

R computing for Business Data Analytics

Homework 2

Due date: 23:59PM November 6 (Sunday), 2016

Please e-mail your homework (.pdf) and the associated R code (.R) to hchuang.om@gmail.com.

The email title must be **R_HW2_GroupName**. Each group submits ONLY ONE copy.

NO late homework will be accepted.

Q1. (15%) The Fibonacci sequence is famous in mathematics. The sequence is defined as

$$F_1 = 1, F_2 = 1, F_n = F_{n-1} + F_{n-2} \text{ for } n > 2$$

(a) Write a *while()* loop to find the first Fibonacci number $k >$ greater than 100.

(b) For the number $F_n=k$ in (a), what is the index n ?

(c) Use *for()* to write a function *print.Fib* that takes an integer k as its input and prints **ALL** Fibonacci numbers $\leq k$. Test $k=100$ and show me the results.

Q2. (10%) Write a function *second.smallest()* that takes a vector x as its input. The function will return the number that is the second smallest (第二小) inside x .

Show me the results of *second.small*($x=c(2, 8, 8, 2, 5, 2, 5, 2)$).

Q3. (10%) Use *while()* and/or *if...else* to write a function *f.exist* that takes an integer z and a vector x as its inputs. The function *f.exist* will return TRUE only if z is inside x .

Test *f.exist*($z=10, x=c(1:10)$) and *f.exist*($z=10, x=c(9, 3, 1)$). Show the answers.

Q4. (10%) Use *while()* and/or *if...else* to write a function *f.divide* that takes an integer z as its input. The function *f.divide* will return how many divisors (除數) z has (other than 1 & z itself).

Test *f.divide*(100) and show me the results.

Q5. (10%) Write a function *UNIQUE()* that takes a vector x as its input. The function will return a new vector with all unique (獨特的) numbers in x with duplicated (重複) elements removed.

Do NOT use *unique()* in R. Show me the results of *UNIQUE*($x=c(2, 8, 8, 2, 5, 2, 5, 2)$).

Q6. (20%) The Babylonian method (巴比倫法) is famous for getting the square root (平方根) of any number. Suppose we have a positive number S , the Babylonian method suggests that

$$x_{n+1} = 0.5(x_n + S / x_n)$$

x_n is your current guess of \sqrt{S} and x_{n+1} is your next guess of \sqrt{S} . You will STOP searching only if $|x_{n+1} - x_n| < \text{tolerance}$.

Now, set $S=125348$, your initial guess $x_0=600$, and $\text{tolerance}=1\text{e-}5$.

Use *while*() to implement the algorithm and show me the square root (\sqrt{S}) you find. The answer should be 354.0452.

How about $S=9527$, initial guess $x_0=87$, and $\text{tolerance}=1\text{e-}5$? What's the answer?

How about $S=5566$, initial guess $x_0=78$, and $\text{tolerance}=1\text{e-}5$? What's the answer?

Q7. (25%) Write a function *Bessell_Gen* that has five arguments ($a, v, z, \text{max}, \text{tolerance}$). The function will compute the **generalized** modified Bessel function of the first kind:

$$I_a^v(z) = \sum_{m=0}^{\infty} \frac{1}{[\Gamma(m+a+1)m!]^v} \left(\frac{z}{2}\right)^{2m+a} \quad \text{where } \Gamma(\bullet) \text{ is the } \textit{gamma}(\) \text{ in } R.$$

As infinite sum is NOT possible in computers, please mimic the example in lecture 2 to compute the value of the function. Also, the *Bessell_Gen* function will continue to add up numbers only if

$$m < \text{max} \ \& \ |I_a^v(z)_{m-1} - I_a^v(z)_m| > \text{tolerance}$$

Once you finish coding, run *Bessell_Gen*(5, 1, 10, 1000, 1e-5) in *R*. What is the value? Then run *bessell*(10, 5) (a built-in function for the modified Bessel function of the first kind) in *R*. Are the two values identical?