How I Have Taught Electricity and Magnetism

Bob Clare

(robert.clare@ucr.edu)

UCR Summer Physics Teacher Academy

June 2017

Copyright Acknowledgements

- Most figures are from
  - Physics for Scientists and Engineers, by Randall Knight, 1st edition, 2004, Pearson Addison Wesley
- Some figures are from
  - Principles of Physics, by Raymond Serway and John Jewett, 4th edition, 2006, Cengage

Triboelectric Series

- Skin
- Rabbit fur
- Glass
- Wool
- Silk
- Cotton
- Hard rubber
- Saran wrap
- Polyester
- PVC
- Teflon

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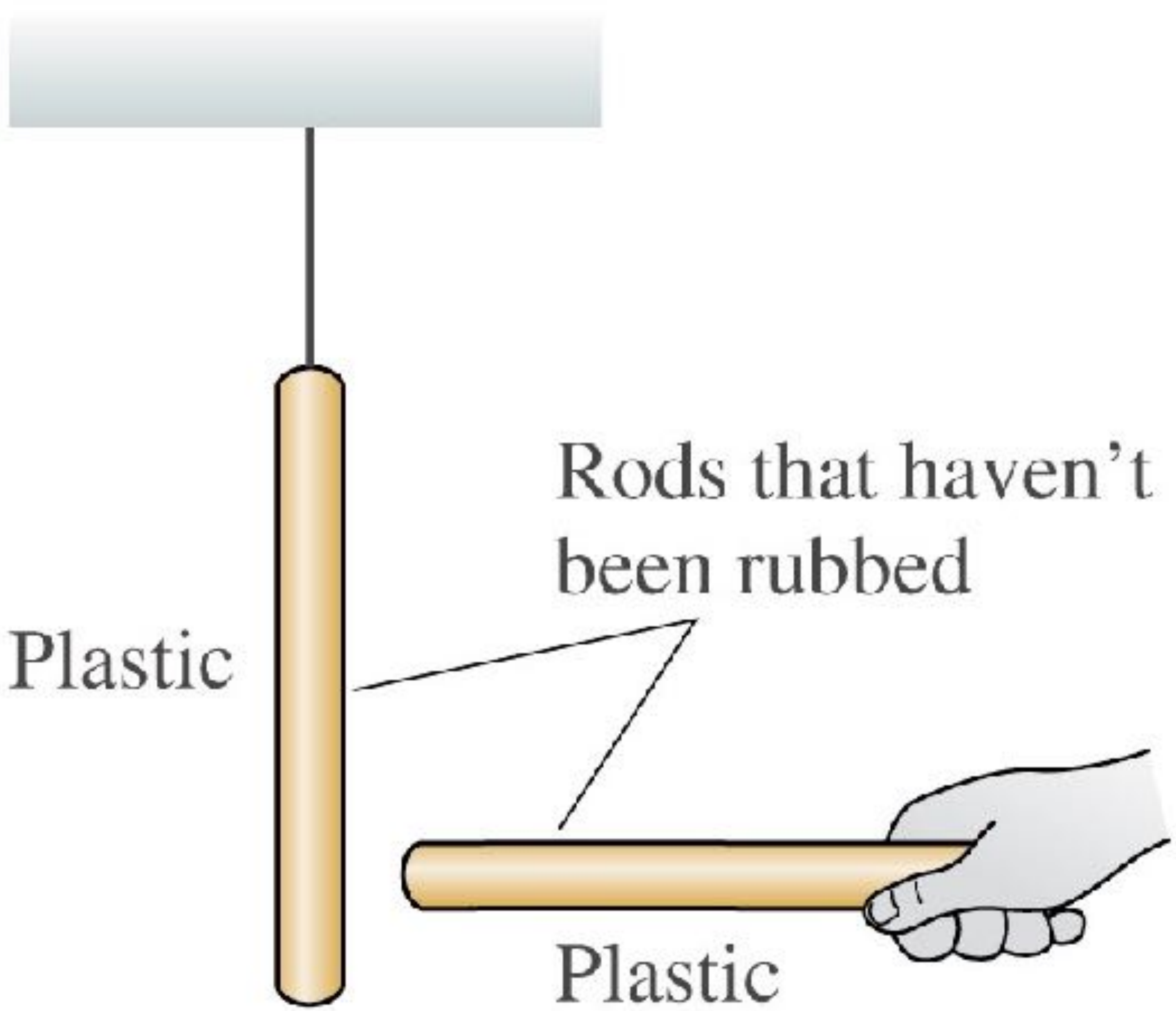
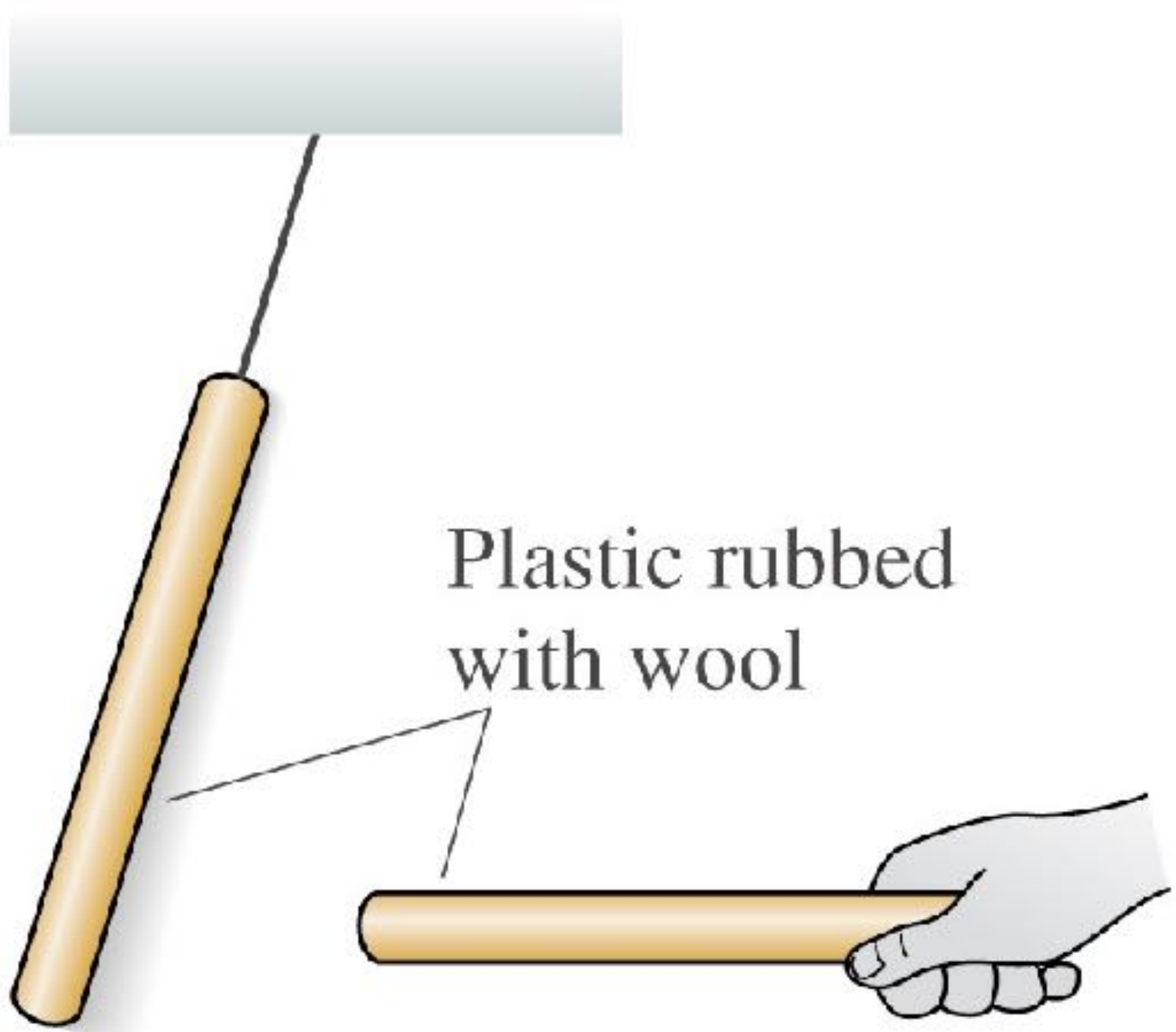
Note: materials are often impure
and surfaces are often not clean!
In other words: caveat emptor!

Experiment 1



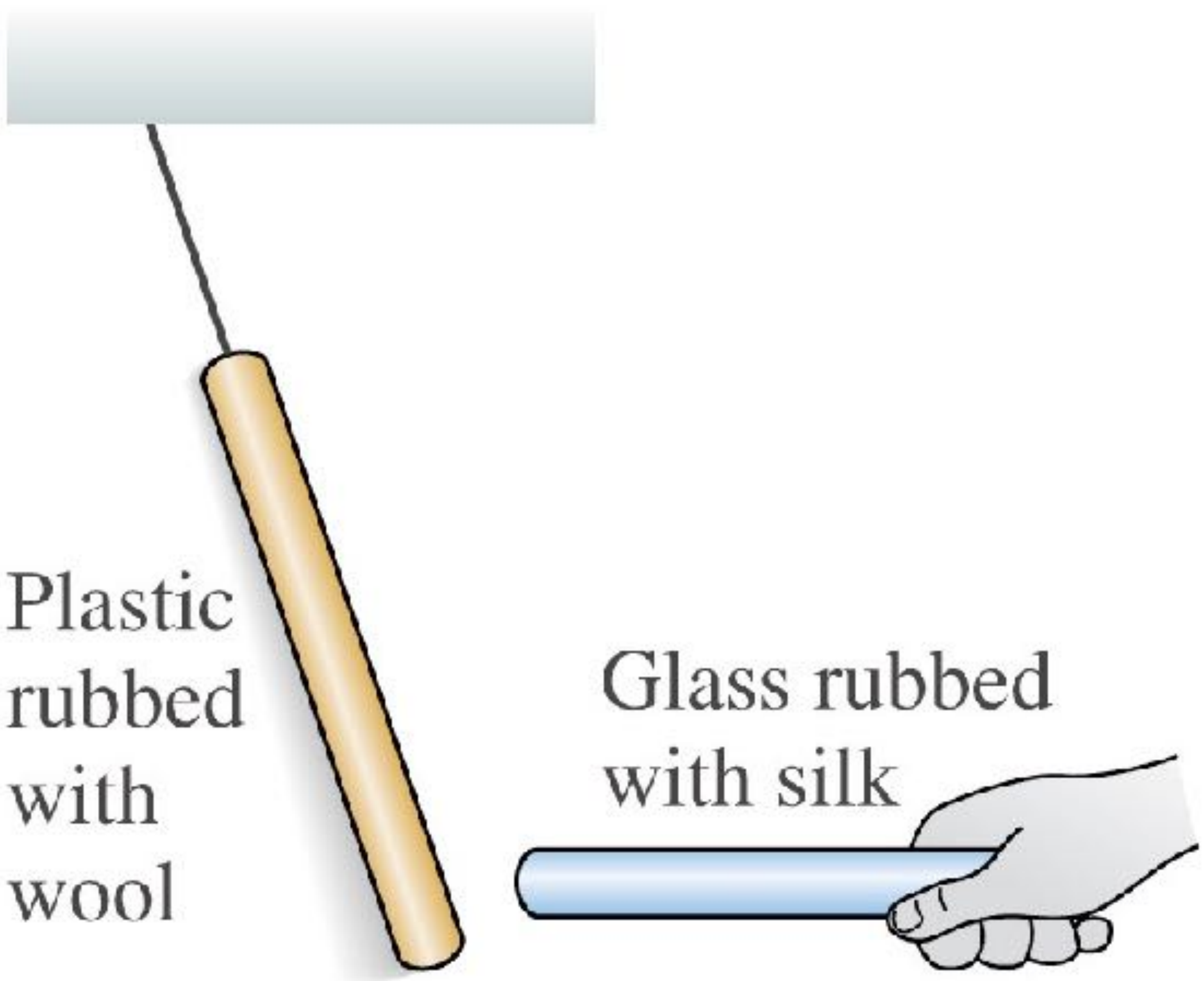
Nothing happens.
We say that the objects are neutral.

Experiment 2



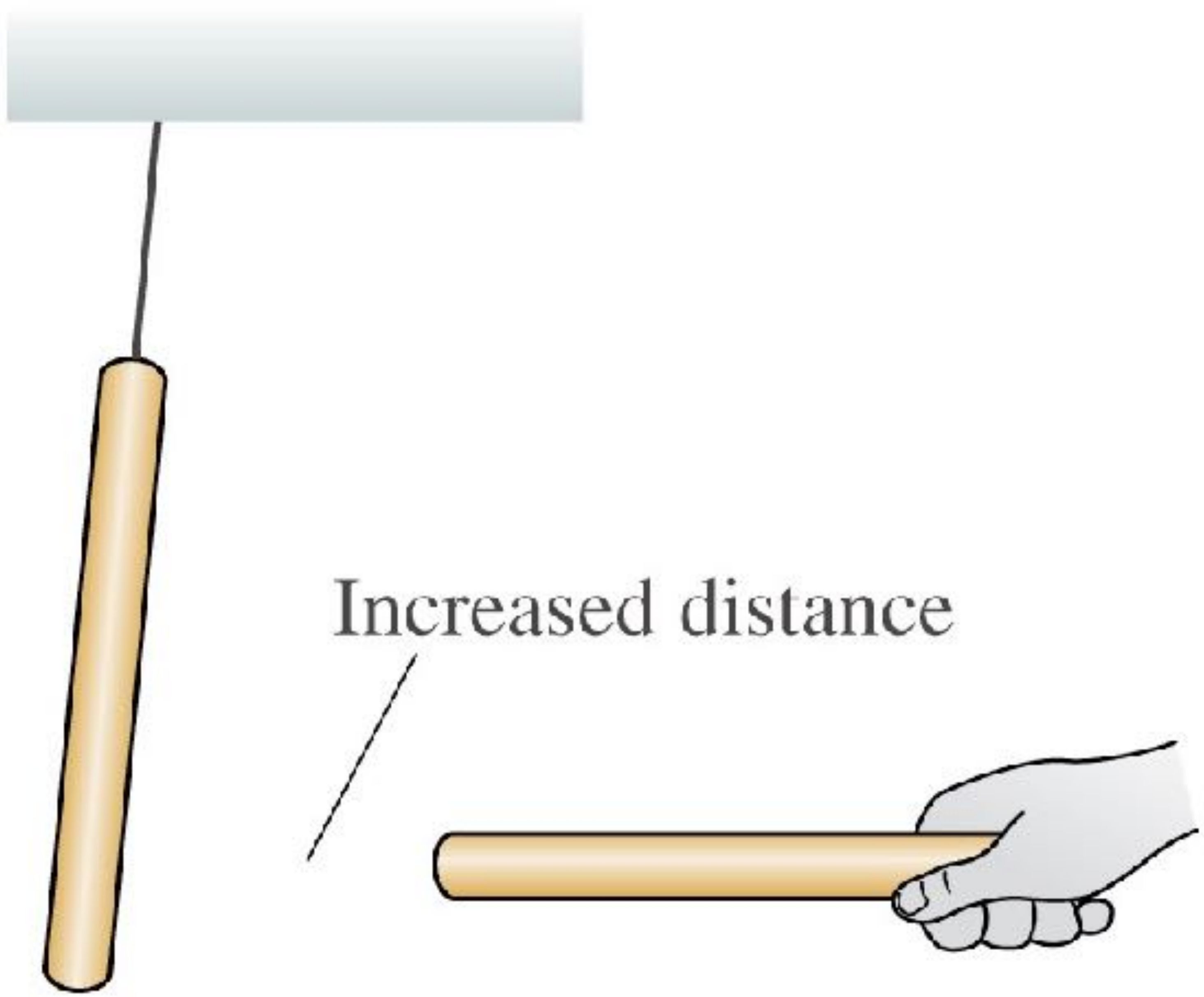
We call these objects charged.
Long range repulsive force!

Experiment 3



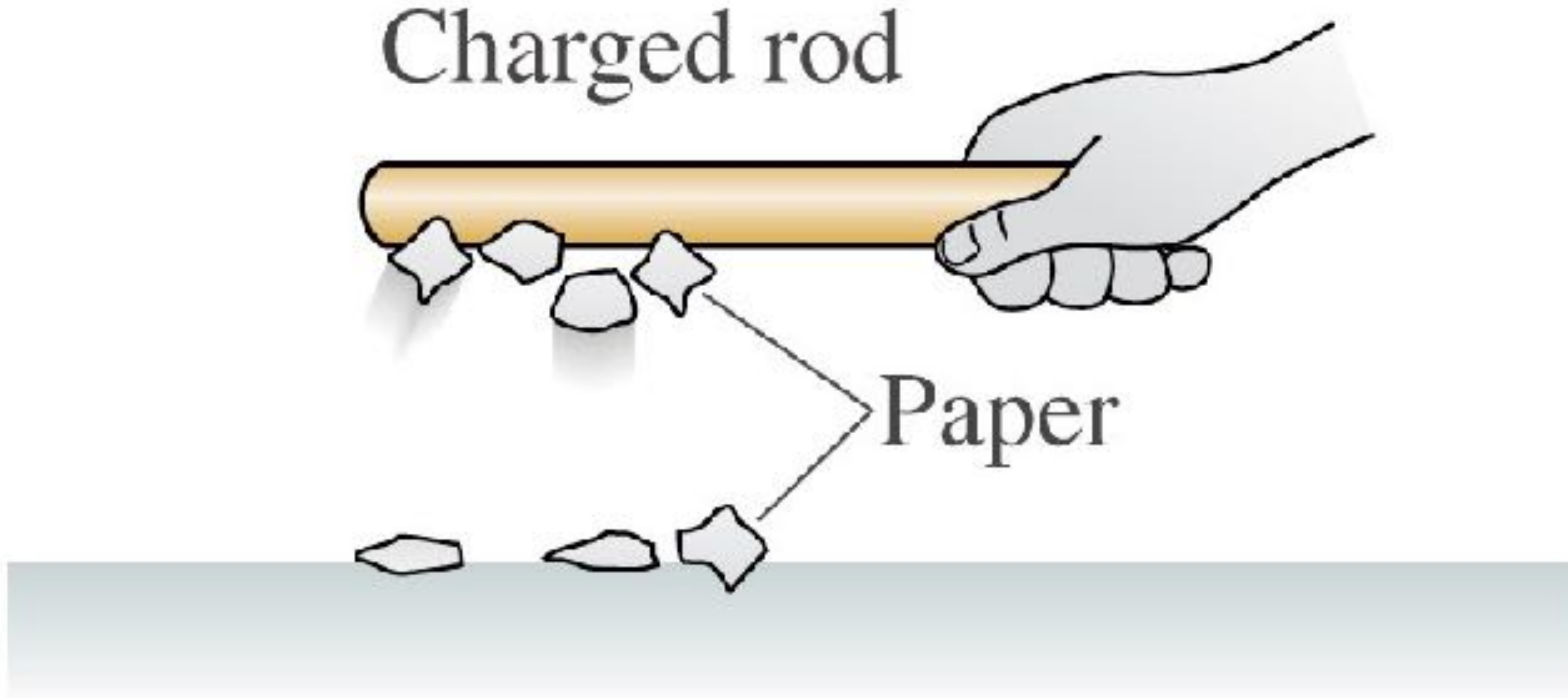
Charged glass and plastic *attract* each other!

Experiment 4



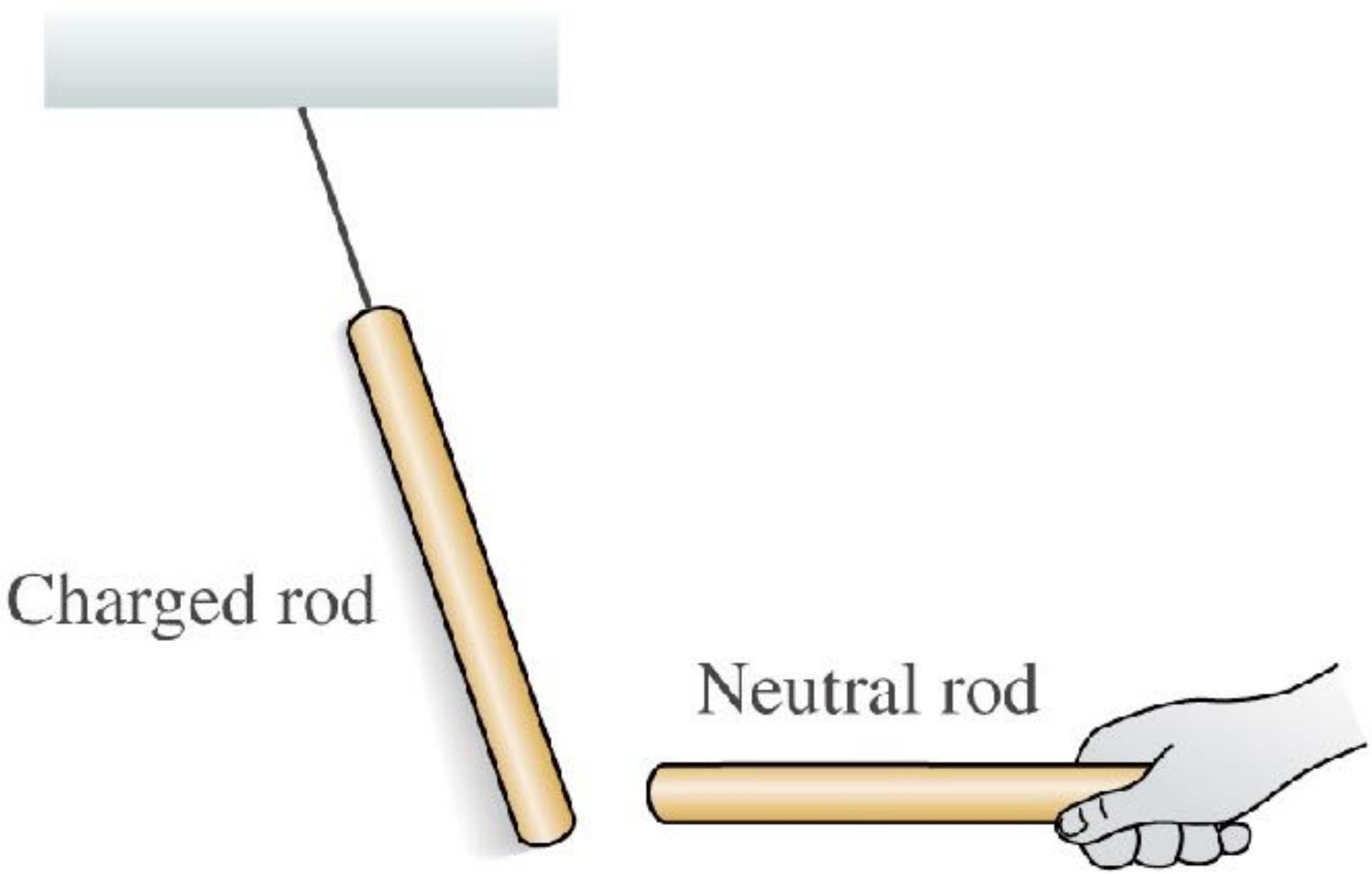
The larger the distance, the weaker the force.

Experiment 5

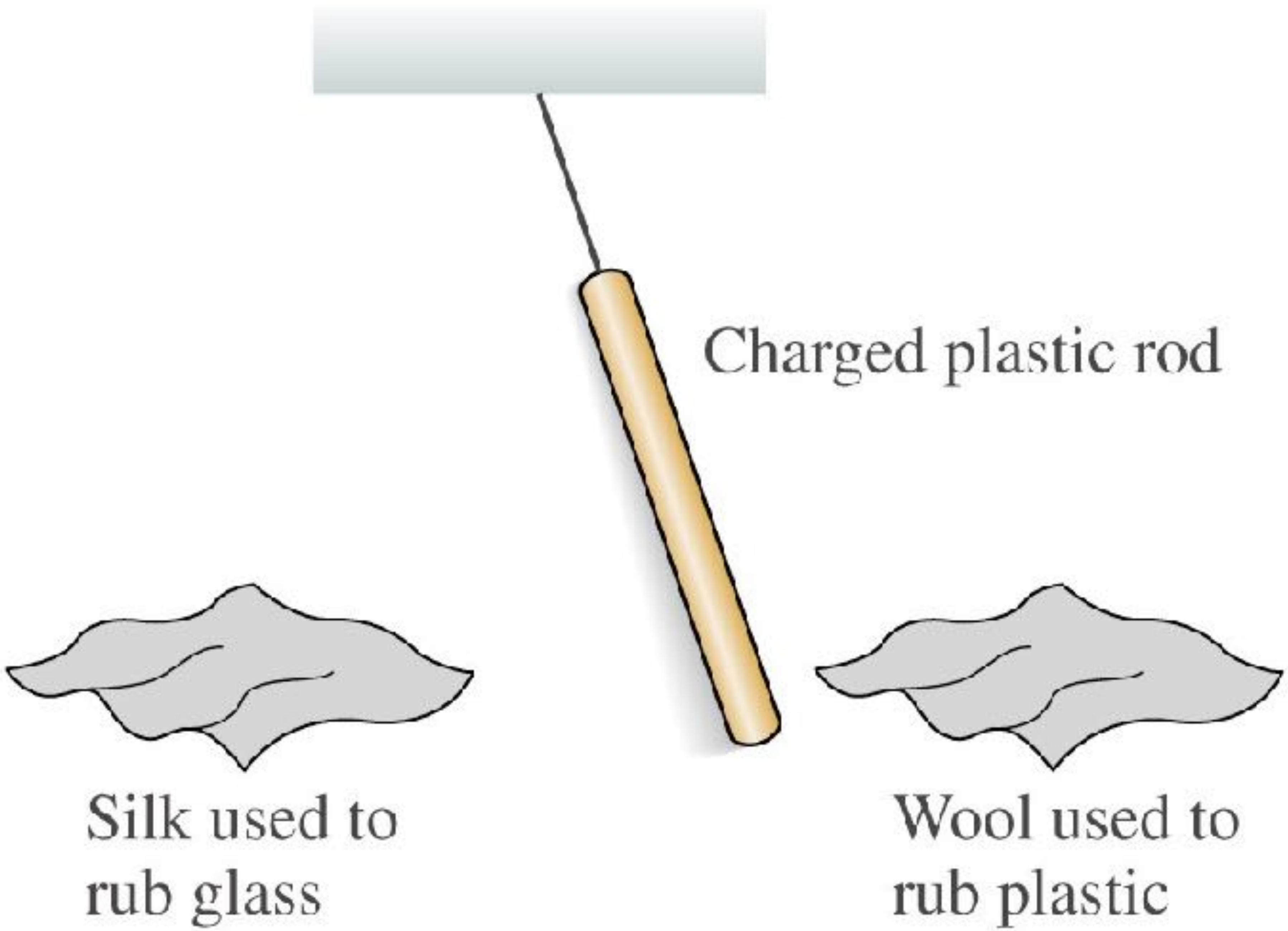


A charged rod picks up small pieces of paper.
A neutral rod does not.

Experiment 6

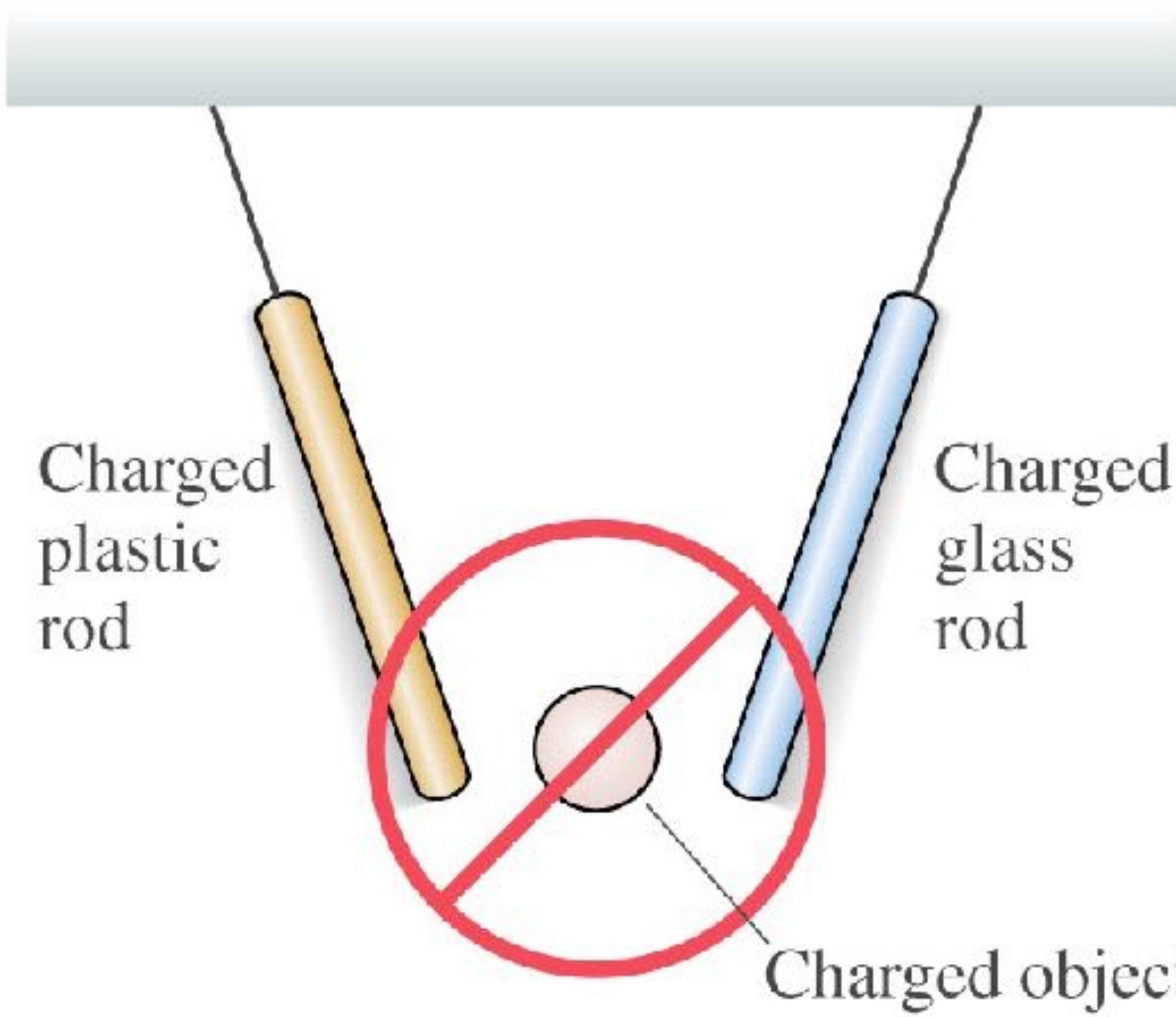


Experiment 7



This experiment is VERY difficult to do...

Experiment 8



We have yet to find an object that, after being charged, attracts *both* a charged plastic rod as well as a charged glass rod.

Conclusions?

1. Frictional charges, such as rubbing, add something called **charge** to an object (or remove it from an object).
2. There are two and only two kinds of charge. We can call them "plastic" and "glass" charge.
3. Two like charges exert repulsive forces on each other. Two unlike charges exert attractive forces on each other.
4. The force is a long range force, like gravity. Also, like gravity, the magnitude of the force decreases as the distance between the two objects increases.
5. Neutral objects have an equal mixture of both "plastic" and "glass" charge. Rubbing somehow manages to separate the two.

Discussion Question

To determine if an object has “glass” charge, you need to

A. see if the object attracts a charged plastic rod.
B. see if the object repels a charged glass rod.
C. do both A and B.
D. do either A or B.

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Experiment 9

- Charge can be transferred from one object to another, but only by contact.
  - Removing charge, also by contact, is called discharging.

Experiment 10

- Charge can be transferred through a conductor.
  - Metals are conductors.
- Charge cannot be transferred through an insulator.
  - Plastics, glass, wood are examples of insulators.

Additional conclusions.

6. There are two types of materials. Conductors are materials through or along which charge easily moves. Insulators are materials on in which charges remain fixed in place.
7. Charge can be transferred from one object to another by contact.

BUT,

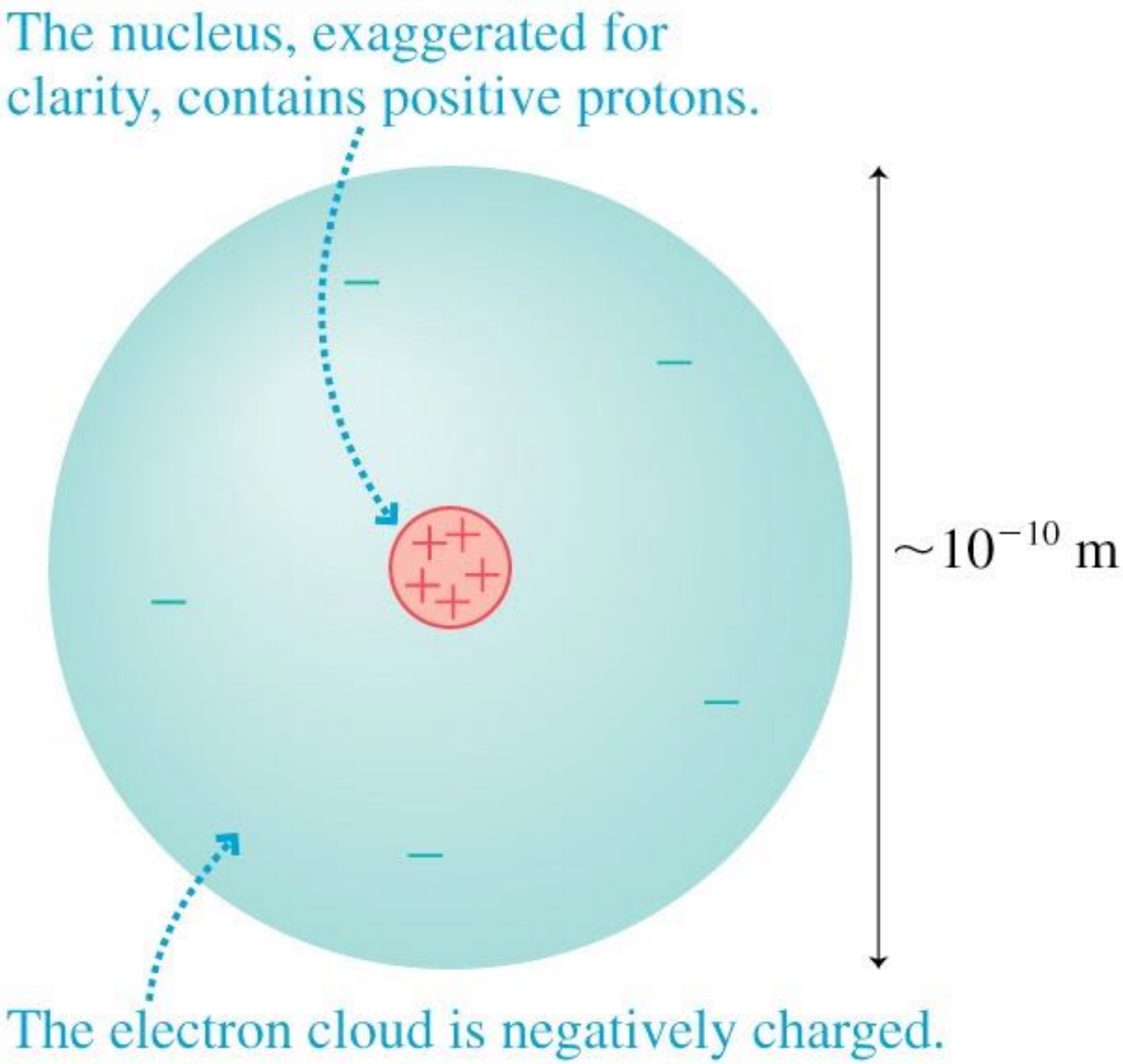
both insulators *and* conductors can be charged!

Charge

- Ben Franklin discovered that charges act like positive and negative numbers.
    - Charges add algebraically:
        - Glass + glass = 2 Glass (1+1 = 2)
        - Glass + plastic = 0 (1 + (-1) = 0)
- He established the ***convention*** that the charge on a glass rod is positive.
- With the discovery of electrons and protons, we thus have the ***convention*** that electrons have a negative charge and protons have a positive charge.

In hindsight it would have been better had he called the glass rod negative. Electrons are the carriers of the electric current in a wire. That they are negative will present us with some signs problems later...

Modern View



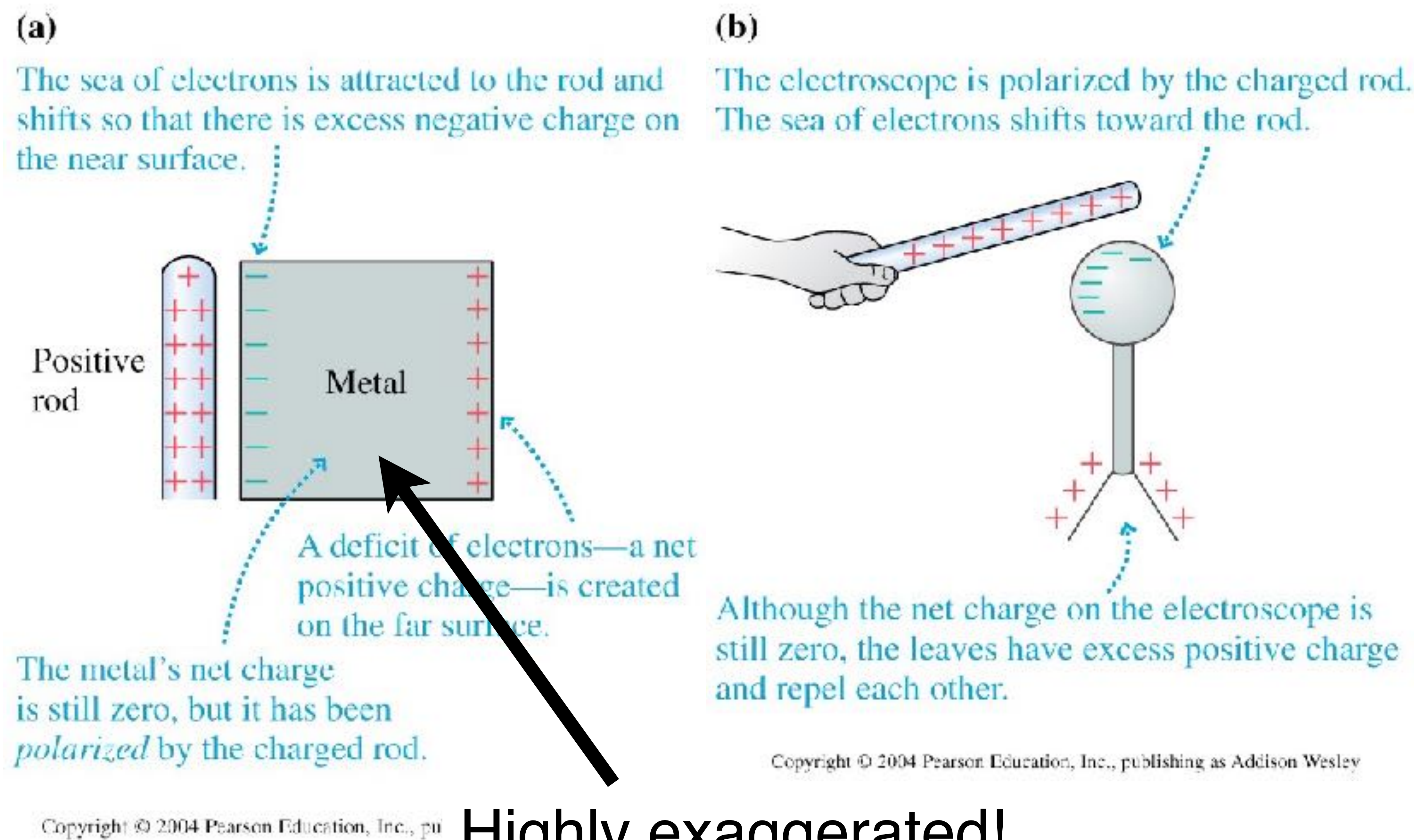
Charge is an *inherent* property, like mass, of electrons and protons.

TABLE 25.1 Protons and electrons

Particle	Mass (kg)	Charge
Proton	1.67×10^{-27}	$+e$
Electron	9.11×10^{-31}	$-e$

As far as we can measure, electrons and protons have charges of opposite sign but **exactly** equal magnitudes.

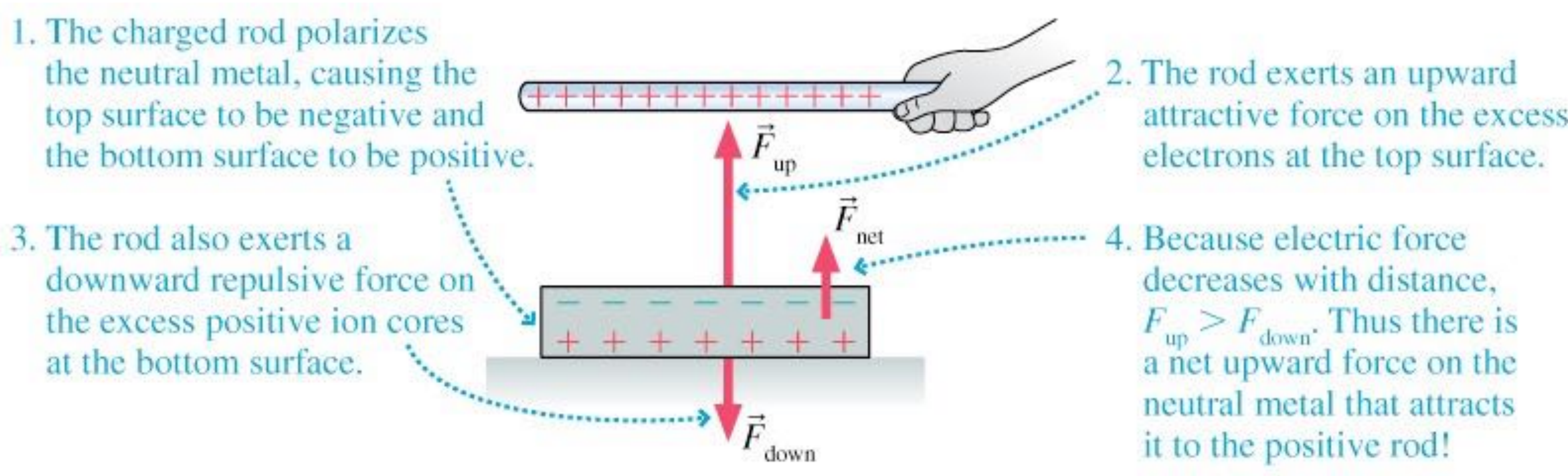
Charge Polarization



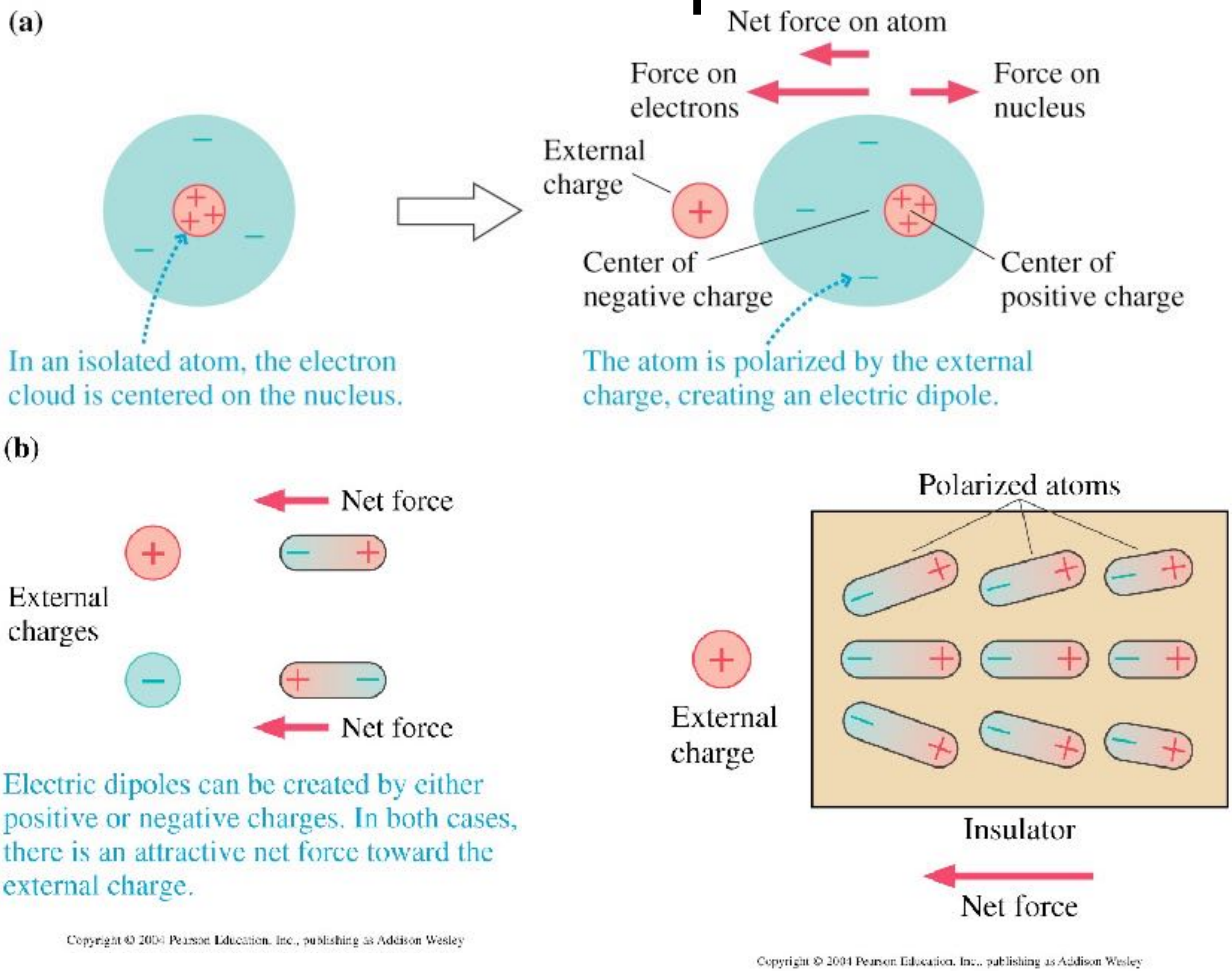
Highly exaggerated!

It is NOT true that ALL of the electrons move to the left! As the electrons move to the left, the remaining positive ions begin to exert a restoring force to the right. The equilibrium position for the sea of electrons is just enough to the left for the forces from the external charges and the positive ions are in balance. In practice the displacement is usually less than 10^{-15} m!

Picking up a piece of foil (or paper!)



Electric dipole



Discussion Question

An electroscope is negatively charged by *touching* it with a negative plastic rod. The electroscope leaves spread apart and the rod is removed. A positively charged glass rod is brought close to the top of the electroscope, but doesn't touch it. What happens to the leaves?

A. The leaves get closer together.
B. The leaves spread further apart.
C. The leaves don't move.
D. Need more info.

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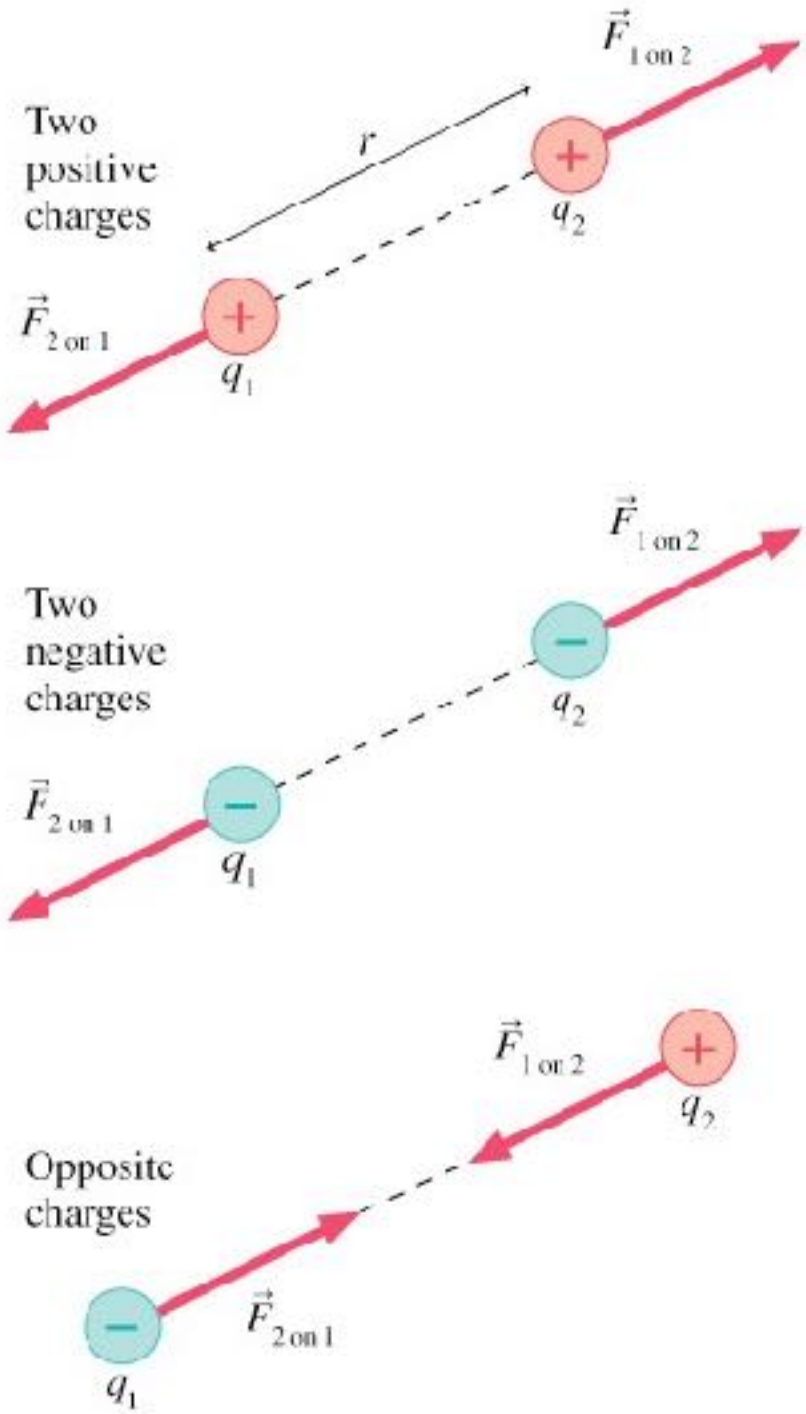
Coulomb's Law



$$F_{1 \text{ on } 2} = F_{2 \text{ on } 1} = k\frac{|q_1||q_2|}{r^2}$$

The force is along a line connecting the two charges.

It is repulsive for like sign charges.

It is attractive for unlike sign charges.

$$e = 1.602 \times 10^{-19}\text{C}$$
$$k = 8.99 \times 10^{9}\text{Nm}^2/\text{C}^2$$

$$k = \frac{1}{4\pi\varepsilon_0} \qquad F = \frac{1}{4\pi\varepsilon_0}\frac{|q_1||q_2|}{r^2}$$

Newton 3rd Law Pair!

Coulomb's Law

- Coulomb's Law applies only to point charges. Two charges can be considered to be point charges if the distance between them is much larger than their size.
- Coulomb's law applies only to static charges.
- Electric forces, like all other forces, can be superimposed. If there are multiple charges, the net force on charge *j* is

$$\vec{F}_{\text{net}} = \vec{F}_{1 \text{ on } j} + \vec{F}_{2 \text{ on } j} + \vec{F}_{3 \text{ on } j} + \cdots$$

Discussion Question

Charges A and B exert repulsive forces on each other. $q_A = 4q_B$. Which statement is true?

A. $F_{\text{A on B}} > F_{\text{B on A}}$

B. $F_{\text{A on B}} = F_{\text{B on A}}$

C. $F_{\text{A on B}} < F_{\text{B on A}}$

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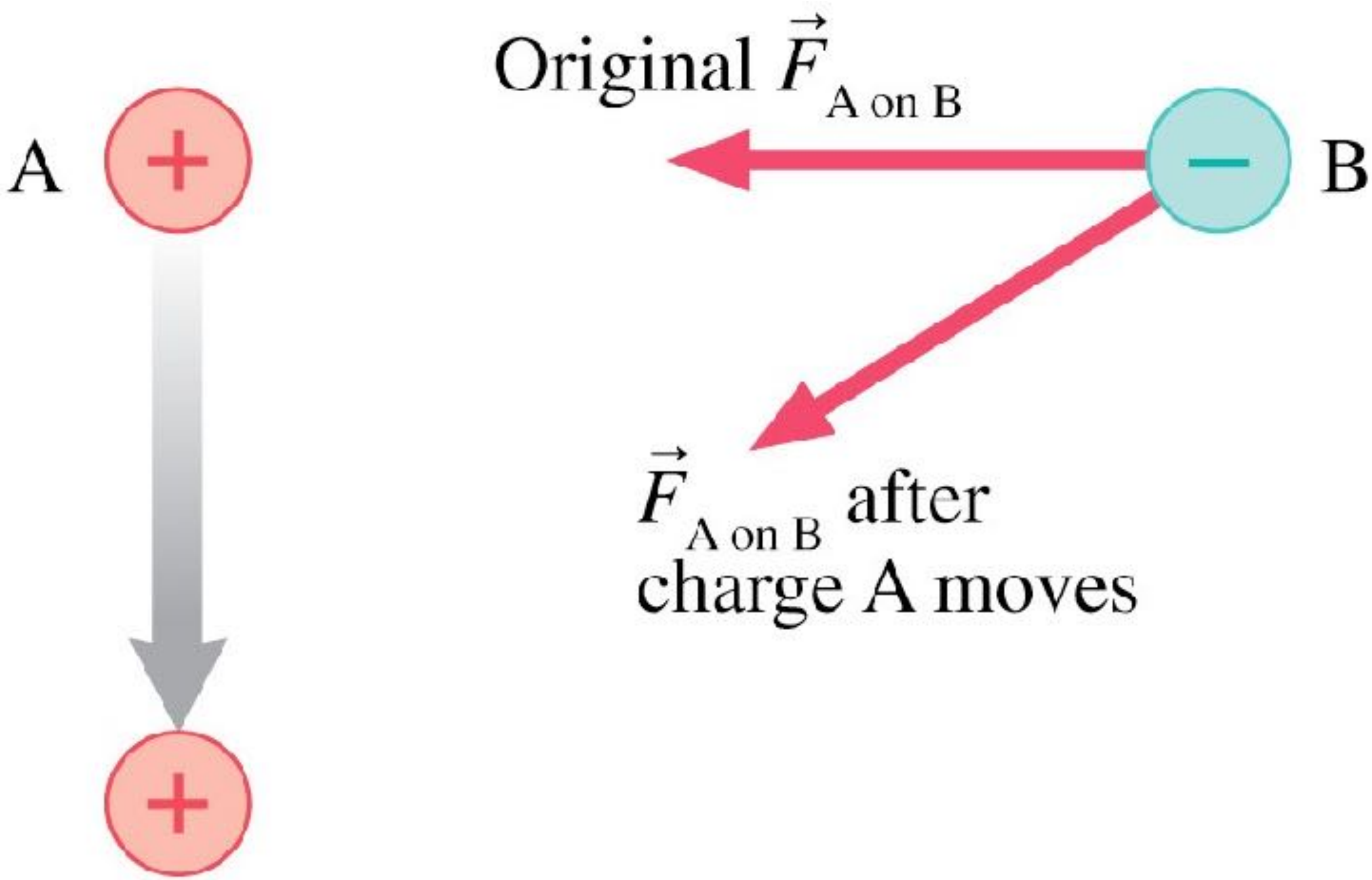
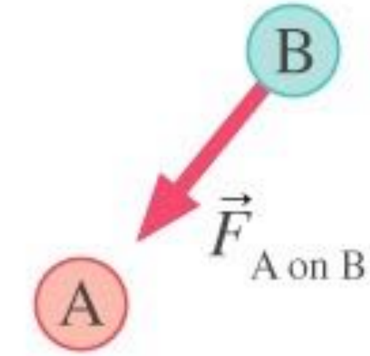
Action at a Distance



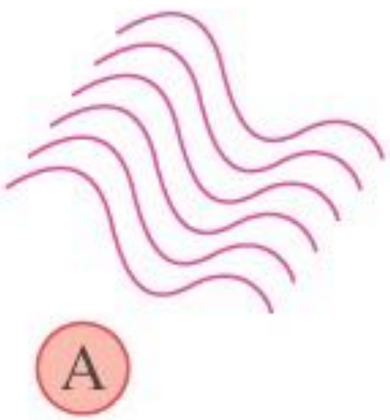
How does B know that A moved?

How long does it take before B notices?

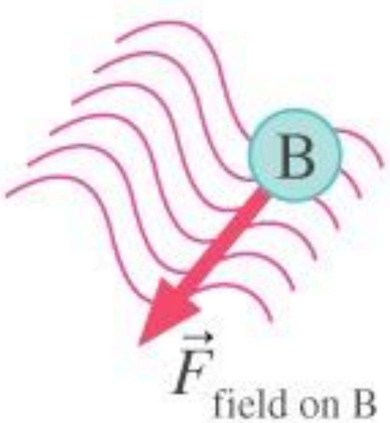
Newton vs Faraday



In the Newtonian view, A exerts a force directly on B.



In Faraday's view, A alters the space around it. (The wavy lines are poetic license. We don't know what the alteration looks like.)



Particle B then responds to the altered space. The altered space is the agent that exerts the force on B.

We call this modification of space a ***field***.

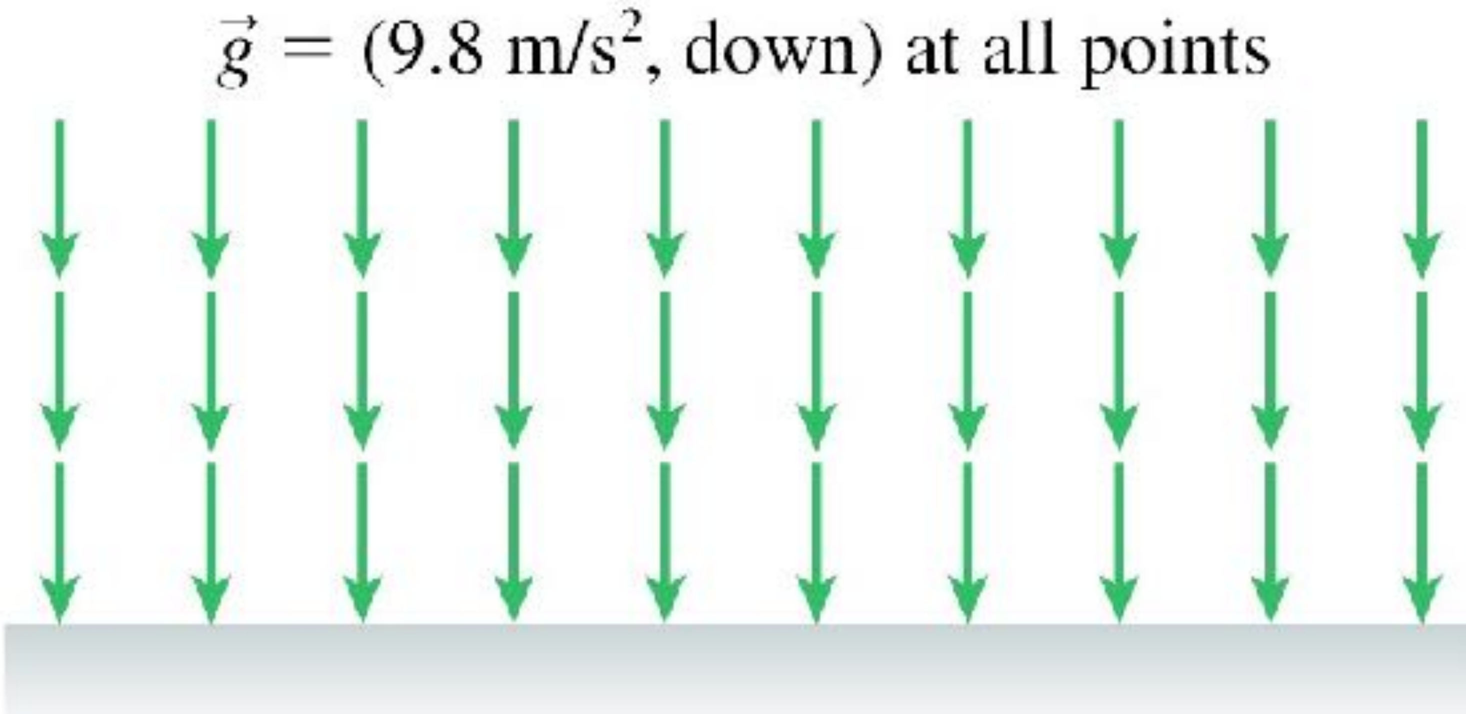
Around a mass it is the gravitational field.

Around a charge it is the electric field

Fields

- The term *field* describes a function $f(x,y,z)$ that assigns a value to every point in space.
- The concept of a field is in sharp contrast to that of a particle.
  - A particle exists at ***one*** point in space.
  - A field exists simultaneously at all points in space.
- An example of a field is the temperature in a room. It has a value at every point in the room, which might even vary. The temperature is a ***scalar*** field.

Gravitational field



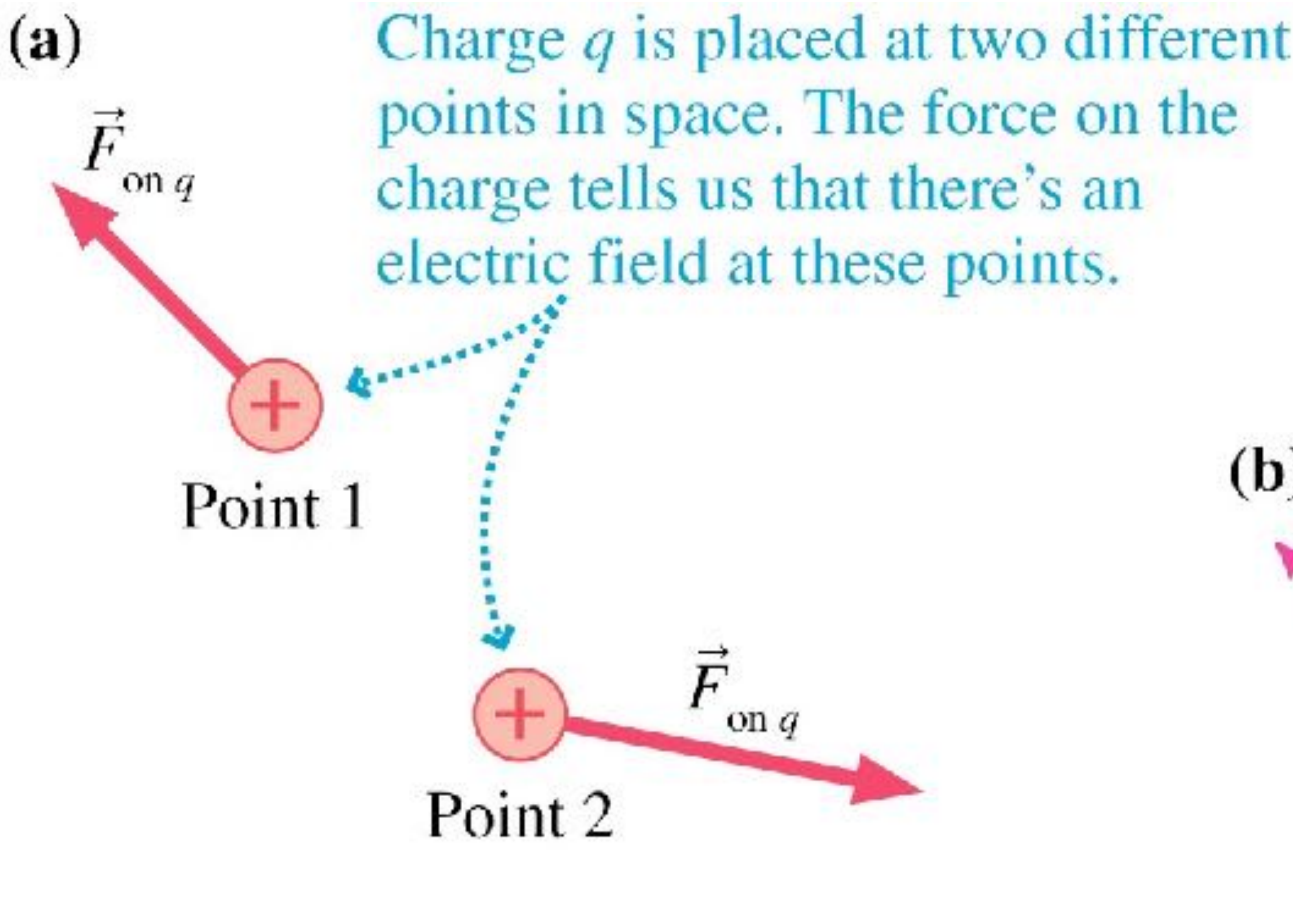
The mass of the earth creates the gravitational field. At the surface of the earth, the field is approximately vertical, with a constant value of 9.8 m/s^2, pointing downward.

The gravitational force on any mass near the surface of the earth is

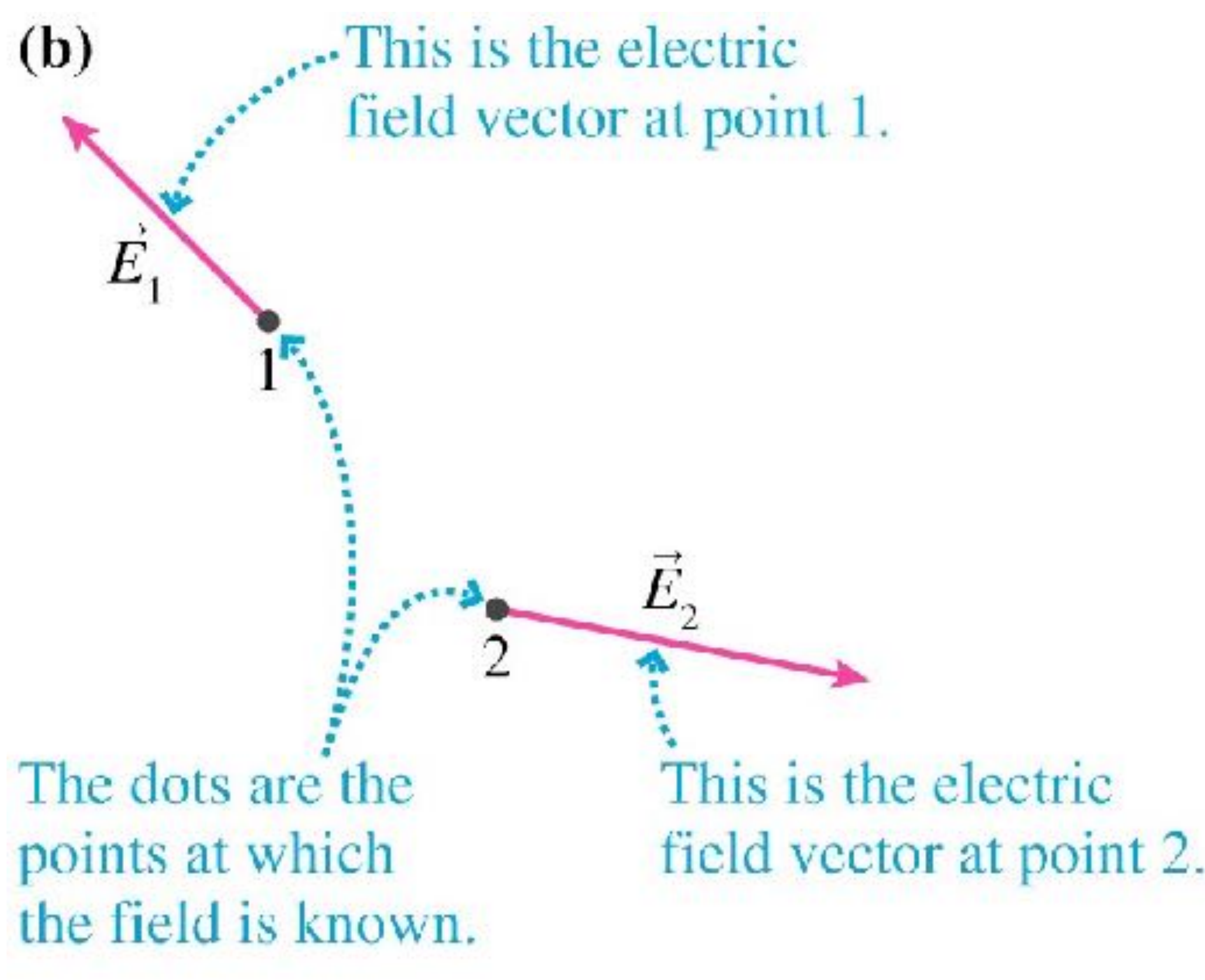
$$\vec{F}_{\text{on } m} = m\vec{g}$$

The gravitational field is a *vector* field, assigning a vector to every point in space.

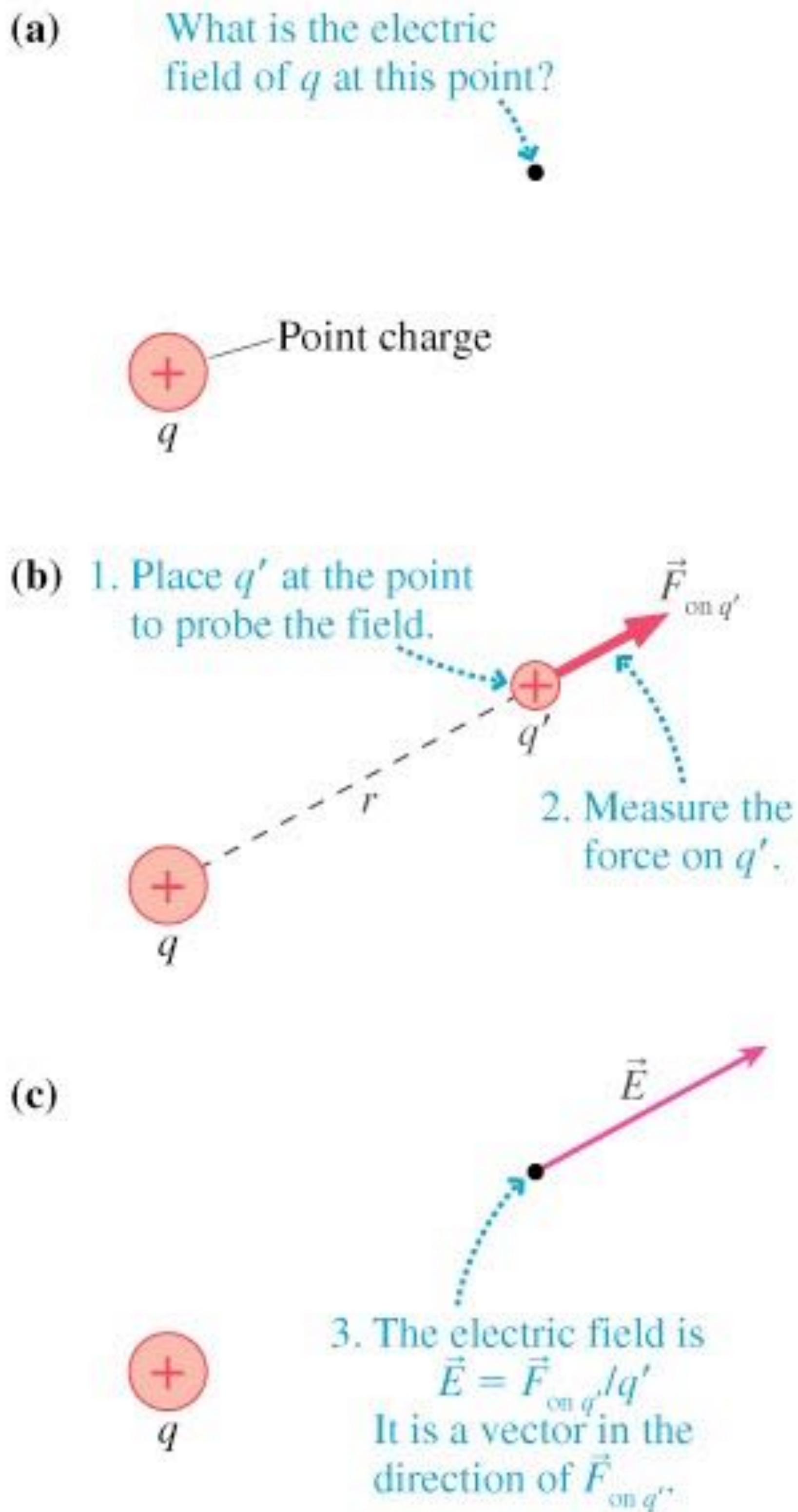
Electric Field



$$\vec{E}(x, y, z) = \frac{\vec{F}_{\text{on } q} \text{ at } (x, y, z)}{q}$$



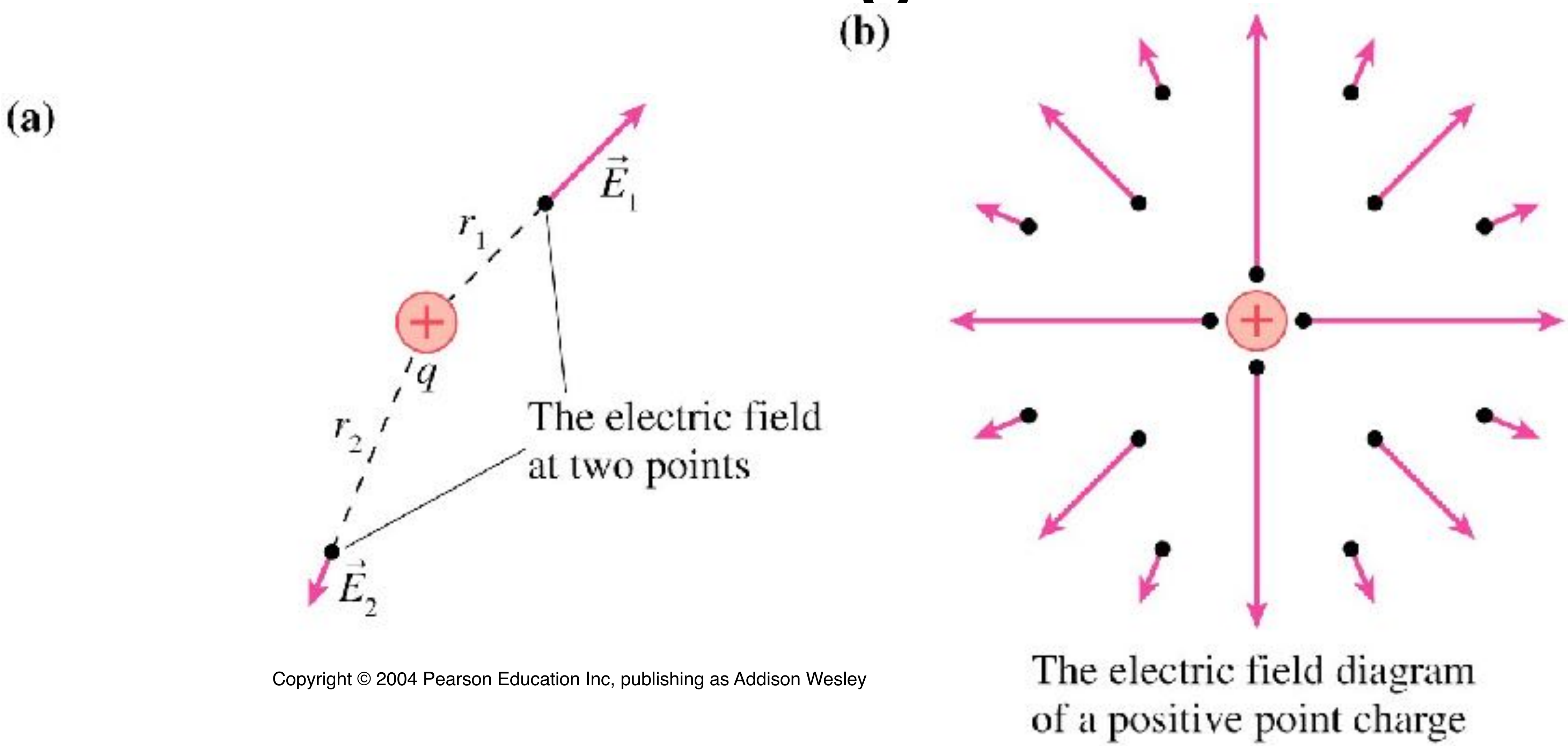
Electric Field of a point charge



$$\vec{F}_{\text{on } q'} = \left(\frac{1}{4\pi\varepsilon_0} \frac{qq'}{r^2}, \text{away from q} \right)$$

$$\vec{E} = \frac{\vec{F}_{\text{on } q'}}{q'} = \left(\frac{1}{4\pi\varepsilon_0} \frac{q}{r^2}, \text{away from q} \right)$$

Field diagrams



The diagram shows only a few representative points. The field exists ***everywhere***.

The arrow indicates the direction and strength of the field *at the point to which it is attached*.

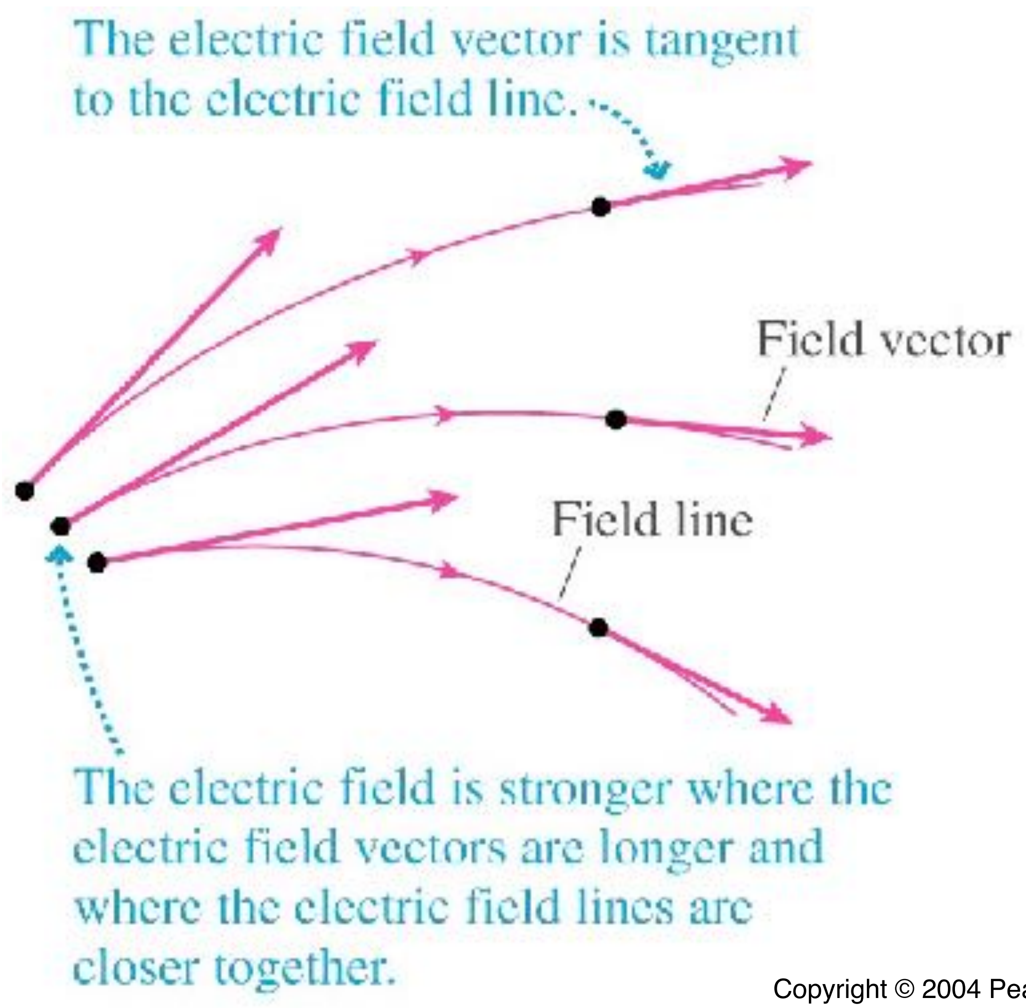
We have to draw the vector across the page. However, it does not stretch across space. Each vector represents the field at ***one point*** in space.

Picturing the Electric Field

It is difficult to picture the electric field. It has a value and a direction at every point in space.

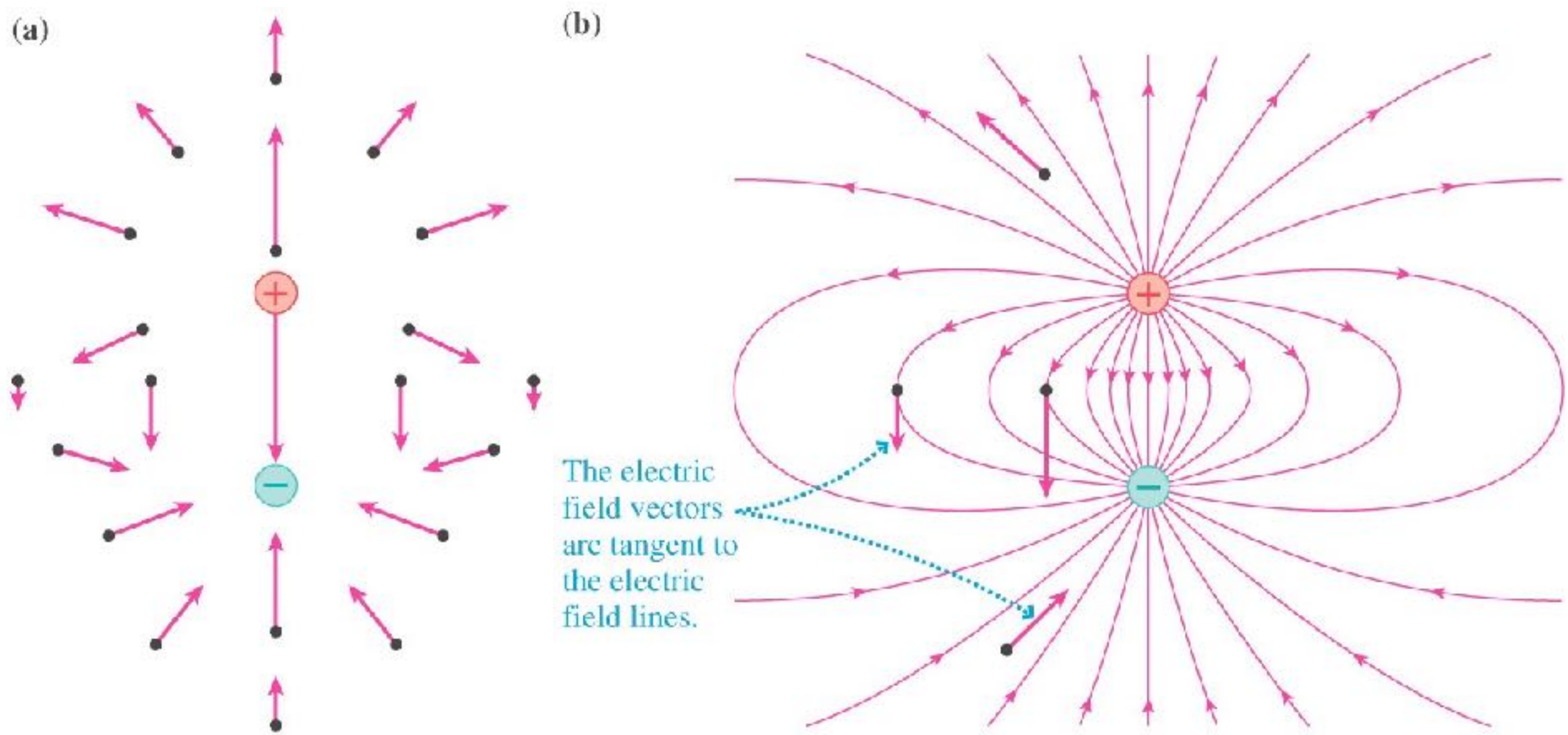
(Temperature is also a field. At every point in this room, the air has a temperature. It varies, and is probably warmer at the top of the room than at the bottom. But this is easier to visualize.)

There are two basic ways of picturing the electric field: with field vectors and with field lines:



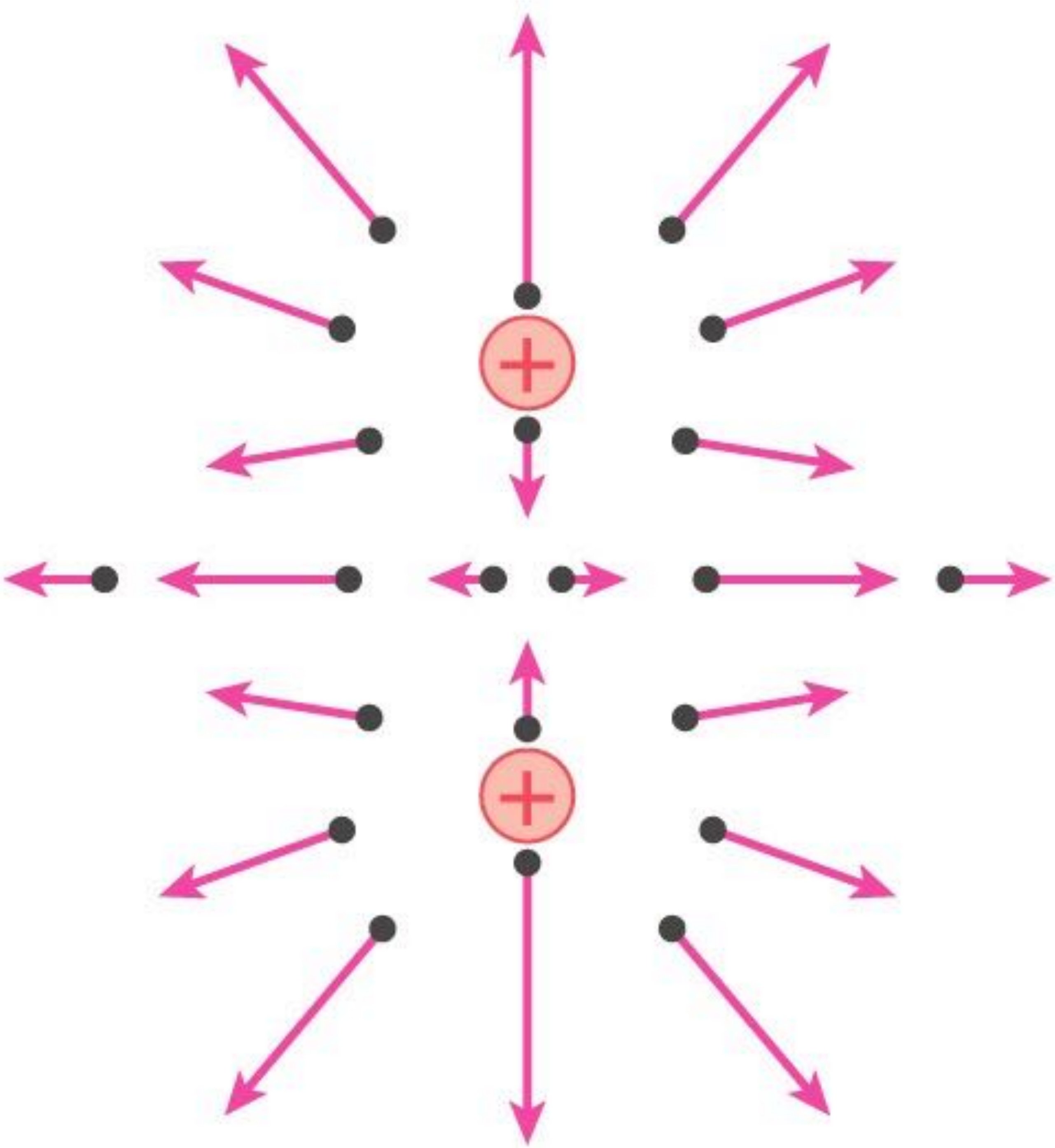
Field lines are like streamlines in fluid mechanics.

Two views of a dipole field

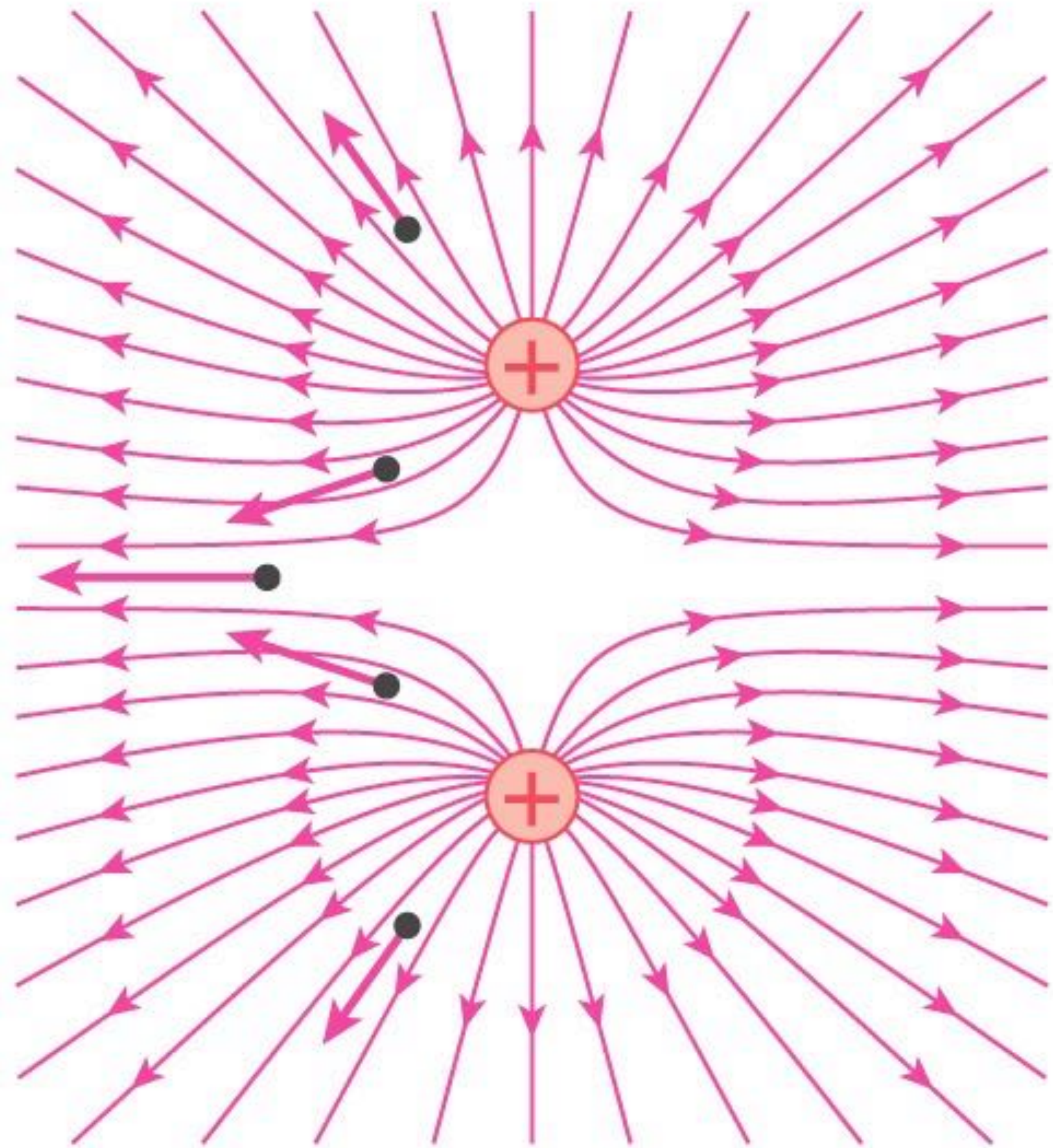


Electric Field of 2 positive charges

(a)



(b)



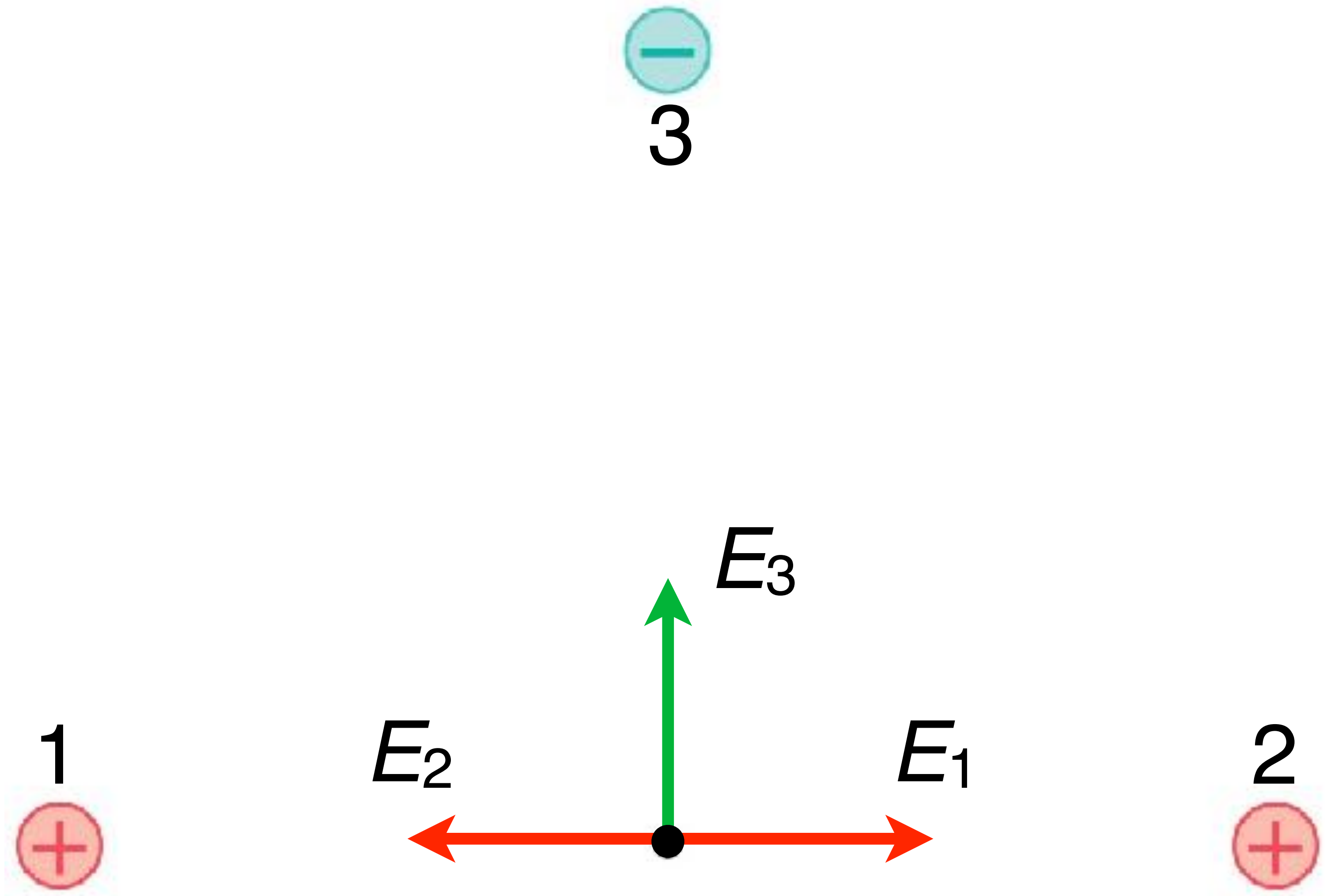
Discussion Question

At the position of the dot, the electric field points

A. left.
B. right.
C. up.
D. down.
E. The electric field is zero.

Discussion Question

At the position of the dot, the electric field points



A. left.
B. right.
C. up.
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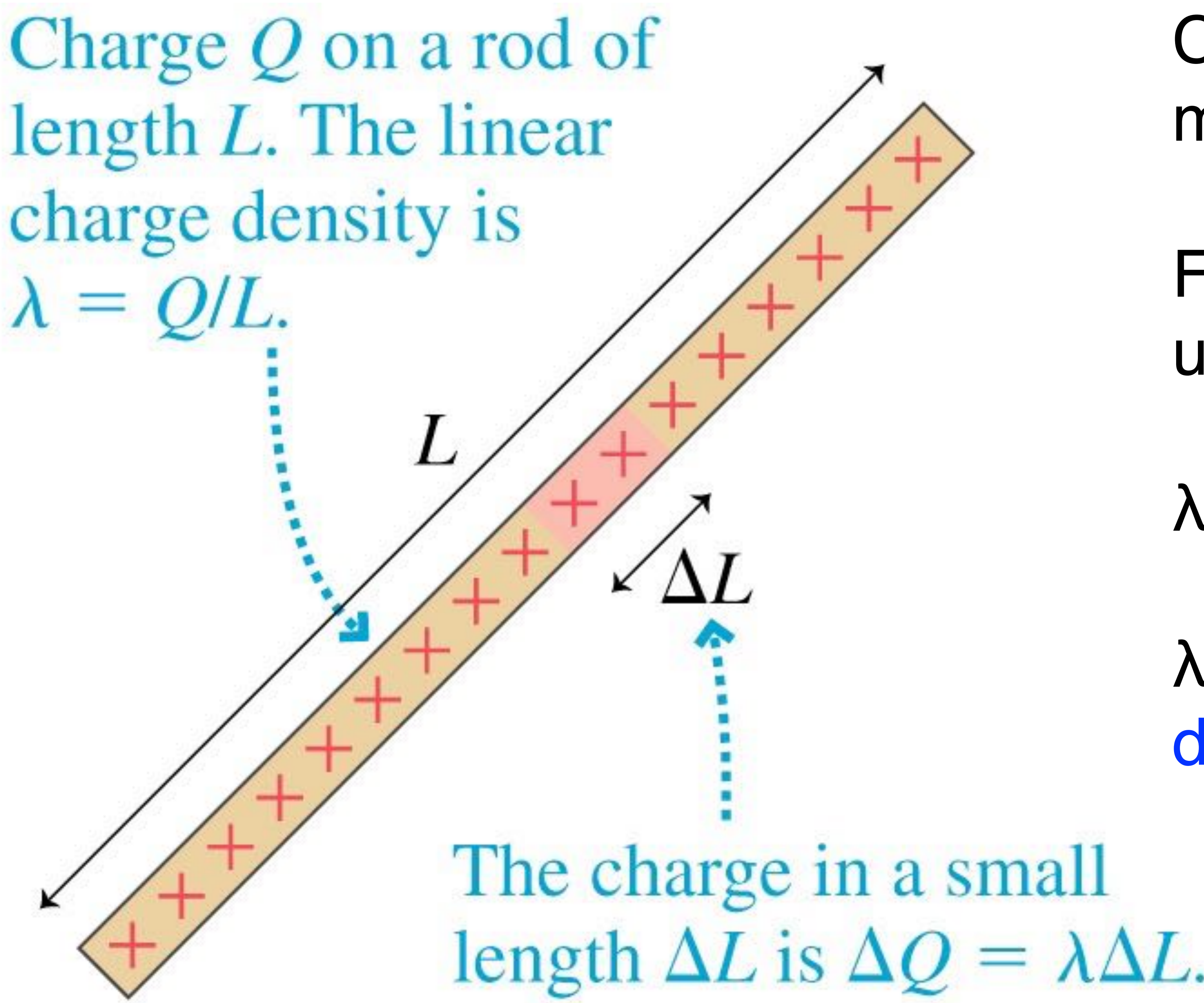
Continuous Charge Distribution



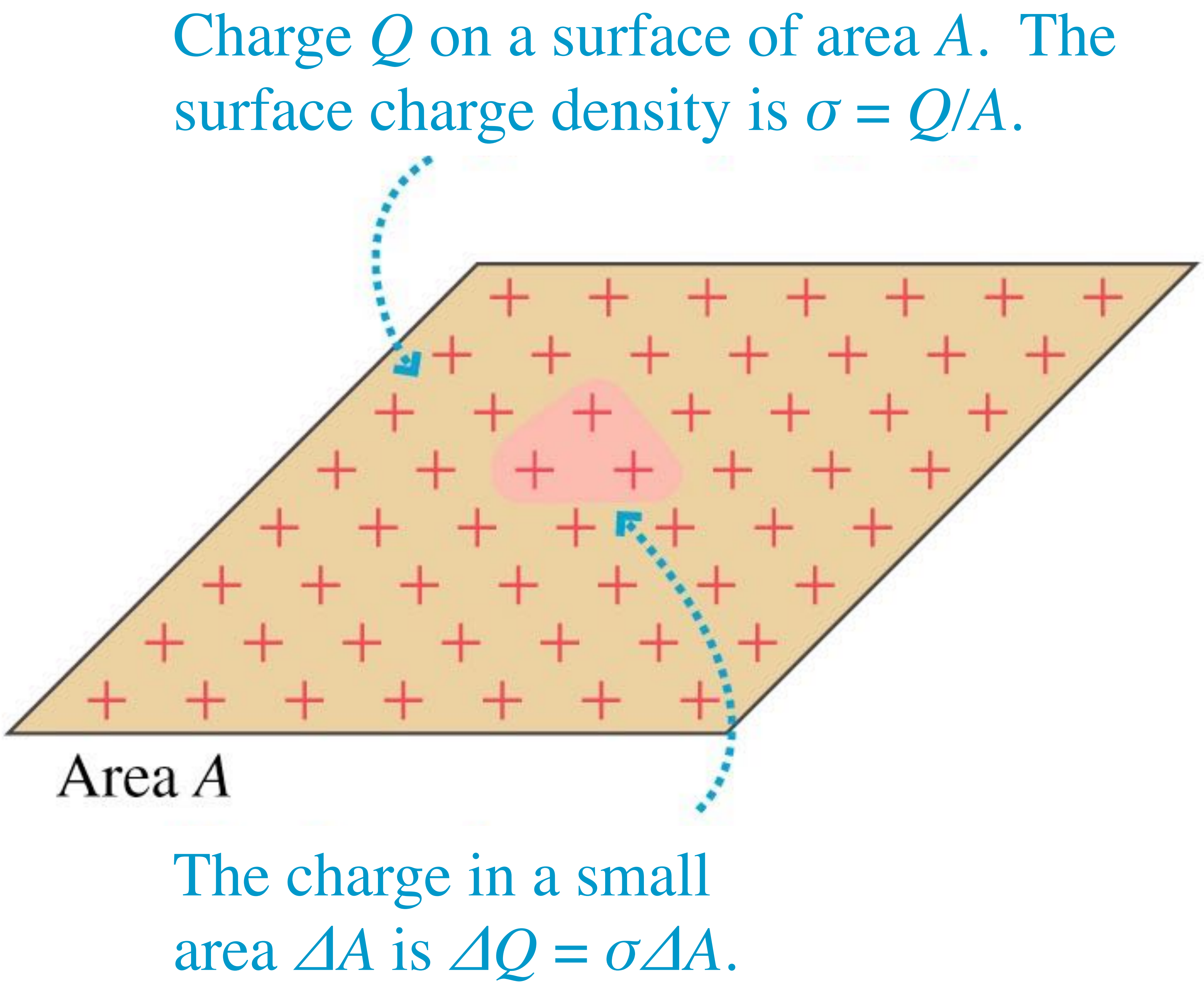
Charge density is like mass density.

For a thin rod with a uniform charge,

$\lambda = Q/L$

λ is the linear charge density

Surface Charge Density



Electric Field

For individual charges we had:

$$\vec{E} = \vec{E}_1 + \vec{E}_2 + \cdots = \sum_i \vec{E}_i$$

For a continuous charge distribution:

Divide the total charge into many small point-like charges ΔQ.

Use our knowledge of the electric field of a point charge to find the electric field of each ΔQ.

Calculate the net electric field $\vec{E}_{\text{net}}$ by summing the fields of all the ΔQ.

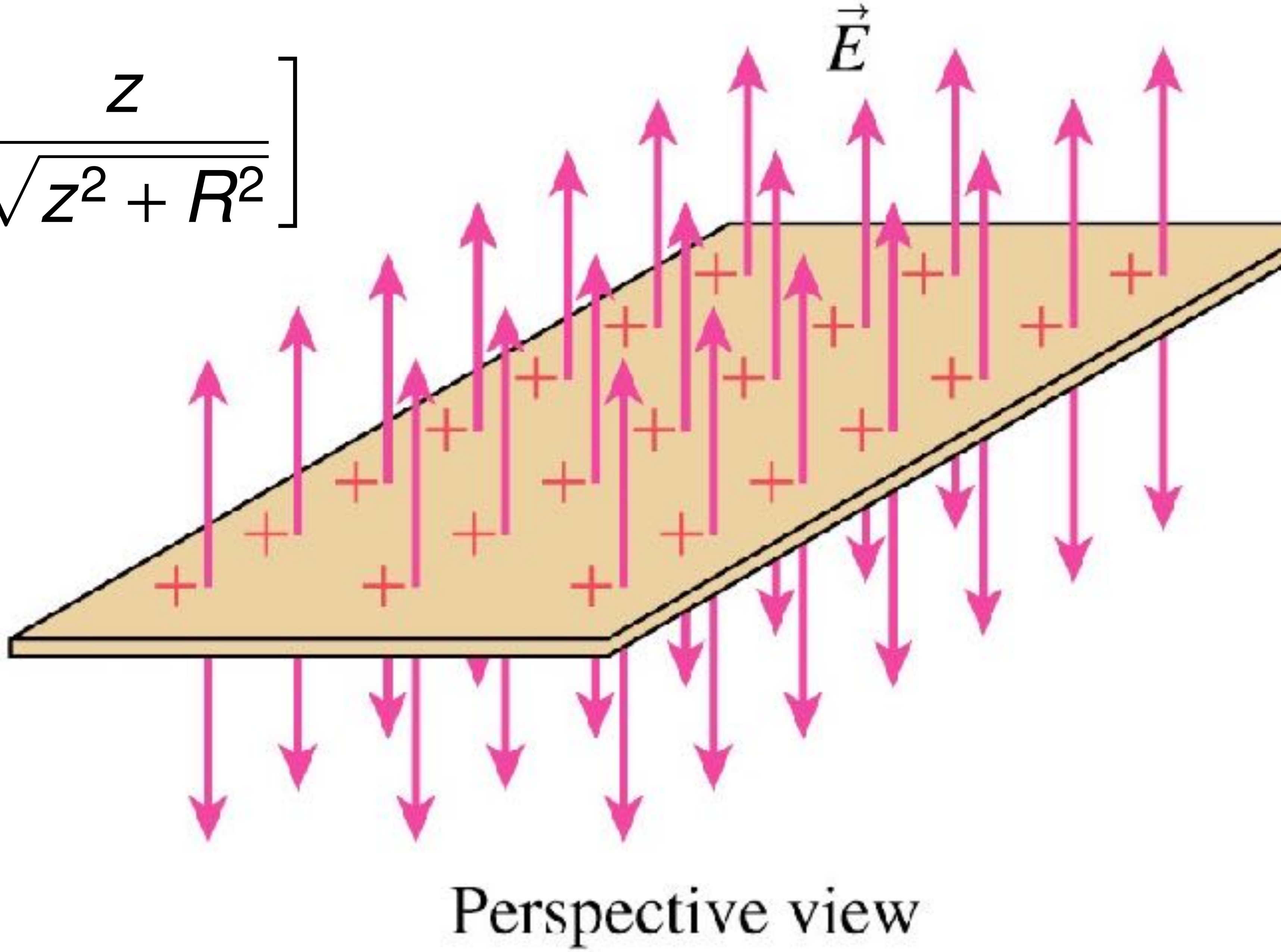
Let the sum become an integral.

Field of an Infinite Plane

$$(E_{\text{disk}})_z = \frac{\sigma}{2\varepsilon_0}\left[1 - \frac{z}{\sqrt{z^2 + R^2}}\right]$$

Take $R \rightarrow \infty$

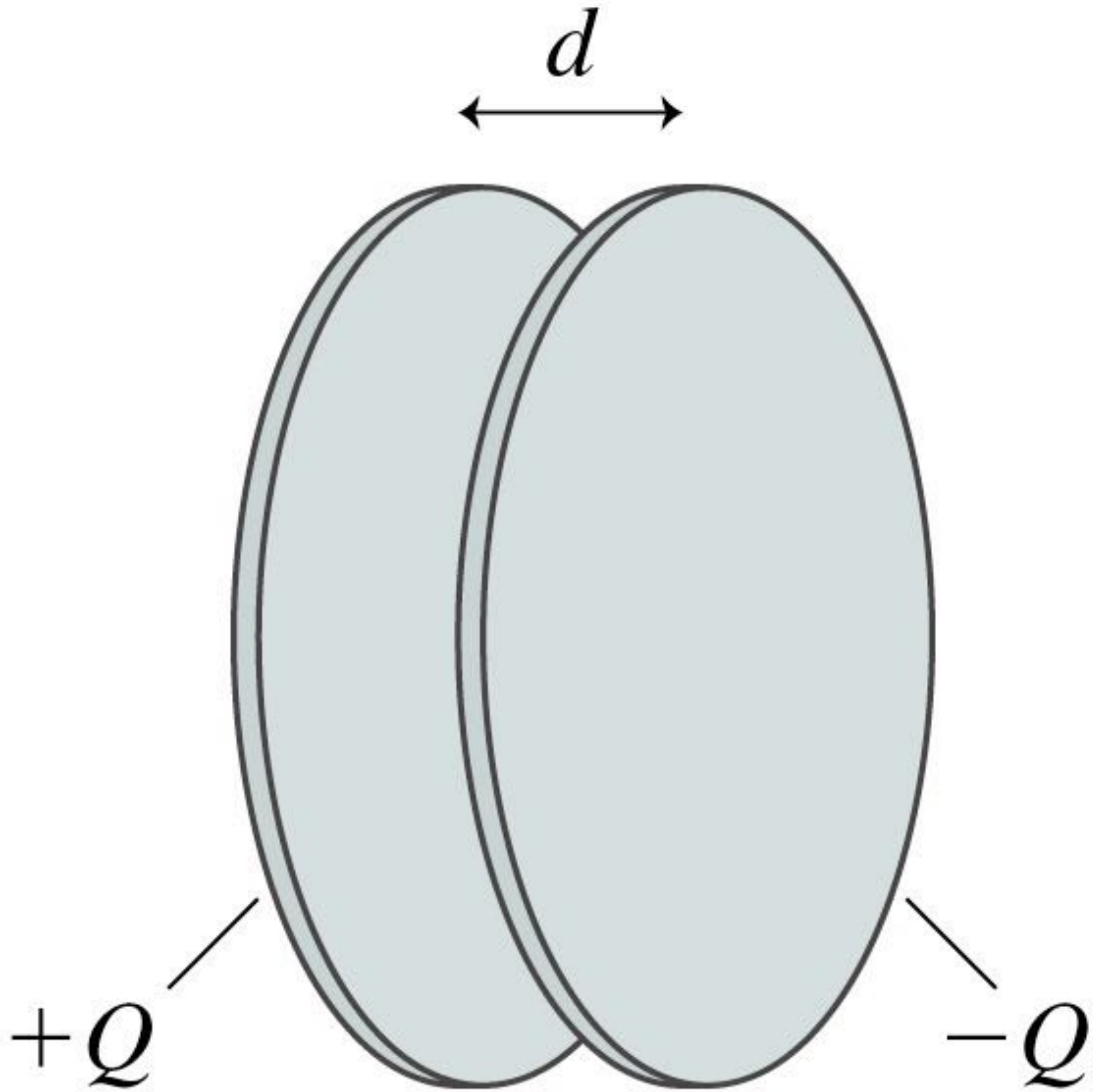
$$(E_{\text{plane}})_z = \frac{\sigma}{2\varepsilon_0}$$

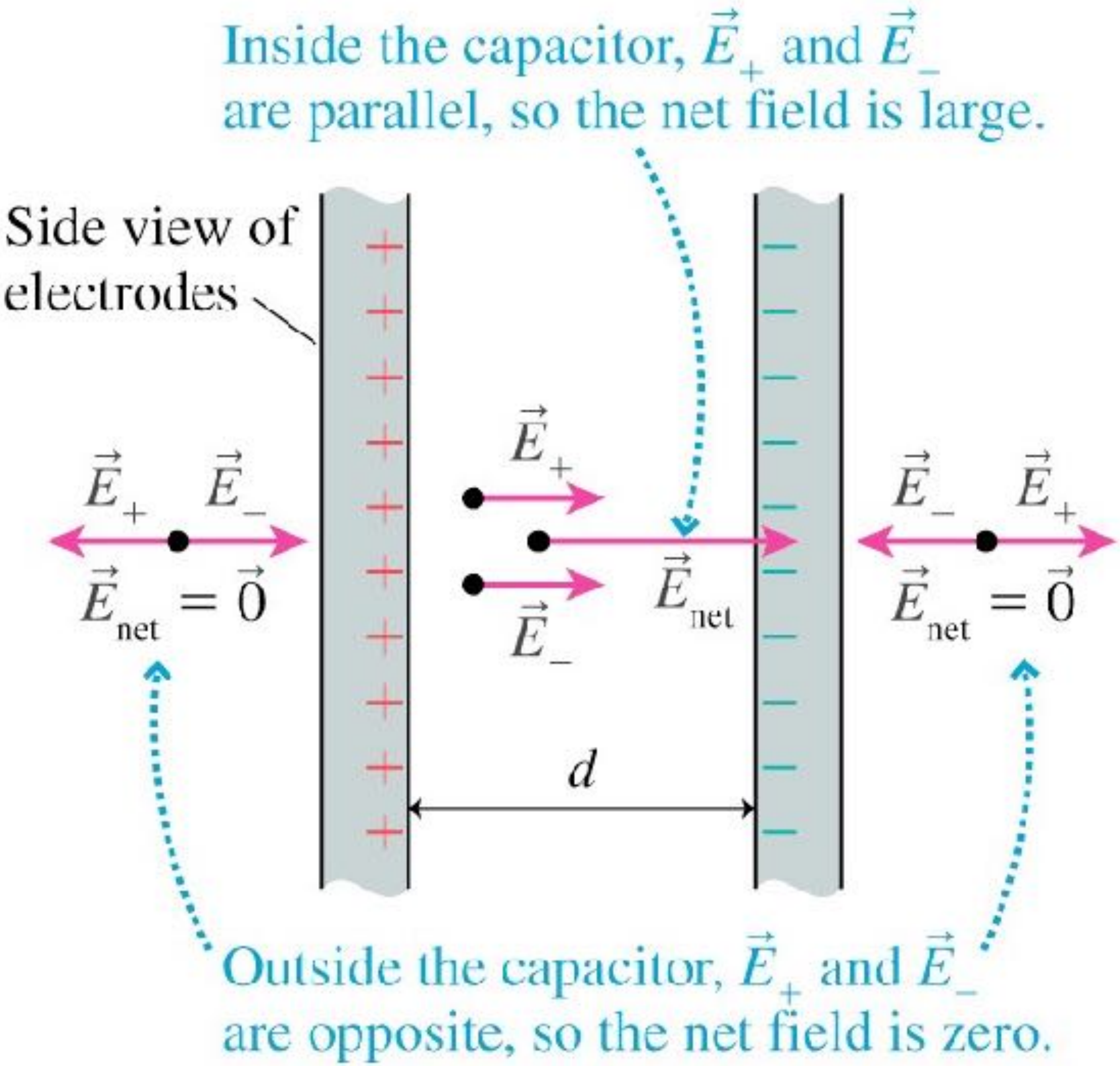


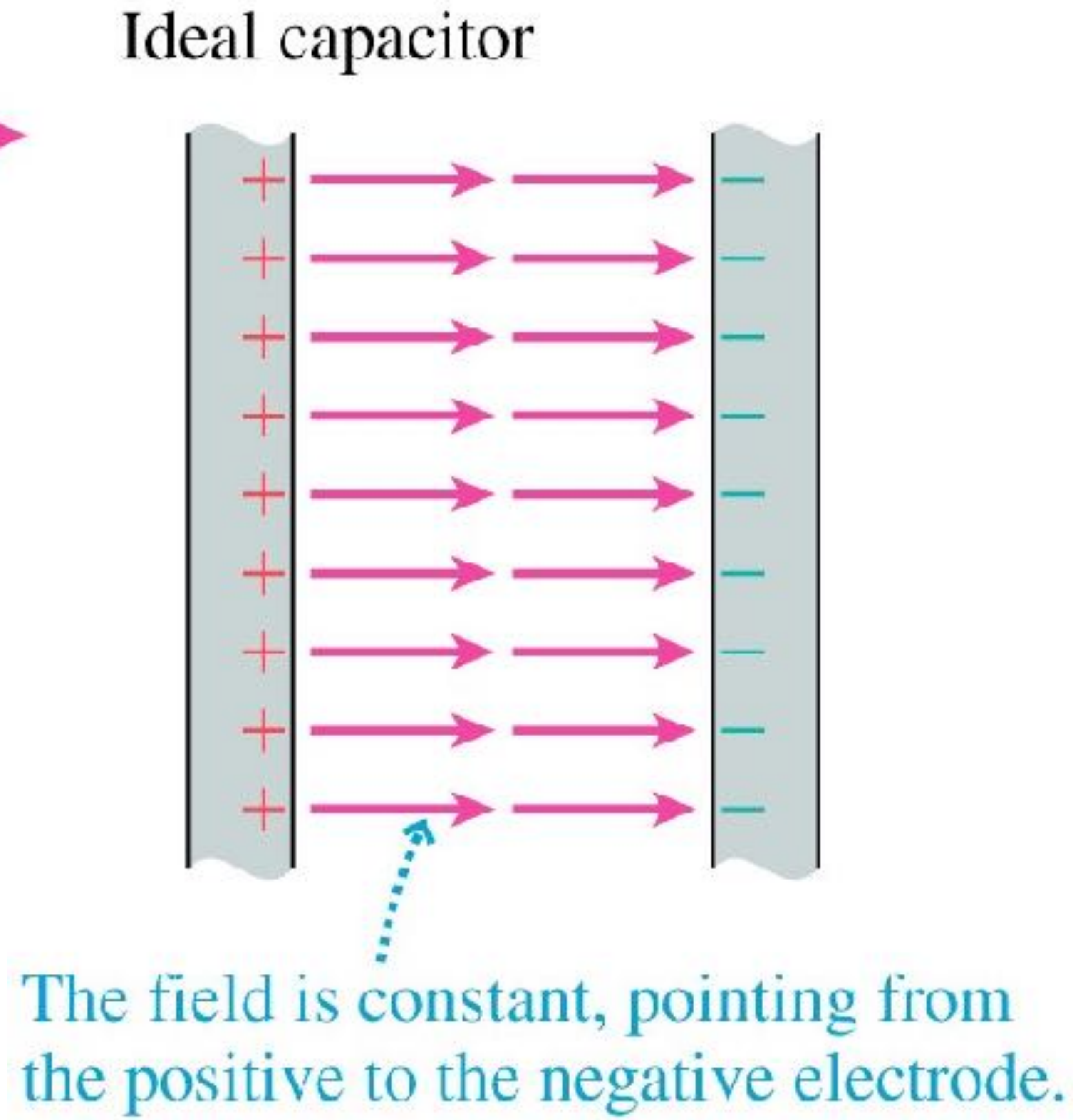
(An infinite plane looks the same no matter how far away you are.)

(A finite plane looks like an infinite plane if you are close to it and far from the edge.)

Parallel Plate Capacitor

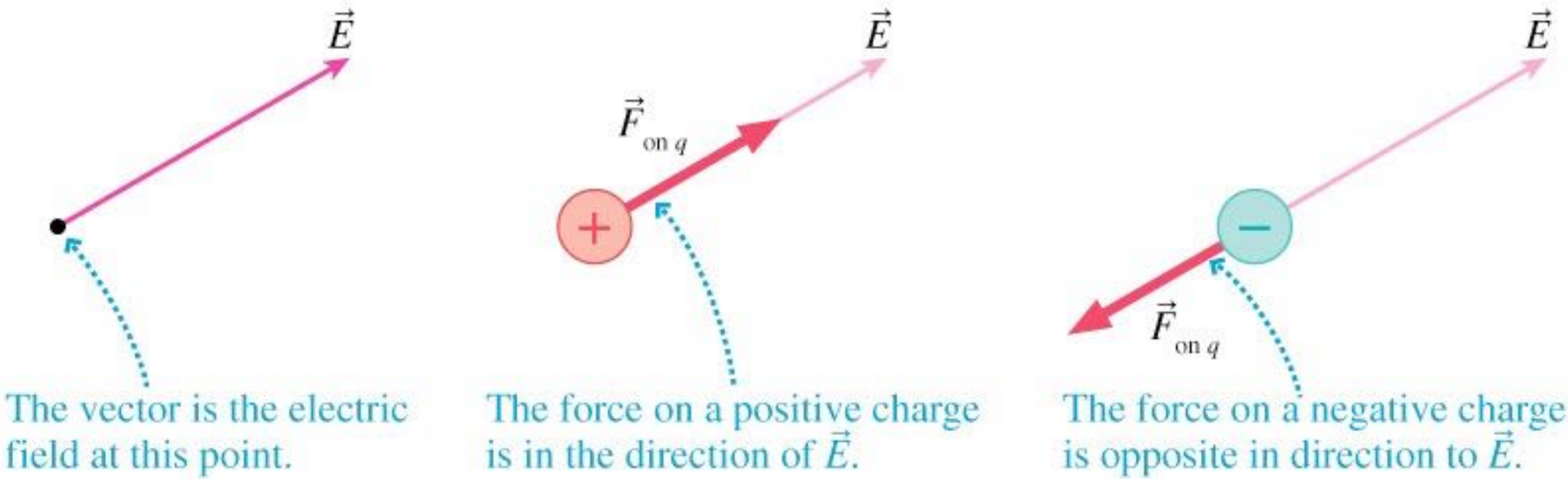



Inside the capacitor, $\vec{E}_+$ and $\vec{E}_-$ are parallel, so the net field is large.
Side view of electrodes
$\vec{E}_+$
$\vec{E}_-$
$\vec{E}_{net} = \vec{0}$
$\vec{E}_+$
$\vec{E}_-$
$\vec{E}_{net}$
$\vec{E}_-$
$\vec{E}_+$
$\vec{E}_{net} = \vec{0}$
d
Outside the capacitor, $\vec{E}_+$ and $\vec{E}_-$ are opposite, so the net field is zero.

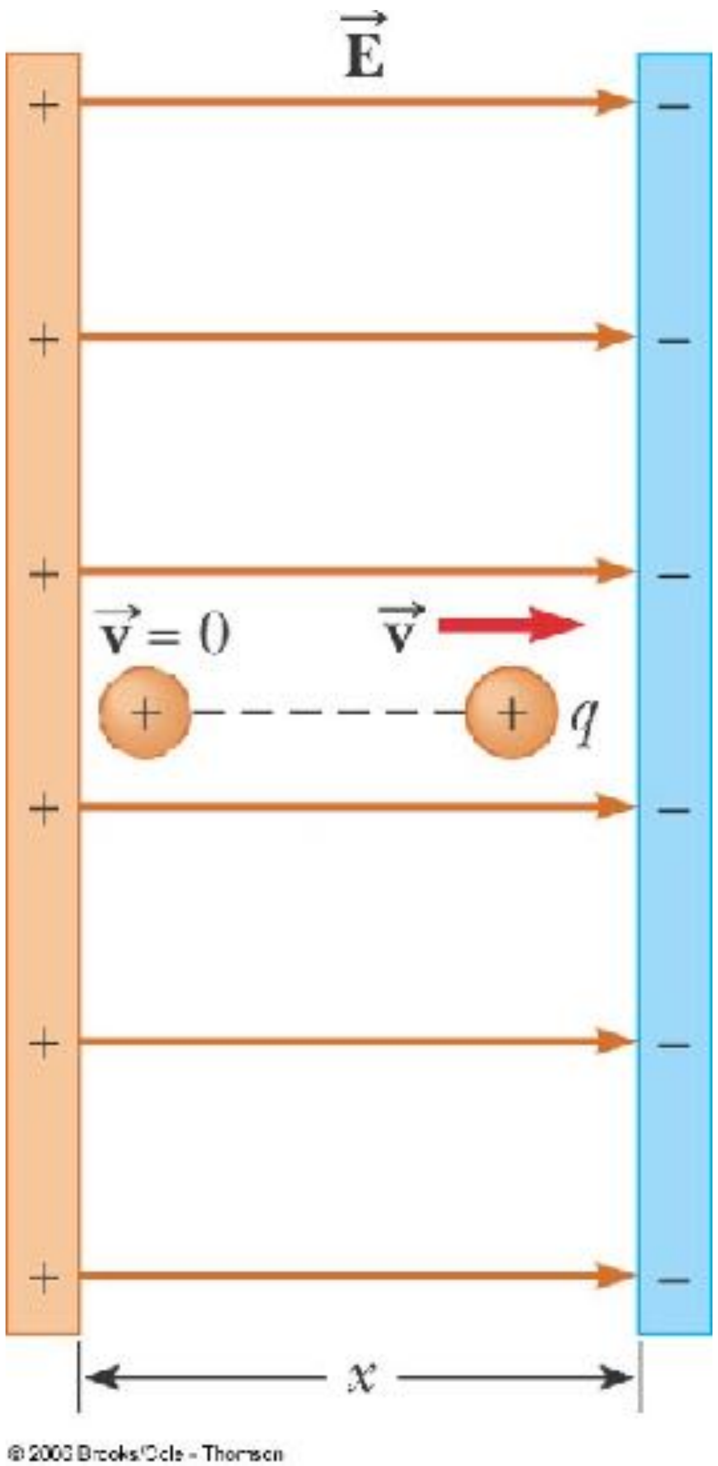

Ideal capacitor
The field is constant, pointing from the positive to the negative electrode.

Forces and Electric Fields

$$\vec{F}_{\text{on } q} = q\vec{E}$$



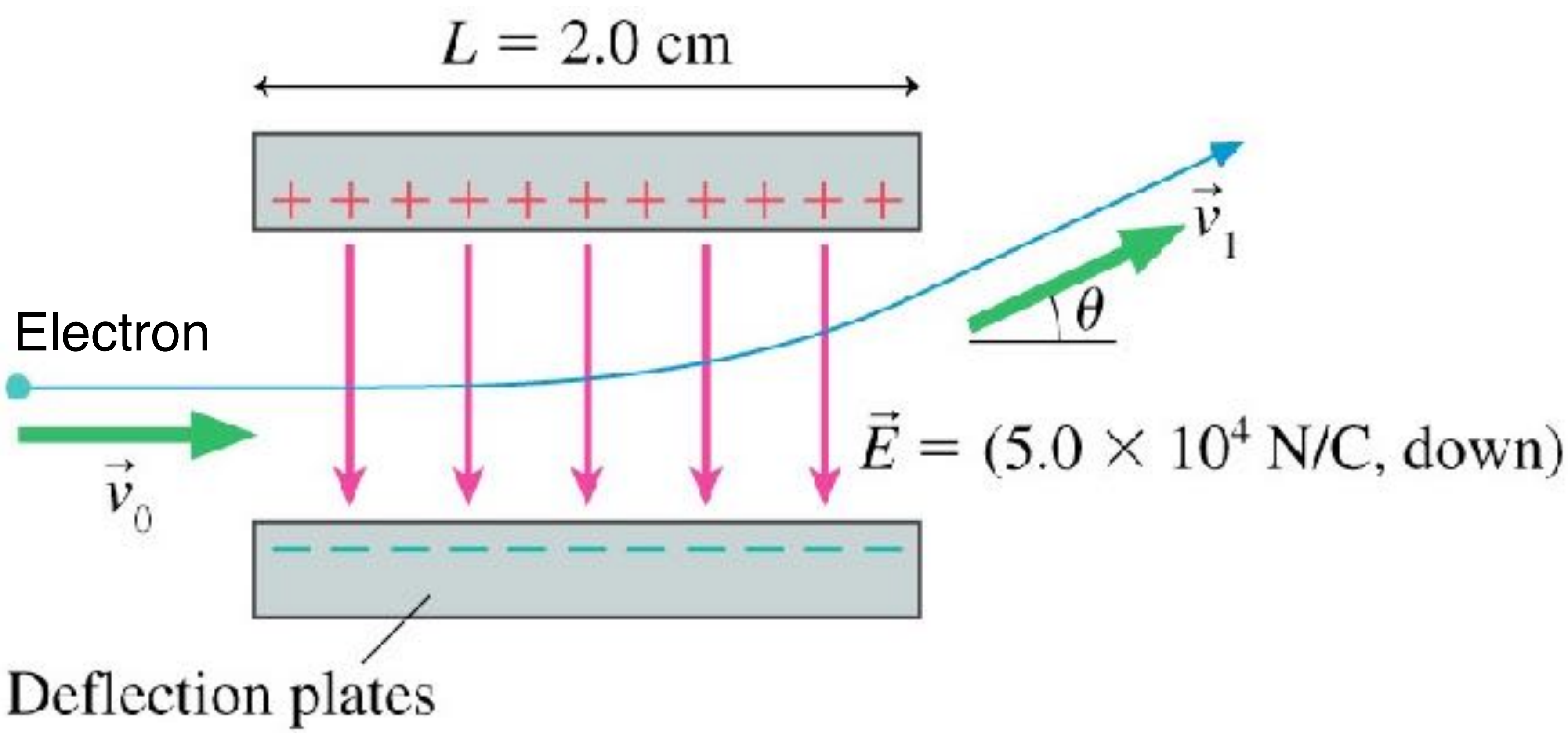
Motion in an Electric Field



$$\vec{F}_{\text{on } q} = q\vec{E} = m\vec{a}$$

$$\vec{a} = \frac{q}{m}\vec{E}$$

If E is constant, then this is just constant acceleration, that we already did in mechanics!



Again, if E is constant, then the electron motion is a parabola, just like ballistic motion.

Recall potential energy and work...

When dealing with *conservative* forces, we had

$\Delta E_{\text{mech}} = \Delta K + \Delta U = 0$

with

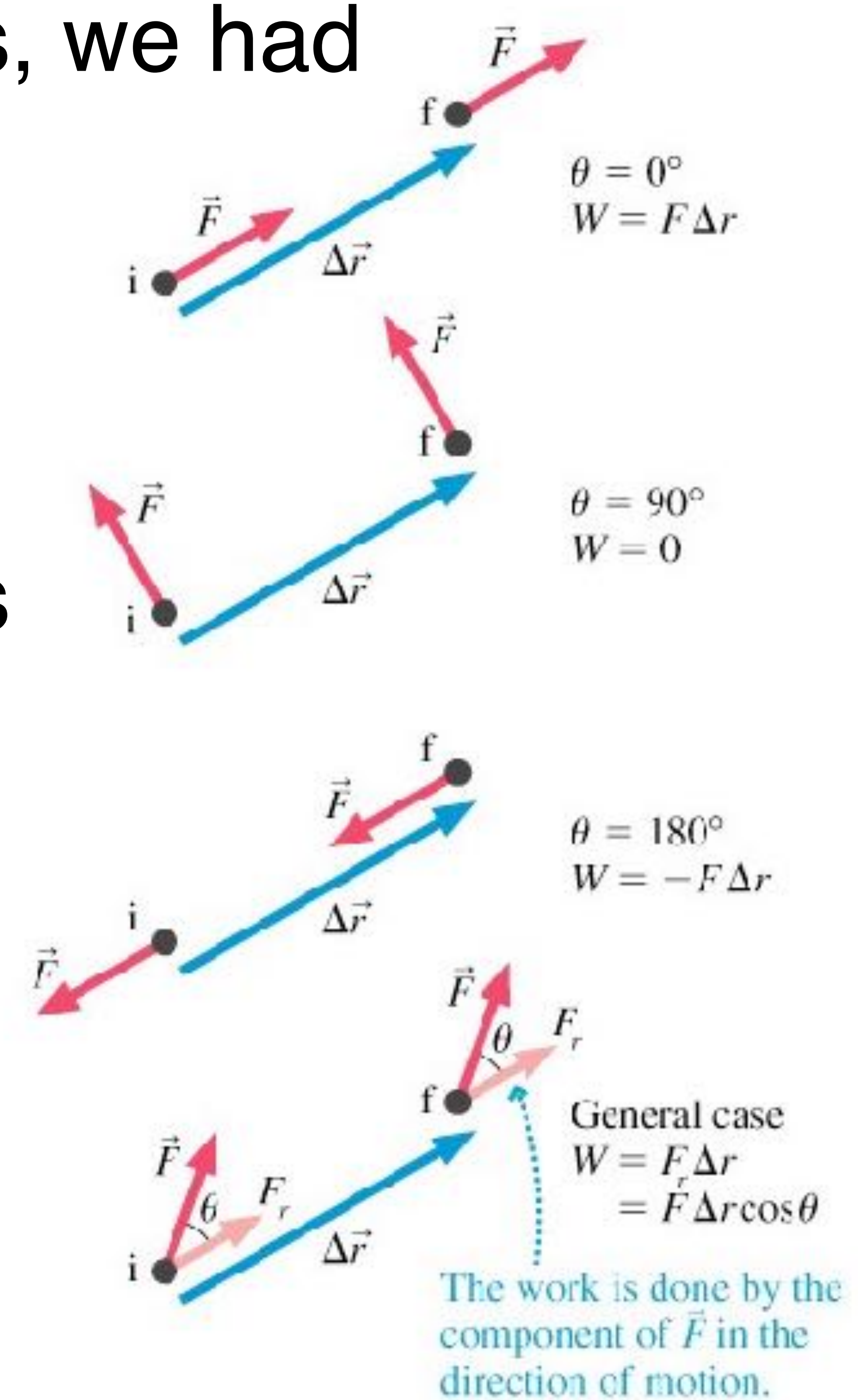
$\Delta U = U_f - U_i = -W_{\text{cons}}$

The work done with a *constant* force is

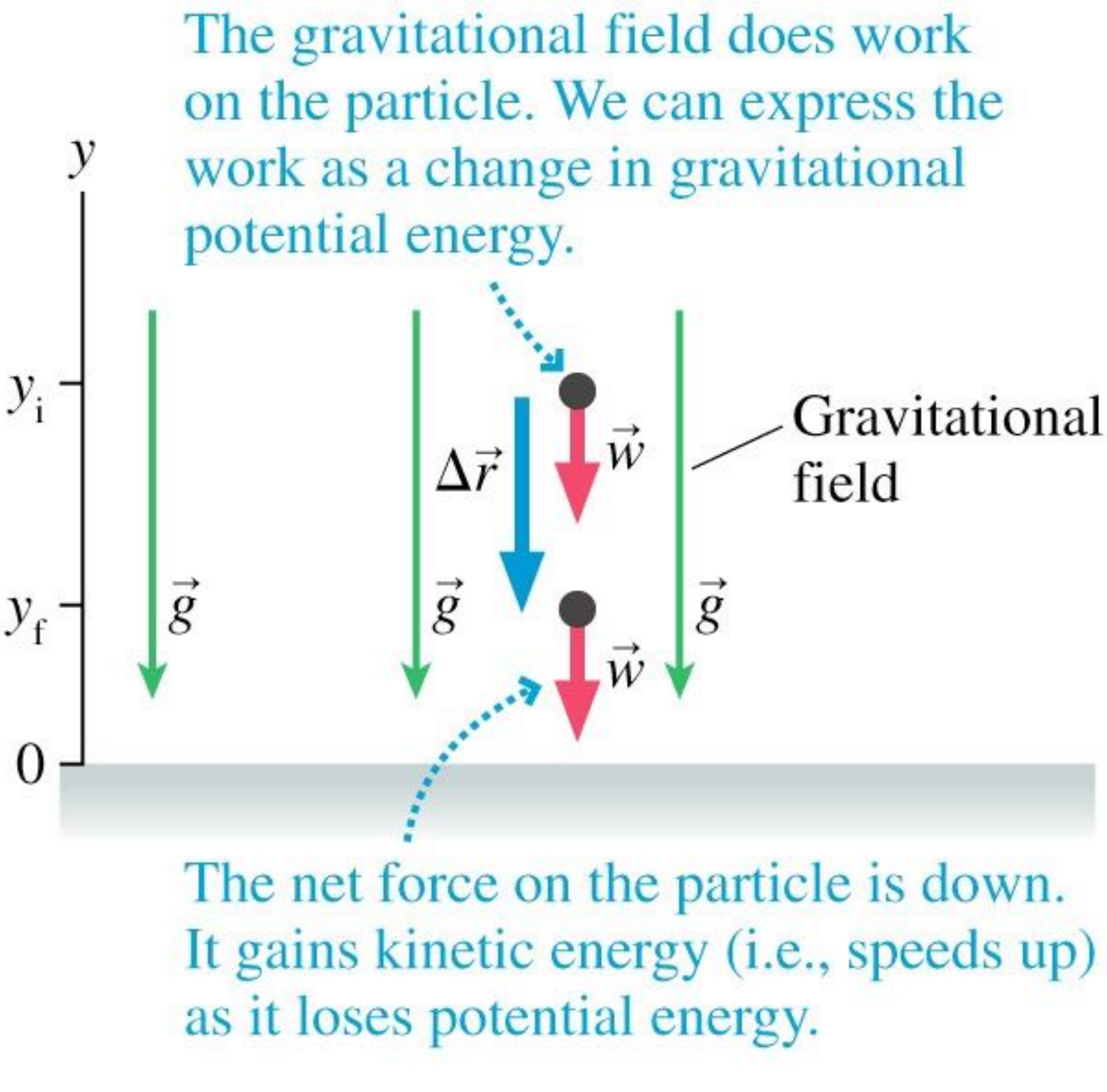
$$W = \vec{F} \cdot \Delta\vec{r} = F\Delta r \cos\theta$$

For a non-constant force, we have to integrate:

$$W = \int_{r_i}^{r_f} \vec{F} \cdot d\vec{r}$$



Uniform Field



$$W_{grav} = w\Delta r \cos 0^o$$
$$= mg\,|y_f - y_i|$$
$$= mgy_i - mgy_f$$

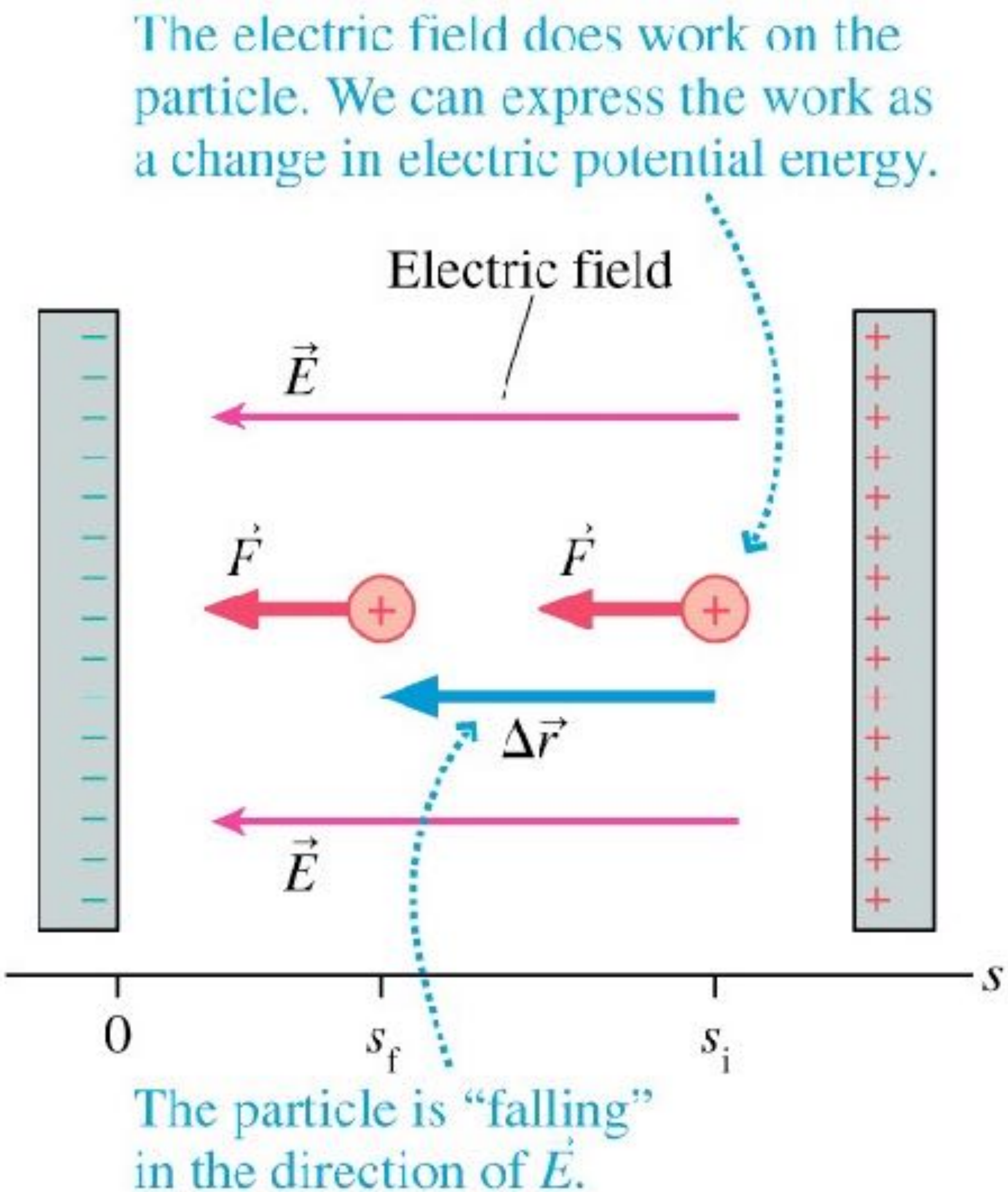
$$\Delta U_{grav} = U_f - U_i = -W_{grav}$$

$$\Delta U_{grav} = mgy_f - mgy_i$$

or

$$U_{grav} = U_0 + mgy$$

Uniform Electric Field



s is the distance from the negative plate!

Gravitation field *g* always points down.
Electric field, *E*, can be in any orientation (depending on orientation of plates), so use generic axis *s.*

$$W_{elec} = F\Delta r \cos 0^o = qE\,|s_f - s_i|$$
$$= qEs_i - qEs_f$$

$$\Delta U_{elec} = qEs_f - qEs_i$$

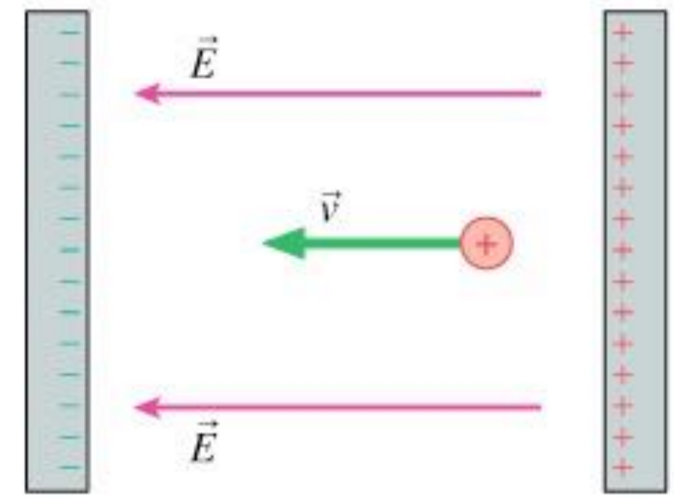
$$U_{elec} = U_0 + qEs$$

This works also for negative *q*!

Electric Potential Energy

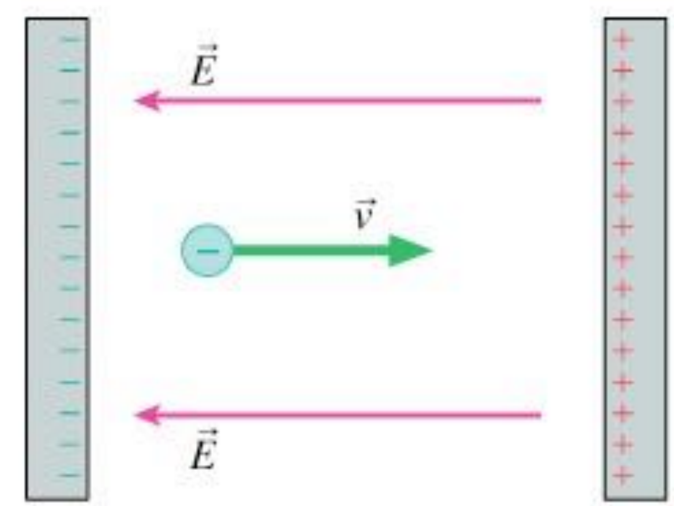
$U_{\text{elec}} = U_0 + qEs$

Potential energy of charge q in a ***uniform*** electric field.



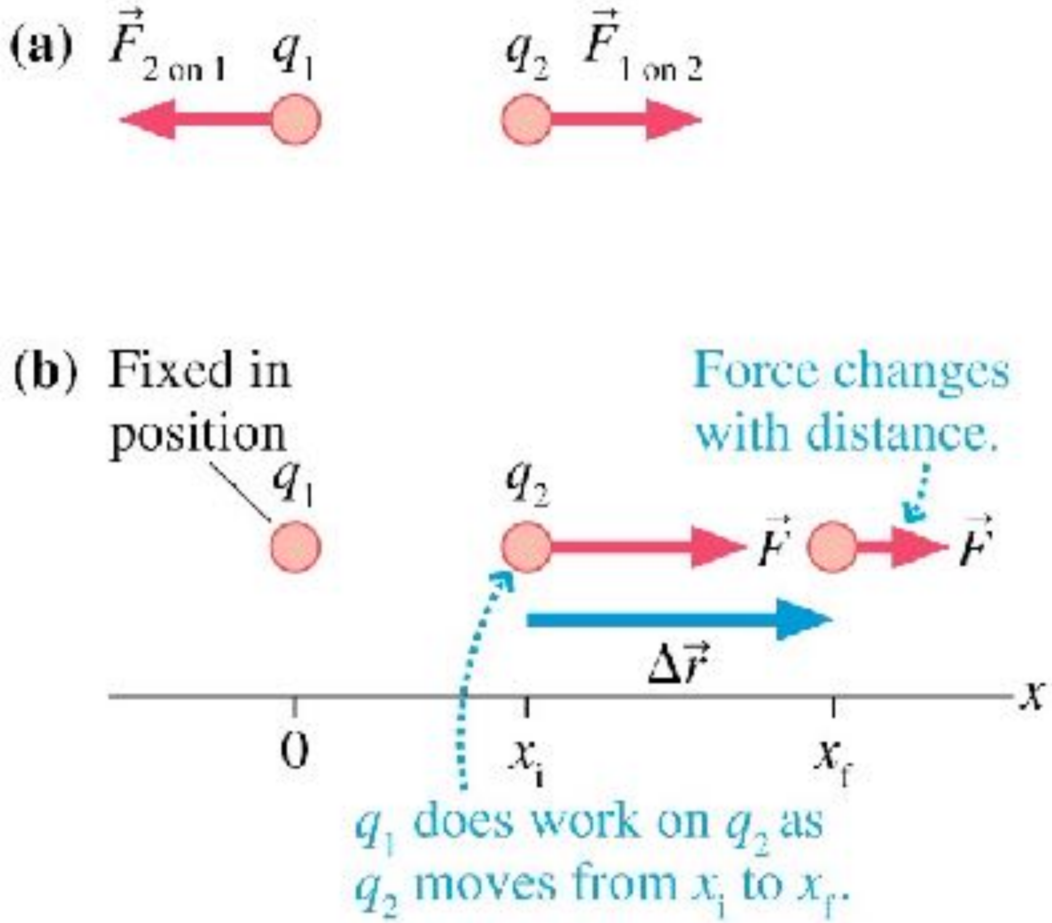
The potential energy of a positive charge decreases in the direction of $\vec{E}$. The charge gains kinetic energy as it moves toward the negative plate.

This works also for negative q!



The potential energy of a negative charge decreases in the direction opposite to $\vec{E}$. The charge gains kinetic energy as it moves away from the negative plate.

Now for point charges



$$W_{\text{elec}} = \int_{x_i}^{x_f} F_{1\text{ on }2}\,dx = \int_{x_i}^{x_f} k_e \frac{q_1 q_2}{x^2}\,dx$$

$$= k_e q_1 q_2 \frac{-1}{x}\bigg|_{x_i}^{x_f} = -\frac{k_e q_1 q_2}{x_f} + \frac{k_e q_1 q_2}{x_i}$$

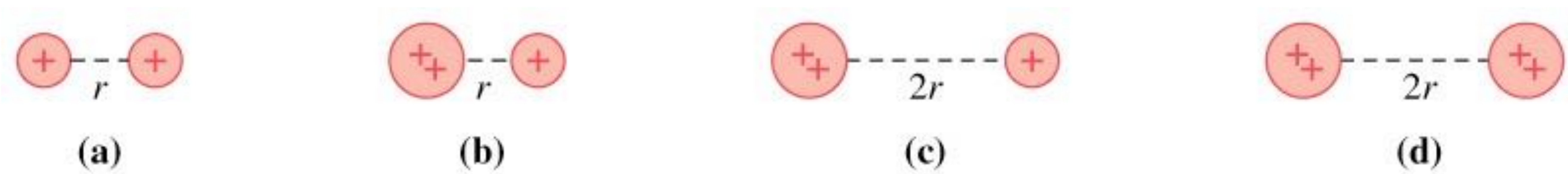
$$\Delta U_{\text{elec}} = U_f - U_i = -W_{\text{elec}}(i \rightarrow f) = \frac{k_e q_1 q_2}{x_f} - \frac{k_e q_1 q_2}{x_i}$$

$$U_{\text{elec}} = \frac{k_e q_1 q_2}{x}$$

What's really important is the distance, so $\boxed{U_{\text{elec}} = \frac{k_e q_1 q_2}{r} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r}}$

Discussion Question

Rank in order, from largest to smallest, the potential energies U_a to U_d of the four pairs of charges. Each + sign represents the same amount of charge.



A. $U_a > U_b > U_c > U_d$

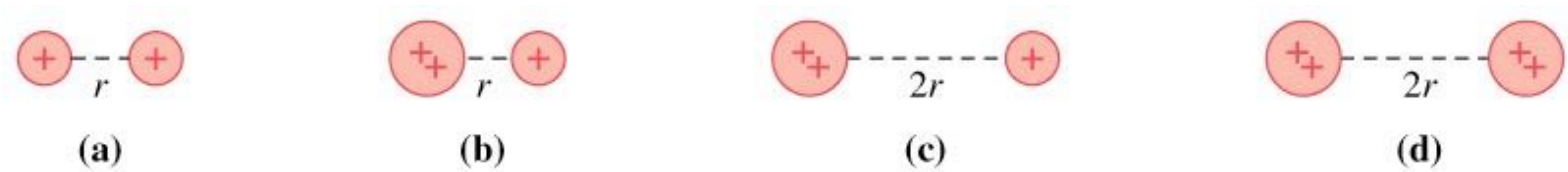
B. $U_b = U_d > U_a = U_c$

C. $U_d > U_b > U_c > U_a$

D. $U_b > U_a > U_d > U_c$

Discussion Question

Rank in order, from largest to smallest, the potential energies U_a to U_d of the four pairs of charges. Each + sign represents the same amount of charge.



A. $U_a > U_b > U_c > U_d$

B. $U_b = U_d > U_a = U_c$

C. $U_d > U_b > U_c > U_a$

D. $U_b > U_a > U_d > U_c$

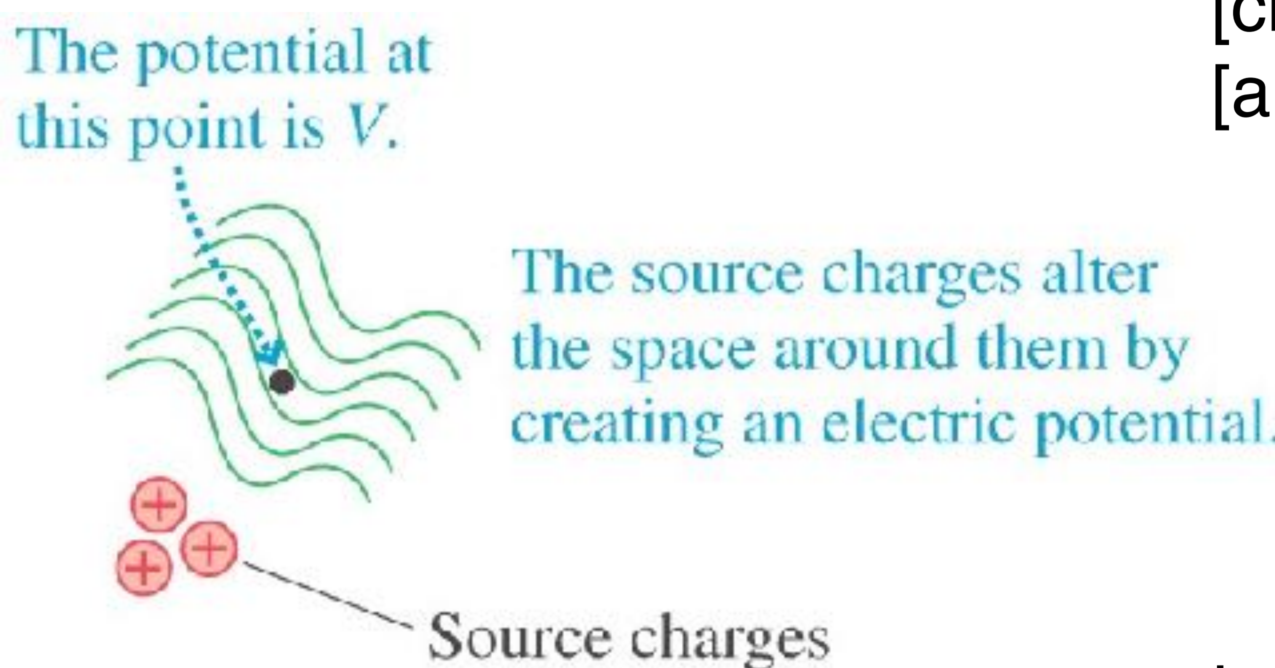
Electric Potential

The concept of the field was useful because of problems with action at a distant (how does one charge know that a distant one has moved?).
A charge somehow alters the space around it by creating an electric field. The second charge then interacts with that field: $\vec{F} = q_2\vec{E}$.

We have the same difficulties understanding how electric potential energy changes.
For a mass on a spring, we can see how the energy is stored in the compressed string.

But where is the potential energy stored when two charges fly apart, converting that potential energy into kinetic?

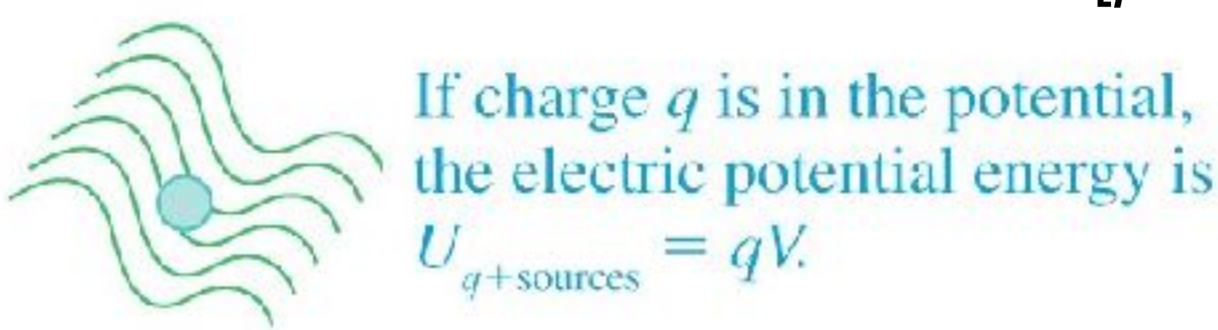
Electric Potential



force on q =
[charge q] X
[alteration of space by the source charges]

$$\vec{F}_{\text{on } q} = q\vec{E}$$

potential energy of q + sources =
[charge q] X
[*potential* for interaction of the source charges]



$$U_{q+\text{sources}} = qV$$

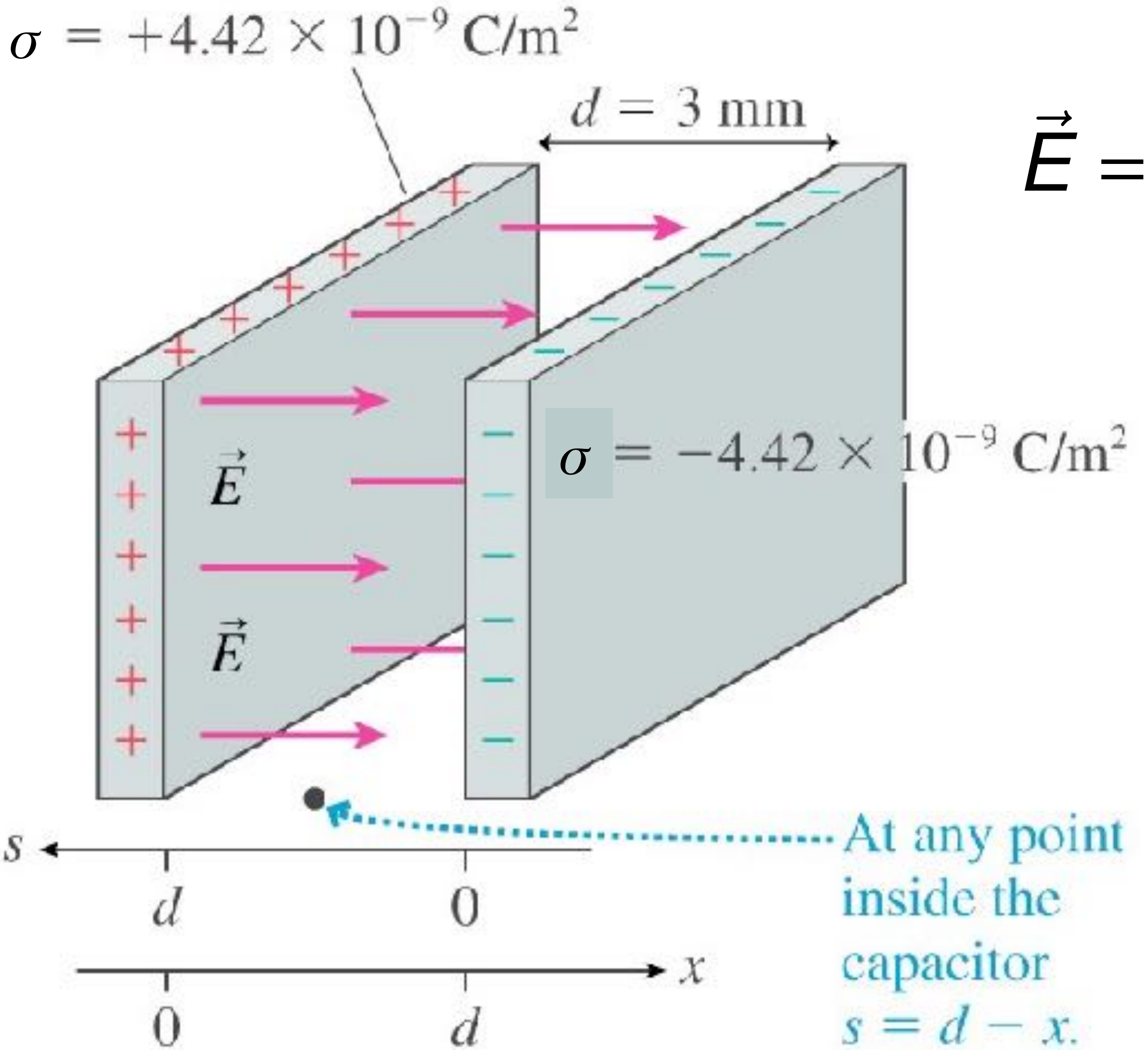
$$\boxed{V \equiv \frac{U_{q+\text{sources}}}{q}}$$

Why bother?

- The electric potential depends only on the source charges and their geometry. The potential is the ‘ability’ of the source charges to have an interaction if a charge q shows up. **The potential is present throughout space, even if charge *q* is not there.**
- If we know the potential, we immediately know the potential energy $U = qV$ of any charge that enters the area.

NB: *potential* and *potential energy* sound very much alike, so it is very easy to confuse the two. They are **not** interchangeable!

Potential of a uniform field



$$\vec{E} = \left(\frac{\sigma}{\varepsilon_0},\ \text{from positive to negative}\right)$$

$$= (500\ \text{N/C, left to right})$$

Earlier we had $U = qEs$ (setting $U_0 = 0$)

Thus $V = Es$

$V_- = 0\text{V}$

$V_+ = (500\text{N/C})*0.003\text{m} = 1.5\text{V}$

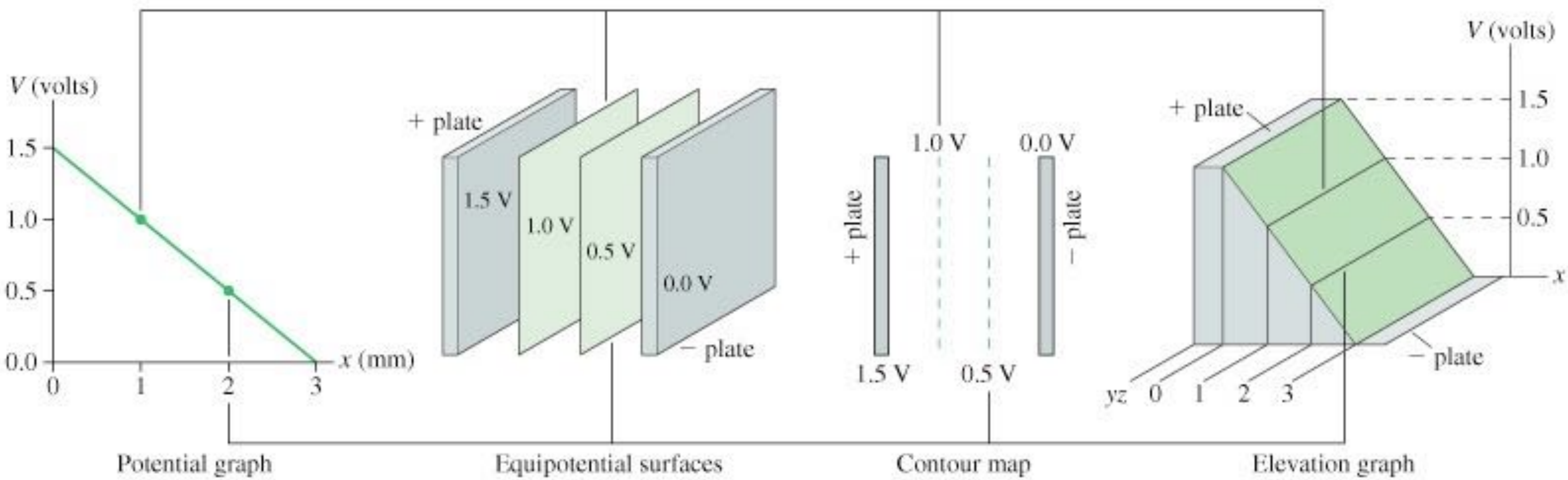
$$\Delta V_C = V_+ - V_- = Ed$$

$$V = Es = \frac{\Delta V_C}{d}(d - x) = \left(1 - \frac{x}{d}\right)\Delta V_C$$

What does it look like?

$$V = Es = \frac{\Delta V_C}{d}(d - x) = \left(1 - \frac{x}{d}\right)\Delta V_C$$

Four different, useful, ways of visualizing the potential:



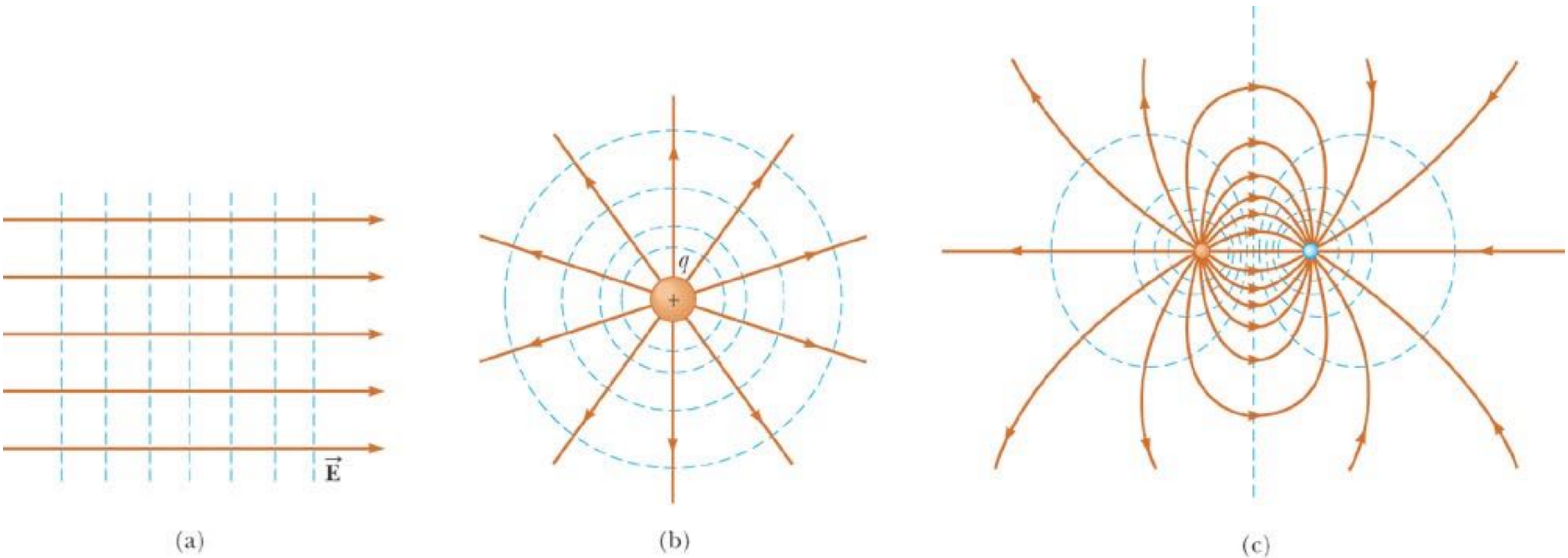


Figure 20.8

Potential of a point charge

We saw already that the potential energy of two point charges is

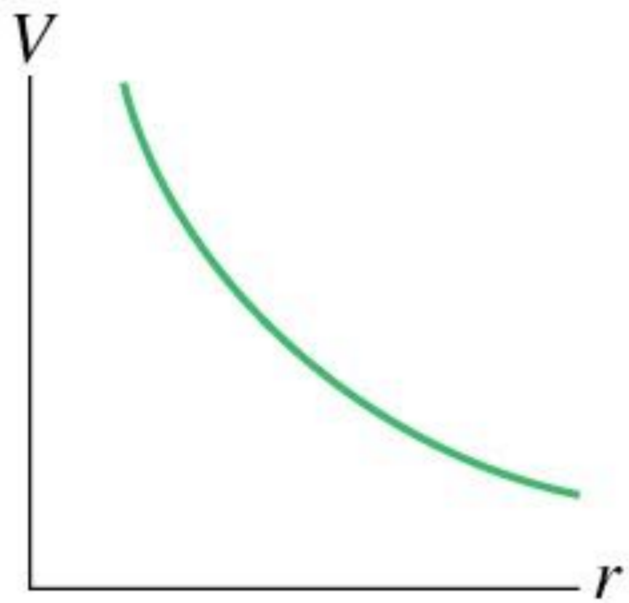
$$U_{q+q'} = \frac{1}{4\pi\varepsilon_0}\frac{qq'}{r}$$

thus, by definition, the electric potential of charge q is

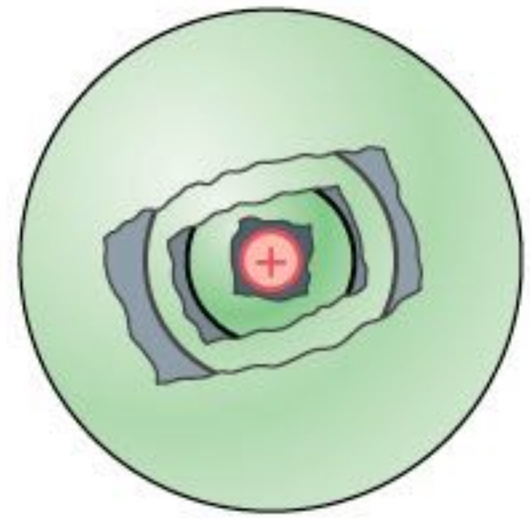
$$V = \frac{U_{q+q'}}{q'} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}$$

The potential extends through all of space, but diminishes with distance like $1/r$. We have chosen $U = 0$ and thus $V = 0$ at $r = \infty$, which makes sense, since that is where the effect of the charge ends.

Visualizing the potential

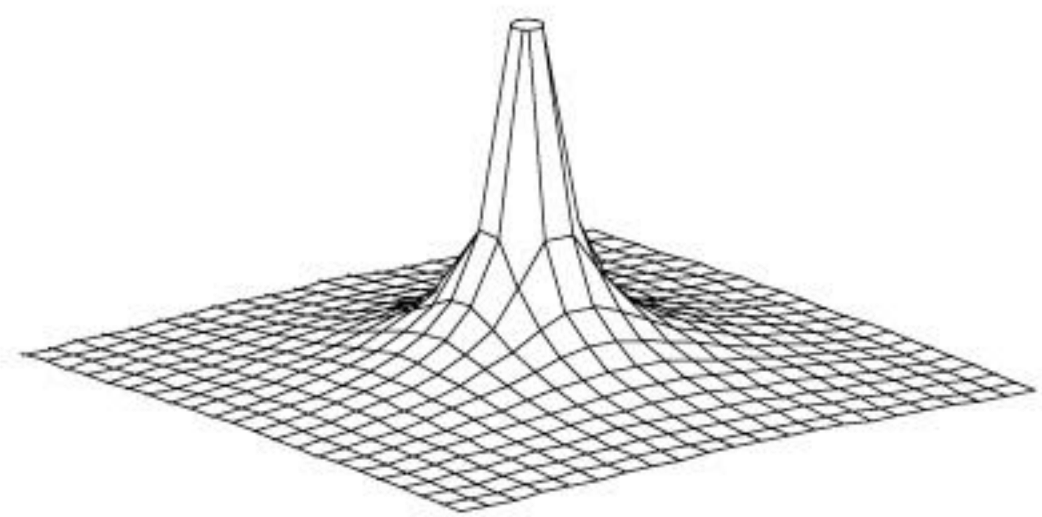


Potential graph



Equipotential surfaces

Contour map



Elevation graph

Potential of a charge distribution

For a continuous charge distribution:

Divide the total charge into many small point-like charges ΔQ.

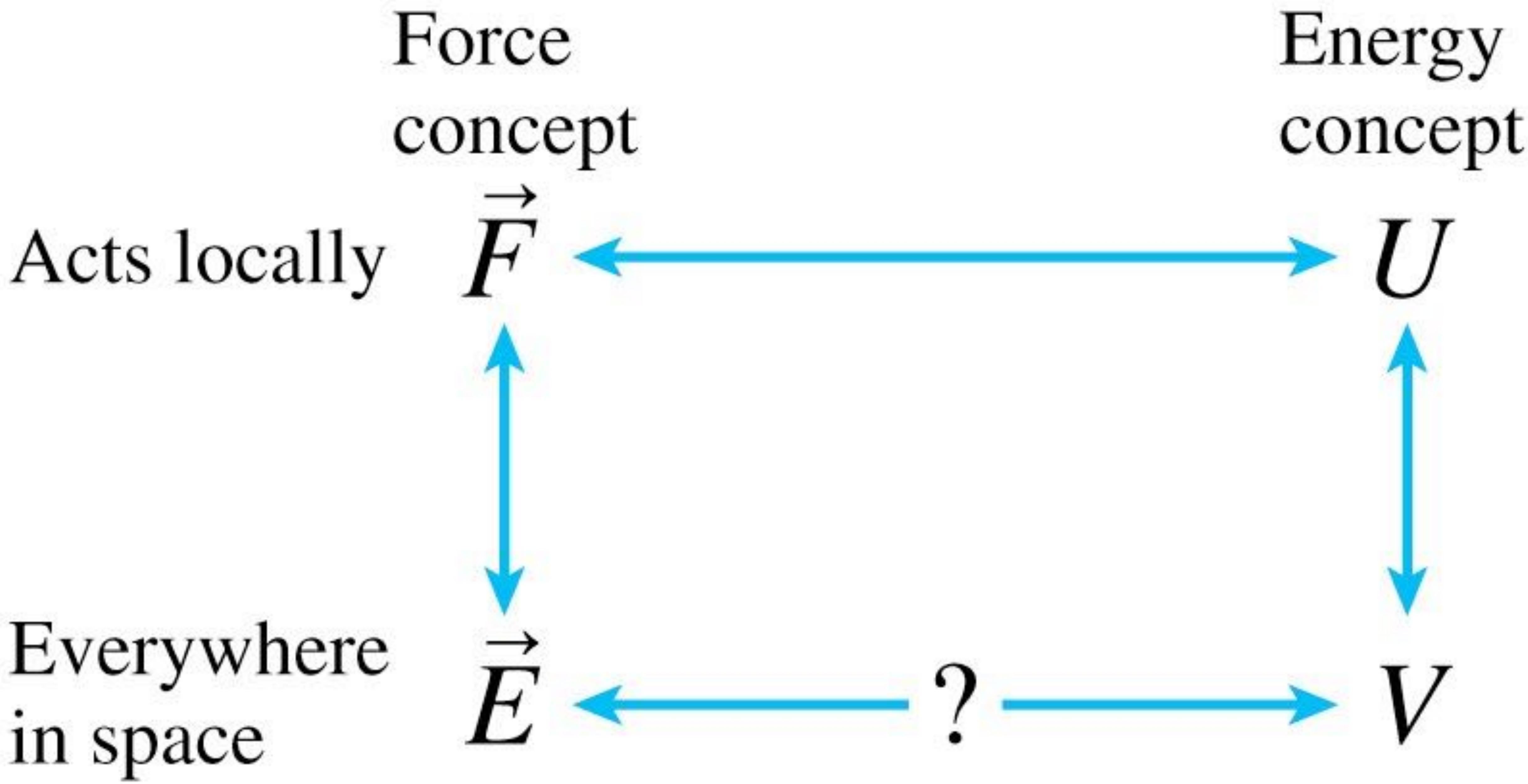
Use our knowledge of the potential of a point charge to find the potential of each ΔQ.

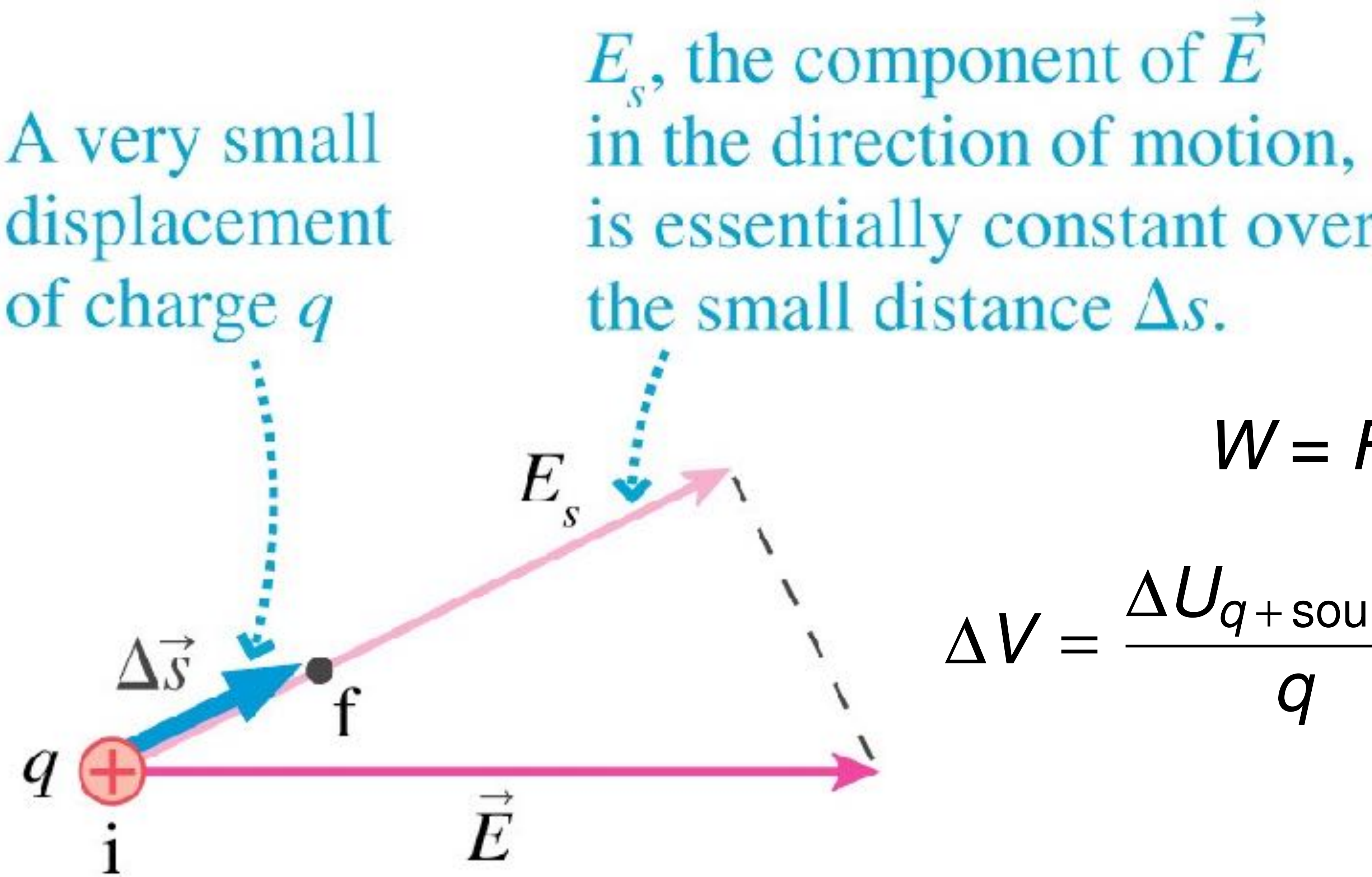
Calculate the net potential by summing the potentials of all the ΔQ.

Let the sum become an integral.

Looks pretty much the same as what we did for the electric field! The **big** advantage: the potential is a scalar, whereas the field is a nasty vector. Scalar addition is much easier than vector addition!

Connecting potential and field



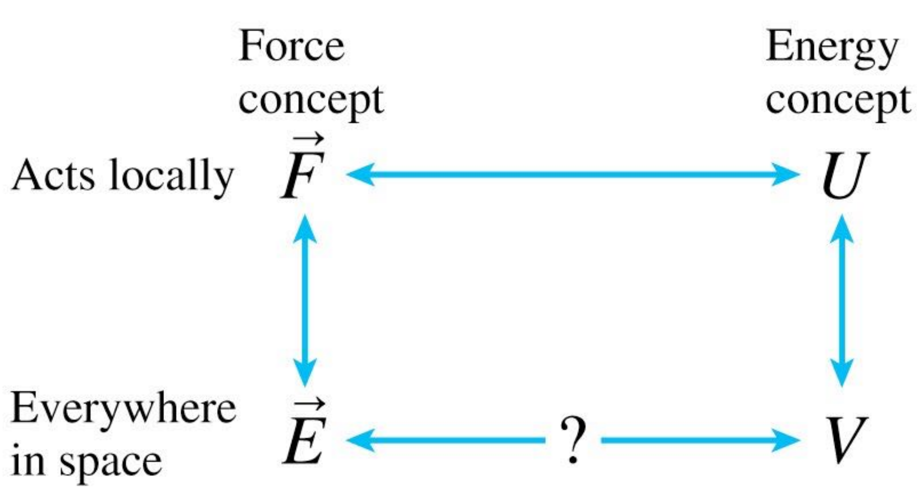


$$W = F\Delta s = qE_s\Delta s$$

$$\Delta V = \frac{\Delta U_{q\,+\,\text{sources}}}{q} = \frac{-W}{q} = -E_s\Delta s$$

$$E_s = -\frac{\Delta V}{\Delta s}$$

$$\boxed{E_s = -\frac{dV}{ds}}$$



Remember: $F_s = -\dfrac{dU}{ds}$

Let's try it!

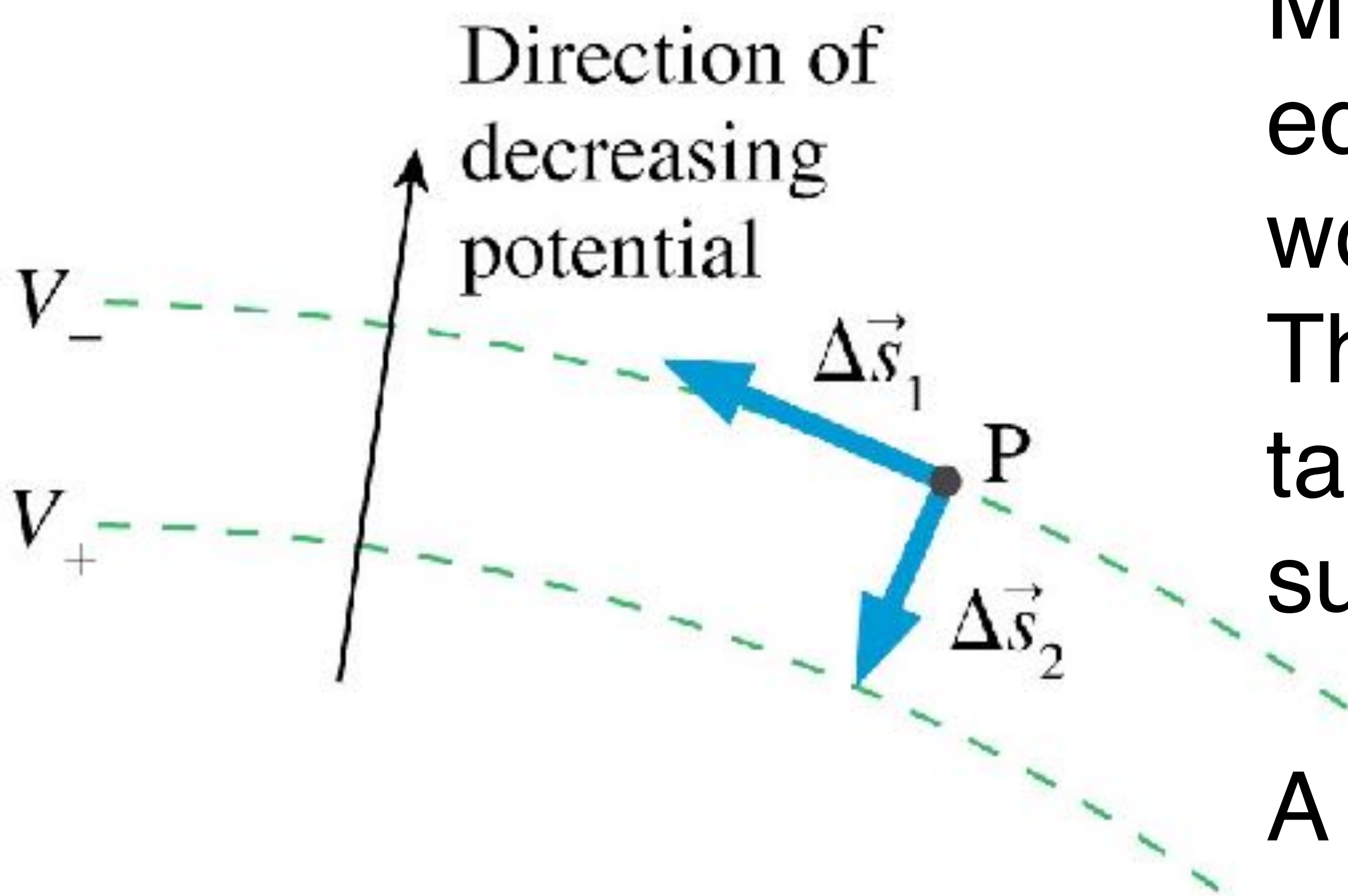
Potential for a point charge: $V = \dfrac{1}{4\pi\varepsilon_0}\dfrac{q}{r}$

Make the *s* axis the radial axis to get:

$$E = E_r = -\frac{dV}{dr} = -\frac{d}{dr}\left(\frac{1}{4\pi\varepsilon_0}\frac{q}{r}\right) = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2} \quad ✔$$

The real utility is in calculating the potential of a continuous charge distribution. The potential is a scalar, so this is easier than calculating the field. Once you have the potential, you can get the field by just taking a derivative.

Connecting potential and field

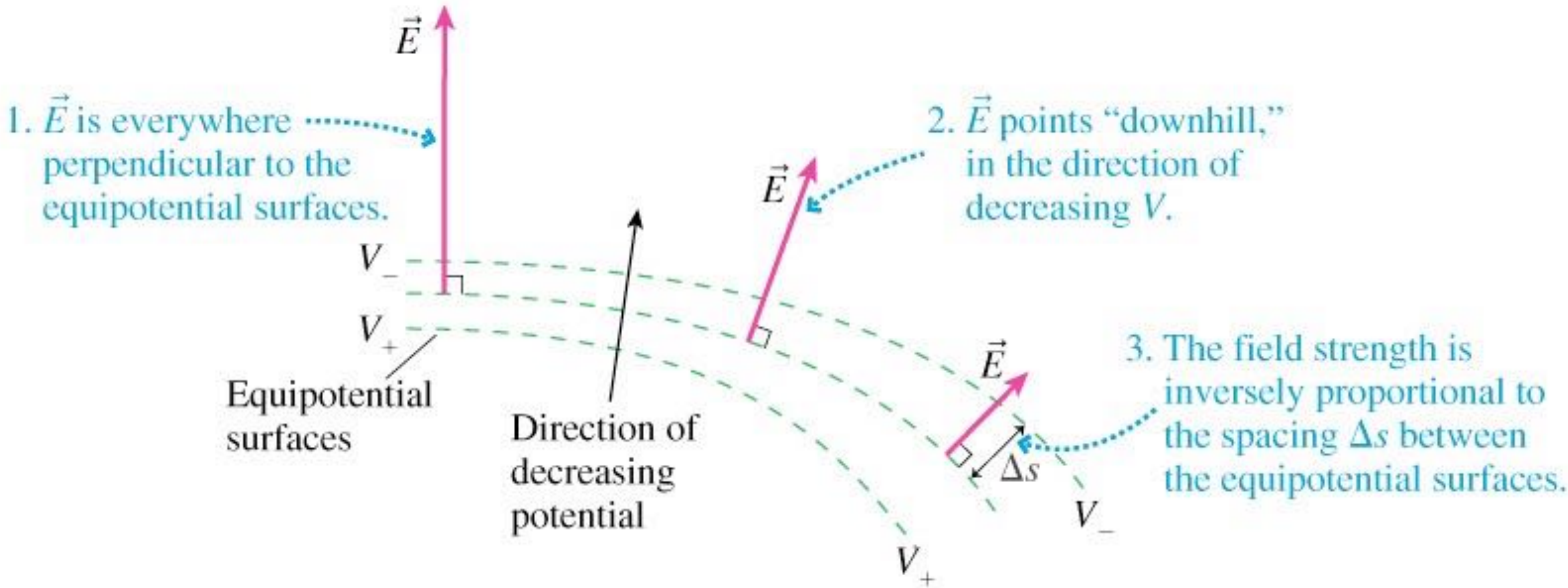


Moving along Δs_1 is along the equipotential surface. A charge would see no change in potential. Thus $E_{s1} = 0$. There is ***no*** field tangent to an equipotential surface.

A displacement along Δs_2 **does** see a potential difference:

$$E_\perp = -\frac{dV}{ds} \approx -\frac{\Delta V}{\Delta s} = -\frac{V_+ - V_-}{\Delta s}$$

The field is inversely proportional to Δs and points in the direction of *decreasing* potential.

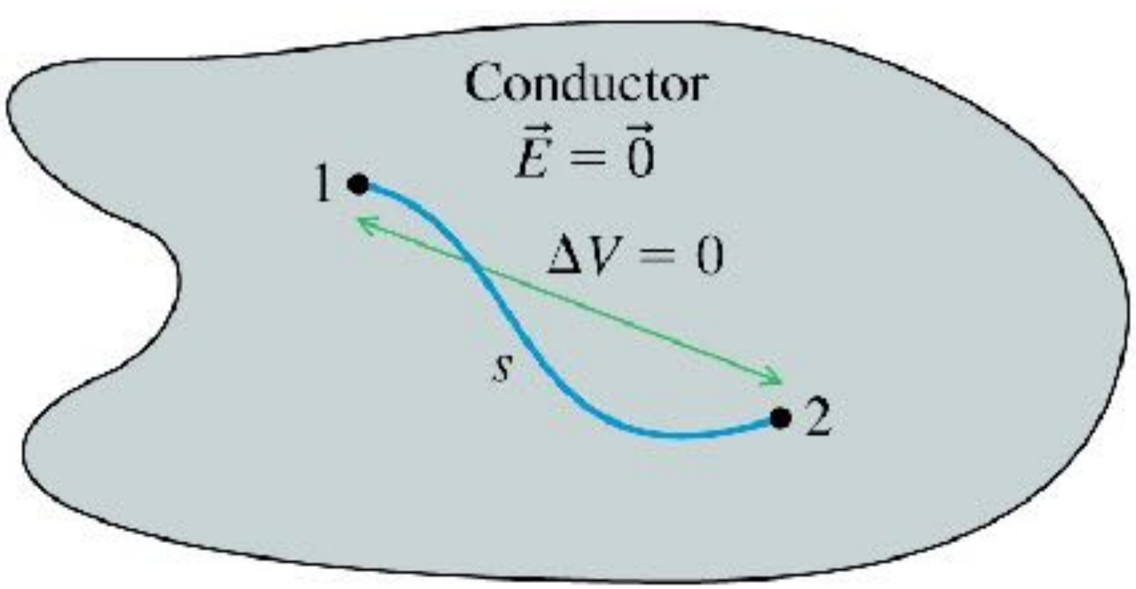

$\vec{E}$
1. $\vec{E}$ is everywhere perpendicular to the equipotential surfaces.
2. $\vec{E}$ points "downhill," in the direction of decreasing V.
$\vec{E}$
V_-
V_+
Equipotential surfaces
Direction of decreasing potential
$\vec{E}$
3. The field strength is inversely proportional to the spacing Δs between the equipotential surfaces.
Δs
V_-
V_+

Conductor in Electrostatic Equilibrium

Any excess charge in a conductor in electrostatic equilibrium will always be at the surface.

Why? If there were an excess electron in the interior, the nearby electrons would feel a force and move, upsetting the equilibrium.

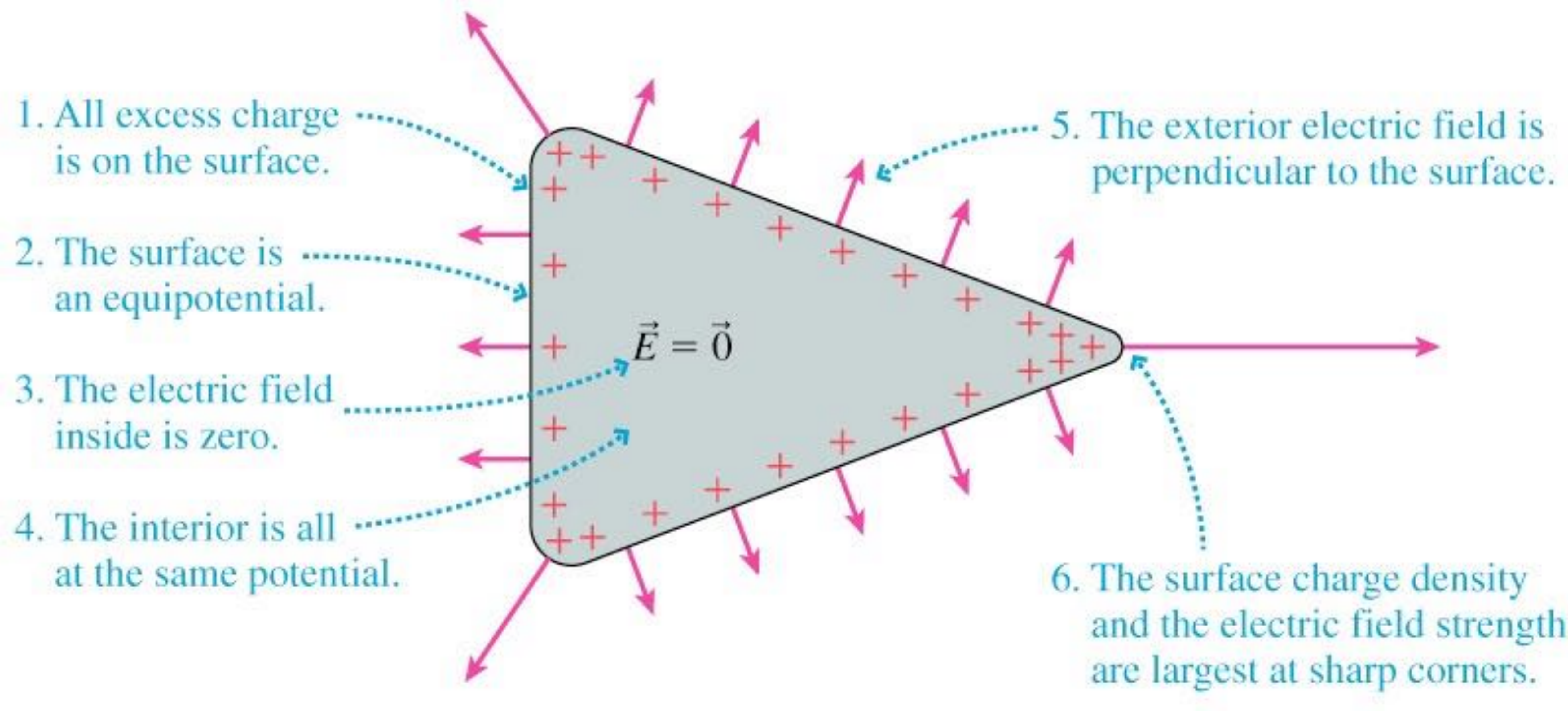
Similarly, the net electric field inside a conductor must be zero. If not, there would be a force $F = qE$, causing the electrons to move and creating a current.



Thus, the potential inside a conductor must be everywhere the same.

The whole conductor is at the same potential.

Summary of field of a conductor



Remember: this applies for a **conductor** in *electrostatic equilibrium*.

Discussion Question

Which set of equipotential surfaces matches this electric field?

A

B

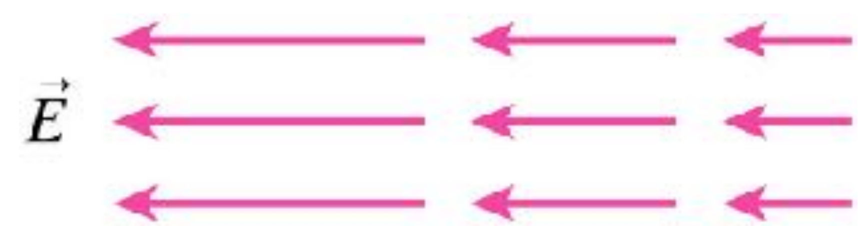
C

D

E

Discussion Question

Which set of equipotential surfaces matches this electric field?



A

B

C

D

E

Capacitors

We saw last week that the potential difference between two plates of a capacitor is related to the electric field:

$$\Delta V_{\mathrm{C}} = Ed$$

In addition, we know what the electric field is:

$$E = \frac{\sigma}{\varepsilon_0} = \frac{Q}{\varepsilon_0 A}$$

And thus,

$$Q = \frac{\varepsilon_0 A}{d} \Delta V_{\mathrm{C}}$$

The charge on the capacitor plates is directly proportional to the potential difference between the plates.

Capacitance

The ratio of Q to ΔV_C is called the ***capacitance***, C:

$$C = \frac{Q}{\Delta V_C}$$

For a parallel plate capacitor,

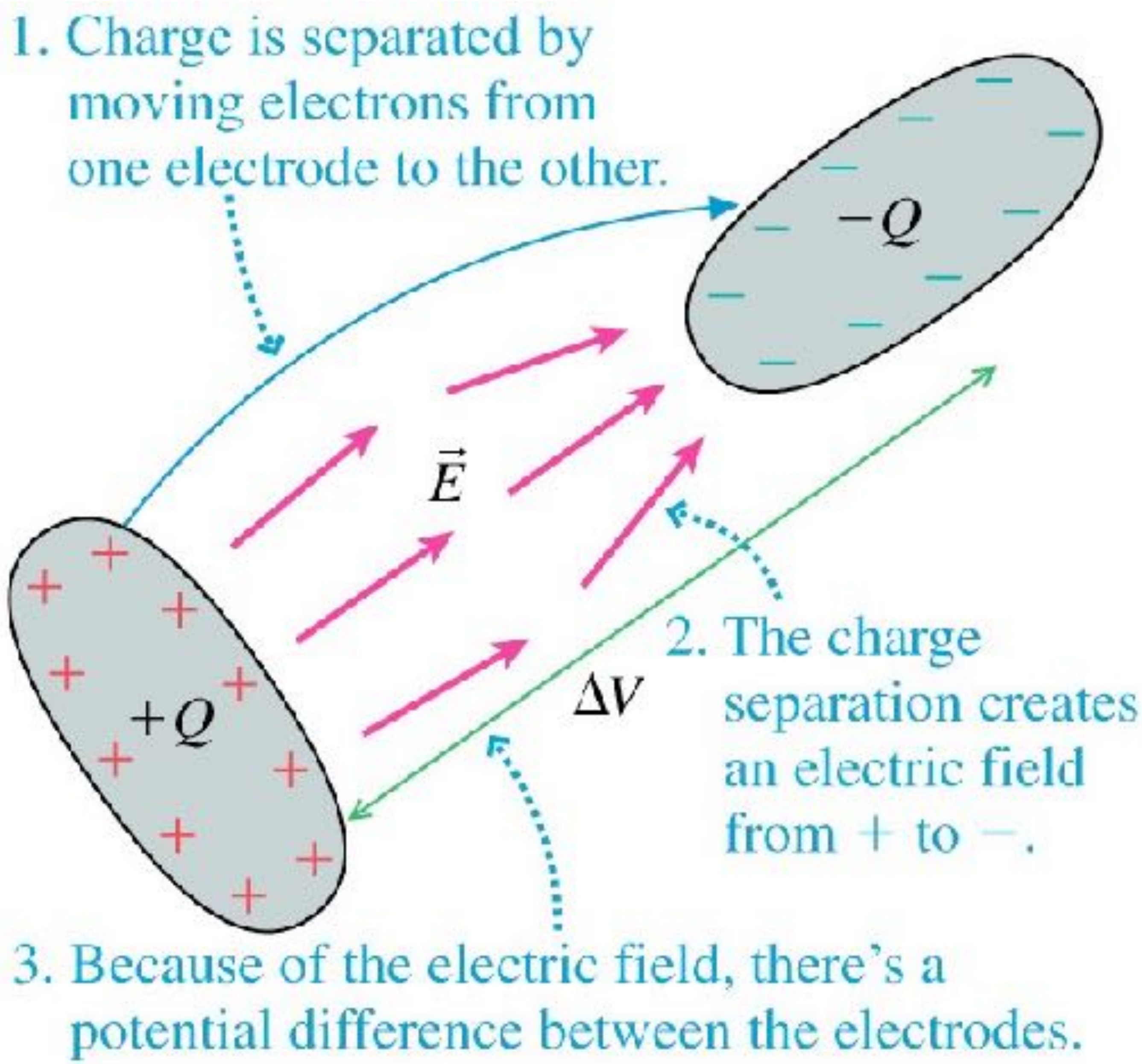
$$C = \frac{Q}{\Delta V_C} = \frac{\varepsilon_0 A}{d}$$

In general, capacitance depends only on the geometry of the electrodes of the capacitor.

The unit of capacitance is the **farad**. 1 F = 1C/V

The farad is a very large unit. Practical capacitors are measured in microfarads (μF) or picofarads (pF = 10^{-12} F).

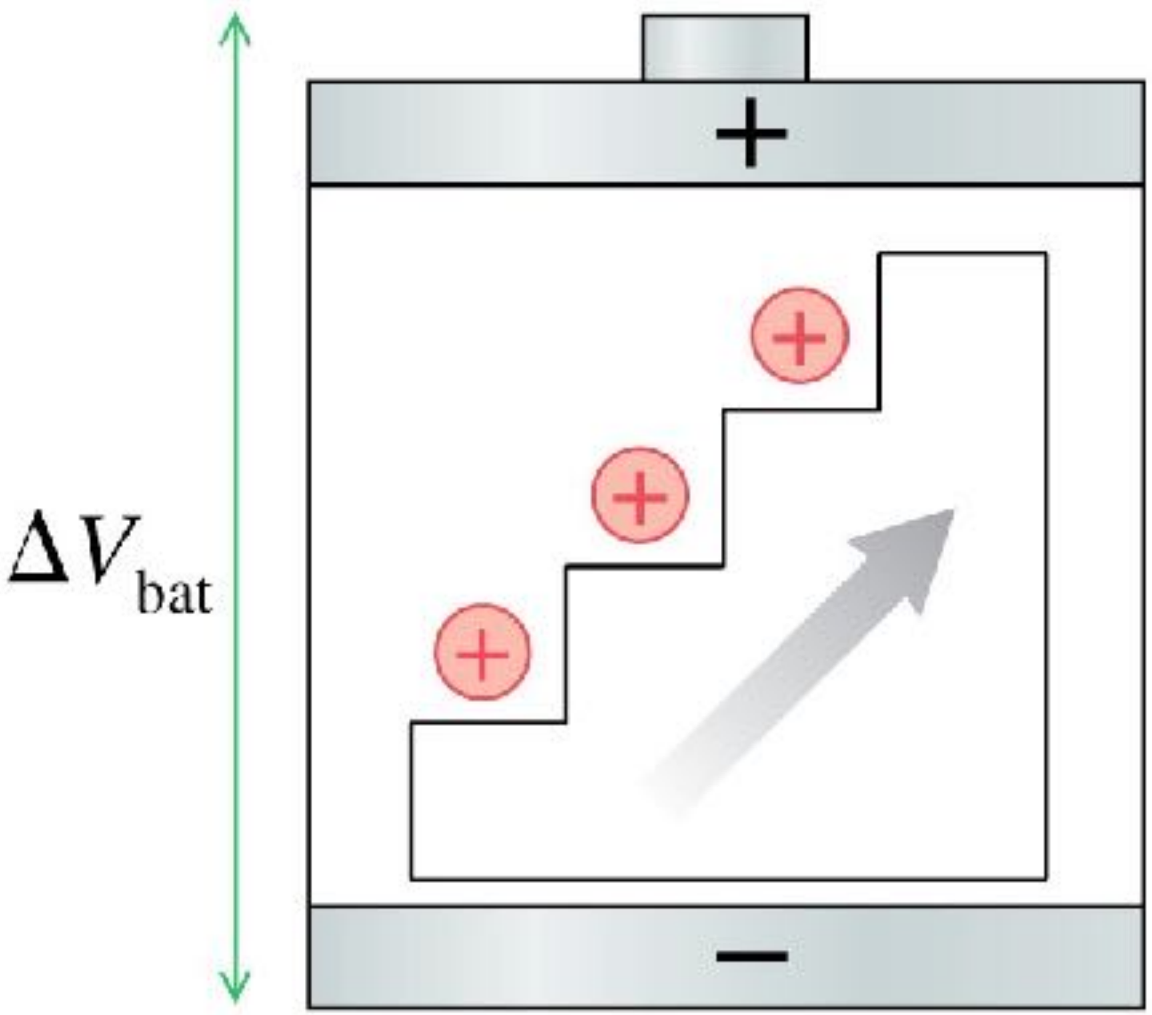
Potential differences



A ***separation of charge*** will produce a ***potential difference***.

One example is shuffling your feet on a carpet. You build up a large potential difference until it is discharged by touching a doorknob.

Batteries

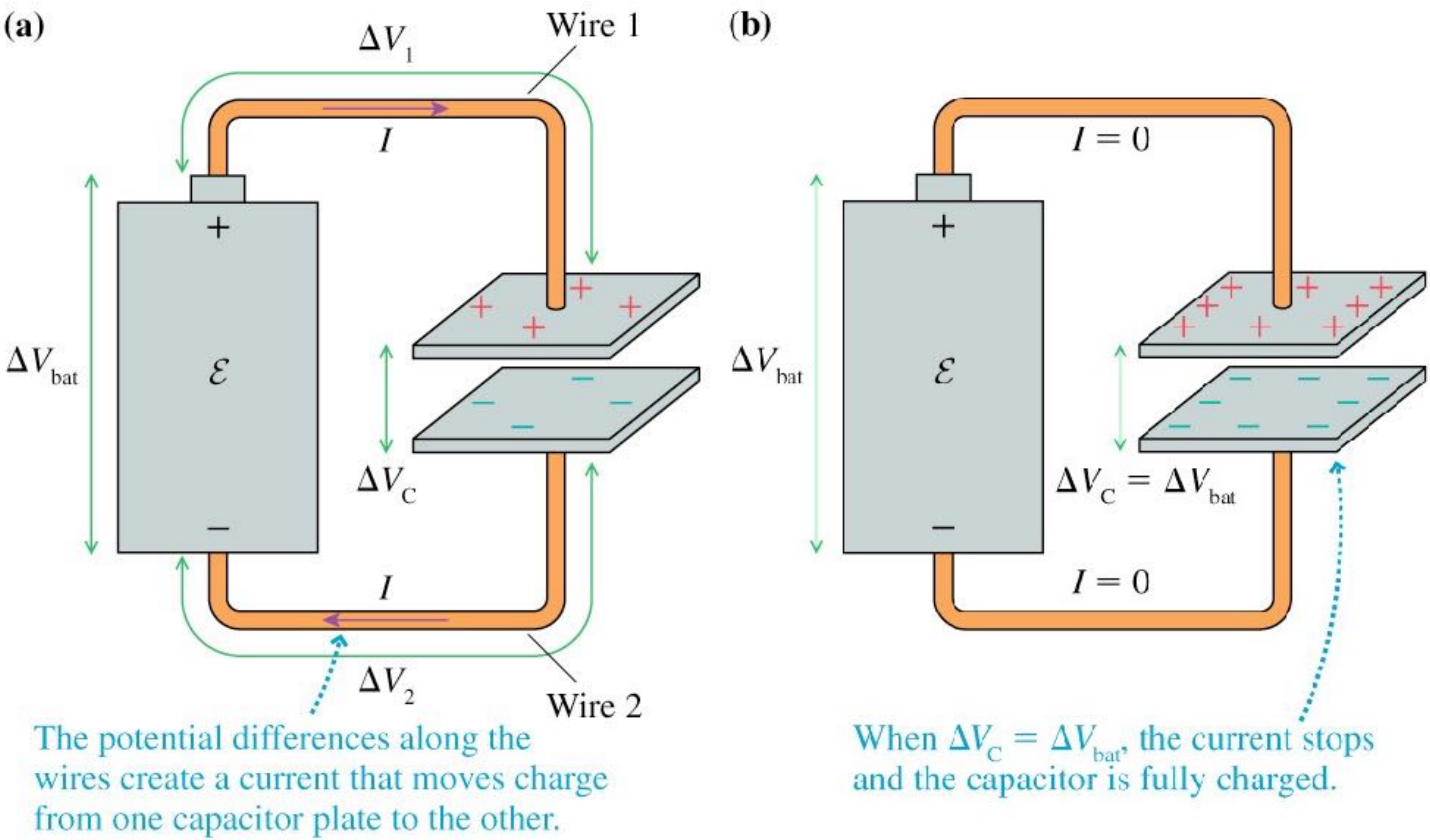


A common source of a potential difference is a battery. In a battery, the charge separation is caused by a chemical reaction.

It can be viewed as a ‘charge escalator’, moving positive ions “up” to the positive terminal. This requires work, and that work is provided by the chemicals in the battery. When the chemicals are used up, the escalator stops, and the battery is dead.

By separating charge, the battery produces a potential difference, ΔV_{bat}. The specific value of ΔV_{bat} depends on the chemicals used.

Charging a capacitor



Capacitors

We saw last time

$$C = \frac{Q}{\Delta V_C}$$

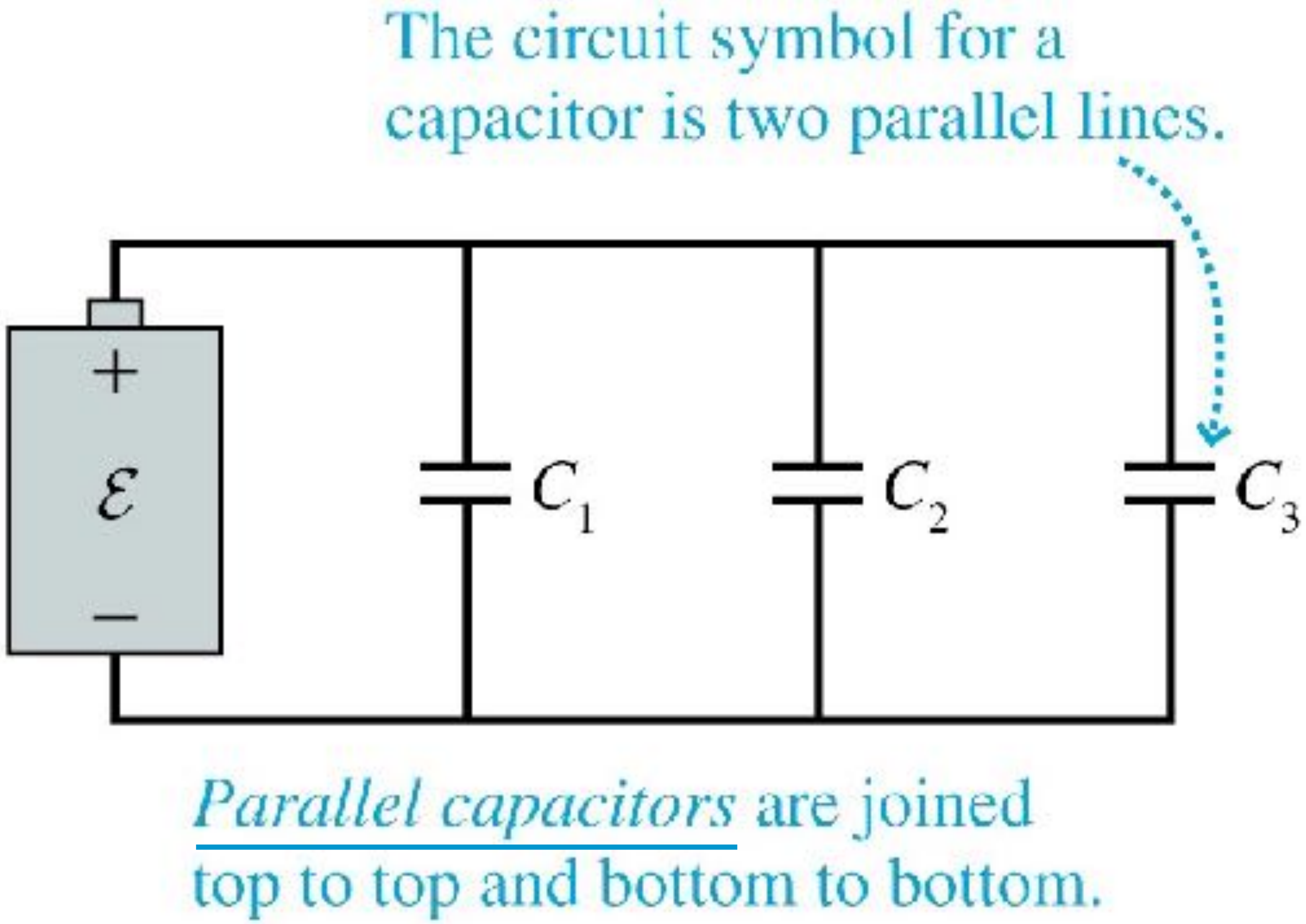
For a parallel plate capacitor,

$$C = \frac{Q}{\Delta V_C} = \frac{\varepsilon_0 A}{d}$$

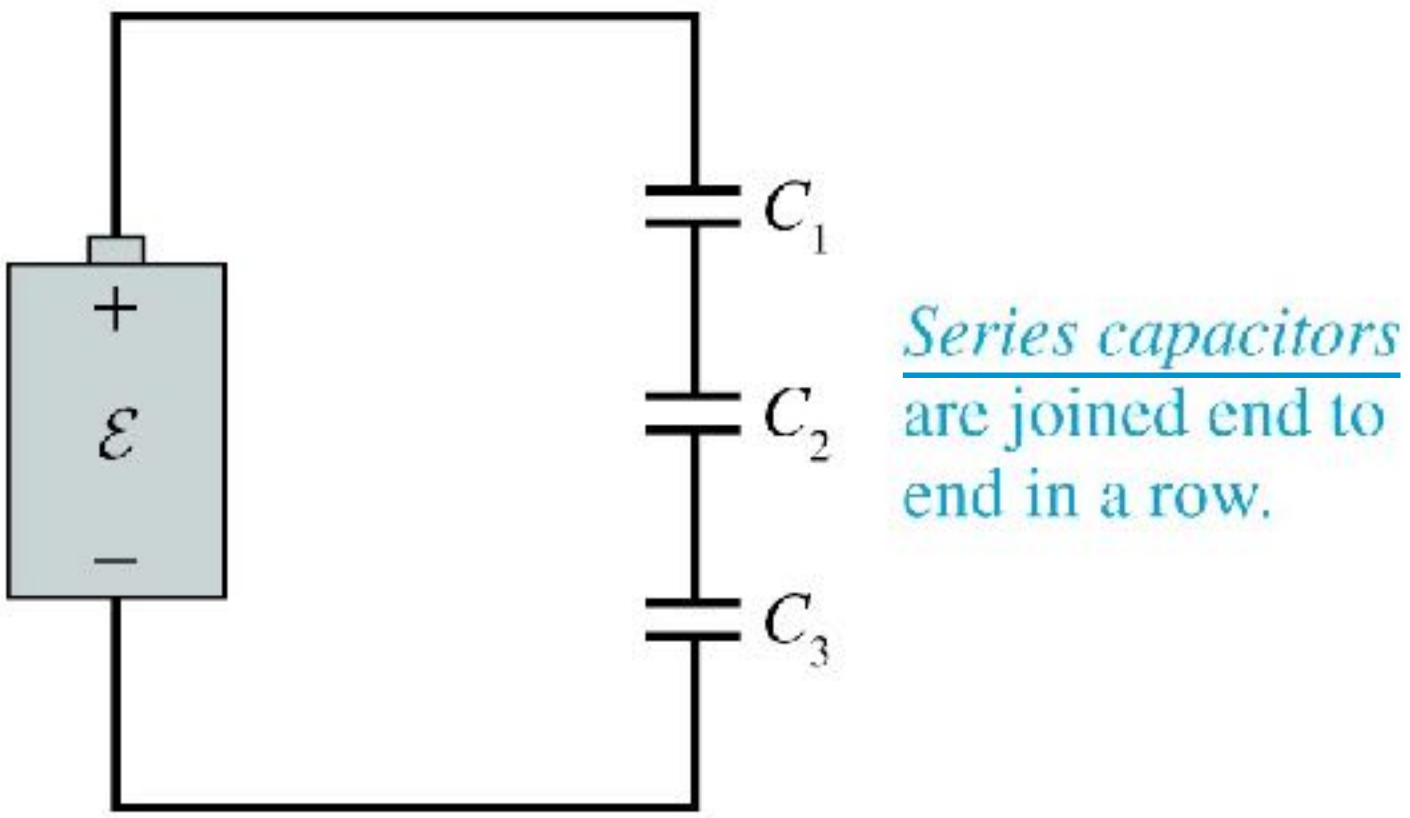
Thus, $\Delta V_C = Q/C = Qd/(\varepsilon_0 A)$

Let's try it!

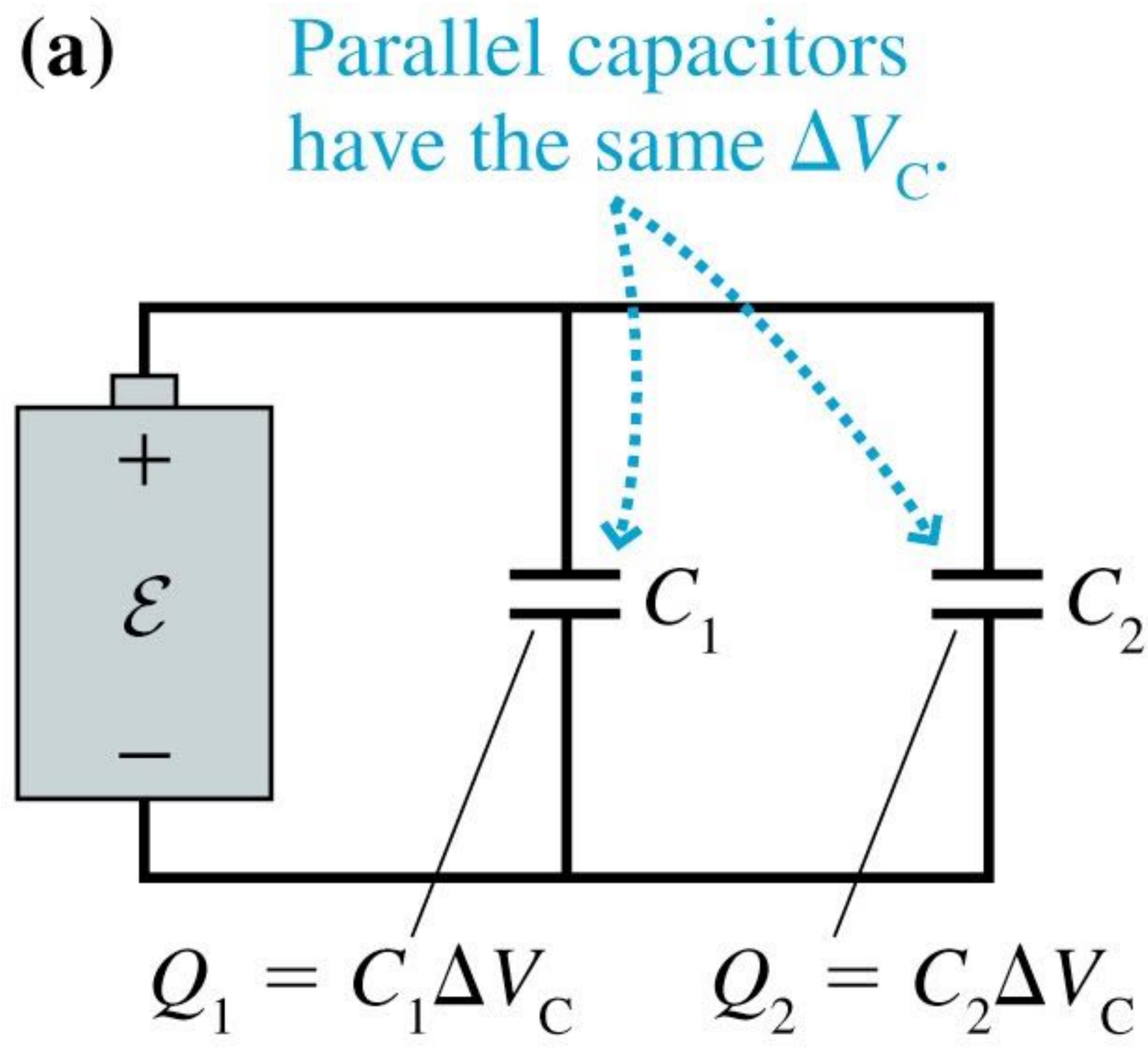
Combinations of capacitors

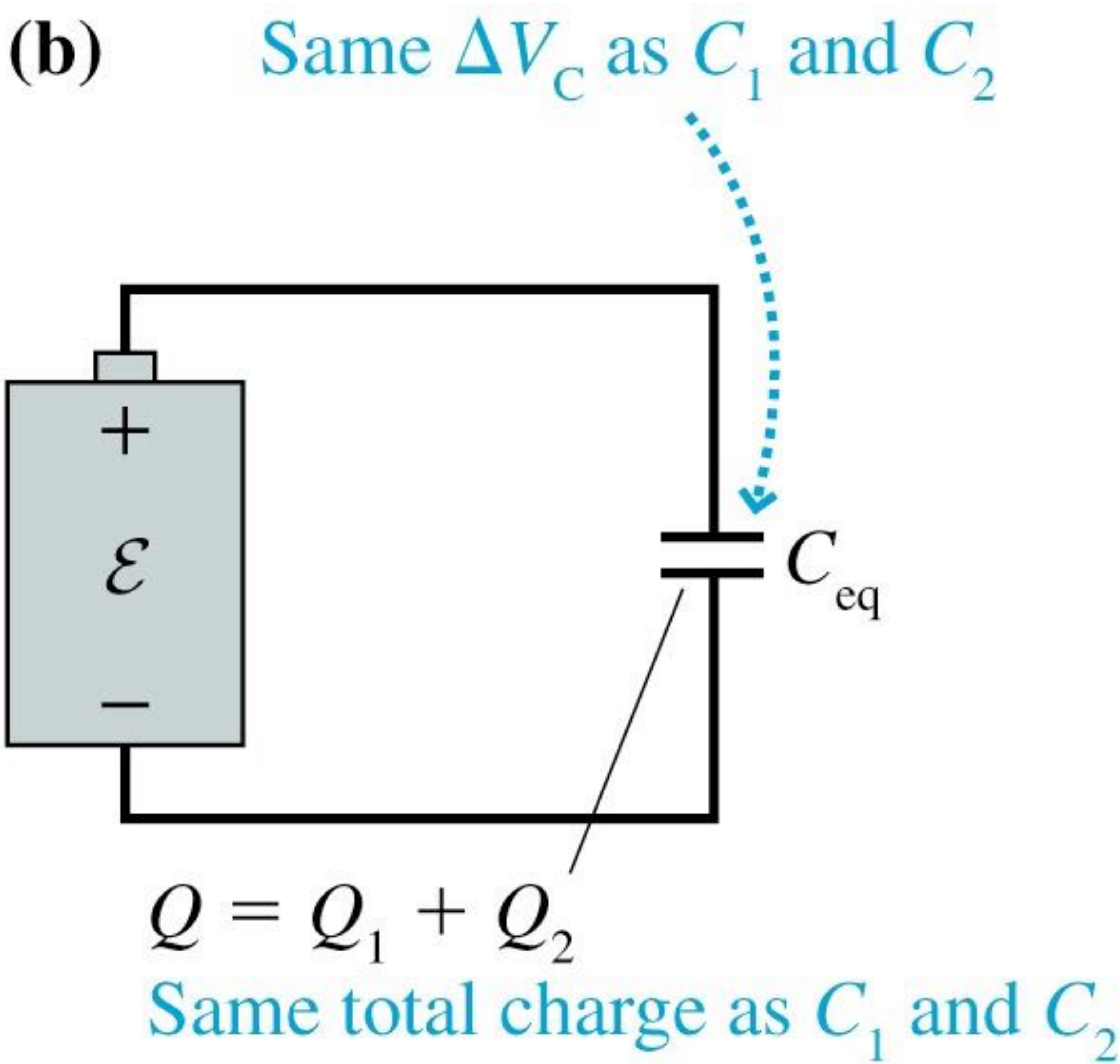


Don't confuse parallel capacitors with parallel-place capacitor!



Parallel Capacitors





$$C_{eq} = \frac{Q}{\Delta V_C}$$

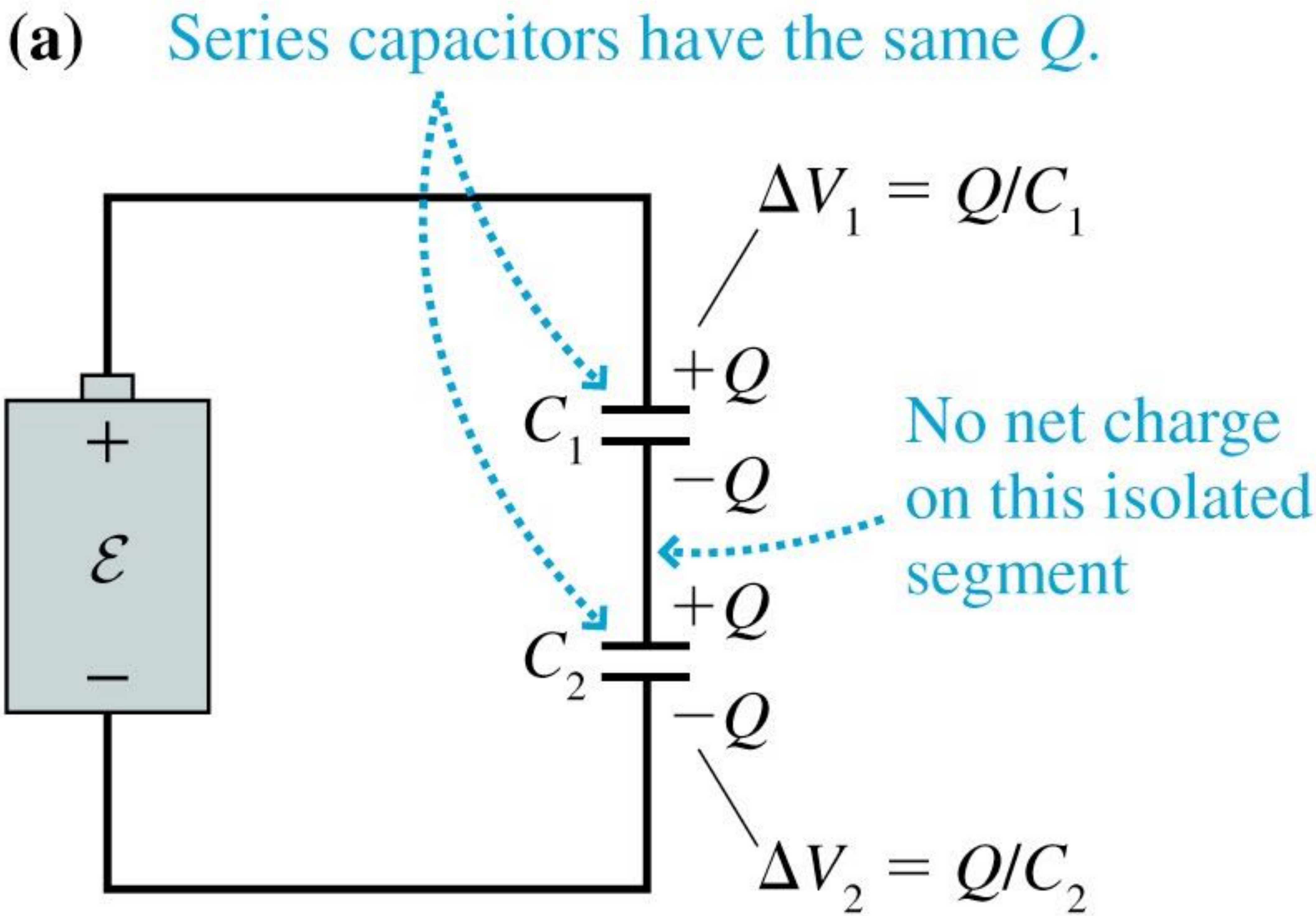
$$C_{eq} = \frac{Q_1 + Q_2}{\Delta V_C}$$

$$C_{eq} = \frac{Q_1}{\Delta V_C} + \frac{Q_2}{\Delta V_C}$$

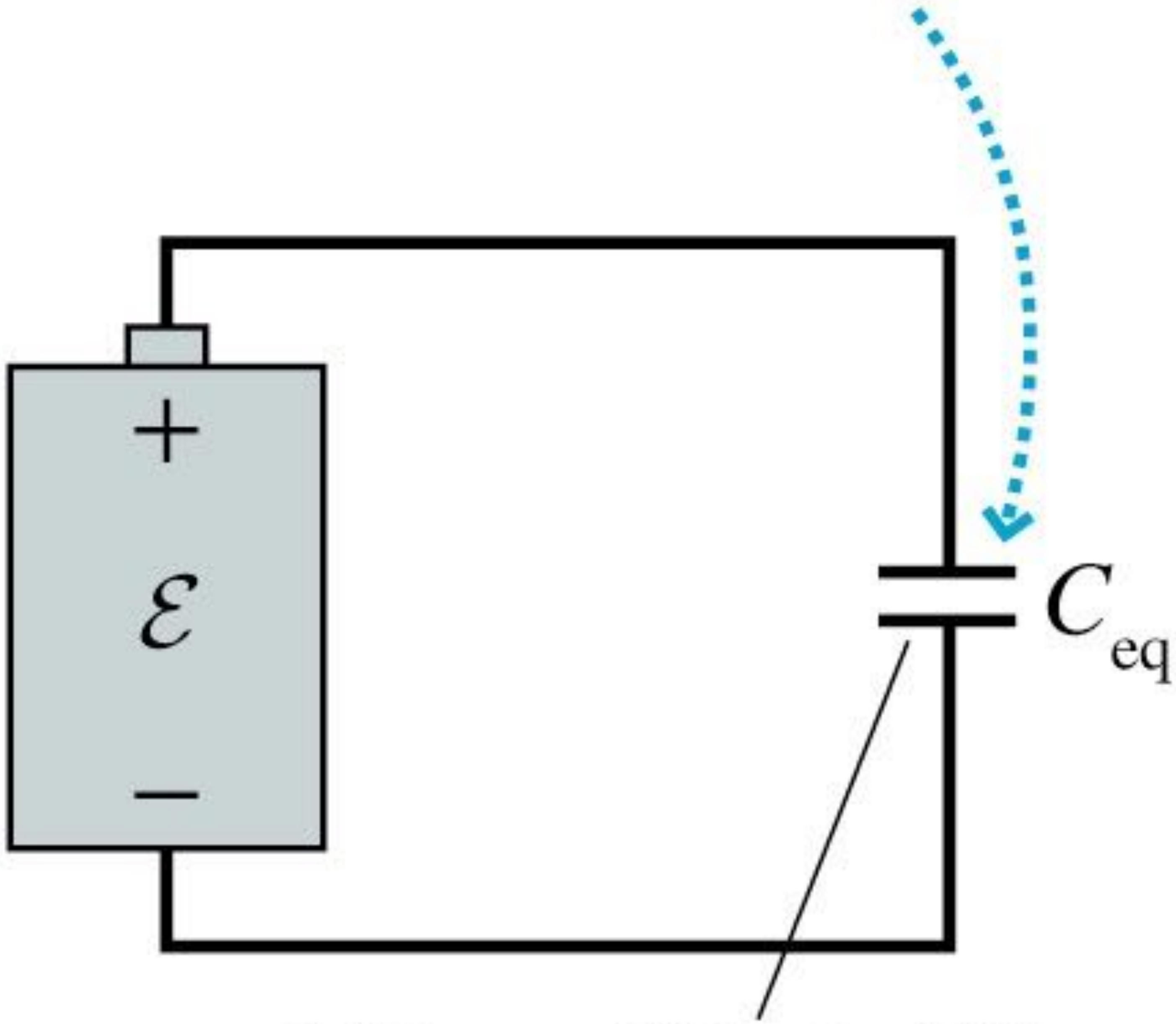
$$C_{eq} = C_1 + C_2$$

$$\boxed{C_{eq} = C_1 + C_2 + C_3 + \cdots}$$

Series capacitors



(b) Same Q as C_1 and C_2



$\Delta V_C = \Delta V_1 + \Delta V_2$

Same total potential difference as C_1 and C_2

$$\frac{1}{C_{eq}} = \frac{\Delta V_C}{Q}$$

$$\frac{1}{C_{eq}} = \frac{\Delta V_1 + \Delta V_2}{Q}$$

$$\frac{1}{C_{eq}} = \frac{\Delta V_1}{Q} + \frac{\Delta V_2}{Q}$$

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2}$$

$$\boxed{\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \cdots}$$

Parallel combination of capacitors:

$$\boxed{C_{\text{eq}} = C_1 + C_2 + C_3 + \cdots}$$

Series combination of capacitors:

$$\boxed{\frac{1}{C_{\text{eq}}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \cdots}$$

Discussion Question

You have two identical capacitors. Each is charged to a potential difference of 10V. If you want the largest potential difference across the capacitors,

A. do you connect them in parallel?

B. do you connect them in series?

C. it doesn't matter how you connect them, the potential difference will be the same.

D. you don't have enough information yet to answer this question.

Discussion Question

You have two identical capacitors. Each is charged to a potential difference of 10V. If you want the largest potential difference across the capacitors,

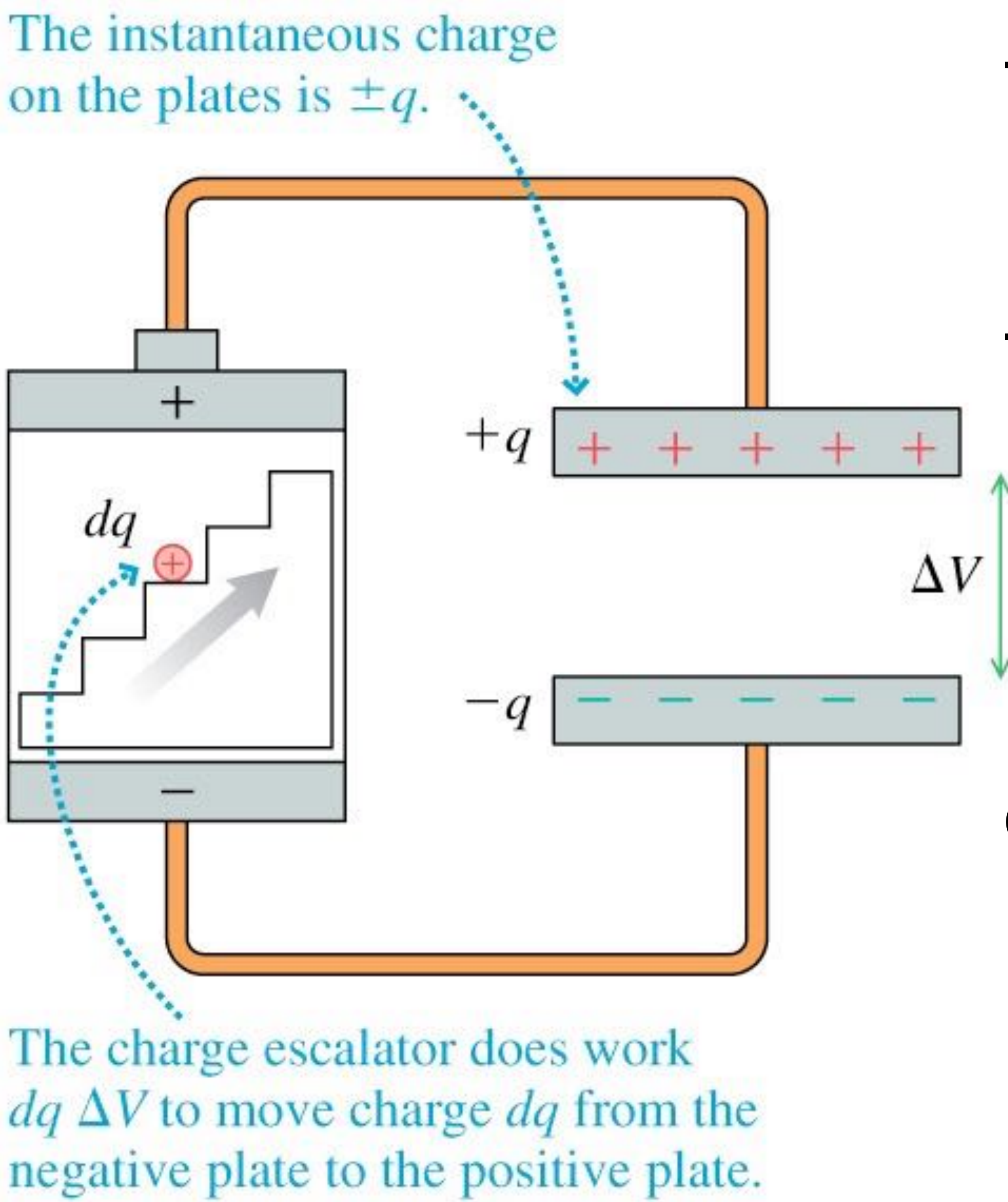
A. do you connect them in parallel?

B. do you connect them in series?

C. it doesn't matter how you connect them, the potential difference will be the same.

D. you don't have enough information yet to answer this question.

Storing energy in a capacitor



The battery must do work to move the charge dq from the negative terminal to the positive. Thus, the system dq + capacitor has gained energy

$$dU = dq\Delta V = \frac{q\,dq}{C}$$

From start to finish, the amount of energy stored in the capacitor is

$$U = \int_0^Q \frac{q\,dq}{C} = \frac{Q^2}{2C}$$

$$U = \frac{1}{2}C(\Delta V)^2$$

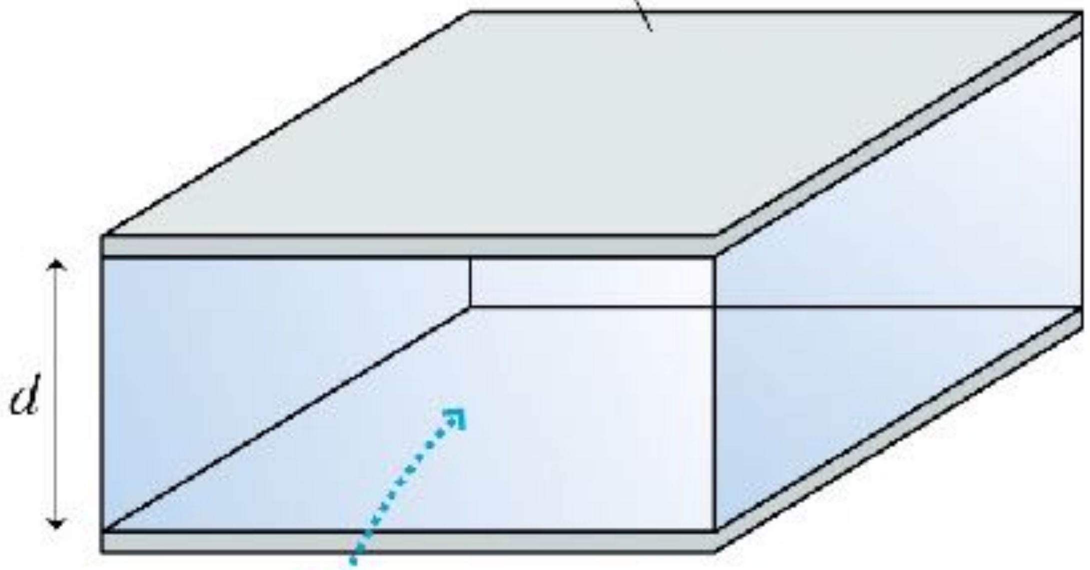
The energy stored in a capacitor is $U = \frac{1}{2}C(\Delta V)^2$

This looks a lot like the energy stored in a spring! It should. As you remember, as you stretch a spring, the force increases linearly. As you charge a capacitor, the field increases linearly, and thus the force needed to add more charge increases linearly!

With a spring, you can see where the energy is stored. The coils of the spring are stretched or compressed.

Where is the energy in a capacitor stored?

In the electric field!



$$\Delta V_C = Ed$$

$$C = \varepsilon_0 A/d$$

$$U = \frac{1}{2}C(\Delta V)^2 = \frac{1}{2}\frac{\varepsilon_0 A}{d}(Ed)^2$$

$$U = \frac{\varepsilon_0}{2}(Ad)(E)^2$$

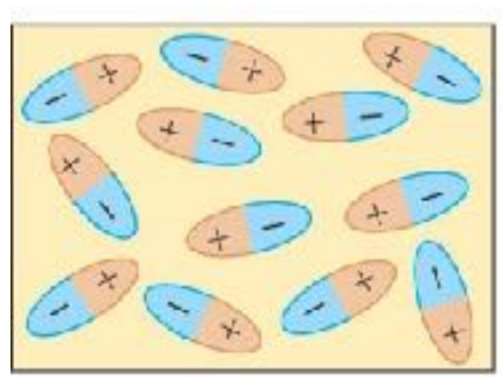
Energy density $$u_E = \frac{\text{energy stored}}{\text{volume in which it is stored}} = \frac{\varepsilon_0}{2}(E)^2$$

The electric field started out to explain a long distance force. But if it can store energy, it must somehow be real and not merely a pictorial device!

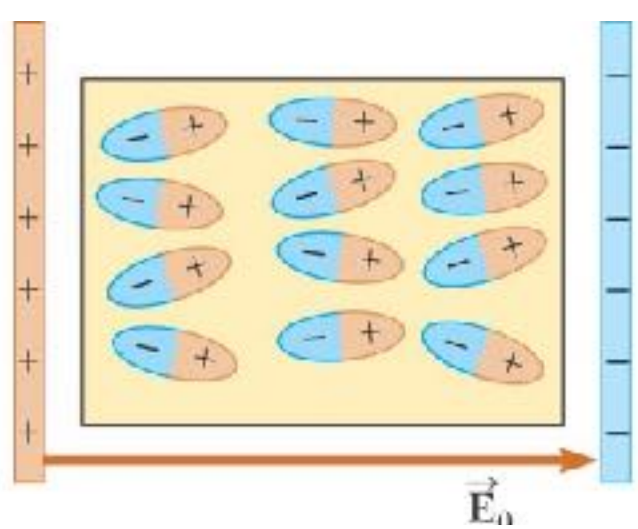
Higher Capacitances

If you calculate the capacitance of the parallel plate capacitor, you'll see, that with a spacing of 1 mm, $C = \varepsilon_0 A/d = 280$ pF.

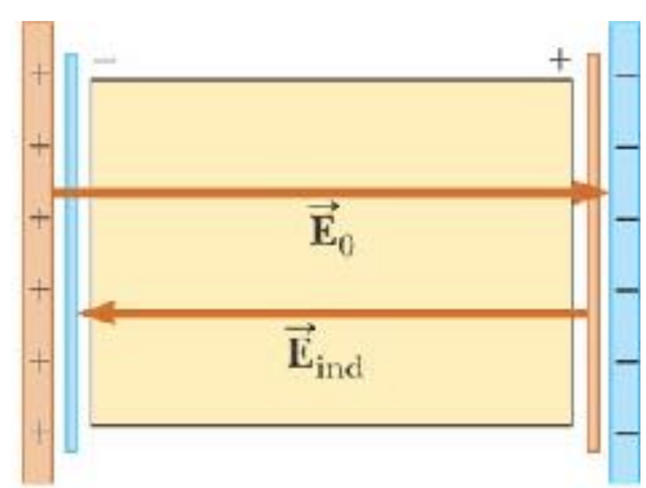
To get a larger capacitance, we can make *d* smaller or *A* larger. Both have problems. The alternative is to introduce a *dielectric* between the plates. This will reduce the field between the plates, and therefore increase the capacitance.



Dielectric

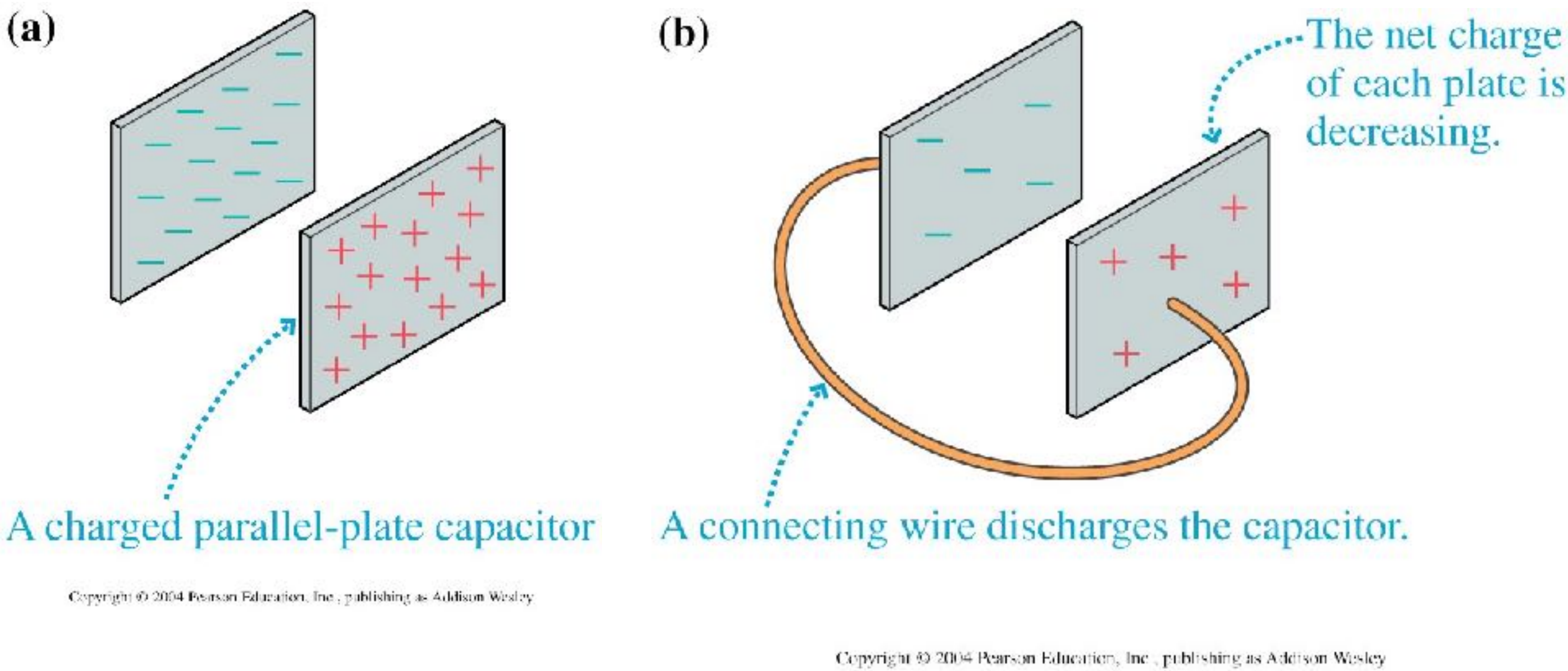


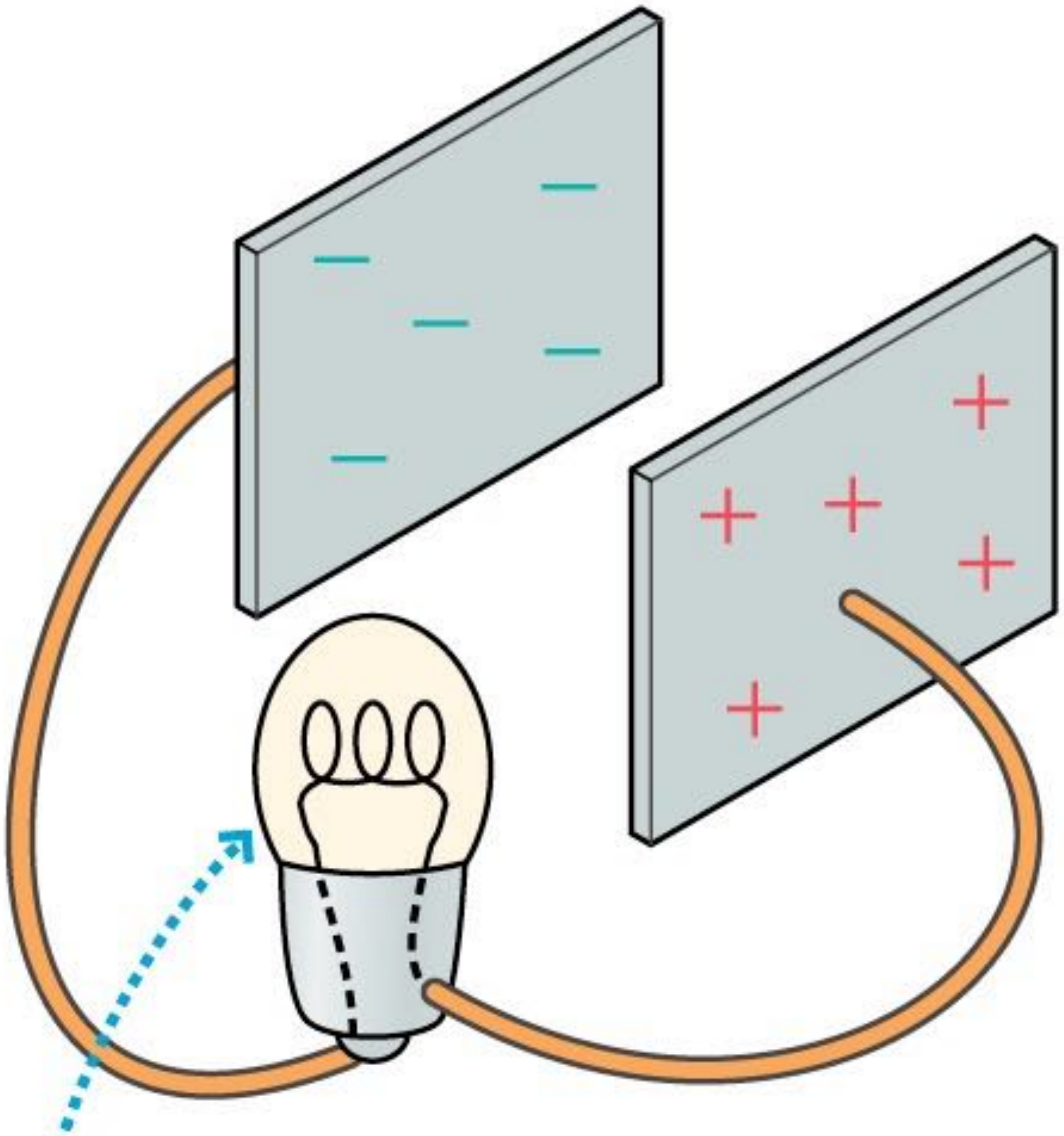
Polarized



Induced field

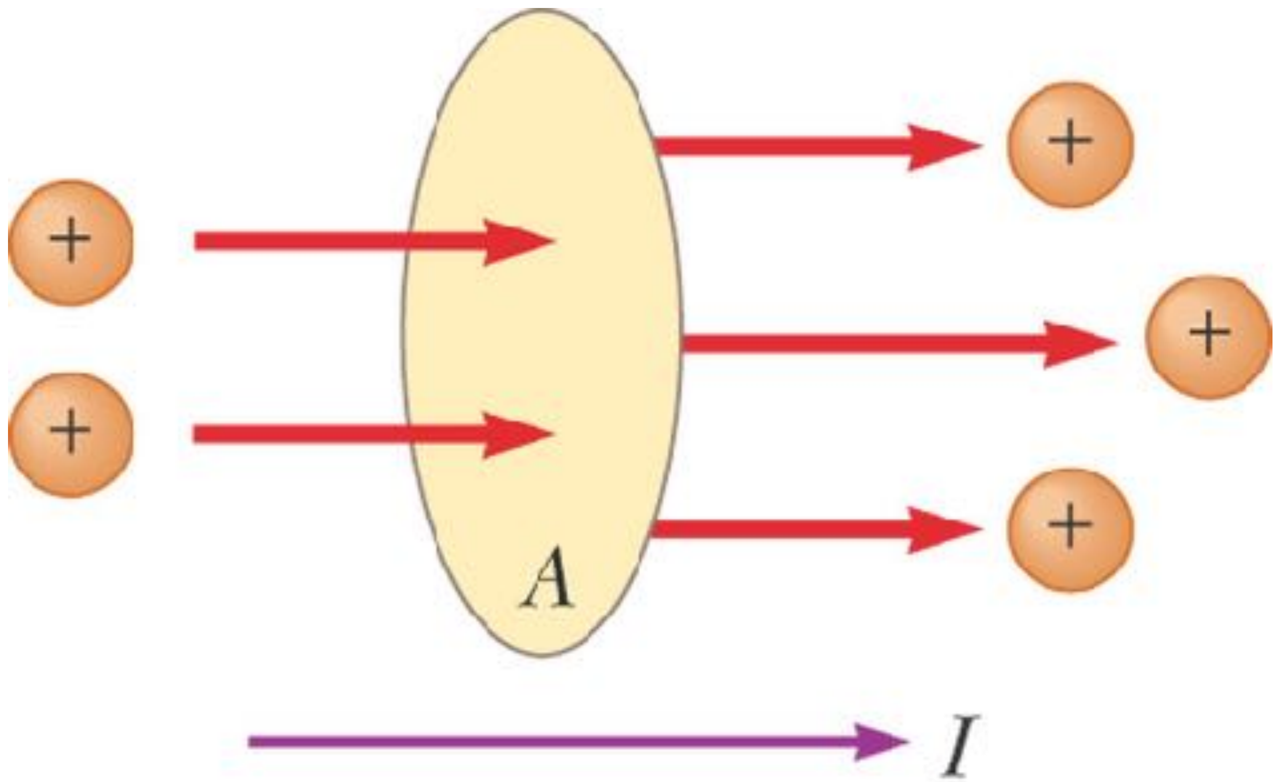
Currents


(a)
A charged parallel-plate capacitor
Copyright © 2004 Pearson Education, Inc., publishing as Addison Wesley
(b)
The net charge of each plate is decreasing.
A connecting wire discharges the capacitor.
Copyright © 2004 Pearson Education, Inc., publishing as Addison Wesley



A light bulb glows. The light bulb filament is part of the connecting wire.

Some definitions

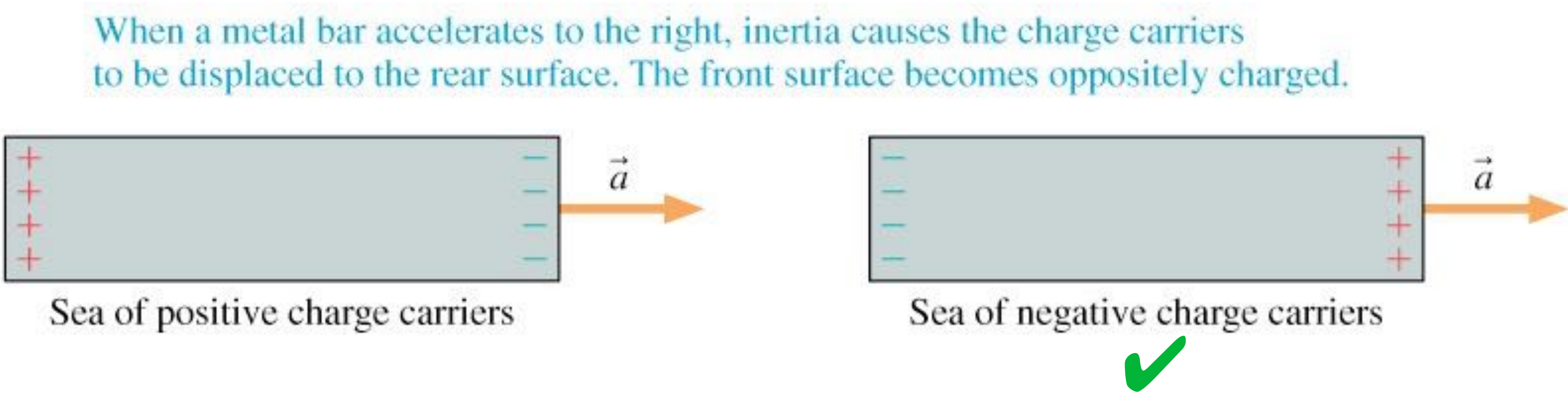


$$I \equiv \frac{dQ}{dt}$$

Unit: ampere (A)
1 A = 1 C/s

Even if electrons are the charge carrier, convention is that direction of current is in the direction of positive flow of charge. For a wire, I is opposite the flow of electrons.

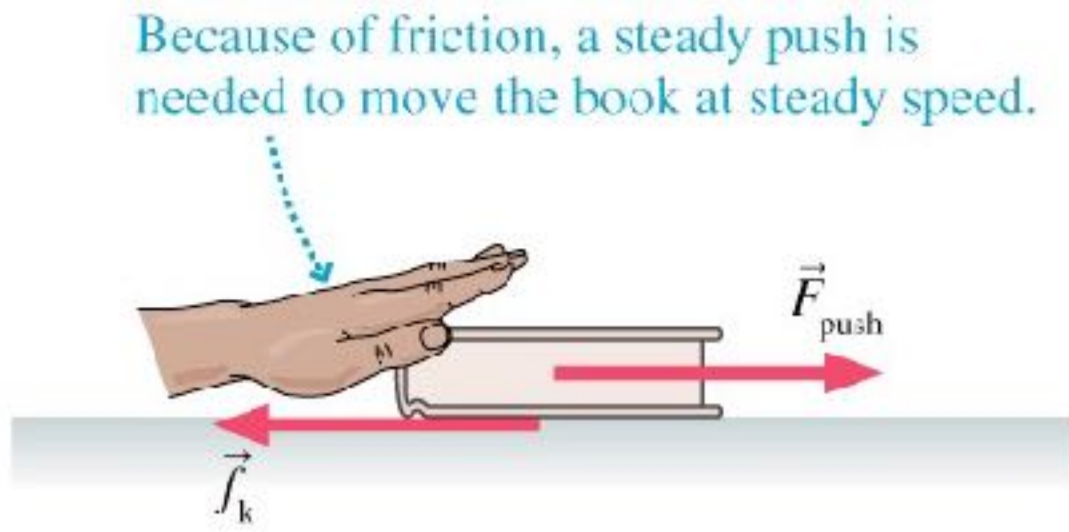
Charge Carriers



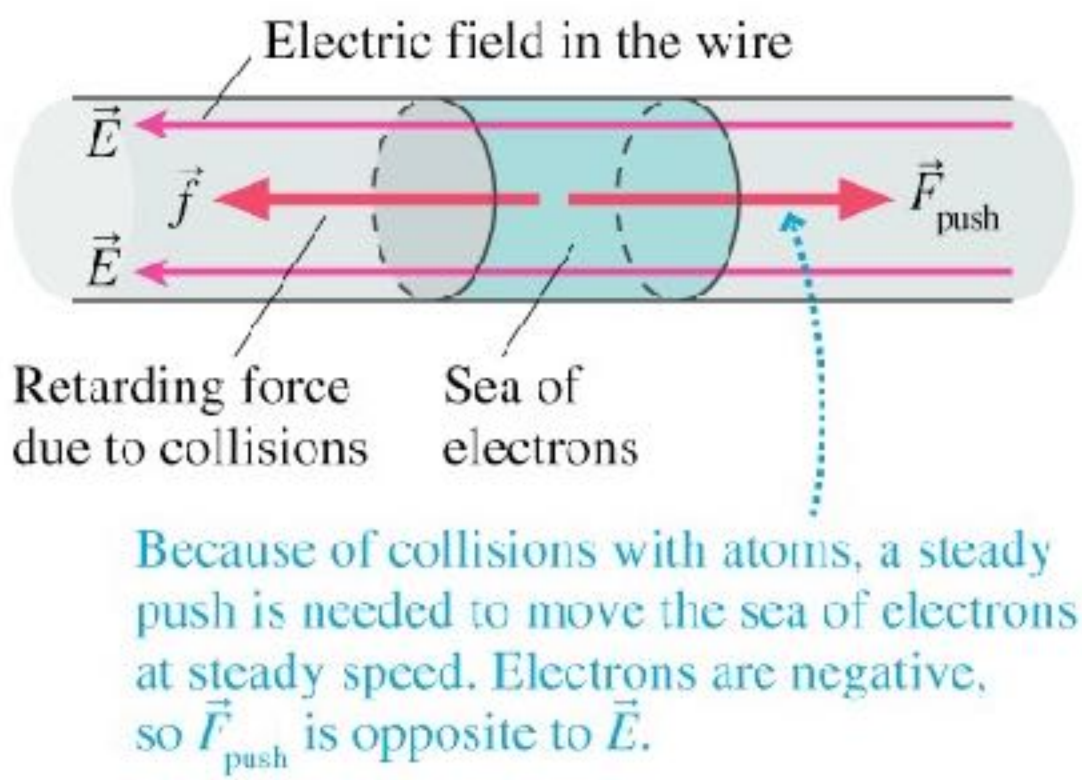
Tolman-Stewart experiment (1916) showed that in metals ***electrons*** are the charge carriers

NB: in, *eg*, saltwater, both + (Na^+) and – (Cl^-) ions are charge carriers!

Creating a current



To keep the book moving, I have to apply a constant force, since I have to counteract the resistive force of friction.

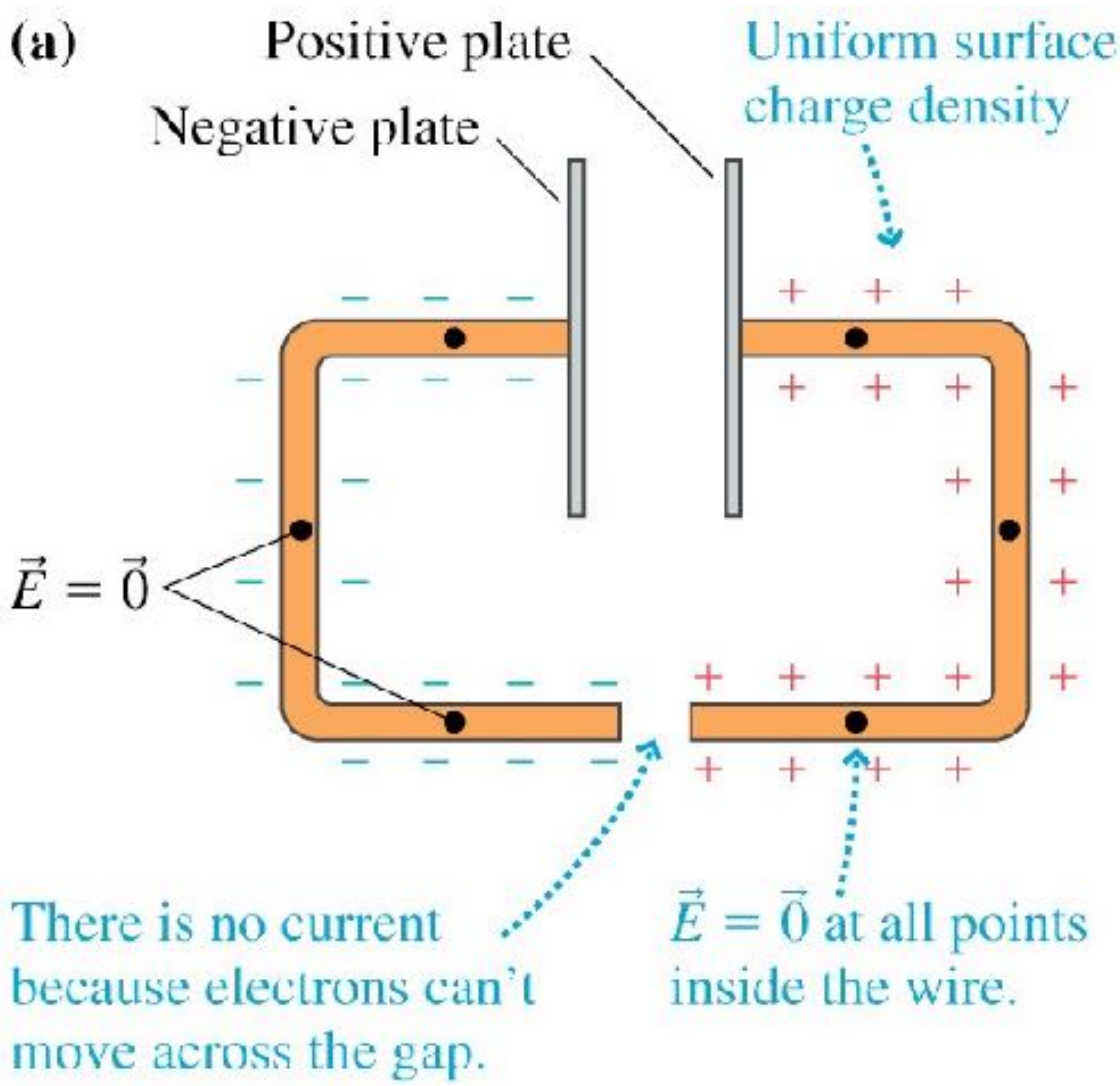


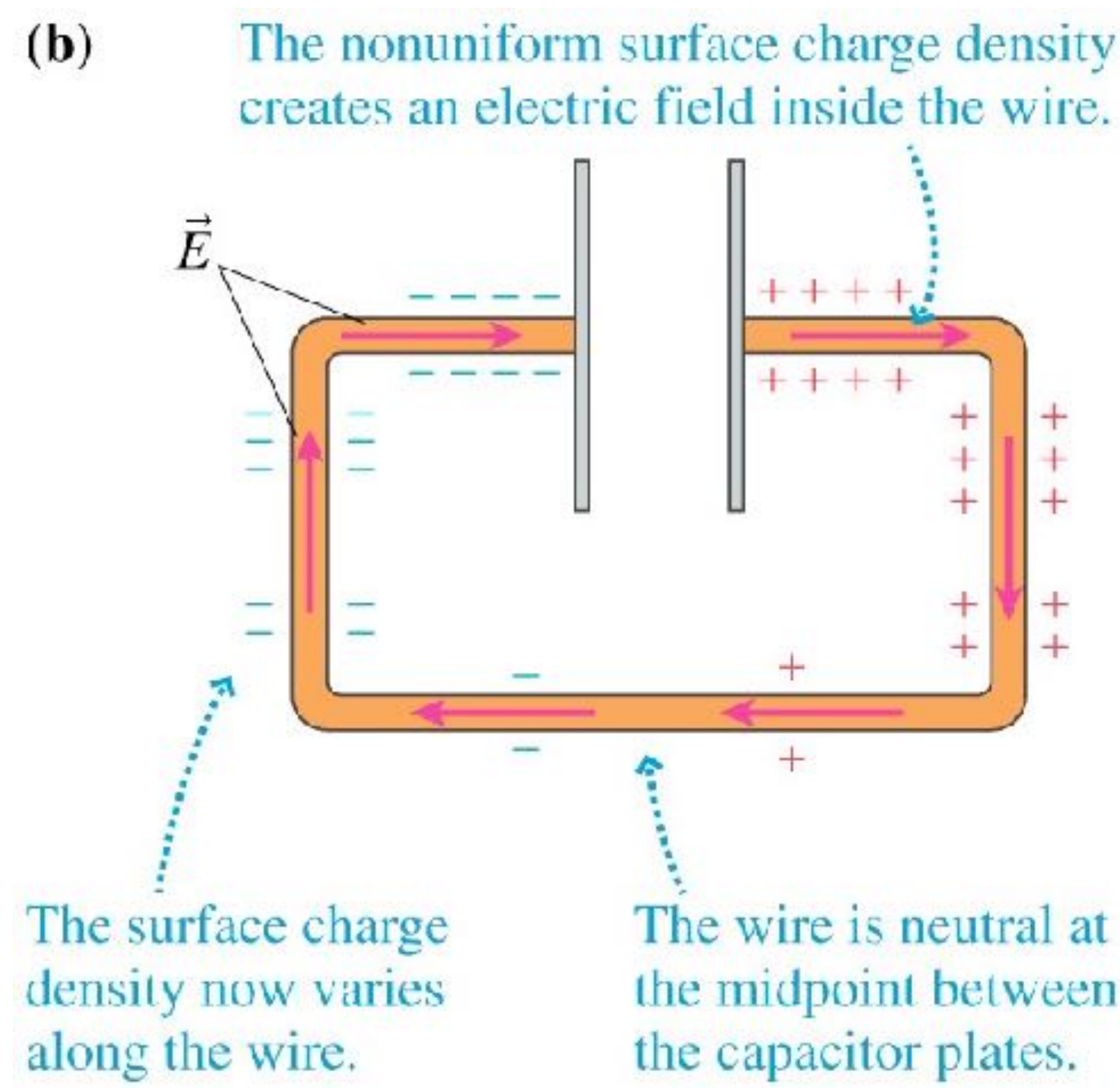
To keep a current flowing in a wire, there has to be a continuous force as well.

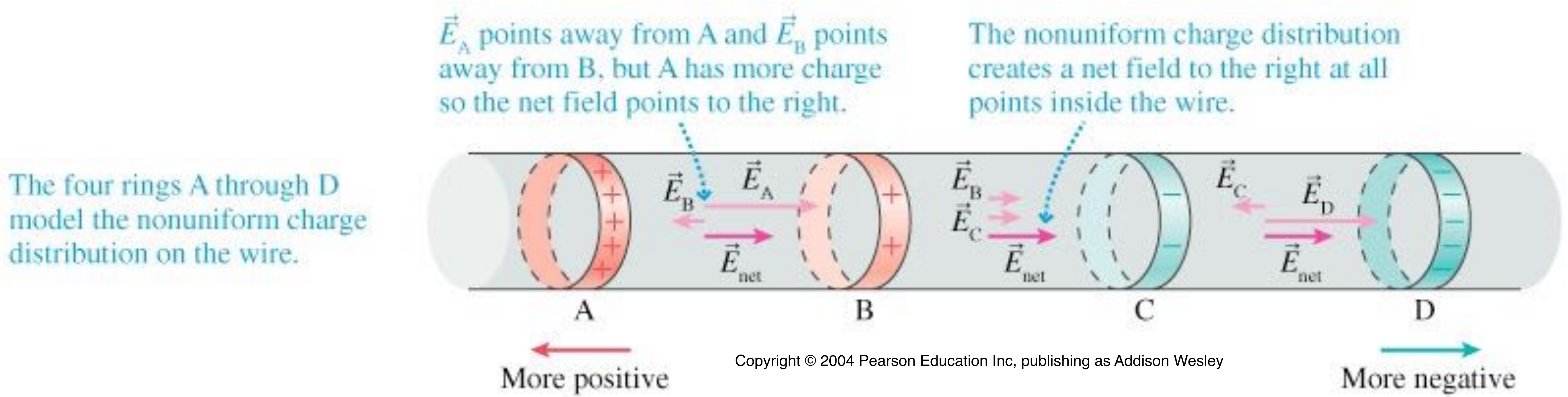
What force? An electric field!

We had seen earlier that E=0 in a conductor. But that was in **electrostatic equilibrium**. A wire with a current is a ***non***-equilibrium situation.

Establishing the field





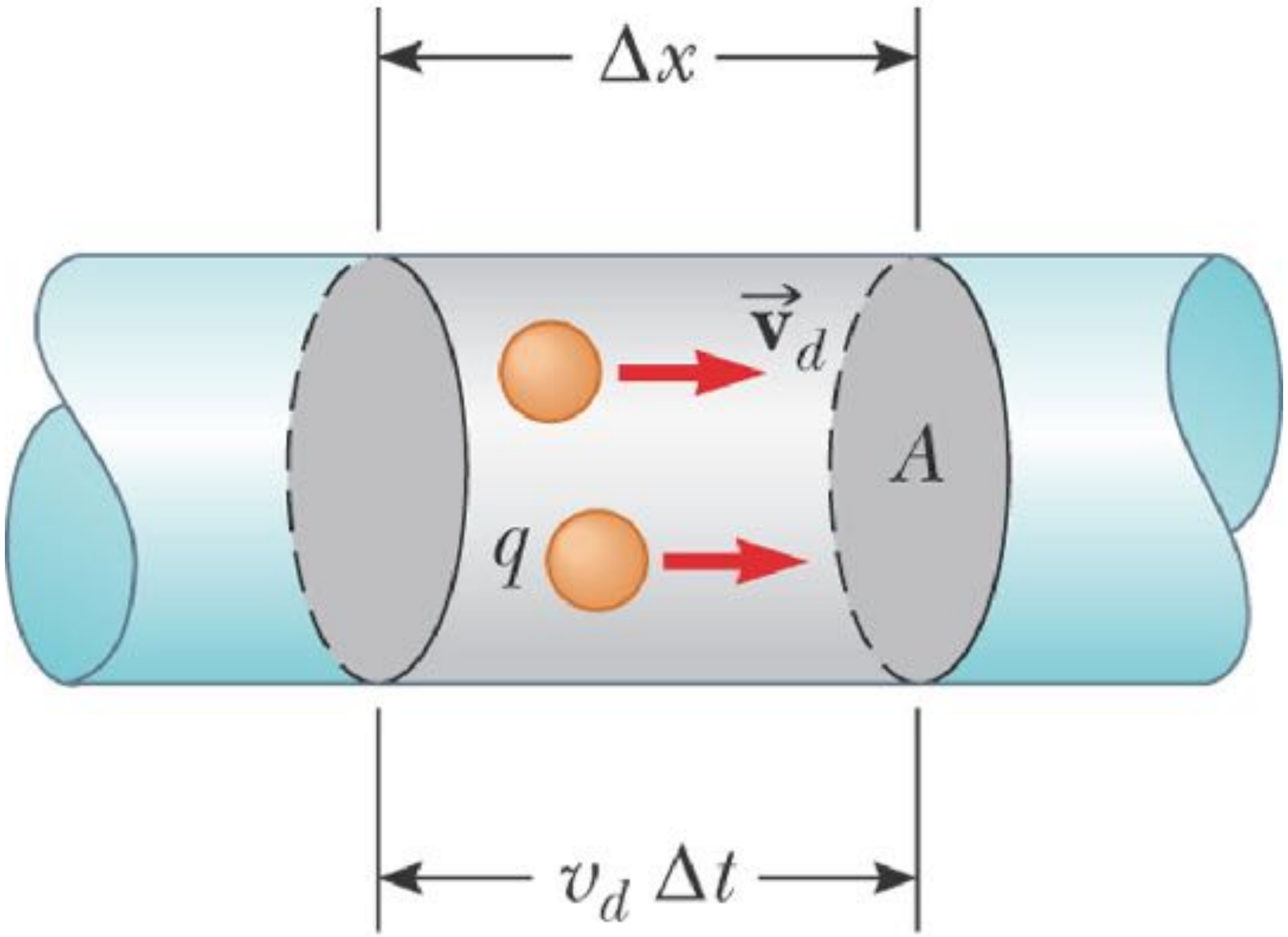


Remember the field of a ring of charge:

1. points away from a positive ring; towards a negative
2. is proportional to the amount of charge on the ring
3. decreases with distance from the ring

The nonuniform distribution of surface charge along a wire creates a net electric field inside the wire that points from the more positive end to the more negative end of the wire. This is the internal field that pushes the charge carriers (electrons in this case) through the wire.

More definitions



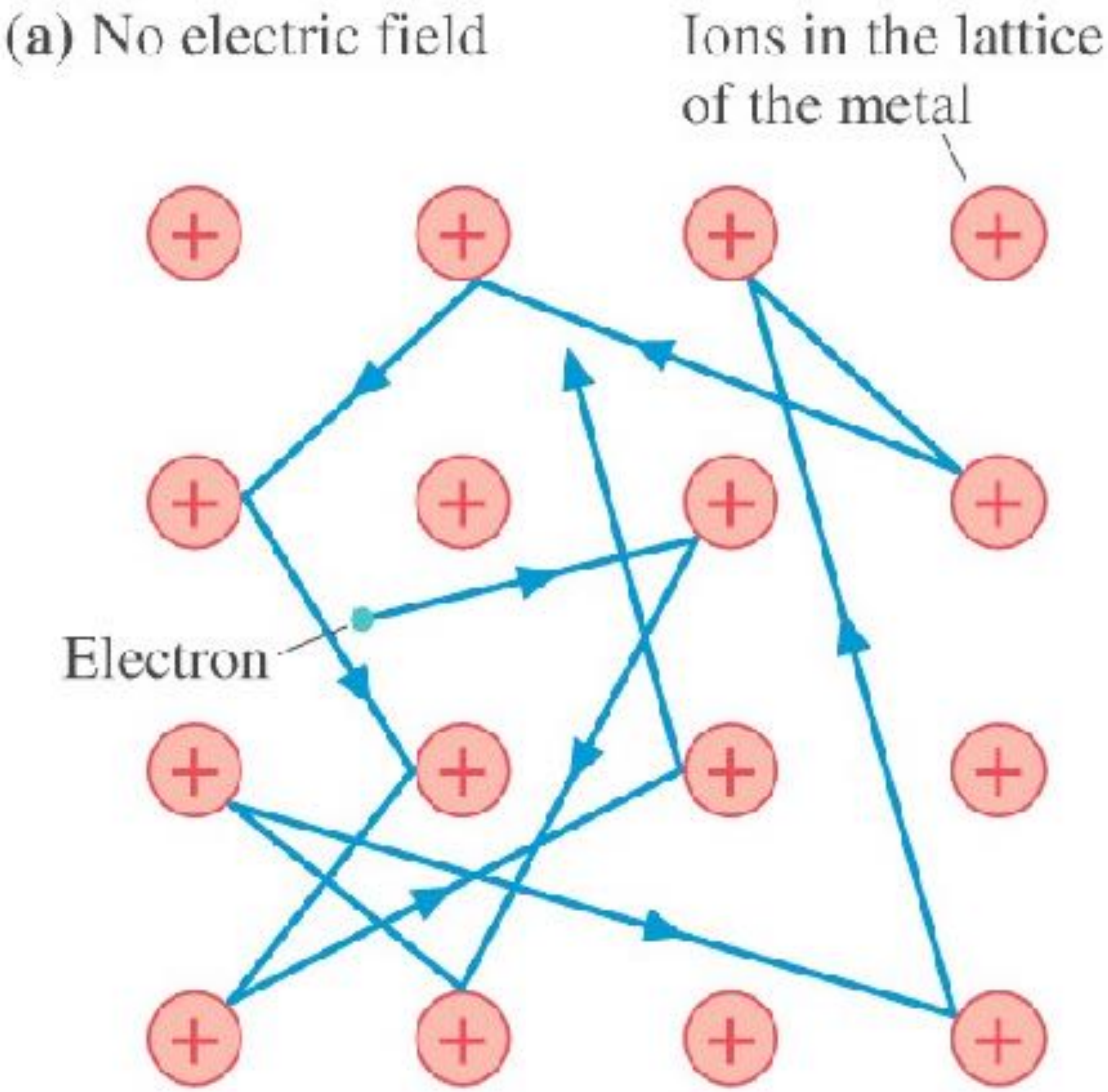
$dQ = (nA\, dx)q = (nAv_d\, dt)q$

$I = dQ/dt = nqv_dA$

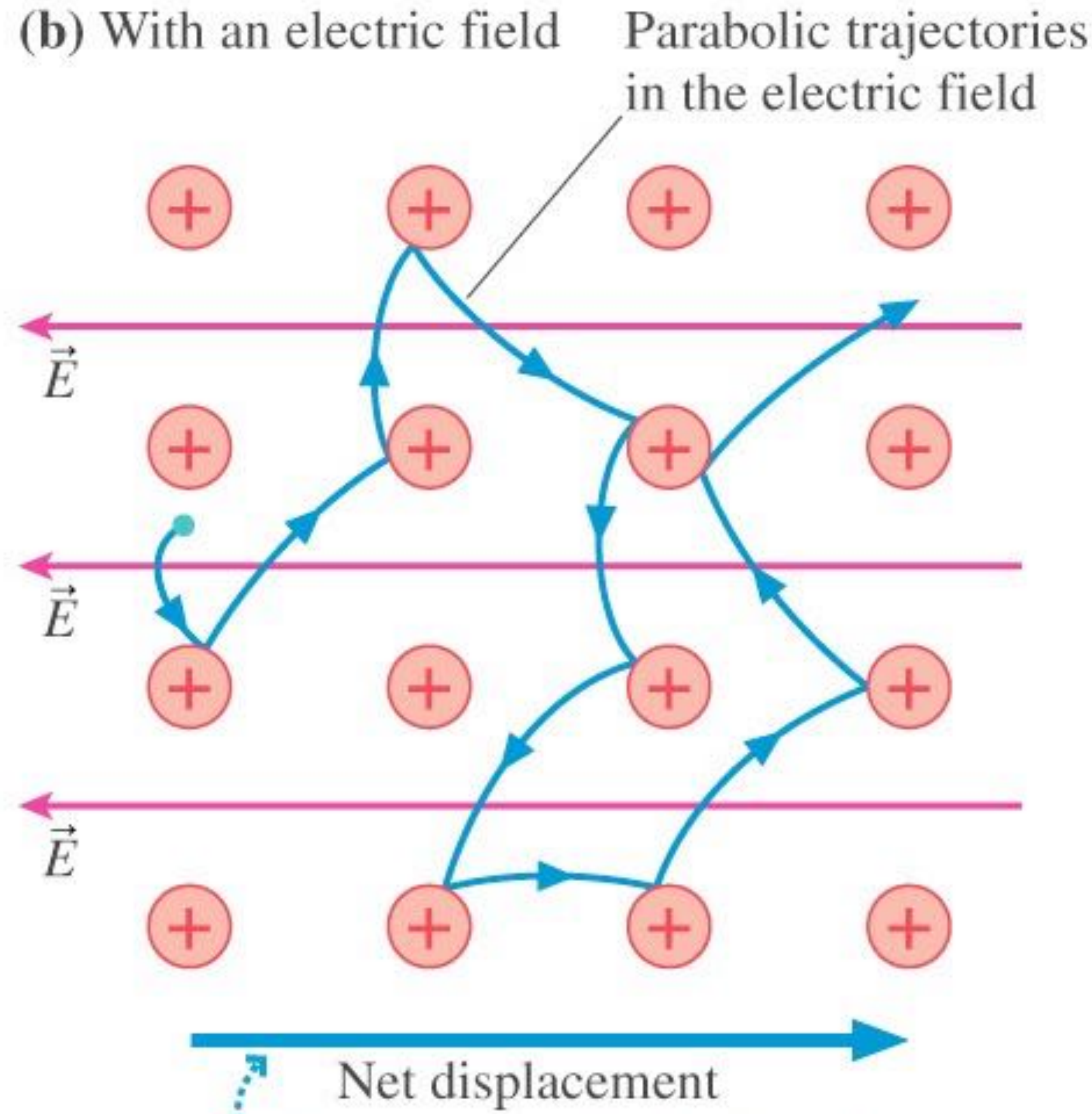
Current density:
$J = I/A = nqv_d$

Typical values for a metal:
n : 6-9 X 10^{28} m^{-3}
v_d : ~ 10^{-4} m/s

Why so slow?

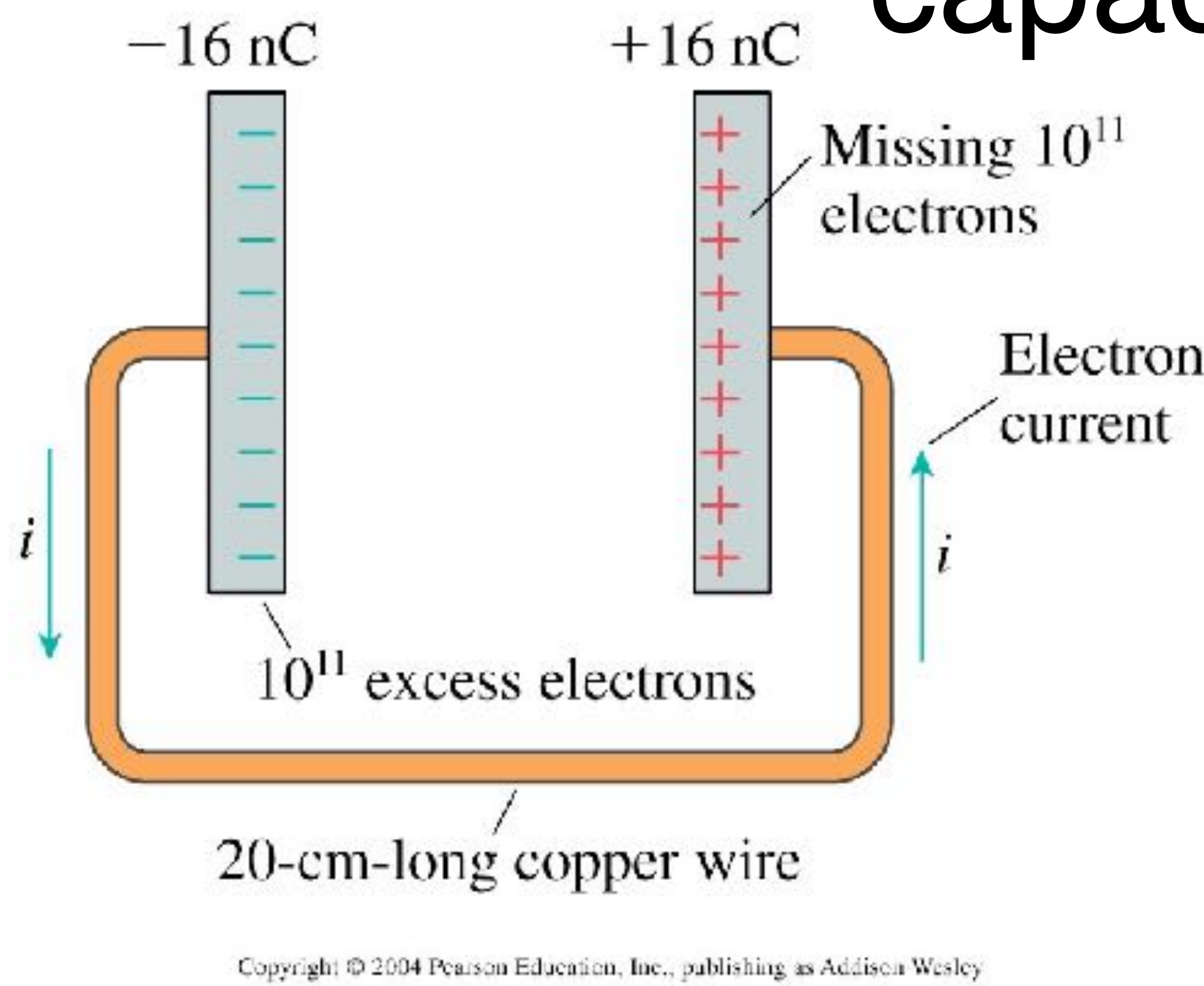


The electron has frequent collisions with ions, but it undergoes no net displacement.

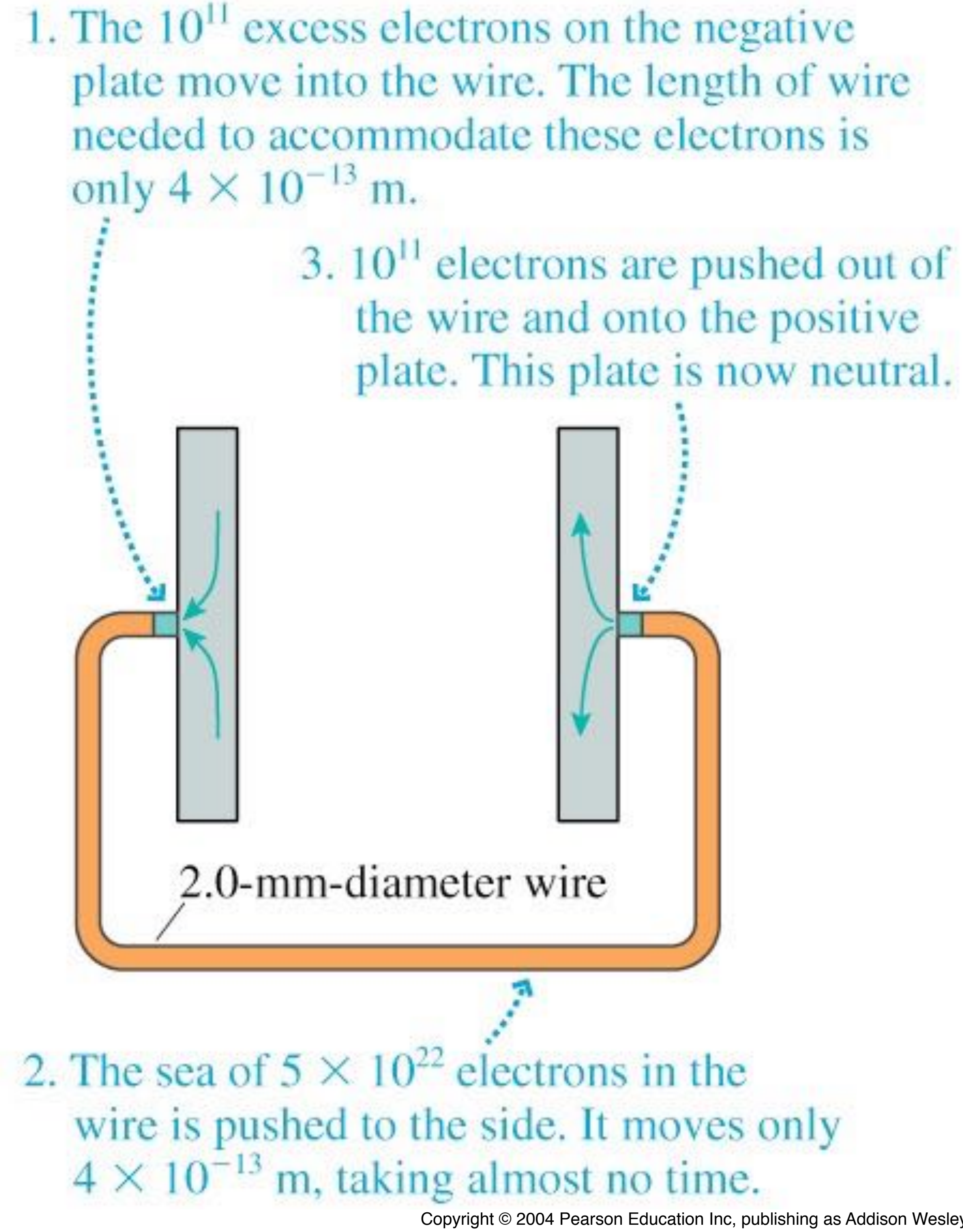


A net displacement in the direction opposite to $\vec{E}$ is superimposed on the random thermal motion.

How long does it take to discharge a capacitor?



If the electrons had to travel from one plate to the other at 10^{-4} m/s, it would take 30 minutes to discharge the capacitor!



It actually takes only a few ns!

Resistance

For many conductors, there is a simple relationship between the applied potential difference, ΔV, and the current, I :

$$I = \frac{\Delta V}{R}$$

R is called the resistance of the conductor, and the unit is called the ohm (Ω). 1 Ω = 1 V/A. This relationship is commonly called Ohm's Law, but it isn't really a law. It applies only to materials where R is relatively constant.

Most metals are ohmic materials. Resistors are devices made with poorly conducting materials, such as carbon, or thin films of metal, that have a high resistance. They are also ohmic devices.

Non-ohmic

Many devices are non-ohmic. Three important ones are:

Batteries, where ΔV is determined by the chemical reaction in the battery and is independent of I

Semiconductors, where the I vs ΔV curve is highly non-linear

Capacitors, where we will see in a while that the relationship between I and ΔV differs from that of a resistor.

Resistivity and Conductivity

The resistance of a wire can be expressed as

$$R = \underbrace{\rho}_{\text{resistivity}} \frac{\overset{\text{length}}{\ell}}{\underset{\text{cross-sectional area}}{A}}$$

Resistivity has the units ohm-meter (Ω-m). Resistivity is a characteristic of the material. A thicker wire has a lower resistance. A longer wire has a higher resistance.

$1/\rho$ is called the conductivity, denoted by σ.
Conductivity also relates the current density to the electric field in the wire:

$$J = \sigma E$$

Common ρ and σ

TABLE 28.2 Resistivity and conductivity of conducting materials

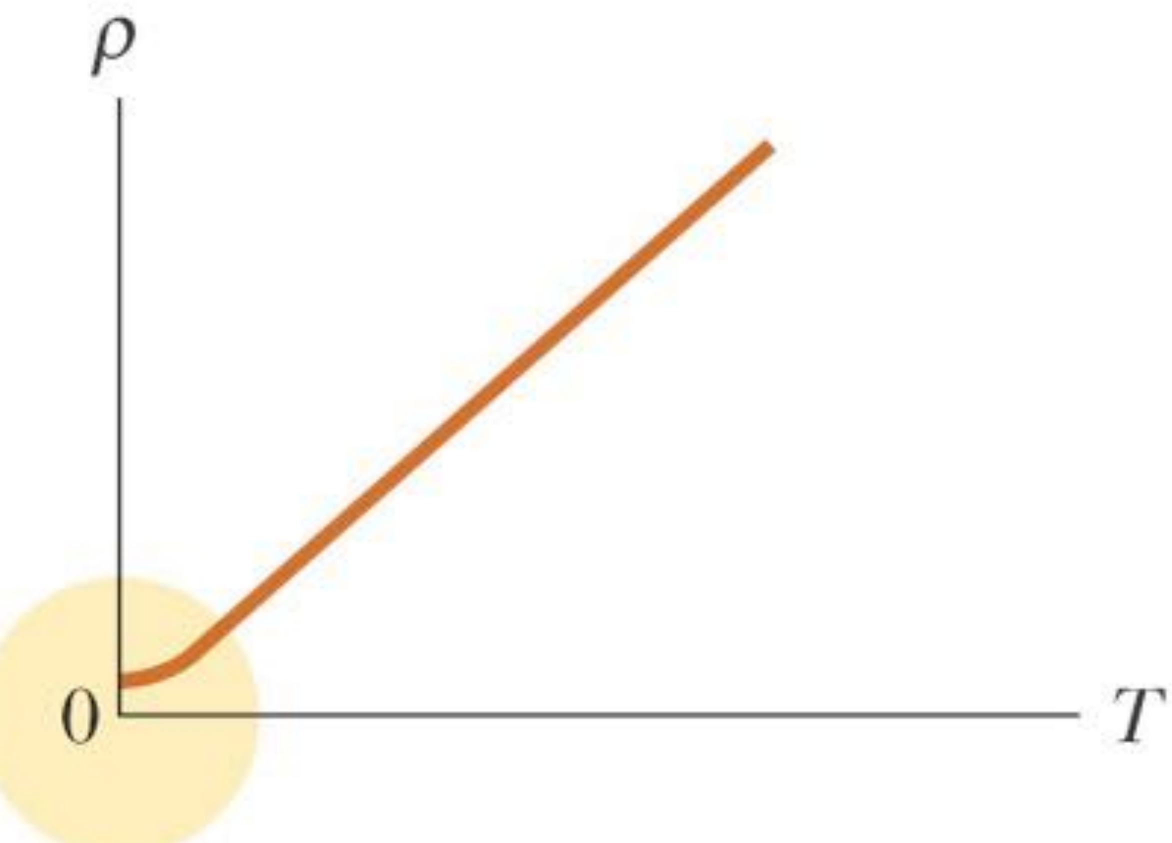
Material	Resistivity (Ω m)	Conductivity (Ω^{-1}m^{-1})
Aluminum	2.8×10^{-8}	3.5×10^{7}
Copper	1.7×10^{-8}	6.0×10^{7}
Gold	2.4×10^{-8}	4.1×10^{7}
Iron	9.7×10^{-8}	1.0×10^{7}
Silver	1.6×10^{-8}	6.2×10^{7}
Tungsten	5.6×10^{-8}	1.8×10^{7}
Nichrome*	1.5×10^{-6}	6.7×10^{5}
Carbon	3.5×10^{-5}	2.9×10^{4}

*Nickel-chromium alloy used for heating wires

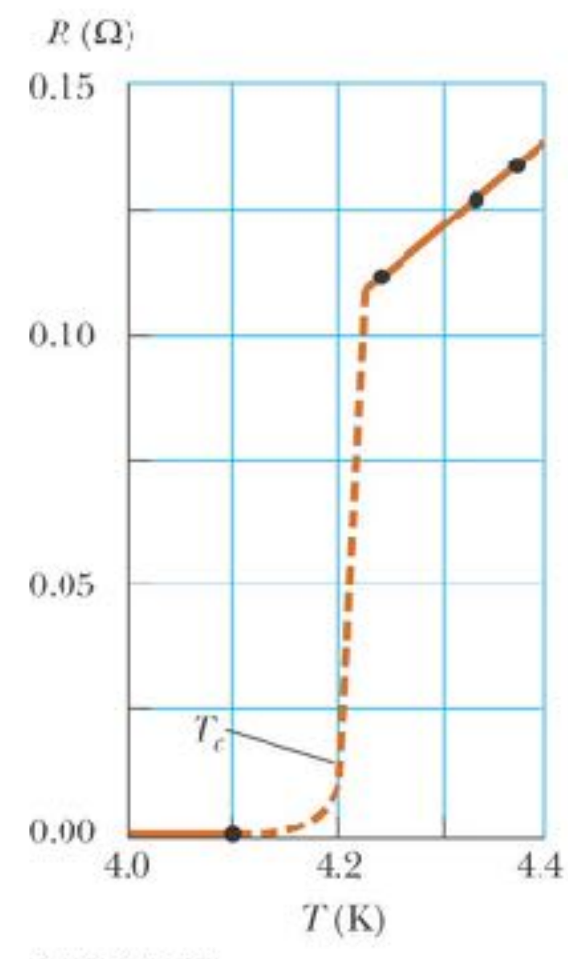
This is why copper is used for wiring.

Superconductivity

Most metals show a decreasing resistivity as the temperature decreases. However, at low temperatures, it does not go to zero.



However, for some metals, the resistivity suddenly drops to zero when a critical temperature is reached.



Hg

Superconductivity

In a superconductor, a current can be sustained *without an applied electric field.* A field (and therefore a potential difference) is needed to get the current started, but after that, the field can be removed.

This is the equivalent of frictionless motion. A force is needed to get an object moving, but without any frictional forces, it will continue to move with no external force.

From all that we can tell, superconductivity is truly a zero resistivity phenomenon. Currents have been maintained without any applied potential difference for years!

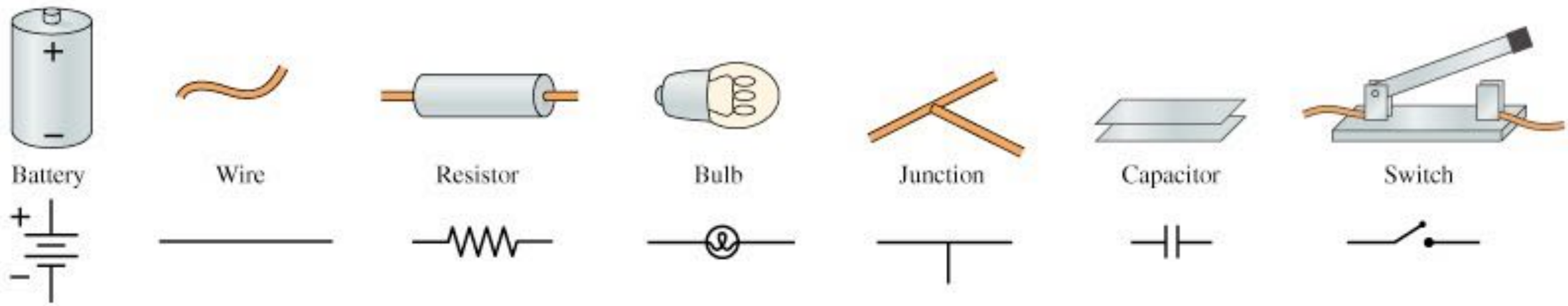
Two types of superconductivity

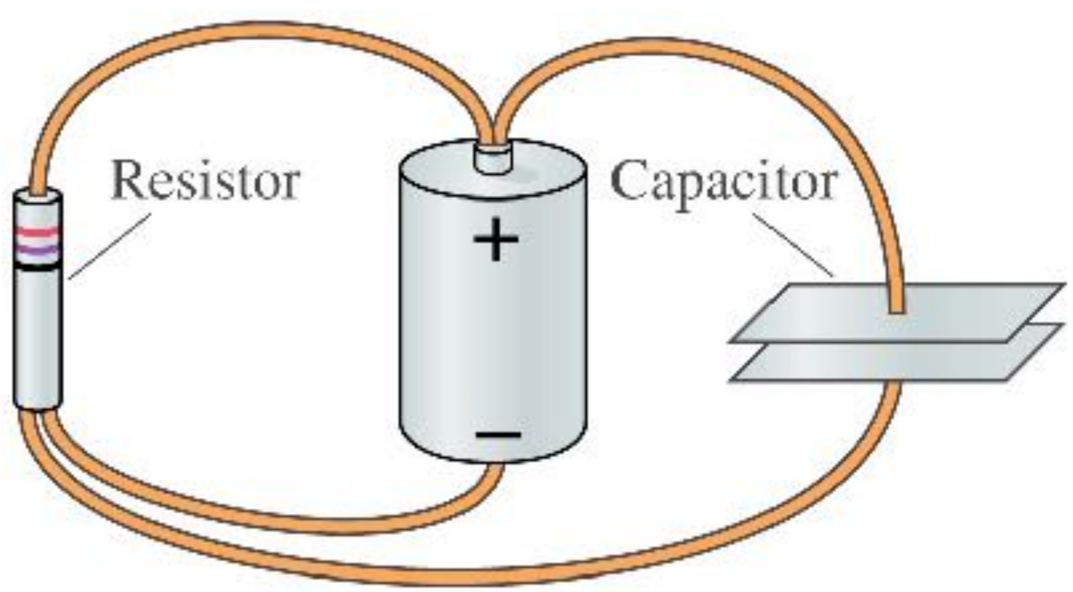
Low temperature: metals, with critical temperatures less than 20 K. The theory of these superconductors was developed in the 1950s.

High temperature: in 1986 a class of ceramics (normally insulators!) were found to be superconductors at relatively high temperatures (>30 K and as high as 135 K). This kind of superconductivity is not very well understood at all right now.

The search continues. There is no reason why there can't be superconductors at room temperature. This would have enormous applications...

Circuit elements





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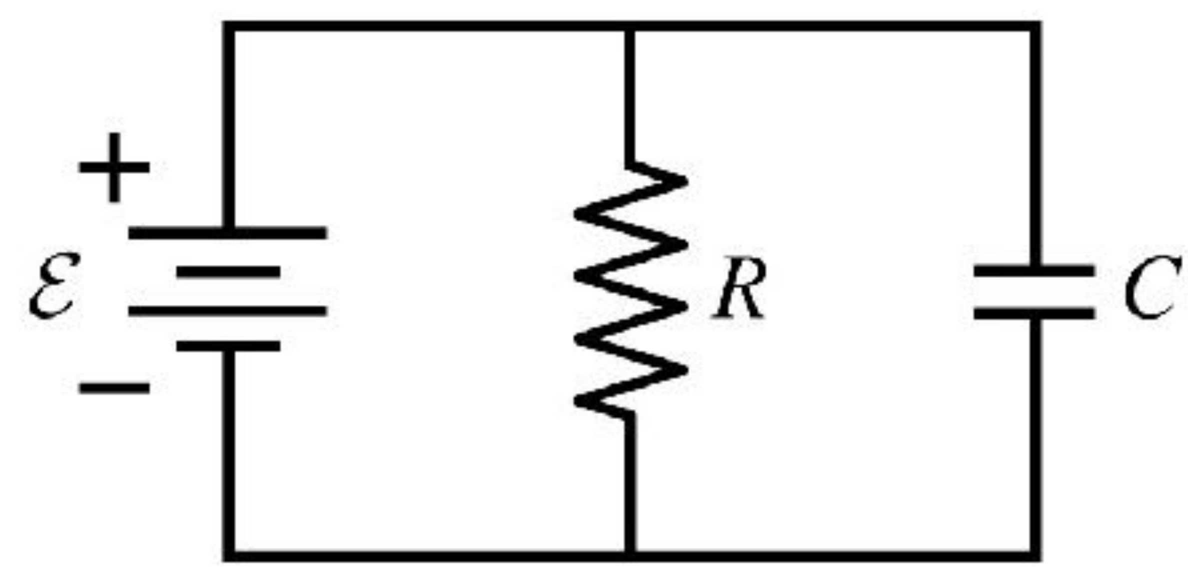
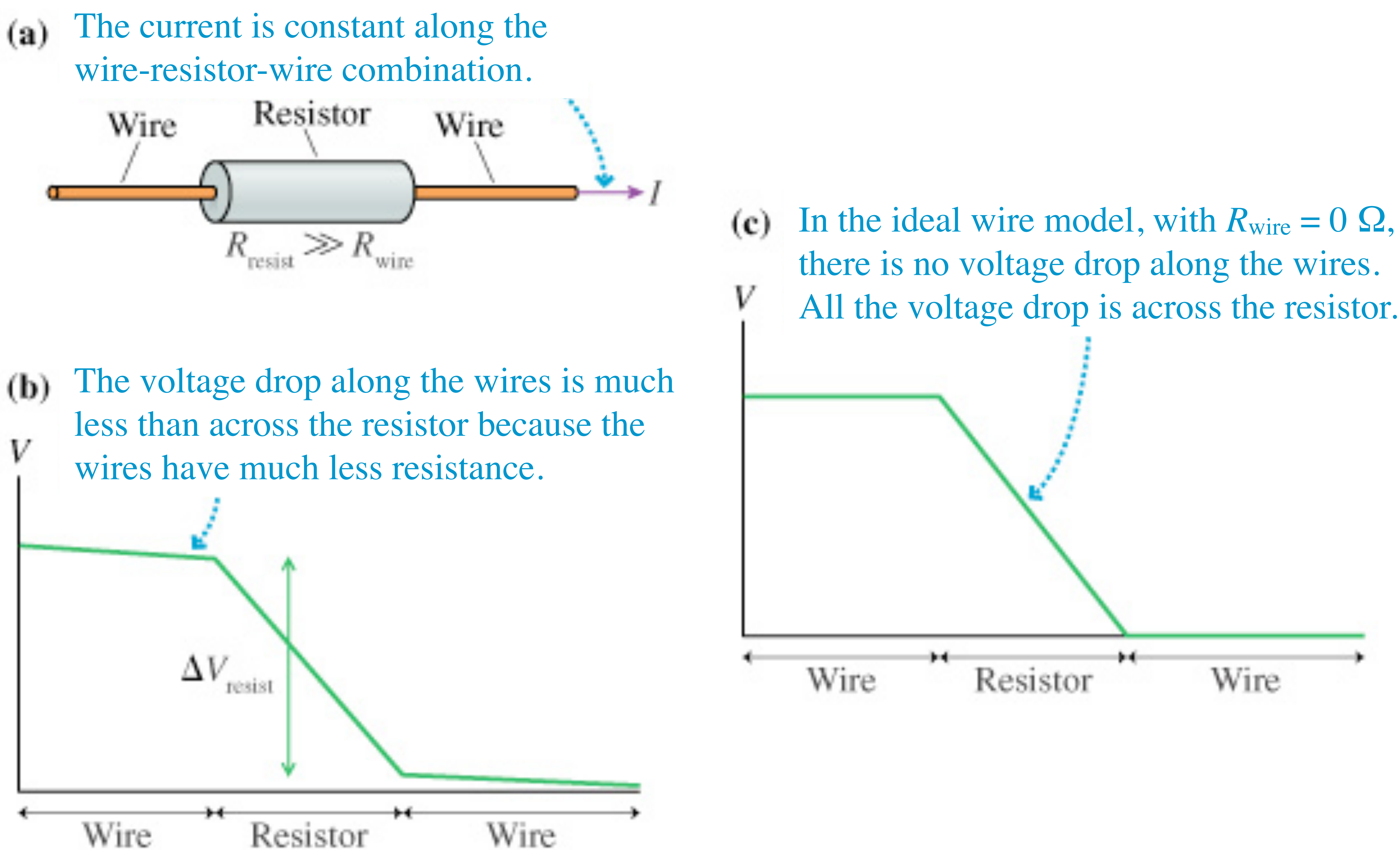
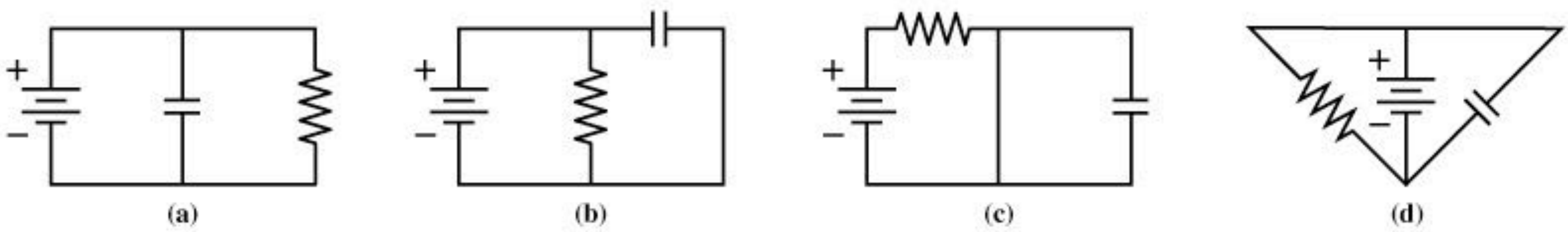


Ideal-wire model



Discussion Question

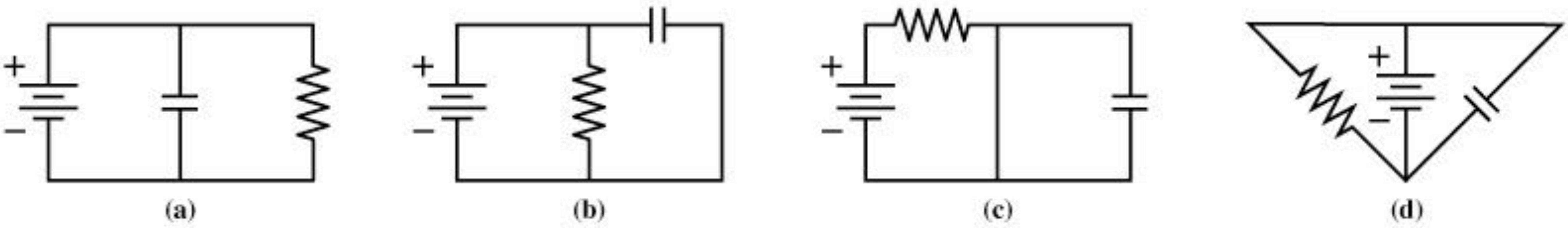
Which of these diagrams represent the same circuit?



A. all of them.
B. a, b, c.
C. a, b, d.
D. a, c, d.
E. b, c, d.

Discussion Question

Which of these diagrams represent the same circuit?



A. all of them.

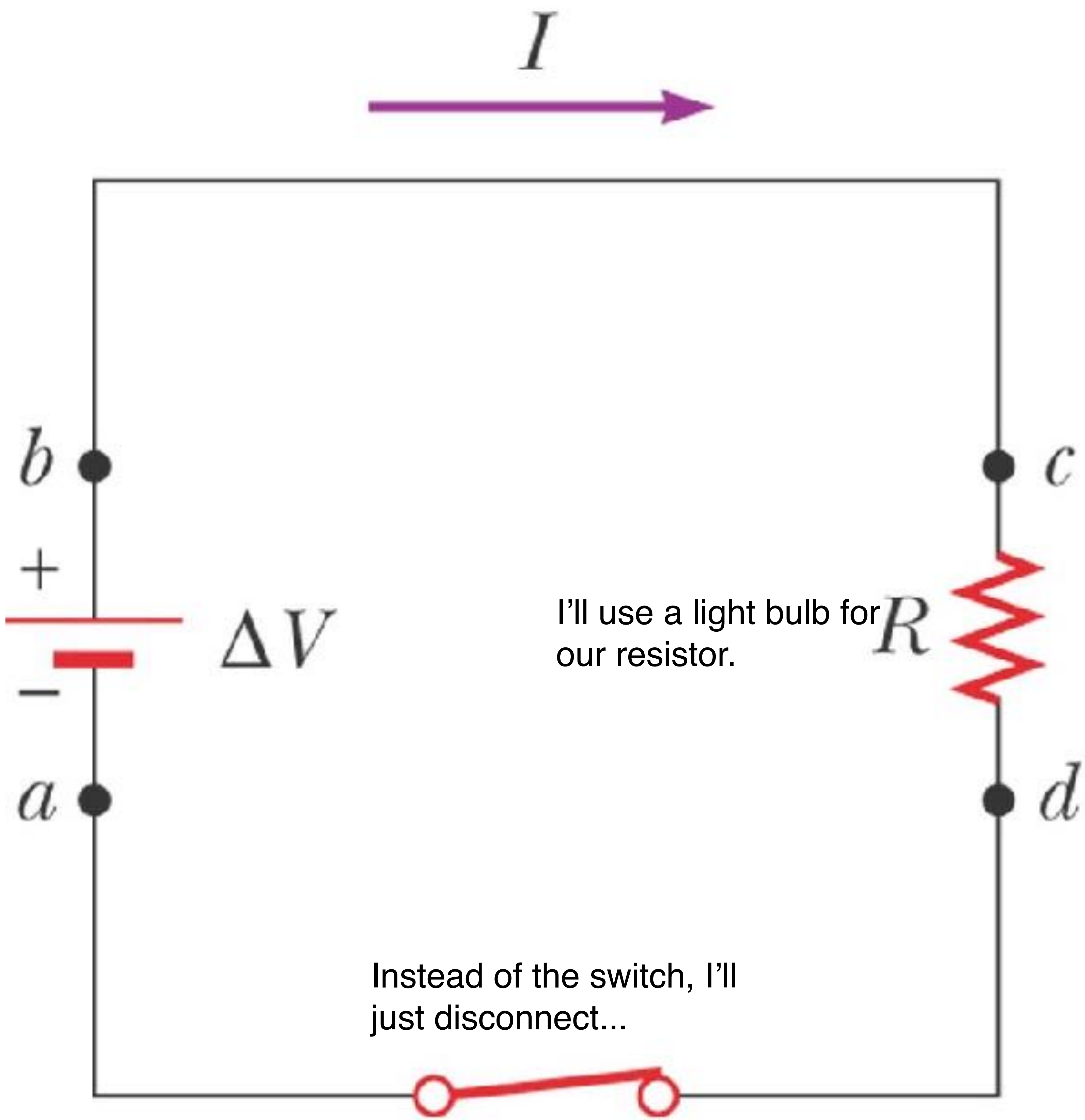
B. a, b, c.

C. a, b, d.

D. a, c, d.

E. b, c, d.

The basic circuit



I'll use a light bulb for our resistor.

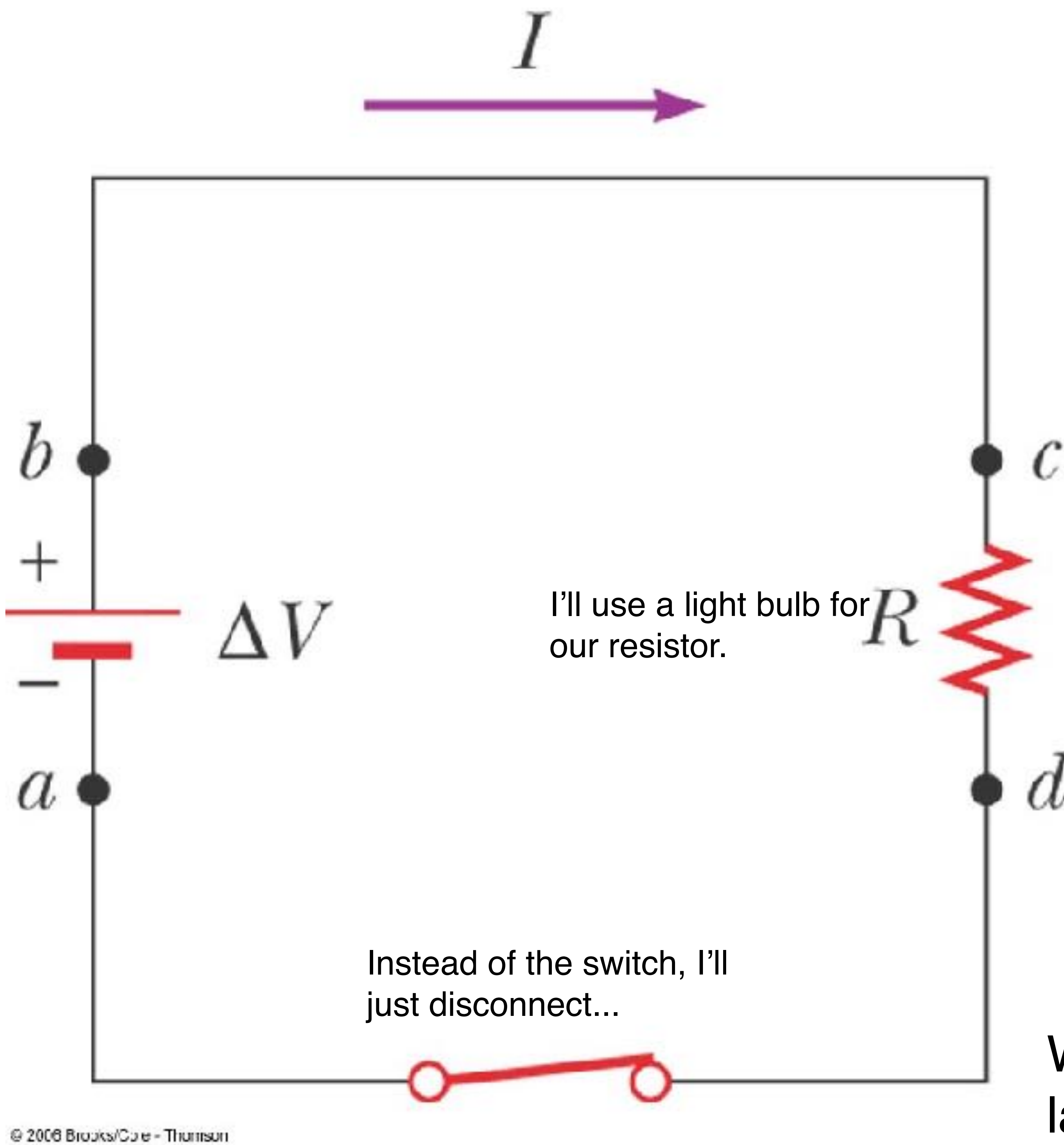
Instead of the switch, I'll just disconnect...

Let's analyze this circuit…

We can measure the potential differences at various points with a *voltmeter*.

We can measure the currents at various points with an *ammeter*.

The basic circuit



We see that the potential difference

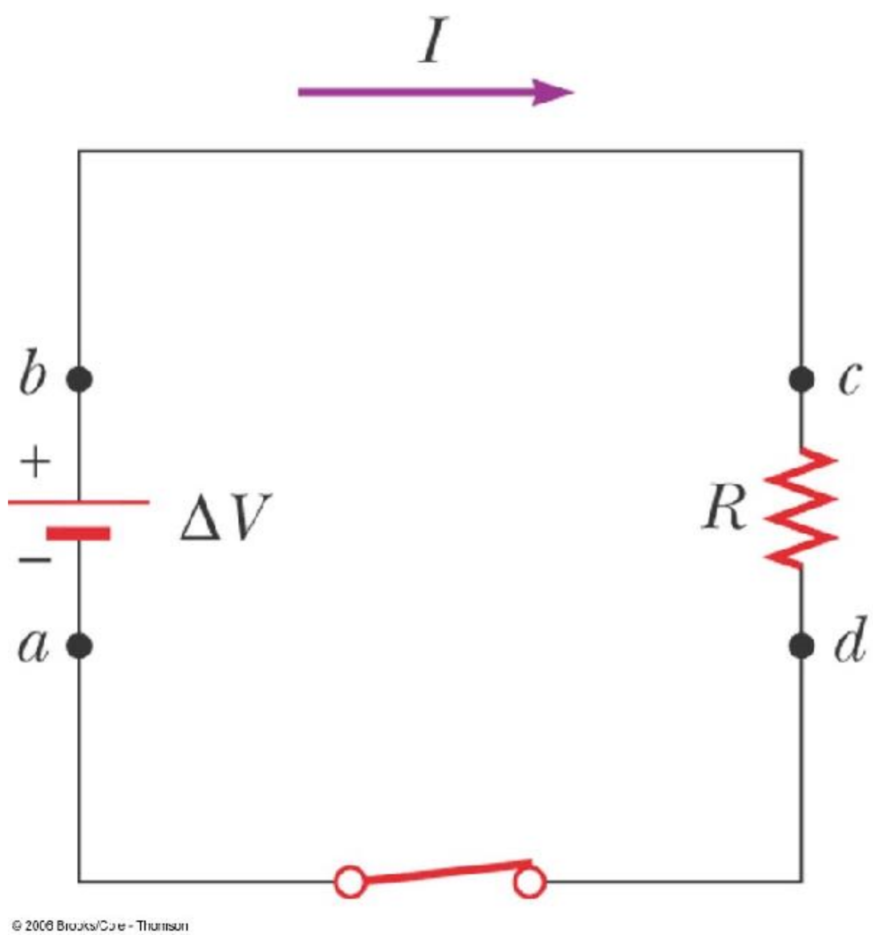
$V_b - V_a = \Delta V_{bat}$
$V_c - V_b = 0\ \text{V}$
$V_d - V_c = -\Delta V_{bat}$
$V_a - V_d = 0\ \text{V}$

In addition,

$I_{bc} = I_{da}$

What gets used up to make the lamp glow?

Energy and Power

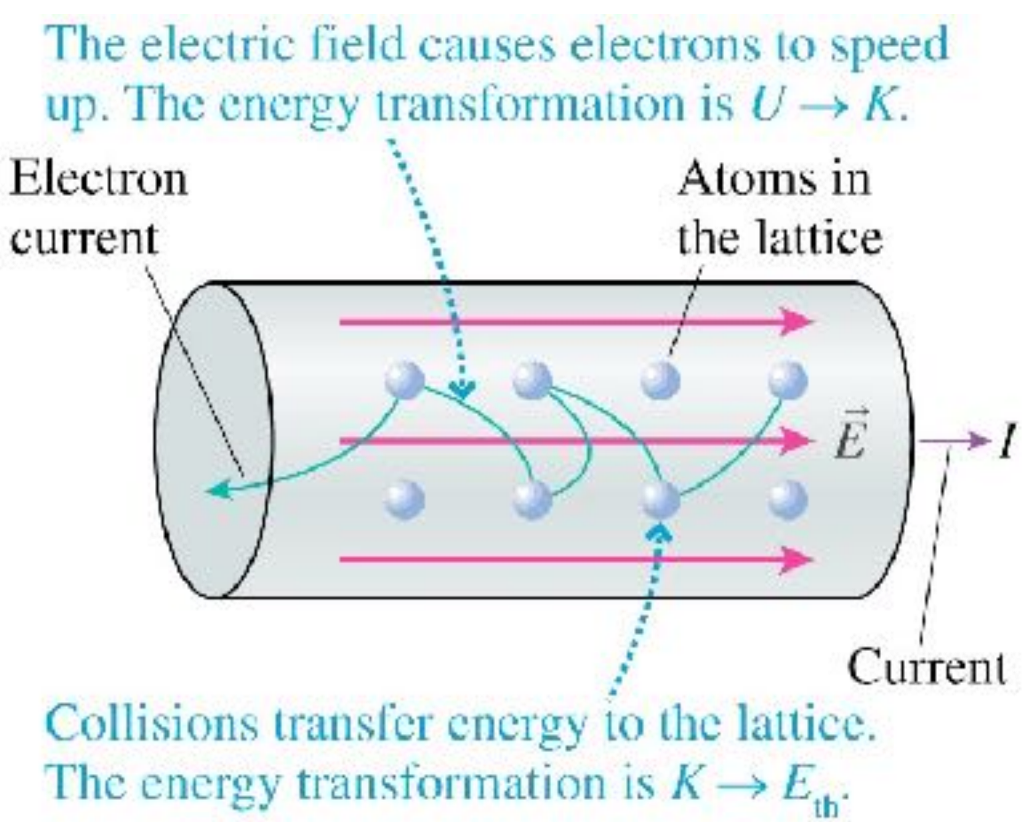


As a charge q is moved up from the - side of the battery to the + side, it gains potential energy $\Delta U = q\Delta V_{\text{bat}}$. This is supplied by the chemicals in the battery.

The rate at which the battery supplies energy is the power:

$\mathscr{P} = dU/dt = dq/dt\, \Delta V_{\text{bat}} = I\, \Delta V_{\text{bat}}$

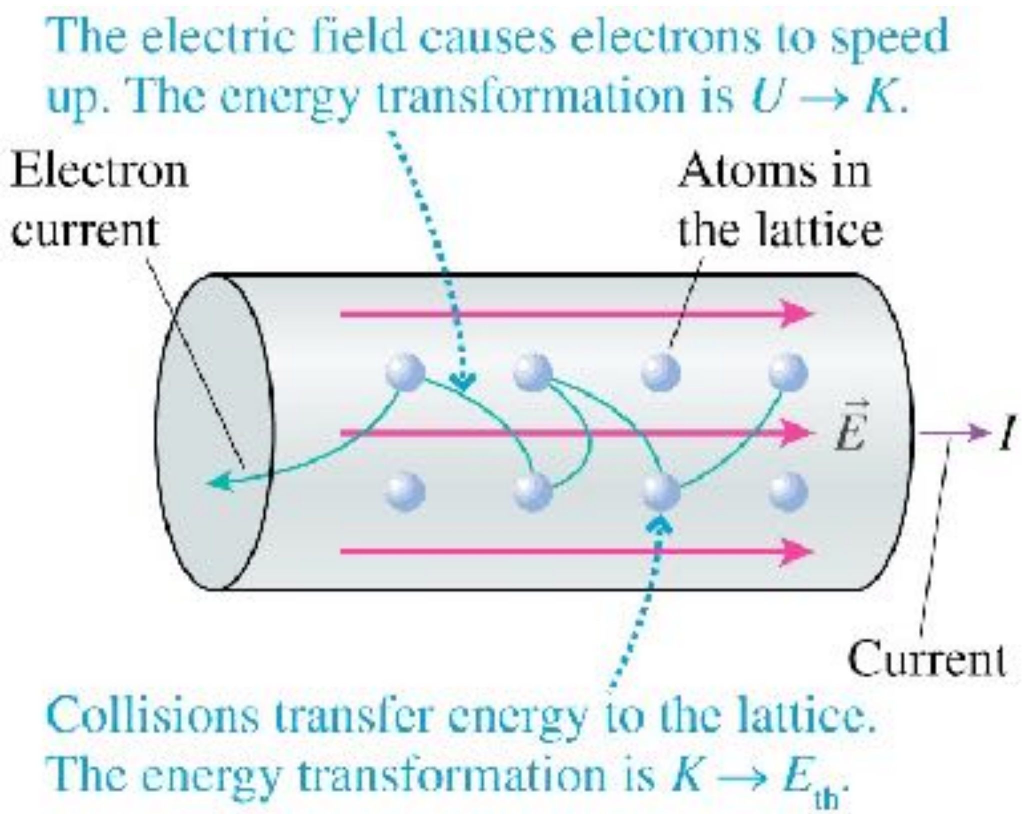
Energy and Power



Now let's look at the resistor.
In the resistor, the electrons are accelerated by the electric field (remember we have a current!) There is thus a transformation of potential energy into kinetic. The electrons collide with the atoms of the lattice of the resistor. This transforms their kinetic energy into thermal energy, and the resistor gets warm (in the case of a light bulb, warm enough to glow).

$$E_{\text{chem}} \rightarrow U \rightarrow K \rightarrow E_{\text{therm}}$$

Energy and Power



The electric force does work on the charge as it moves through the resistor. $W = F\Delta s = qEd$, assuming d is the distance between collisions.
This work increases the kinetic energy of charge q by $\Delta K = W = qEd$. This energy is then lost by the charge when it hits the lattice. The total energy transformed to the lattice is thus $E_{th} = qEL$, where L is the length of the resistor. But EL is just the potential difference across the resistor, ΔV_R. Thus $E_{th} = q\,\Delta V_R$.
The rate of energy transfer is $\mathscr{P}_R = dE_{th}/dt = dq/dt\,\Delta V_R = I\,\Delta V_R$.

But we saw earlier that $\Delta V_R = -\Delta V_{\mathrm{bat}}$, so the amount of power that the resistor dissipates is equal to the amount of power that the battery supplies.

Other power formulas

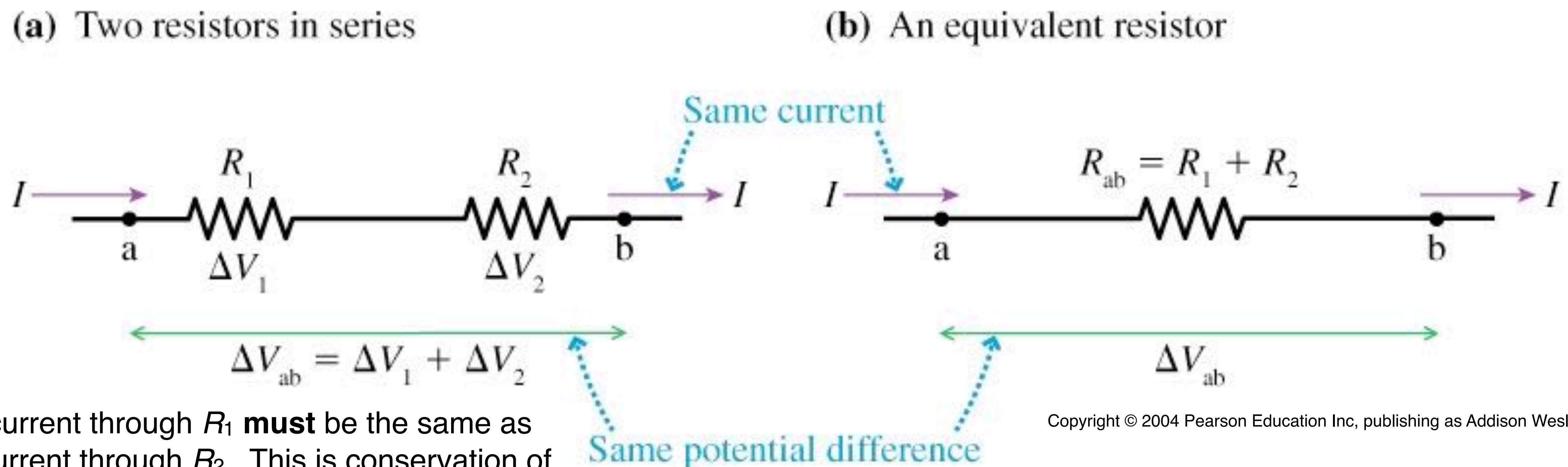
$$I = \frac{\Delta V}{R}$$

$$\mathscr{P} = I\Delta V = I^2 R = \frac{(\Delta V)^2}{R}$$

If the same amount of current is flowing through several resistors in series, the most power will be dissipated by the highest resistance. That is why the light bulb glows (high resistance) and the wires do not (low resistance).

This analysis is yet another example of conservation of energy.

Resistors in Series



The current through R_1 **must** be the same as the current through R_2. This is conservation of charge. Remember charge does ***NOT*** get "used up".

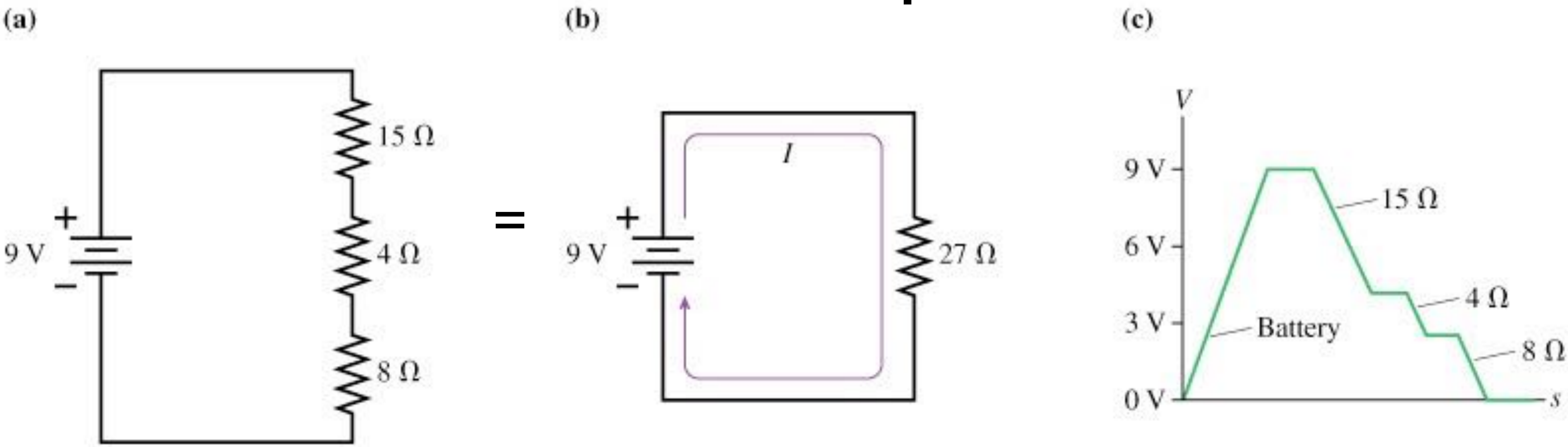
$$\Delta V_{ab} = \Delta V_1 + \Delta V_2 = IR_1 + IR_2 = I(R_1+R_2)$$

if we replace the two resistors by an equivalent single resistor with the same current and voltage difference we get

$$R_{eq} = \frac{\Delta V_{ab}}{I} = \frac{I(R_1 + R_2)}{I} = R_1 + R_2$$

$$\boxed{R_{eq} = R_1 + R_2 + R_3 + \cdots}$$

An example



$$R_{eq} = 15\Omega + 4\Omega + 8\Omega = 27\Omega$$

$$I = \frac{\Delta V_{bat}}{R_{eq}} = \frac{9\text{V}}{27\Omega} = 0.333\text{A}$$

$$\Delta V_{R1} = -IR_1 = -5.00\text{V}$$

$$\Delta V_{R2} = -IR_2 = -1.33\text{V}$$

$$\Delta V_{R3} = -IR_3 = -2.67\text{V}$$

$$\Delta V_{R_1} + \Delta V_{R_2} + \Delta V_{R_3} = -9.00\text{V}$$

emf and Real batteries

So far, we have been talking about the potential of a battery and the chemical reaction being the same. This is not completely true.

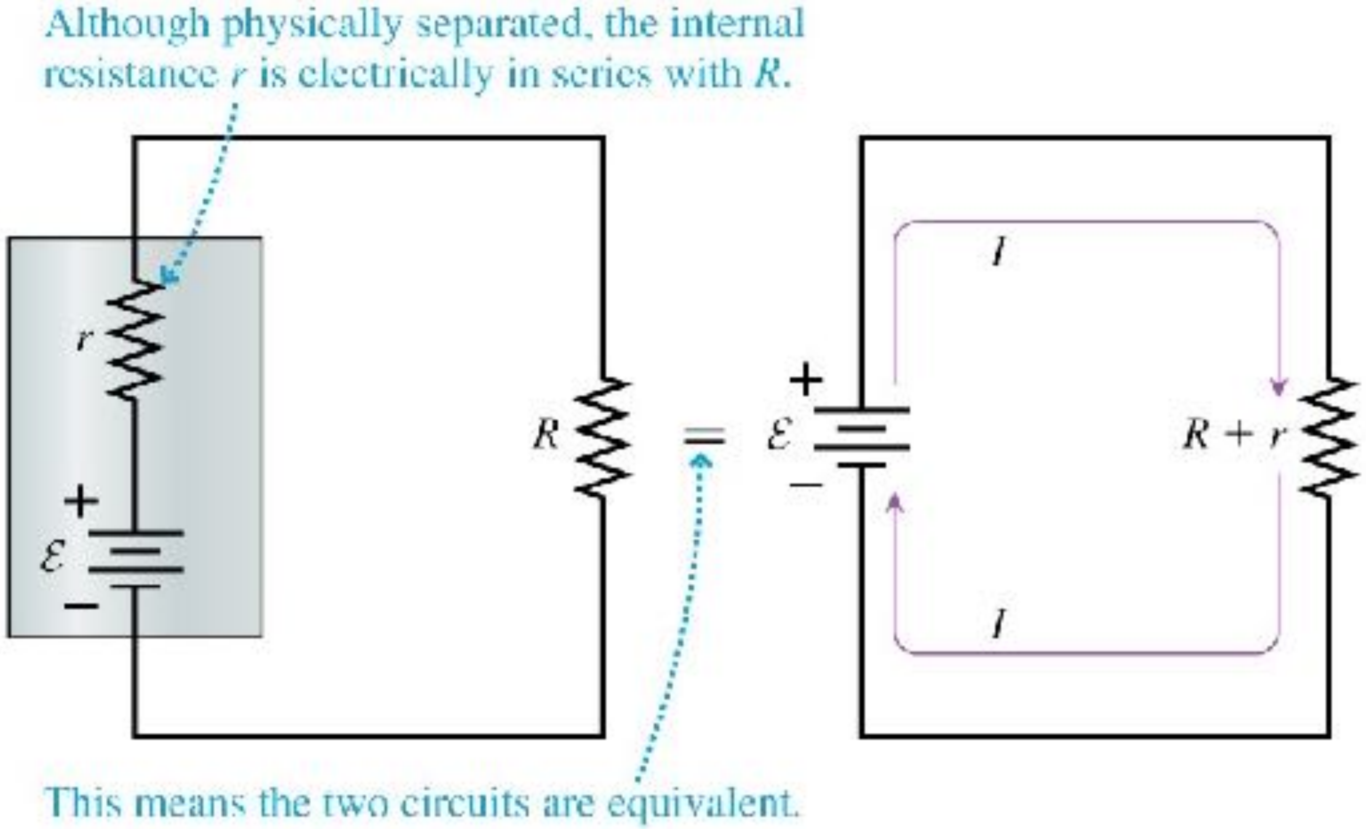
What is true is that $\Delta U = W_{\text{chem}}$. This is the work needed to get a charge from the minus terminal to the plus terminal. The amount of work per charge is W_{chem}/q, and this is called the ***emf*** of the battery with the symbol $\mathcal{E}$.

By definition, $\Delta V = \Delta U/q$, and hence, for an *ideal* battery,

$$\mathcal{E} = \frac{W_{\text{chem}}}{q} = \Delta V_{\text{bat}}$$

But real batteries also have *internal resistance*.

Real Batteries



The current in this circuit is

$$I = \frac{\mathcal{E}}{R_{\text{eq}}} = \frac{\mathcal{E}}{R + r}$$

The potential difference across the resistor R is

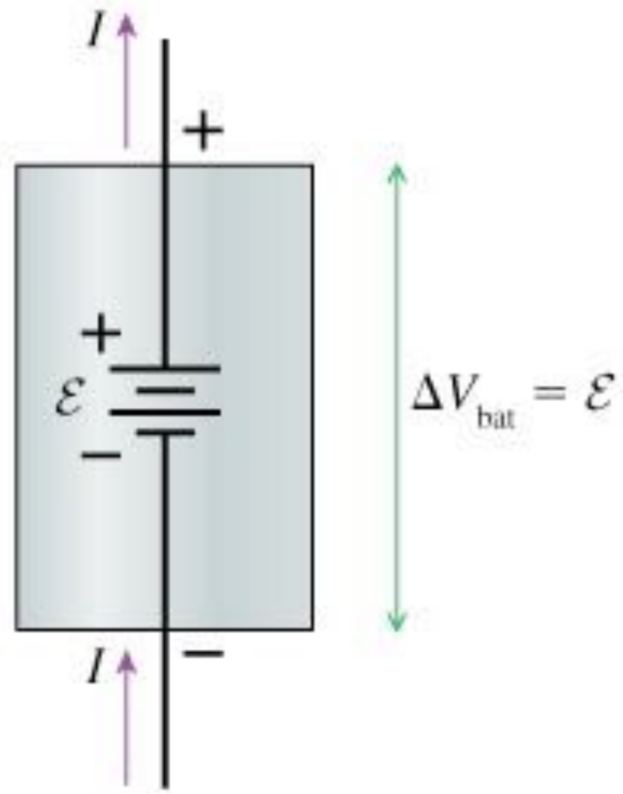
$$\Delta V_R = IR = \frac{R}{R + r}\mathcal{E}$$

Similarly, the potential difference across the terminals of the battery is

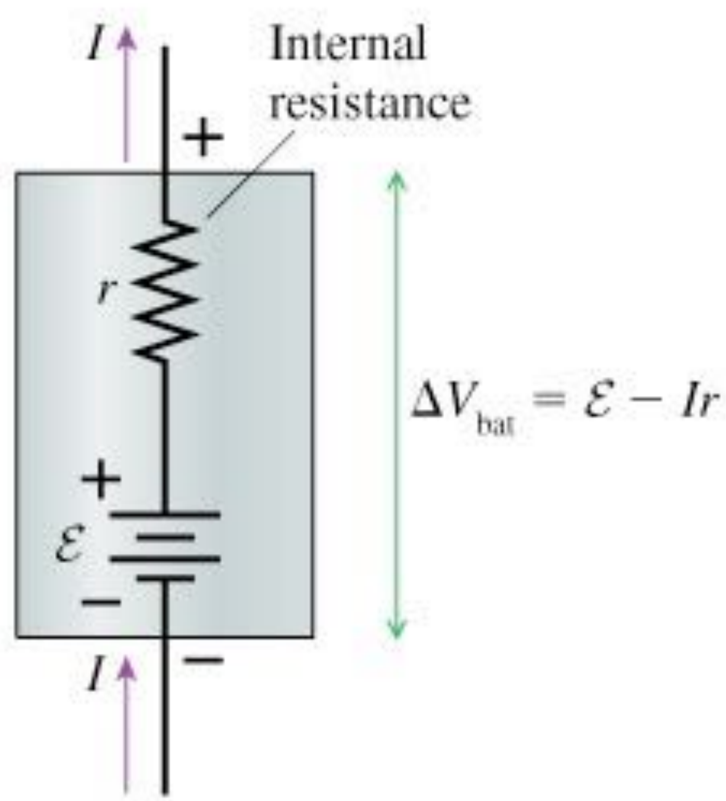
$$\Delta V_{\text{bat}} = \mathcal{E} - Ir = \mathcal{E} - \frac{r}{R + r}\mathcal{E} = \frac{R}{R + r}\mathcal{E}$$

The voltage of the battery is between the terminals, not the emf!

Real batteries



Ideal battery



Real battery

A real battery has internal resistance, r. Suppose there is a current in the battery, I. As the charges travel from the negative to the positive terminal they gain potential $\mathcal{E}$, but they lose potential $\Delta V_{int} = -Ir$

$\Delta V_{bat} = \mathcal{E} - Ir < \mathcal{E}$

Only when $I = 0$, is $\Delta V_{bat} = \mathcal{E}$

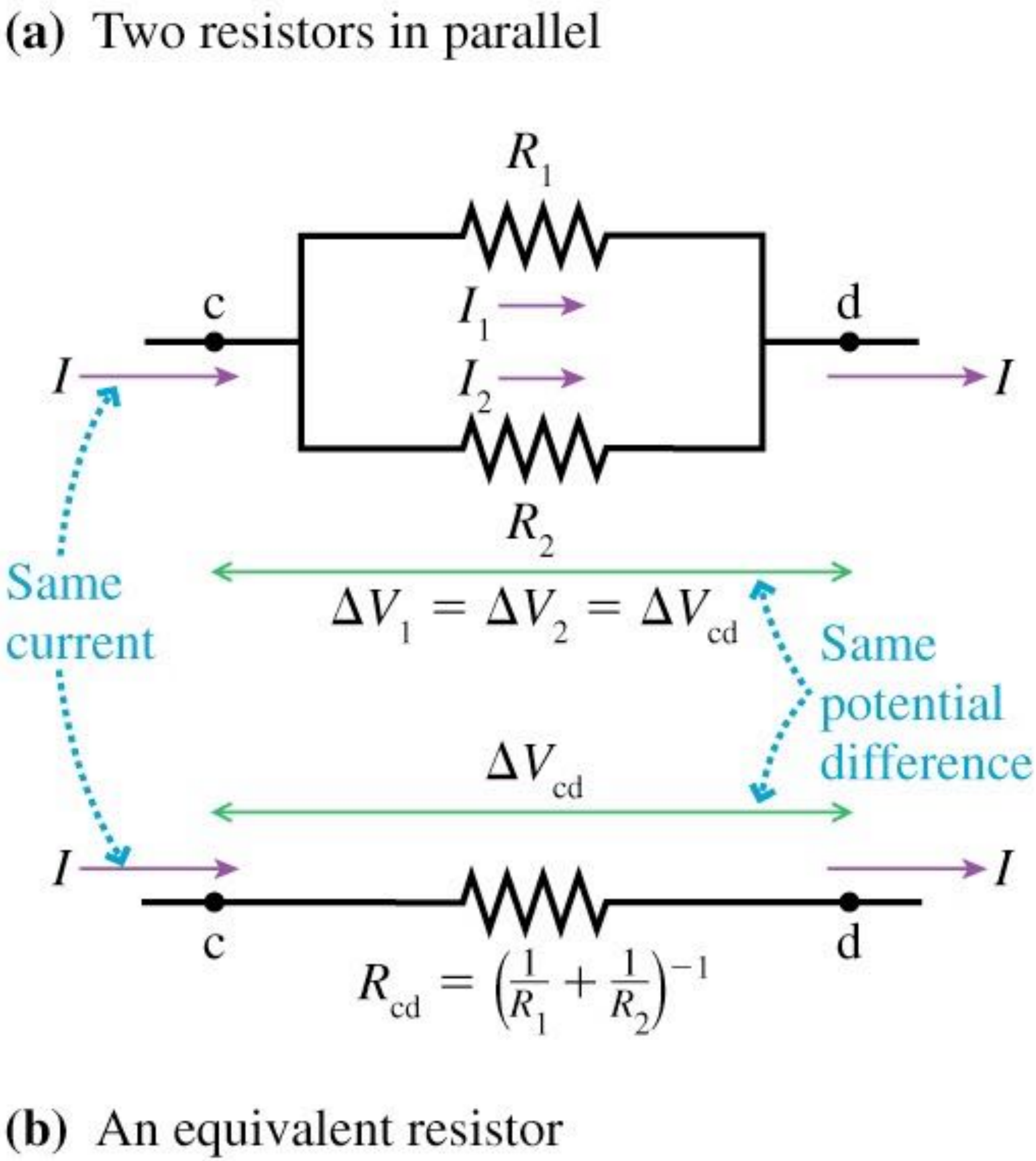
Weak batteries

In most cases, $r \ll R$, and we can treat batteries as ideal. However, as a battery dies (as the chemicals are used up), the internal resistance goes up.

This is why you can check a battery with a voltmeter (which has a very high resistance) and the battery voltage might look OK, but when you put it in your flashlight, it doesn't work.

Without a current (checking with a voltmeter), the battery will show full voltage. But with a current, the voltage drop across the internal resistance becomes very large, and little voltage is available for the flashlight bulb.

Resistors in parallel



Current must be conserved, so $I = I_1 + I_2$

$$I = \frac{\Delta V_1}{R_1} + \frac{\Delta V_2}{R_2}$$

but also, $\Delta V_1 = \Delta V_2 = \Delta V_{cd}$

$$I = \Delta V_{cd}\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

$$\boxed{\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots}$$

Why is that? Think of water pipes. Put many water pipes in parallel, and there is more places for the water to go!

Handy reminder for resistors

	I	ΔV	R
Series	Same	Add	$R_1 + R_2 + ...$
Parallel	Add	Same	$\left(\frac{1}{R_1} + \frac{1}{R_2} + \cdots\right)^{-1}$

Discussion Question

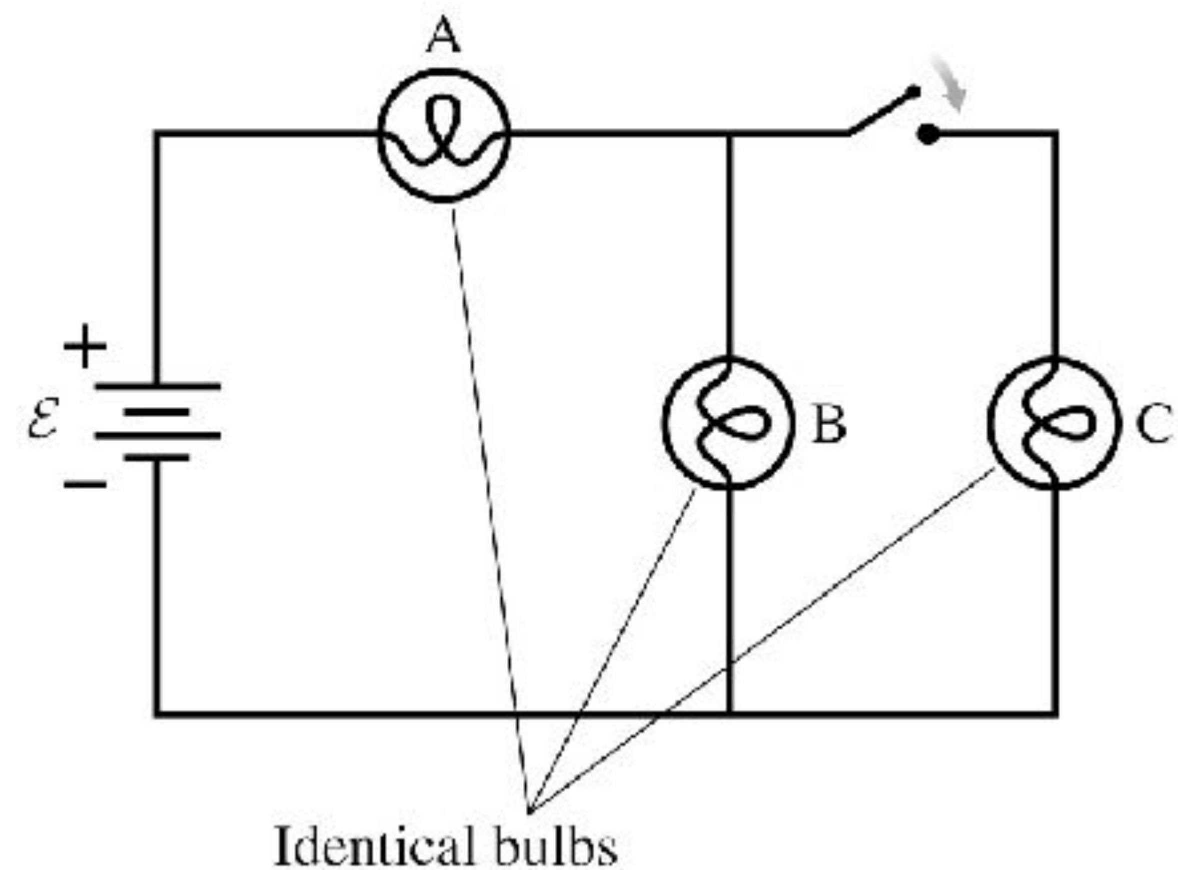
What will happen to bulb A when the switch is closed?

A. It won’t change brightness.

B. It will get brighter.

C. It will get dimmer.

D. Need more information.



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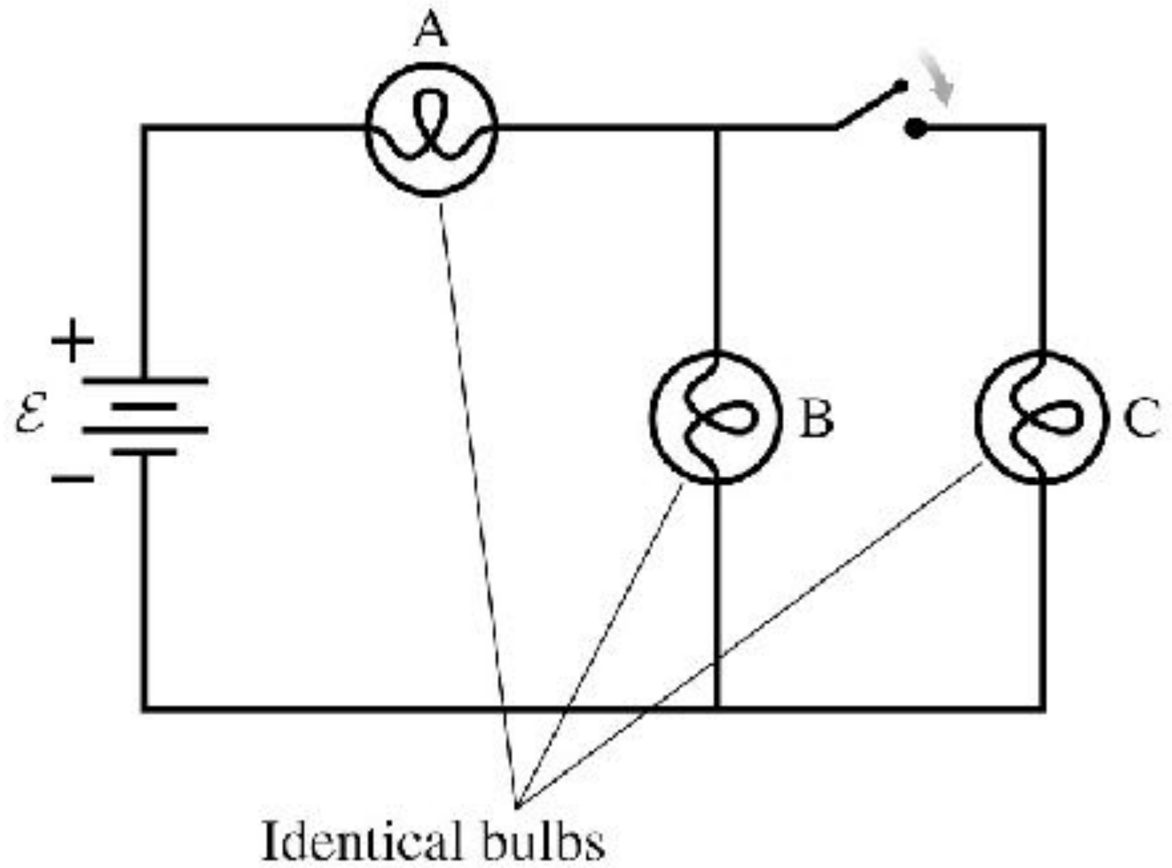

Closing the switch puts B and C in parallel, *reducing* the resistance. With a reduced resistance, the total current through the circuit increases, making A *brighter*.

Discussion Question

What will happen to bulb A when the switch is closed?

A. It won't change brightness.

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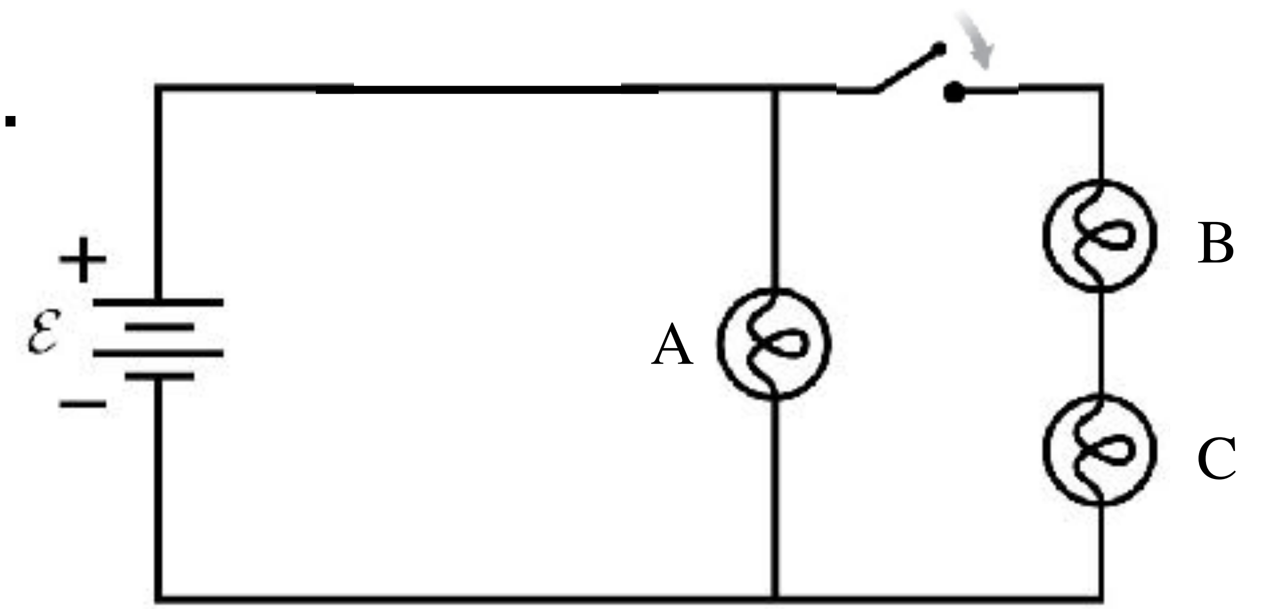
C. It will get dimmer.

D. Need more information.



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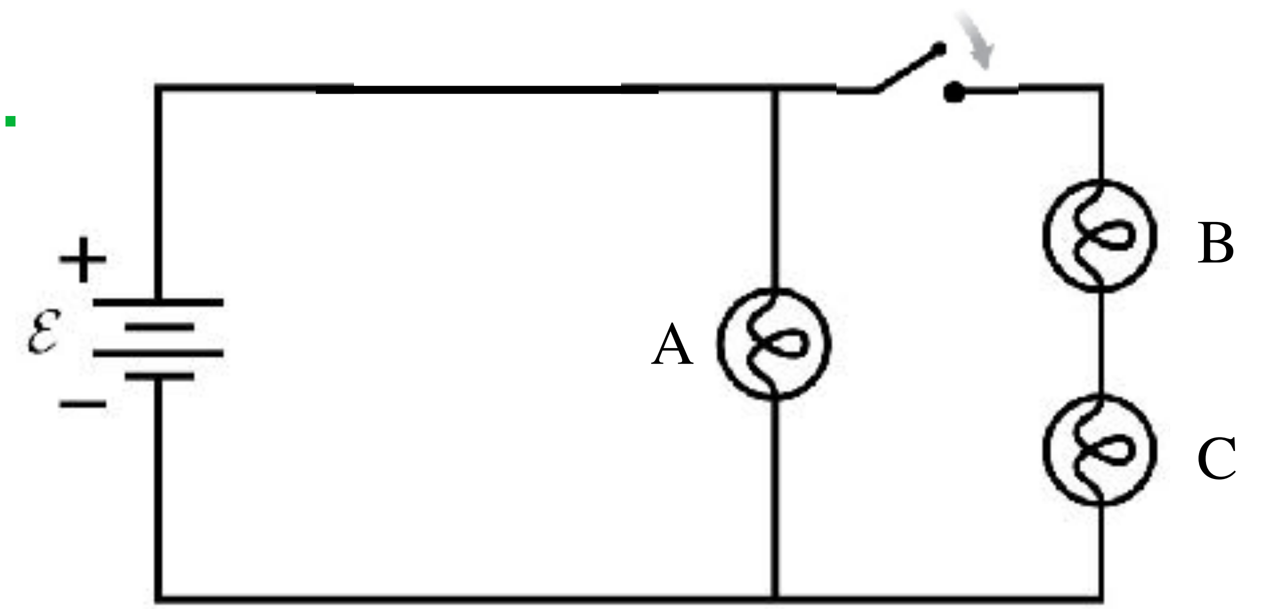
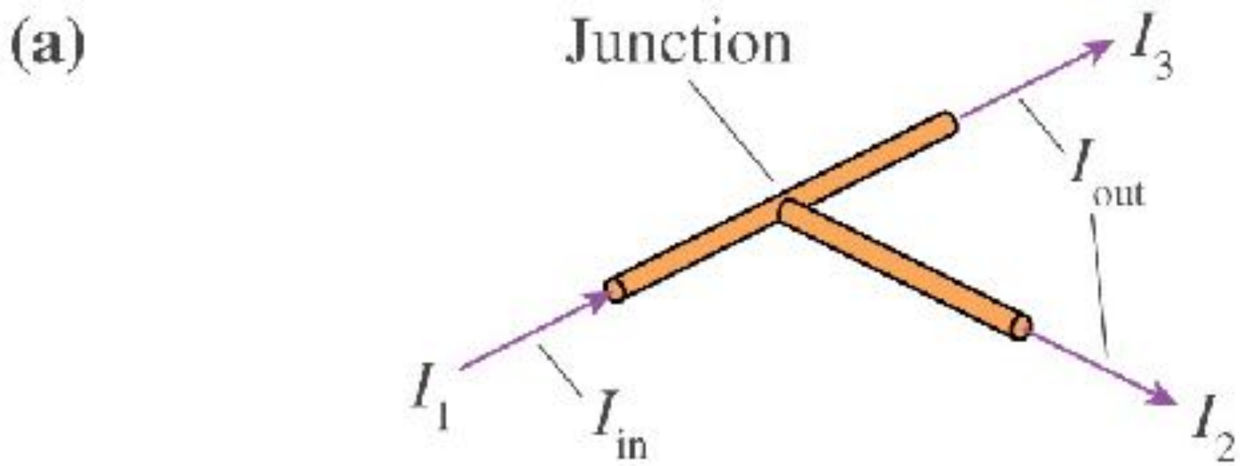
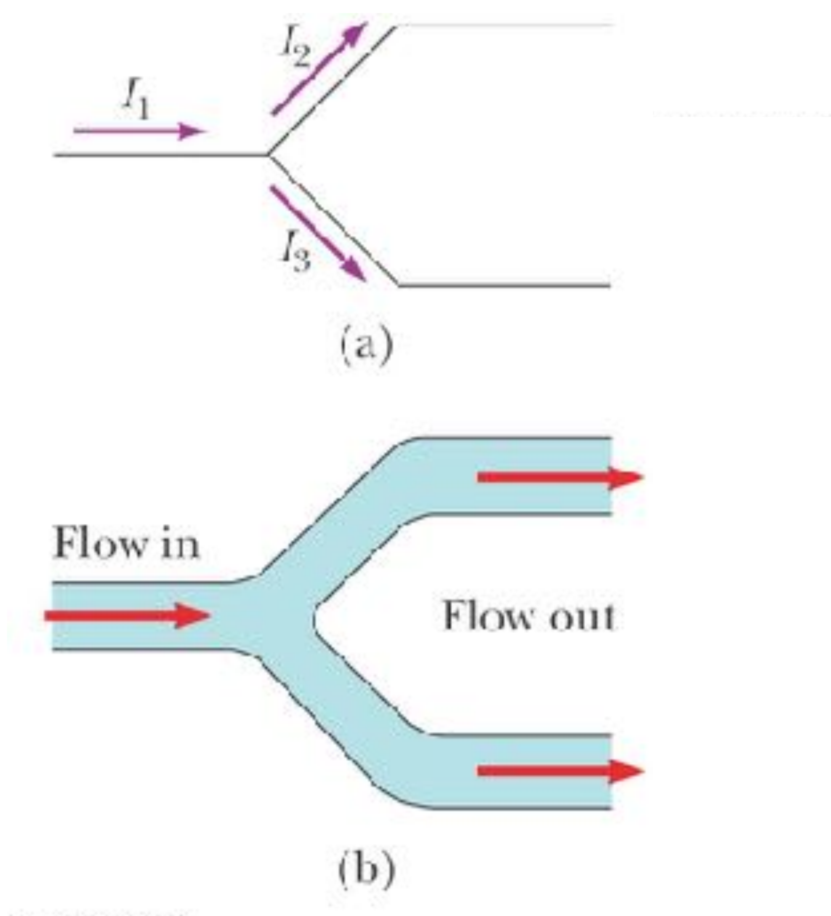


A battery is a voltage source. Adding BC in parallel with A does not change the potential difference across A, so it does not change the current through A, nor the power dissipated by A.

Kirchhoff’s junction rule



Junction law: $I_1 = I_2 + I_3$



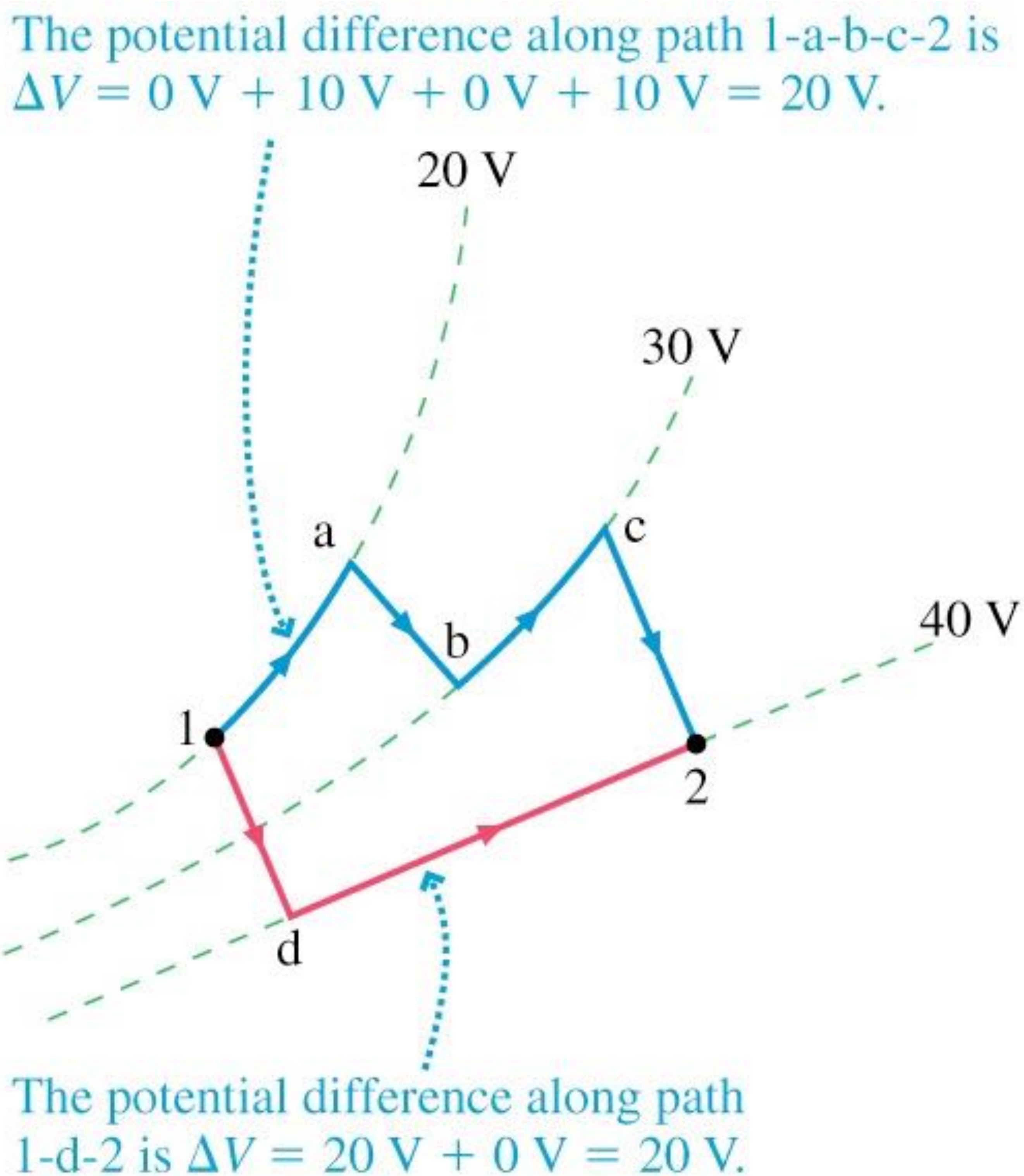
At any junction, the current going in to the junction must equal the current going out:

$$\sum I_{\text{in}} = \sum I_{\text{out}}$$

(I find this much easier to understand than what your textbook uses. The textbook actually mixes things up a bit… Sorry about that!)

This is nothing more than conservation of charge:
“What goes in, must come out!”

Kirchhoff's loop rule



The potential difference along path 1-a-b-c-2 is 20 V.

The potential difference along path 1-d-2 is 20 V.

The potential difference along any path from 1 back to 1 is 0 V.

This is nothing more than conservation of energy.

The electric force is a conservative force.

Remember conservative forces and closed paths from mechanics!

Kirchhoff’s loop rule

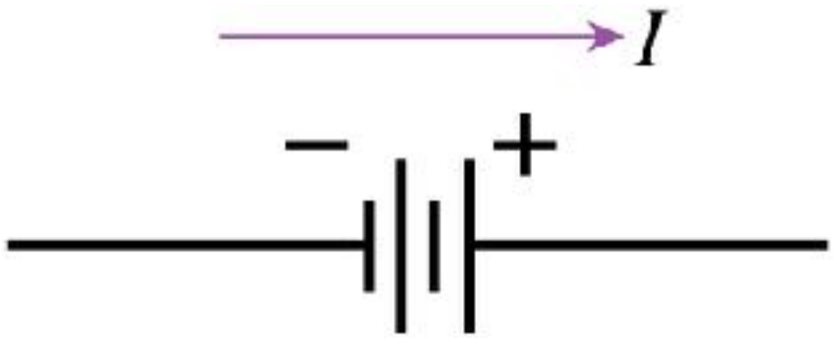
The sum of potential differences around a closed loop must be zero.

$$\Delta V_{\text{loop}} = \sum_{1}^{N} \Delta V_i = 0$$

This rule can only work if at least one of the potential differences is negative! You have to be very careful identifying the signs of the potential differences in using Kirchhoff’s loop rule.

Signs on ΔV

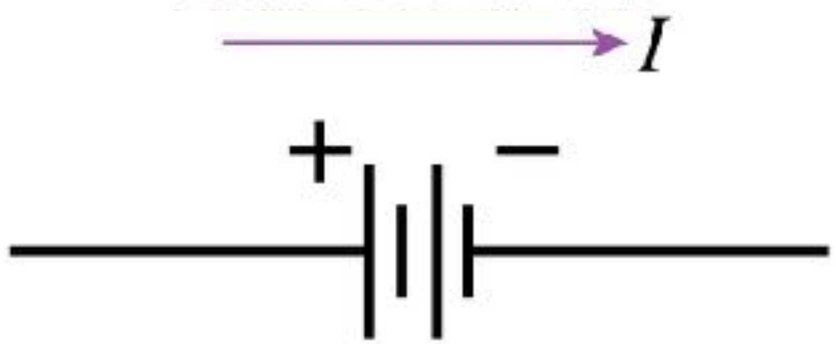
$$\Delta V = V_{\text{downstream}} - V_{\text{upstream}}$$



Potential increases

Ideal battery – to +: $\Delta V = +\mathcal{E}$

(Normal case)



Potential decreases

Ideal battery + to –: $\Delta V = -\mathcal{E}$

(Reversed case; this is how rechargeable batteries are recharged.)

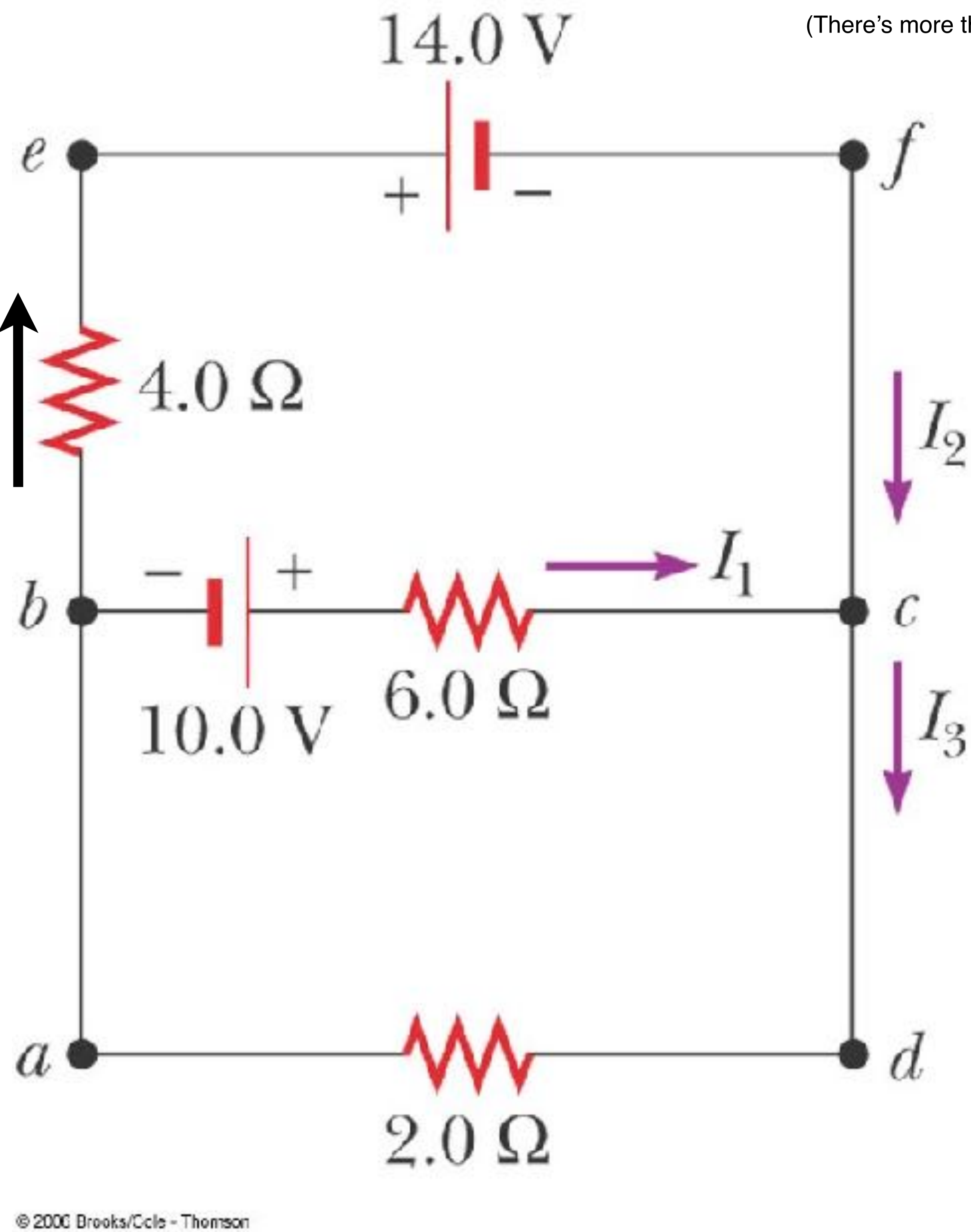
Potential decreases

Resistor: $\Delta V = -IR$

(But only if you are going in the same direction as the current!)

Example

(There's more than one way to skin a cat.)



Loop bcfeb:

$+10.0\text{V} - 6.0\Omega I_1 + 14.0\text{V} + 4.0\Omega I_2 = 0$

Loop aefda:

$-4.0\Omega I_2 - 14.0\text{V} - 2.0\Omega I_3 = 0$

Junction c:

$I_1 + I_2 = I_3$

Note: junction b would give the same equation!

Simplify Loop bcfeb:

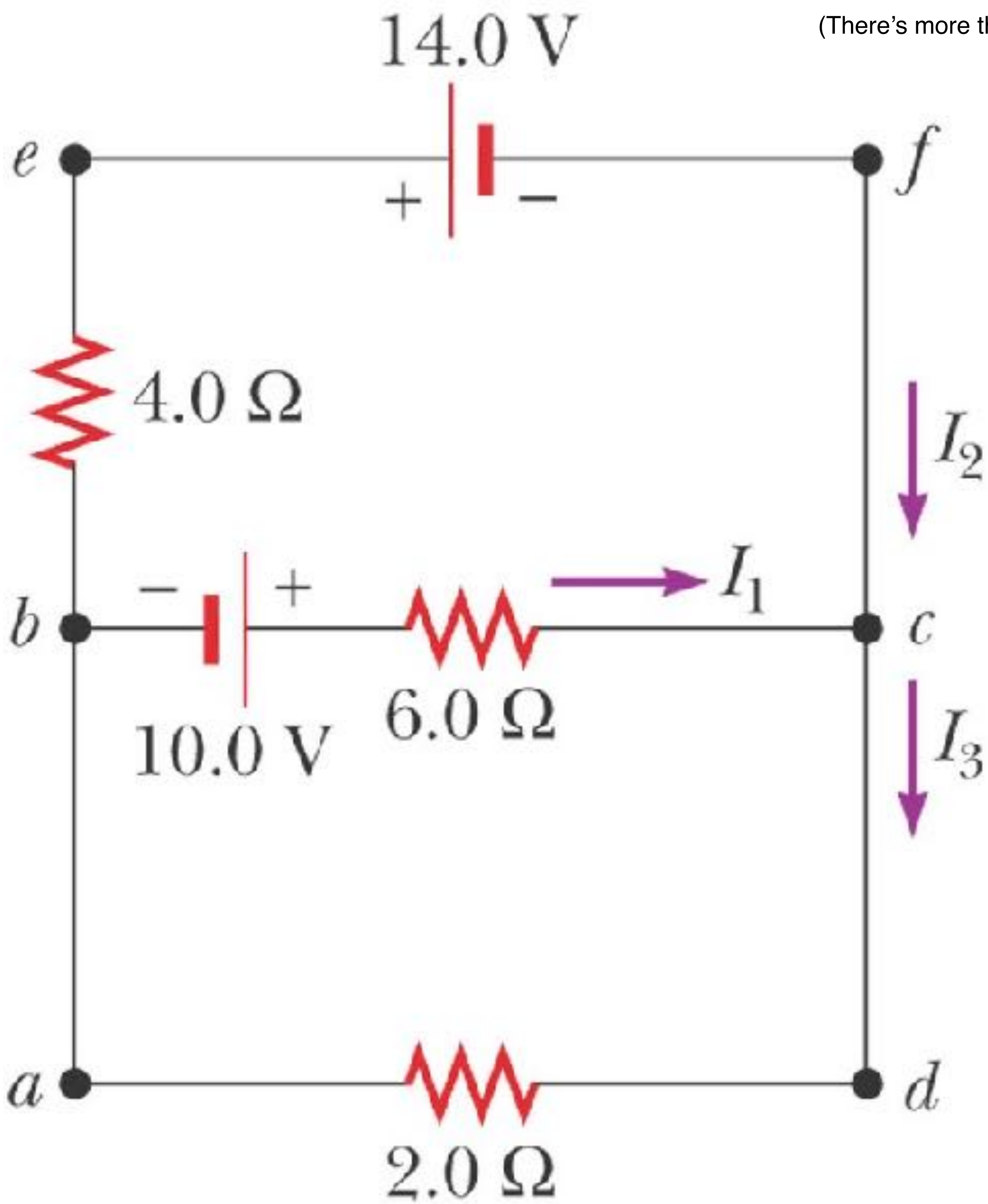
$+24.0\text{V} - 6.0\Omega I_1 + 4.0\Omega I_2 = 0$

$+12.0\text{V} - 3.0\Omega I_1 + 2.0\Omega I_2 = 0$

$+12.0\text{V} = +3.0\Omega I_1 - 2.0\Omega I_2$

Example

(There's more than one way to skin a cat.)



Loop bcfeb:

$+12.0 = +\,3.0I_1 - 2.0I_2$

Loop aefda:

$-\,4.0I_2 - 14.0 - 2.0I_3 = 0$

Junction c:

$I_1 + I_2 = I_3$

Put J_c into L_{aefda}:

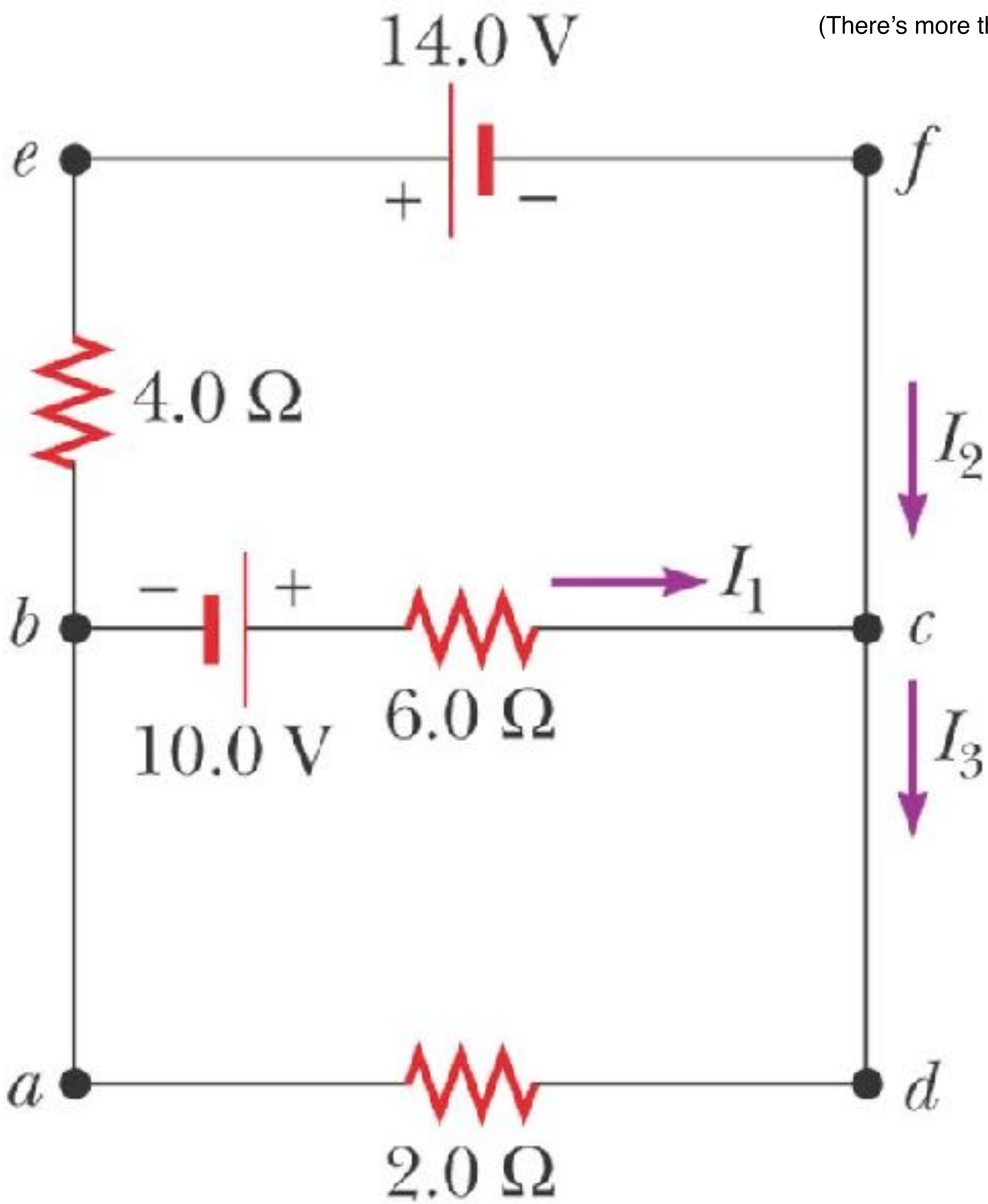
$-\,4.0I_2 - 14.0 - 2.0I_1 - 2.0I_2 = 0$

$-\,6.0I_2 - 14.0 - 2.0I_1 = 0$

$2.0I_1 + 6.0I_2 = -\,14.0$

Example

(There's more than one way to skin a cat.)



Loop bcfeb (1):

$$+12.0 = +3.0I_1 - 2.0I_2$$

Loop aefda + Junction c (2):

$$2.0I_1 + 6.0I_2 = -14.0$$

Take 3X(1) and add to (2):

$$+36.0 = +9.0I_1 - 6.0I_2$$

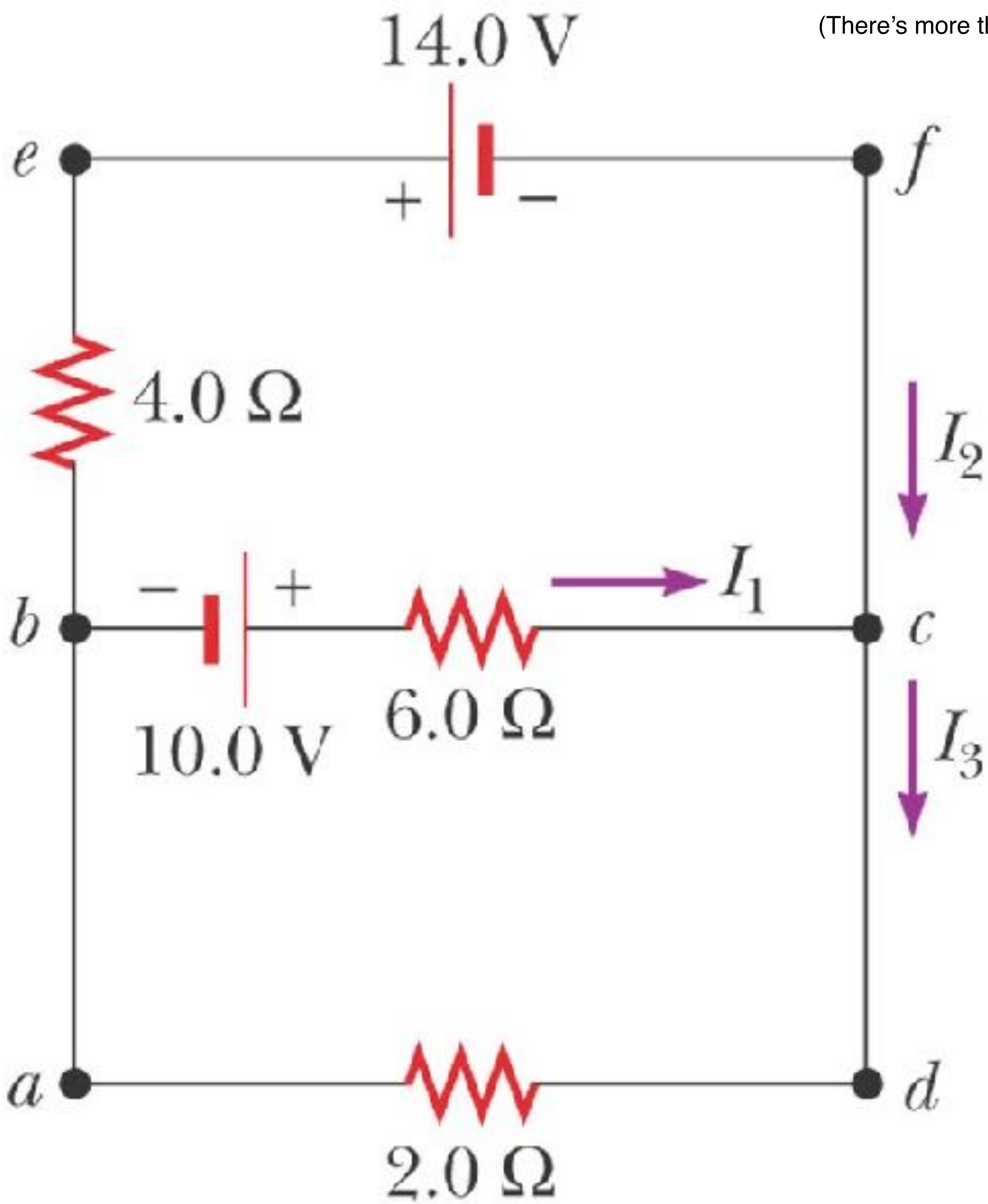
$$-14.0 = +2.0I_1 + 6.0I_2$$

$$+22.0 = +11.0I_1$$

$$I_1 = +2.0 \text{ A}$$

Example

(There's more than one way to skin a cat.)



Put:

$I_1 = +2.0$ A

Into loop bcfeb (1):

$+12.0 = +\ 3.0 I_1 - 2.0 I_2$

$+12.0 = +\ 6.0 - 2.0 I_2$

$-2.0 I_2 = 6.0$

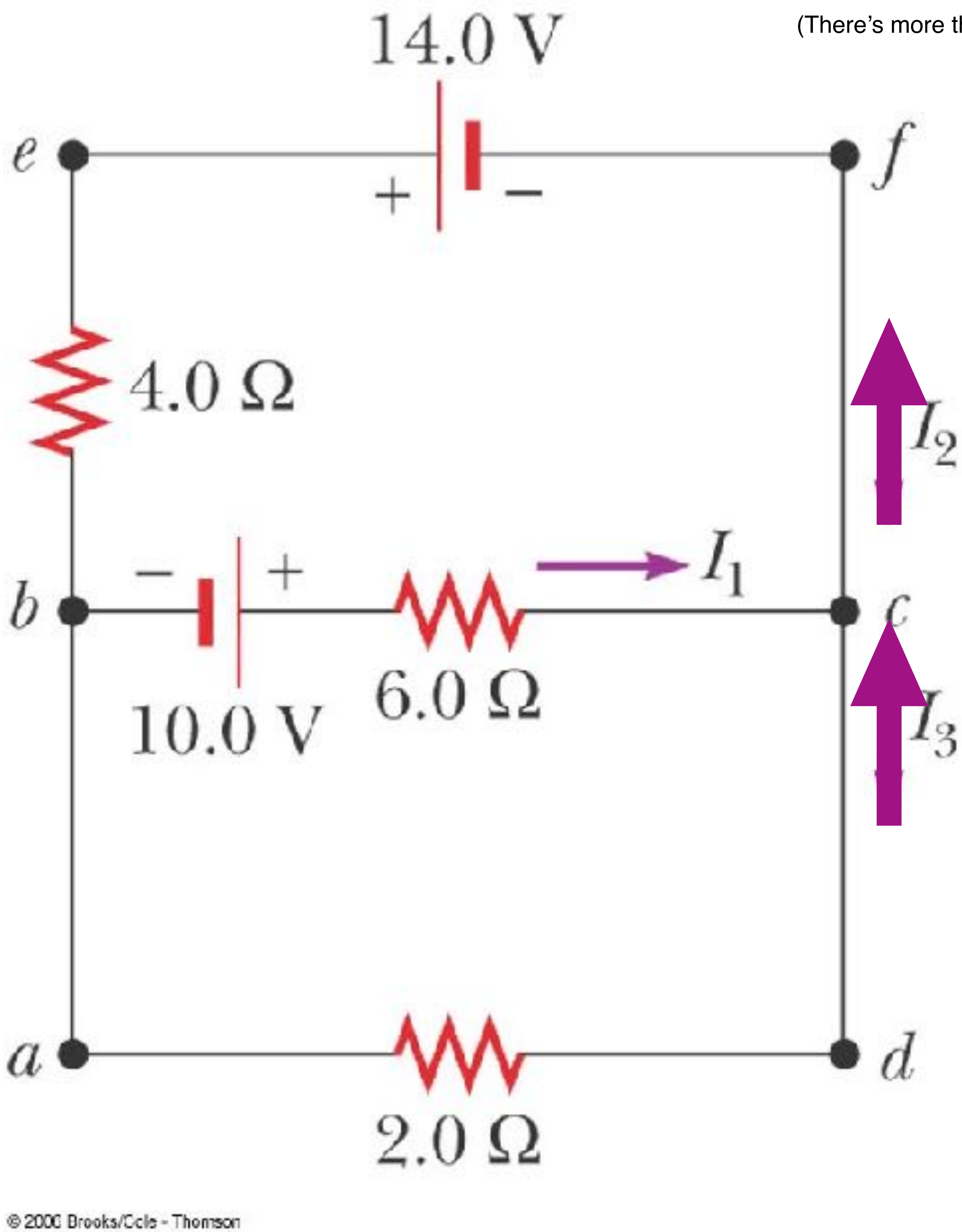
$I_2 = -3.0$A

And finally, $I_1 + I_2 = I_3$, so

$I_3 = -1.0$A

Example

(There's more than one way to skin a cat.)



Final result:

$I_1 = +2.0$ A

$I_2 = -3.0$A

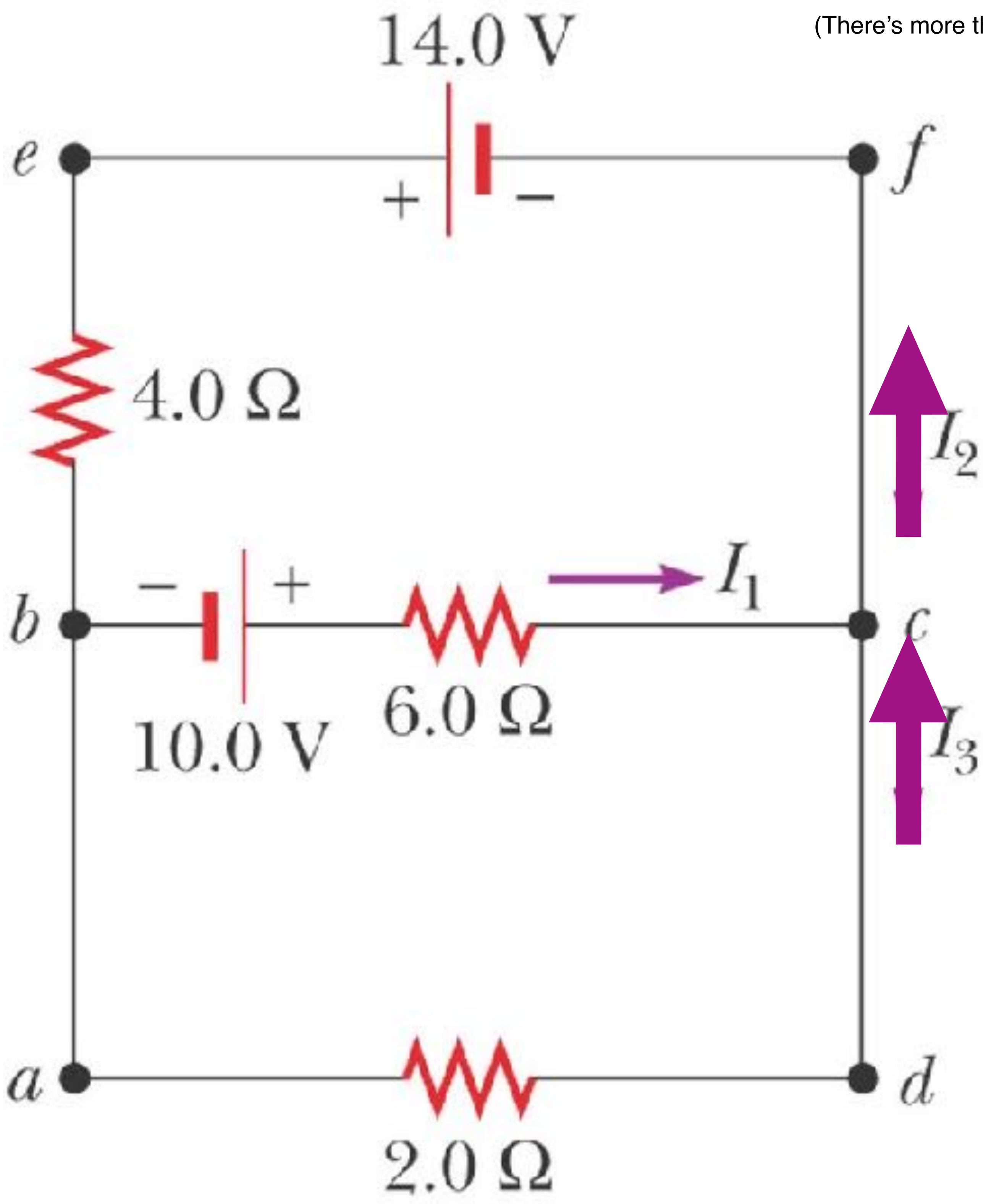
$I_3 = -1.0$A

What do the negative currents mean?

We chose the wrong initial guess on the direction!

Cross-check

(There's more than one way to skin a cat.)



Final result:

$I_1 = +2.0$ A

$I_2 = +3.0$A

$I_3 = +1.0$A

Loop febcf:

$+14.0 - 4.0 X 3.0 + 10.0 - 6.0 X 2.0 = 0$

Loop bcdab:

$+10.0 - 6.0 X 2.0 + 2.0 X 1.0 = 0$

Problem Solving for Resistor Circuits

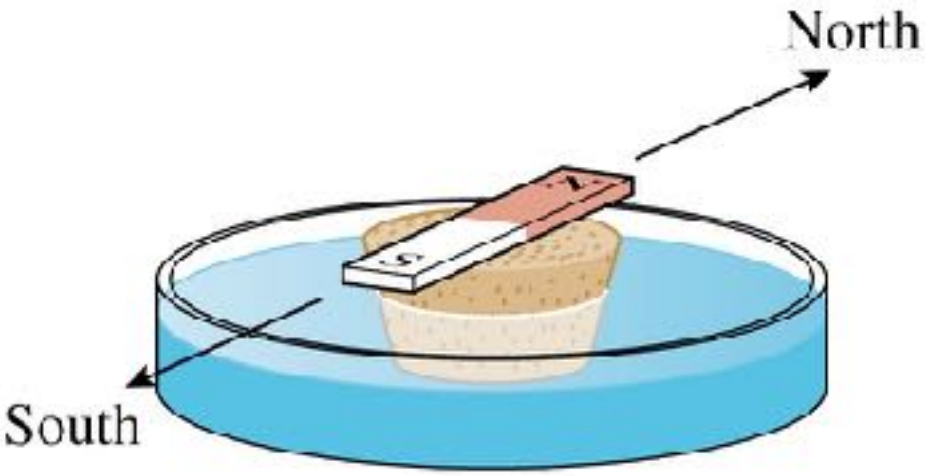
Model: Assume ideal wires and ideal batteries (unless told otherwise!)
Visualize: Draw a circuit diagram. Label all known and unknown quantities.
Solve: Use Kirchhoff's rules and the series and parallel rules for resistors.

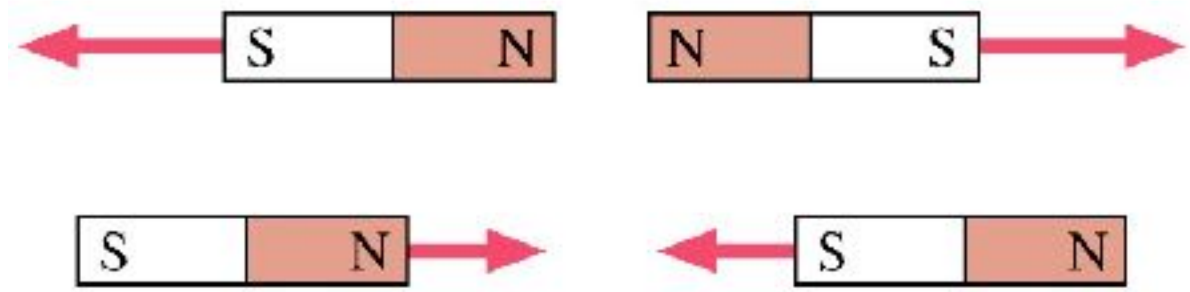
- Step by step, reduce the circuit to the smallest number of equivalent resistors
- Determine the current through and potential difference across the equivalent resistors
- Rebuild the circuit, using the facts that the current is the same through all resistors in series and that the potential difference is the same across all resistors in parallel.

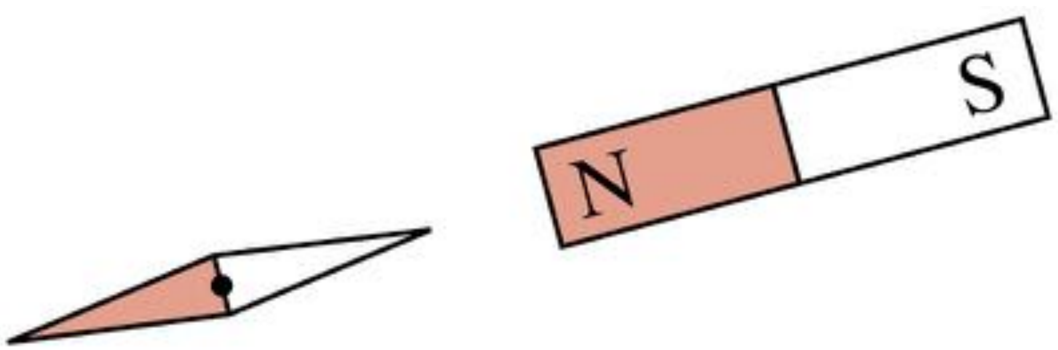
Assess: Use two important checks as you rebuild the circuit.

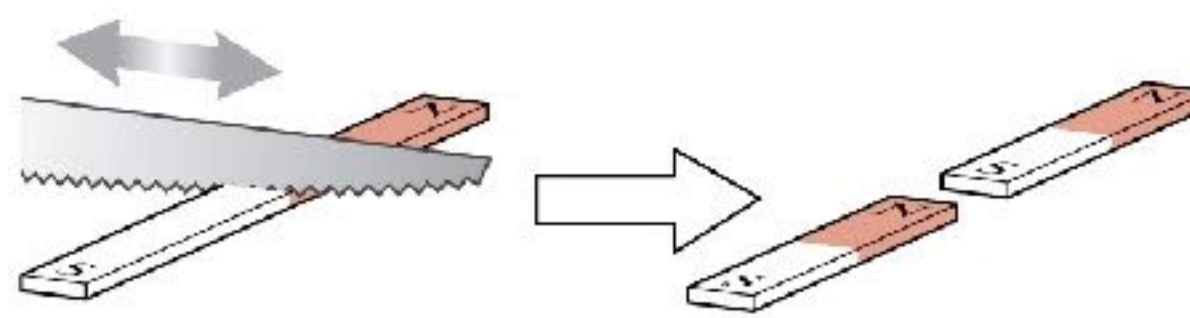
- Verify that the sum of potential differences across series resistors matches that for the equivalent resistor.
- Verify that the sum of the currents through parallel resistors matches the current through the equivalent resistor.

Discovering Magnetism

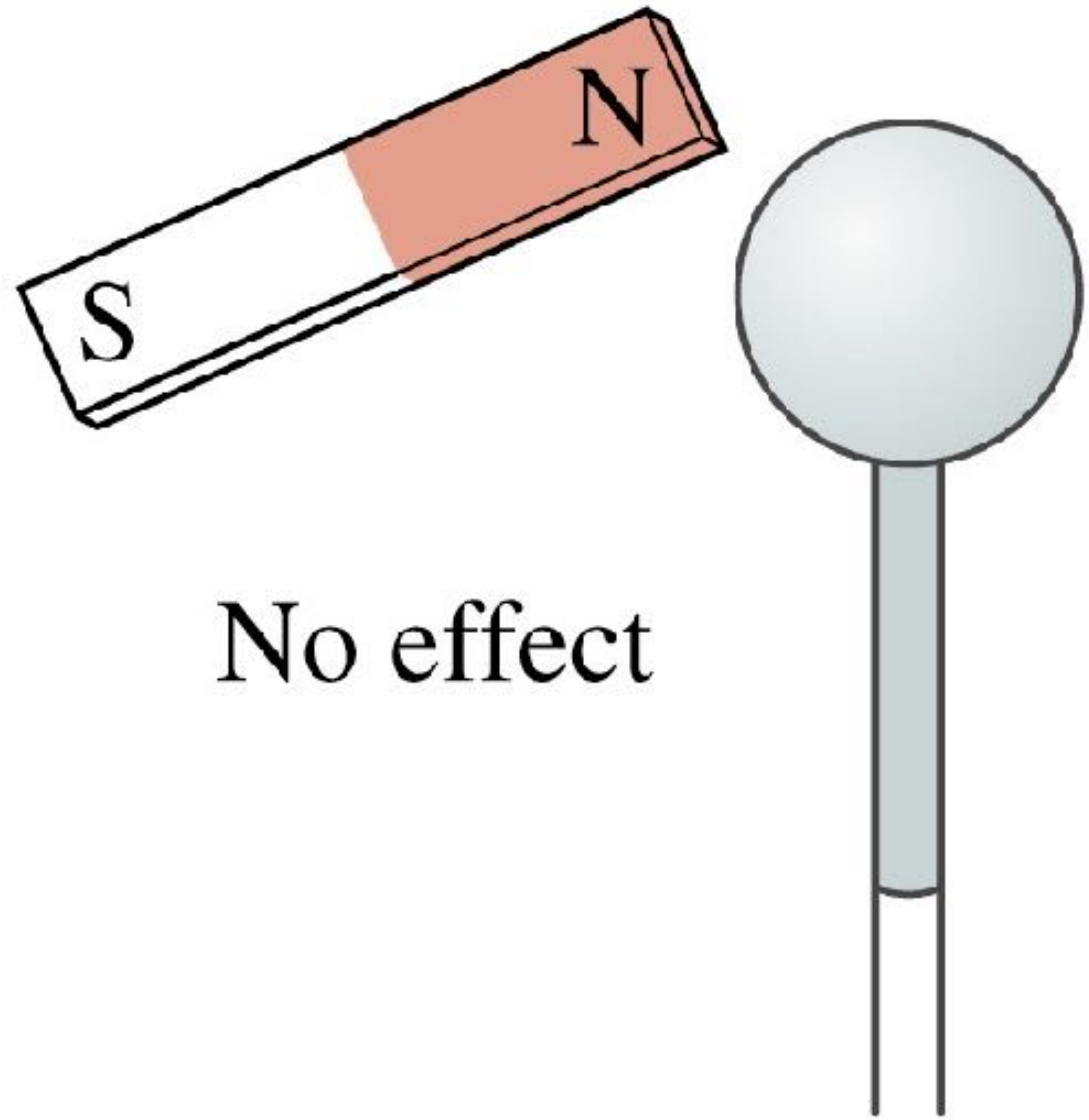








Discovering Magnetism



Some Conclusions

1. **Magnetism is not the same as electricity.** Magnetic poles and electric charges share some similar behavior, but they are **not** the same. The magnetic force is a **new** force of nature that we have not yet explored.
2. Magnetism is a long range force. Paper clips leap up to the magnet. You can feel the pull as you bring a fridge magnet close to the refrigerator.
3. Magnets have two poles, which we call the north and south poles. Two like poles repel; two opposite pole attract. This is *analogous* to electric charges but magnetic poles and electric charges are **not** the same.

More Conclusions

4. The poles of a bar magnet can be identified using it as a compass. Other magnets, such as fridge magnets or horseshoe magnets can't readily be used as compasses, but we can identify their poles using a bar magnet. A pole that attracts a known north pole and repels a known south pole must be a south pole.

5. Materials that are attracted to a magnet, or that a magnet sticks to, are called **magnetic materials**. The most common magnetic material is iron. Magnetic materials are attracted to both poles of a magnet. This is similar to how neutral objects are attracted to both positive and negative charges. The difference is that all neutral objects are attracted to a charged object, whereas only a few materials are attracted to a magnet.

Dipoles and Monopoles

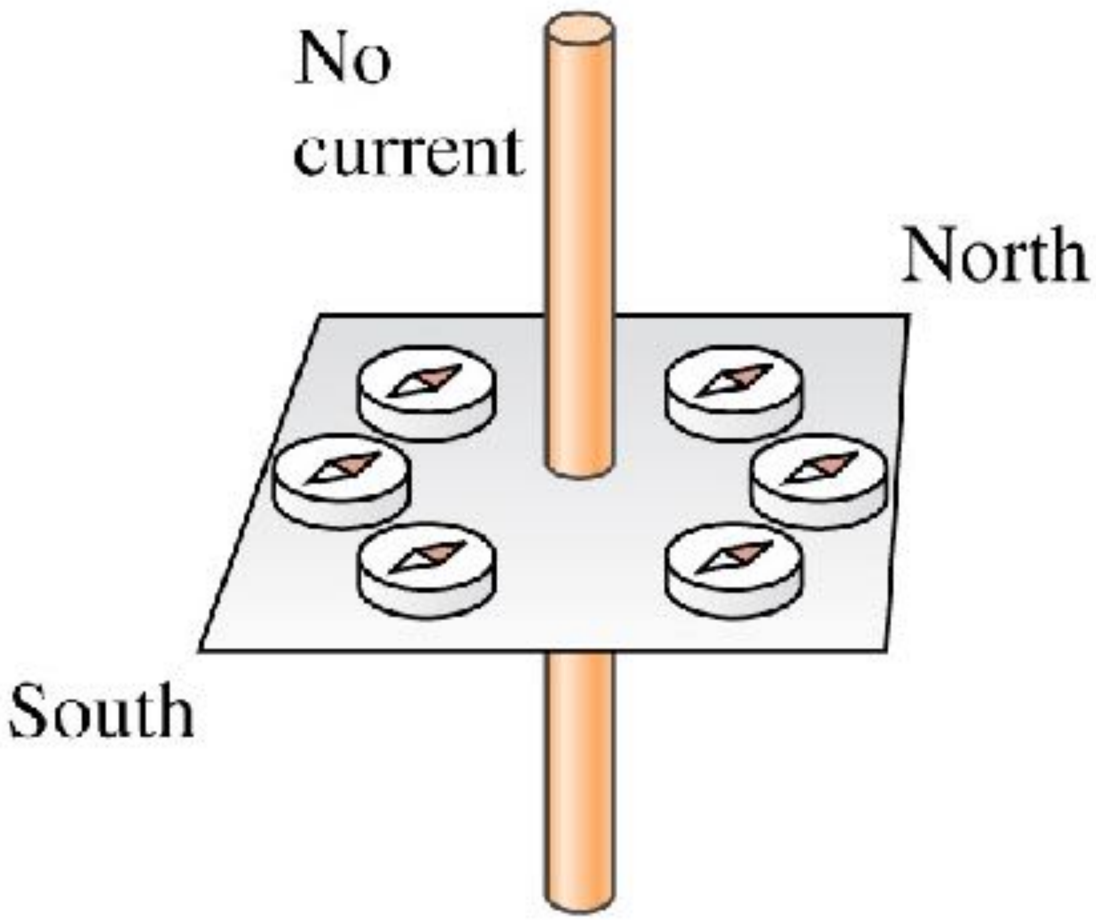
It is strange that whenever you cut a magnet in half, you get two smaller, weaker, but still complete magnets both with a north and south pole. Every magnet ever observed has both poles, forming a **magnetic dipole**, similar to an electric dipole (two opposite charges separated by a distance). An electric dipole can be separated and the charges used independently. This appears not to be the case for a magnetic dipole.

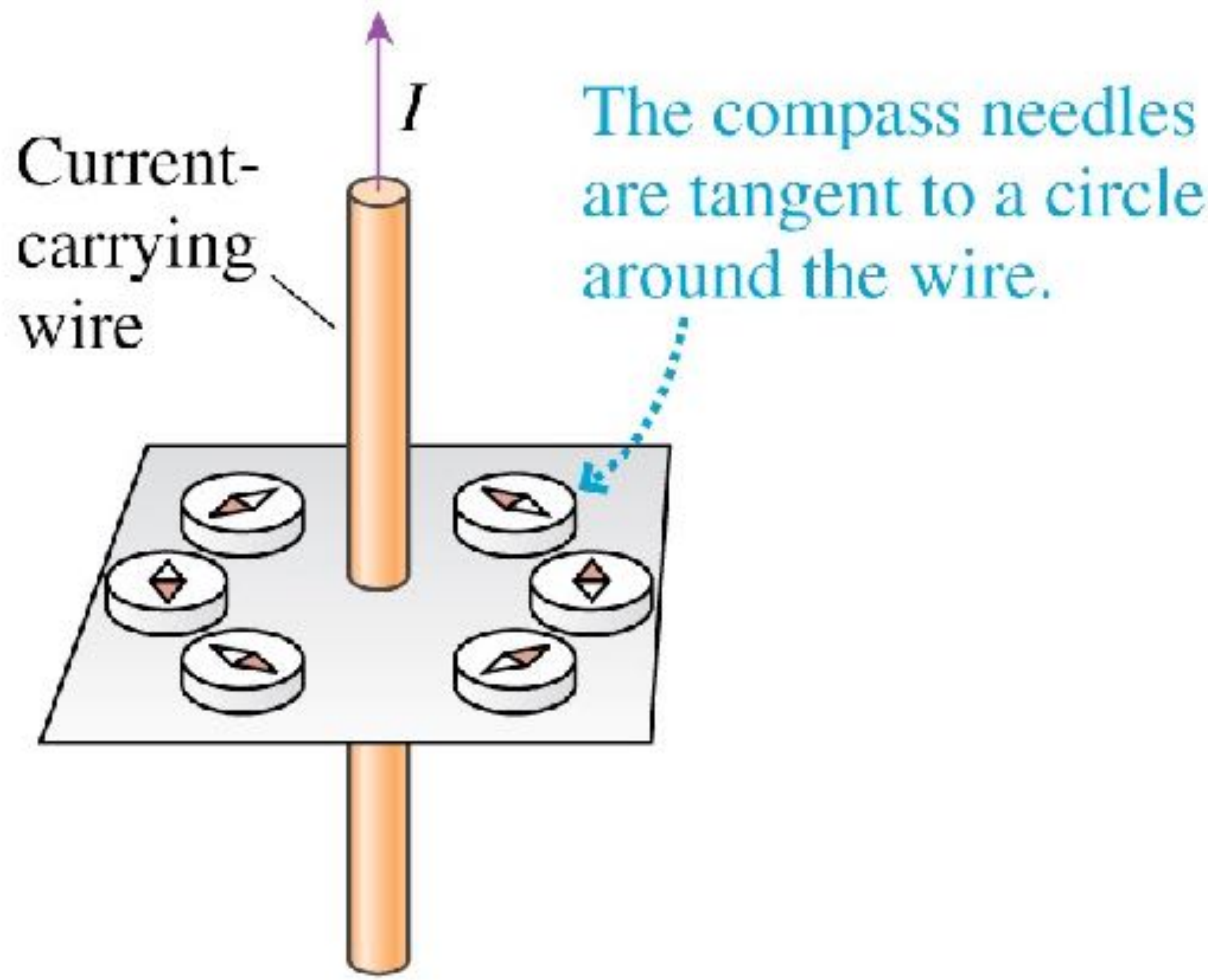
A single north or south pole all by itself would be called a **magnetic monopole**. None have ever been observed. On the other hand, we don't really know why not. In fact, some theories of particle physics predict their existence. So the existence of magnetic monopoles is a question at the most fundamental level of physics.

Discovery of the Magnetic Field

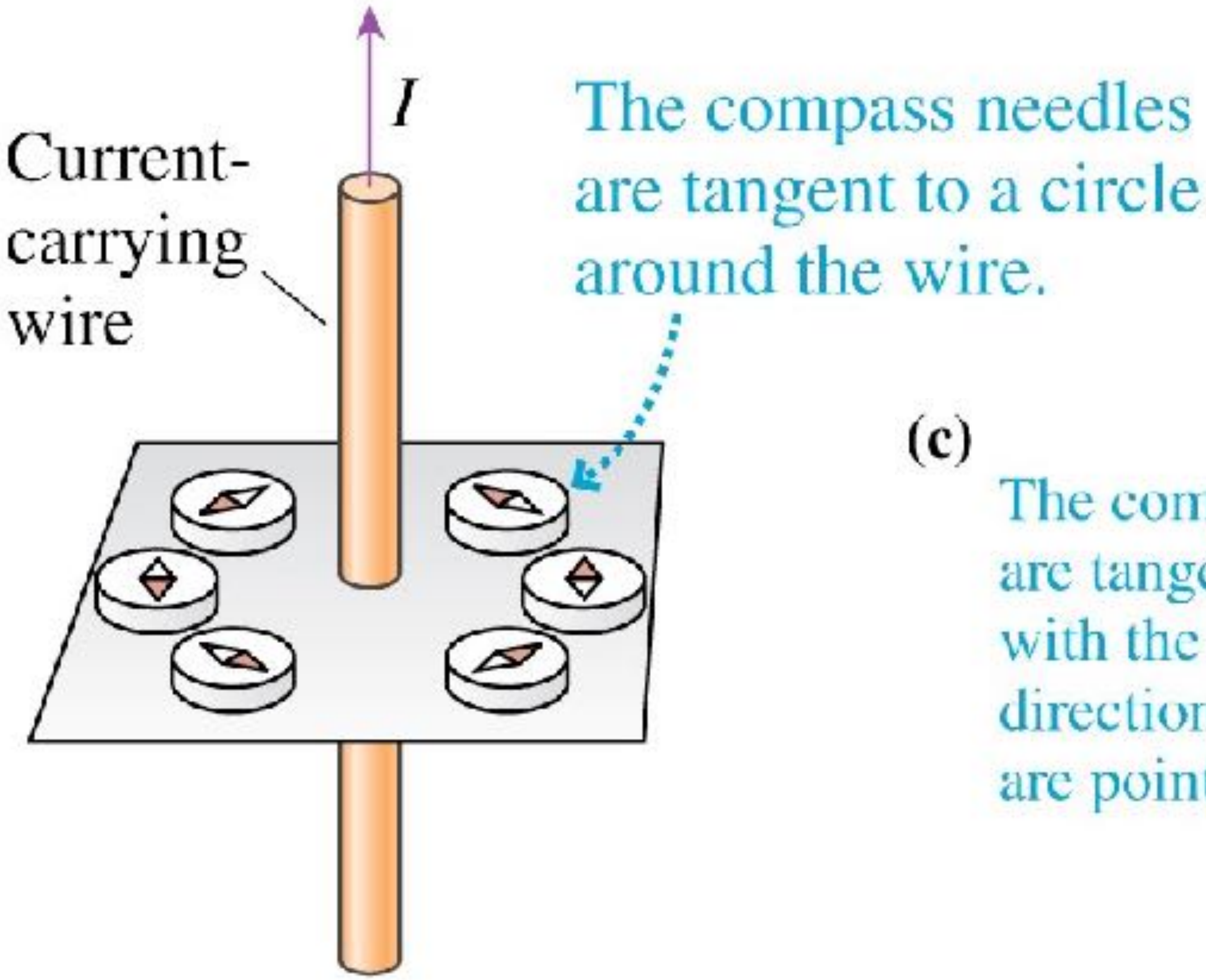
Although there was some speculation that there might be a connection between electricity and magnetism, it wasn't until 1819, when Hans Christian Oersted discovered a link in the ***middle of a classroom lecture demonstration***. He discovered that a current in a wire caused a compass needle to deflect.

Effect of a Current on a Compass

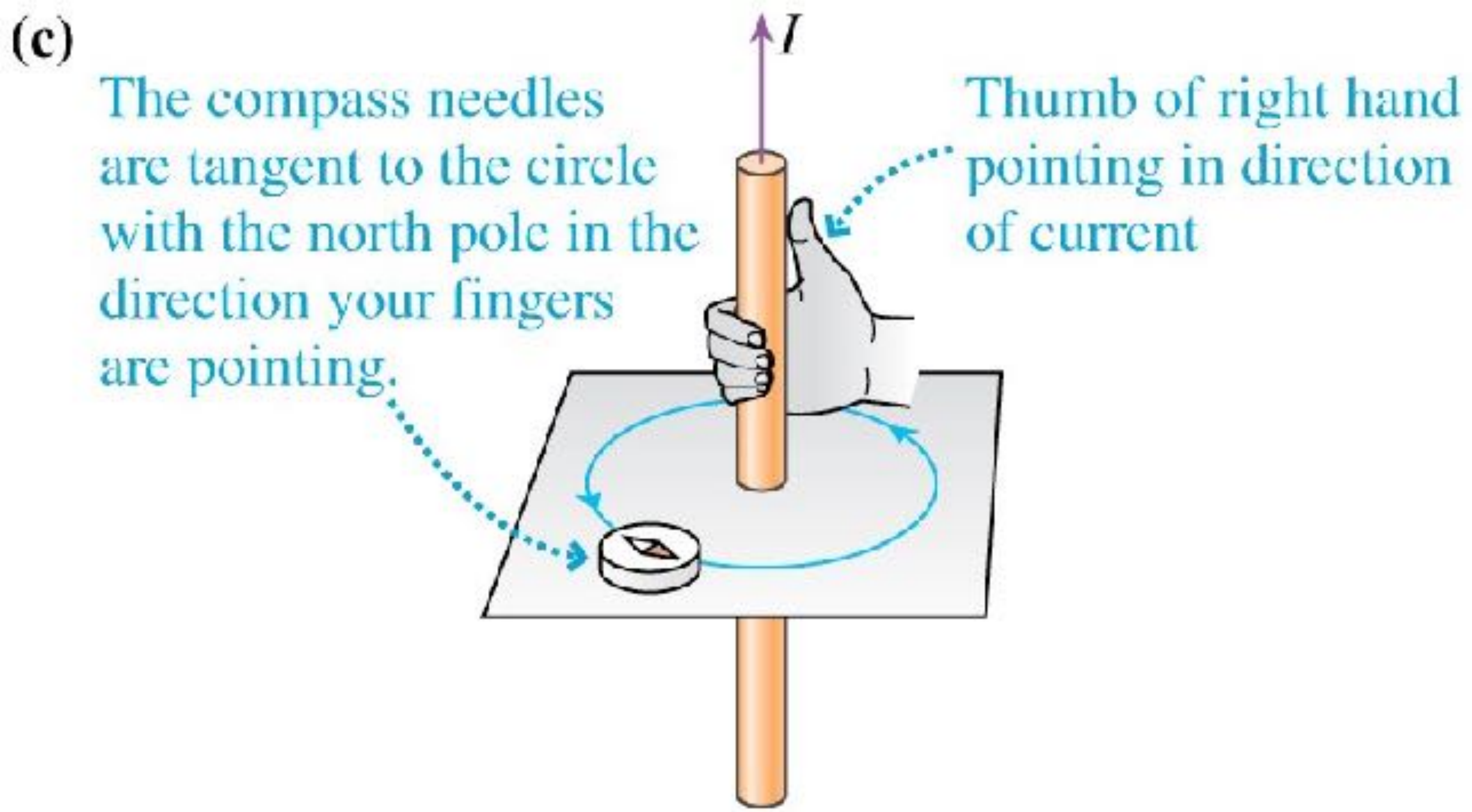




Effect of a Current on a Compass



Right-hand rule



Conventions

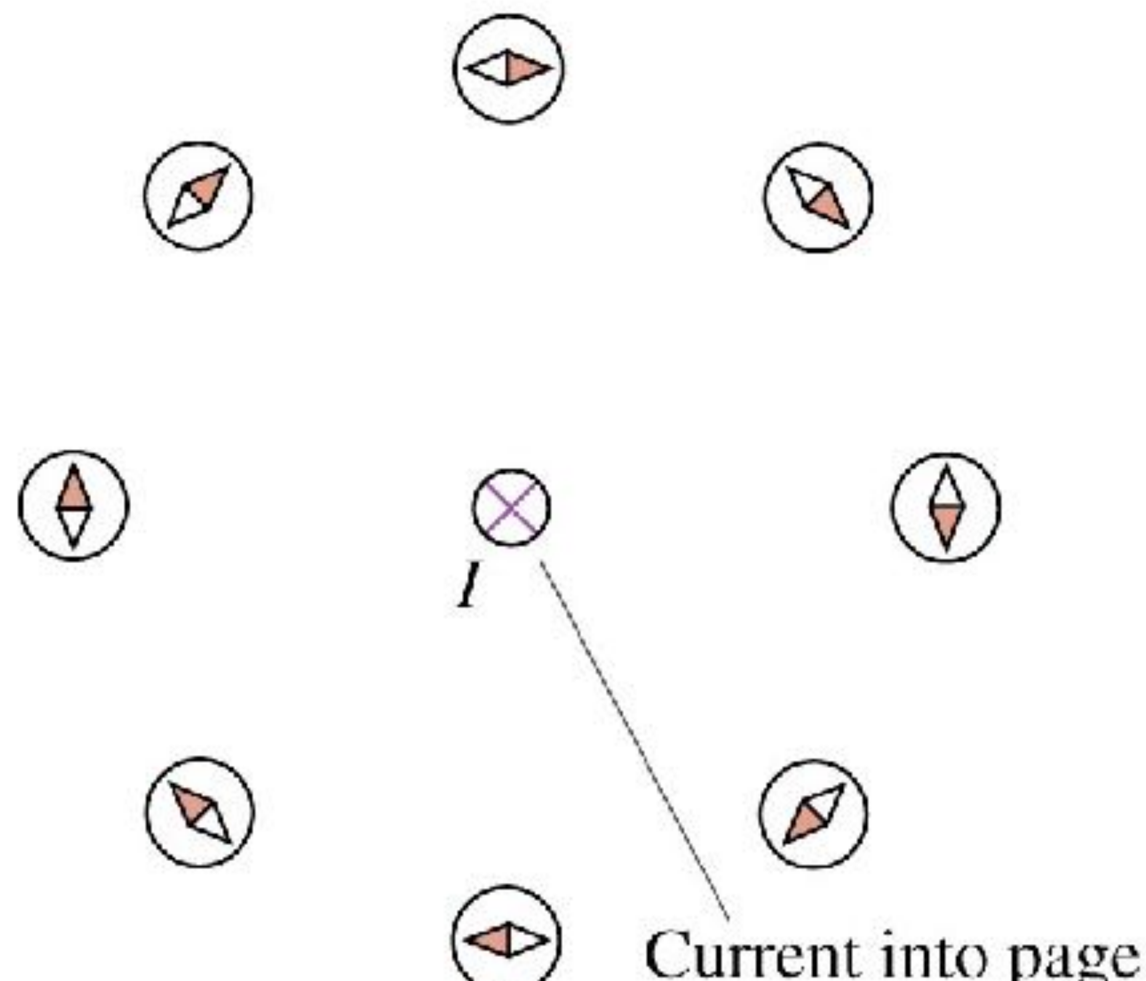
Vectors into page

Current into page

Vectors out of page

Current out of page

Wire with current



Magnetic Field

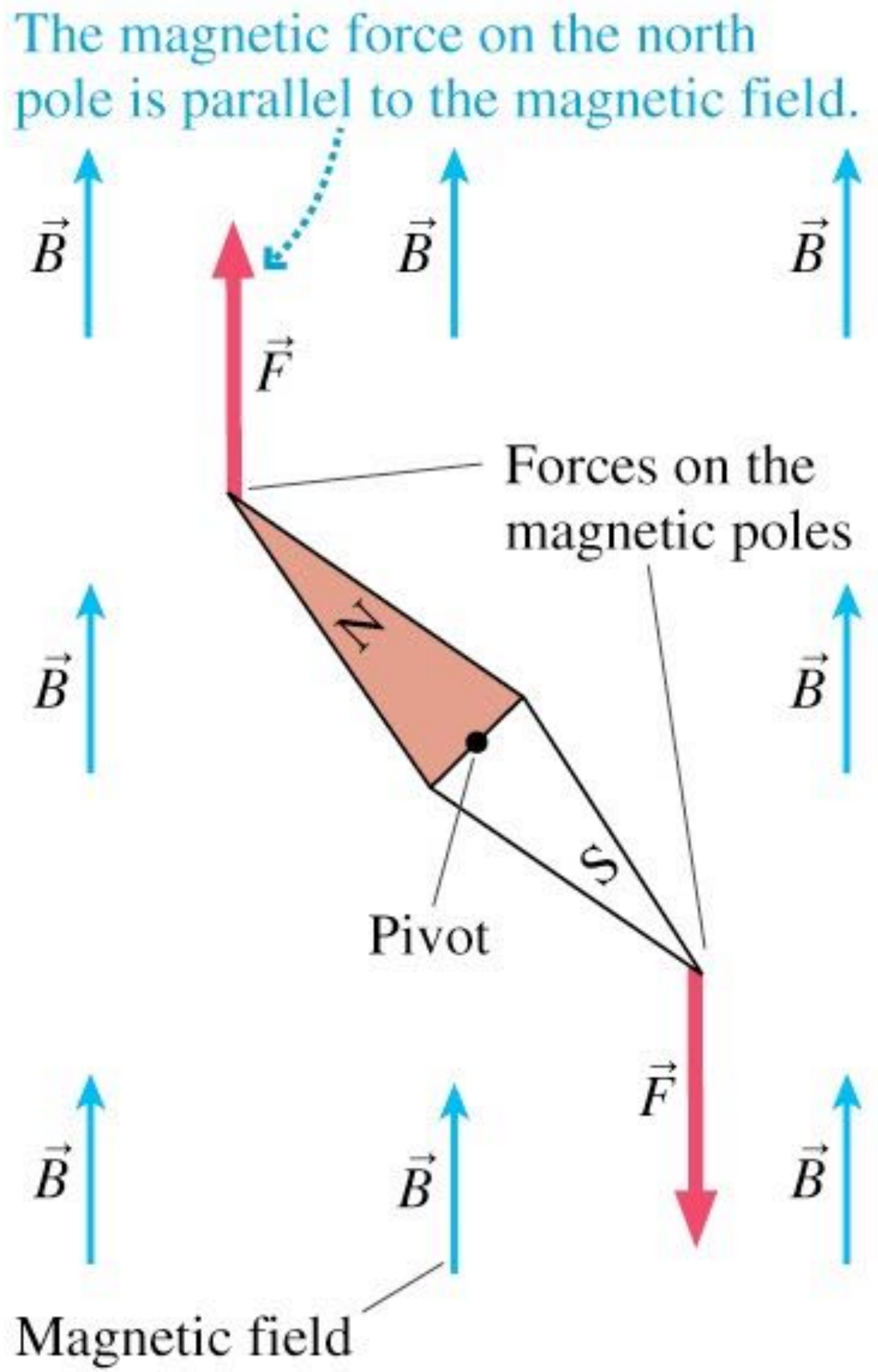
Similar to what we did for the electric field… Define a magnetic field $\vec{B}$ with the following properties:

1. A magnetic field is created at all points in space surrounding a current-carrying wire.

2. The magnetic field at each point is a vector. It has both a magnitude which is called the magnetic field strength B, and a direction.

3. The magnetic field exerts forces on magnetic poles. The force on a north pole is parallel to $\vec{B}$; the force on a south pole is opposite to $\vec{B}$.

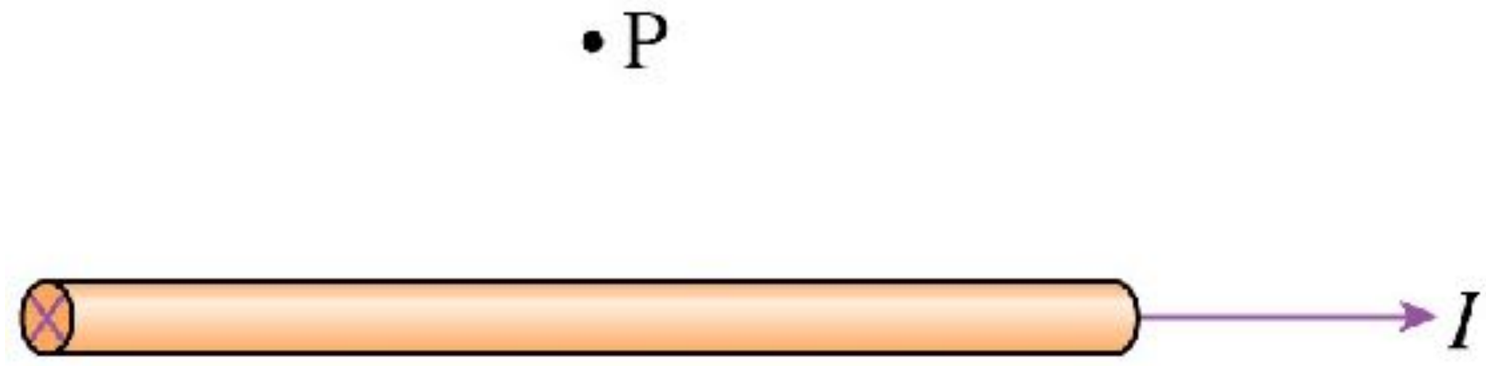
Magnetic Field and Compass



A compass needle can be used as a probe of the magnetic field, just as a charge was used as a probe of the electric field.

Discussion Question

The magnetic field at position P points



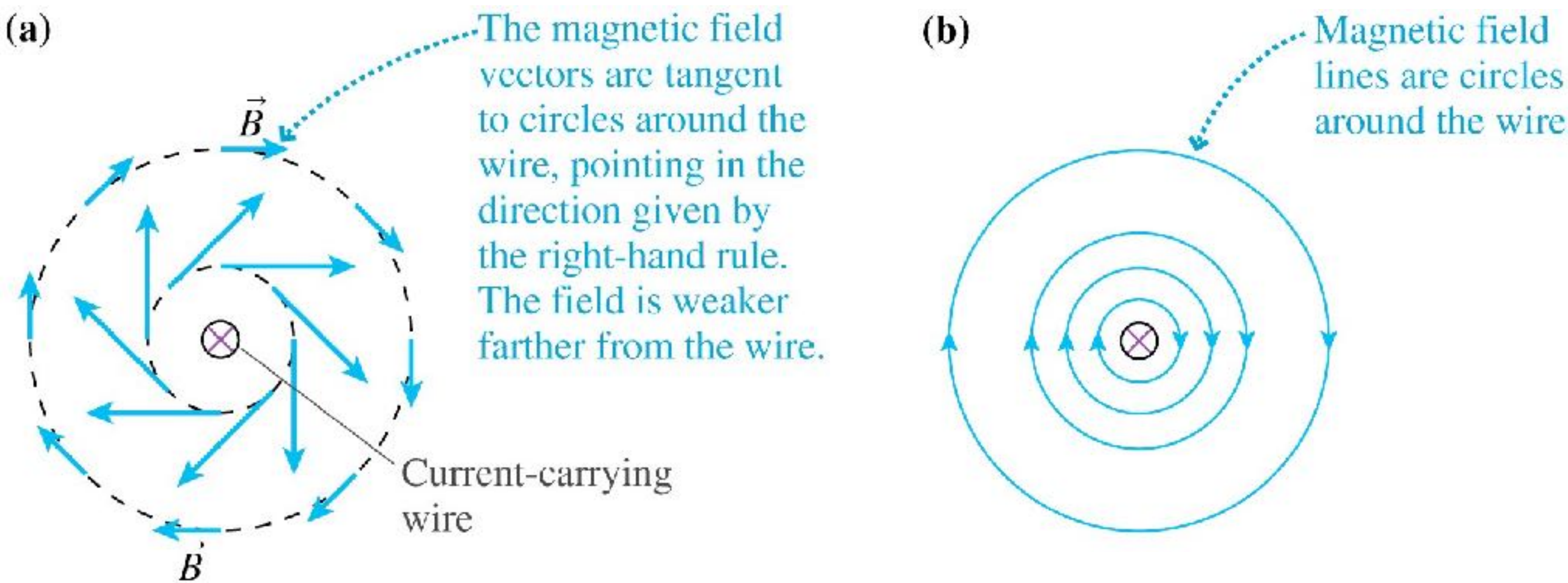
A. up.

B. down.

C. into the page.

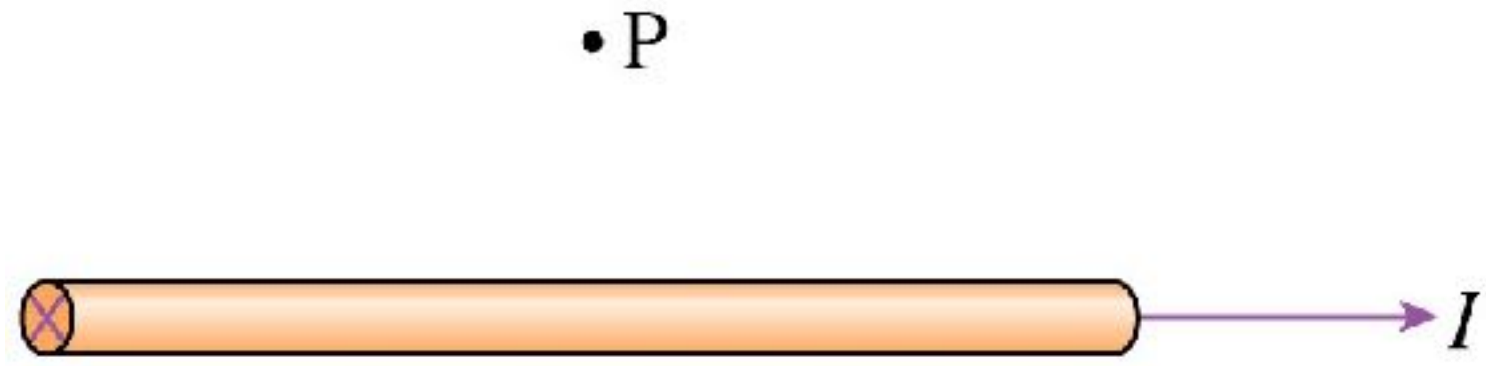
D. out of the page.

Two Views of the Magnetic Field



Discussion Question

The magnetic field at position P points



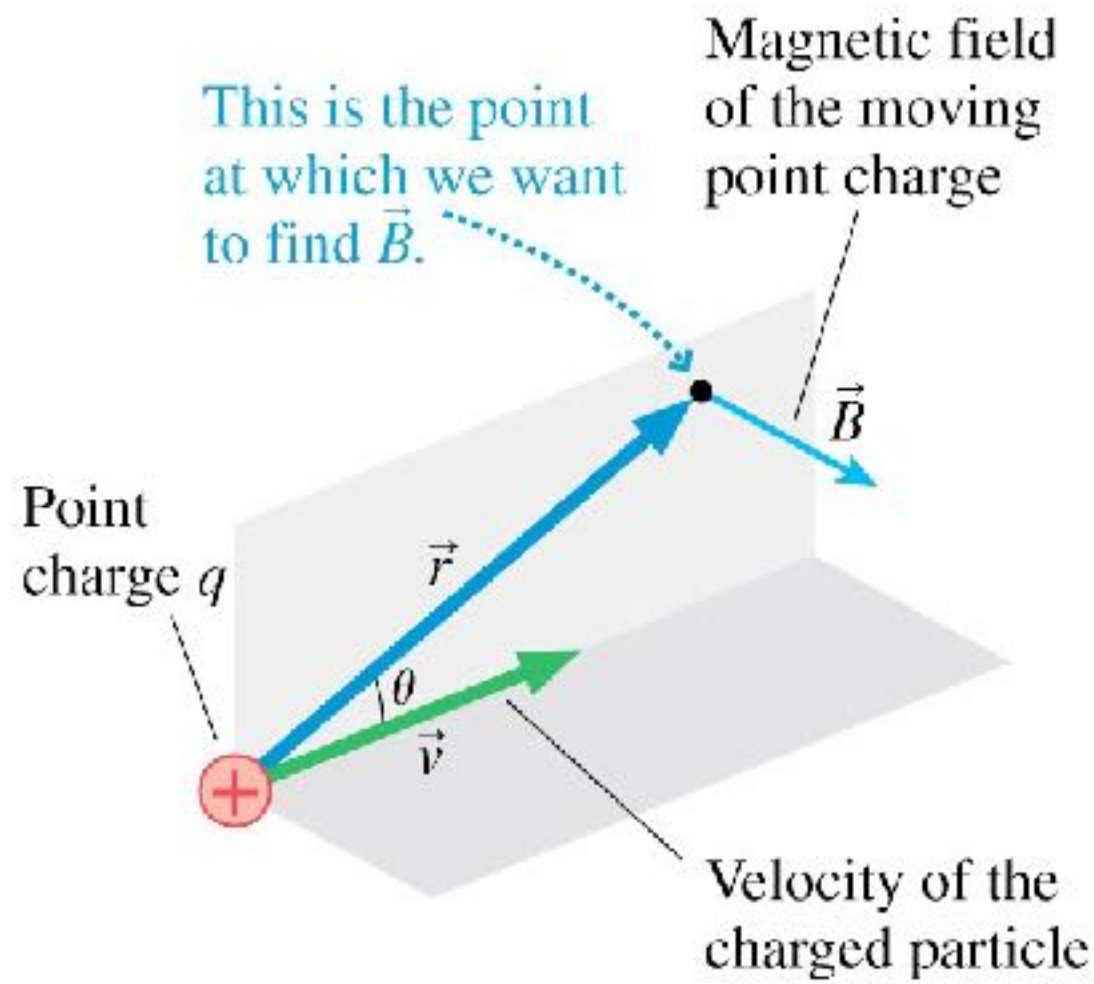
A. up.

B. down.

C. into the page.

D. out of the page.

Source of the Magnetic Field: Moving Charges

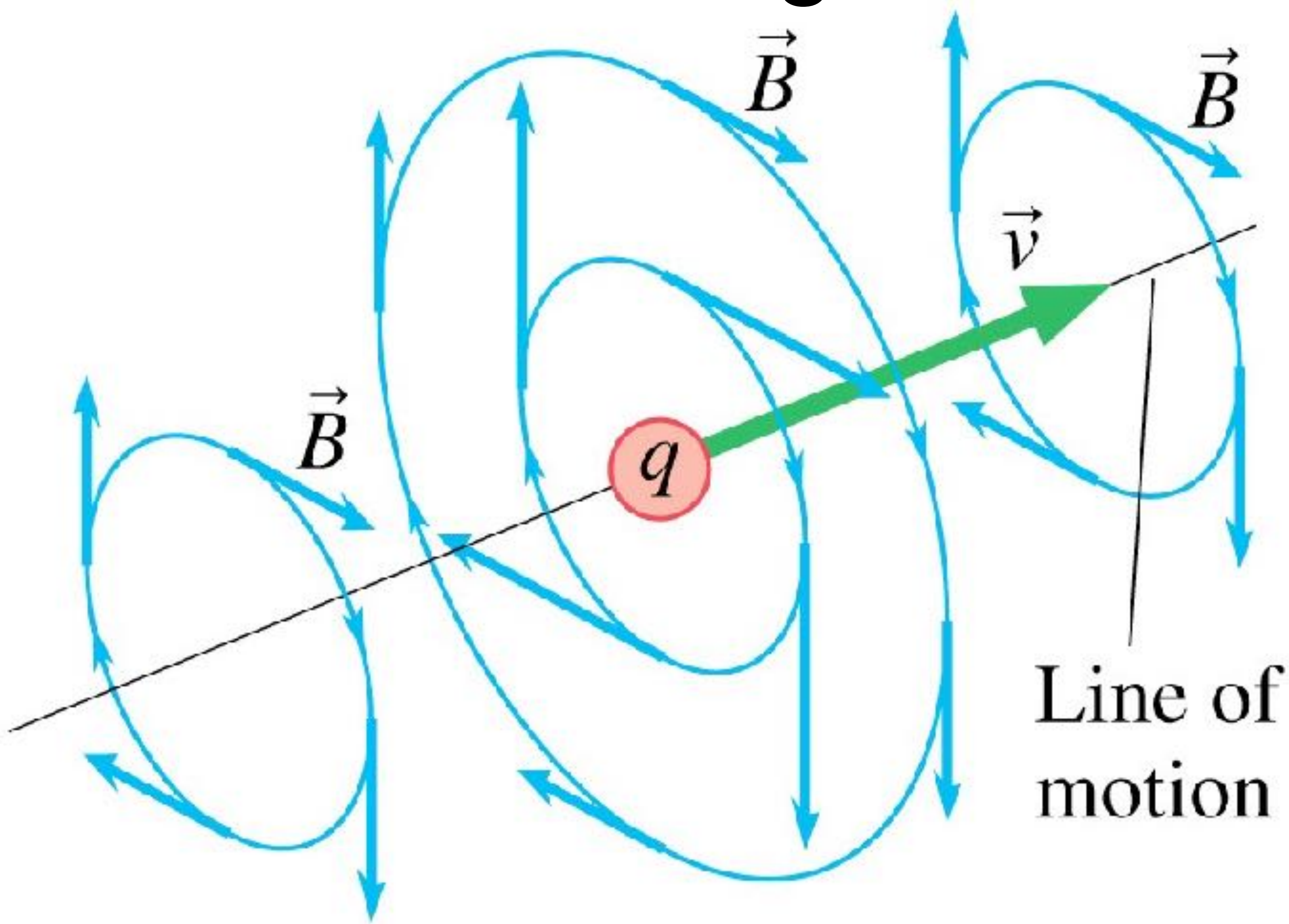


$$\vec{B} = \left(\frac{\mu_0}{4\pi} \frac{qv \sin\theta}{r^2}, \text{direction given by right-hand rule} \right)$$

Biot-Savart Law

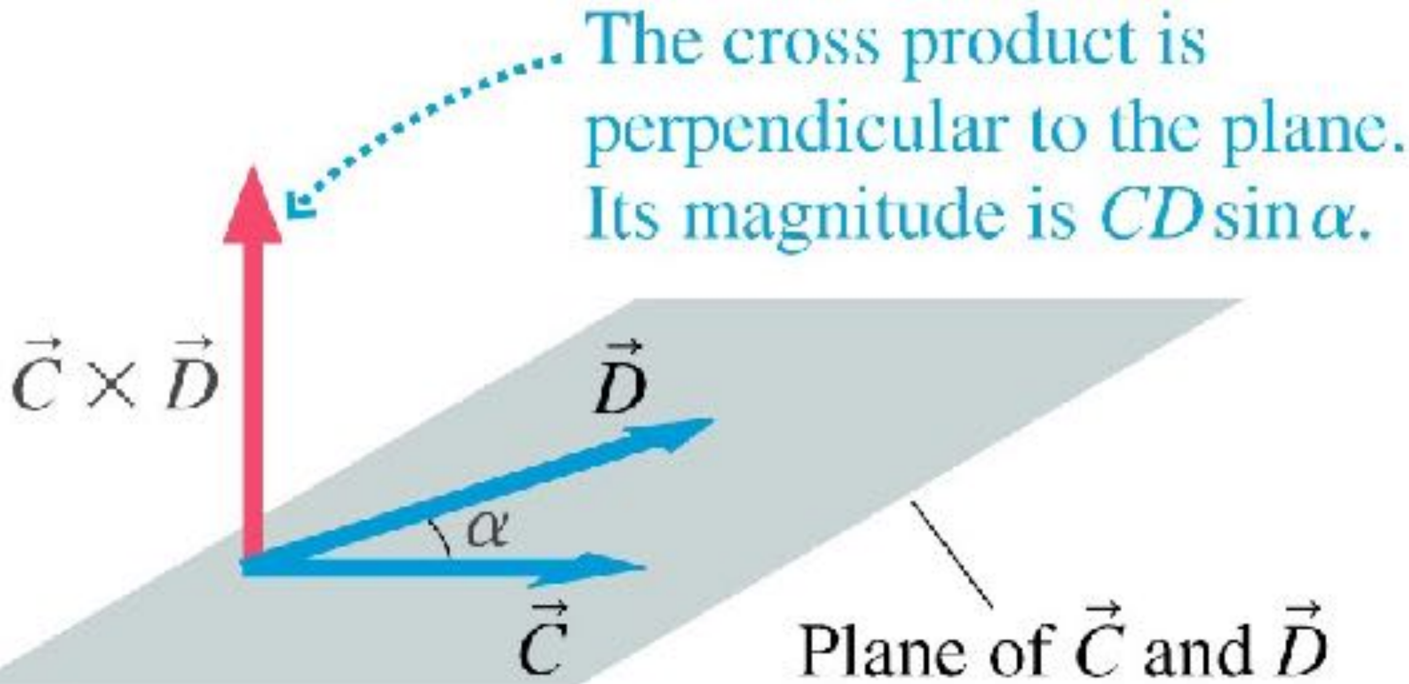
Unit: 1 tesla = 1T = 1 N/Am $\quad \mu_0 = 4\pi \text{ X } 10^{-7}$ T-m/A

Source of the Magnetic Field



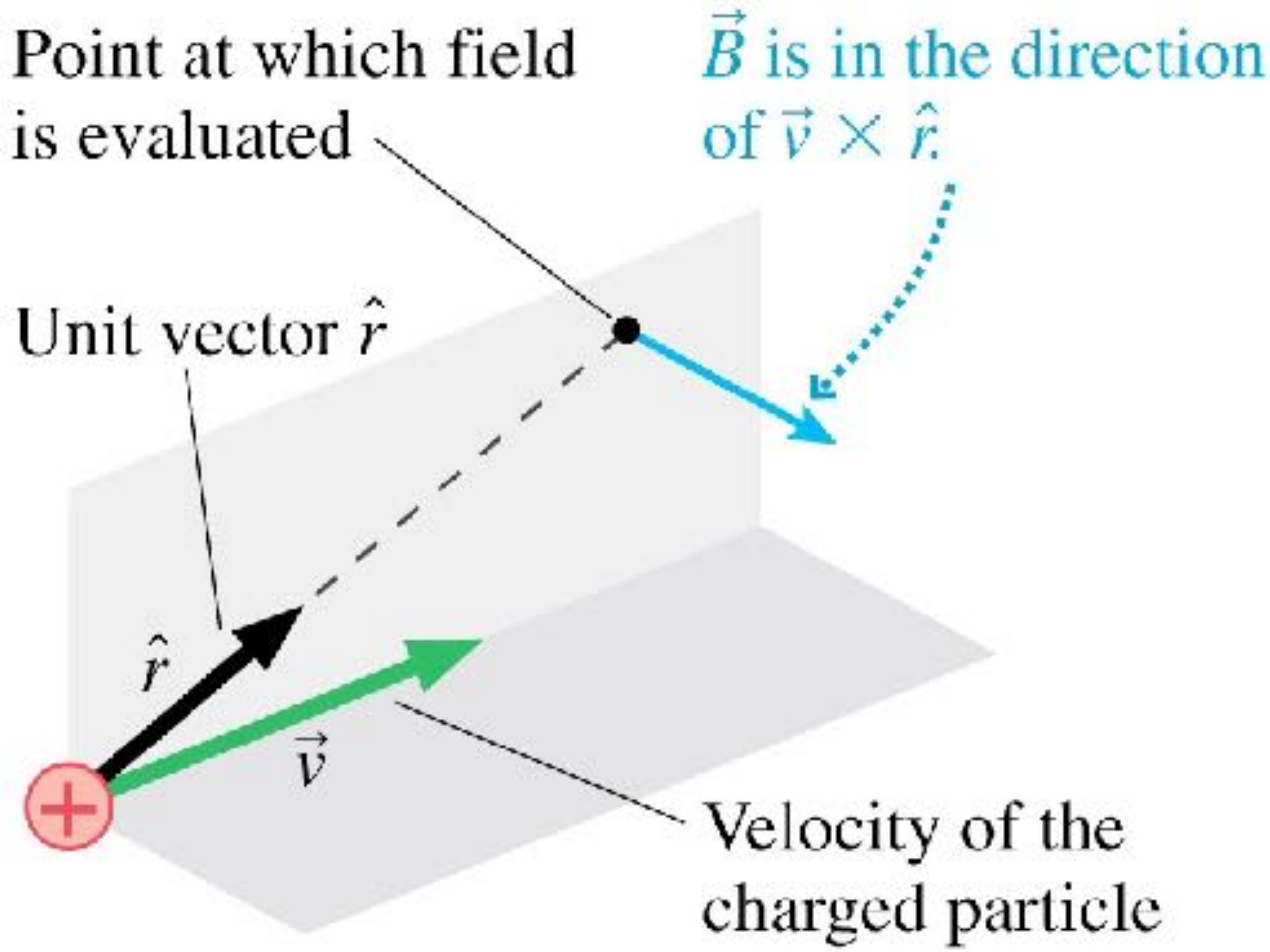
$$\vec{B} = \left(\frac{\mu_0}{4\pi} \frac{qv \sin\theta}{r^2}, \text{direction given by right-hand rule} \right)$$

Vector Cross Product



$$\vec{B} = \frac{\mu_0}{4\pi}\frac{q\vec{v}\times\hat{r}}{r^2}$$

Biot-Savart



Discussion Question

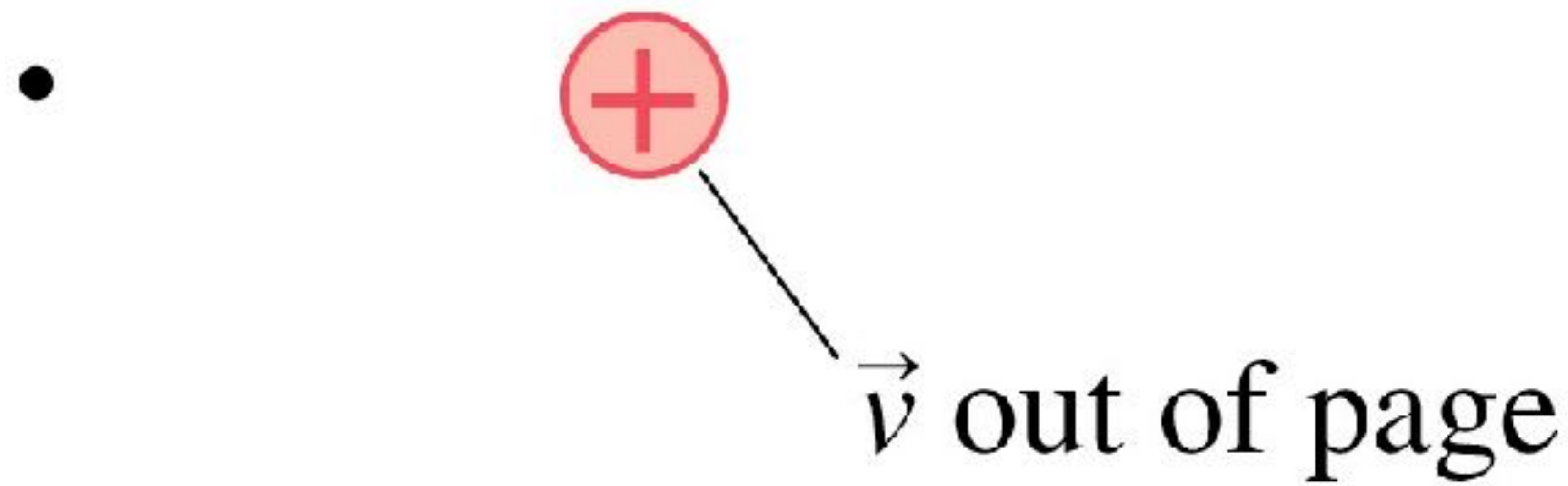
The positive charge is moving straight out of the page. What is the direction of the magnetic field at the position of the dot?

A. up.

B. down.

C. left.

D. right.



Discussion Question

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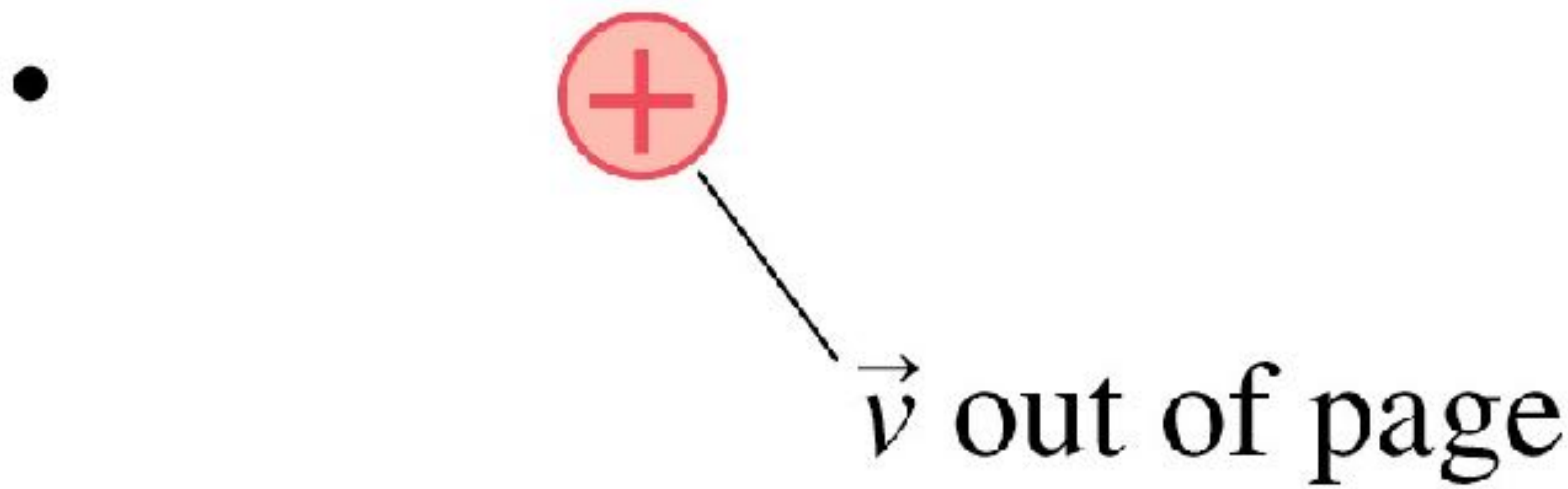
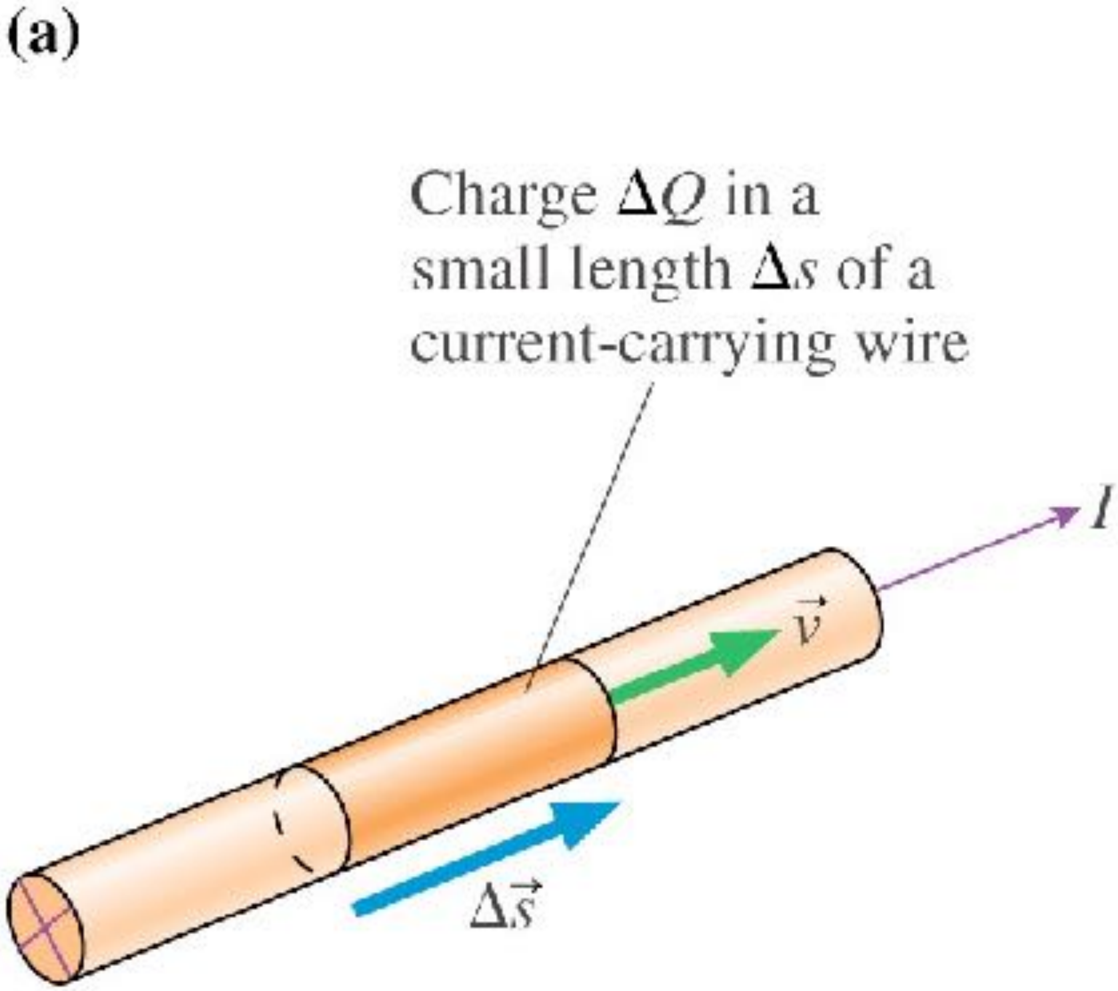
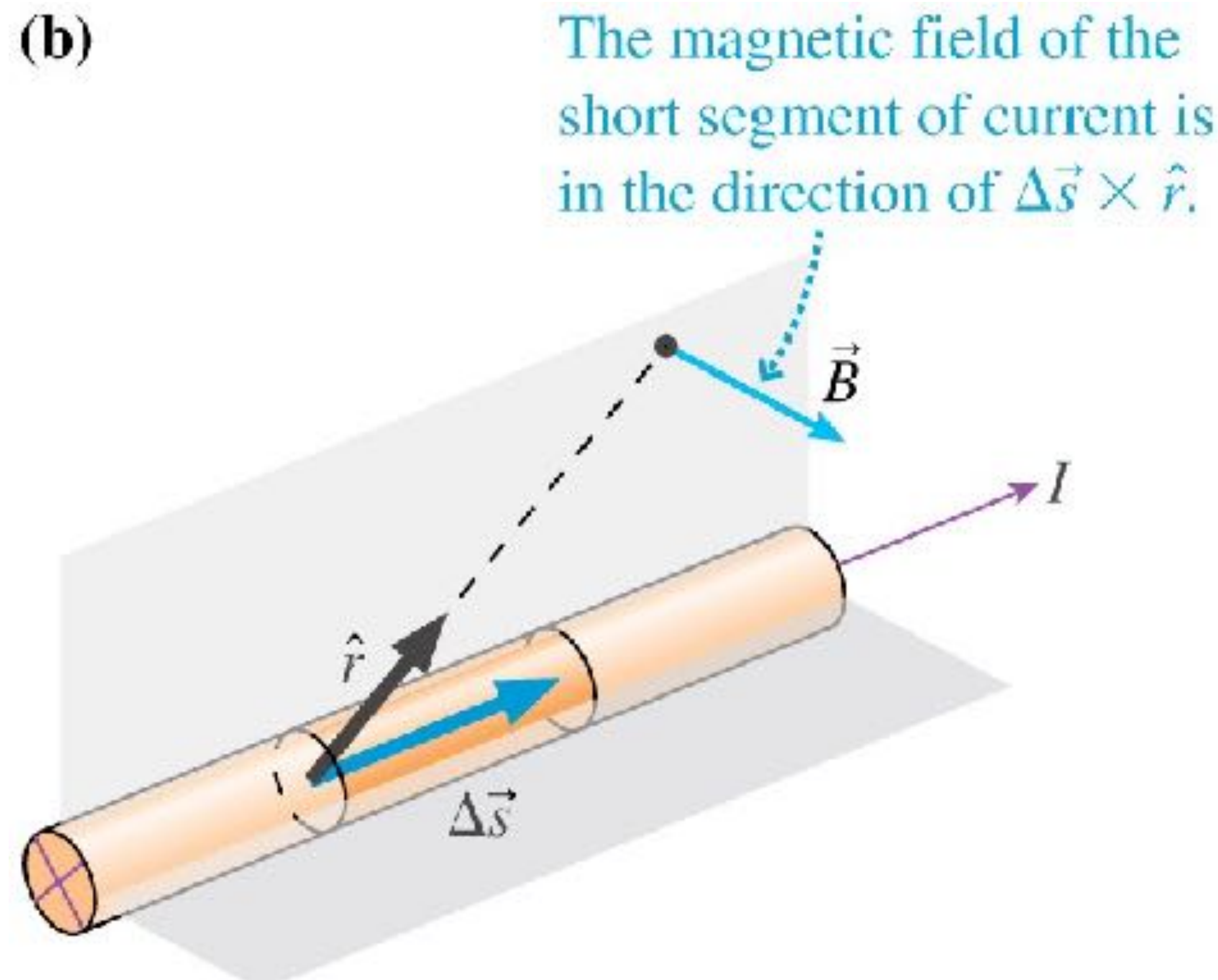


Magnetic Field of a Current



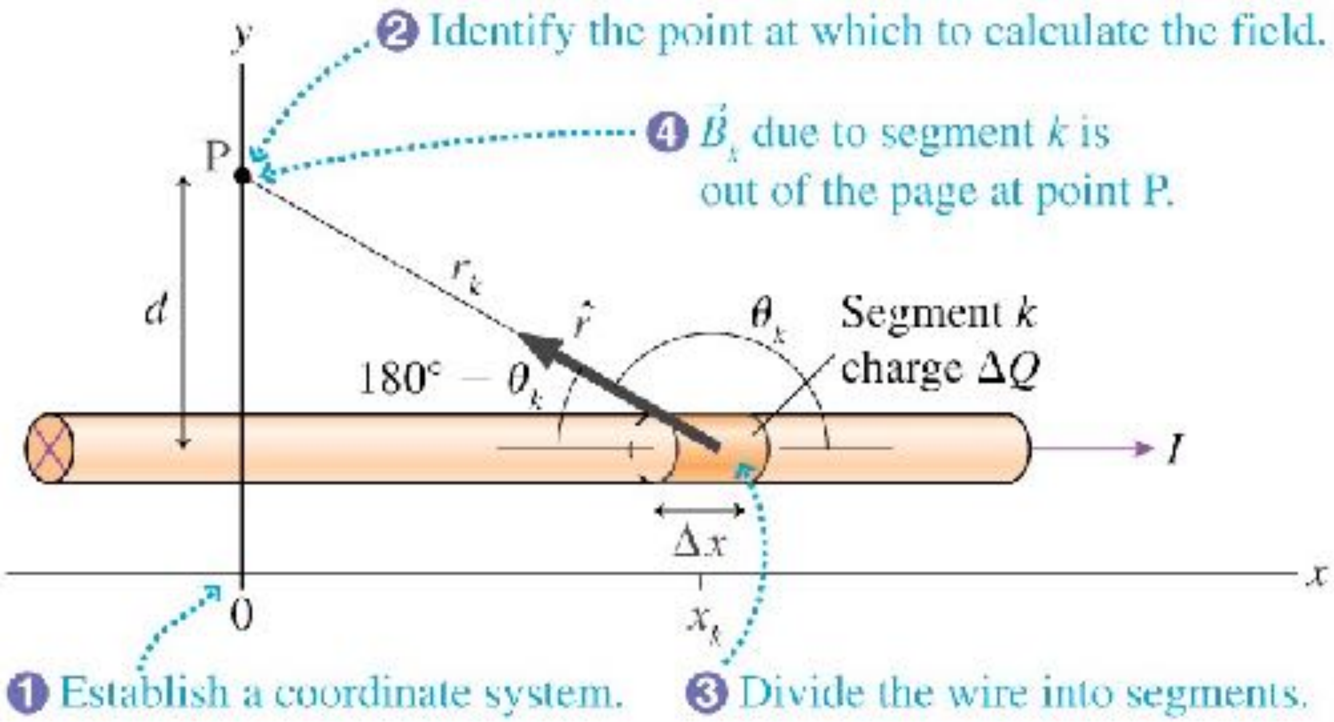


$$\vec{v} = \Delta\vec{s}/\Delta t$$

$$(\Delta Q)\vec{v} = \Delta Q\frac{\Delta\vec{s}}{\Delta t} = \frac{\Delta Q}{\Delta t}\Delta\vec{s} = I\Delta\vec{s}$$

$$\vec{B} = \frac{\mu_0}{4\pi}\frac{I\Delta\vec{s} \times \hat{r}}{r^2}$$

Field of a very long wire



$$(B_k)_z = \frac{\mu_0}{4\pi}\frac{I\Delta x \sin\theta_k}{r_k^2}$$

$$(B_k)_z = \frac{\mu_0}{4\pi}\frac{I\sin\theta_k}{x_k^2 + d^2}\Delta x$$

$$\sin\theta_k = sin(180^\circ - \theta_k) = \frac{d}{r_k} = \frac{d}{\sqrt{x_k^2 + d^2}}$$

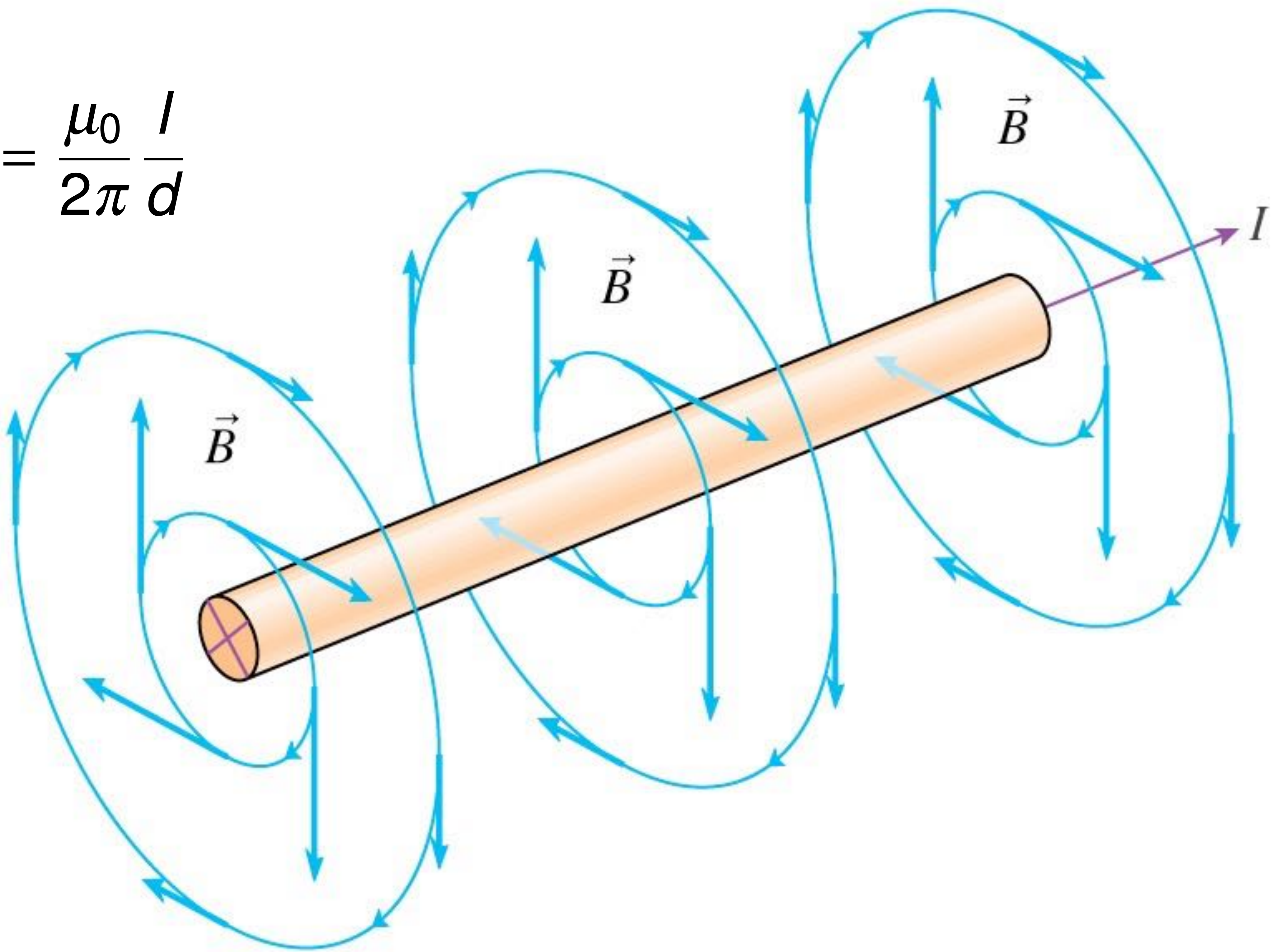
$$(B_k)_z = \frac{\mu_0}{4\pi}\frac{Id}{(x_k^2 + d^2)^{3/2}}\Delta x$$

$$B_{\text{wire}} = \sum_k (B_k)_z = \frac{\mu_0 Id}{4\pi}\sum_k \frac{\Delta x}{(x_k^2 + d^2)^{3/2}} \rightarrow \frac{\mu_0 Id}{4\pi}\int_{-\infty}^{+\infty}\frac{dx}{(x_k^2 + d^2)^{3/2}}$$

$$B_{\text{wire}} = \frac{\mu_0 Id}{4\pi}\left.\frac{x}{d^2(x_k^2 + d^2)^{1/2}}\right|_{-\infty}^{+\infty} = \frac{\mu_0}{2\pi}\frac{I}{d}$$

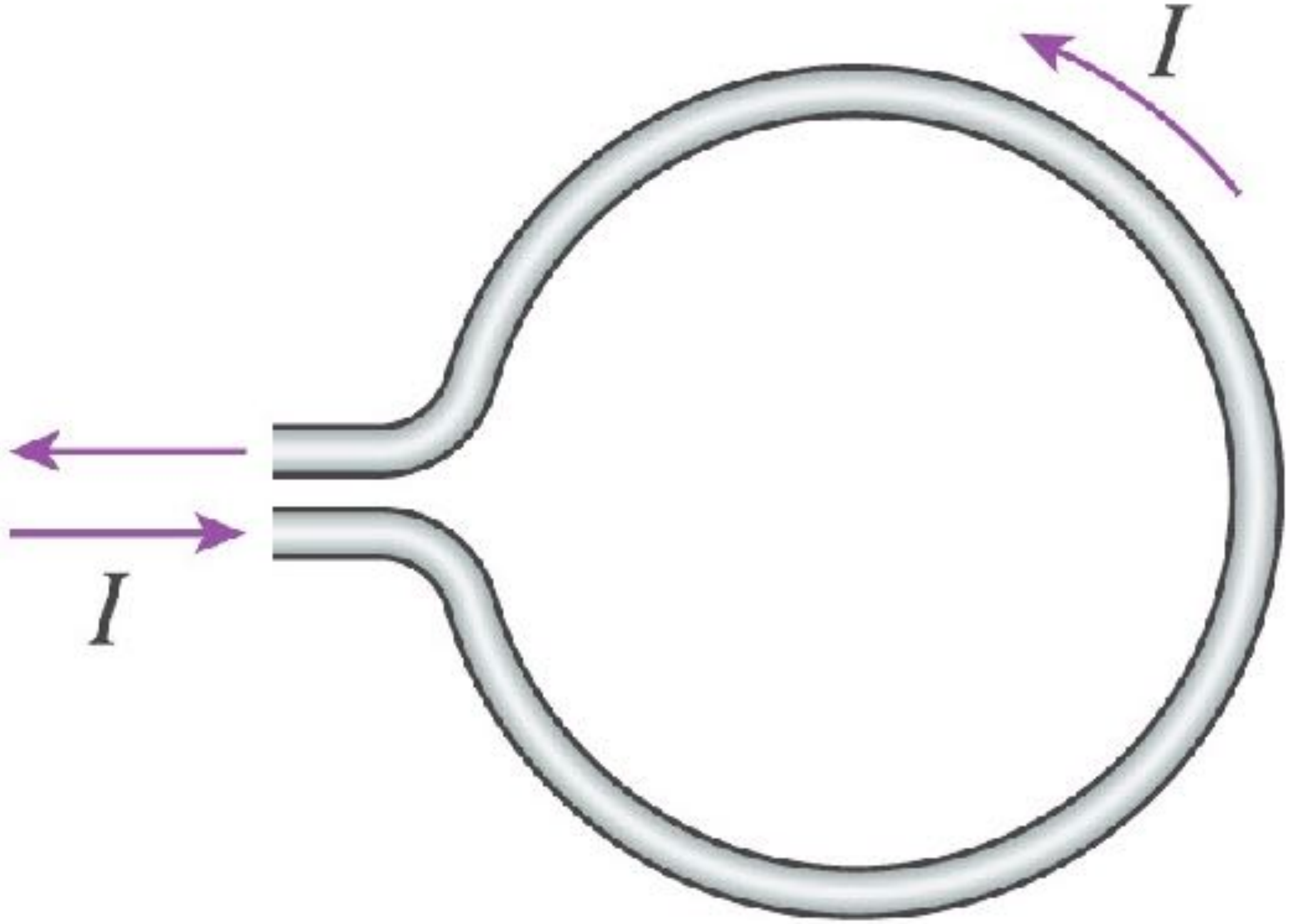
Field of a very long wire

$$B_{\text{wire}} = \frac{\mu_0}{2\pi}\frac{I}{d}$$

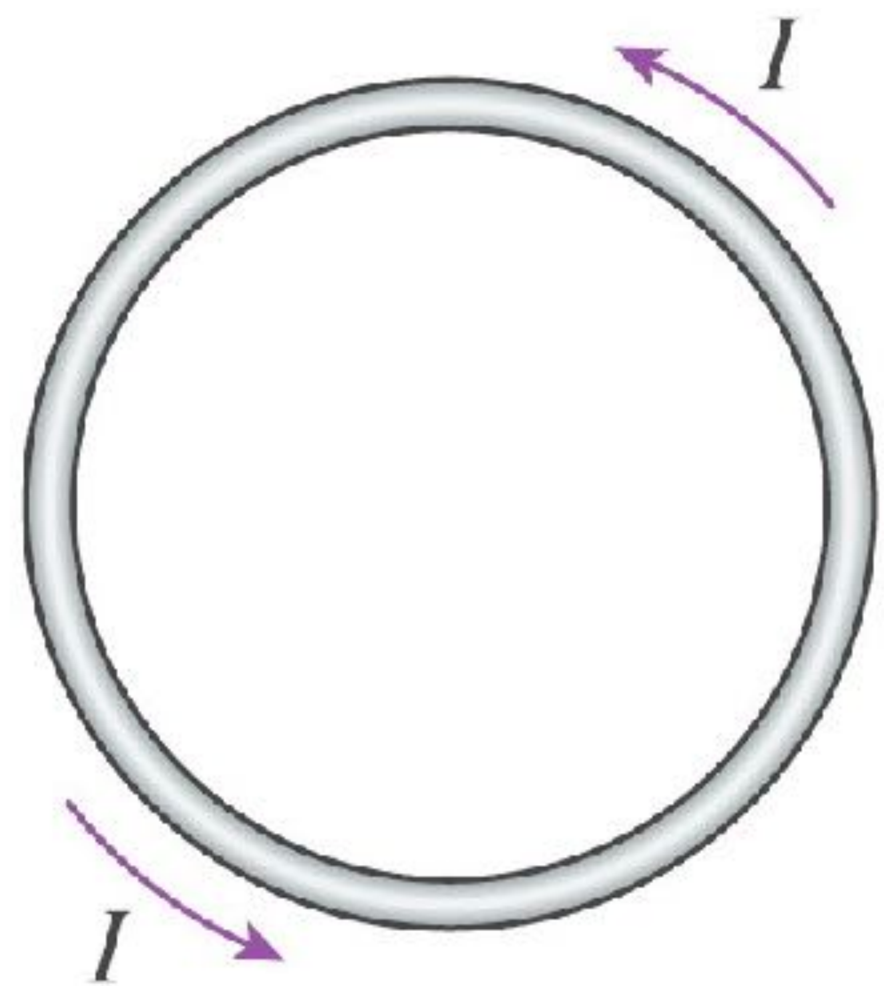


Field of a Current Loop

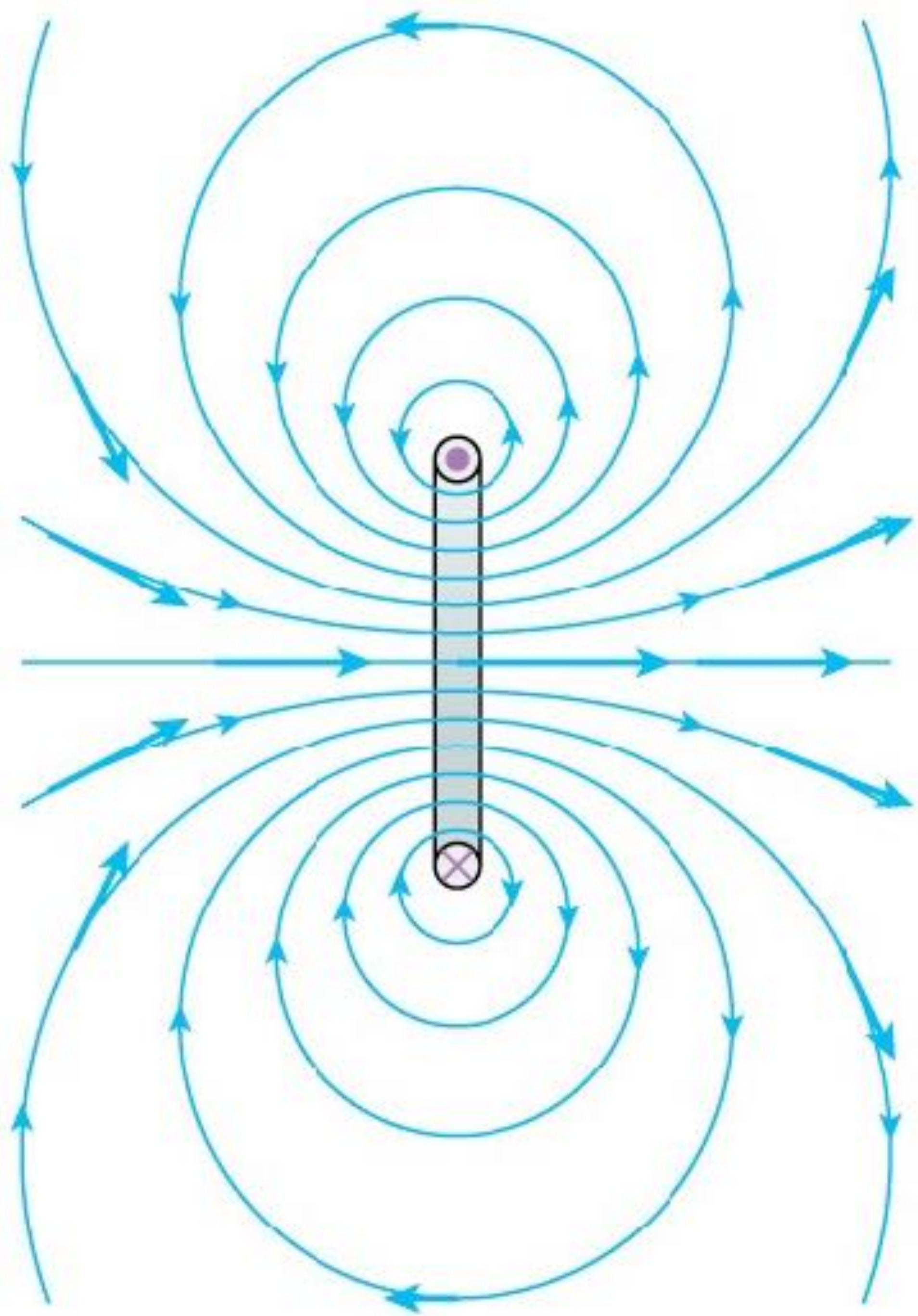
(a) A practical current loop

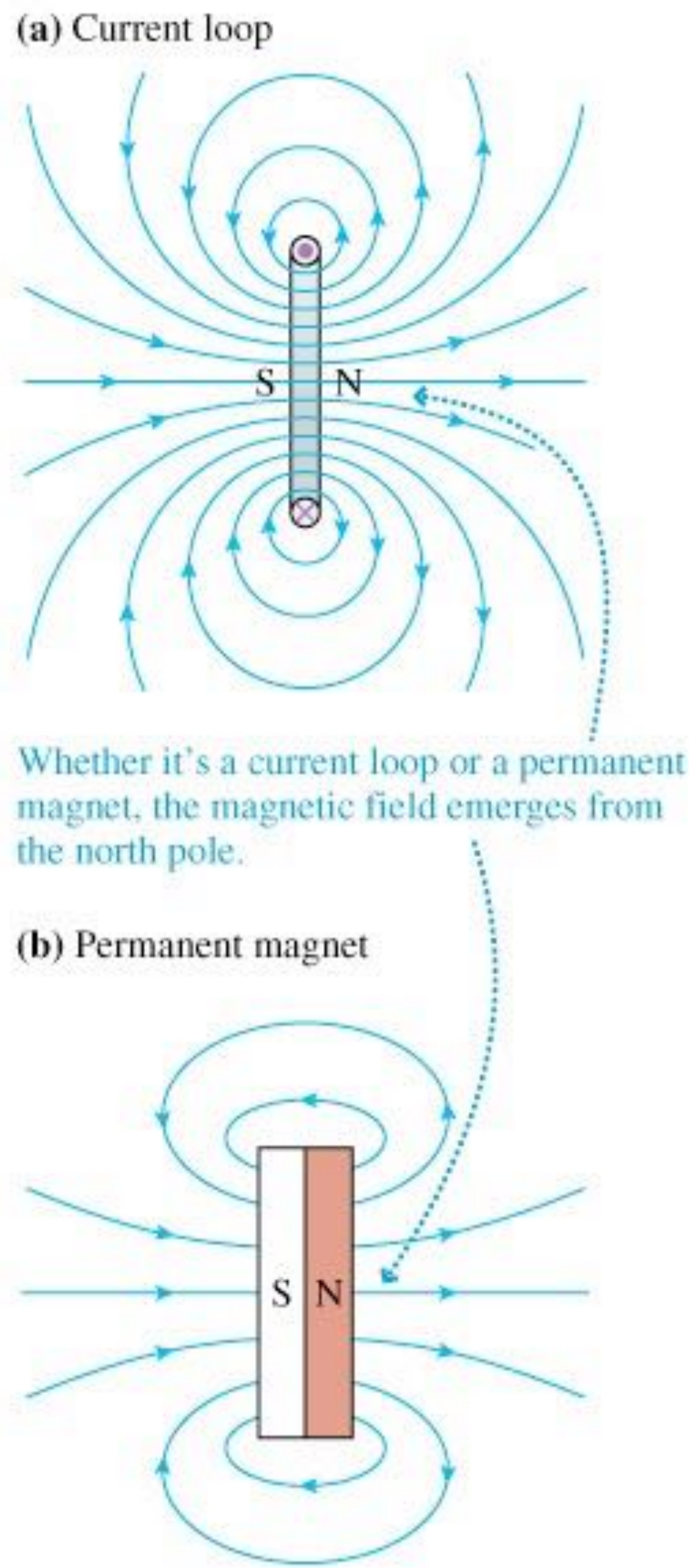


(b) An ideal current loop



Current Loops

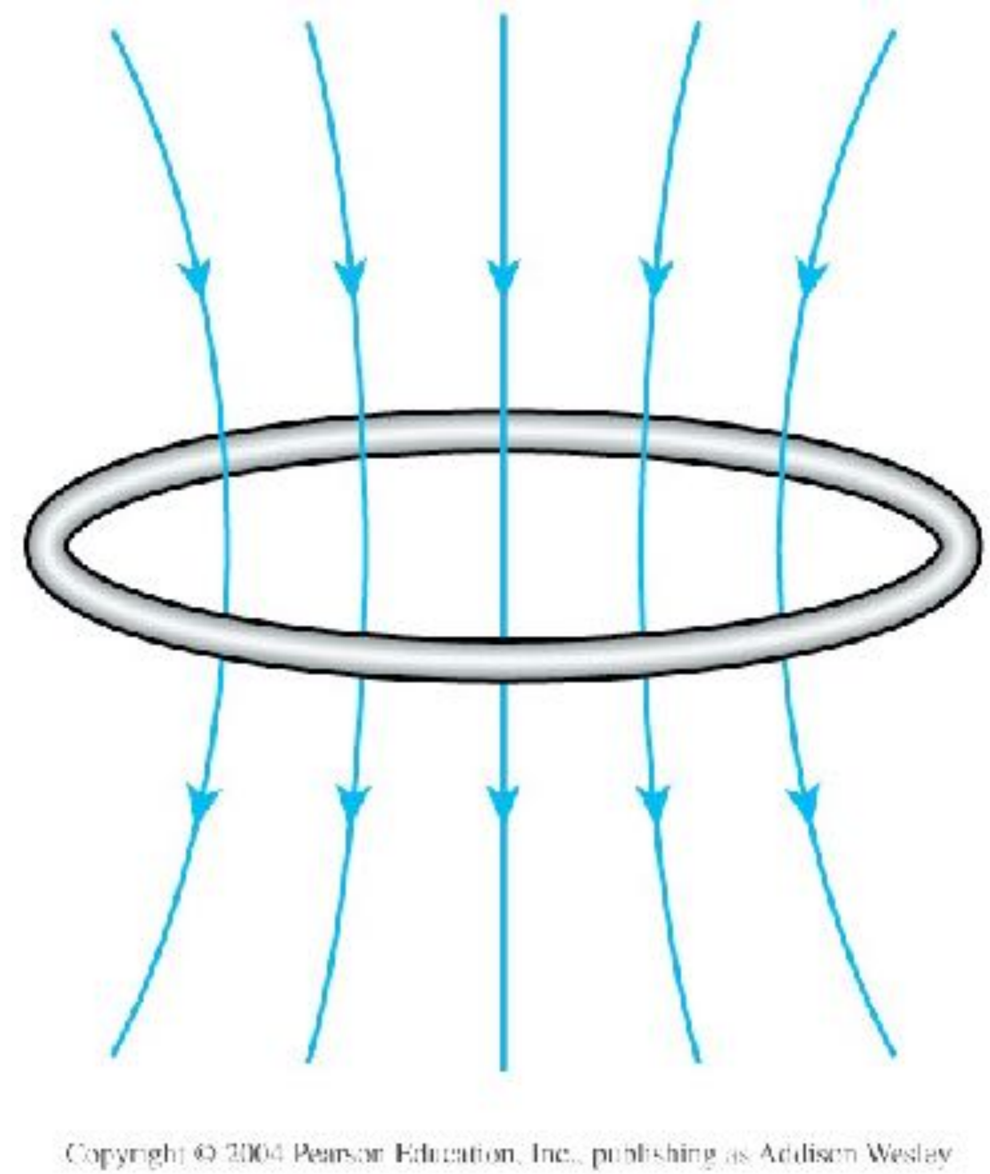




Note that the magnetic field *inside* a permanent magnet differs from the field at the center of a current loop

Discussion Question

What is the current direction in this loop? And which side of the loop is the north pole?



A. Current cw; north pole on top

B. Current cw; north pole on bottom

C. Current ccw; north pole on top

D. Current ccw; north pole on bottom

Discussion Question

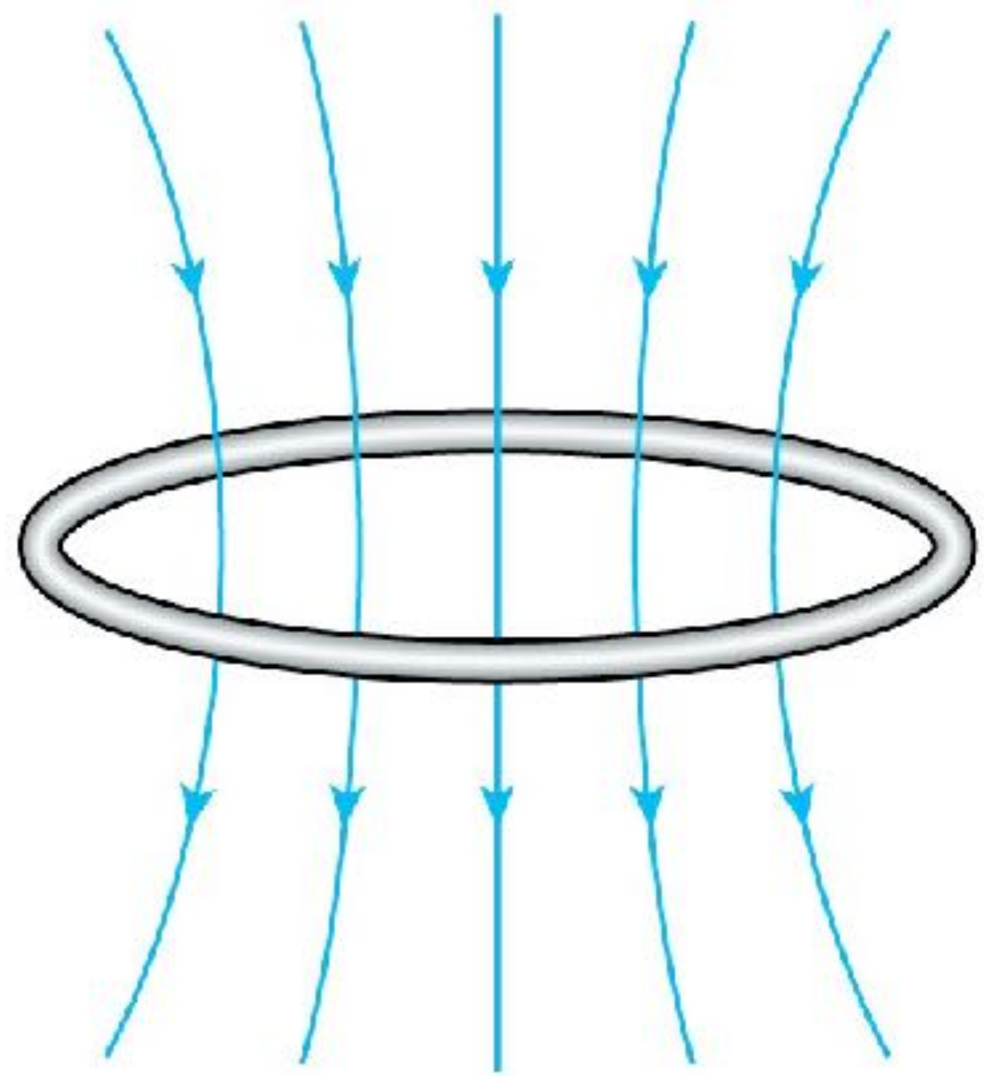
What is the current direction in this loop? And which side of the loop is the north pole?

A. Current cw; north pole on top

B. Current cw; north pole on bottom

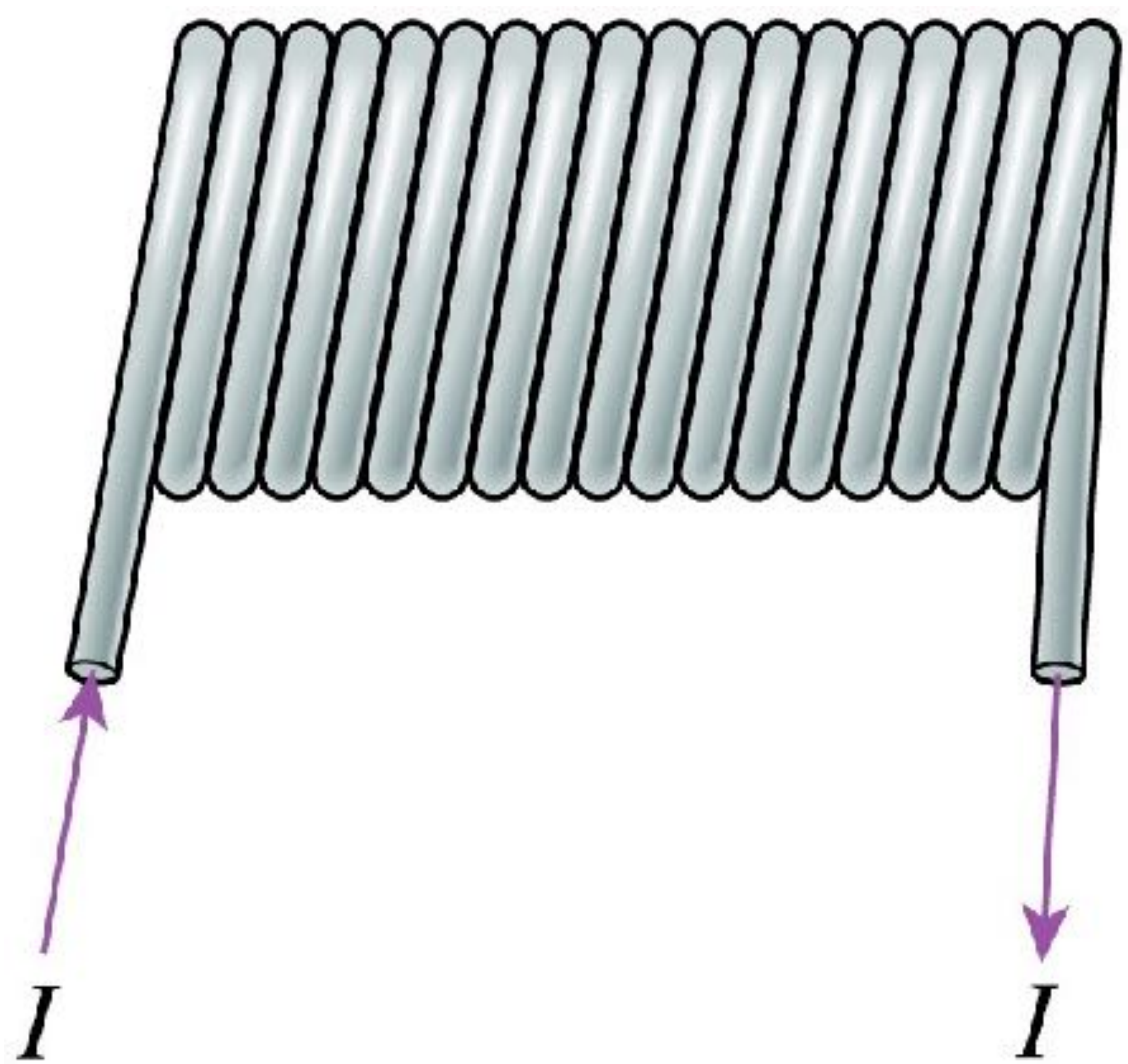
C. Current ccw; north pole on top

D. Current ccw; north pole on bottom

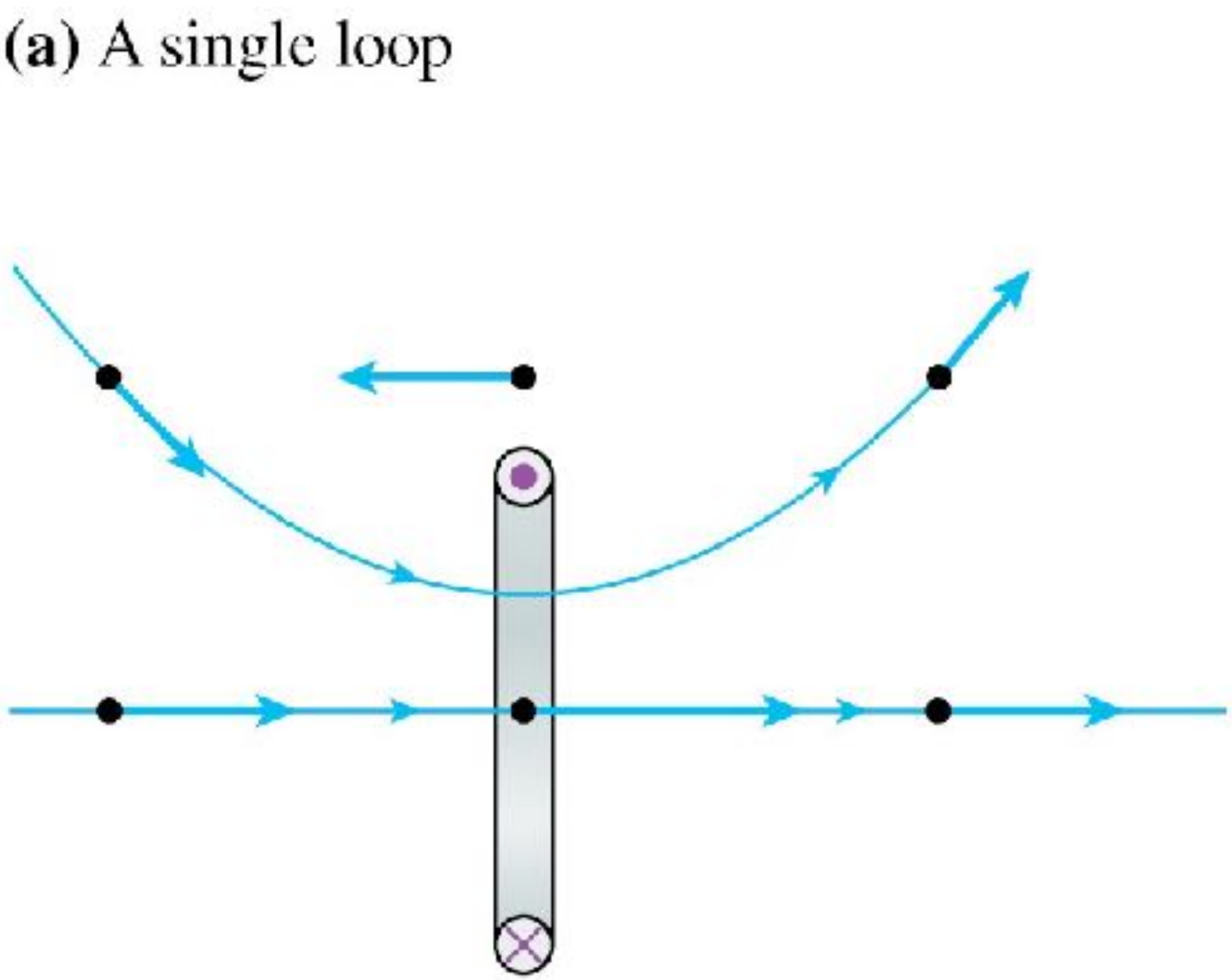


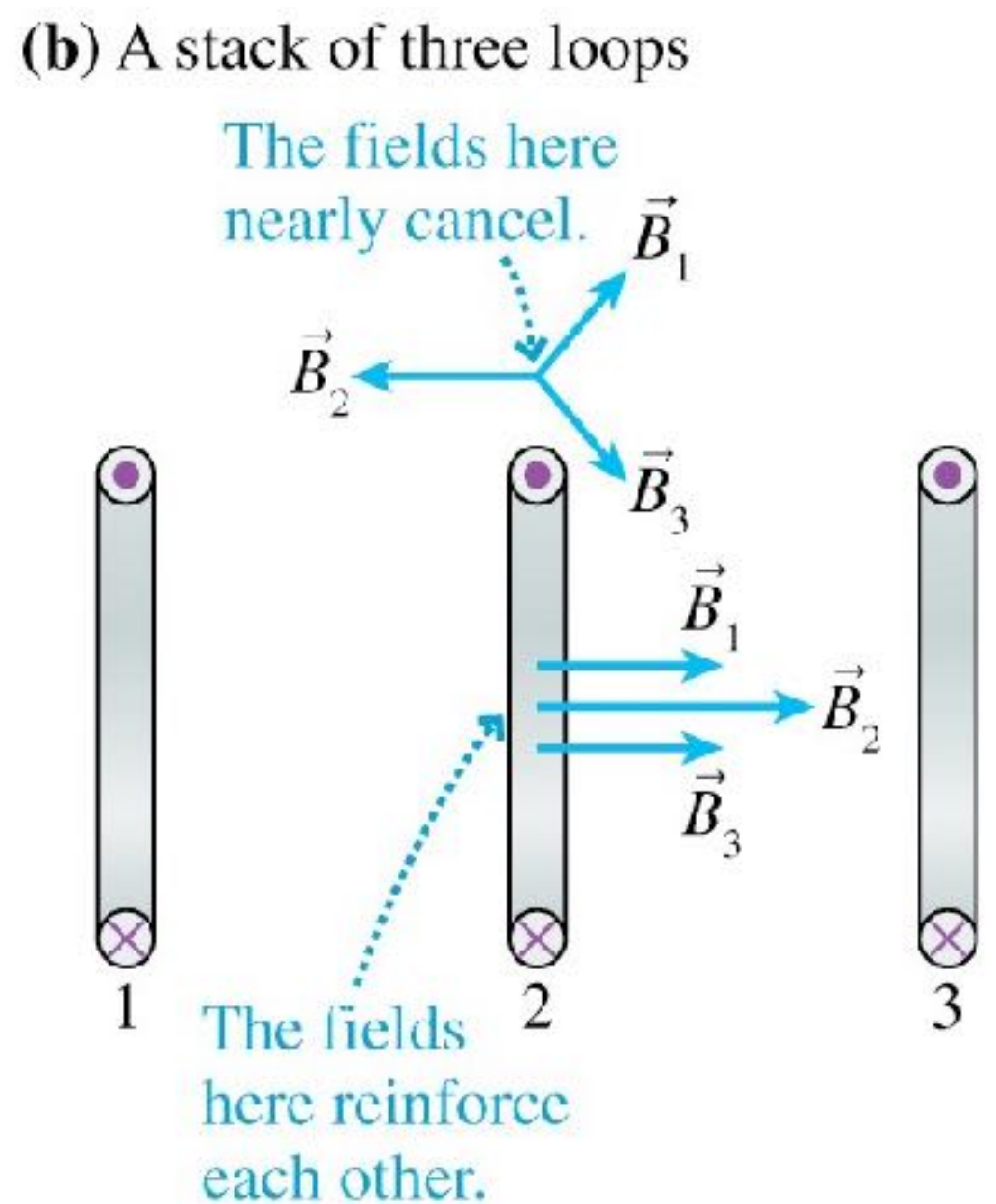
Solenoids

A solenoid is a helical coil of wire with a current passing through it. It can be used to create a relatively uniform magnetic field.

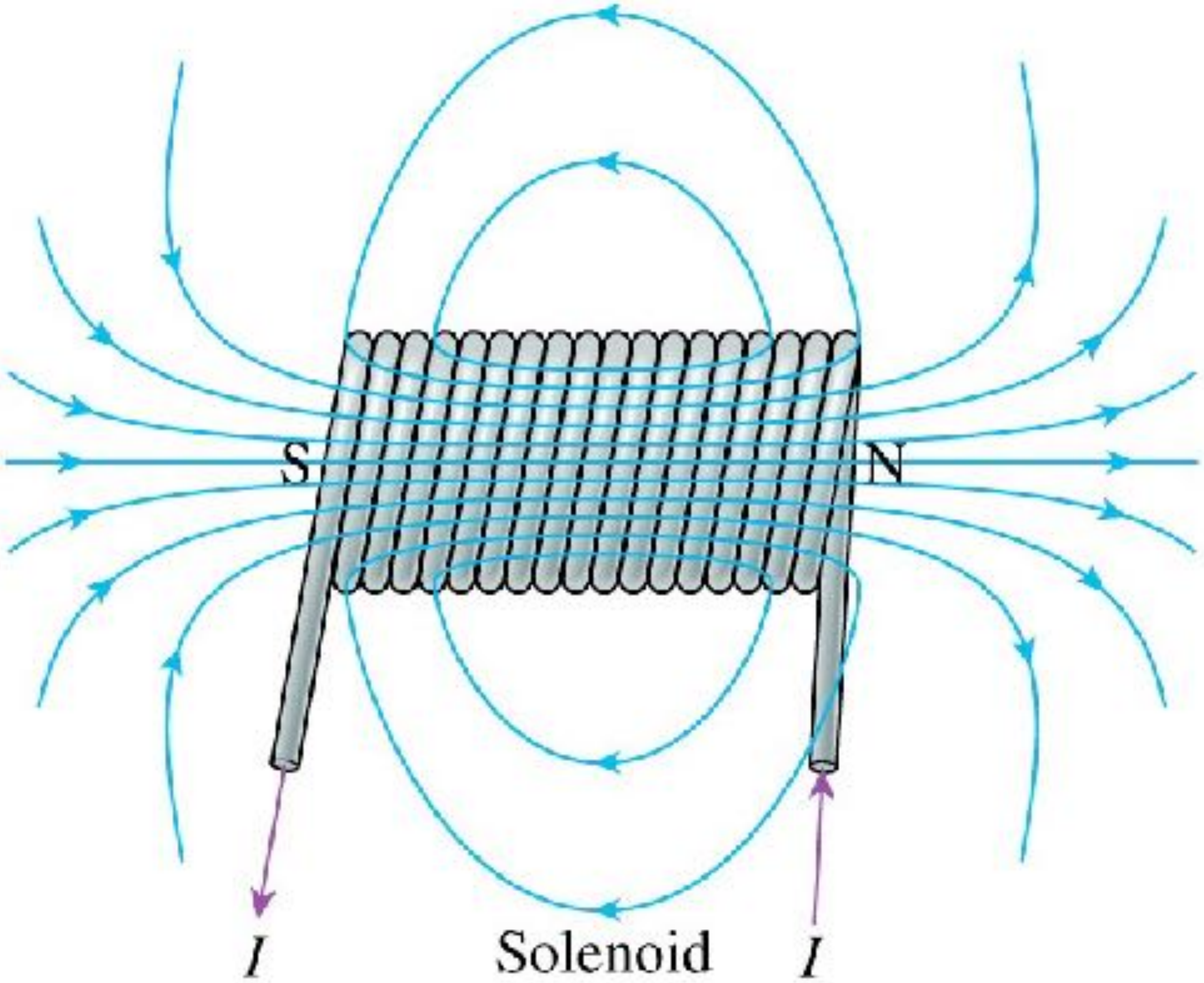


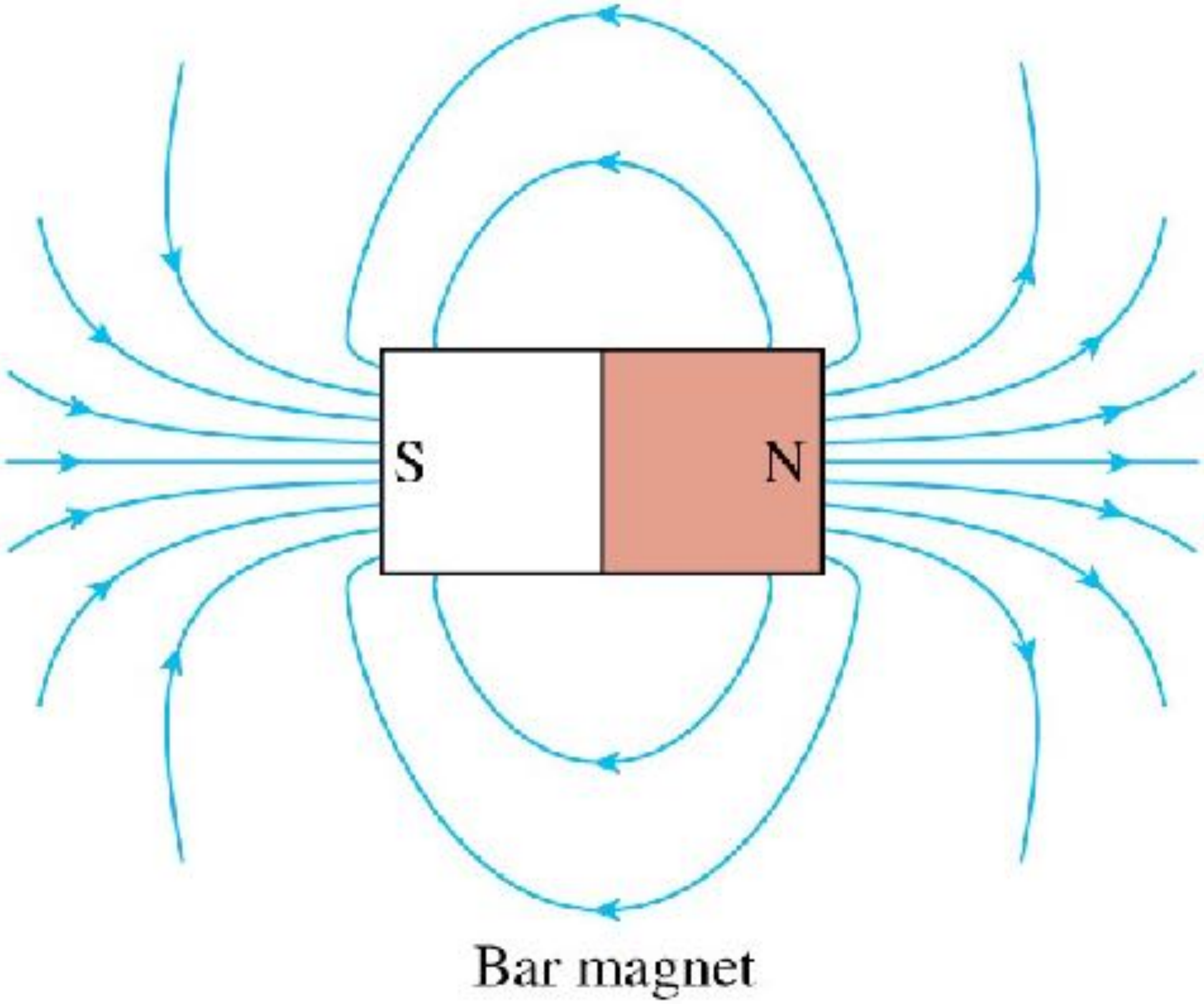
Magnetic Field of a Solenoid





Solenoid vs Bar Magnet





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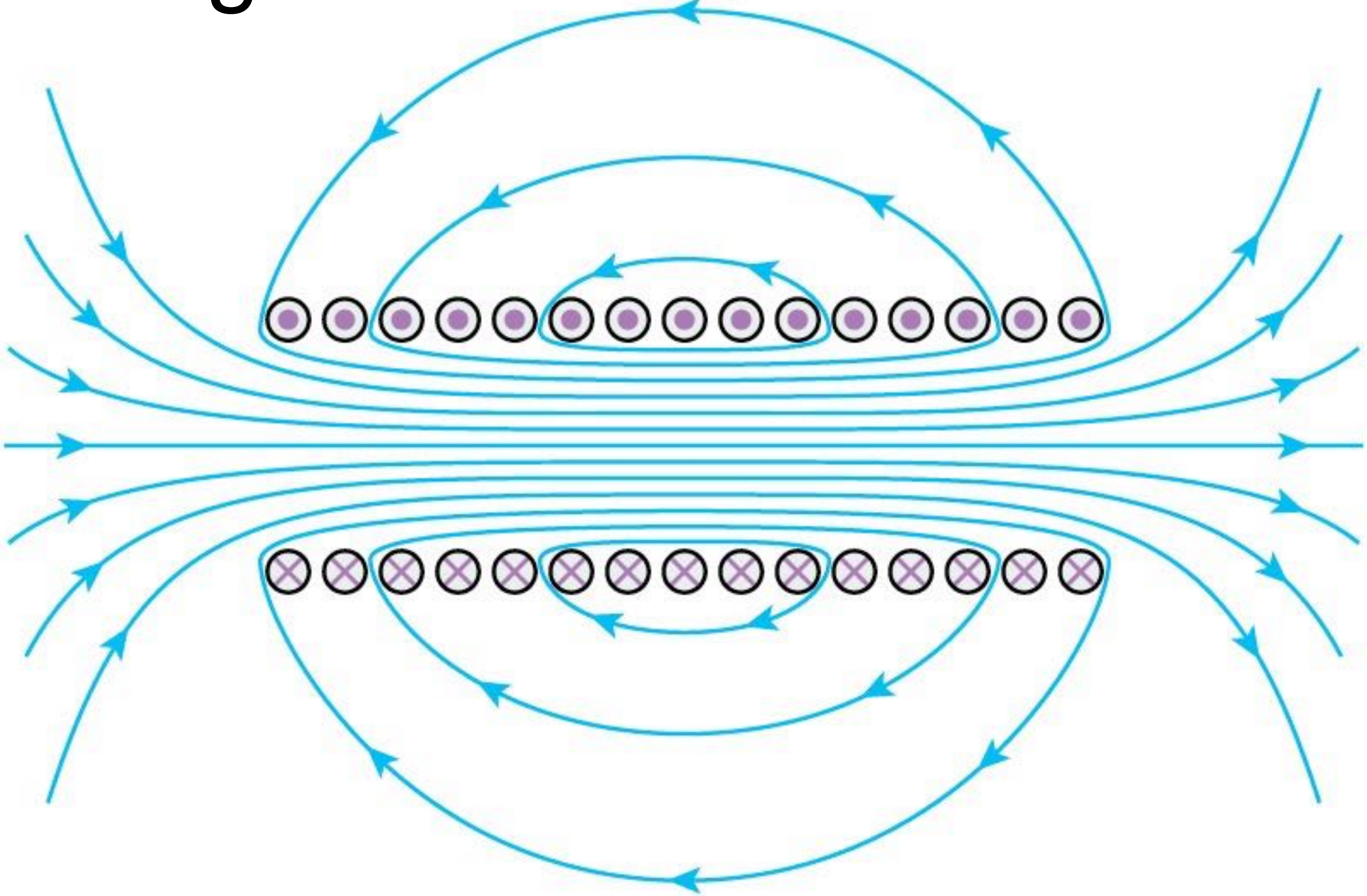
Magnetic Field of a Solenoid

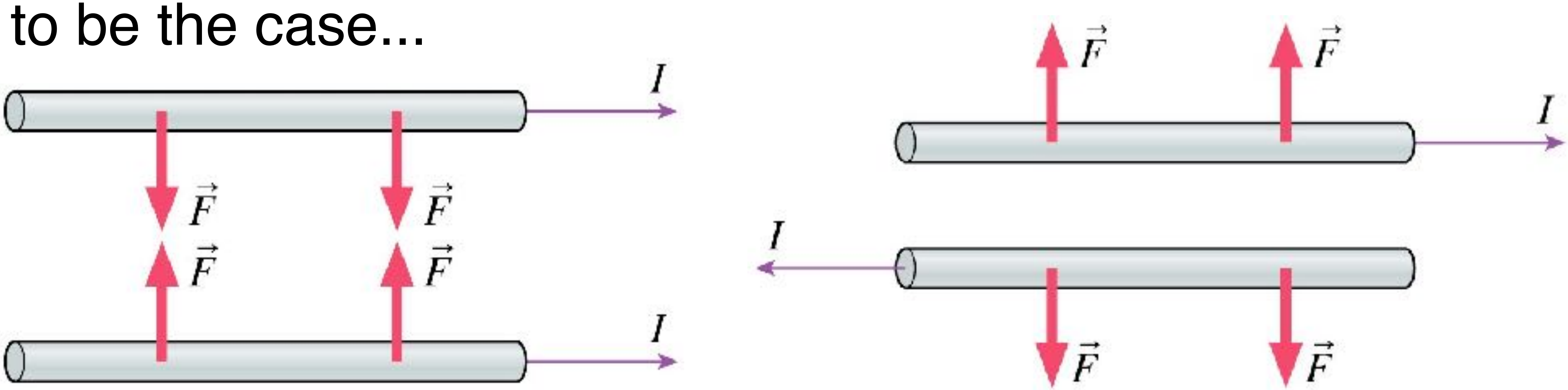


$$B_{\text{solenoid}} = \frac{\mu_0 N I}{L} = \mu_0 n I$$

$$n = \frac{N}{L}$$

Magnetic Force on a Moving Charge

After Oersted discovered the force on a compass due to a current, Ampère reasoned that the current was acting like a magnet, and thus two current carrying wires should exert magnetic forces on each other. His experiment showed that to be the case...



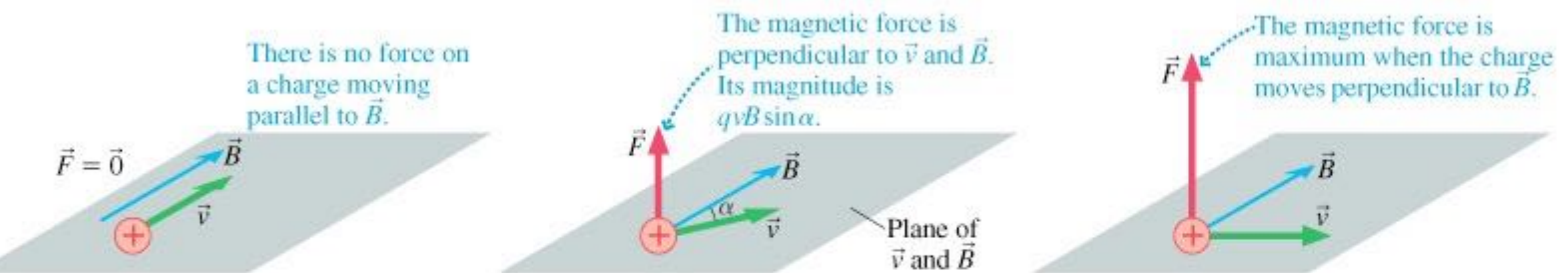
"Like" currents attract.

"Opposite" currents repel.

But first let's take a look at the magnetic force on a single charged particle...

Magnetic Force on a Moving Charge

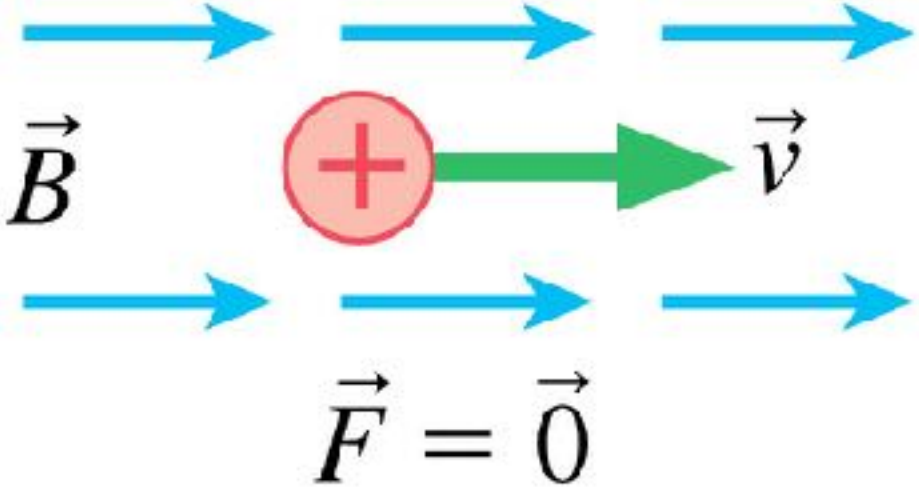
Ampère showed that a magnetic field exerts a force on a moving charge. But it is complicated! The force depends not just on the charge and the charge's velocity, but also on the *orientation* of the velocity vector relative to the field:

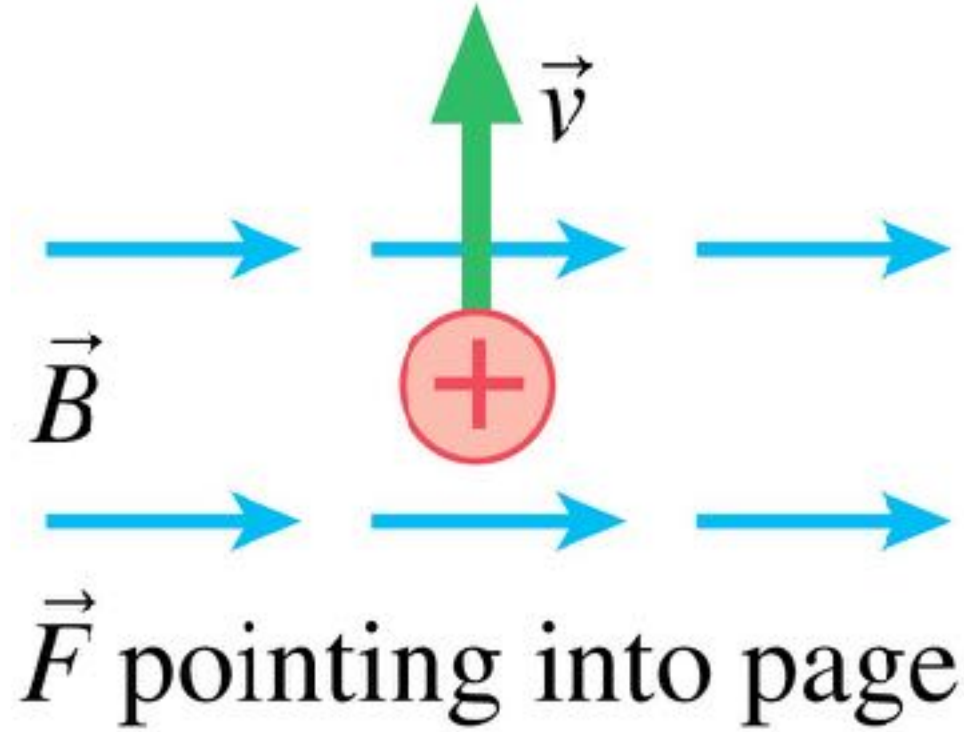


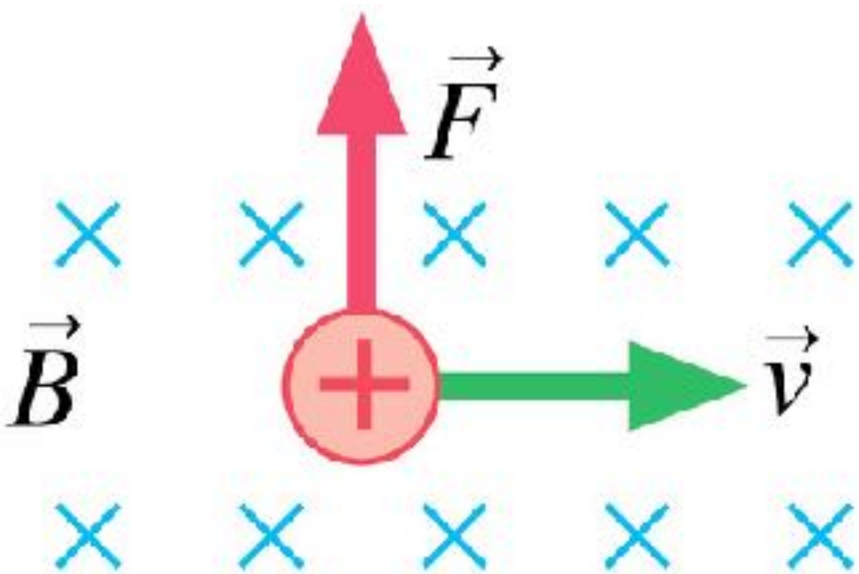
$$\vec{F}_{\text{on } q} = q\vec{v} \times \vec{B} = (qvB\sin\alpha, \text{direction of right-hand rule})$$

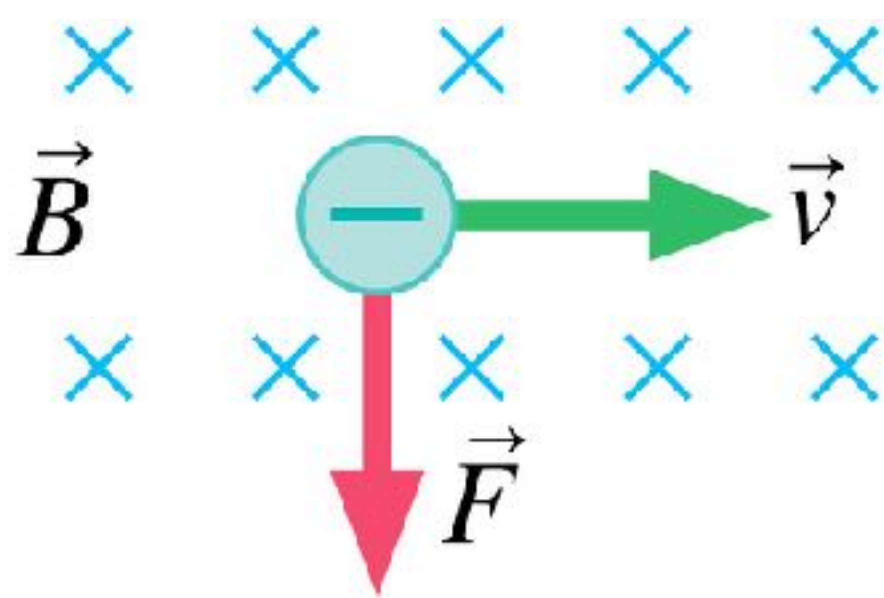
Magnetic Forces on a Moving Charge

1. Only a *moving* charge experiences a magnetic force.
2. There is no force on a charge moving parallel or anti-parallel to a magnetic field.
3. Where there is a force, the force is perpendicular to *both* $\vec{v}$ and $\vec{B}$.
4. The force on a negative charge is in the direction *opposite* to $\vec{v} \times \vec{B}$.
5. For a charge moving perpendicular to $\vec{B}$, the magnitude of the magnetic force is $F = |q|vB$.

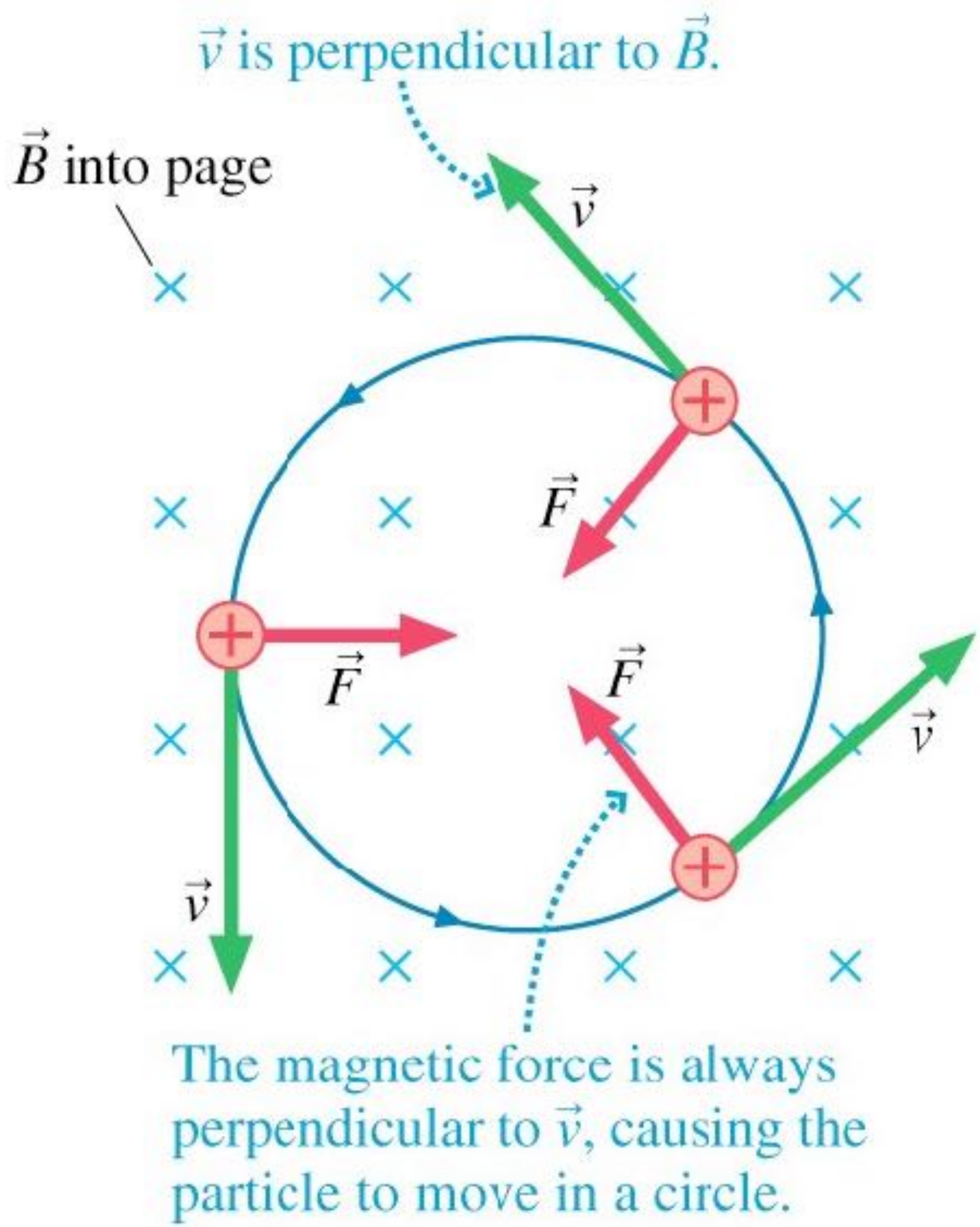

$\vec{B}$
$\vec{v}$
$\vec{F} = \vec{0}$


$\vec{v}$
$\vec{B}$
$\vec{F}$ pointing into page


$\vec{F}$
$\vec{B}$
$\vec{v}$


$\vec{B}$
$\vec{v}$
$\vec{F}$

Cyclotron Motion



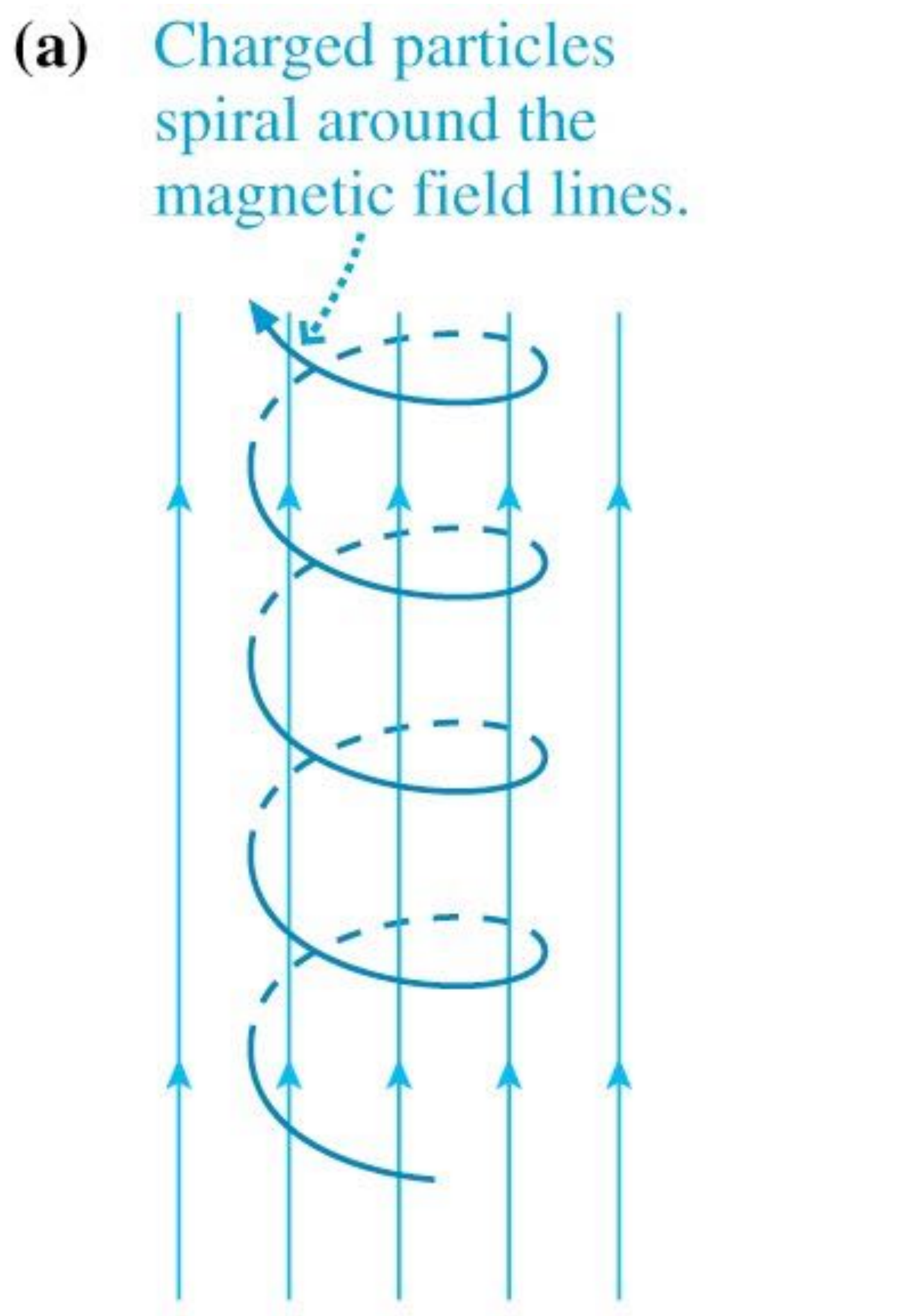
$$F = qvB = ma_r = \frac{mv^2}{r}$$

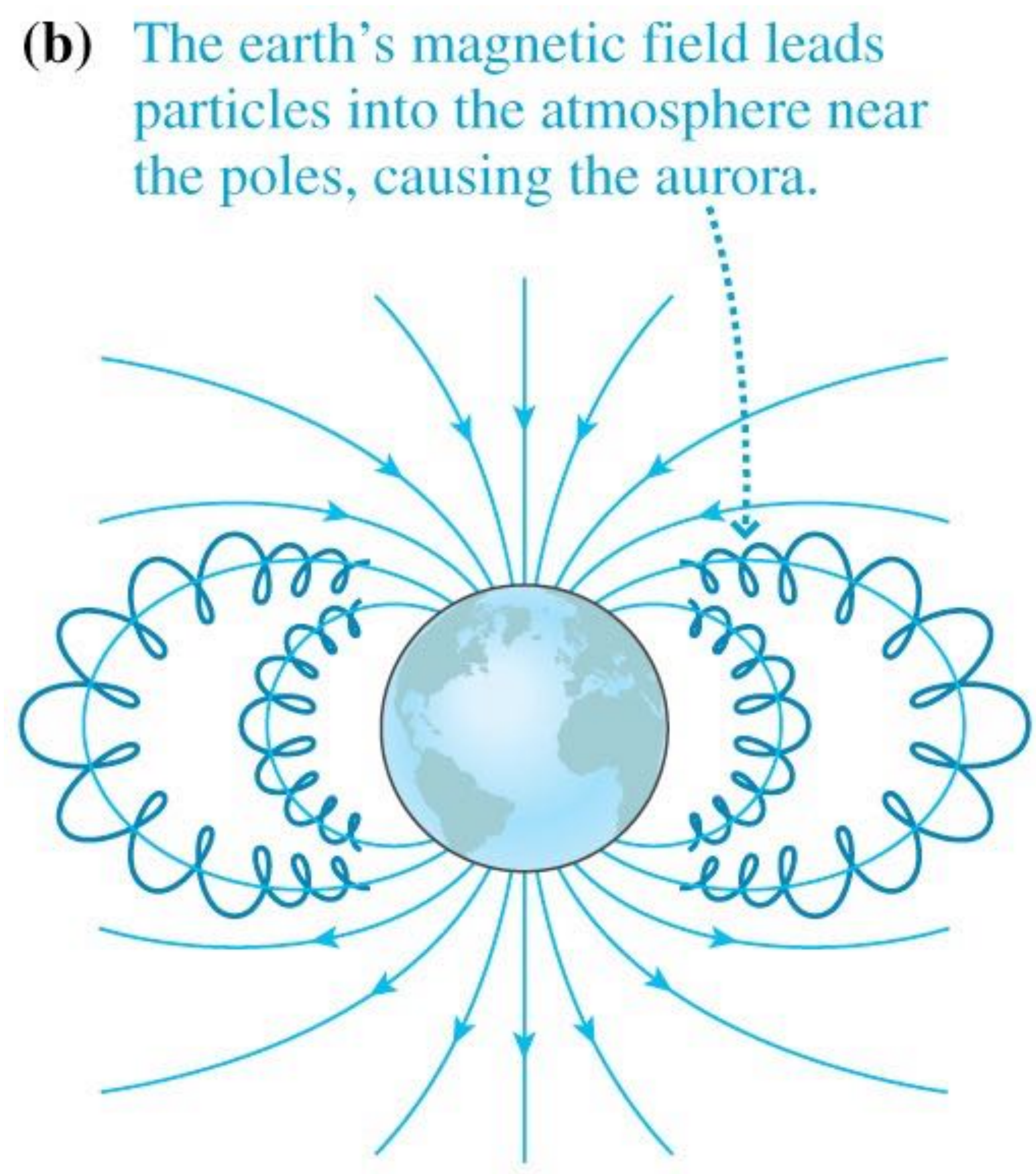
$$r = \frac{mv}{qB}$$

$$\omega = \frac{v}{r} = \frac{qB}{m}$$

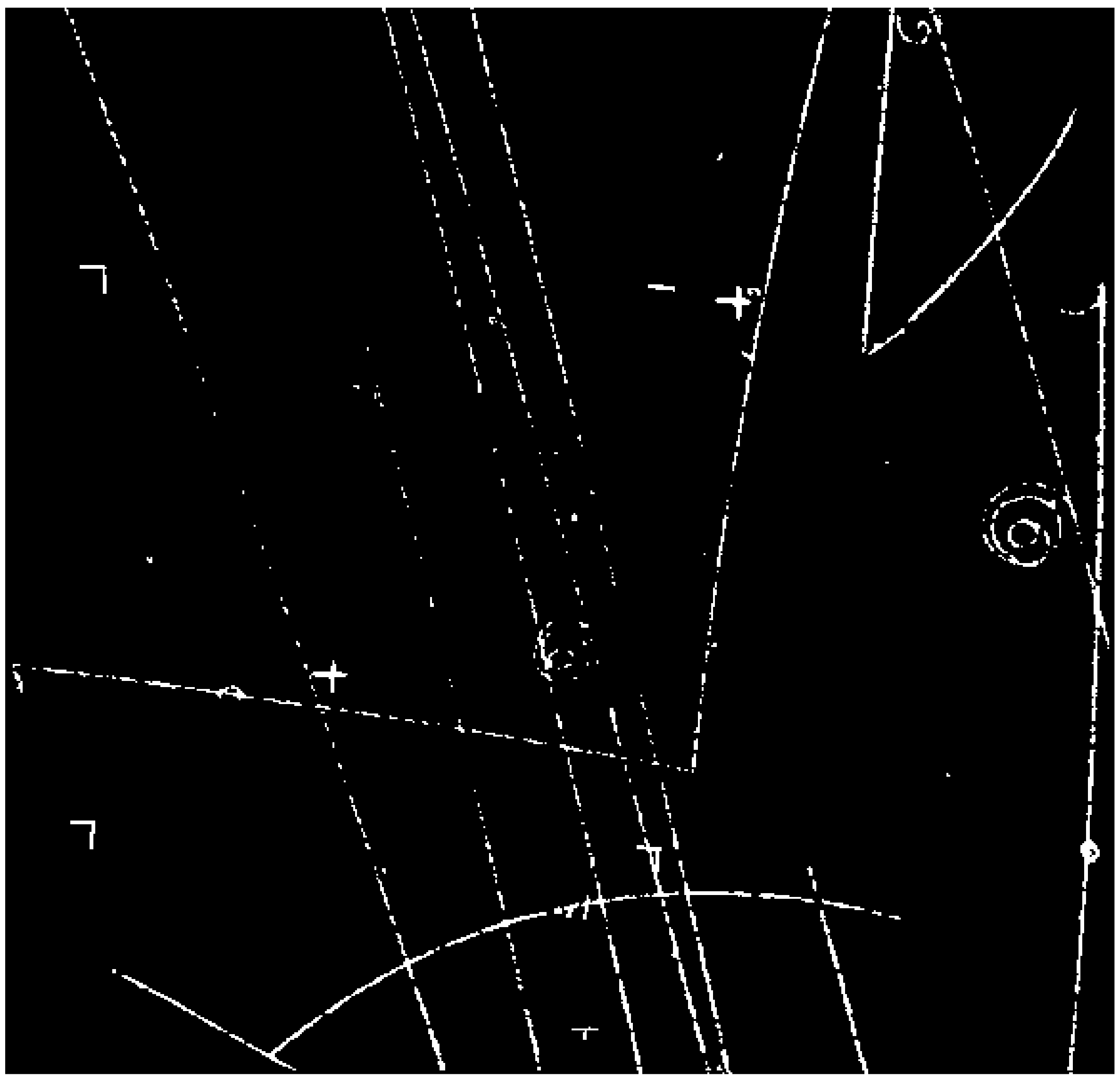
Cyclotron frequency

Cyclotron Motion





Applications...



$$r = \frac{mv}{qB}$$

$$mv = p = qBr$$

This technique is used all the time in particle physics to determine the momentum of particles produced in interactions.

Discussion Question

An electron moves perpendicular to a magnetic field. What is the direction of $\vec{B}$?

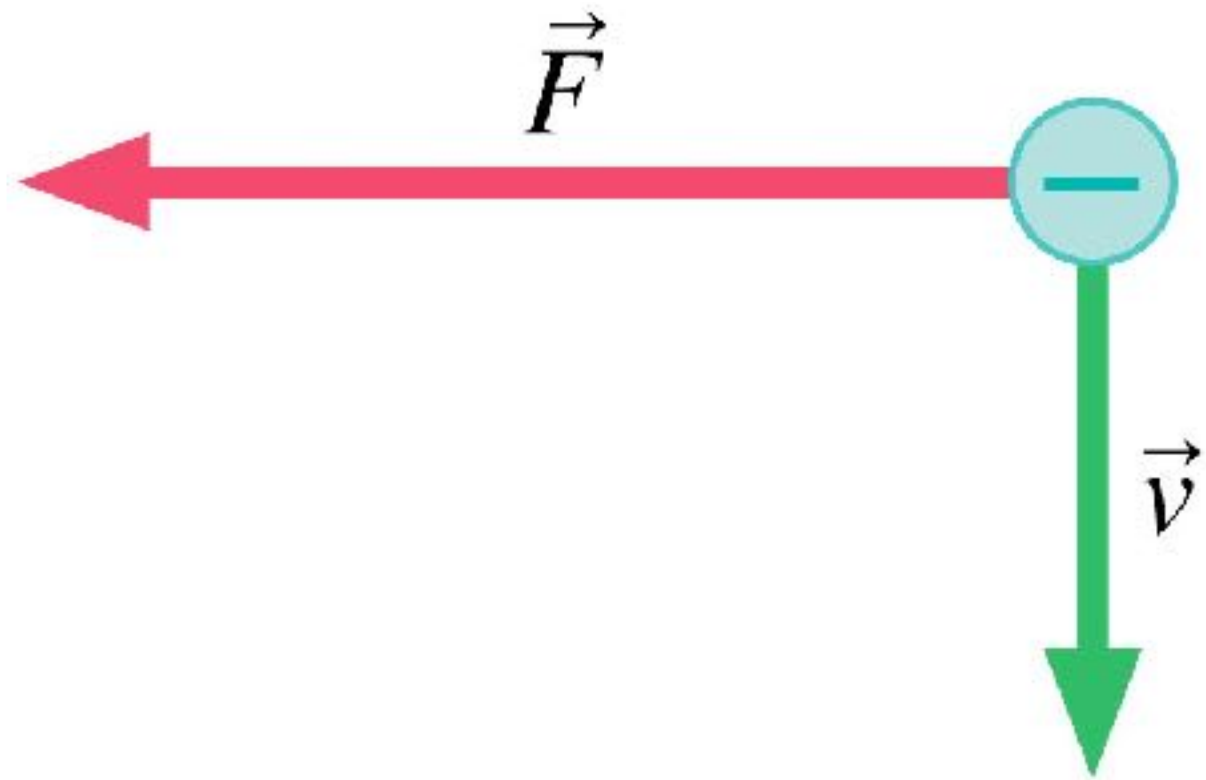
A. Left

B. Right

C. Up

D. Down

E. Into the page



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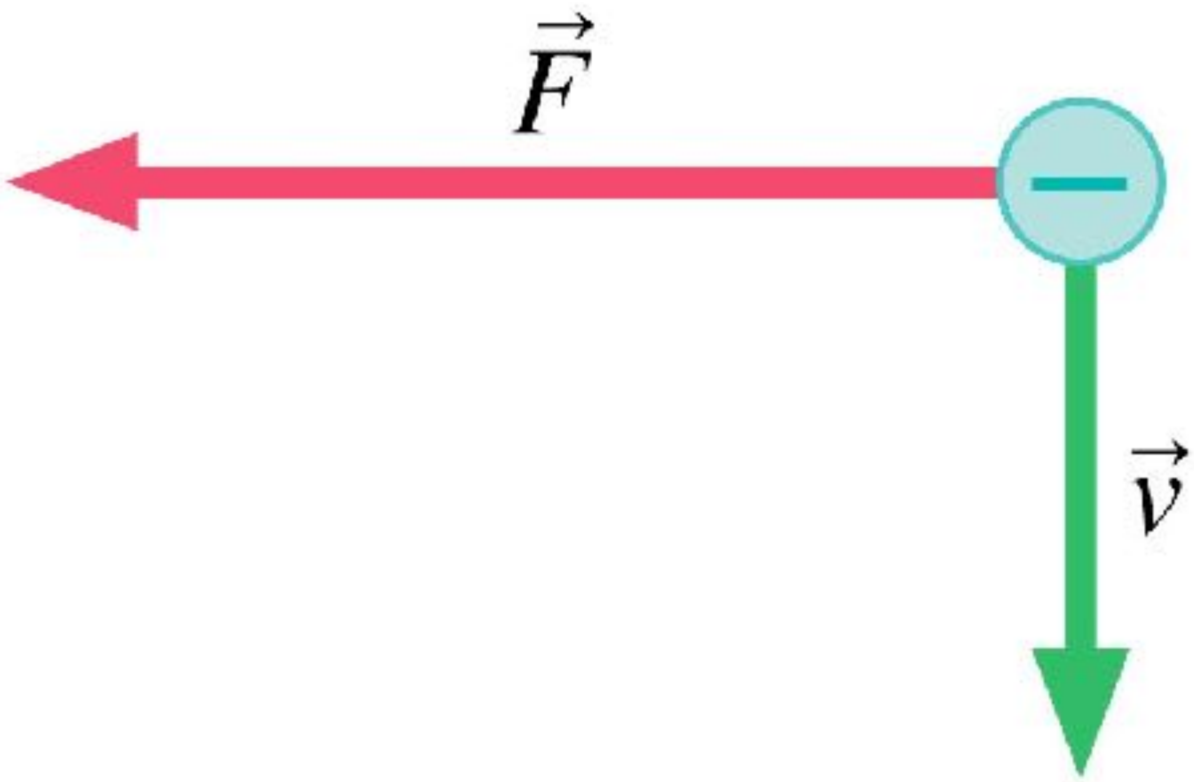
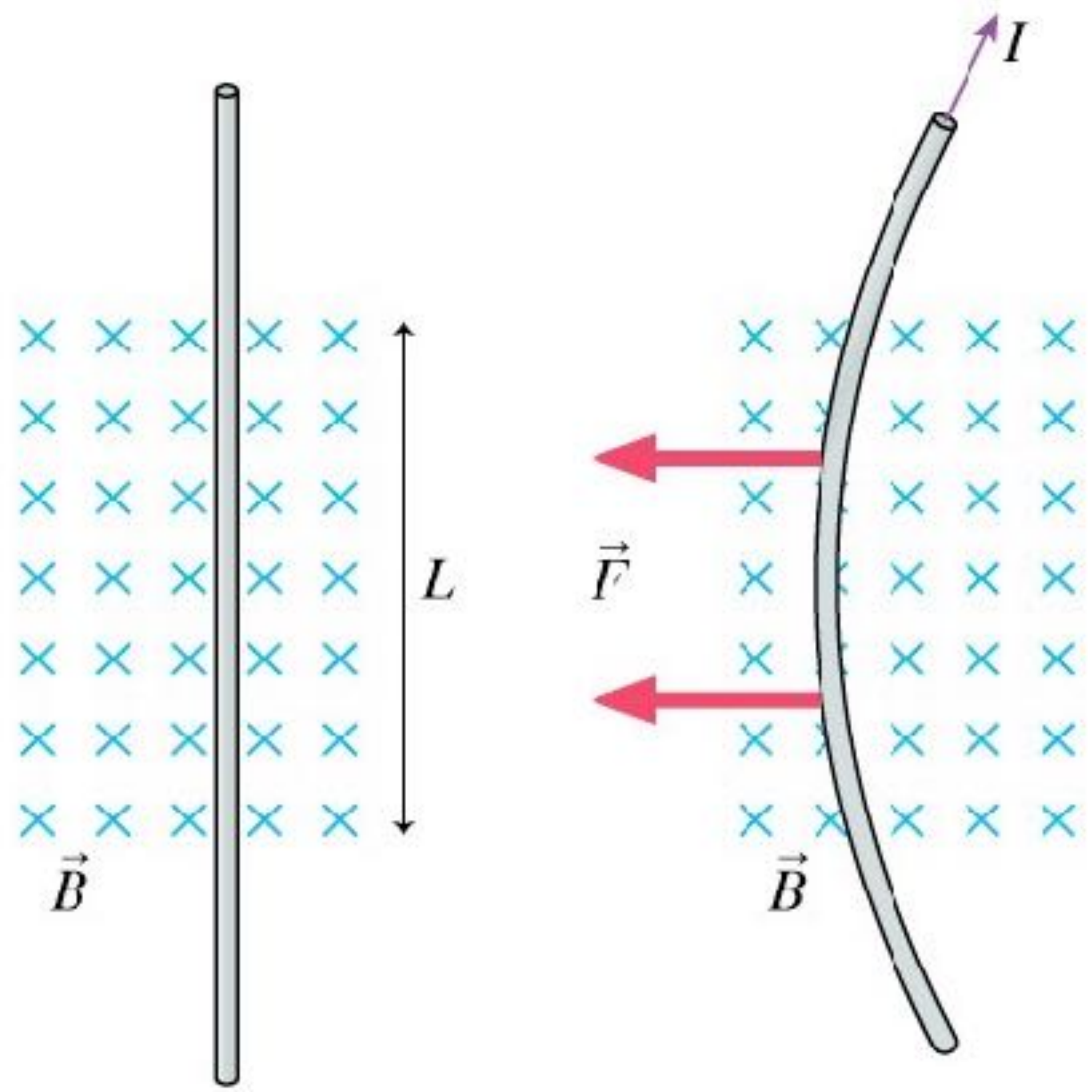


Magnetic Forces on a Current-Carrying Wire



A wire is perpendicular to an externally created magnetic field.

A current through a wire that is fixed at the ends causes the wire to be bent sideways.

$I = q/\Delta t \qquad \Delta t = L/v$

$$I = \frac{q}{\Delta t} = \frac{q}{L/v} = \frac{qv}{L}$$

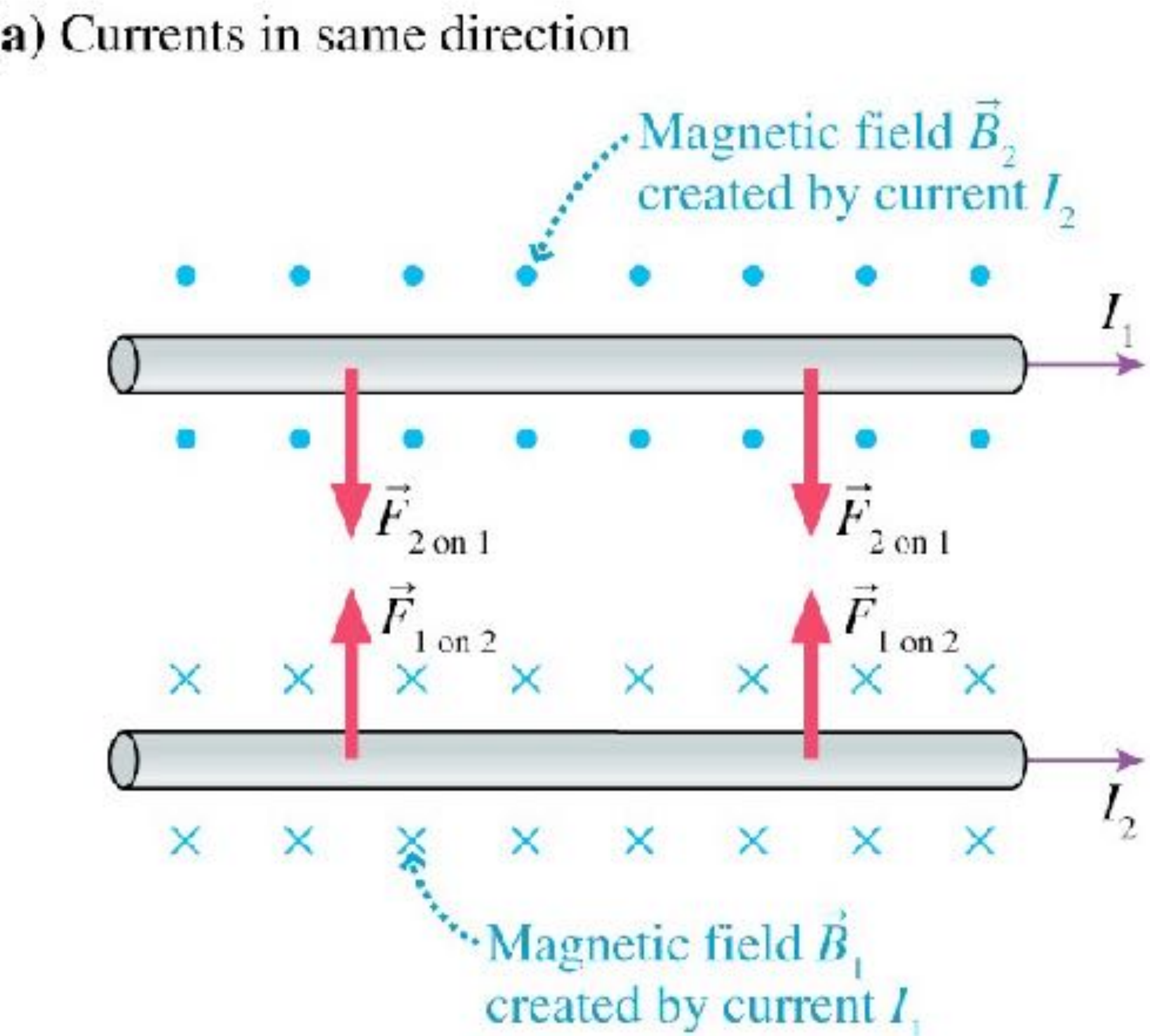
$qv = IL$

$F = qvB$

$F_{\text{wire}} = ILB$

Force on a current perpendicular to the field

Force Between Two Parallel Wires

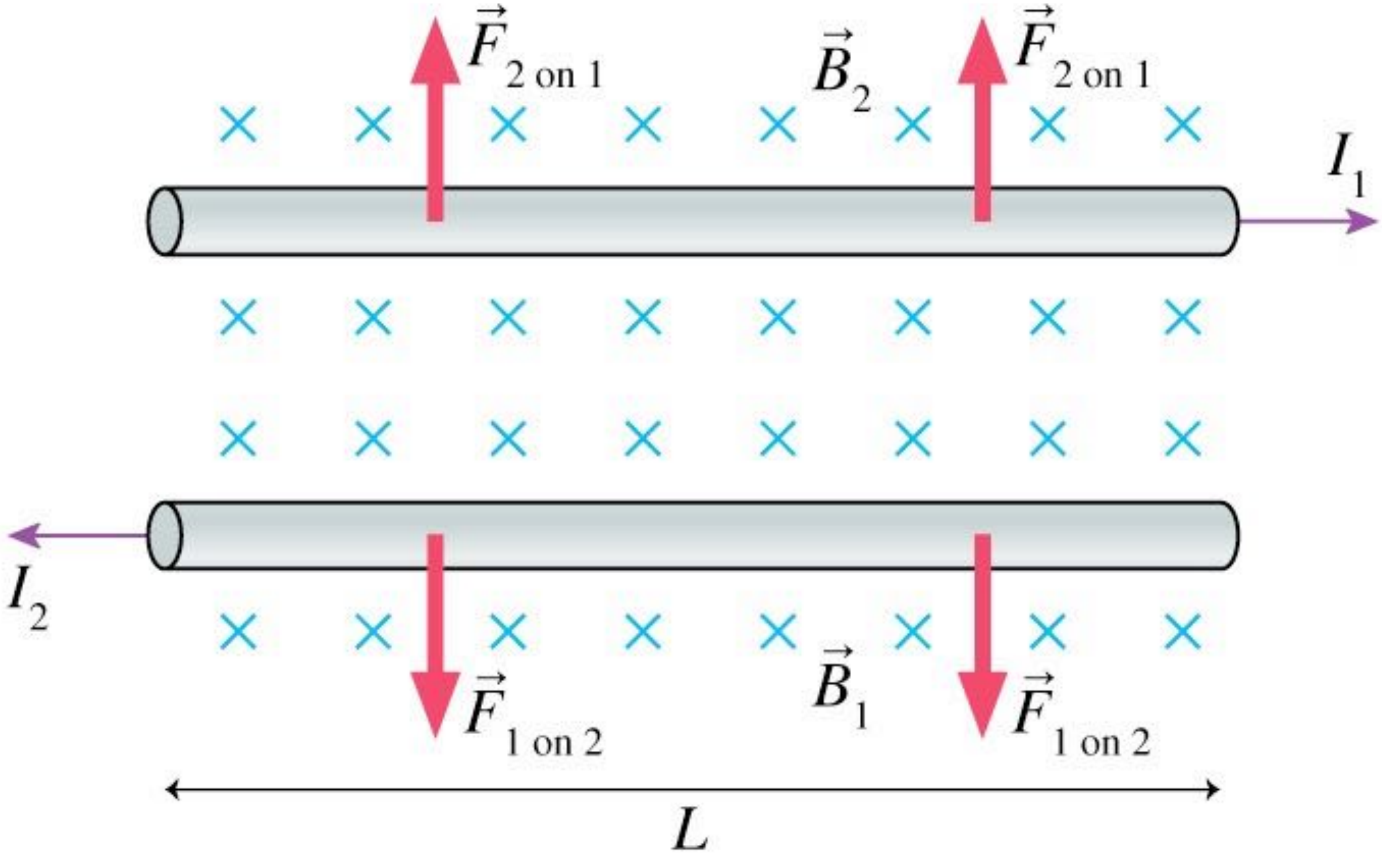


$$B_2 = \frac{\mu_0 I_2}{2\pi d}$$

The magnetic field of the lower wire exerts a force on the current in the upper wire.

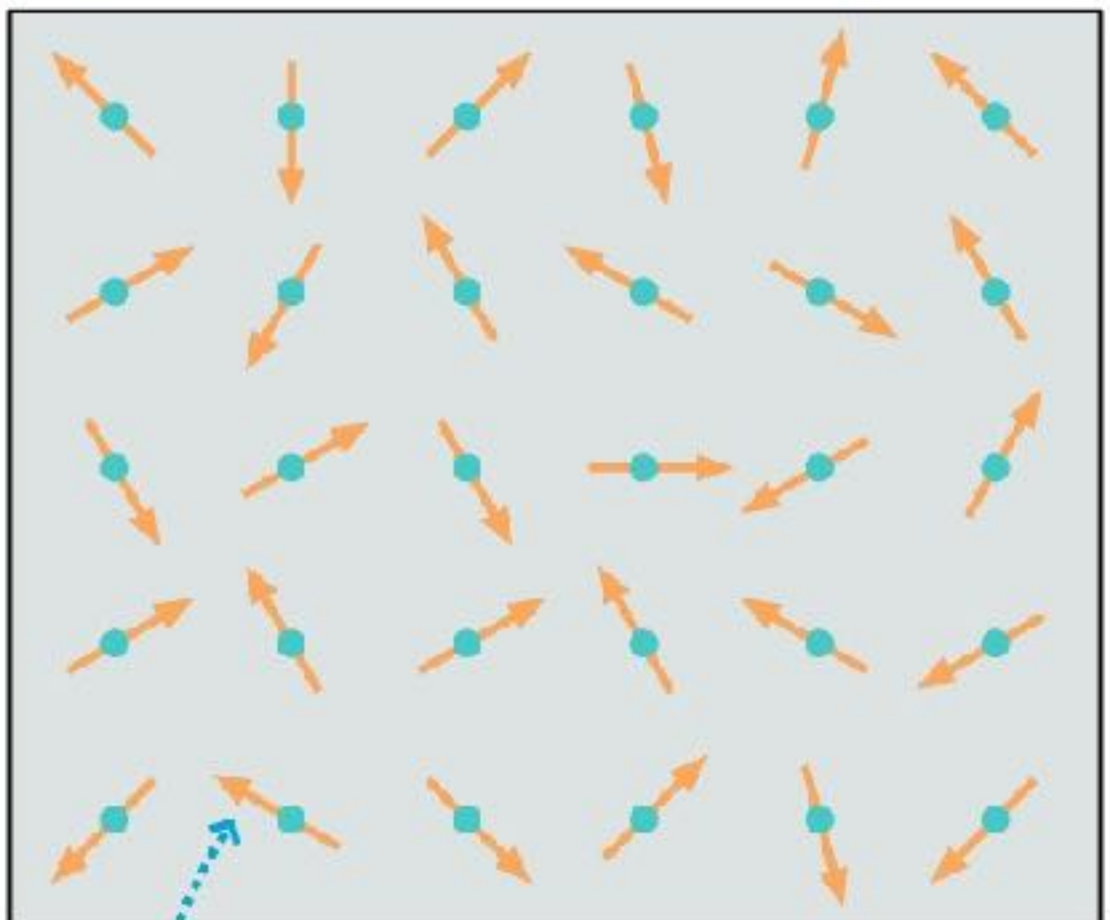
$$F_{\text{parallel wires}} = I_1 L B_2 = I_1 L \frac{\mu_0 I_2}{2\pi d} = \frac{\mu_0 L I_1 I_2}{2\pi d}$$

(b) Currents in opposite directions

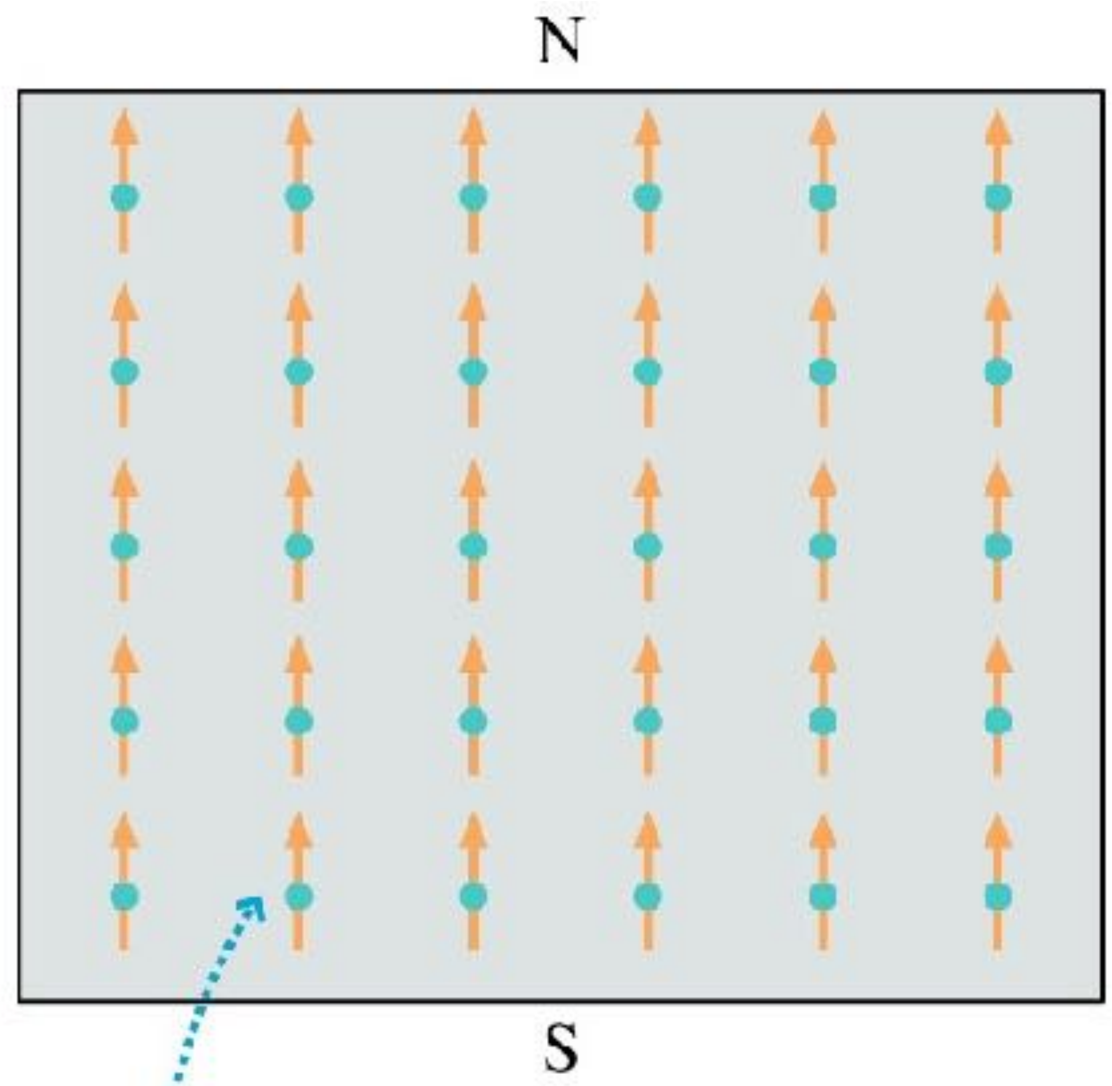


Full Circle!

Electrons have *spin* and thus behave like microscopic magnets

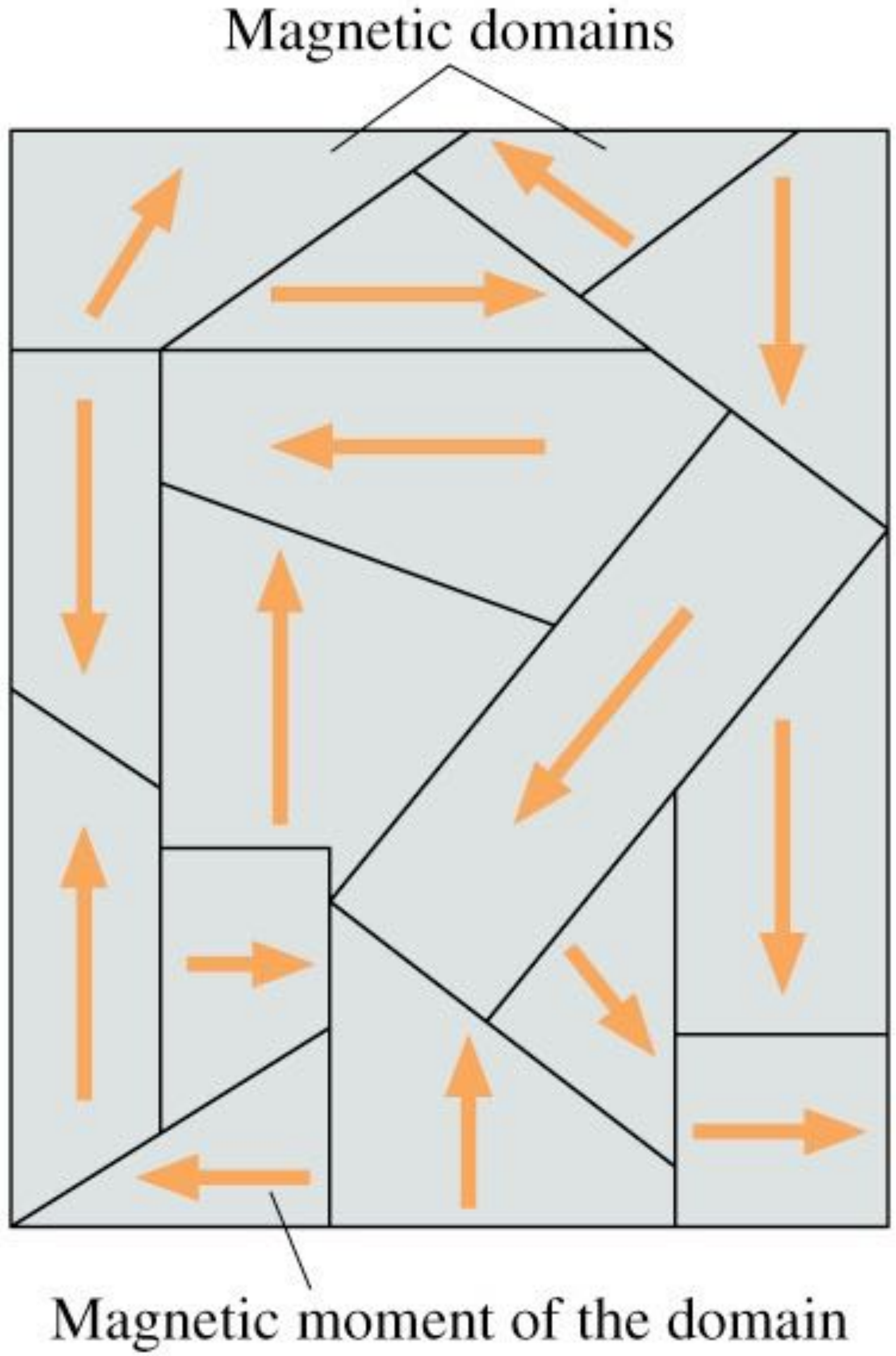


The atomic magnetic moments due to unpaired spins point in random directions. The sample has no net magnetic moment.



The atomic magnetic moments are aligned. The sample has a north and south magnetic pole.

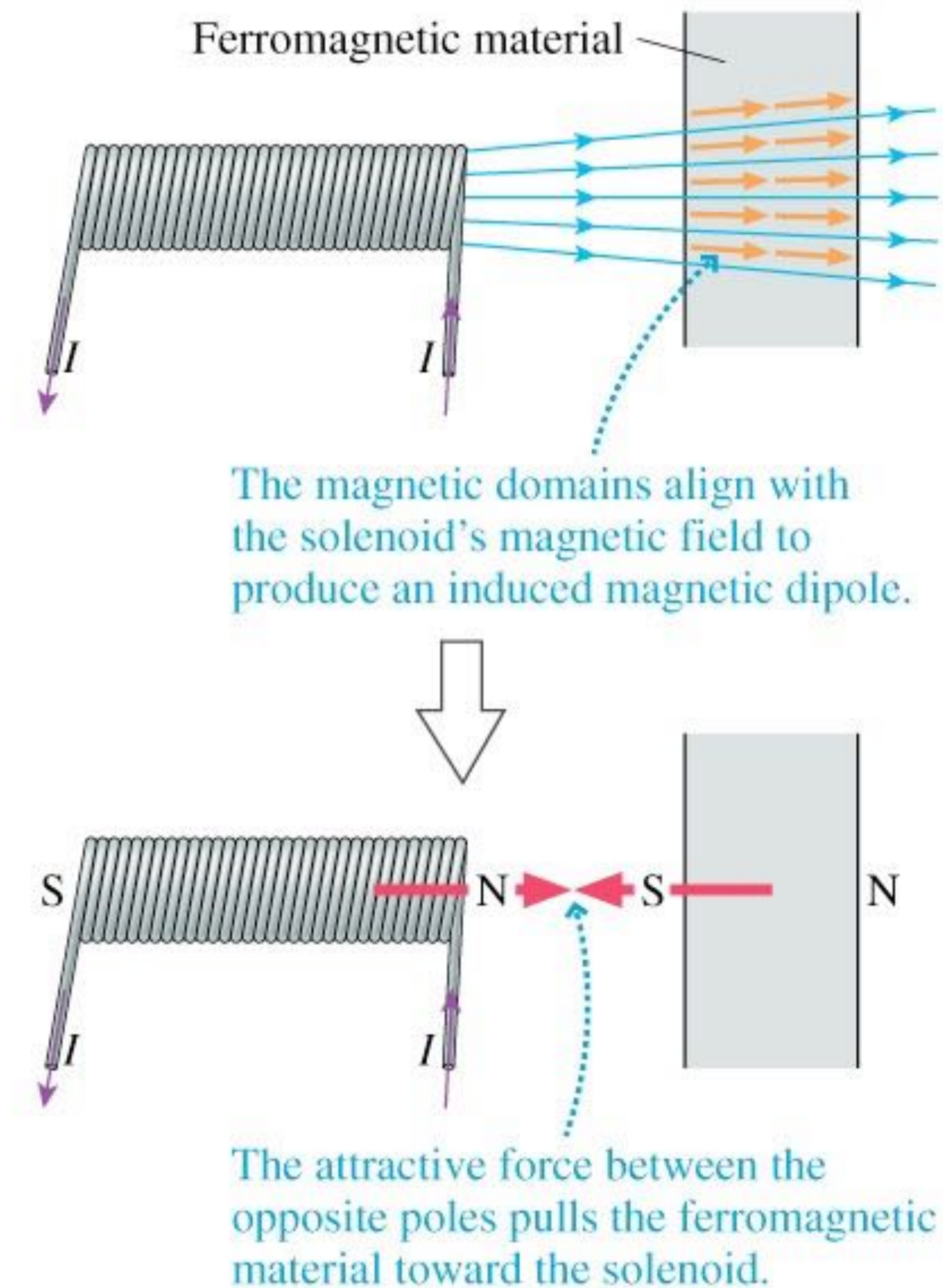
Ferromagnetic Materials



Induced Magnetic Dipole

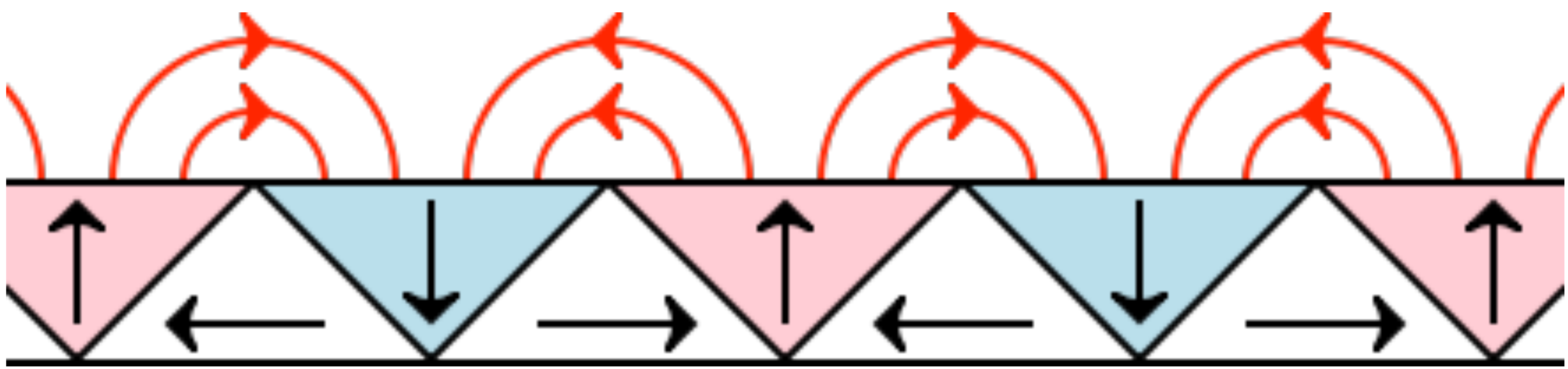
Any magnet will do this, not just an electro-magnet.

This induced magnetic dipole is very much like the induced electrostatic dipole that an electric field can produce (causing a neutral object to be attracted to a charged object).



Fridge Magnets

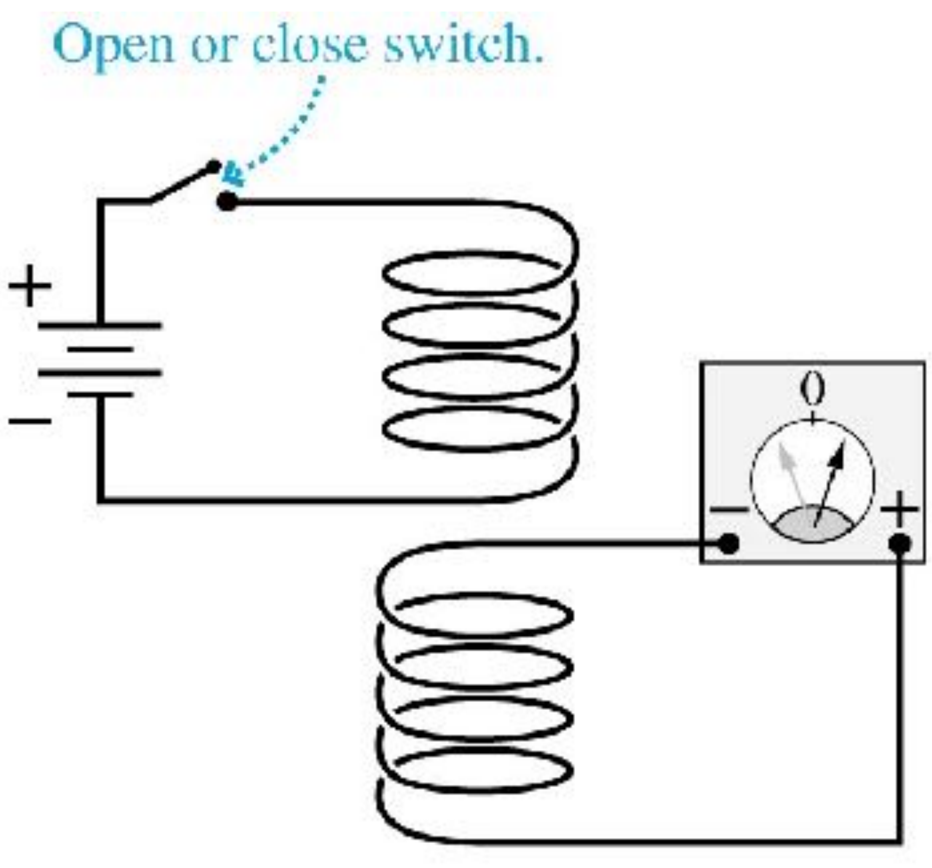
- Unusual since only one side sticks to the fridge!
- How does that work, since every magnet has a north and south pole?
- Ingenious layout of magnets:

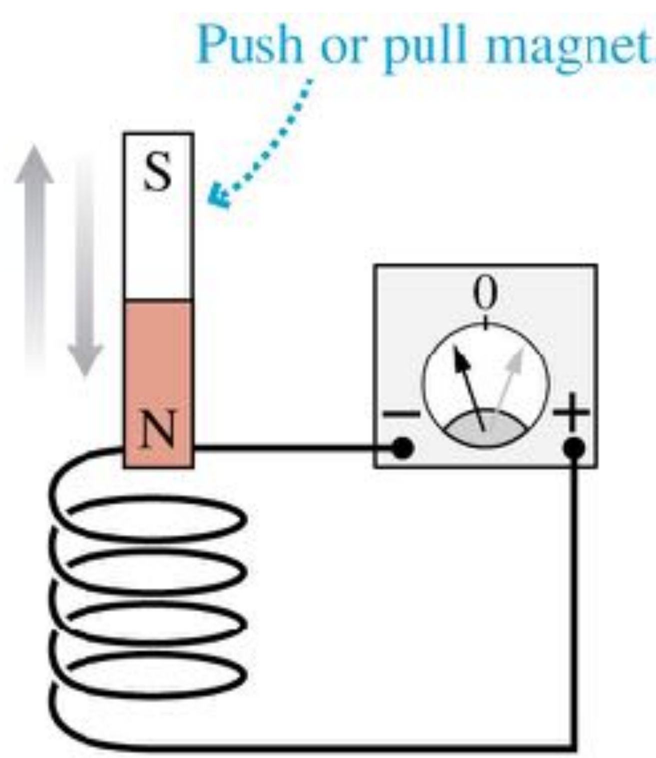


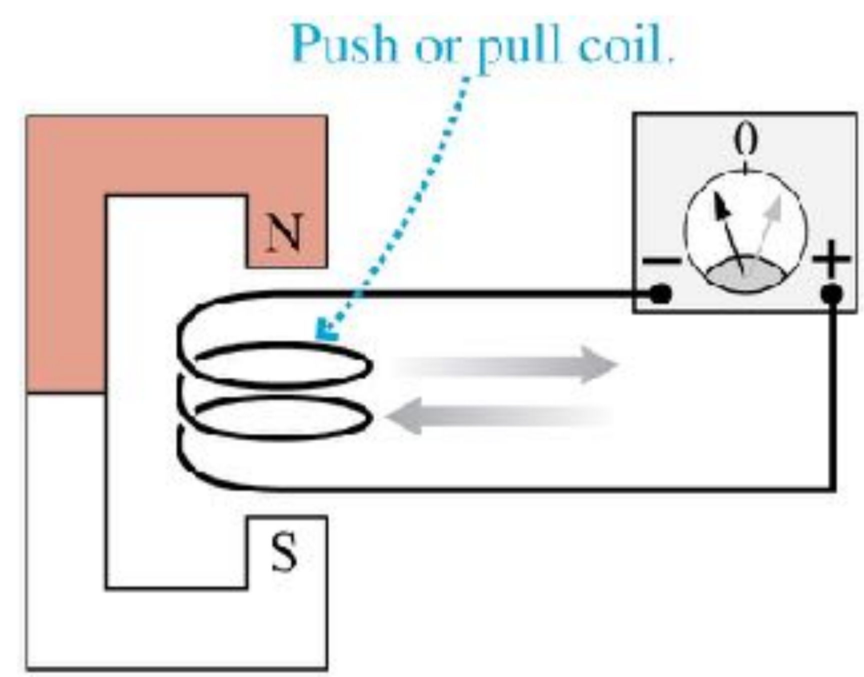
Invented by Klaus Halbach in the 1980s (Livermore)

Electromagnetic Induction

A series of experiments made by Faraday:



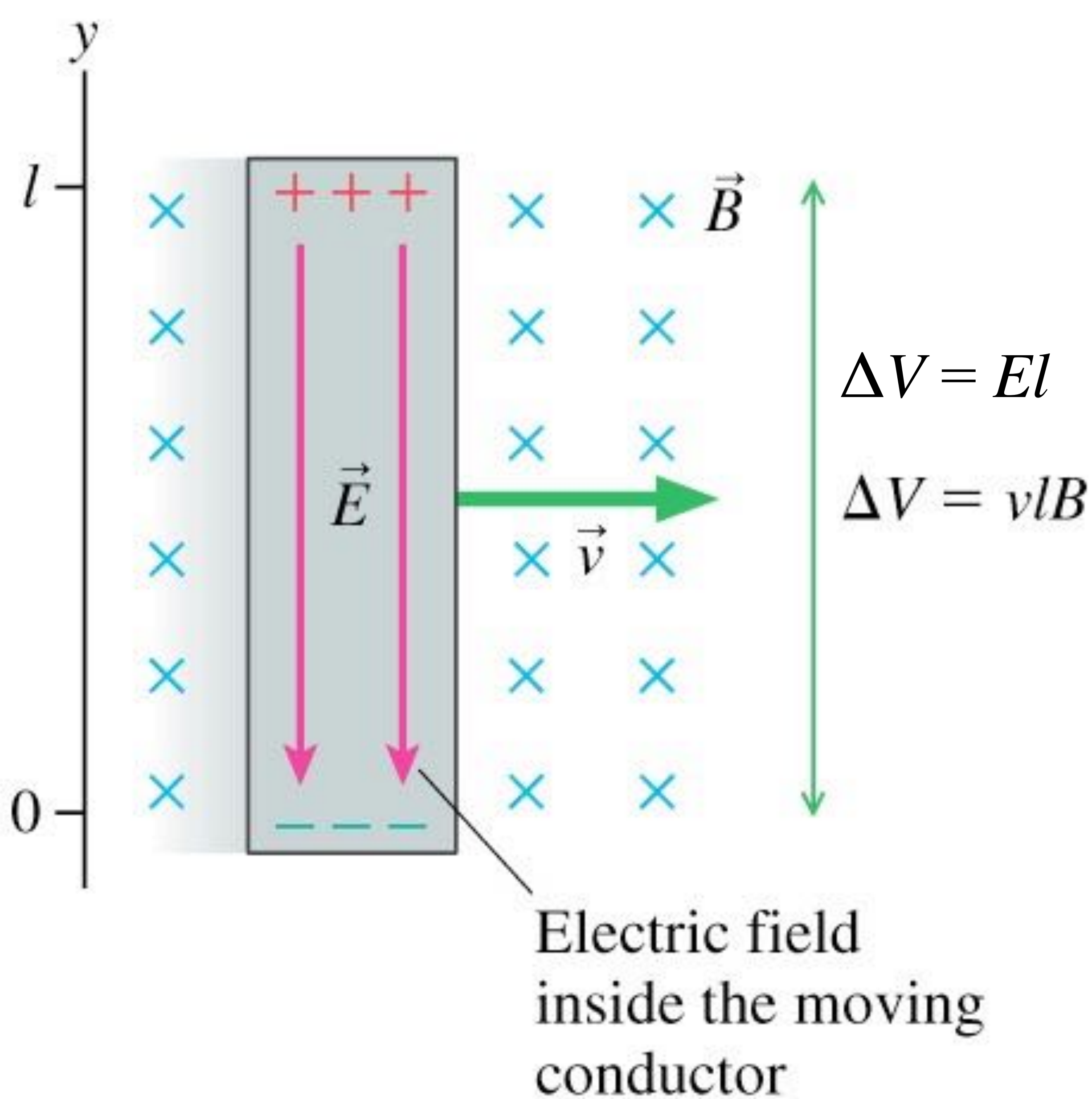




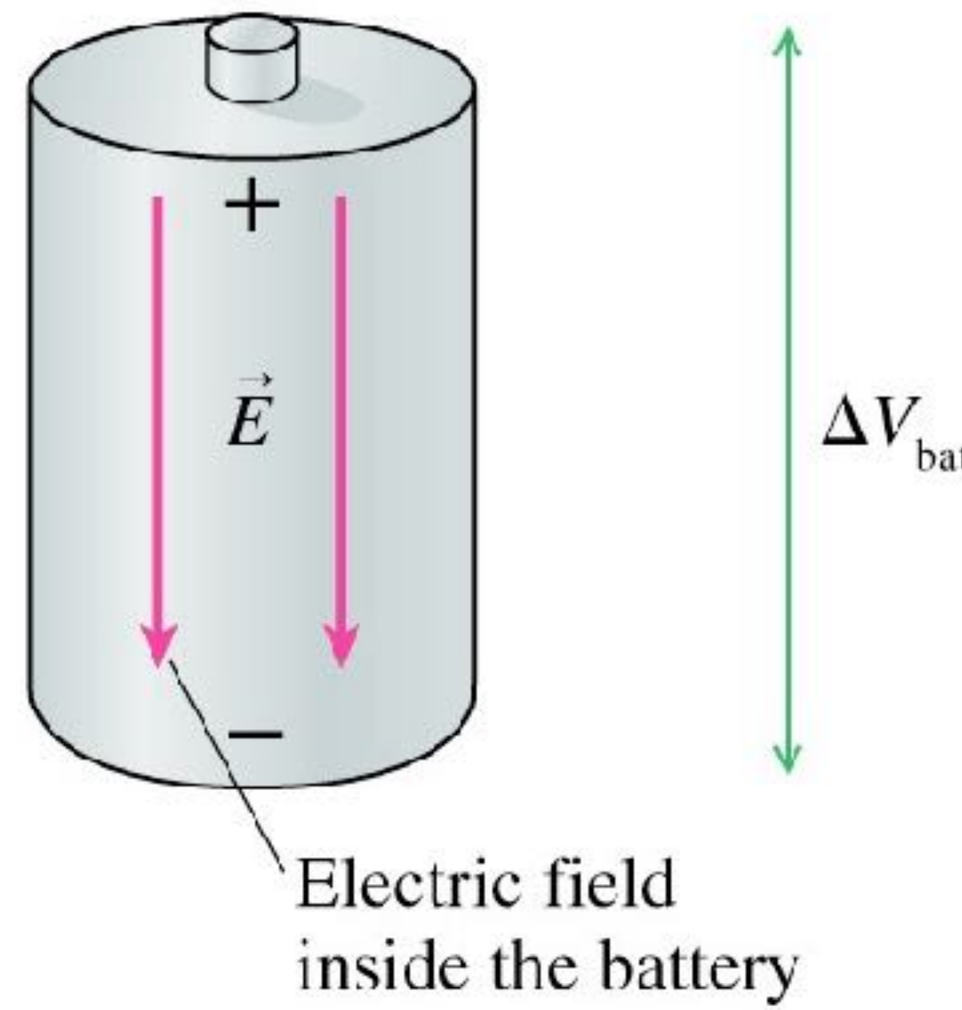
Conclusion: There is a current in a coil of wire if and only if the magnetic field passing through the coil is ***changing***.

Motional vs chemical emf

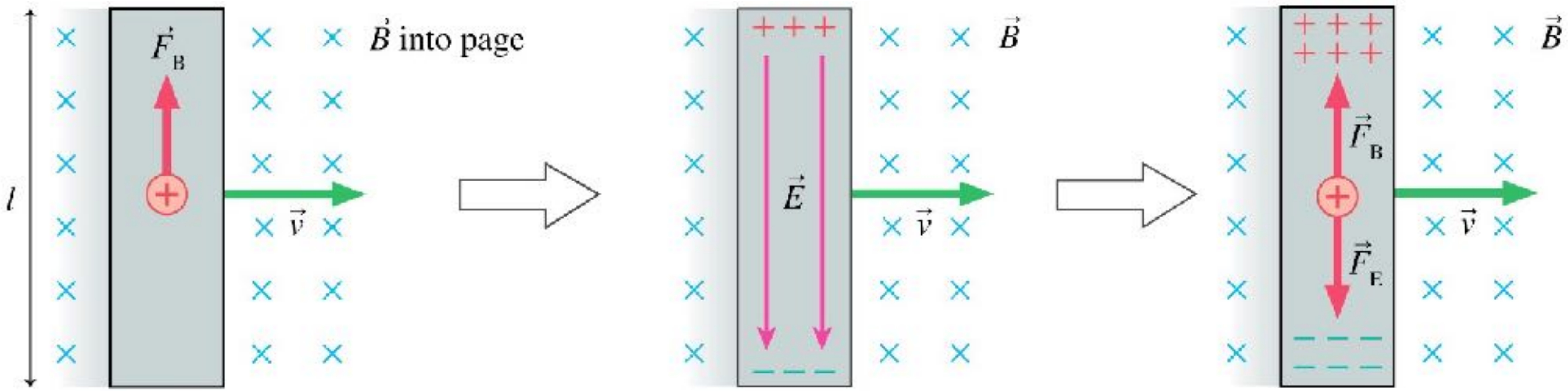
(a) Magnetic forces separate the charges and cause a potential difference between the ends. This is a motional emf.



(b) Chemical reactions separate the charges and cause a potential difference between the ends. This is a chemical emf.



Motional emf



The charges in the wire experience a force, since they are moving in a magnetic field.

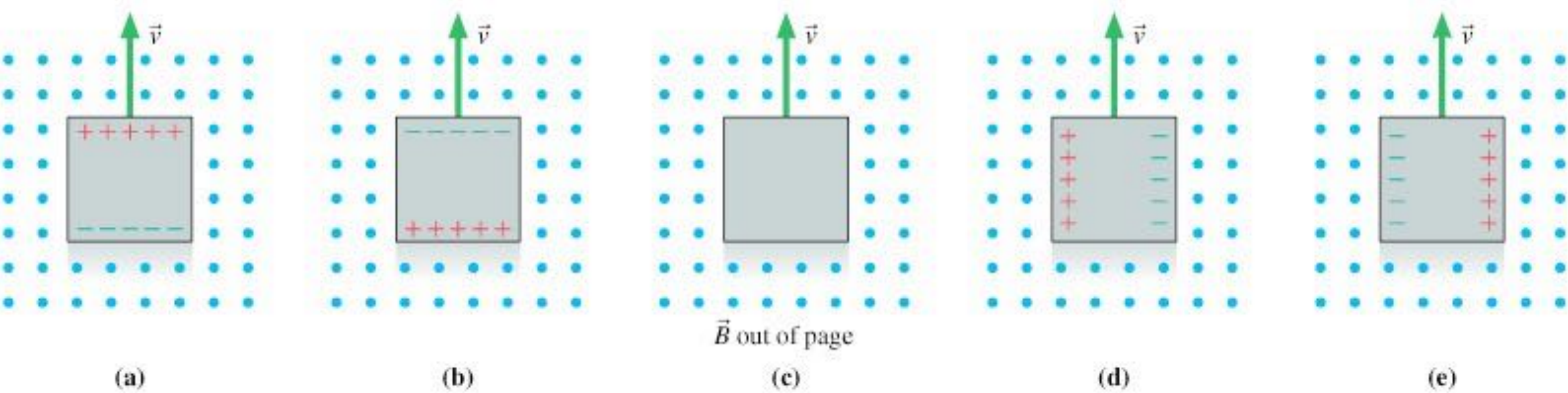
The charge separation creates an electric field.

The charge flow continues until the electric force on a charge is balanced by the magnetic force.

$$F_E = F_B \Rightarrow qE = qvB \Rightarrow E = vB$$

Discussion Question

A square conductor moves through a uniform magnetic field. Which of the figures shows the correct charge distribution on the conductor?



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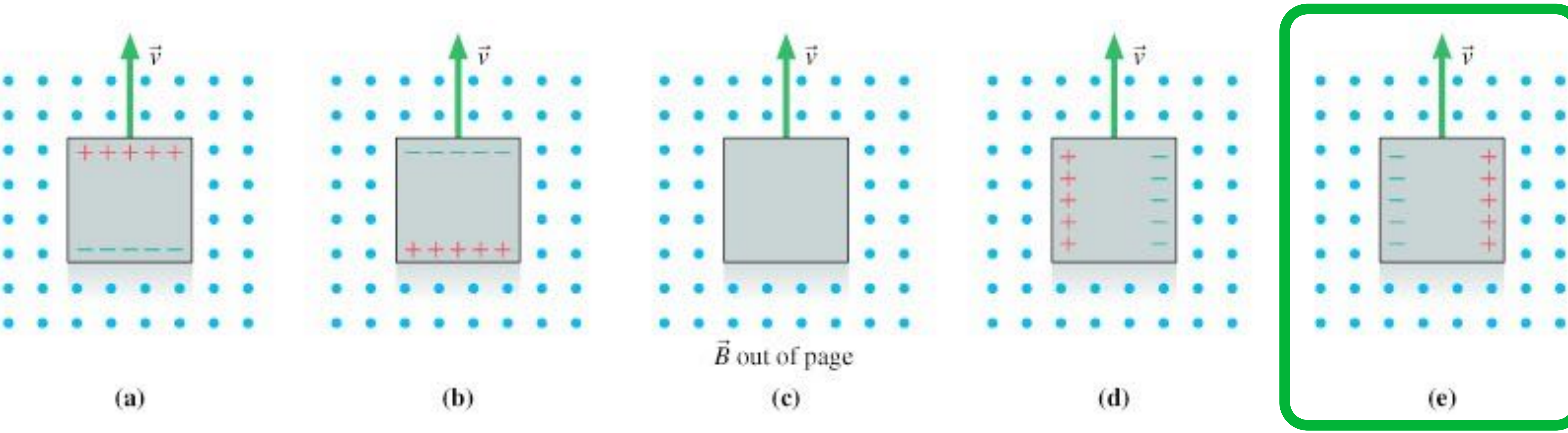
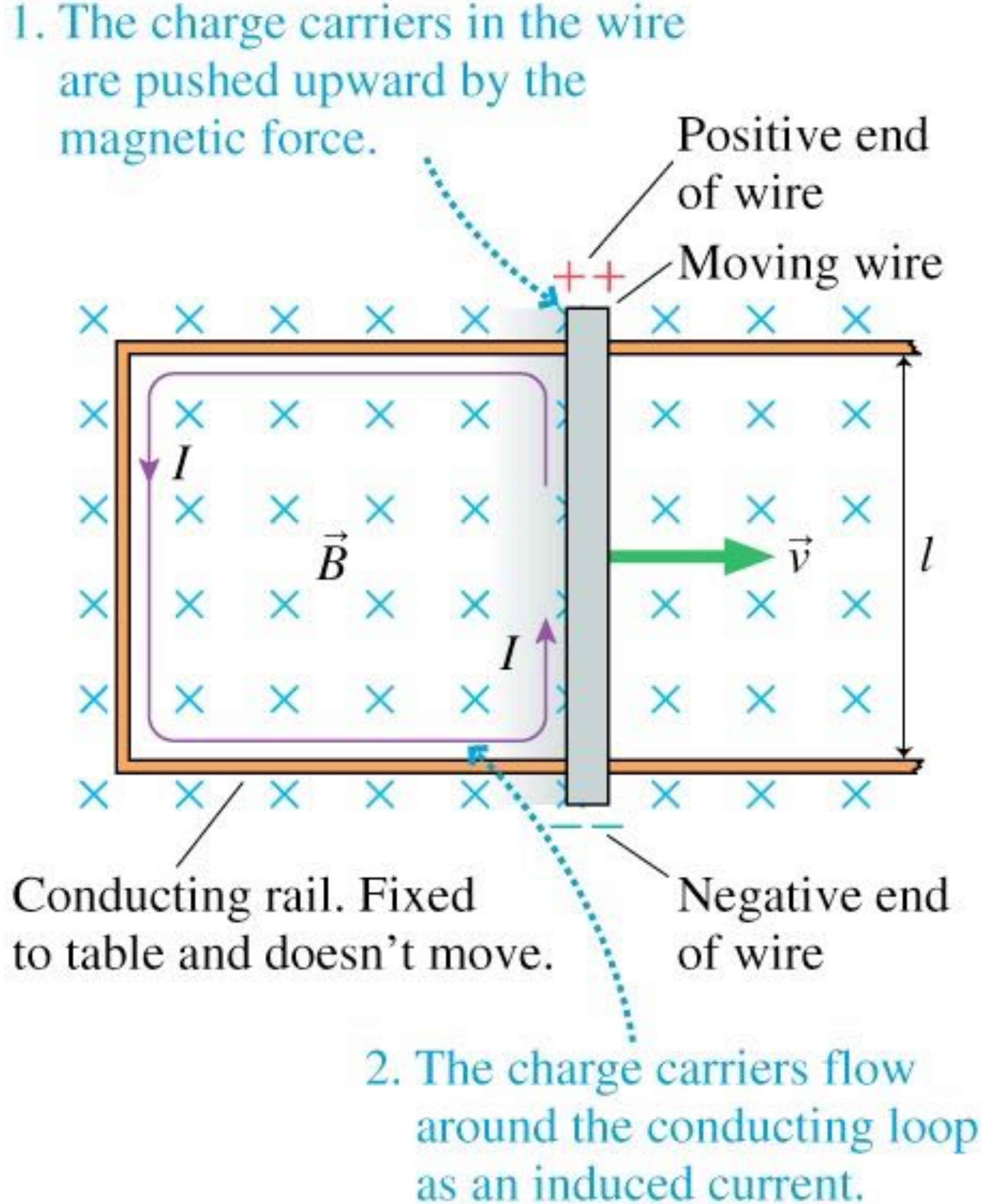


Induced Currents



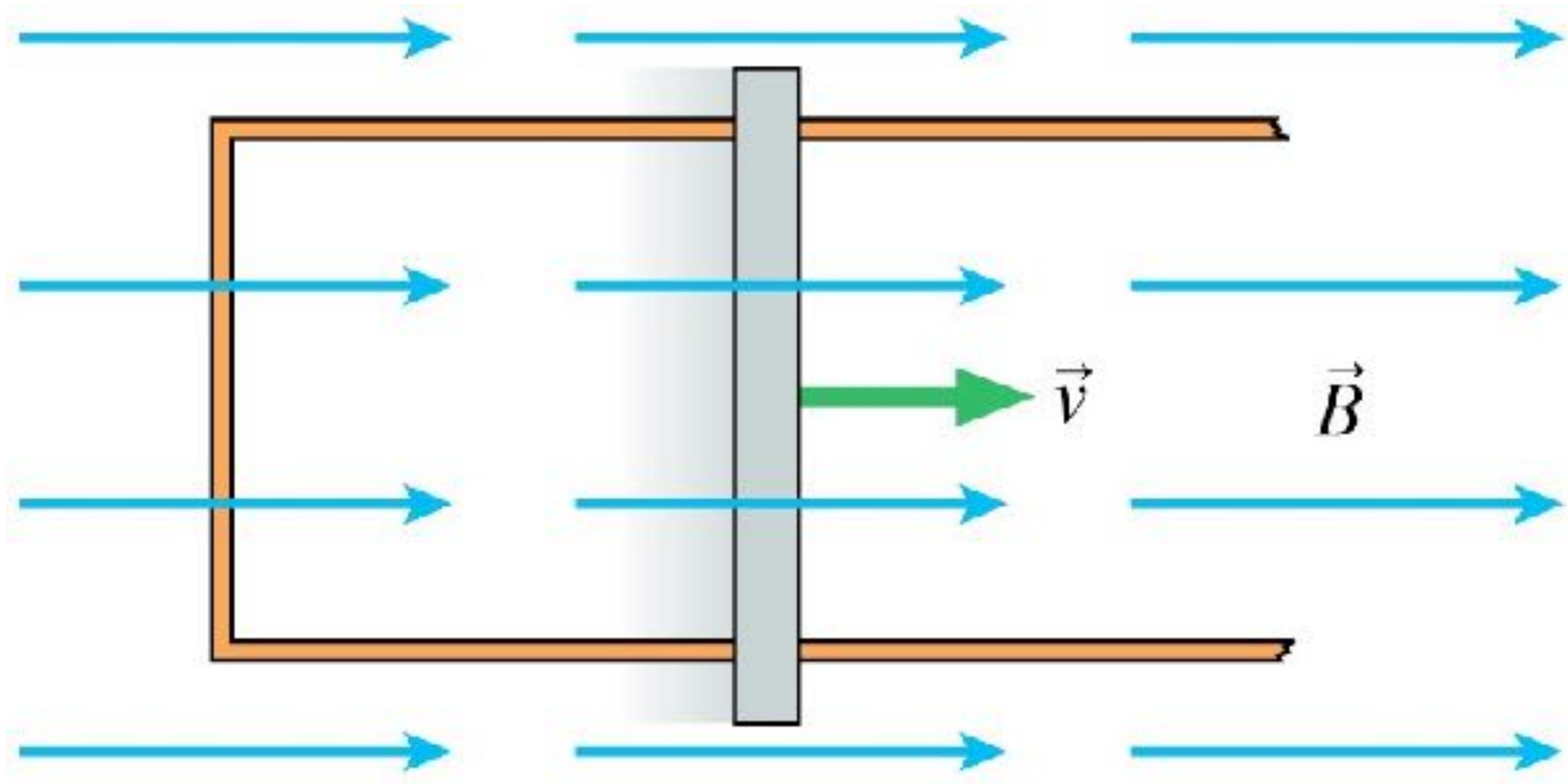
Discussion Question

Is there an induced current in this circuit? If so, in which direction?

A. No.

B. Yes, cw.

C. Yes, ccw.



Discussion Question

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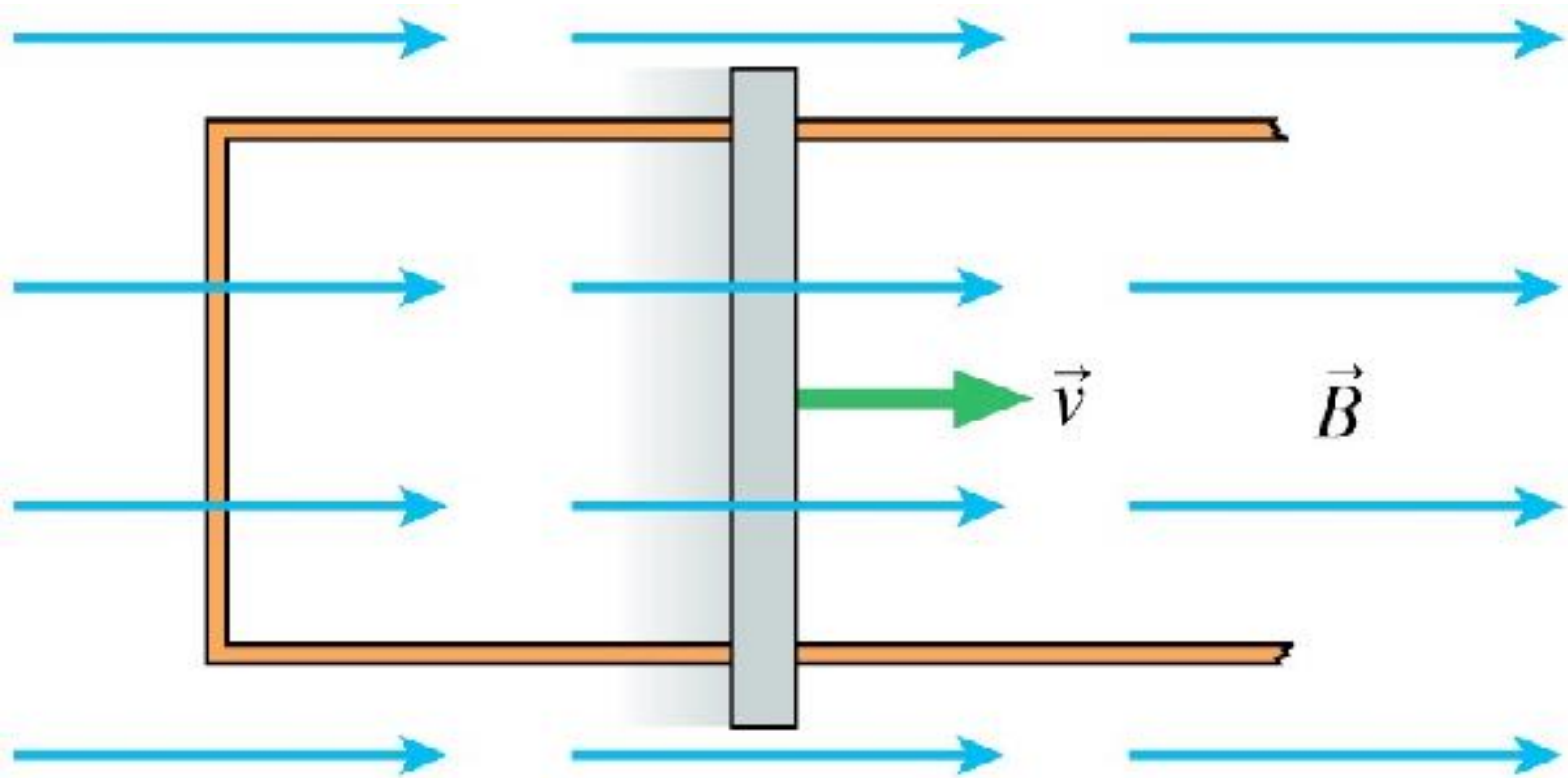


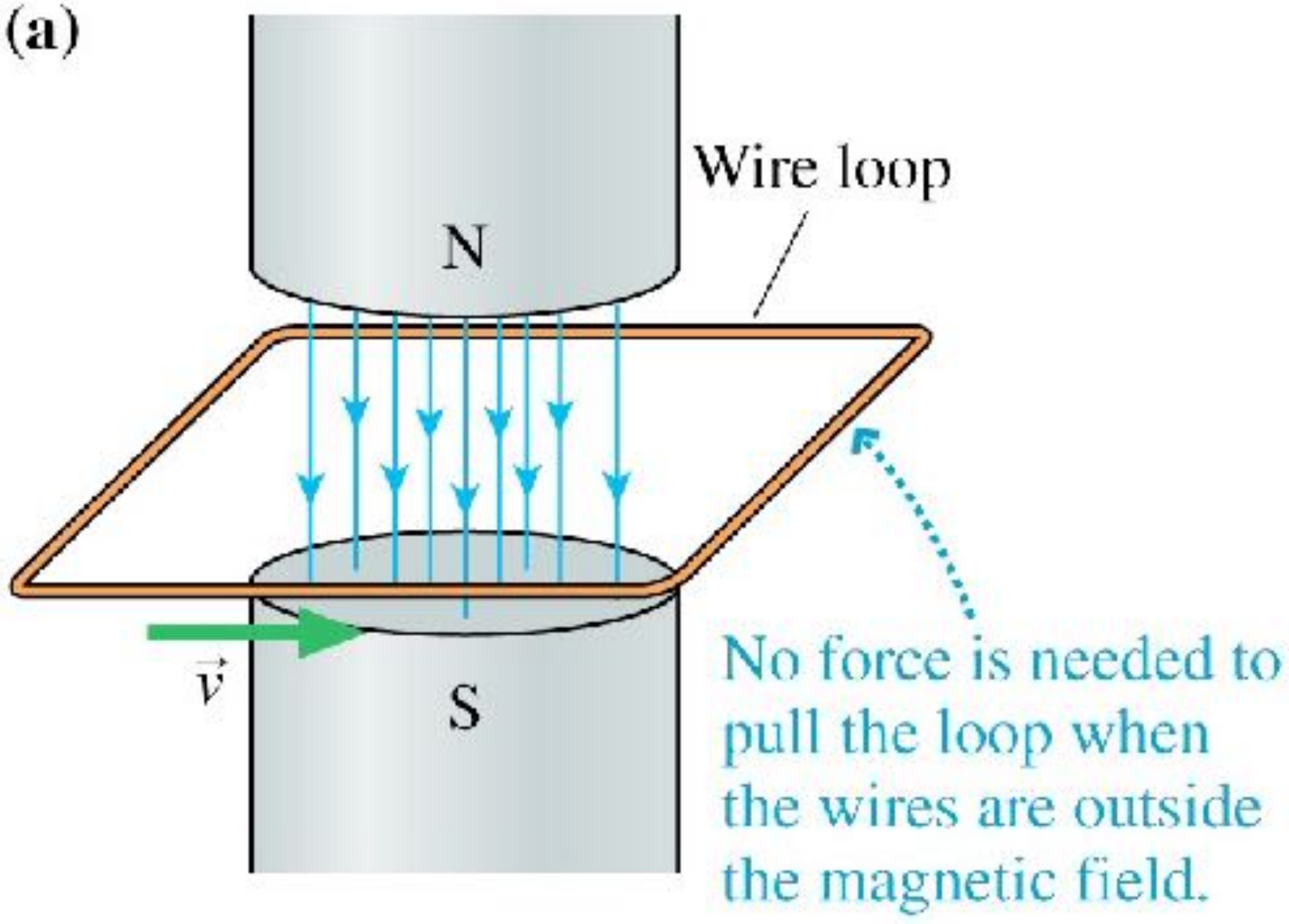
A. No.

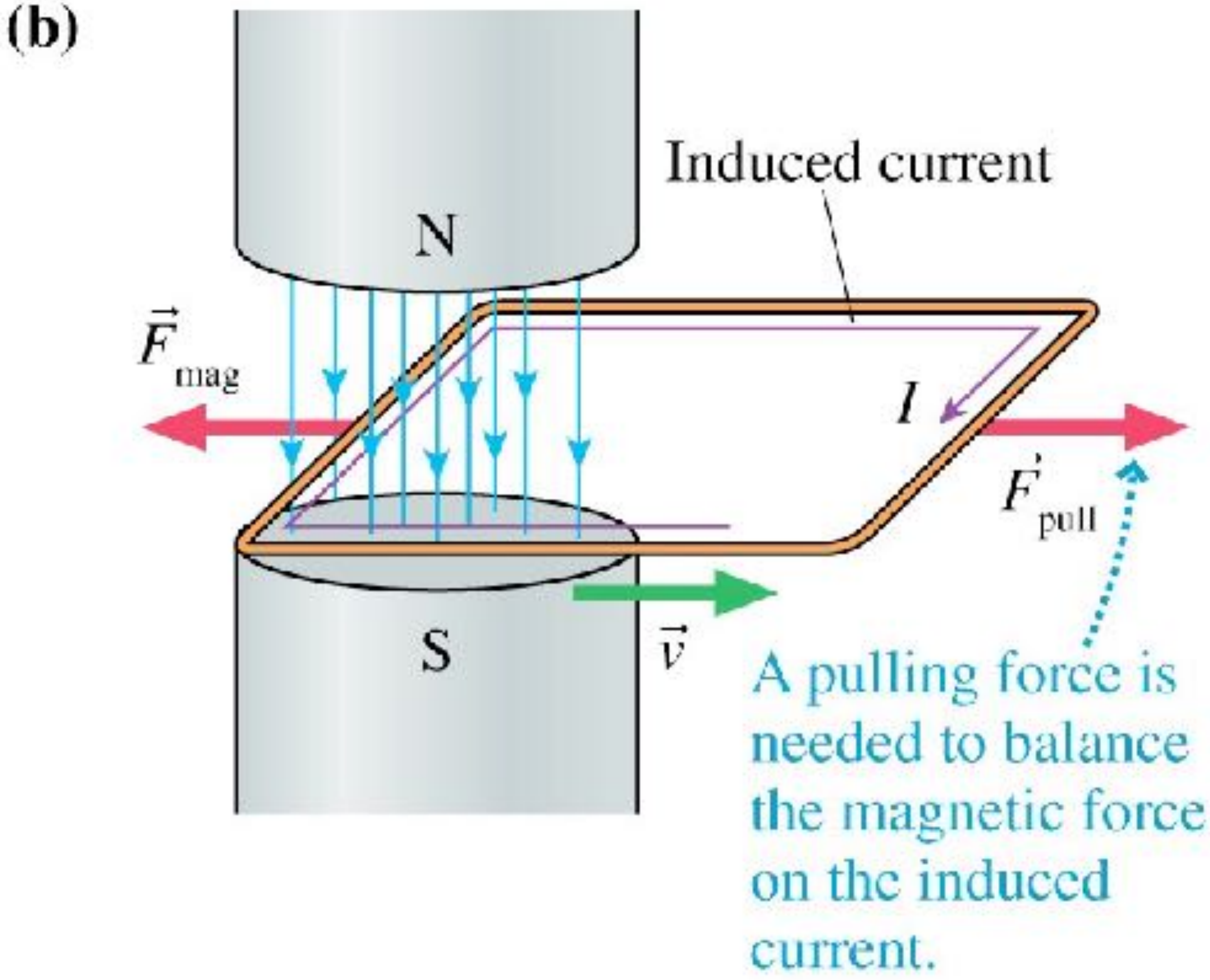
B. Yes, cw.

C. Yes, ccw.

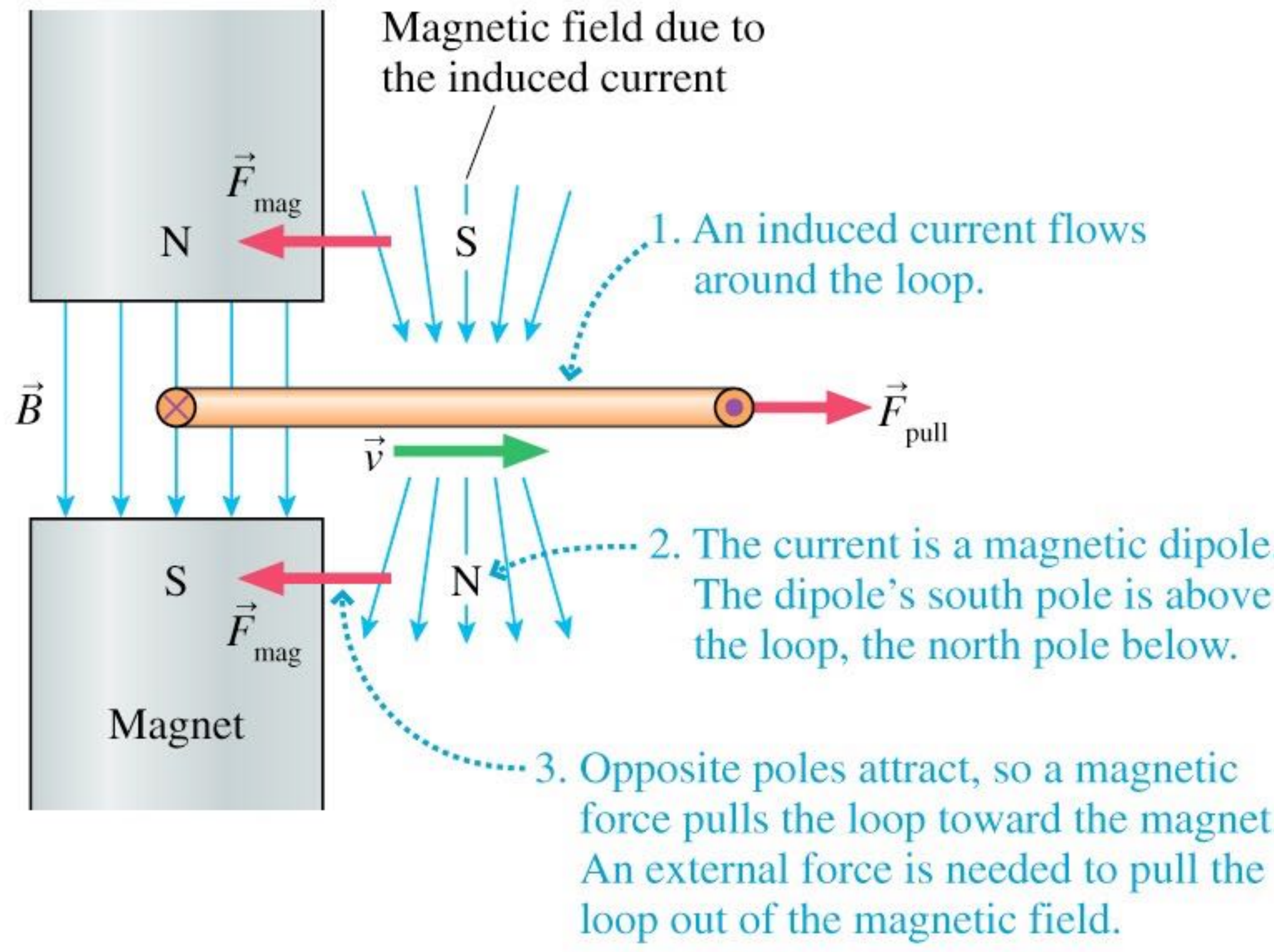
$\vec{v}$ is parallel to $\vec{B}$, so there is ***no*** magnetic force!

Eddy Currents

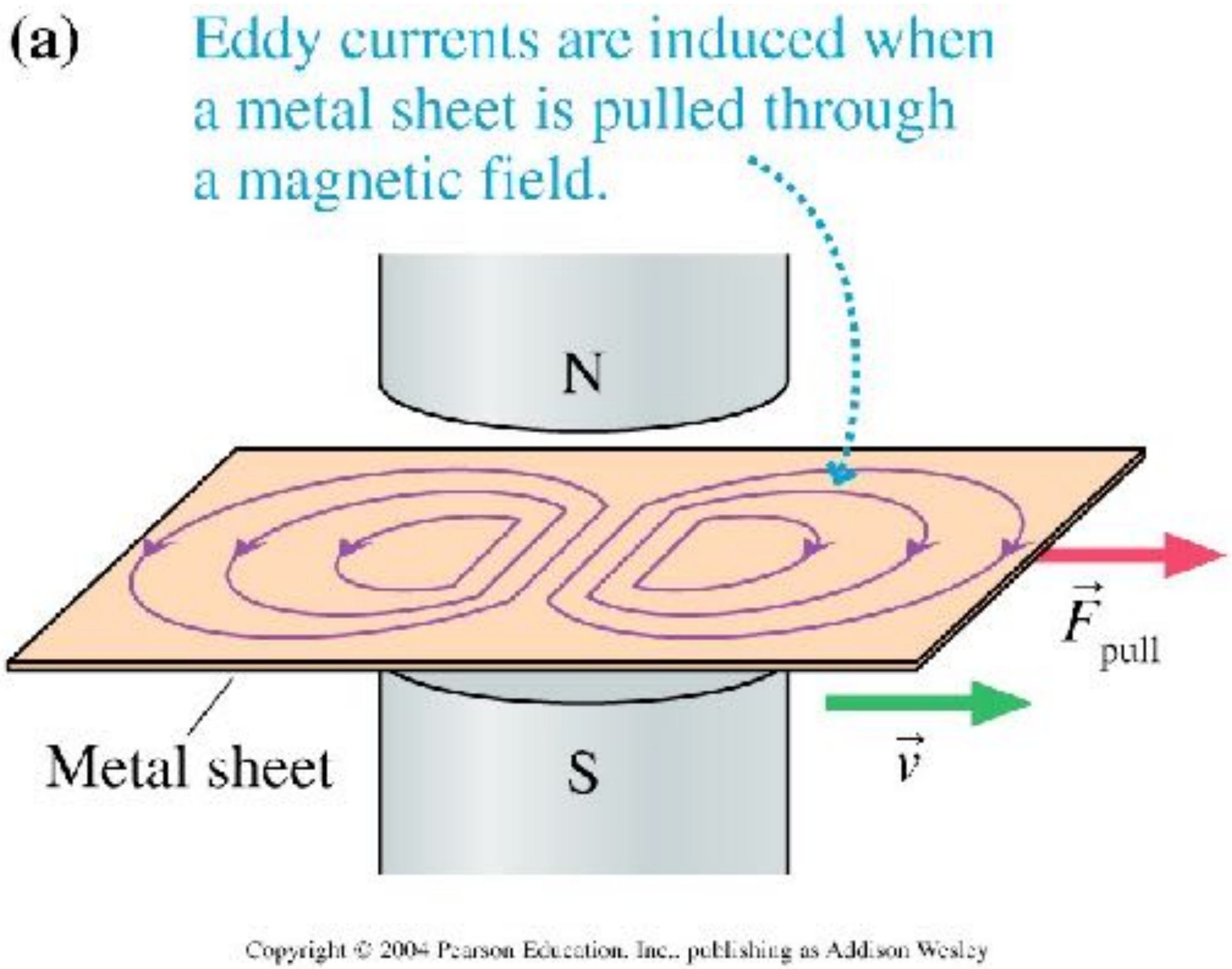


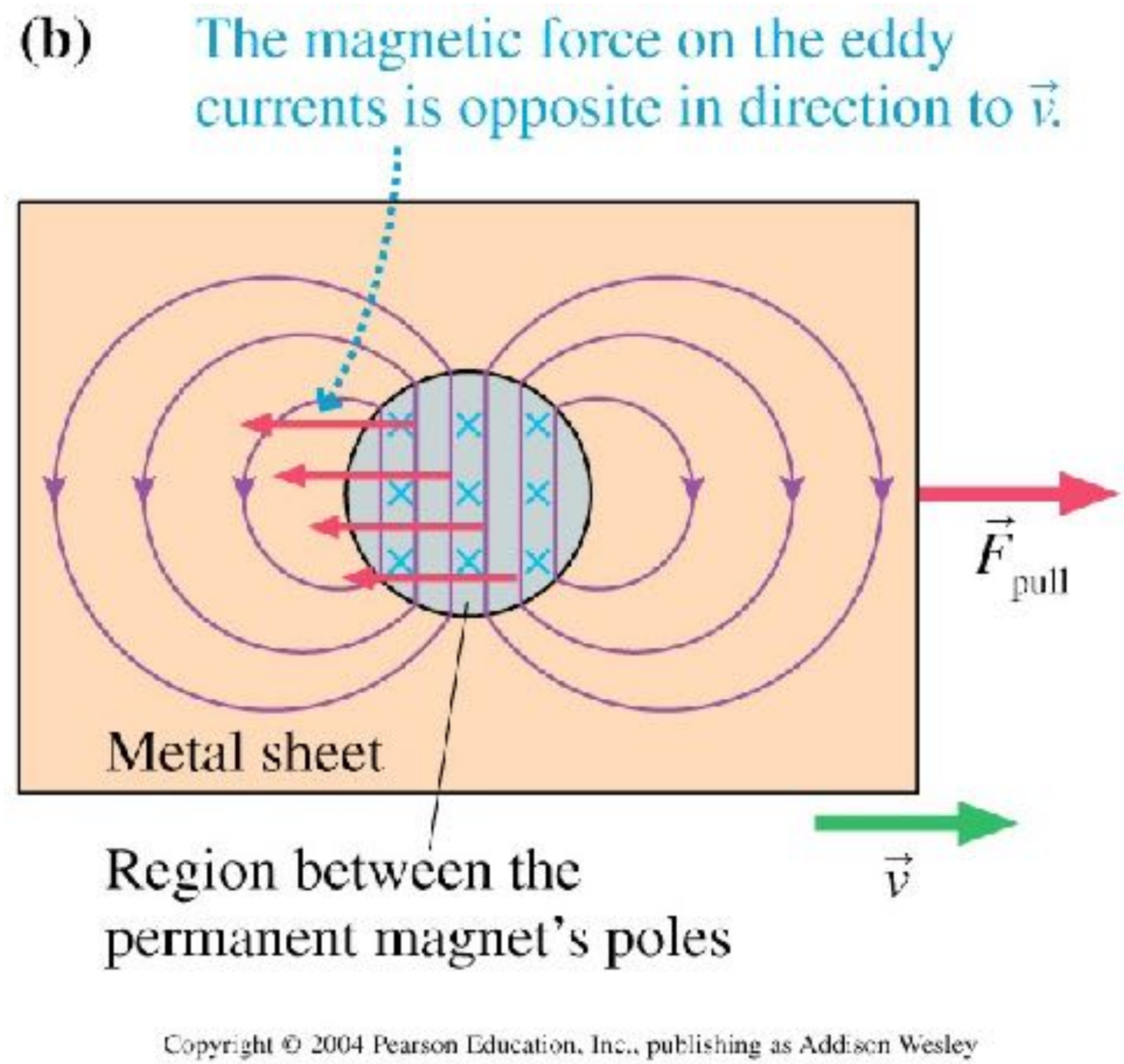


Another View



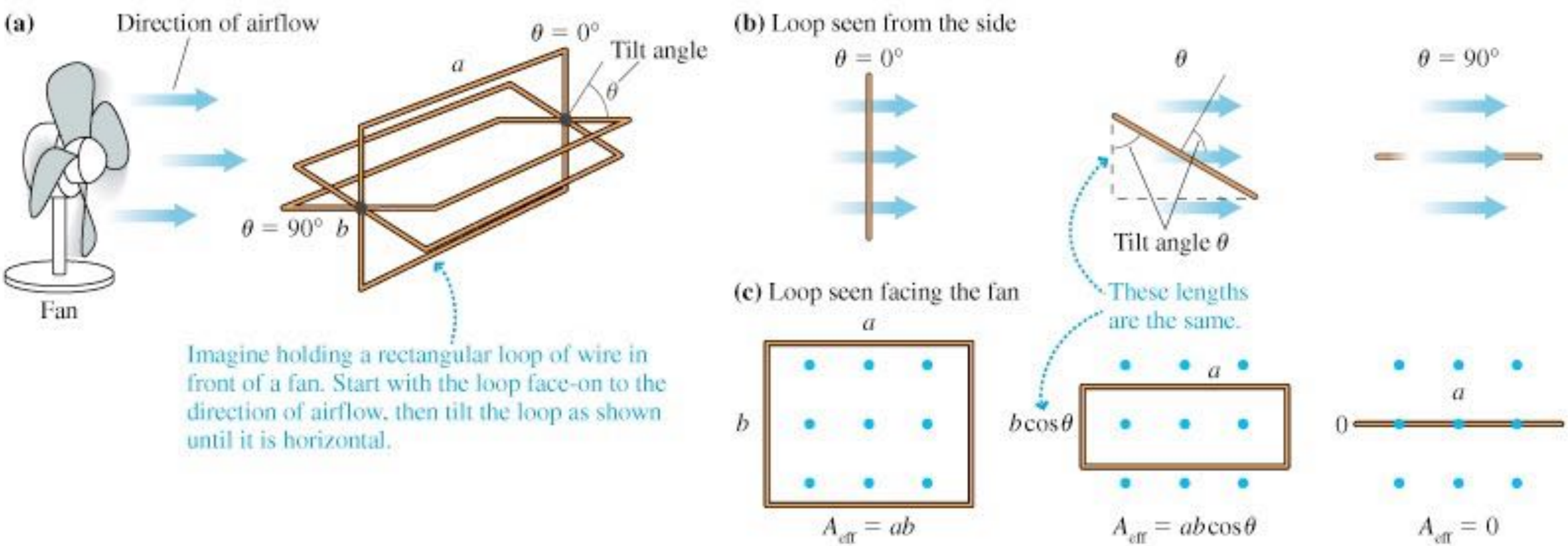
Eddy Current Braking



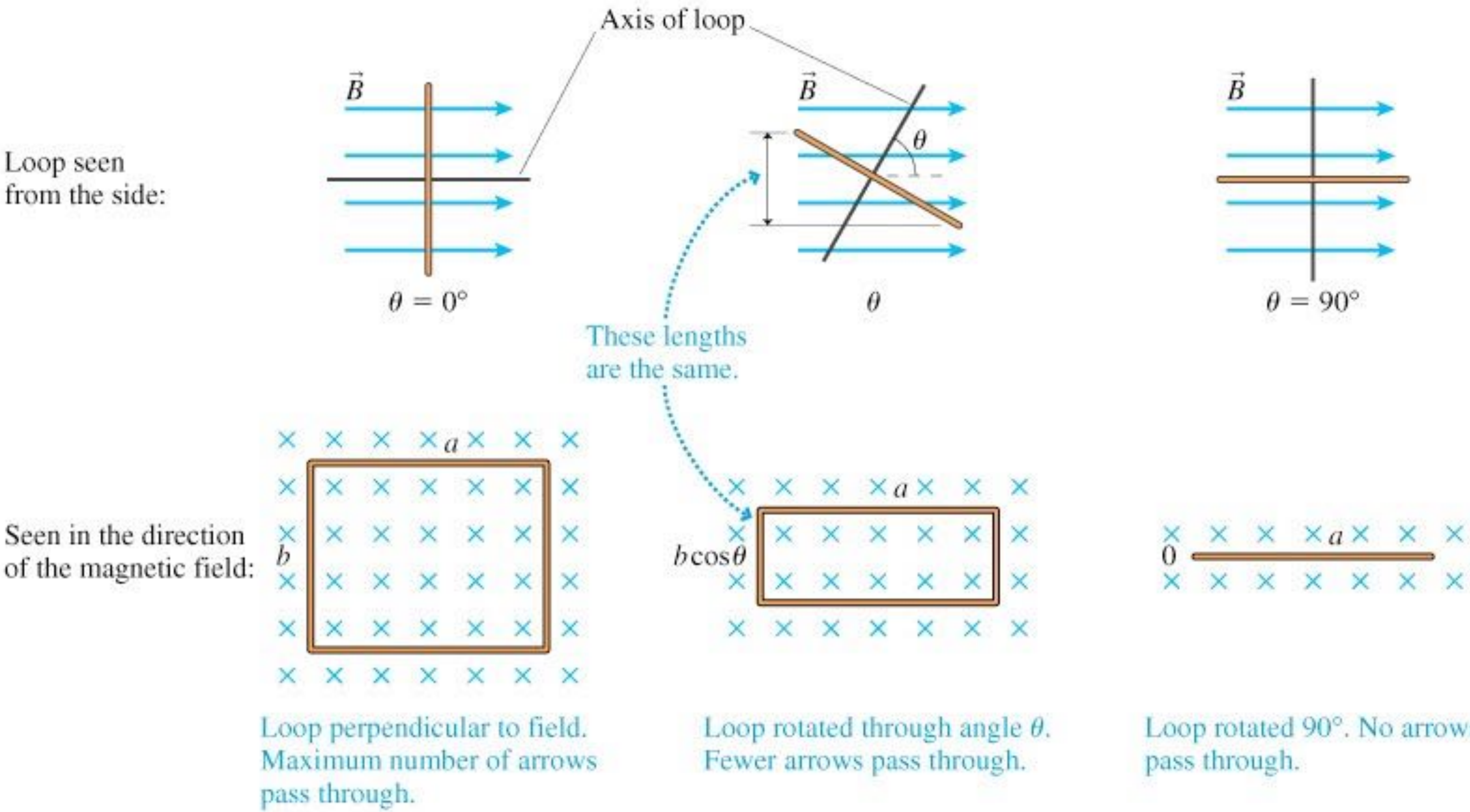


Magnetic Flux

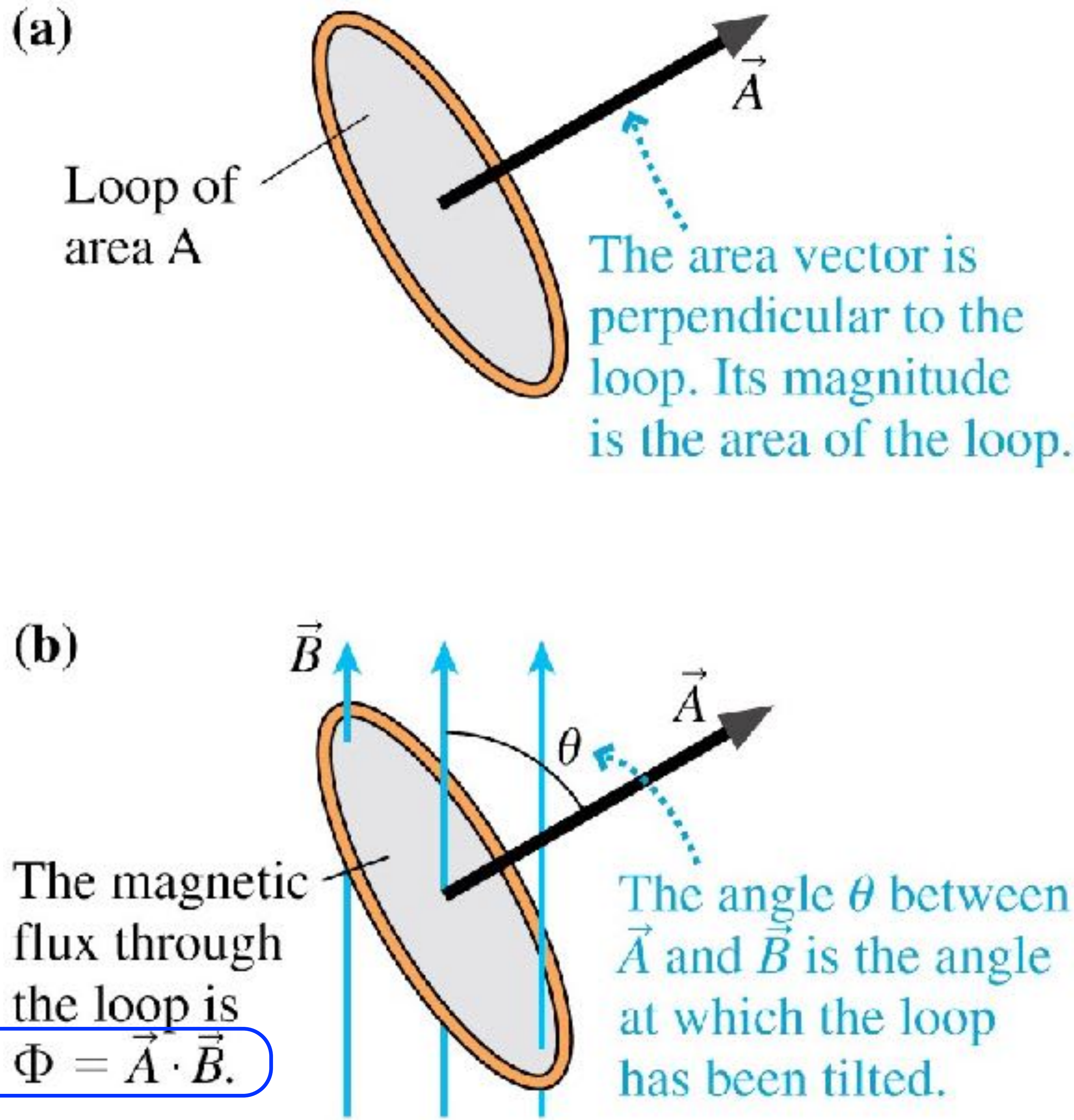
Faraday discovered that a current is induced when the amount of magnetic field passing through a loop changes. What does this mean?



Magnetic Flux



Definition of Flux

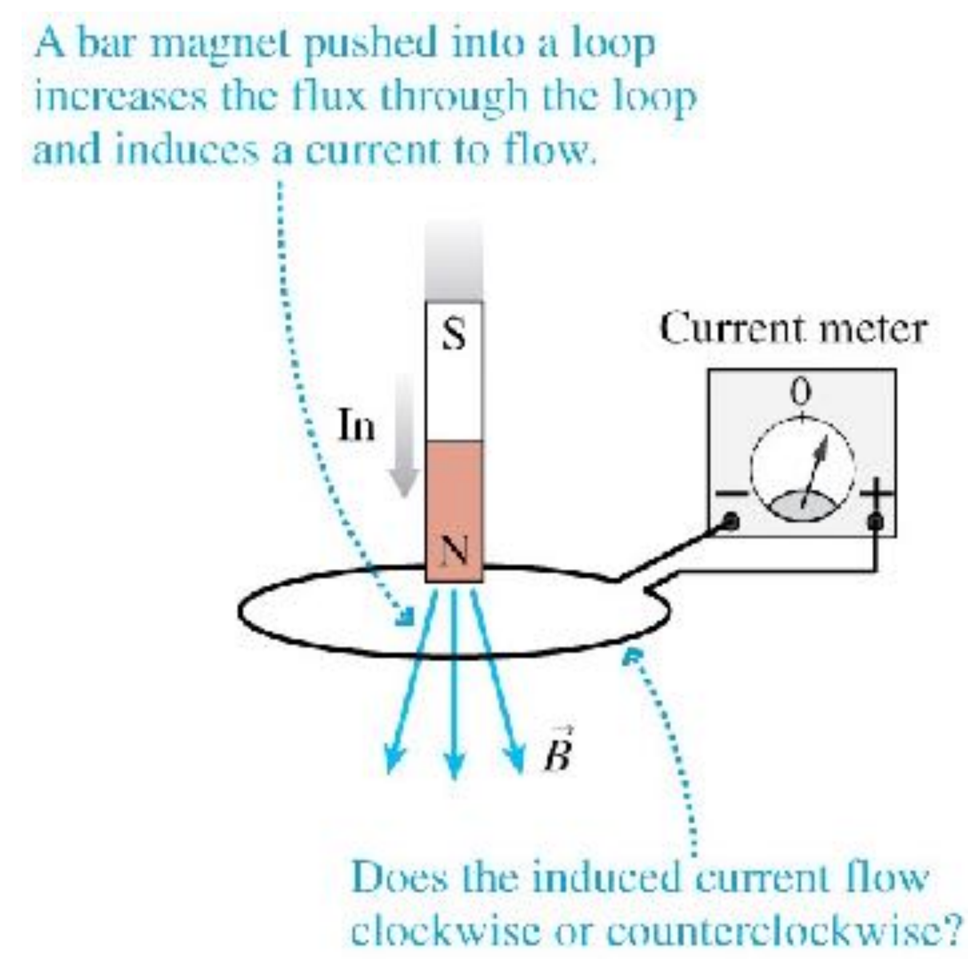


So?

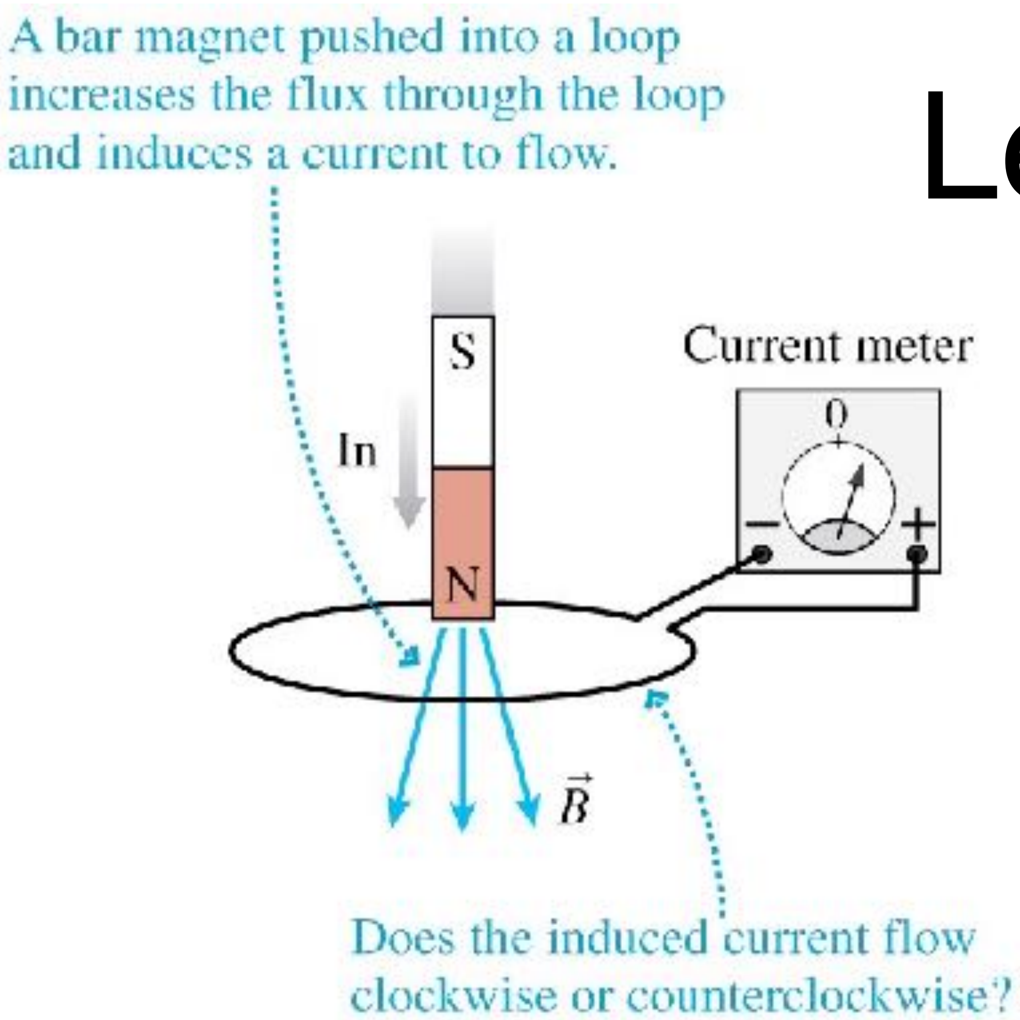
Faraday discovered that a current is induced whenever the magnetic flux through a loop changes, no matter how it changes.

We've seen already the case of a moving wire or moving loop. That is motional emf.

In this case, the loop is not moving, but the induced current is nevertheless quite real. Here, as the magnet gets closer to the loop, the magnetic field gets larger, and thus the flux increases.



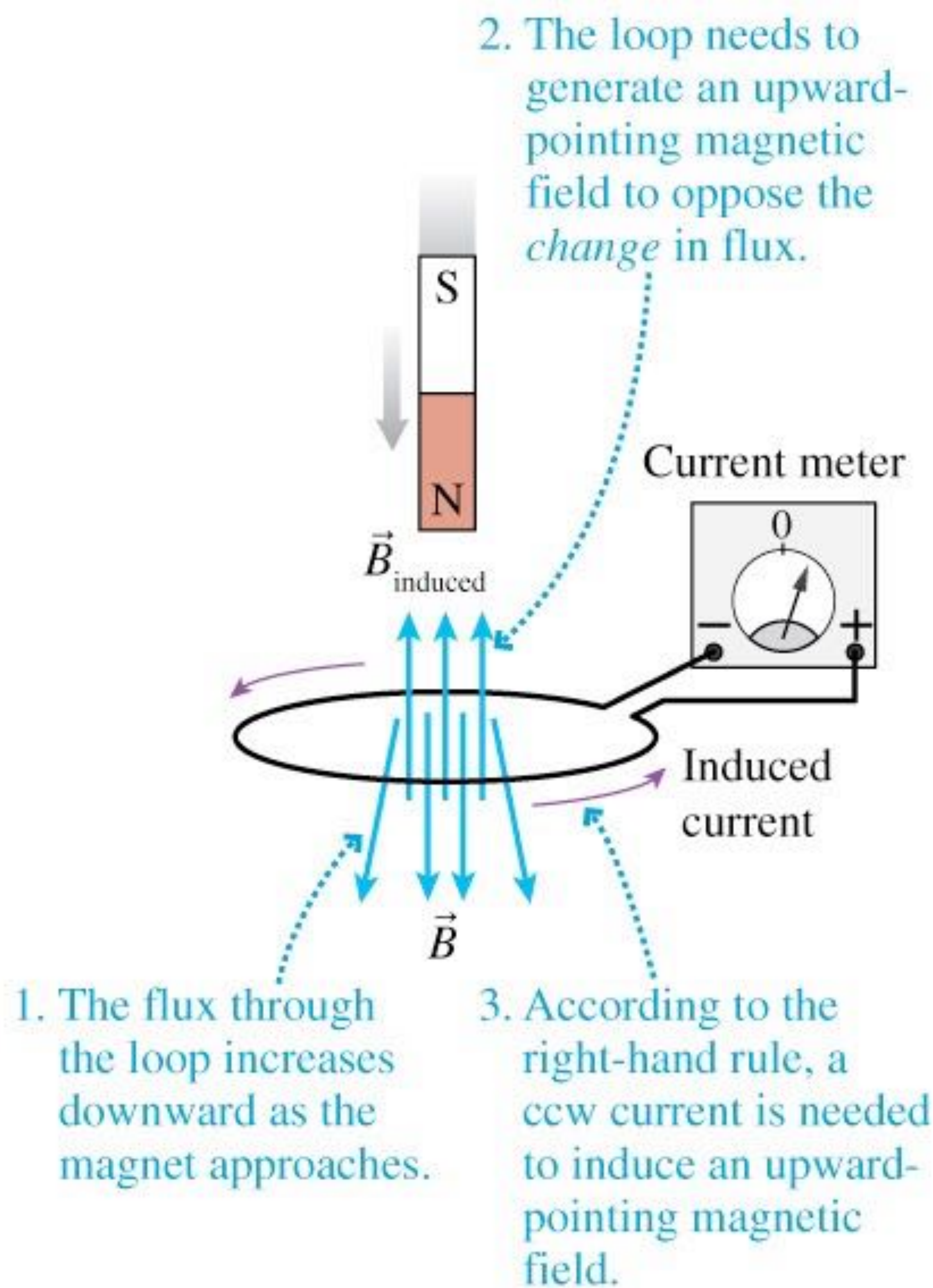
Lenz's Law



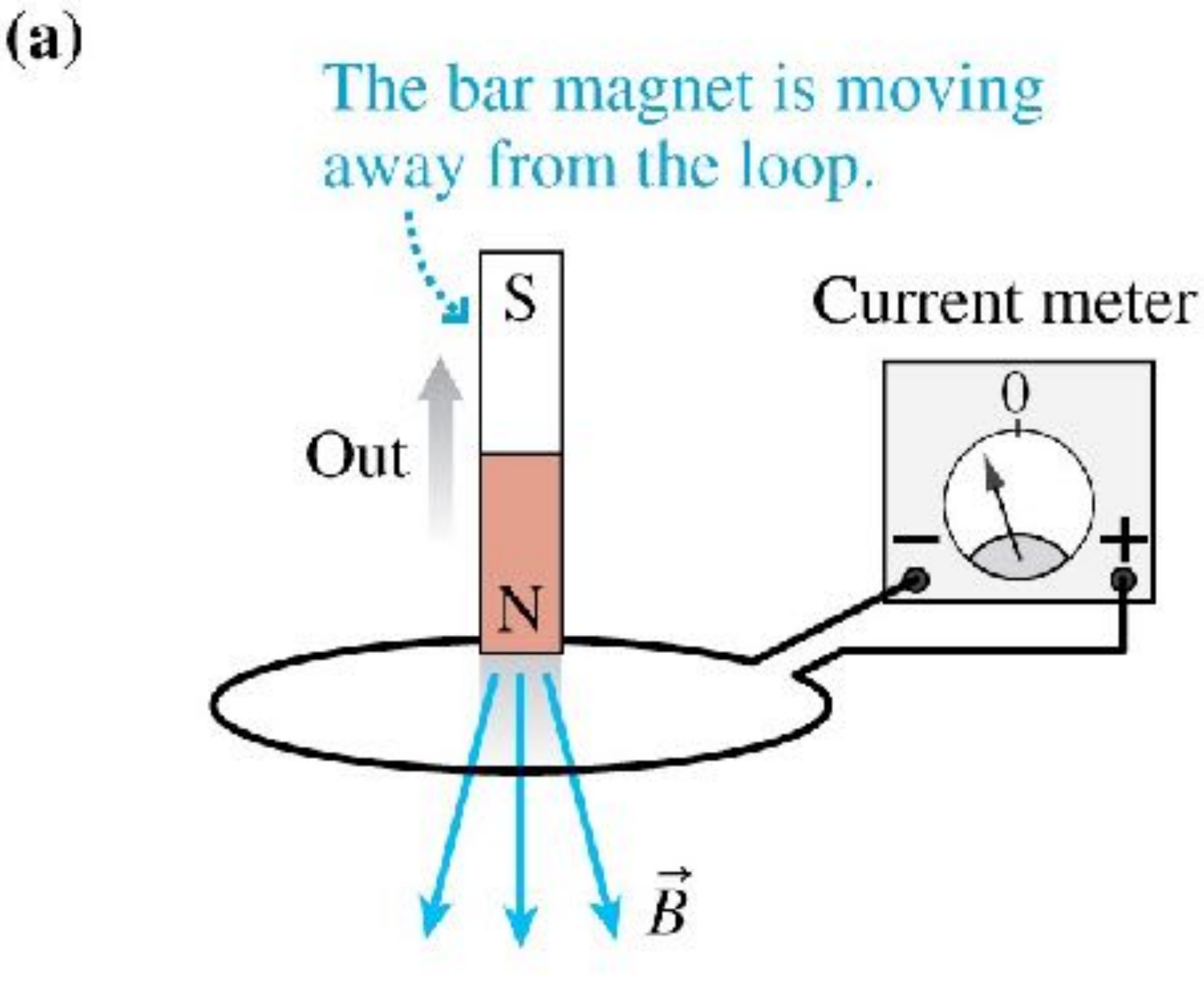
Which way does the current flow?

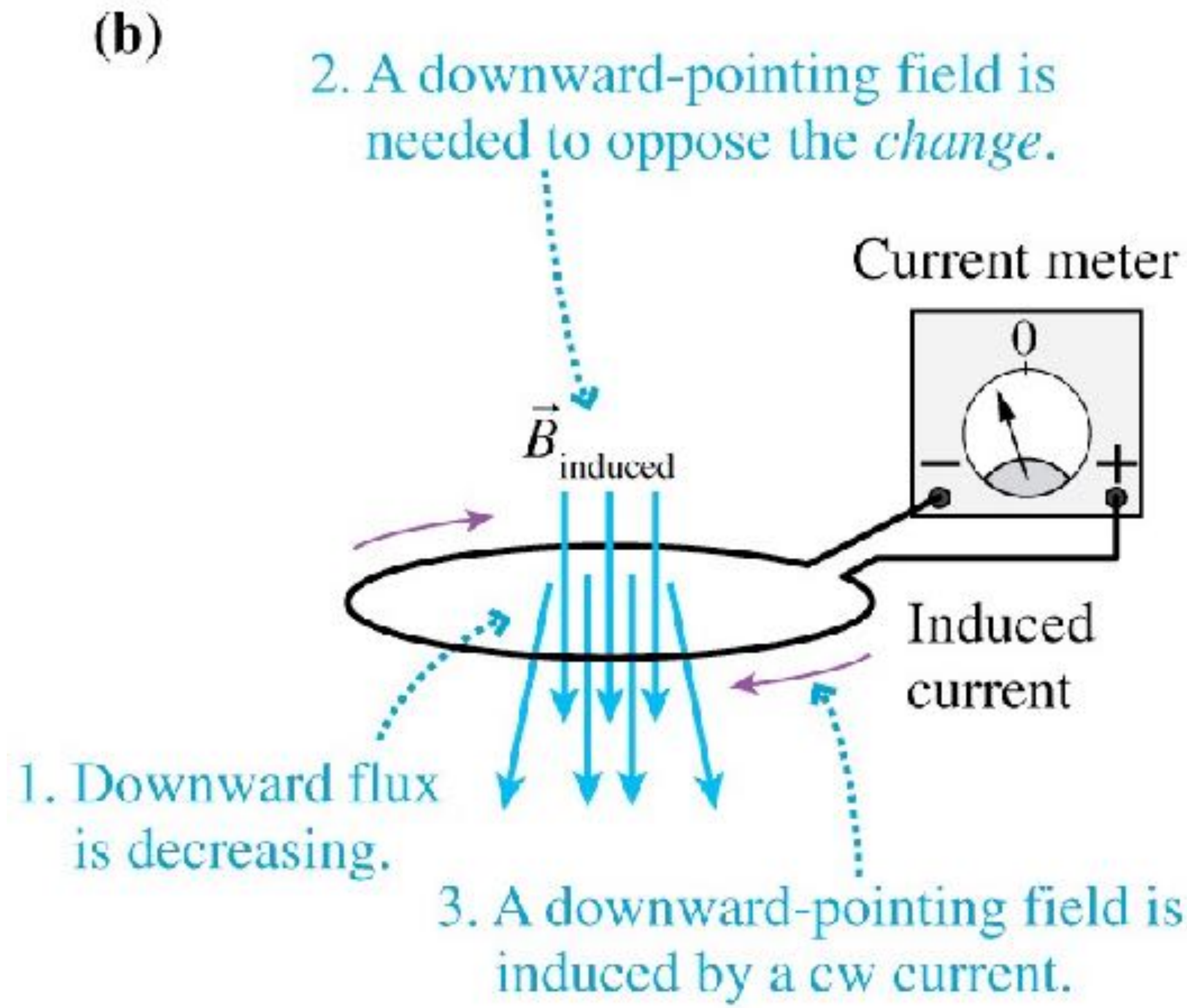
Lenz discovered in 1834 how to determine the direction.

Lenz's law states: The direction of the induced current is such that the induced magnetic field opposes the ***change*** in the magnetic flux through the loop.

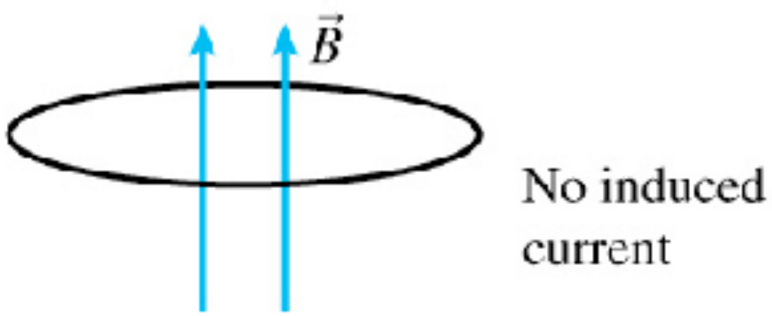

2. The loop needs to generate an upward-pointing magnetic field to oppose the *change* in flux.
S
N
Current meter
0
−
+
$\vec{B}_{\text{induced}}$
Induced current
$\vec{B}$
1. The flux through the loop increases downward as the magnet approaches.
3. According to the right-hand rule, a ccw current is needed to induce an upward-pointing magnetic field.

Now pull the magnet out of the loop

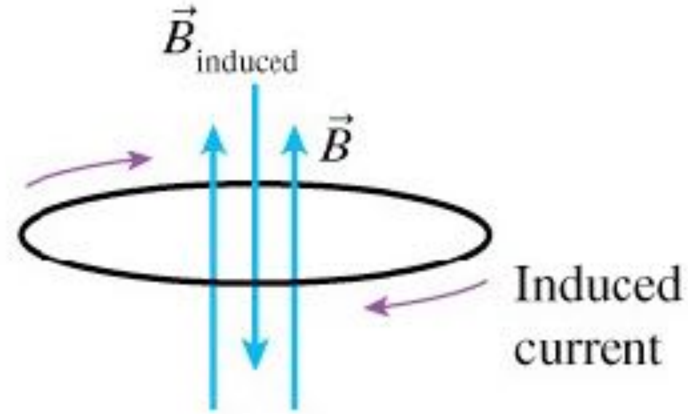




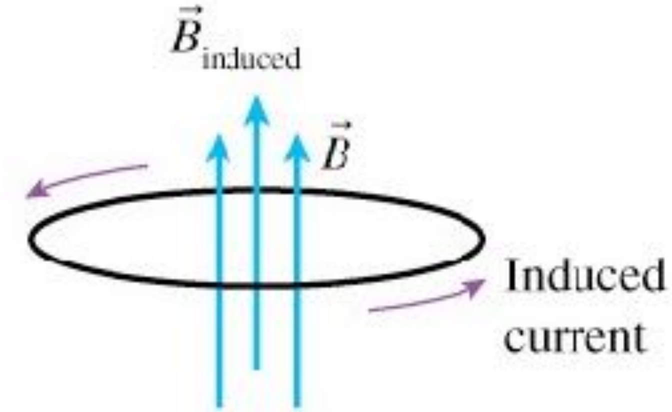
Six examples



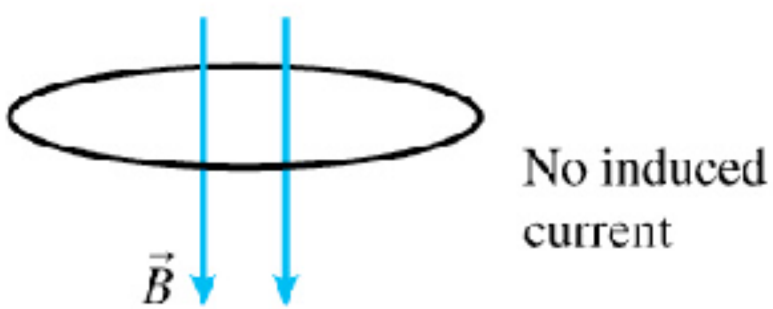
$\vec{B}$ up and steady
- No change in flux
- No induced field
- No induced current



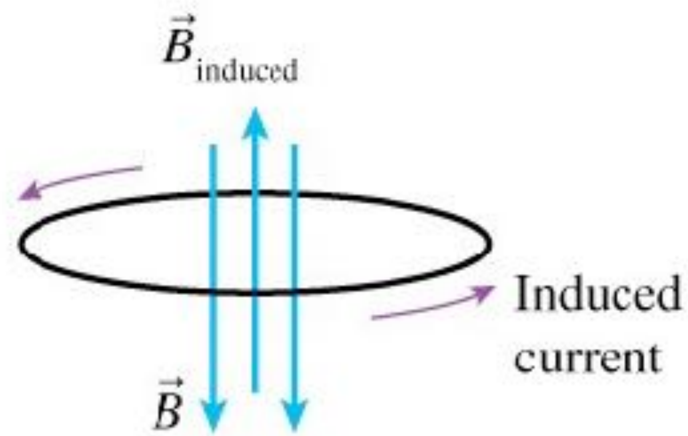
$\vec{B}$ up and increasing
- Change in flux ↑
- Induced field ↓
- Induced current cw



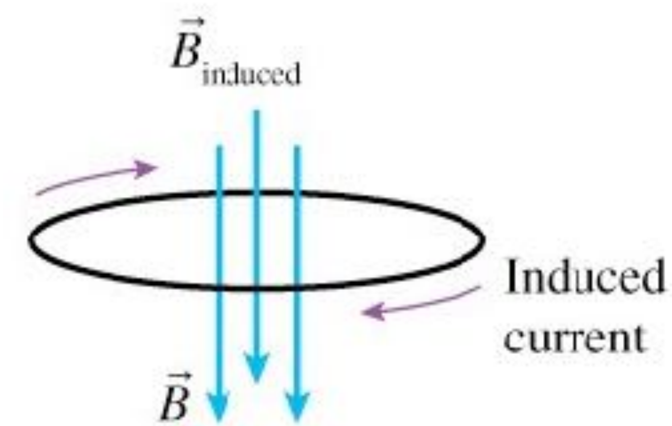
$\vec{B}$ up and decreasing
- Change in flux ↓
- Induced field ↑
- Induced current ccw



$\vec{B}$ down and steady
- No change in flux
- No induced field
- No induced current



$\vec{B}$ down and increasing
- Change in flux ↓
- Induced field ↑
- Induced current ccw



$\vec{B}$ down and decreasing
- Change in flux ↑
- Induced field ↓
- Induced current cw

Discussion Question

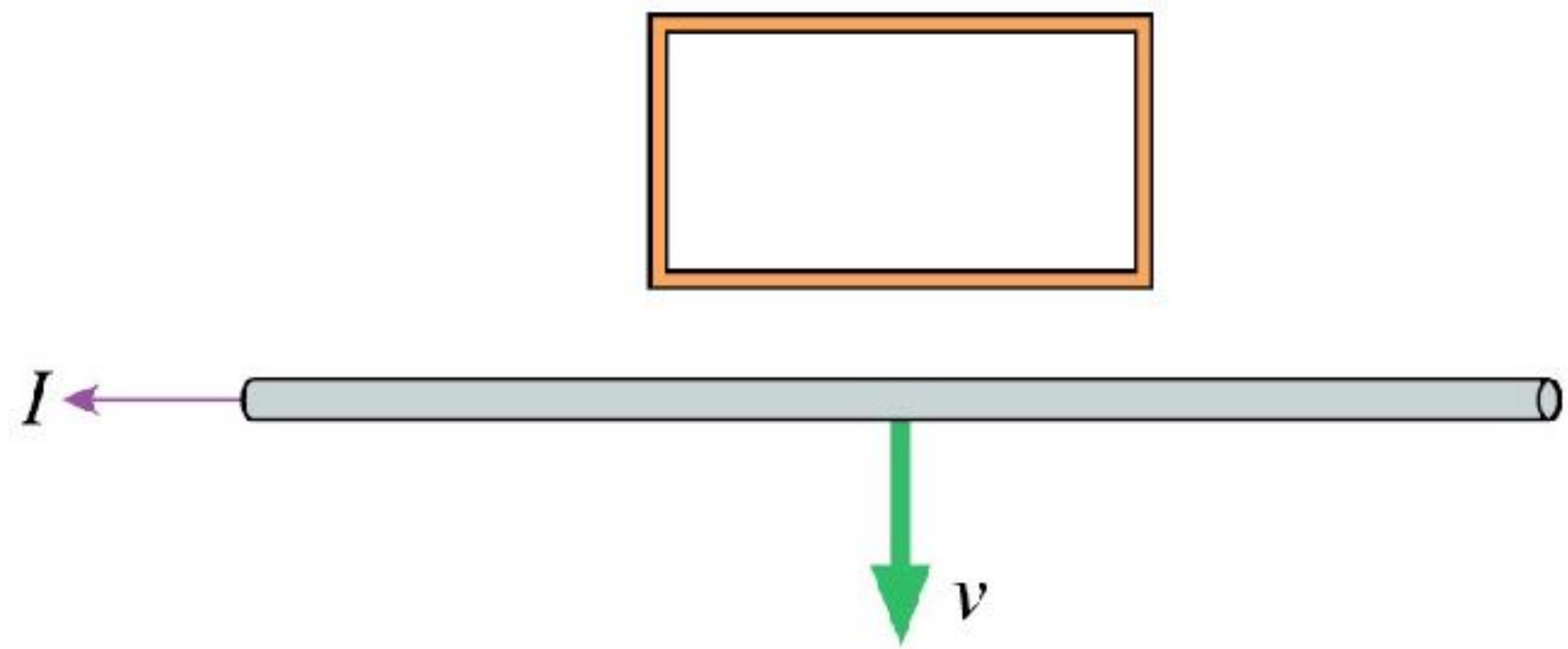
A current-carrying wire is pulled away from a conducting loop in the direction shown. As the wire is moving, is there

A. a cw current

B. a ccw current

C. no current

in the loop?



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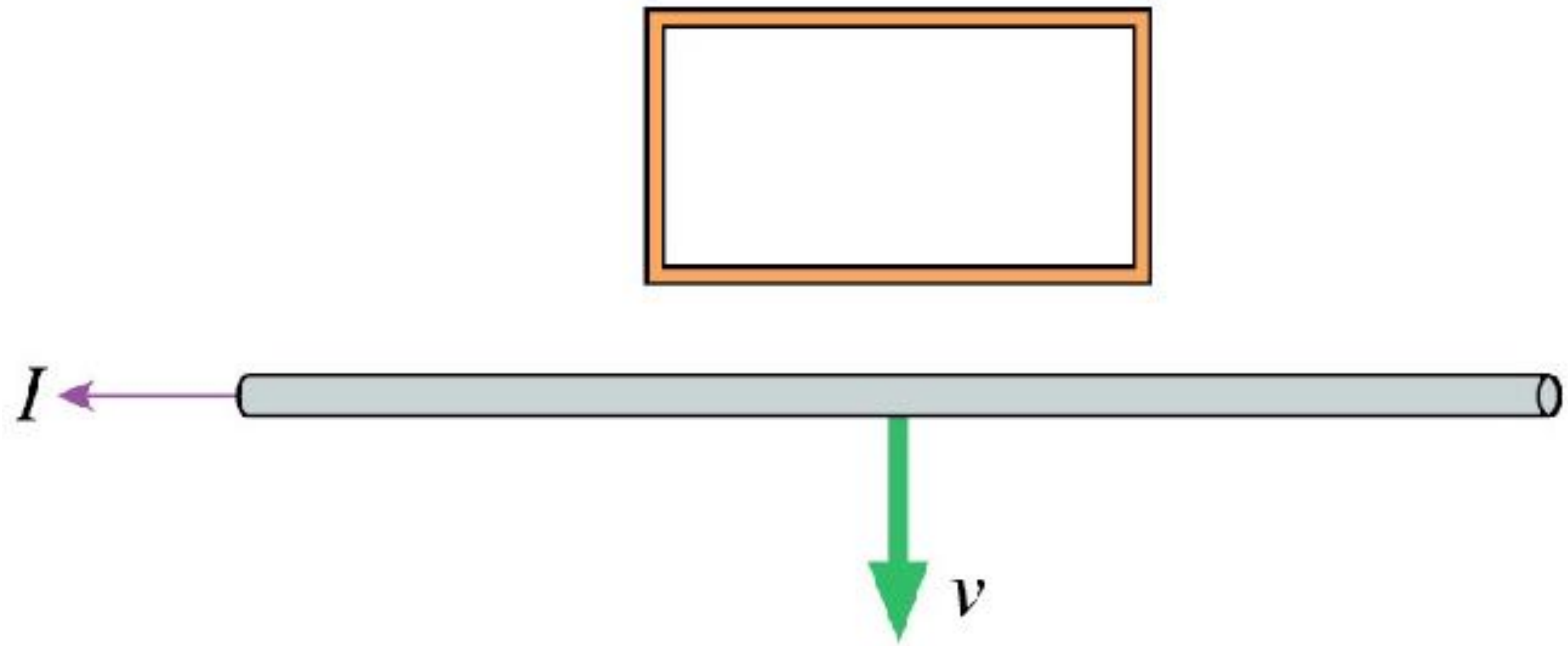
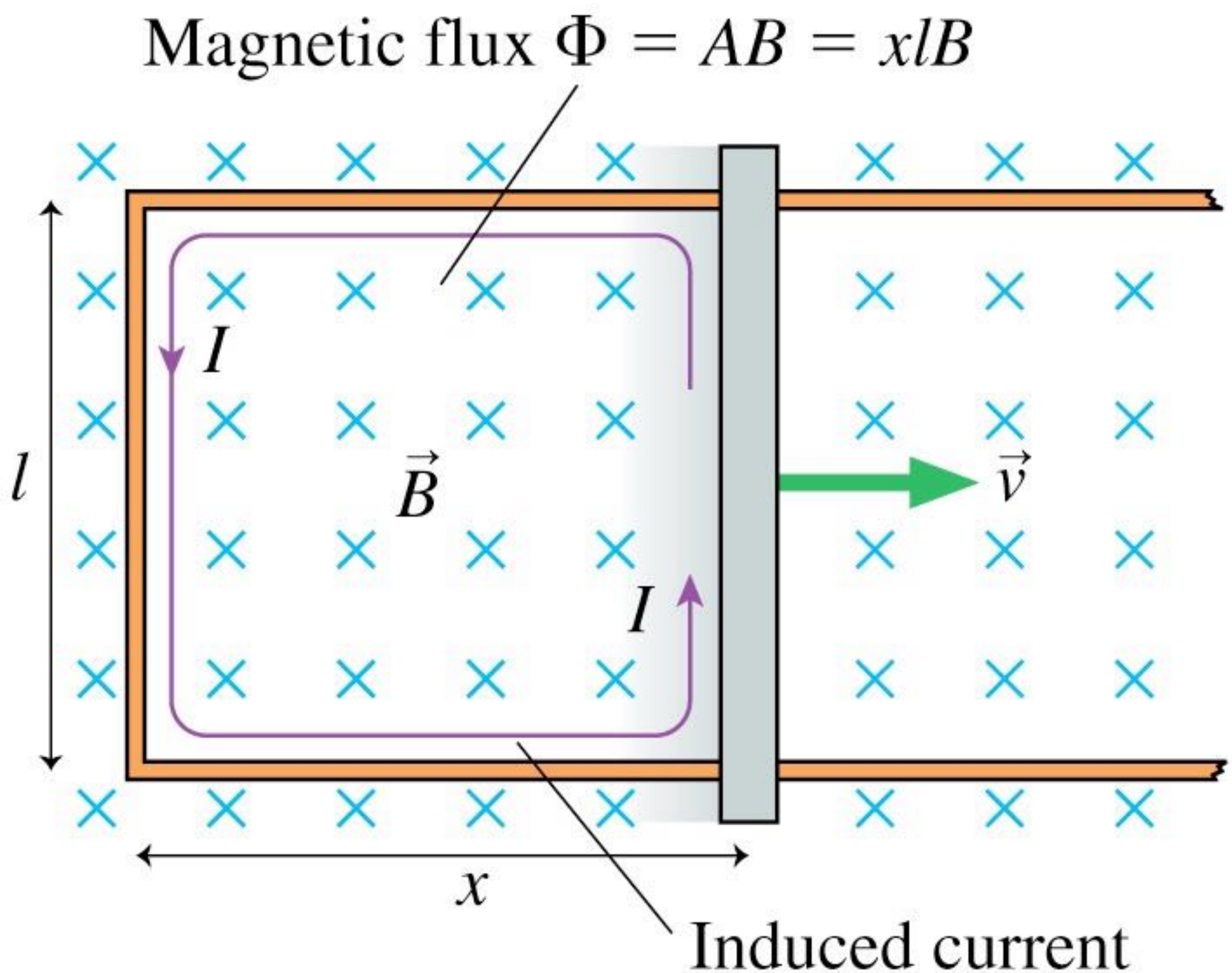


Moving Wire Revisited



As the sliding wire moves to the right, the flux through the loop increases. A ccw current is induced to oppose the change in flux. (A ccw current makes a field out of the plane.)

Faraday's Law

Two fundamentally different ways to change the magnetic flux through a conducting loop:

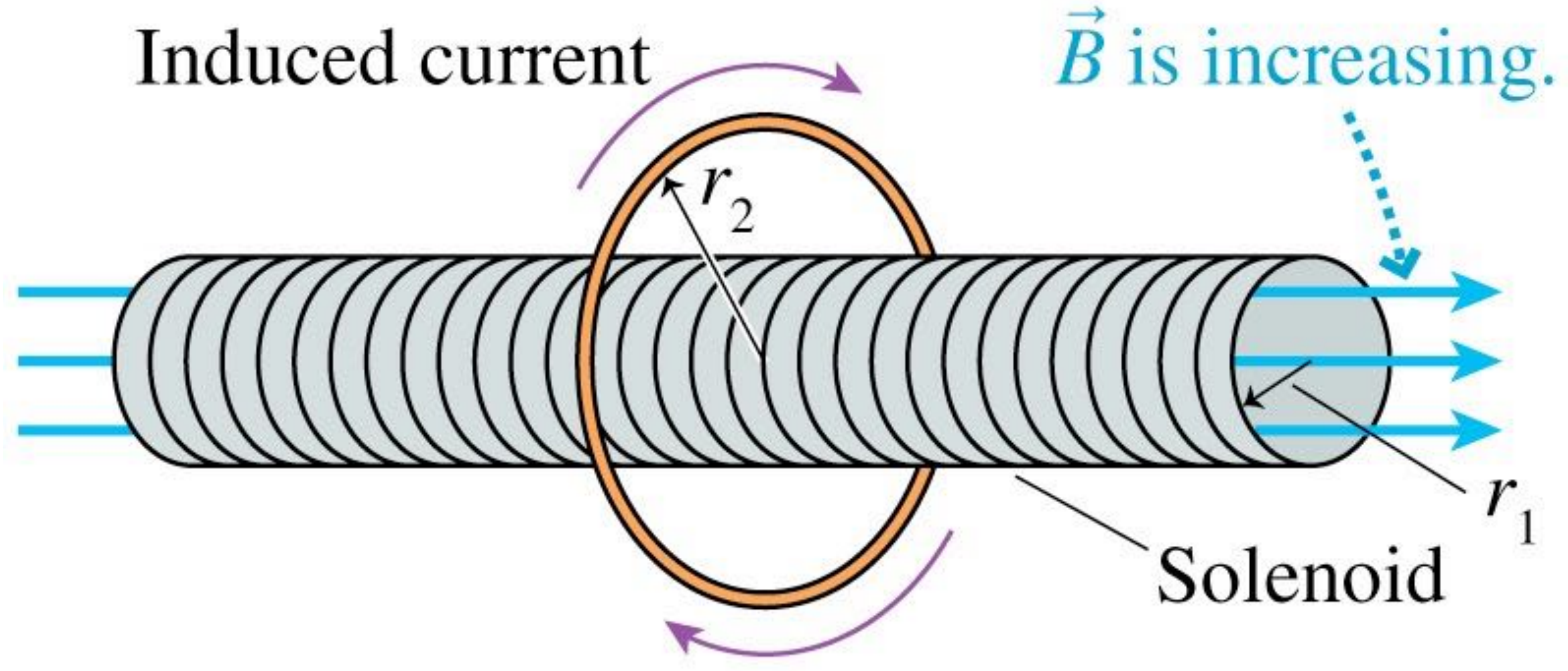
1. The loop can expand or move or rotate, creating a motional emf
2. The magnetic field can change.

$$\mathcal{E} = \left| \frac{d\Phi_m}{dt} \right| = \left| \vec{B} \cdot \frac{d\vec{A}}{dt} + \vec{A} \cdot \frac{d\vec{B}}{dt} \right|$$

Motional emf

New physics!

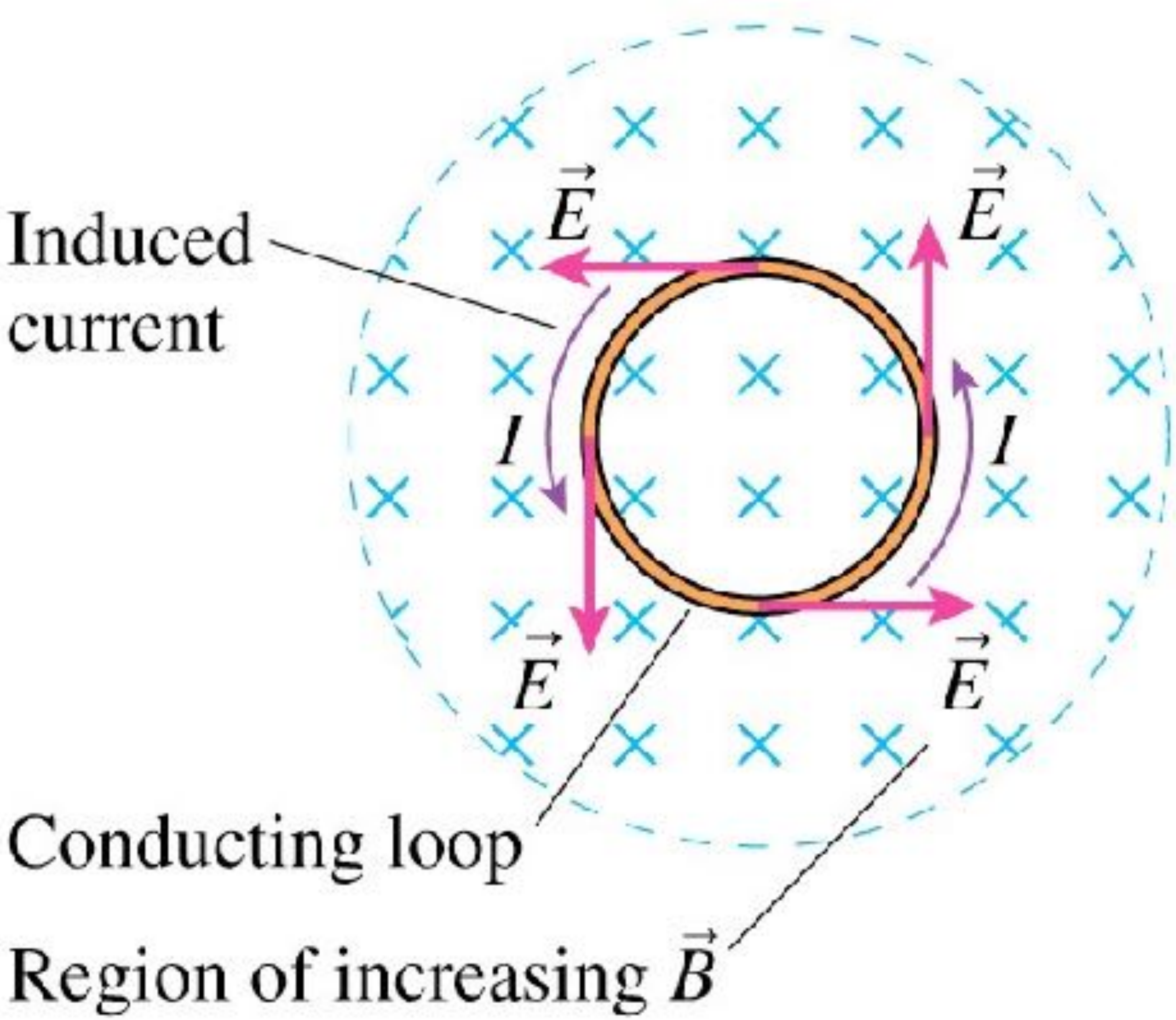
How does this work?

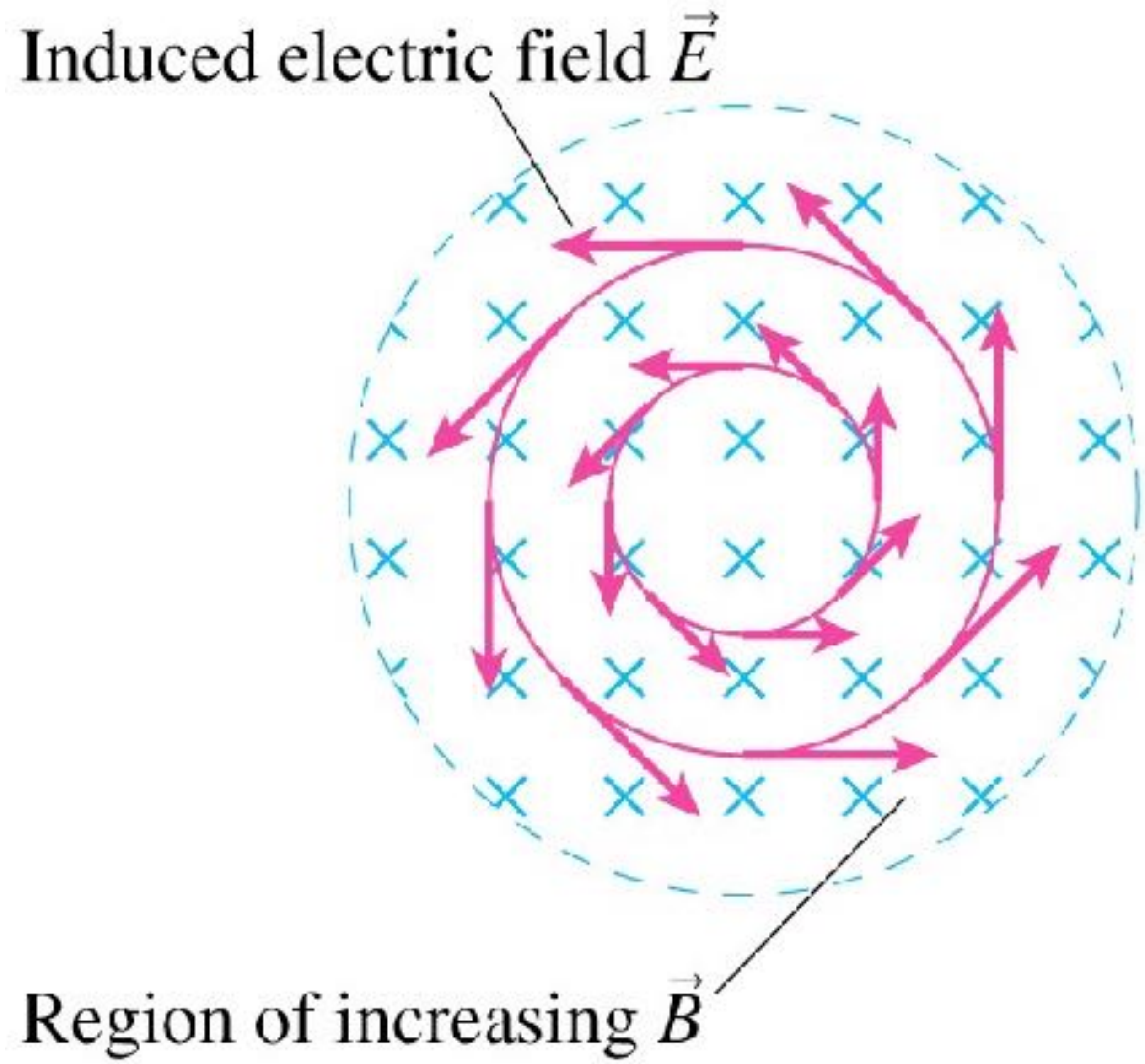


The field outside the solenoid is essentially zero. So how does the loop ‘know’ that the flux ***inside*** the solenoid is changing?

Induced Electric Field

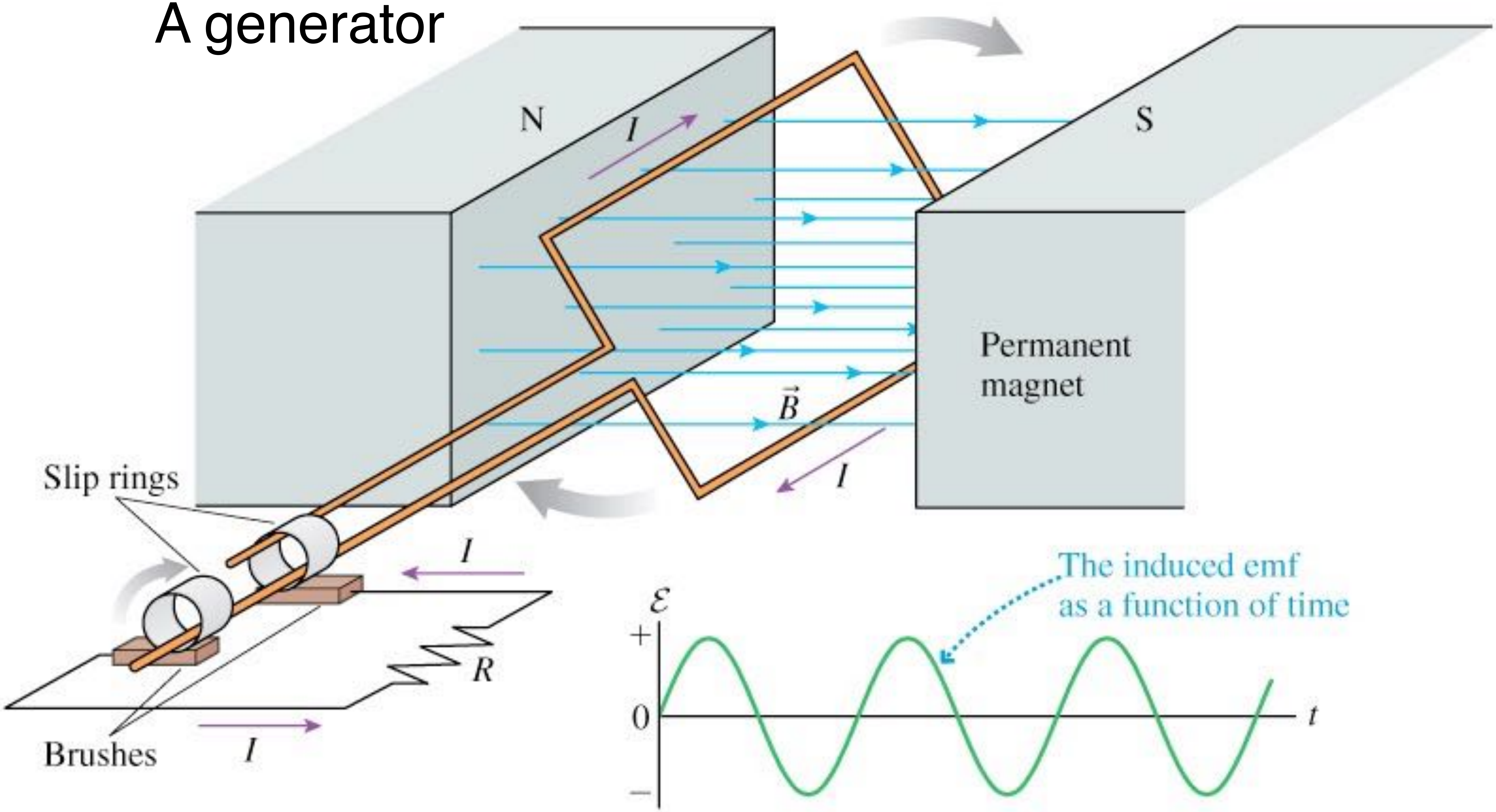
We need *something* to get the charges moving to create the induced current. The only thing that can do it with *static* charges is an *electric field*. Therefore, a ***changing*** magnetic field *induces* an electric field!





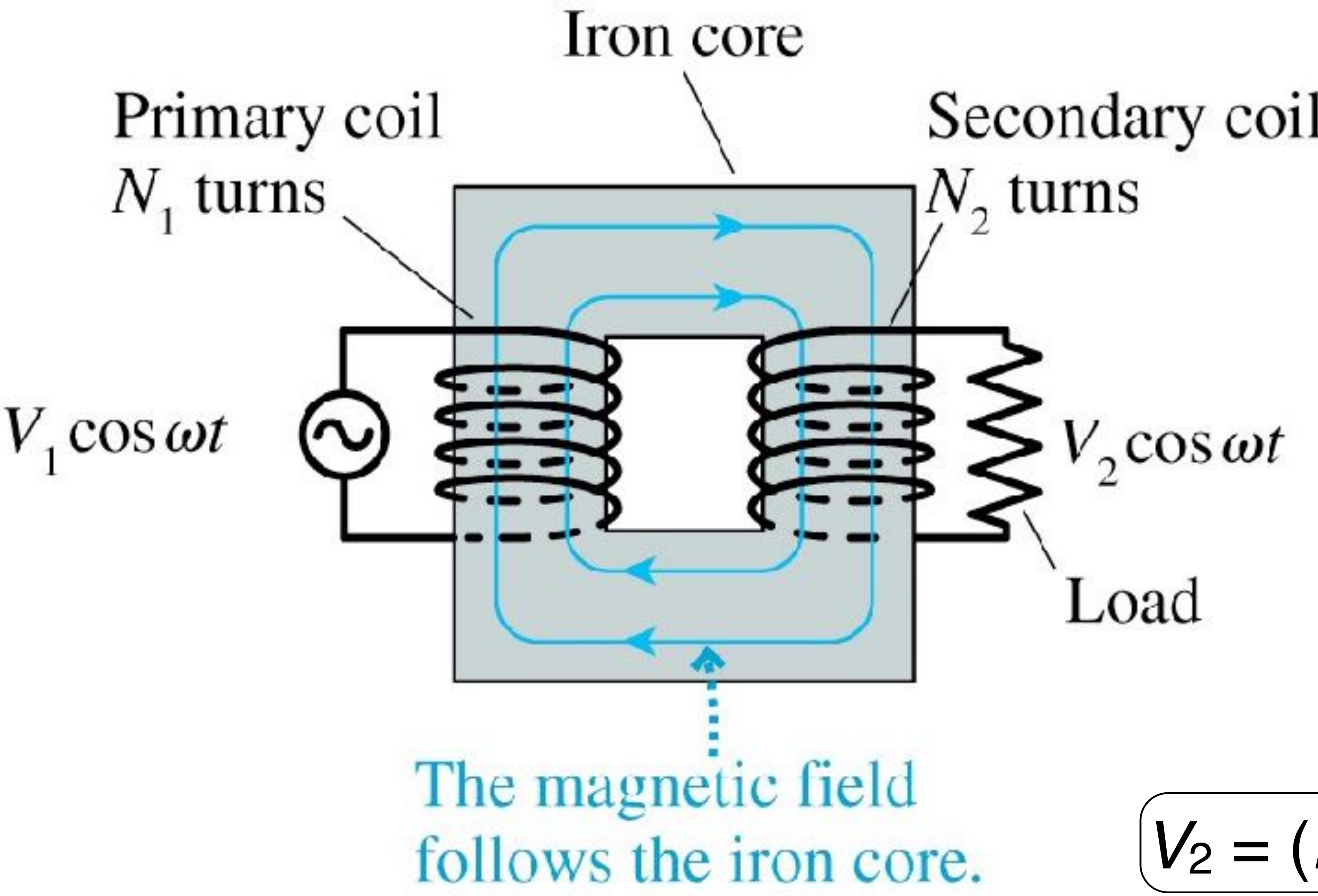
Some Applications

A generator



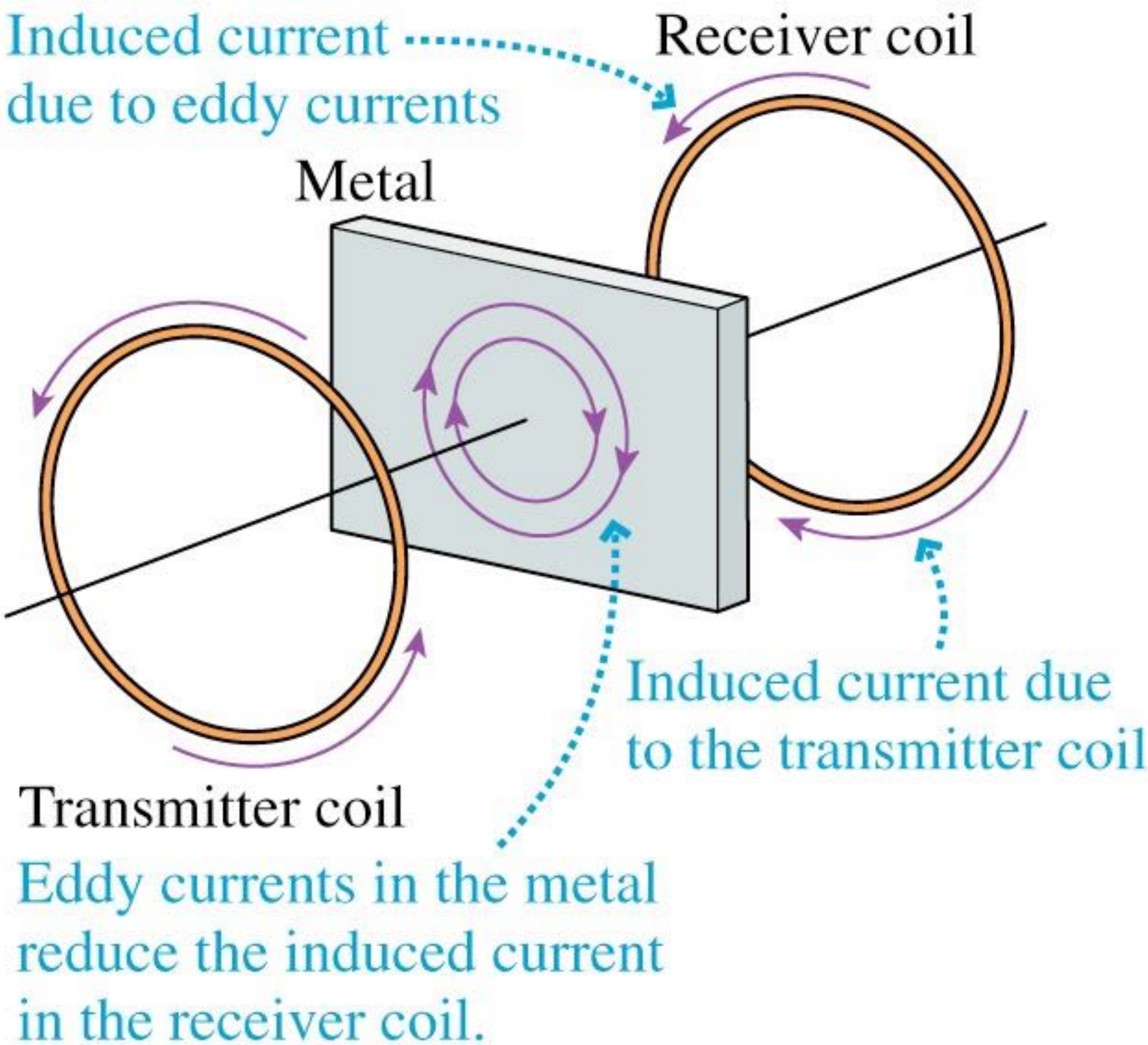
Some Applications

A transformer



$$\boxed{V_2 = (N_2/N_1)\,V_1}$$

Some Applications



A metal detector

The transmitter coil has a rapidly changing current and thus a rapidly changing magnetic field. This induces a current in the receiving coil. A piece of metal will get eddy currents induced, which will induce currents in the opposite direction in the receiver coil. Thus a piece of metal will ***reduce*** the current seen in the receiver.