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# Tainter Inspired Model of Survival and Collapse of Simple Hierarchical Society Networks

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A PREPRINT

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April 27, 2019

## ABSTRACT

In the past twenty years several events disrupted global economics and social well-being and generally shook the confidence in the stability of western societies. Popular examples are, the financial crisis, bankruptcy of multiple developed states, populism, war and climate refugees or Brexit. With this background we aimed to identify drivers of societal instability or even collapse. For this purpose a model was developed inspired by the theory of the collapse of complex societies. A simple network model simulated the development of complexity in terms of an administration body as a response to stresses affecting the productivity of the network agents.

We were able to illustrate societal collapse as a function of complexity measured in the share of administration in a network. Furthermore, we identified minimum requirements of the administration and the societal network topology to improve well-being of the society, estimated in terms of produced energy per capita. Finally we provide a mechanism for improving well-being and survival of the modeled society by enabling agents to randomly change between labor and administration, which is effective at very low rates.

**Keywords** First keyword · Second keyword · More

## 1 Tainter today

1. Observations of heavy administration bodies and bureaucracy in contemporary societies and associated problems

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2. Theory of tainters marginal productivity Economic explanation for collapse diminishing marginal returns (a) reduced advantages of complexity (b) increased costliness of complexity ((c) increasing disadvantages of complexity)
3. Combination of economic theory and runaway model of a society which will not change course. (Positive feedback loops)

## 2 Model description

### 2.1 Tainter inspired network of a runaway society steering into deminishing marginal returns and collapse

We considered an Erdos Renyi network with a fixed density of links ( $\rho$ ) and stable number of  $N$  nodes, representing social entities participating in a simple hierarchical society, which is made up of three classes of different ability to produce energy ( $E$ ). Initially the network consists only of working class nodes ( $W = N$ ), which harvest an arbitrary energy resource ( $R$ ) to fulfill the energy requirement of the society ( $\epsilon$ ) with an efficiency  $\phi_w = 1 \text{ e capita}^{-1}$ .

$$E = R(W^{\phi_w} + C^{\phi_c}) \quad (1)$$

At the beginning of each time step, the availability of  $R$  is drawn from a beta distribution with parameters  $\alpha = 15$  and  $\beta = 1$ , mostly resulting in  $R \cong 1$ , with a low chance of  $R \ll 1$ . When  $E < \epsilon$ , the network reacts by selecting the node with the highest degree and changes its class to Administrator ( $A$ ). Nodes of class  $A$  do not produce energy but instead increase the efficiency of the neighboring nodes ( $C$ ) to  $\phi_c$ . In each following time step the node with the highest degree out of all  $C$  is recruited whenever  $E < \epsilon$ .

### 2.2 random class exploration as a countermeasure to collapse on a individual basis

Additionally we implemented a random class exploration, in which a random probability ( $p_e$ ) is assigned to all nodes. At the beginning of each time step for each node a sample ( $s$ ) is drawn from a uniform distribution  $[0,1]$ . When  $s < p_e$ , the node changes its class from  $W, C$  to  $A$  and vice versa.

### 2.3 Analytic approximation to the mechanistic probabilistic models

Model separated into two dynamics

1. Reaction mechanism to stress:

- network reacts to

1. Exploration part

$$A = p_e * (N - 2x) + F\left(\frac{\epsilon N}{(N - x)(1 - \rho)^x + ((N - x)(1 - (1 - \rho)^x))^\phi}, \beta, \alpha\right) \quad (2)$$

### 3 Results and Discussion

#### 3.1 original Tainter dynamics

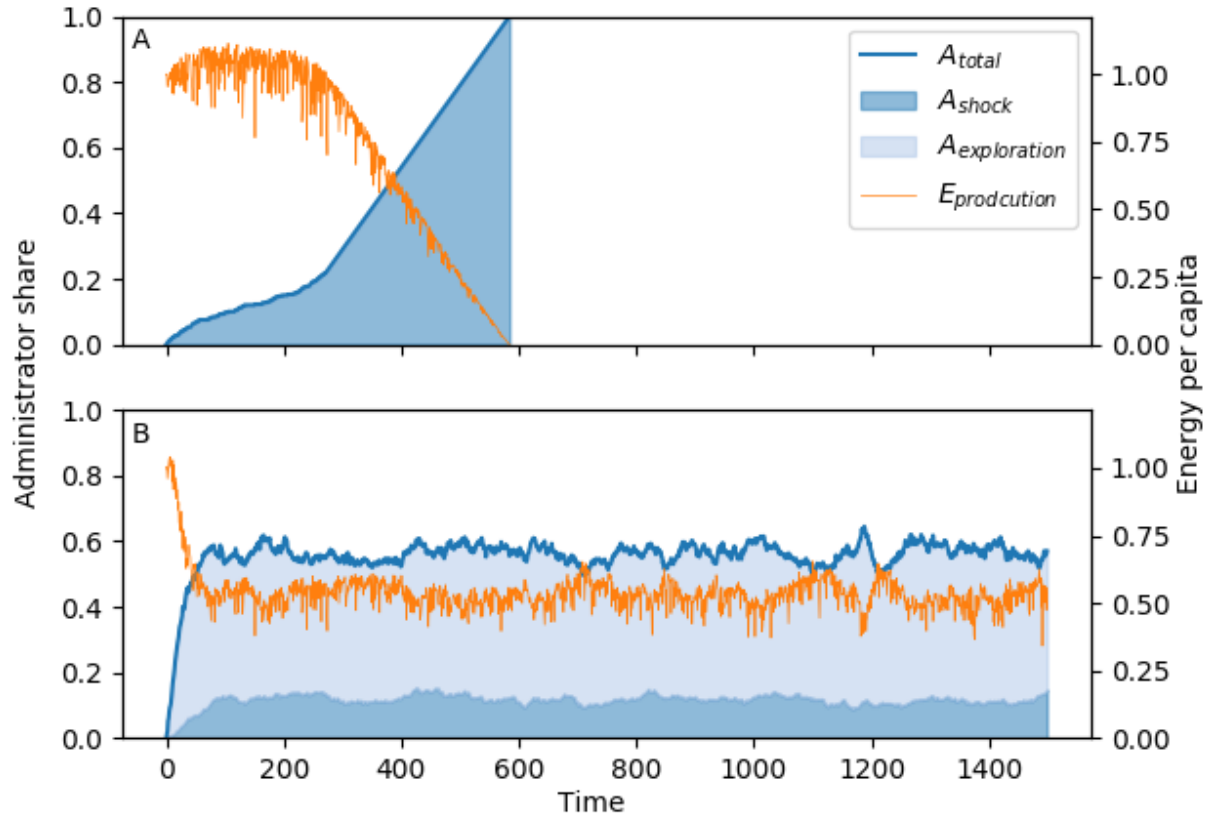


Figure 1: Exemplary network simulation of a Tainter like model according to the model description in ?? . Both simulations were run for a maximum of 1500 time steps. Blue curves show the share of administration in the network (light blue: Administration as result of decreased resource availability, dark blue: Administration resulting from exploration). Orange curves show the average energy produced per node. A) shows the typical development of a network reacting to shocks by changing one node from C to A. As a result, initially the share of administration rises slowly, due to high marginal returns on investments (shown by the orange curve) until each further node changed to administration, negatively affects the energy produced by the network. B) shows the development of a network with the interaction of tainter dynamic and exploration dynamic. After a short period of rapid increase in administration, the network reaches a stable fixpoint upon which the share of administration and energy production remain stable and fluctuates only as a result of random beta distributed reductions of energy access.

#### Exemplary development of a tainter inspired society

Interplay between network characteristics. Conditions for a beneficial administration Find out realistic ranges of efficiency and link density in simplistic societies, in order to be able to discuss the graphic

#### Macroscopic approximation of original Tainter Dynamics

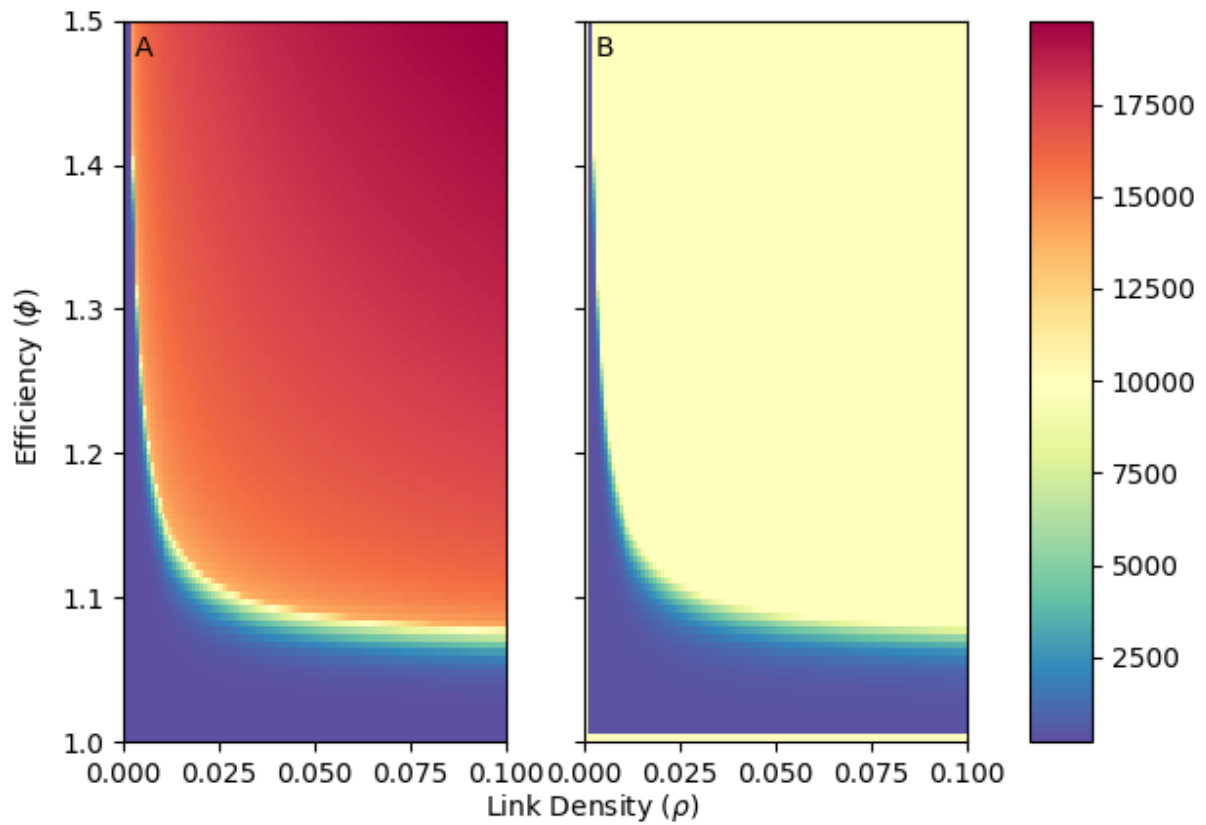


Figure 2: Survival of a network with  $N = 400$  as a function of exploration, link density and efficiency.

### 3.2 modified Tainter dynamics. Exploration

model run description

Low exploration results in highly increased survival times

extension of macroscopic approximation

## 4 Conclusion

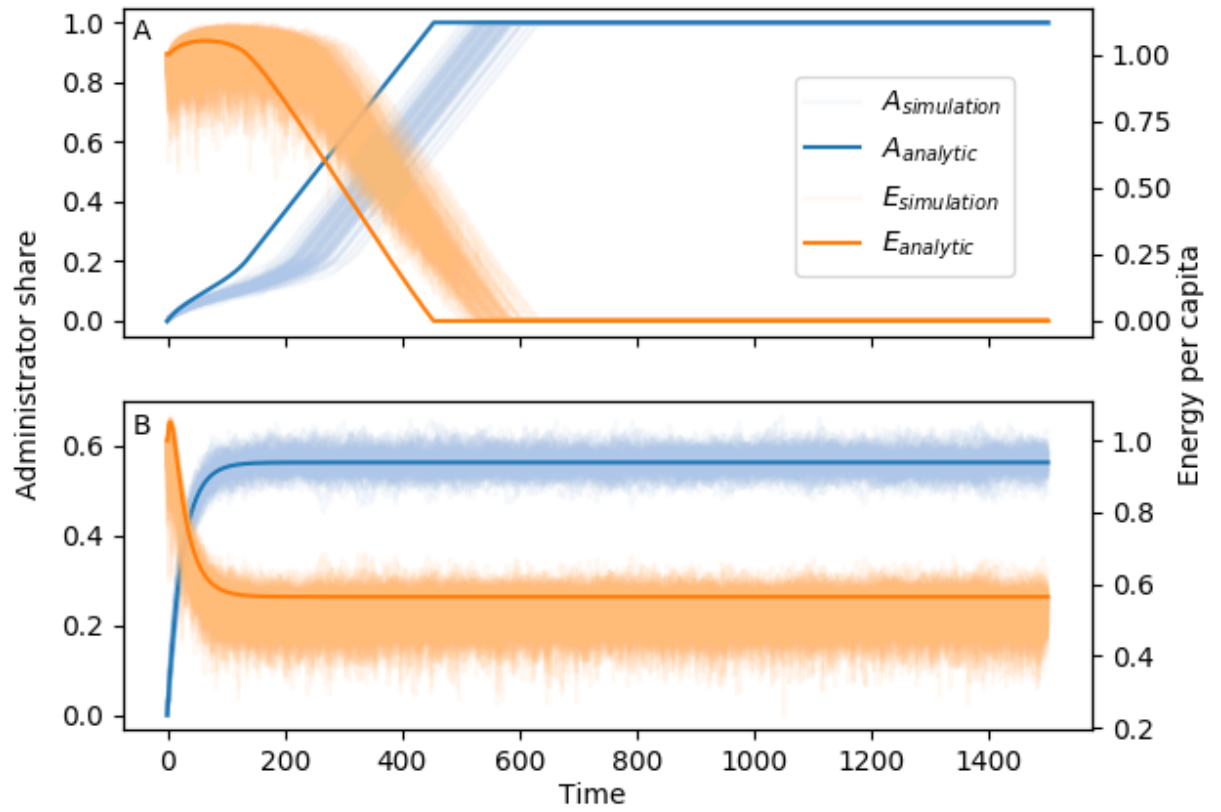


Figure 3: Survival of a network with  $N = 400$  as a function of exploration, link density and efficiency.