Cycloidal Gear Equations

Frances O'Leary

January 2021

This brief write-up deals with the equations necessary to derive the profiles of a cycloidal gear.

1 Necessary Inputs

- Diameter of the pins in cm, D_p
- Number of lobes, N_l
- Radius of the cam in cm, R_c
- Radius of the center pin in cm, R_{cp}

Note that N_l corresponds to the reduction achieved by the mechanism; $N_l = 10$ corresponds to a reduction of rotation by 10.

2 Construction of the Disk

From these inputs, we can derive the diameter of the inner plate, D_{ip}, which is given by the equation below:

$$D_{ip} = \frac{2}{3} D_p N_l$$

In addition to the diameter of the inner plate, the parametric equations for the cycloidal disk profile also require an eccentricity, e. As a default, we define e as the following:

$$e = \frac{1}{2} D_p$$

Now, we have all the necessary components to employ the parametric equations necessary to draw the profile of the cycloidal disk. They are given below:

$$\begin{cases} x(t) = D_{ip}\cos(t) - D_p\cos(t + \arctan(\sin((1-N_l)t)/((D_{ip}/eN_l) - \cos((1-N_l)t)))) - e\cos(N_l t) \\ y(t) = -D_{ip}\sin(t) + D_p\sin(t + \arctan(\sin((1-N_l)t)/((D_{ip}/eN_l) - \cos((1-N_l)t)))) + e\sin(N_l t) \end{cases} \quad t \in [0, 2\pi]$$

After plotting this curve from 0 to 2π, the last step is to offset it by D_p inwards. The final step is to cut out a circle of radius R_c, to house the cam. This circle is simply cut out about the origin. Thus completes the generation of the profile of your cycloidal disk.

3 Construction of the Base

The base consists of a large circle that can accommodate the cycloidal disk and cam within its breadth. As a result, we define R_b, the radius of the base, by the following equation:

$$R_b = \text{minDist(parCurve, origin)} + D_p + 0.5$$

Note that in this instance, minDist(parCurve, origin) denotes the minimum distance between the parametric curve we drew in the portion prior. We then add the diameter of the pins, along with 0.5(5 mm for Fusion) to ensure that the base is large enough. This distance is rather arbitrary; the base can be of any size such that it can house the disk and pins; the location of the latter will be detailed next.

Now that we have the base, we must construct the pins. Given that there are N_l lobes, we can deduce that there are $N_p = N_l + 1$ pins. As a result, we add N_p circles around the origin, each with diameter D_p, and with a distance from the origin of $d = $minDist(parCurve, origin)$+\frac{1}{2}D_p$. The simplest method of construction is to draw a circle of diameter D_p at $(x, y, z) = (0, d, 0)$ and then rotate this coordinate by $\frac{360}{N_p}$ and draw another circle of diameter D_p. This process is repeated N_p times.

The final step is to draw the center pin, of radius R_{cp} about the origin. Thus concludes the base.

4 Construction of the Cam

Finally, we must construct our cam, which consists merely of two circles; one drawn with respect to the origin and the other drawn from an offset.

The offset(aka radius at which the center of the hole for the center pin should be drawn) for the center pin in the cam is given by the following:

$$\text{offset} = \frac{2}{3}D_p$$

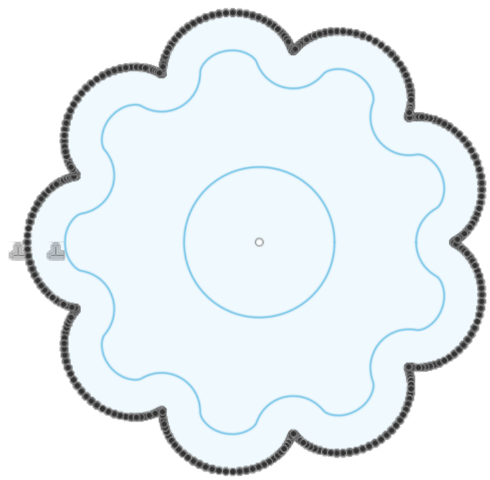
Thus, if your outer circle, of radius R_c was drawn about $(x, y, z) = (0, 0, 0)$, you would draw an inner circle of radius R_{cp} about $(0, \frac{2}{3}D_p, 0)$.

5 Example

For this example, we will employ the techniques given above, with the inputs below:

- $D_p = 0.50$ cm
- $N_l = 10$
- $R_c = 1$ cm
- $R_{cp} = 0.15$ cm

Following the steps for the construction of the disk, we generate the following profile:

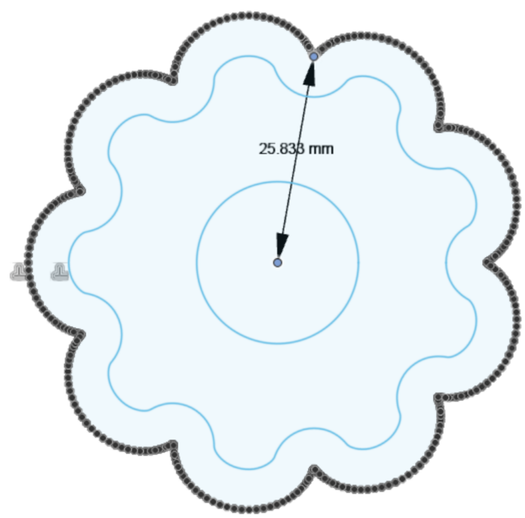


The outer curve denotes our parametric curve, while the curve in the middle denotes our outer curve offset by a distance of D_p inwards. Finally, a circle of R_c was drawn about the origin.

Now, we are ready to construct the base. First, we find the radius of our base:

$$R_b = \text{minDist}(\text{parCurve, origin}) + 0.50 + 0.5$$

From the figure below, we deduce minDist(parCurve, origin) to have a value of 25.833, thus resulting in $R_b = 26.833$ cm. We draw a circle of radius R_b about the origin as our base.

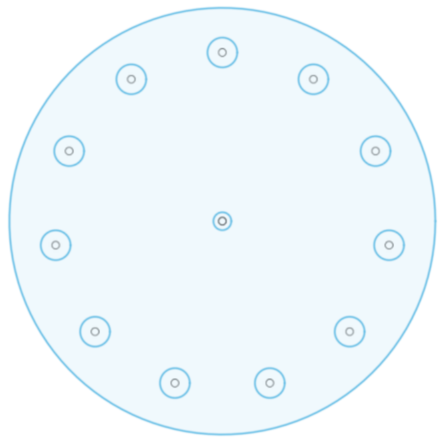


Next, we need our pins, of which we know there are $N_l + 1 = 11$. We find our distance to be

$$d = 25.833 + 0.5 * 0.50 = 26.083$$

Thus we are tasked with drawing 11 pins of diameter 0.5 cm, each 26.083 cm from the origin, and spaced $\frac{360}{11}$ degrees apart.

Finally, we must draw our center pin of radius 0.15 cm about the origin, which results in the following profile:



Now the only component that remains is the cam. We begin by drawing our circle of radius 1 cm about the origin. Then, we must figure out our offset:

$$\text{offset} = \frac{2}{3} * 0.5 = 0.3333333$$

Thus we can conclude by drawing our final circle of radius 0.15 cm with an offset of 0.3333333 cm from the origin: