Origin of $\mathcal{CP}$ Violation

Michael Ratz

SoCal BSM 2017 workshop, Riverside, April 2, 2017

Based on:

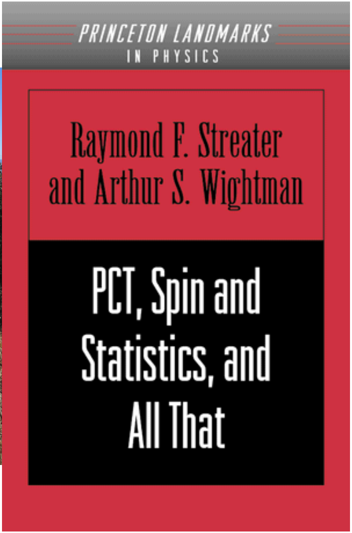
M.-C. Chen, M. Fallbacher, K.T. Mahanthappa, M.R. & A. Trautner, Nucl. Phys. B883 (2014) 267

M.R. & A. Trautner, JHEP02(2017)103

PCT

$\mathcal{PCT}$


PACIFIC CREST TRAIL
LAKE MORENA 13.5 MI.
SO. DRURY. RD. 14.4 MI.


PRINCETON LANDMARKS
IN PHYSICS
Raymond F. Streater
and Arthur S. Wightman
PCT, Spin and
Statistics, and
All That

$\mathcal{CP}$ violation in Nature

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huge literature

$\mathcal{CP}$ violation in Nature

- ☞ $\cancel{\mathcal{CP}}$ so far only observed in flavor sector
- ➥ it appears natural to seek connection between flavor physics & $\cancel{\mathcal{CP}}$
- ☞ flavor structure may be partially explained by (non–Abelian discrete) flavor symmetries

huge literature

this talk:

non–Abelian discrete (flavor) symmetry $G \leftrightarrow \cancel{\mathcal{CP}}$

Strong $\mathcal{CP}$ problem

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axion

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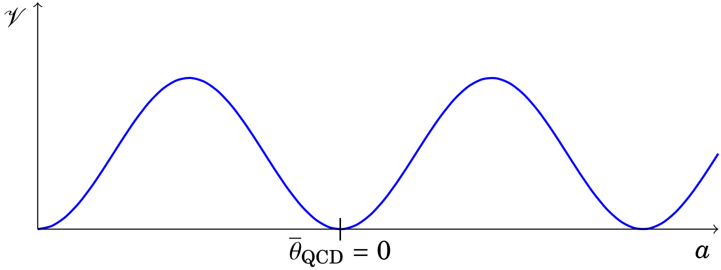
$$\bar{\theta}_{\mathrm{QCD}} = \theta_{\mathrm{QCD}} - \arg\left(\det Y_u\right) - \arg\left(\det Y_d\right)$$

- effective QCD angle $\bar{\theta}_{\mathrm{QCD}}$ vanishes at the minimum of the axion potential



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this talk:

proposal for a symmetry that kills $\overline{\theta}_{\mathrm{QCD}}$ but allows for $\cancel{\mathcal{CP}}$ in the flavor sector

The canonical $\mathcal{CP}$ transformation

- scalar field operator

$$\widehat{\phi}(x) = \int \mathrm{d}^3 p \, \frac{1}{2E_{\vec{p}}} \left[\widehat{a}(\vec{p})\, \mathrm{e}^{-\mathrm{i}p\cdot x} + \widehat{b}^\dagger(\vec{p})\, \mathrm{e}^{\mathrm{i}p\cdot x}\right]$$

annihilates particle

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creates anti–particle

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phase factor

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- $\mathcal{CP}$ transformation of (scalar) fields

$$\phi(x) \xmapsto{\widehat{C}\,\widehat{\mathcal{P}}} \underbrace{\eta_{\mathcal{CP}}}_{\text{freedom of re–phasing fields}}\ \phi^*(\mathcal{P}x)$$

$\mathcal{CP}$ vs. outer automorphisms

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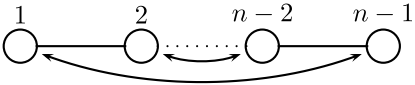

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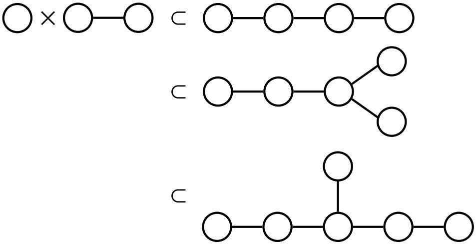
  $G_{\mathrm{SM}} = \mathrm{SU}(3)_{\mathrm{C}} \times \mathrm{SU}(2)_{\mathrm{L}} \times \mathrm{U}(1)_Y \subset \mathrm{SU}(5) \subset \mathrm{SO}(10) \subset \mathrm{E}_6$:
  $\mathcal{CP}$ always involves outer automorphism



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this talk:

not at all true

Generalized $\mathcal{CP}$ transformations

☞ setting w/ discrete symmetry G

Generalized $\mathcal{CP}$ transformations

- setting w/ discrete symmetry G
- generalized $\mathcal{CP}$ transformation

vector of creation operators

$$\left(\widehat{C}\,\widehat{\mathcal{P}}\right)^{-1}\,\widehat{\boldsymbol{a}}(\vec{p})\,\widehat{C}\,\widehat{\mathcal{P}} = U_\mathrm{CP}\,\widehat{\boldsymbol{b}}(-\vec{p}) \quad \& \quad \left(\widehat{C}\,\widehat{\mathcal{P}}\right)^{-1}\,\widehat{\boldsymbol{a}}^\dagger(\vec{p})\,\widehat{C}\,\widehat{\mathcal{P}} = \widehat{\boldsymbol{b}}^\dagger(-\vec{p})\,U_\mathrm{CP}{}^\dagger$$

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vector of annihilation operators

unitary matrix

Generalized $\mathcal{CP}$ transformations

- setting w/ discrete symmetry G
- generalized $\mathcal{CP}$ transformation
- invariant contraction/coupling in A_4 or T'

Holthausen, Lindner, and Schmidt (2013)

$$\left[\phi_{\mathbf{1}_2} \otimes (x_{\mathbf{3}} \otimes y_{\mathbf{3}})_{\mathbf{1}_1}\right]_{\mathbf{1}_0} \propto \phi \left(x_1\,y_1 + \omega^2\,x_2\,y_2 + \omega\,x_3\,y_3\right)$$

$\omega = \mathrm{e}^{2\pi\,\mathrm{i}/3}$

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➥ need generalized $\mathcal{CP}$ transformation $\widetilde{\mathcal{CP}}$: $\phi \xmapsto{\widetilde{\mathcal{CP}}} \phi^*$ as usual but

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} \xmapsto{\widetilde{\mathcal{CP}}} \begin{pmatrix} x_1^* \\ x_3^* \\ x_2^* \end{pmatrix} \quad \& \quad \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} \xmapsto{\widetilde{\mathcal{CP}}} \begin{pmatrix} y_1^* \\ y_3^* \\ y_2^* \end{pmatrix}$$

Constraints on generalized $\mathcal{CP}$ transformations

☞ generalized $\mathcal{CP}$ transformation

$$\Phi(x) \quad \stackrel{\widetilde{\mathcal{CP}}}{\longmapsto} \quad U_{\mathrm{CP}}\,\Phi^*(\,\mathcal{P}\;\,x)$$

fields of the theory/model

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$$\mathcal{P}\,(t,\vec{x}) = (t,-\vec{x})$$

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$$\Phi(x) \quad \xmapsto{\widetilde{\mathcal{CP}}} \quad U_{\mathrm{CP}}\,\Phi^*(\,\mathcal{P}\ x)$$

$$\begin{pmatrix} \uparrow \\ \phi_{\boldsymbol{r}_{i_1}} \\ \downarrow \\ \hline \uparrow \\ \phi_{\boldsymbol{r}_{i_2}} \\ \downarrow \\ \hline \vdots \end{pmatrix} \quad \xmapsto{\widetilde{\mathcal{CP}}} \quad \left(\begin{array}{c|c|c} \begin{matrix} \nwarrow & & \nearrow \\ & U_{\boldsymbol{r}_{i_1}} & \\ \swarrow & & \searrow \end{matrix} & & \\ \hline & \begin{matrix} \nwarrow & & \nearrow \\ & U_{\boldsymbol{r}_{i_2}} & \\ \swarrow & & \searrow \end{matrix} & \\ \hline & & \ddots \end{array}\right) \begin{pmatrix} \uparrow \\ \phi^*_{\boldsymbol{r}_{i_1}} \\ \downarrow \\ \hline \uparrow \\ \phi^*_{\boldsymbol{r}_{i_2}} \\ \downarrow \\ \hline \vdots \end{pmatrix}$$

field transforming in representation $\boldsymbol{r}_{i_2}$

- $\widetilde{\mathcal{CP}}$ depends on symmetry, not on model

$\lightning$ disagreement w/ Holthausen, Lindner, and Schmidt (2013)

Constraints on generalized $\mathcal{CP}$ transformations

- generalized $\mathcal{CP}$ transformation

$$\Phi(x) \quad \xrightarrow{\widetilde{\mathcal{CP}}} \quad U_{\mathrm{CP}}\, \Phi^*(\,\mathcal{P}\ x)$$

- consistency condition

Holthausen, Lindner, and Schmidt (2013)

$$\rho\bigl(u(g)\bigr) \quad = \quad U_{\mathrm{CP}}\ \rho(g)^*\, U_{\mathrm{CP}}{}^\dagger \qquad \forall\, g \in G$$

automorphism $u \ : \ G \to G$

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representation matrix

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☞ further properties:

- u has to be class–inverting
  the consistency condition
  $$\rho_{\boldsymbol{r}_i}\big(u(g)\big) \;=\; U_{\boldsymbol{r}_i}\,\rho_{\boldsymbol{r}_i}(g)^*\,U_{\boldsymbol{r}_i}^\dagger \quad \forall\, g\in G \text{ and } \forall\, i$$
  implies

$$\begin{aligned}\chi_{\boldsymbol{r}_i}\,(u(g)) &= \operatorname{tr}\left[\rho_{\boldsymbol{r}_i}(u(g))\right] \;=\; \operatorname{tr}\left[U_{\boldsymbol{r}_i}\,\rho_{\boldsymbol{r}_i}(g)^*\,U_{\boldsymbol{r}_i}^\dagger\right]\\ &= \operatorname{tr}\left[\rho_{\boldsymbol{r}_i}(g)\right]^* \;=\; \chi_{\boldsymbol{r}_i}(g)^* \;=\; \chi_{\boldsymbol{r}_i}(g^{-1}) \quad \forall\, i\end{aligned}$$

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- further properties:
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  - in all known cases, u is equivalent to an automorphism of order two

we have scanned all groups of orders up to 150 (with a few exceptions of order 128) & did not find a single example of a class–inverting automorphism of higher order that is not equivalent to another u of order 2

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bottom–line:

u has to be a class–inverting (involutory) automorphism of G

How (not) to generalize $\mathcal{CP}$

proper $\mathcal{CP}$ transformations

- ☞ map field operators to *their own* Hermitean conjugates

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How (not) to generalize $\mathcal{CP}$

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- violation of physical $\mathcal{CP}$ is prerequisite for a non–trivial

anti–particles

$$\varepsilon_{i\to f} \;=\; \frac{\left|\Gamma\left(i\to f\right)\right|^2-\left|\Gamma\left(\bar{\imath}\to\bar{f}\right)\right|^2}{\left|\Gamma\left(i\to f\right)\right|^2+\left|\Gamma\left(\bar{\imath}\to\bar{f}\right)\right|^2}$$

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➥ connection to observed $\cancel{\mathcal{CP}}$, baryogenesis & . . .

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- explicit example later

The Bickerstaff–Damhus automorphism (BDA)

Bickerstaff and Damhus (1985)

☞ Bickerstaff–Damhus automorphism (BDA) u

$$\rho_{\boldsymbol{r}_i}(u(g)) \quad = \quad U_{\boldsymbol{r}_i} \;\rho_{\boldsymbol{r}_i}(g)^* \, U_{\boldsymbol{r}_i}^\dagger \quad \forall\, g \in G \text{ and } \forall\, i \qquad (\,\star\,)$$

unitary & symmetric

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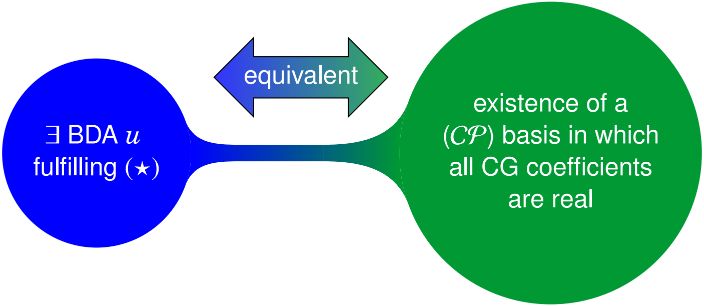

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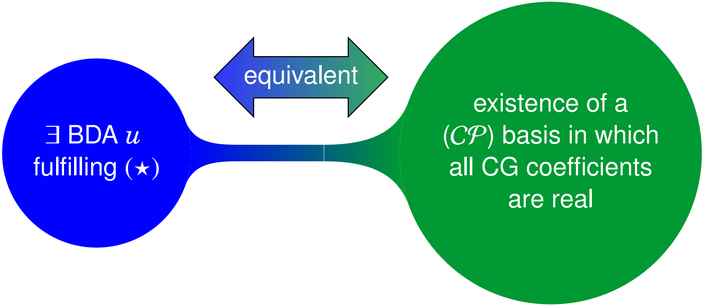

$$\mathcal{CP}\text{ basis}\;:\;\rho_{\boldsymbol{r}_i}(u(g)) \;=\; \rho_{\boldsymbol{r}_i}(g)^* \quad \forall\, g\in G \text{ and } \forall\, i$$

The twisted Frobenius–Schur indicator

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- Frobenius–Schur indicator

$$\mathrm{FS}(\boldsymbol{r}_i) := \frac{1}{|G|}\sum_{g\in G}\chi_{\boldsymbol{r}_i}(g^2) = \frac{1}{|G|}\sum_{g\in G}\mathrm{tr}\left[\rho_{\boldsymbol{r}_i}(g)^2\right]$$

$$\mathrm{FS}(\boldsymbol{r}_i) = \begin{cases} +1, & \text{if } \boldsymbol{r}_i \text{ is a real representation,} \\ 0, & \text{if } \boldsymbol{r}_i \text{ is a complex representation,} \\ -1, & \text{if } \boldsymbol{r}_i \text{ is a pseudo–real representation.} \end{cases}$$

The twisted Frobenius–Schur indicator

- how can one tell whether or not a given automorphism u is a BDA?
- Frobenius–Schur indicator

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Bickerstaff and Damhus (1985); Kawanaka and Matsuyama (1990)

- twisted Frobenius–Schur indicator

$$\mathrm{FS}_u(\boldsymbol{r}_i) \;=\; \frac{1}{|G|}\sum_{g\in G}\left[\rho_{\boldsymbol{r}_i}(g)\right]_{\alpha\beta}\left[\rho_{\boldsymbol{r}_i}(u(g))\right]_{\beta\alpha}$$

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- crucial property

$$\mathrm{FS}_u(\boldsymbol{r}_i) \;=\; \begin{cases} +1 \quad \forall\, i, & \text{if } u \text{ is a BDA},\\ +1 \text{ or } -1 \quad \forall\, i, & \text{if } u \text{ is class–inverting \& involutory},\\ \text{different from } \pm 1, & \text{otherwise.}\end{cases}$$

Extended twisted Frobenius–Schur indicator

- extended twisted Frobenius–Schur indicator

$$\mathrm{FS}_{u}^{(n)}(\boldsymbol{r}_i) := \frac{(\dim \boldsymbol{r}_i)^{n-1}}{|G|^n} \sum_{g_1,\dots,g_n \in G} \chi_{\boldsymbol{r}_i}(g_1\, u(g_1) \cdots g_n\, u(g_n))$$

$$n = \begin{cases} \mathrm{ord}(u)/2 & \text{if } \mathrm{ord}(u) \text{ is even} \\ \mathrm{ord}(u) & \text{if } \mathrm{ord}(u) \text{ is odd} \end{cases}$$

Extended twisted Frobenius–Schur indicator

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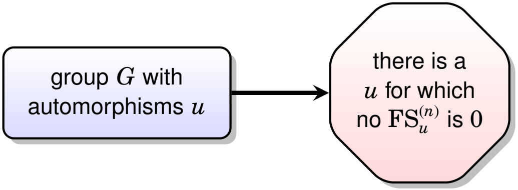
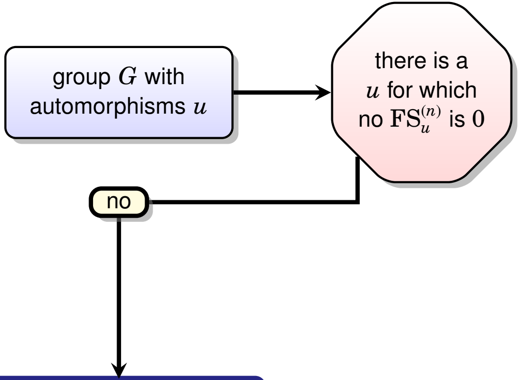
Three types of groups

group G with automorphisms u

Three types of groups



Three types of groups



Type I groups G_{I}:

generic settings based on G_{I} do not allow for a physical $\mathcal{CP}$ transformation

Three types of groups

group G with automorphisms u

there is a u for which no $\mathrm{FS}_u^{(n)}$ is 0

yes

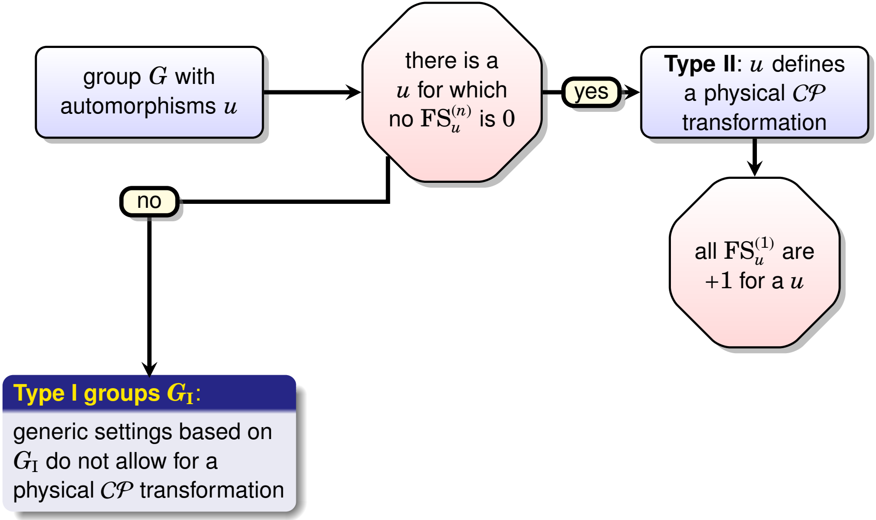
Type II: u defines a physical $\mathcal{CP}$ transformation

no

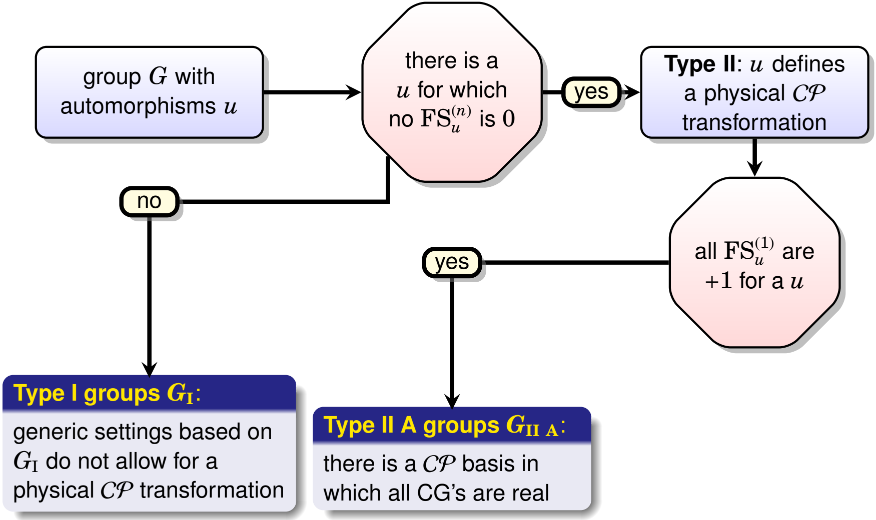
Type I groups G_{I}:

generic settings based on G_{I} do not allow for a physical $\mathcal{CP}$ transformation

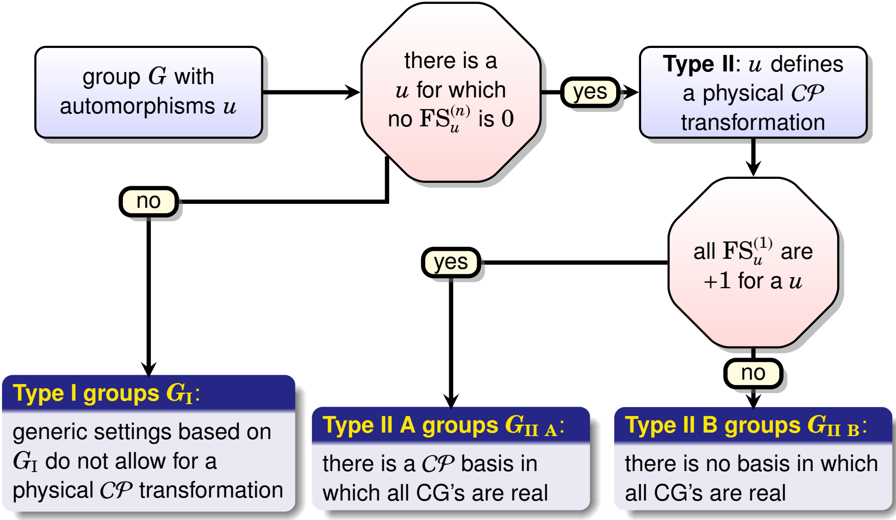
Three types of groups



Three types of groups



Three types of groups



Examples

will be discussed in detail later

- ☞ type I : all odd order non–Abelian groups

group	$\mathbb{Z}_5 \rtimes \mathbb{Z}_4$	T_7	$\Delta(27)$	$\mathbb{Z}_9 \rtimes \mathbb{Z}_3$
SG	(20,3)	(21,1)	(27,3)	(27,4)

Examples

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- type II A : dihedral & all Abelian groups

group	S_3	Q_8	A_4	$\mathbb{Z}_3 \rtimes \mathbb{Z}_8$	T'	S_4	A_5
SG	(6,1)	(8,4)	(12,3)	(24,1)	(24,3)	(24,12)	(60,5)

Examples

- type I : all odd order non–Abelian groups

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- type II B

group	$\Sigma(72)$	$((\mathbb{Z}_3 \times \mathbb{Z}_3) \rtimes \mathbb{Z}_4) \rtimes \mathbb{Z}_4$
SG	(72,41)	(144,120)

$\mathcal{CP}$ violation with an unbroken $\mathcal{CP}$ transformation

- type I groups can be embedded in $\mathrm{SU}(N)$

no $\mathcal{CP}$ transformation

has $\mathcal{CP}$ transformation

$\mathcal{CP}$ violation with an unbroken $\mathcal{CP}$ transformation

- ☞ type I groups can be embedded in $\mathrm{SU}(N)$

- ➥ question: at which stage gets $\mathcal{CP}$ broken?

$\mathcal{CP}$ violation with an unbroken $\mathcal{CP}$ transformation

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➥ question: at which stage gets $\mathcal{CP}$ broken?

☞ possible options include:

- $\mathcal{CP}$ gets broken by the VEV that breaks $\mathrm{SU}(N)$ to G

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  - the resulting setting always has additional symmetries and does not violate $\mathcal{CP}$

$\mathcal{CP}$ violation with an unbroken $\mathcal{CP}$ transformation

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- ☞ possible options include:
  - $\mathcal{CP}$ gets broken by the VEV that breaks $\mathrm{SU}(N)$ to G
  - the resulting setting always has additional symmetries and does not violate $\mathcal{CP}$

- ☞ surprisingly the answer is none of the above

Example: $\mathrm{SU}(3) \to \mathrm{T}_7$

- starting point: $\mathrm{SU}(3)$ gauge theory with

$$\mathscr{L} = \left(D_\mu \phi\right)^\dagger \left(D^\mu \phi\right) - \frac{1}{4} G^a_{\mu\nu} G^{\mu\nu,a} - \mathscr{V}(\phi)$$

$D_\mu = \partial_\mu - \mathrm{i} g A_\mu$

field strength

Example: $\mathrm{SU}(3) \to \mathrm{T}_7$

☞ starting point: $\mathrm{SU}(3)$ gauge theory with

$$\mathscr{L} \;=\; \left(D_\mu\,\phi\right)^\dagger \left(D^\mu\,\underbrace{\phi}_{\mathbf{15}\text{–plet}}\right) - \frac{1}{4}\,G_{\mu\nu}^a\,G^{\mu\nu,a} - \mathscr{V}(\phi)$$

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quartic $\mathrm{SU}(3)$ invariants

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- action invariant under $\mathcal{CP}$ transformation

$$A^a_\mu(x) \xmapsto{\mathrm{SU}(3)-\mathcal{CP}} R^{ab}\,\mathcal{P}_\mu^{\ \nu}\,A^b_\nu(\mathcal{P}\,x)$$

$$\phi_i(x) \xmapsto{\mathrm{SU}(3)-\mathcal{CP}} U_{ij}\,\phi_j^*(\mathcal{P}\,x)$$

$$\mathcal{P} = \mathrm{diag}(1,-1,-1,-1)$$

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$\mathrm{SU}(3) \rightarrow \mathsf{T}_7$

see e.g. Luhn (2011) & Merle and Zwicky (2012)

☞ $\langle\phi\rangle$ breaks $\mathrm{SU}(3)$ to T_7

$$\mathrm{SU}(3) \rtimes \mathbb{Z}_2 \xrightarrow{\langle\phi\rangle} \mathsf{T}_7 \rtimes \mathbb{Z}_2$$

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- $\langle\phi\rangle$ breaks $\mathrm{SU}(3)$ to T_7

$$\mathrm{SU}(3) \rtimes \mathbb{Z}_2 \xrightarrow{\langle\phi\rangle} \mathsf{T}_7 \rtimes \mathbb{Z}_2$$

- physical fields before and after symmetry breaking

name	$\mathrm{SU}(3) \xrightarrow{\langle\phi\rangle}$	name	T_7
A_μ	$\mathbf{8}$	Z_μ	$\mathbf{1}_1$
		W_μ	$\mathbf{3}$
ϕ	$\mathbf{15}$	$\mathrm{Re}\,\sigma_0, \mathrm{Im}\,\sigma_0$	$\mathbf{1}_0$
		σ_1	$\mathbf{1}_1$
		τ_1	$\mathbf{3}$
		τ_2	$\mathbf{3}$
		τ_3	$\mathbf{3}$

$\mathrm{SU}(3) - \mathcal{CP}$ vs. $\mathrm{Out}(\mathrm{T}_7)$

- $\mathrm{SU}(3) - \mathcal{CP}$ breaks to unique $\mathbb{Z}_2$ outer automorphism of T_7

$$\mathrm{Out}(\mathrm{T}_7) \ : \qquad \mathbf{1_1} \longleftrightarrow \mathbf{1_1}\ , \quad \overline{\mathbf{1}}\mathbf{_1} \longleftrightarrow \overline{\mathbf{1}}\mathbf{_1}\ , \quad \mathbf{3} \longleftrightarrow \overline{\mathbf{3}}$$

SU(3) – $\mathcal{CP}$ vs. Out(T_7)

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- T_7 character table

T_7	C_{1a}	C_{3a}	C_{3b}	C_{7a}	C_{7b}
	e	b	b^2	a	a^3
$\mathbf{1_0}$	1	1	1	1	1
$\mathbf{1_1}$	1	ω	ω^2	1	1
$\overline{\mathbf{1}}_\mathbf{1}$	1	ω^2	ω	1	1
$\mathbf{3}$	3	0	0	η	η^*
$\overline{\mathbf{3}}$	3	0	0	η^*	η

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$\overline{\mathbf{3}}$	3	0	0	η^*	η

$\omega := \mathrm{e}^{2\pi\mathrm{i}/3}$

$\eta = \rho + \rho^2 + \rho^4$ with $\rho := \mathrm{e}^{2\pi\mathrm{i}/7}$

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$\mathbf{3}$	3	0	0	η	η^*
$\overline{\mathbf{3}}$	3	0	0	η^*	η

- $\mathbf{1}_1$ and $\overline{\mathbf{1}}_1$ do **not** get swapped!

T_7

- T_7 can be generated by two elements with the presentation

$$\left\langle \mathsf{a}, \mathsf{b} \;\middle|\; \mathsf{a}^7 = \mathsf{b}^3 = \mathsf{e}\,, \mathsf{b}^{-1}\,\mathsf{a}\,\mathsf{b} = \mathsf{a}^4 \right\rangle$$

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$$\left\langle \mathsf{a},\mathsf{b} \;\middle|\; \mathsf{a}^7 = \mathsf{b}^3 = \mathsf{e}\,,\mathsf{b}^{-1}\,\mathsf{a}\,\mathsf{b} = \mathsf{a}^4 \right\rangle$$

- triplet representation

$$A = \begin{pmatrix} \rho & 0 & 0 \\ 0 & \rho^2 & 0 \\ 0 & 0 & \rho^4 \end{pmatrix} \quad \text{and} \quad B = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}$$

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- embedding into $\mathrm{SU}(3)$

$$X^{(\boldsymbol{r})} = \exp\left(\mathrm{i}\,\alpha_a\,\mathsf{t}_a^{(\boldsymbol{r})}\right)$$

$$\vec{\alpha}^{(A)} = \frac{2\pi}{7}\left(0,0,0,0,0,0,\sqrt{3},5\right)$$

T_7

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$$\left\langle \mathsf{a}, \mathsf{b} \;\middle|\; \mathsf{a}^7 = \mathsf{b}^3 = \mathsf{e}\,, \mathsf{b}^{-1}\,\mathsf{a}\,\mathsf{b} = \mathsf{a}^4 \right\rangle$$

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- embedding into $\mathrm{SU}(3)$

$$X^{(\boldsymbol{r})} = \exp\left(\mathrm{i}\,\alpha_a\,\mathsf{t}_a^{(\boldsymbol{r})}\right)$$

$$\vec{\alpha}^{(B)} = \frac{4\pi}{3\sqrt{3}}\,(0,0,1,1,1,0,0,0)$$

T_7

- T_7 can be generated by two elements with the presentation

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- embedding into $\mathrm{SU}(3)$

$$X^{(\boldsymbol{r})} = \exp\left(\mathrm{i}\,\alpha_a\,\mathsf{t}_a^{(\boldsymbol{r})}\right)$$

- work in `SusyNo` basis

Fonseca (2012)

T_7 scalar states

☞ branchings: $\mathbf{8} \to \mathbf{1_1} \oplus \overline{\mathbf{1}}_\mathbf{1} \oplus \mathbf{3} \oplus \overline{\mathbf{3}}$

$$\mathbf{15} \to \mathbf{1_0} \oplus \mathbf{1_1} \oplus \overline{\mathbf{1}}_\mathbf{1} \oplus \mathbf{3} \oplus \mathbf{3} \oplus \overline{\mathbf{3}} \oplus \overline{\mathbf{3}}$$

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- physical scalar fields (would–be Goldstone bosons subtracted)

$$\phi \;=\; \left(v + \phi_1, \frac{\phi_2}{\sqrt{2}}, \frac{\phi_2^*}{\sqrt{2}}, \phi_4, \phi_5, \phi_6, \frac{\phi_7}{\sqrt{2}}, \frac{\phi_8}{\sqrt{2}}, \frac{\phi_9}{\sqrt{2}}, \phi_{10}, \phi_{11}, \phi_{12}, \frac{\phi_7^*}{\sqrt{2}}, \frac{\phi_8^*}{\sqrt{2}}, \frac{\phi_9^*}{\sqrt{2}}\right)$$

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- T_7 representations

$$\phi_1 \;\widehat{=}\; \mathbf{1}_0\,, \qquad \phi_2 \;\widehat{=}\; \mathbf{1}_1\,,$$

$$T_1 \;:=\; (\phi_4, \phi_5, \phi_6) \;\widehat{=}\; \mathbf{3}\,, \qquad T_2 \;:=\; (\phi_7, \phi_8, \phi_9) \;\widehat{=}\; \mathbf{3}\,,$$

$$\overline{T}_3 \;:=\; (\phi_{10}, \phi_{11}, \phi_{12}) \;\widehat{=}\; \overline{\mathbf{3}}$$

T_7 scalar states

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- T_7 representations

- no physical CP trafo allowed by T_7!

$$\mathbb{Z}_2-\mathrm{Out}: \quad \begin{array}{c} \mathbf{15} \;\to\; \mathbf{1_0} \oplus \mathbf{1_1} \oplus \overline{\mathbf{1}}_\mathbf{1} \oplus \mathbf{3} \oplus \mathbf{3} \oplus \overline{\mathbf{3}} \oplus \overline{\mathbf{3}} \;\& \\ \downarrow \\ \overline{\mathbf{15}} \;\to\; \mathbf{1_0} \oplus \overline{\mathbf{1}}_\mathbf{1} \oplus \mathbf{1_1} \oplus \overline{\mathbf{3}} \oplus \overline{\mathbf{3}} \oplus \mathbf{3} \oplus \mathbf{3} \end{array}$$

T_7 scalar states

- branchings: $\mathbf{8} \;\to\; \mathbf{1_1} \oplus \mathbf{\overline{1}_1} \oplus \mathbf{3} \oplus \mathbf{\overline{3}}$

$$\mathbf{15} \;\to\; \mathbf{1_0} \oplus \mathbf{1_1} \oplus \mathbf{\overline{1}_1} \oplus \mathbf{3} \oplus \mathbf{3} \oplus \mathbf{\overline{3}} \oplus \mathbf{\overline{3}}$$

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$$\phi \;=\; \left(v + \phi_1,\, \frac{\phi_2}{\sqrt{2}},\, \frac{\phi_2^*}{\sqrt{2}},\, \phi_4,\, \phi_5,\, \phi_6,\, \frac{\phi_7}{\sqrt{2}},\, \frac{\phi_8}{\sqrt{2}},\, \frac{\phi_9}{\sqrt{2}},\, \phi_{10},\, \phi_{11},\, \phi_{12},\, \frac{\phi_7^*}{\sqrt{2}},\, \frac{\phi_8^*}{\sqrt{2}},\, \frac{\phi_9^*}{\sqrt{2}}\right)$$

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Scalar masses

- VEV

$$|v| \;=\; \mu \times 3\,\sqrt{\frac{7}{2}}\left(-7\sqrt{15}\,\lambda_1 + 14\sqrt{15}\,\lambda_2 + 20\sqrt{6}\,\lambda_4 + 13\sqrt{15}\,\lambda_5\right)^{-1/2}$$

Scalar masses

- VEV

$$|v| = \mu \times 3\,\sqrt{\frac{7}{2}}\left(-7\sqrt{15}\,\lambda_1 + 14\sqrt{15}\,\lambda_2 + 20\sqrt{6}\,\lambda_4 + 13\sqrt{15}\,\lambda_5\right)^{-1/2}$$

- T_7 $\mathbf{1}$–plet representations

$$\mathrm{Re}\,\sigma_0 = \frac{1}{\sqrt{2}}\left(\phi_1 + \phi_1^*\right)\;, \qquad \mathrm{Im}\,\sigma_0 = -\frac{\mathrm{i}}{\sqrt{2}}\left(\phi_1 - \phi_1^*\right)\;,$$

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can be eliminated gauging accidental $\mathrm{U}(1)$

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- masses

$$\begin{aligned}m^2_{\mathrm{Re}\,\sigma_0} &= 2\,\mu^2\,, \qquad m^2_{\mathrm{Im}\,\sigma_0} = 0\\ m^2_{\sigma_1} &= -\,\mu^2 + \sqrt{15}\,\lambda_5\,v^2\end{aligned}$$

Gauge fields

- gauge fields

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\begin{aligned}
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$$m_Z^2 = \frac{7}{3}g^2\,v^2 \quad \text{and} \quad m_W^2 = g^2\,v^2$$

Triplet mass eigenstates

- mass eigenstates

$$\begin{pmatrix}\tau_1\\ \tau_2\\ \tau_3\end{pmatrix} = \underbrace{\begin{pmatrix}V_{11} & V_{12} & V_{13}\\ V_{21} & V_{22} & V_{23}\\ V_{31} & V_{32} & V_{33}\end{pmatrix}}_{=V}\begin{pmatrix}T_2\\ \overline{T}_3^*\\ T_1\end{pmatrix}$$

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- masses and mixing matrix depend on potential parameters

T_7 outer automorphism vs. $\mathcal{CP}$

- $\mathrm{Out}(\mathrm{T}_7)$

$$\begin{aligned} Z_\mu(x) &\mapsto -\mathcal{P}_\mu^{\ \nu} Z_\nu(\mathcal{P}x)\,, & \sigma_0(x) &\mapsto \sigma_0(\mathcal{P}x)\,, \\ W_\mu(x) &\mapsto \mathcal{P}_\mu^{\ \nu} W_\nu^*(\mathcal{P}x)\,, & \sigma_1(x) &\mapsto \sigma_1(\mathcal{P}x)\,, \quad \tau_i(x) \mapsto \tau_i^*(\mathcal{P}x) \end{aligned}$$

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- mode expansion

$$\widehat{\boldsymbol{\sigma}}_1(x) = \int \widetilde{\mathrm{d}p}\,\left\{\widehat{\boldsymbol{a}}(\vec{p})\,\mathrm{e}^{-\mathrm{i}\,p\,x} + \widehat{\boldsymbol{b}}^\dagger(\vec{p})\,\mathrm{e}^{\mathrm{i}\,p\,x}\right\}$$

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\mathrm{Out}(\mathrm{T}_7)\ :\quad \widehat{\boldsymbol{a}}(\vec{p}) \mapsto \widehat{\boldsymbol{a}}(-\vec{p}) \qquad \text{and} \qquad \widehat{\boldsymbol{b}}^\dagger(\vec{p}) \mapsto \widehat{\boldsymbol{b}}^\dagger(-\vec{p})
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- QFT $\mathcal{CP}$ not a symmetry of the action

$$\mathcal{CP}\ :\quad \widehat{\boldsymbol{a}}(\vec{p}) \mapsto \widehat{\boldsymbol{b}}(-\vec{p}) \quad \text{and} \quad \widehat{\boldsymbol{b}}^\dagger(\vec{p}) \mapsto \widehat{\boldsymbol{a}}^\dagger(-\vec{p})$$

$\mathcal{CP}$ violation in the T_7 phase

☞ decay asymmetry

$$\varepsilon_{\sigma_1\to W\,W^*} \;:=\; \frac{\left|\mathscr{M}(\sigma_1\to W\,W^*)\right|^2-\left|\mathscr{M}(\sigma_1^*\to W\,W^*)\right|^2}{\left|\mathscr{M}(\sigma_1\to W\,W^*)\right|^2+\left|\mathscr{M}(\sigma_1^*\to W\,W^*)\right|^2}$$

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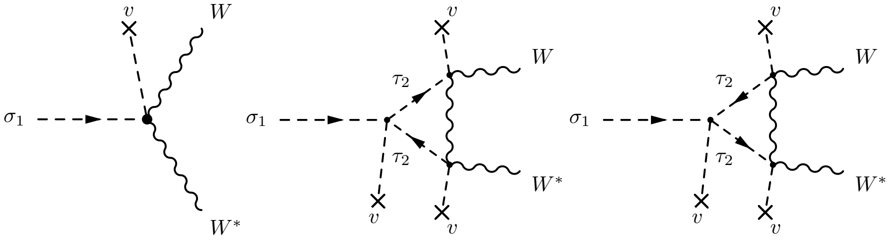
- $\mathcal{CP}$ violation from interference between tree–level and 1–loop



Application to the strong $\mathcal{CP}$ problem

- starting point: $\mathcal{CP}$ conservating theory based on

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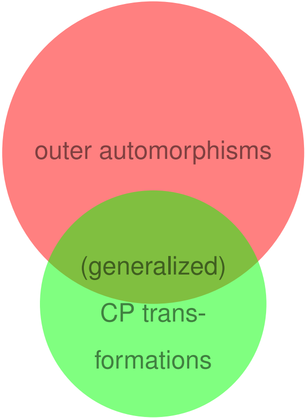
- main problem: find a realistic model based on T_7 allowing for outer automorpism

Summary

Summary

☞ not every outer automorphism defines a physical CP transformation!



proper CP transformations:

class–inverting automorphisms of G

Summary

- not every outer automorphism defines a physical CP transformation!
- three types of groups

Discrete (flavor) symmetry G

Type I groups G_{I}:

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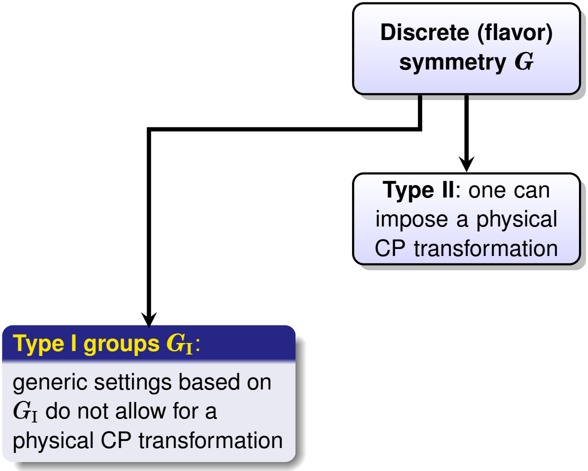

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Type II: one can impose a physical CP transformation

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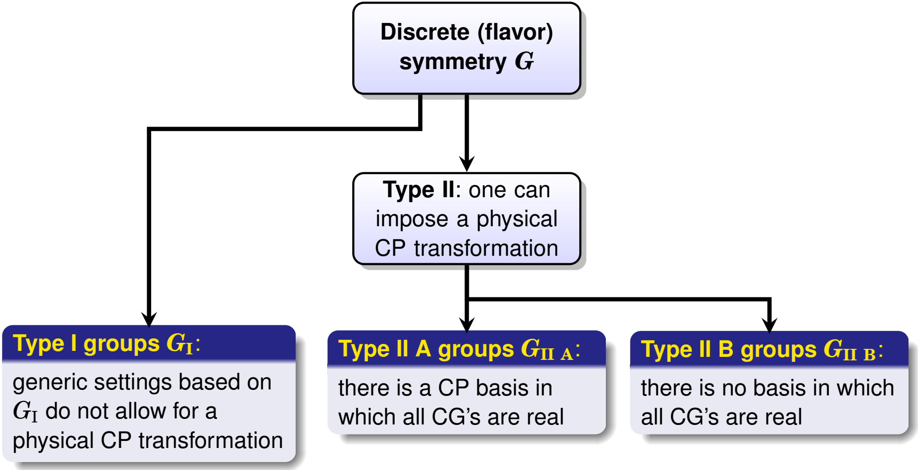

Summary

- not every outer automorphism defines a physical CP transformation!
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CP conserving models can be constructed by:

- introducing only special subsets of representations
- enlarging the symmetry non–trivially beyond the type I symmetry

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Discrete (flavor) symmetry G

Type II: one can impose a physical CP transformation

- the CP basis may not be the most convenient choice for performing computations
- CP invariance restricts the phases of coupling coefficients

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Discrete (flavor) symmetry G

Type II: one can impose a physical CP transformation

- CP may enlarge the group (trivially) to $G_{\mathrm{II\ B}} \rtimes \mathbb{Z}_N$
- CP invariance may forbid certain coupling coefficients

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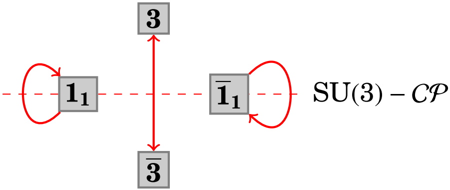

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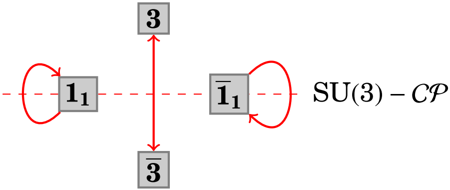

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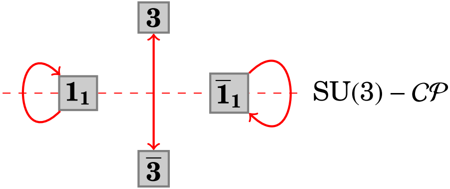

- ➥ physical $\mathcal{CP}$ violated in T_7 sector!
- ➥ $\mathrm{SU}(3)$–$\mathcal{CP}$ survives as (unique) outer automorphism of T_7

Summary

Three examples:

☞ type I group: T_7

- generic settings based on T_7 violate $\mathcal{CP}$!
- spontaneous breaking of type II A group $\mathrm{SG}(54,5) \to \Delta(27)$
  $\curvearrowright$ prediction of $\mathcal{CP}$ violating phase from group theory!

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☞ type II B group: $\Sigma(72)$

- absence of $\mathcal{CP}$ basis but generalized $\mathcal{CP}$ transformation ensures physical $\mathcal{CP}$ conservation
- $\mathcal{CP}$ forbids couplings

Outlook

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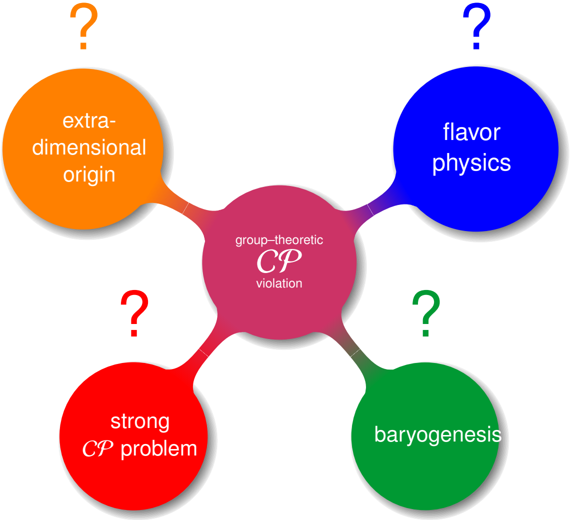

26th International Workshop on Weak Interactions and Neutrinos (WIN 2017)

University of California, Irvine, June 19 - 24, 2017

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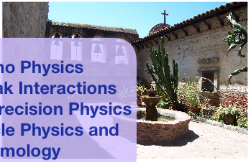

Just steps away...

Local Organizers:

Mu-Chun Chen (muchunc@uci.edu)
Michael Smy (msmy@uci.edu)

http://sites.uci.edu/win2017

Thank you

very much!

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