup_{k\geq 1} \text{NSPACE}(2^{n^k})$$

Polynomial Hierarchy

$$\Delta_0^P = \Sigma_0^P = \Pi_0^P = P$$

$$\begin{array}{lll}
\Delta_{k+1}^P = P^{\Sigma_k^P} & \Delta_1^P = P & \Delta_2^P = P^{NP} \\
\Sigma_{k+1}^P = NP^{\Sigma_k^P} & \Sigma_1^P = NP & \Sigma_2^P = NP^{NP} \\
\Pi_{k+1}^P = co\text{–}NP^{\Sigma_k^P} & \Pi_1^P = co\text{–}NP & \Pi_2^P = co\text{–}NP^{NP}
\end{array}$$

$$PH = \bigcup_{k\geq 0} \Delta_k^P = \bigcup_{k\geq 0} \Sigma_k^P = \bigcup_{k\geq 0} \Pi_k^P$$

Probabilistic Complexity Classes

$$
\begin{aligned}
P = \{A \in \Sigma^* \mid \exists \text{ PDTM } T: \quad & \forall x \in A : \mathrm{P}(T(x) = 1) = 1 \wedge \\
& \forall x \notin A : \mathrm{P}(T(x) = 1) = 0 \;\} \\
NP = \{A \in \Sigma^* \mid \exists \text{ PNTM } T: \quad & \forall x \in A : \mathrm{P}(T(x) = 1) > 0 \wedge \\
& \forall x \notin A : \mathrm{P}(T(x) = 1) = 0 \;\} \\
PP = \{A \in \Sigma^* \mid \exists \text{ PPT } T: \quad & \forall x \in A : \mathrm{P}(T(x) = 1) > 0.5 \wedge \\
& \forall x \notin A : \mathrm{P}(T(x) = 1) \leq 0.5 \;\} \\
BPP = \{A \in \Sigma^* \mid \exists \text{ PPT } T: \quad & \forall x \in A : \mathrm{P}(T(x) = 1) > 0.5 + \epsilon \wedge \\
& \forall x \notin A : \mathrm{P}(T(x) = 1) \leq 0.5 - \epsilon \;\} \\
RP = \{A \in \Sigma^* \mid \exists \text{ PPT } T: \quad & \forall x \in A : \mathrm{P}(T(x) = 1) > 0.5 \wedge \\
& \forall x \notin A : \mathrm{P}(T(x) = 1) = 0 \;\} \\
ZPP = \{A \in \Sigma^* \mid \exists \text{ PPT } T: \quad & \forall x \in A : \mathrm{P}(T(x) = 1) > 0.5 \wedge \\
& \qquad\qquad \mathrm{P}(T(x) = ?\,) < 0.5 \wedge \\
& \forall x \notin A : \mathrm{P}(T(x) = 1) = 0 \wedge \\
& \qquad\qquad \mathrm{P}(T(x) = ?\,) < 0.5 \;\}
\end{aligned}
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