Linear Time Periodic Modelling of Single-Phase Elementary Phase-Locked-Loop

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INTRODUCTION

➢MOTIVATION

- Due to the current need of the environment, use of renewable energy sources (RES) like PV, Wind etc. is rapidly increasing.
- These RESs are being used with the help of switch-based power electronic converters, with multilevel controls.
- Such complex interaction with the grid, often causes some stability issues, unwanted harmonic events, frequency coupling.
- Traditionally, these systems were modelled using Linear time invariant (LTI) approach, which only considers fundamental frequency.
- Linear Time Periodic modelling framework is now being adopted to study harmonic interaction between power electronics systems and the grid.

INTRODUCTION

➢OBJECTIVES

- The focus is on Single-Phase Elementary Phase-Locked-Loop (PLL).
- The objective is to:
  1. To model the PLL with traditional LTI modelling approach.
  2. To derive the LTP state- space model system.
  3. To formulate LTI model from the LTP system obtained.
  4. Validate the two models with the non-linear model of PLL.

SINGLE-PHASE ELEMENTARY PHASE-LOCKED-LOOP

- The input signal v_α is given by,

$$v_\alpha(t) = \hat{V}\sin(\theta + \phi_0)$$

- The PI controller is the loop filter with gains k_p and k_i.

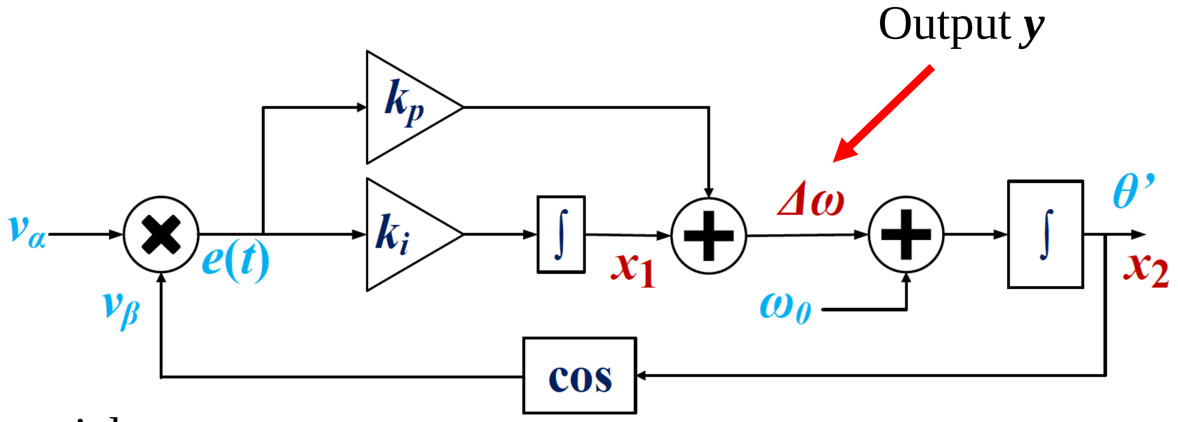


Fig. 2 Block diagram for Single-Phase Elementary PLL adopted for study, and is referred as Non-Liner Model

- There are two states present in the model, and their differential equations are represented as:

$$\dot{x}_1(t) = k_i\, e(t)$$
$$\dot{x}_2(t) = \omega_0\, t + x_1 + k_p\, e(t)$$

- The expression for the error signal is given by

$$e(t) = \frac{1}{2}\left[\underbrace{\sin(2\,\omega_0\, t + \Delta\,\theta + \Delta\,\theta')} + \sin(\Delta\,\theta - \Delta\,\theta')\,\right]$$

LINEAR TIME PERIODIC (LTP) MODELLING

- Expanding the for-error signal $e(t)$, using Taylor's series:

$$e(t) \approx \frac{1}{2}[\sin(2\,\omega_0\,t) + (\cos(2\,\omega_0\,t) + 1)\,(\Delta\,\theta) + (\cos(2\,\omega_0\,t) - 1)\,(\Delta\,\theta')]$$

- The LTP model is represented as:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \overbrace{\begin{bmatrix} 0 & k_i\,\frac{\hat{V}}{2}\,[\cos(2\,\omega_0\,t) - 1] \\ 1 & k_p\,\frac{\hat{V}}{2}\,[\cos(2\omega_0\,t) - 1] \end{bmatrix}}^{A(t)} \overbrace{\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}}^{x(t)} + \overbrace{\frac{\hat{V}}{2}\,[\cos(2\,\omega_0\,t) + 1]\begin{bmatrix} k_p \\ k_i \end{bmatrix}}^{B(t)} u + \overbrace{\frac{\hat{V}}{2}\,[\sin(2\,\omega_0\,t)]\begin{bmatrix} k_p \\ k_i \end{bmatrix}}^{r_1(t)}$$

$$y(t) = \overbrace{\left[1 \quad k_p\frac{\hat{V}}{2}\,(\cos(2\,\omega_0 t) - 1)\right]}^{C(t)} + \overbrace{k_p[\frac{\hat{V}}{2}\,(\cos(2\,\omega_0 t) + 1)]}^{D(t)} + \overbrace{k_p\frac{\hat{V}}{2}\,[\sin(2\,\omega_0\,t)]}^{r_2(t)}$$

- It is observed that ***A*(*t*)**, ***B*(*t*)**, ***C*(*t*)**, ***D*(*t*)**, $r_1(t)$ and $r_2(t)$ are timer periodic with time period of $T_0/2$.

LINEAR TIME INVARIANT (LTI) MODELLING

- Next step is to formulate **LTI model** from the **LTP state space model**.

- We expand the time periodic quantities using complex Fourier Series, considering only ± 120 Hz, and 0 Hz.

$$x(t) = X_0 + X_2\, e^{j2\omega_0 t} + X_{-2}\, e^{-j2\omega_0 t}$$

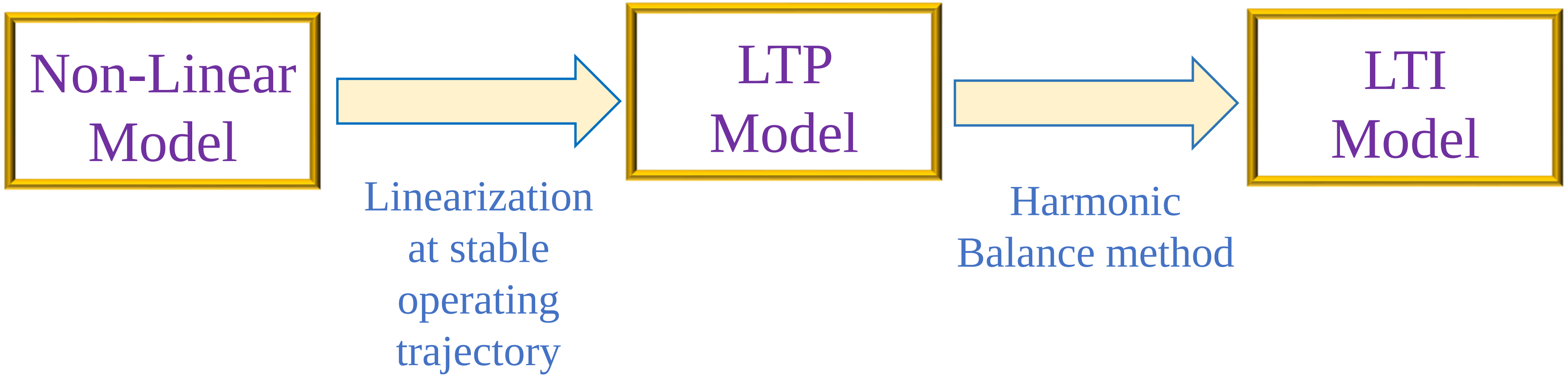
$$A(t) = A_0 + A_2\, e^{j2\omega_0 t} + A_{-2}\, e^{-j2\omega_0 t}$$

- The LTI model then can be formulated as,

$$s\begin{bmatrix} X_{-2} \\ X_0 \\ X_2 \end{bmatrix} = \left(\overbrace{\begin{bmatrix} A_0 & A_{-2} & 0 \\ A_2 & A_0 & A_{-2} \\ 0 & A_2 & A_0 \end{bmatrix}}^{\mathbf{A}} - \overbrace{\begin{bmatrix} -2j\omega_0 & & \\ & 0 & \\ & & 2j\omega_0 \end{bmatrix}}^{\mathbf{N}} \right) \begin{bmatrix} X_{-2} \\ X_0 \\ X_2 \end{bmatrix} + \overbrace{\begin{bmatrix} B_{-2} \\ B_0 \\ B_2 \end{bmatrix}}^{\mathbf{B}} U_0 + \overbrace{\begin{bmatrix} R_{1,(-2)} \\ R_{1,(0)} \\ R_{1,(2)} \end{bmatrix}}^{R_1}$$

$$\begin{bmatrix} Y_{-2} \\ Y_0 \\ Y_2 \end{bmatrix} = \left(\overbrace{\begin{bmatrix} C_0 & C_{-2} & 0 \\ C_2 & C_0 & C_{-2} \\ 0 & C_2 & C_0 \end{bmatrix}} \right) \begin{bmatrix} X_{-2} \\ X_0 \\ X_2 \end{bmatrix} + \overbrace{\begin{bmatrix} D_{-2} \\ D_0 \\ D_2 \end{bmatrix}} U_0 + \overbrace{\begin{bmatrix} R_{2,(-2)} \\ R_{2,(0)} \\ R_{2,(2)} \end{bmatrix}}$$

SUMMARY OF THE DERIVATION PROCESS



SIMULATION AND BENCHMARKING

For simulations and benchmarking a step change in input phase angle ($\Delta\theta$) is applied at **t = 0.5s of 10°**

Benchmarking of LTP model with Non-Linear Model

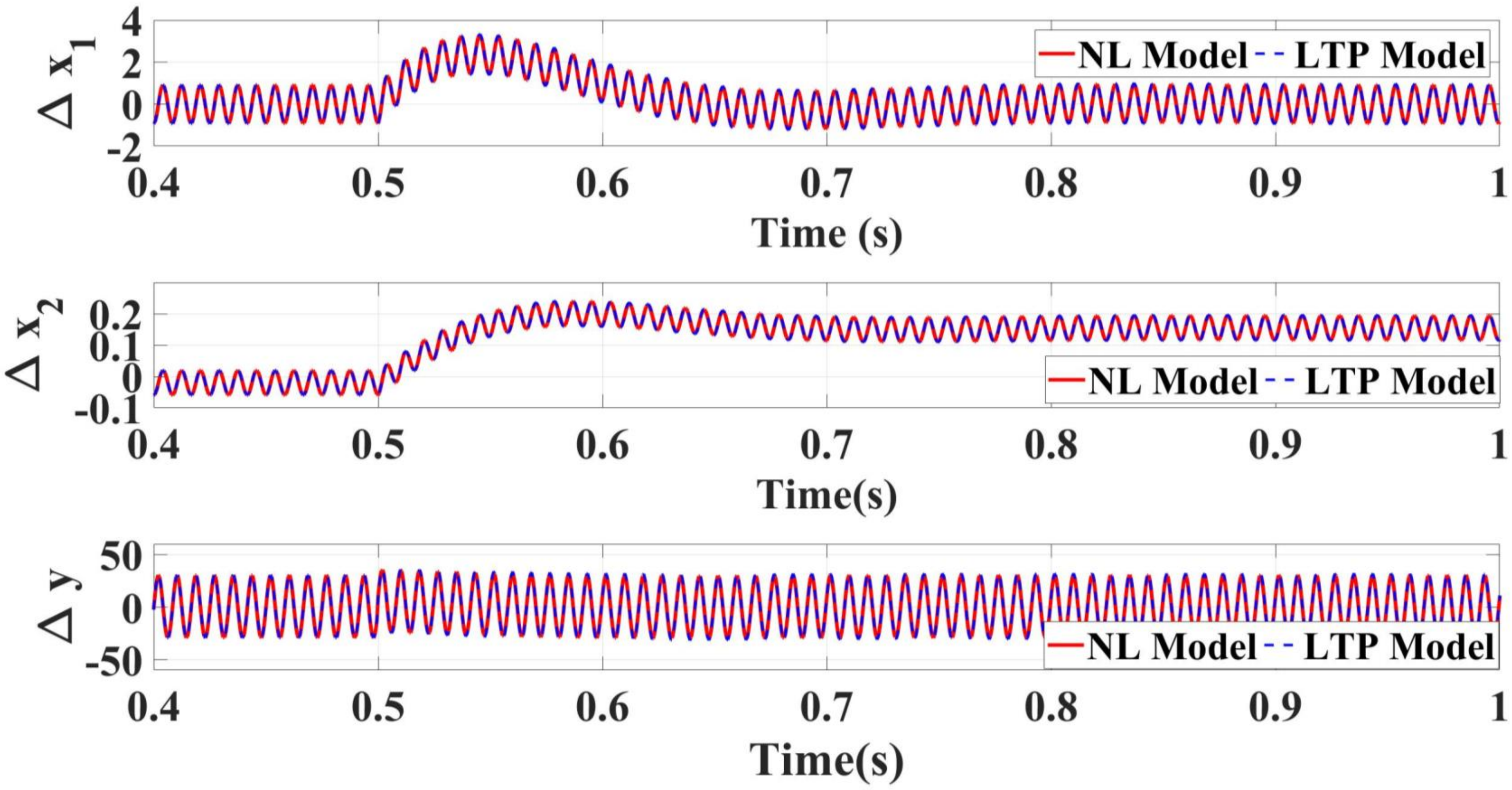


Fig. 7 Benchmarking of results obtained from LTP model with Non-Linear model

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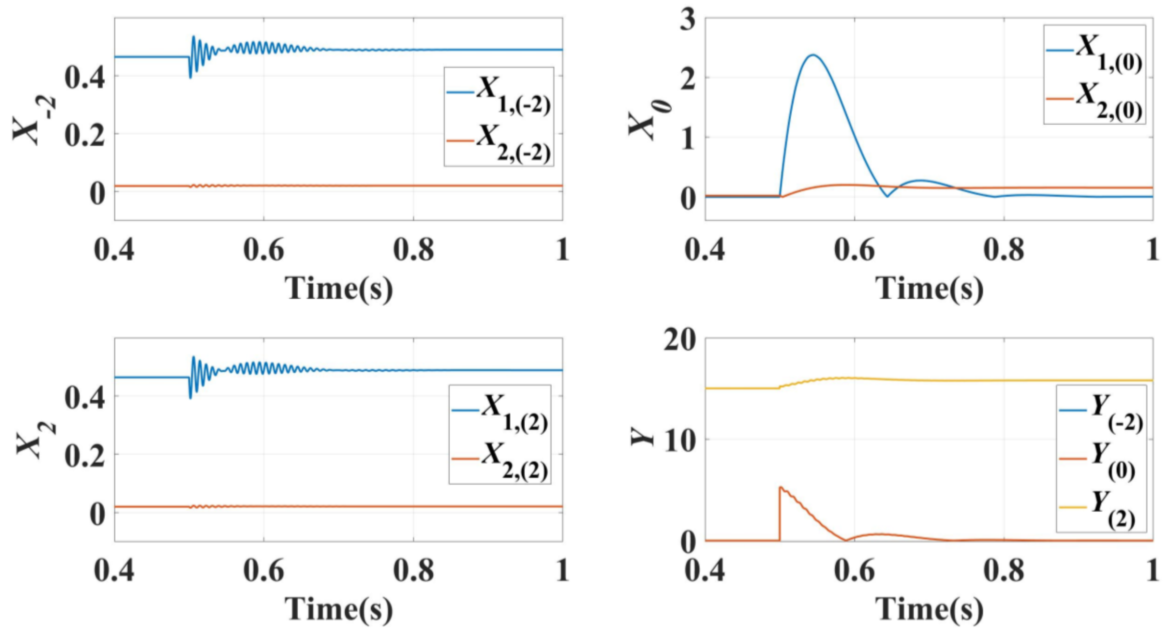
SIMULATION AND BENCHMARKING

- From LTI model we obtain the complex Fourier coefficients.

- Similar, complex Fourier coefficients were obtained from the nonlinear model by performing FFT on the time domain signals.

- Comparison results are tabulated below:

Harmonics	Fourier Coefficients	LTI Model	Nonlinear Model
-120 Hz	X_1 X_2 Y	$0.488\angle 159.5^\circ$ $0.021\angle 161.27^\circ$ $15.806\angle 71.269^\circ$	$0.464\angle 157.8^\circ$ $0.0199\angle 159.8^\circ$ $15.01\angle 69.55^\circ$
0 Hz	X_1 X_2 Y	0 $0.154\angle 0^\circ$ 0	0 $0.1546\angle 0^\circ$ 0
120 Hz	X_1 X_2 Y	$0.488\angle -159.5^\circ$ $0.021\angle 161.27^\circ$ $15.806\angle -71.269^\circ$	$0.464\angle -157.8^\circ$ $0.0199\angle -159.8^\circ$ $15.01\angle -69.55^\circ$



Simulation results from derived LTI model, when a step change in applied to input phase angle, at t =0.5s of 10°. Absolute value of complex Fourier coefficients are presented.

SIMULATION AND BENCHMARKING

- Input-Output relationship between $\boldsymbol{Y}(\boldsymbol{s})$, and $\boldsymbol{U}(\boldsymbol{s})$

$$Y(s) = \overbrace{\left[C\,(s\,I - A + N)^{-1}\,B + D\right)]}^{G(s)}\,U_0 + \overbrace{C\,(s\,I - A + N)^{-1}\,R_1 + R_2}^{Y_2(s)}$$

- Defining $\boldsymbol{Y_1(s) = G(s)\,U_0}$
- To obtain value of $Y(s)$, values of magnitude and phase for $\boldsymbol{G}$ and $\boldsymbol{Y_2}$ are noted for $\boldsymbol{\omega \rightarrow 0}$ **Hz**

$$|Y(j\,\omega)|\measuredangle Y(j\,\omega) = |Y_1(j\,\omega)|\measuredangle Y_1(j\,\omega) + |Y_2(j\,\omega)|\measuredangle Y_2(j\,\omega)$$

SIMULATION AND BENCHMARKING

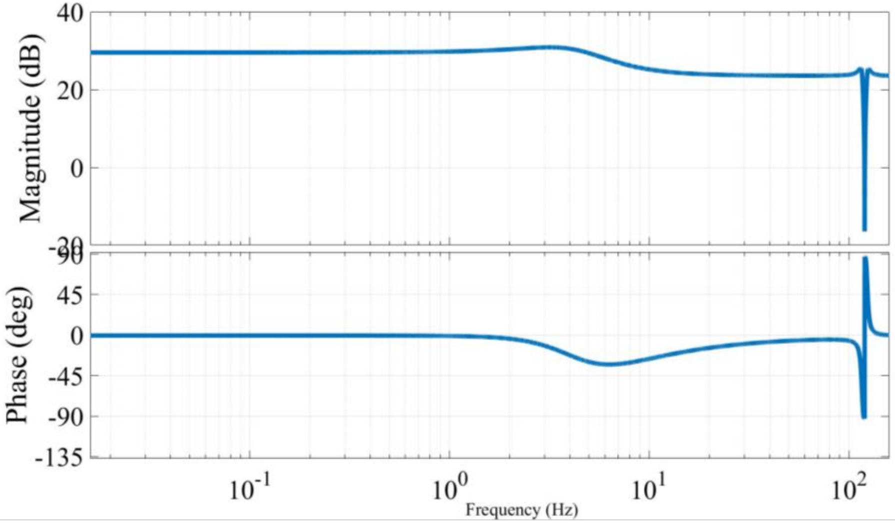
- Bode diagrams of ***G(s)*** and $\boldsymbol{Y}_2$ (s) are plotted.

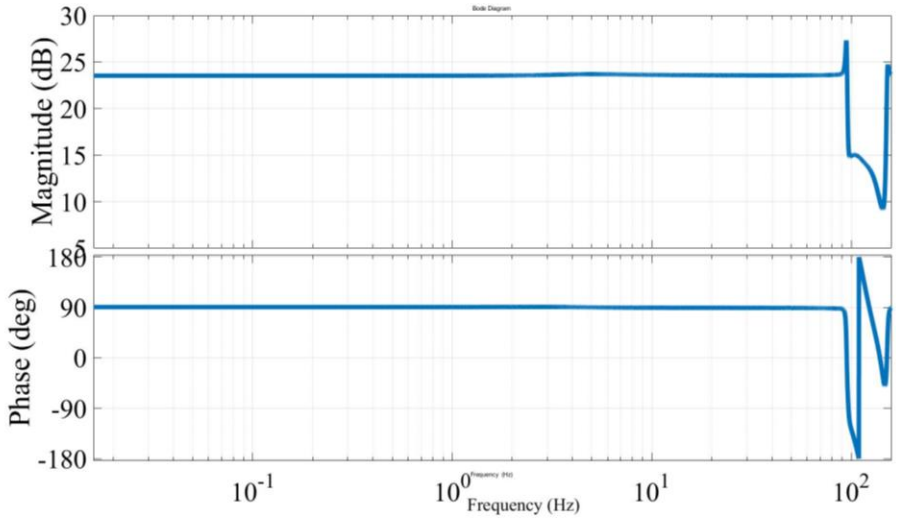
- Phasor expression of Y is thus formed.

- Comparison of complex Fourier Coefficients of Y obtained from Bode plots and Non-Linear model is tabulated

	Bode Plot	Non-Linear Model
-120 Hz	$15.89\angle 70.58°$	$15.01\angle 69.55°$
0 Hz	0	0
120 Hz	$15.89\angle -70.58°$	$15.01\angle -69.55°$

G(**s**) for -120 Hz component.





$\mathbf{Y}_2$(**s**) for -120 Hz component.

CONCLUSIONS

1. Linear time periodic modelling framework was adopted to model single-phase elementary PLL .
2. Instead of linearizing the system at stable operating point, LTP modelling framework linearizes the system around stable operating periodic trajectory.
3. First, LTP system was derived, and then LTI state-space model was obtained using harmonic balance method.
4. ±120 Hz and 0 Hz components were considered for LTI state-space model.
5. Validation of the two models obtained with the Non-Linear Model.
    a. For LTP model the output were time domain signals.
    b. Output of LTI model is complex Fourier coefficients.
    c. Bode plots for were also used for validations.

Thank You
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