When does clockwork not work?

based on 1704.02??? by Nathaniel Craig, Isabel García-García and DS

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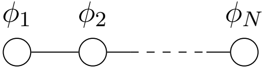
The original clockwork mechanism

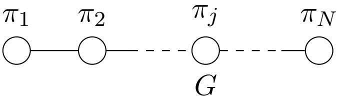
Choi & Im 1511.00132; Kaplan & Rattazzi 1511.01827

Is a quiver theory wherein $U(1)^N \to U(1)$:



$$\mathcal{L} = \sum_{i=1}^{N} |\partial \phi_i|^2 + \sum_{i=1}^{N-1} (\epsilon \phi_i^\dagger \phi_{i+1}^q + \text{h.c.}) \tag{1}$$

Linearise $\phi_i = f e^{i \frac{\pi_i}{\sqrt{2} f}}$:



$$\mathcal{L} = \frac{1}{2} \sum_{i=1}^{N} (\partial \pi_i)^2 + \frac{1}{2} \epsilon f^2 \sum_{i=1}^{N-1} (\pi_i - q \pi_{i+1})^2 + \frac{1}{4} G^2 + \frac{1}{f} \pi_j G \tilde{G} \tag{2}$$

Find the massless mode

$$\mathcal{L} = \frac{1}{2}\sum_{i=1}^{N}(\partial\pi_i)^2 + \frac{1}{2}\epsilon f^2 \sum_{i=1}^{N-1}(\pi_i - q\pi_{i+1})^2 + \frac{1}{4}G^2 + \frac{1}{f}\pi_j G\tilde{G} \quad (3)$$

Sub. in $\pi_i = \frac{\pi_{\text{zero}}}{q^i}$

$$\mathcal{L} = \frac{1}{2}\left(\sum_{i=1}^{N}\frac{1}{q^i}\right)(\partial\pi_{\text{zero}})^2 + \frac{1}{4}G^2 + \frac{1}{q^j f}\pi_{\text{zero}} G\tilde{G} \quad (4)$$

Depending on our choice of j, the coupling to gluons is **exponentially suppressed**, *i.e.* the effective decay constant is exponentially enhanced.

Two model building directions

1. Can we do a general $G^N \to G$ quiver with a hierarchy of couplings of the massless mode?
2. Can we get a clockwork model by deconstructing (discretising) an extra dimension?

*Answers:

1. Almost certainly not. $U(1)$ is special.
2. Yes, with some machinations to localise the zeroth KK mode appropriately.

$G^N \to G$

Let G_i have generators T_i^a, with Lie bracket $[T_i^a, T_j^b] = f^{abc} T_i^c \delta_{ij}$.
Assume non-Abelian.
Suppose there exists a 'clockwork-like' Lie subalgebra

$$T^a_{\text{zero}} = \sum a_i T_i^a \tag{5}$$

Require

$$[T^a_{\text{zero}}, T^b_{\text{zero}}] = f^{abc} T^c_{\text{zero}} \tag{6}$$

$$\implies \sum f^{abc} a_i^2 T_i^c = \sum f^{abc} a_i T_i^c \tag{7}$$

$$\implies a_i^2 = a_i \implies a_i = 0, 1 \tag{8}$$

This is even more obvious in words

If there were a

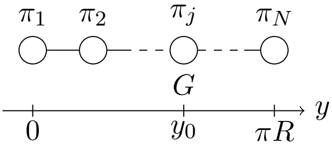
$$\begin{cases} \textbf{clockwork non-Abelian gauge theory} \\ \textbf{clockwork graviton} \end{cases},$$

it would couple differently to matter localised at different sites. This would break

$$\begin{cases} \textbf{gauge invariance} \\ \textbf{the equivalence principle} \end{cases}.$$

Clockwork from 5d?

$$\mathrm{d}s^2 = X(y)\eta_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu + Y(y)\mathrm{d}y^2$$



$$S = \int \mathrm{d}^4x\mathrm{d}y\sqrt{|g|}\left((\partial_M\pi)(\partial^M\pi) + \left[G_{MN}G^{MN} + \frac{\pi}{f^{\frac{3}{2}}}G_{MN}\tilde{G}^{MN}\right]\frac{\delta(y-y_0)}{\sqrt{g_{55}}}\right) \tag{9}$$

$$S = \int \mathrm{d}^4x\mathrm{d}y\left(XY^{\frac{1}{2}}(\partial\pi)^2 + X^2Y^{-\frac{1}{2}}(\partial_y\pi)^2 + \left[G^2 + \frac{\pi}{f^{\frac{3}{2}}}G\tilde{G}\right]\delta(y-y_0)\right) \tag{10}$$

Sub in $\pi(x,y) = \pi_{\text{zero}}(x)$

$$S = \int \mathrm{d}^4x\left(\left(\int \mathrm{d}yXY^{\frac{1}{2}}\right)(\partial\pi_{\text{zero}})^2 + \left[G^2 + \frac{\pi_{\text{zero}}}{f^{\frac{3}{2}}}G\tilde{G}\right]\right) \tag{11}$$

Brane-position-independent coupling that is **not super-Planckian**:

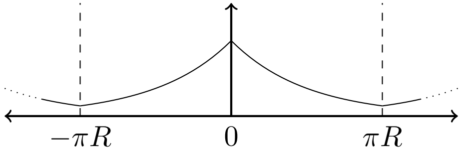
$$\left(\int \mathrm{d}yXY^{\frac{1}{2}}\right) = \frac{M_4^2}{M_5^3} \implies \left(\frac{f_{\text{zero}}}{M_4}\right)^2 = \left(\frac{f}{M_5}\right)^3 \tag{12}$$

Localise the zero mode with bulk and brane localised masses *in flat space*

$$S = \int \mathrm{d}^4x\mathrm{d}y\left((\partial\pi)^2 + (\partial_y\pi)^2 + \pi^2[M^2 + m\delta(y) - m\delta(y-\pi R)]\right) \qquad (13)$$

If $m = -2M$, sub in $\pi(x,y) = \pi_{\text{zero}}(x)e^{-M|y|}$ to get

$$S = \int \mathrm{d}^4x\left(\left(\int \mathrm{d}y e^{-2M|y|}\right)(\partial\pi_{\text{zero}})^2 + \left[G^2 + \frac{\pi_{\text{zero}}e^{-M|y_0|}}{f^{\frac{3}{2}}}G\tilde{G}\right]\right) \qquad (14)$$



Brane-position-dependent coupling that is **super-Planckian**:

$$\left(\int \mathrm{d}y e^{-2M|y|}\right) \lesssim \frac{M_4^2}{M_5^3} \implies \left(\frac{f_{\text{zero}}}{M_4}\right)^2 \gg \left(\frac{f}{M_5}\right)^3 \qquad (15)$$

In summary

A non-Abelian Lie subgroup $G < G^N$ acts equally on all lattice sites (at the algebra level). Therefore, clockwork is a $U(1)$ phenomenon, and is not meant for YM theories, composite Higgs, gravitons, ...

In extra dimensions, zero modes of massless bulk fields are flat, regardless of metric/geometry. Therefore, to mimic clockwork, you need bulk and brane masses to localise the zero mode. We write down one such formulation in flat space which deconstructs *precisely* to the original clockwork theory.