Operations Research I (Lecture 1)

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This is a note I took while studying the Coursera course: Operations Research I taught by Prof. Ling-Chieh Kung at National Taiwan University. You can click here for more notes.

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1 Motivation

Operations research has a very close relationship with management. Daft, R. L. (2014) said Management is the attainment of organizational **goals** in an effective and efficient manner through planning, organizing, leading, and controlling organizational **resources**.

There are two key-words in this statement, goals and resources. Basically these means when you are running an organization, you will want to do somethings, then these somethings is your goals. You have your goals with limited resources, thus you need to do some research about how to do this, this is **Operation Research**.

There are all kinds of things that you want to do, but you just have limited resources. That is why you need to do some careful allocation about them. That is basically the need of operation research. In other words, Operation Research (OR) is the methodology that help you allocate limited resources effectively to achieve your goals. Thus OR has the following other names:

- Management Science
- Decision Science
- Optimization Method
- Mathematical Programming

There has an example: Two people are going to hold an event, and they need to complete some tasks. There are assignment rules: One task musts be assigned to exactly one person, and one person can work one task at a time. Then, how to assign the tasks so that they can complete all tasks fasteastly?

ID	task	processing time (min)
1	t1	20
2	t2	30
3	t3	60
4	t4	15
5	t5	25

In the case the resource is that only two persons; and the objective (goal) is that minimize the total amount of time you need to complete everything.

2 Business analytics

OR is an important part of Business Analytics (BA), thus we need to introduce it. Collectively, BA has three levels: Descriptive Analytics, Predictive Analytics and Prescriptive Analytics.

Descriptive Analytics focus on **what has happened** with historical data. Predictive Analytics focus on **what will happen** by prediction model with historical data. Prescriptive Analytics focus on **what should we do**, it help you make decisions. i.e. here is the field where OR plays.

The first two levels constitute the first step of decision-making through BA – Data Analysis. They collect information and understand the problem. The third level, Prescriptive Analytics, acts as the second step, allocates resources and solve the problem.

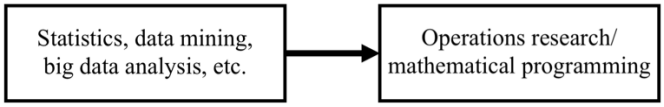

Figure 1: A typical process of decision making with BA

The three levels seem coherent and explanatory, but Analytics is not everything. Analytics describes a problem in a precise and concise way, it facilitates the use of computers to solve problem whose model has been formulated. But sometimes, the problems are too complicated to be formulated into a mathematical model. Or sometimes, Some critical information is missing. In this case, Analytics can not help. Here is a typical process of conducting an OR study:

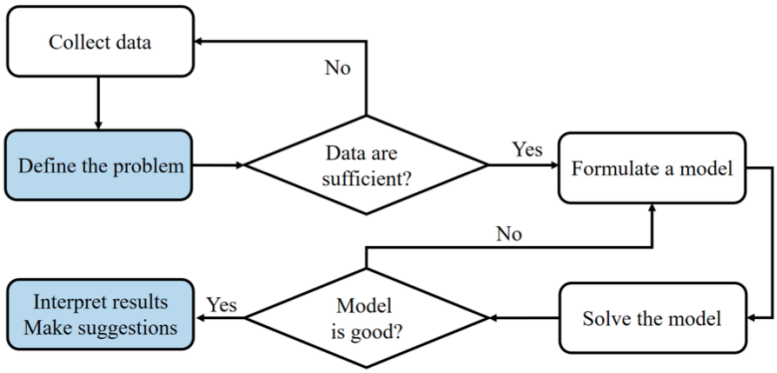

Figure 2: The process of conducting an OR study

3 Mathematical Programming

Mathematical Programming is the form of OR study, we implement it with an Example: You are preparing for hiking. There are some useful items, but your backpack can only carry 5kg. Each item can not be split, then which items should you bring to maximize the total value?

Item	Weight	Value
I1	0.5	6
I2	1.5	5
I3	0.4	4
I4	1.0	4
I5	1.1	3
I6	1.6	4
I7	0.8	1

To formulate this problem, we need identify three objects: **Decision Variable**, **Objective function** and **Constraints**.

Define that

$$x_i = \begin{cases} 1, & \text{if item } i \text{ is chosen,} \\ 0, & \text{otherwise,} \end{cases}$$

then x_i $(i = 1, 2, \cdots, 7)$ is Decision Variables. Let w_i, v_i be the weight and the value of item i. Let n be the number of items and B be the maximum allowable weight (in this example $n = 7$, $B = 5$), then the Objective function is that

$$\sum_{i=1}^{n} v_i x_i,$$

and the Constraints is that

$$\sum_{i=1}^{n} w_i x_i \leq B.$$

Collectively, a typical form of mathematical program model is

$$\begin{aligned} \max \quad & \sum_{i=1}^{n} v_i x_i \\ \text{s.t.} \quad & \sum_{i=1}^{n} w_i x_i \leq B \\ & x_i \in \{0, 1\} \quad \forall i = 1, 2, \cdots, n. \end{aligned}$$

We call this type of mathematical programming model **integer programming**. Correspondingly, other kind of model like

$$\begin{aligned} \max \quad & \sum_{i=1}^{n} v_i x_i \\ \text{s.t.} \quad & \sum_{i=1}^{n} w_i x_i \leq B \\ & x_i \in [0, 1] \quad \forall i = 1, 2, \cdots, n. \end{aligned}$$

is called **linear program**.