DTM - Deterministic Turing Machine
NTM - Non-deterministic Turing Machine
PT - Probabilistic Turing Machine
PDTM - Deterministic Turing Machine *running in polynomial time*
PNTM - Non-deterministic Turing Machine *running in polynomial time*
PPT - Probabilistic Turing Machine *running in polynomial time*

Time Complexity Classes

$\mathrm{DTIME}(f) = \{A \in \Sigma^* \mid \exists \text{ DTM } T : T \text{ decides } A \wedge T \text{ runs in } \mathcal{O}(f(n))\}$
$\mathrm{NTIME}(f) = \{A \in \Sigma^* \mid \exists \text{ NTM } T : T \text{ decides } A \wedge T \text{ runs in } \mathcal{O}(f(n))\}$

$$\begin{aligned} P &= \bigcup_{k\geq 1} \mathrm{DTIME}(n^k) & NP &= \bigcup_{k\geq 1} \mathrm{NTIME}(n^k) \\ E &= \bigcup_{k\geq 1} \mathrm{DTIME}(2^{kn}) & NE &= \bigcup_{k\geq 1} \mathrm{NTIME}(2^{kn}) \\ EXP &= \bigcup_{k\geq 1} \mathrm{DTIME}(2^{n^k}) & NEXP &= \bigcup_{k\geq 1} \mathrm{NTIME}(2^{n^k}) \end{aligned}$$

Polynomial Hierarchy

$$\Delta_0^P = \Sigma_0^P = \Pi_0^P = P$$

$$\begin{aligned} \Delta_{k+1}^P &= P^{\Sigma_k^P} & \Delta_1^P &= P & \Delta_2^P &= P^{NP} \\ \Sigma_{k+1}^P &= NP^{\Sigma_k^P} & \Sigma_1^P &= NP & \Sigma_2^P &= NP^{NP} \\ \Pi_{k+1}^P &= co\text{–}NP^{\Sigma_k^P} & \Pi_1^P &= co\text{–}NP & \Pi_2^P &= co\text{–}NP^{NP} \end{aligned}$$

$PH = \bigcup_{k\geq 0} \Delta_k^P = \bigcup_{k\geq 0} \Sigma_k^P = \bigcup_{k\geq 0} \Pi_k^P$

Probabilistic Complexity Classes

$$
\begin{aligned}
P = \{A \in \Sigma^* \mid \exists \text{ PDTM } T: \quad & \forall x \in A : \mathrm{P}(T(x) = 1) = 1 \wedge \\
& \forall x \notin A : \mathrm{P}(T(x) = 1) = 0 \,\} \\
NP = \{A \in \Sigma^* \mid \exists \text{ PNTM } T: \quad & \forall x \in A : \mathrm{P}(T(x) = 1) > 0 \wedge \\
& \forall x \notin A : \mathrm{P}(T(x) = 1) = 0 \,\} \\
PP = \{A \in \Sigma^* \mid \exists \text{ PPT } T: \quad & \forall x \in A : \mathrm{P}(T(x) = 1) > 0.5 \wedge \\
& \forall x \notin A : \mathrm{P}(T(x) = 1) \leq 0.5 \,\} \\
BPP = \{A \in \Sigma^* \mid \exists \text{ PPT } T: \quad & \forall x \in A : \mathrm{P}(T(x) = 1) > 0.5 + \epsilon \wedge \\
& \forall x \notin A : \mathrm{P}(T(x) = 1) \leq 0.5 - \epsilon \,\} \\
RP = \{A \in \Sigma^* \mid \exists \text{ PPT } T: \quad & \forall x \in A : \mathrm{P}(T(x) = 1) > 0.5 \wedge \\
& \forall x \notin A : \mathrm{P}(T(x) = 1) = 0 \,\} \\
ZPP = \{A \in \Sigma^* \mid \exists \text{ PPT } T: \quad & \forall x \in A : \mathrm{P}(T(x) = 1) > 0.5 \wedge \\
& \qquad\qquad\; \mathrm{P}(T(x) = ?\,) < 0.5 \wedge \\
& \forall x \notin A : \mathrm{P}(T(x) = 1) = 0 \wedge \\
& \qquad\qquad\; \mathrm{P}(T(x) = ?\,) < 0.5 \,\}
\end{aligned}
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