Low Thrust Transfer

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The goal is to transfer our 15-tonne satellite from low Earth orbit (300 km altitude, zero inclination) to geostationary orbit (35786 km, zero inclination) with electric thrusters. Due to the low thrust of our satellite, this transfer cannot be done in the conventional way (in two impulsive manoeuvres). It is required to continuously thrust, in the prograde direction, to slowly increase the opposite point on the orbit while keeping the eccentricity near zero. This transfer will be time consuming.

Hypothesis

- The J2 term is not considered since the orbit remains in the plane of the equator.
- Atmospheric drag is calculated between 0 and 1000 km
- Solar radiation pressure is calculated as a function of Earth shadow
- Earth shadow is determined with a fixed Sun relative to the Earth. As the energy required for the motors to operate comes from the solar panels, no thrust can be provided when the satellite is not illuminated by the Sun. The transfer time is lengthened by this intermittent thrust.

Notations

$\vec{R}$ absolute position vector
$\vec{r}$ relative position vector of a satellite in relation to the Earth
$\vec{r} = \vec{R}_{Earth} - \vec{R}_{satellite}$

In an Earth-centred frame of reference, the relative position vector of the satellite is
$\vec{r}(t) = x(t)\vec{i} + y(t)\vec{j} + z(t)\vec{k}$

The norm or magnitude of $\vec{r}$ is $||\vec{r}|| = r = \sqrt{x^2 + y^2 + z^2}$

Tow body problem - Equation of relative motion with thrust

The equation of relative motion in the two body problem, obtained with the second law of Newton, is:

$$\ddot{\vec{r}} = -\frac{\mu}{r^3}\,\vec{r} + \frac{T}{m_s \dot{r}}\,\dot{\vec{r}}$$

with $\vec{r}$ the satellite position vector, r the satellite position norm, μ Earth gravitational parameter, T the satellite thrust and m_s the satellite mass.

This ordinary differential equation will be solved with a Range-Kutta 45 solver on Matlab.

Tow body problem - Circularisation

Circularisation consists of applying a thrust, in the prograde direction, around the apoapsis (between θ_{min} and θ_{max}) to raise the periapsis and bring it closer to the apoapsis radius. The true anomaly θ of the satellite on its orbit is used to determine if thrust should be applied.

Specific angular momentum h (m^2/s)

$$\vec{h} = \vec{r} \wedge \vec{v} = \vec{r} \wedge \dot{\vec{r}}$$

The semi-major axis can be determined with the formula for conservation of energy on an elliptical orbit:

$$-\frac{\mu}{2a} = \frac{\dot{r}^2}{2} - \frac{\mu}{r}$$

$$a = \frac{-\mu r}{\dot{r}^2 r - 2\mu}$$

Semi-latus rectum l is:

$$l = a(1-e^2) = \frac{h}{\mu}$$

Eccentricity e is:

$$e = \sqrt{1 - \frac{l}{a}}$$

True anomaly θ is:

$$r = \frac{l}{1 + e\cos\theta}$$

$$\theta = \arccos\left(\frac{h^2 - \mu r}{e\mu r}\right)$$

Circularisation thrust will be applied between $\theta_{min} = -45\,\mathrm{deg}$ and $\theta_{max} = +45\,\mathrm{deg}$ around the apoapsis.

Apoapsis and periapsis positions (r_{apo} and r_{peri}) are computed to stop the simulation when circularisation is completed.

$$r_{apo} = \frac{l}{1-e} = a(1+e)$$

$$r_{peri} = \frac{l}{1+e} = a(1-e)$$

Orbital perturbations - Atmospheric drag

The presence of the atmosphere slows down the satellite according to its altitude. This force is predominant between 0 and 1000 km.

$$F_{atm} = \frac{1}{2}\ \rho\ v_{rel}^2\ C_d\ A$$

with ρ atmospheric density at a specific altitude,
C_d satellite drag coefficient,
A satellite impact surface,
and v_{rel} satellite relative velocity in relation to the atmosphere,

$$v_{rel} = v_{sat} - v_{atm} = \sqrt{\frac{\mu}{r}} - \frac{2\pi\ r}{T_{Terre}}$$

with the Earth rotation period $T_{Earth} = 86\ 164s$

Earth atmospheric density as a function of altitude is computed with the NRLMSISE-00 Model 2001 available on Matlab (*atmosnrlmsise00*). This model takes into account the solar radiative flux to calculate the atmospheric density. The 10.7cm Solar Flux is measured by NRC Space Weather Canada and provided freely online. The Solar flux varies between 50 and 300 sfu (solar flux unit, 1 sfu $= 10^{-22}$.m^{-2}.Hz^{-1}). The worst case of 300 sfu will be used.

The following figures represent the air density, from the NRLMSISE-00 model, over the altitude with different solar flux and the atmospheric drag on the satellite with the maximum air density (solar flux of 300 sfu).

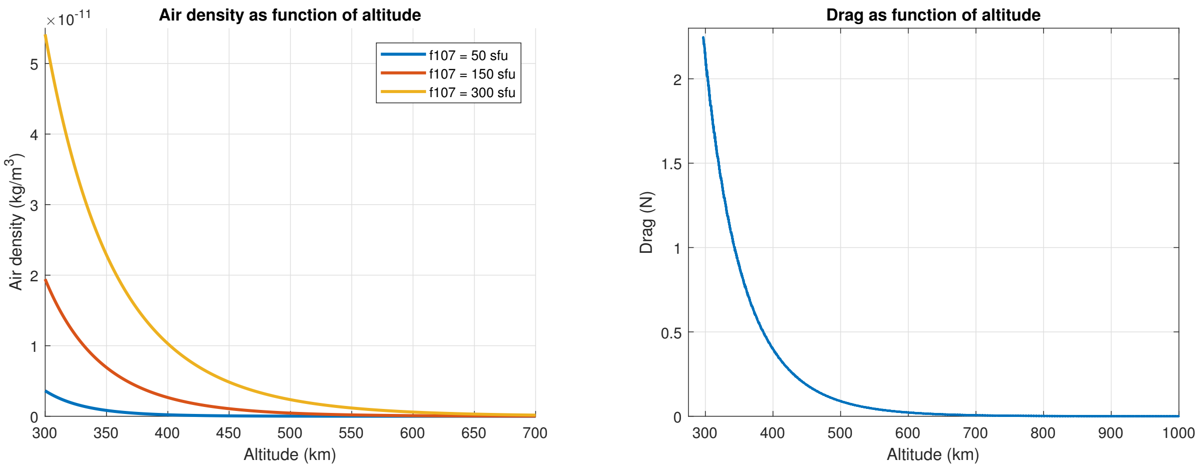

Figure 1 – Air density as function of altitude and solar flux (left) and Satellite drag as function of altitude (right)

Results without Earth shadow and Solar radiation pressure

The transfer of the 15-tonne satellite from low Earth orbit (300 km) to geostationary orbit with 30 electric thrusters is completed in 153 days. The ergol mass required is 1902 kg. The electric thrusters used are the 702HP XIPS from Boeing (0.165 N thrust and 3500 s specific impulsion). The satellite impact surface is 750 m^2.

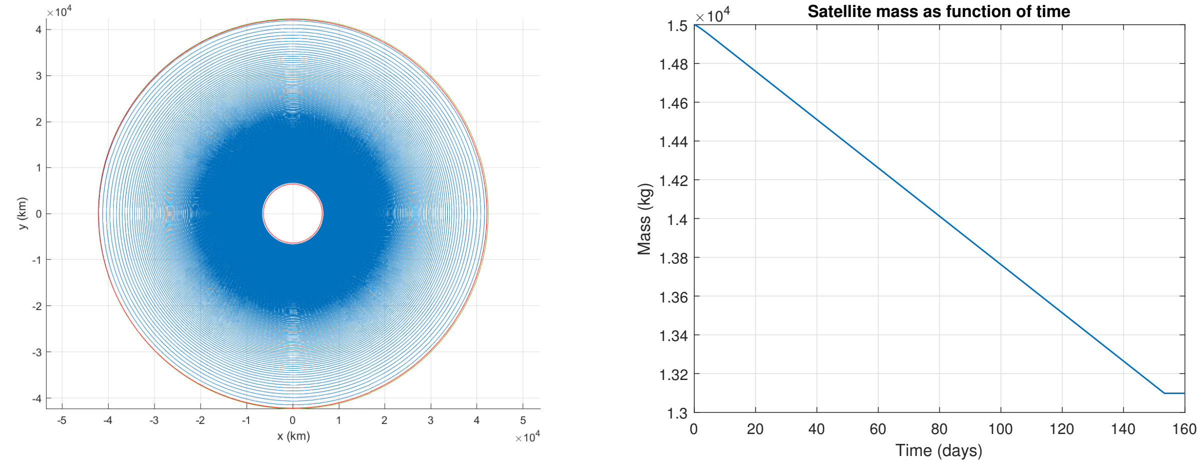

Figure 2 – Satellite trajectory (left) and Satellite mass over time (right)

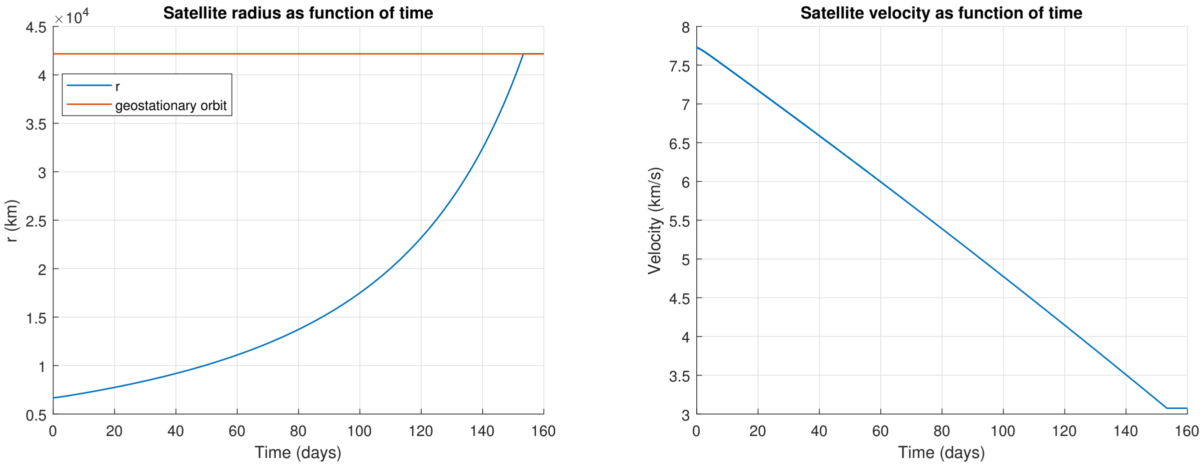

Figure 3 – Satellite radius over time (left) and Satellite velocity over time (right)

Orbital perturbations - Earth umbra and penumbra

Electric thrusters require a large electric power to work. Satellite solar panels are used to provide this power. However, when the satellite is in Earth shadow, no power can be provided and the satellite motion is impacted by this lack of thrust. The shadow function need to be determined. The shadow function returns 0 in umbra, 1 in complete illumination and between 0 and 1 in penumbra.

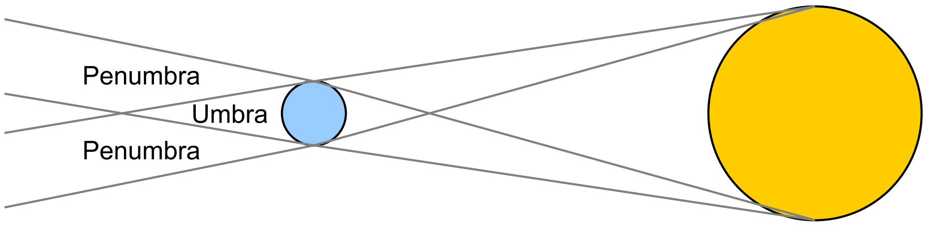

Figure 4 – Earth umbra and penumbra

Shadow function

The following method comes from the book "Orbital Mechanics for Engineering Students", H. Curtis, 2014. In order to determine when a satellite is in Earth shadow, we will use the following procedure (Vallado, 2007). First, consider two spacecraft A and B orbiting a central body of radius R. The two position vectors $\vec{r_A}$ and $\vec{r_B}$ define a plane. That plane contains the circular profile C of the central body. The angle α between the two position vectors is:

$$\alpha = \arccos\left(\frac{\vec{r_A} \cdot \vec{r_B}}{r_A r_B}\right)$$

T_1 and T_2 are points of tangency C of lines drawn from A and B, respectively. The radii OT_1 and OT_2 along with the tangent lines AT_1 and BT_2 and the position vectors $\vec{r_A}$ and $\vec{r_B}$ comprise the two right triangles OAT_1 and OBT_2. The angles at the vertex O of these two triangles are obtained from:

$$\alpha_1 = \arccos\left(\frac{R_C}{r_A}\right) \qquad \alpha_2 = \arccos\left(\frac{R_C}{r_B}\right)$$

The line AB intersects the central body, which means there is no line of sight, then $\alpha_1 + \alpha_2 < \alpha$. If the line AB is tangent to C or lies outside it, there is a line of sight, then $\alpha_1 + \alpha_2 \geq \alpha$.

This method is used to determine if there is a line of sight between the satellite and one Sun point. The Sun will be represented by 9 points to compute a shadow function with penumbra values.

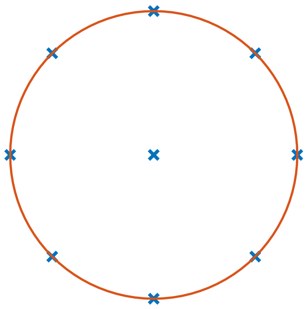

Figure 5 – Sun points

Earth shadow results

The following figures represent the Earth shadow (umbra and penumbra) seen by a satellite orbiting Earth with zero inclination. The four values of the longitude of the ascending node Ω represented correspond to the four seasons of the Earth. The satellite spends the most time in the Earth shadow during the spring and winter equinoxes. The launch date should be chosen to avoid the satellite passing in the Earth's shadow during the equinoxes.

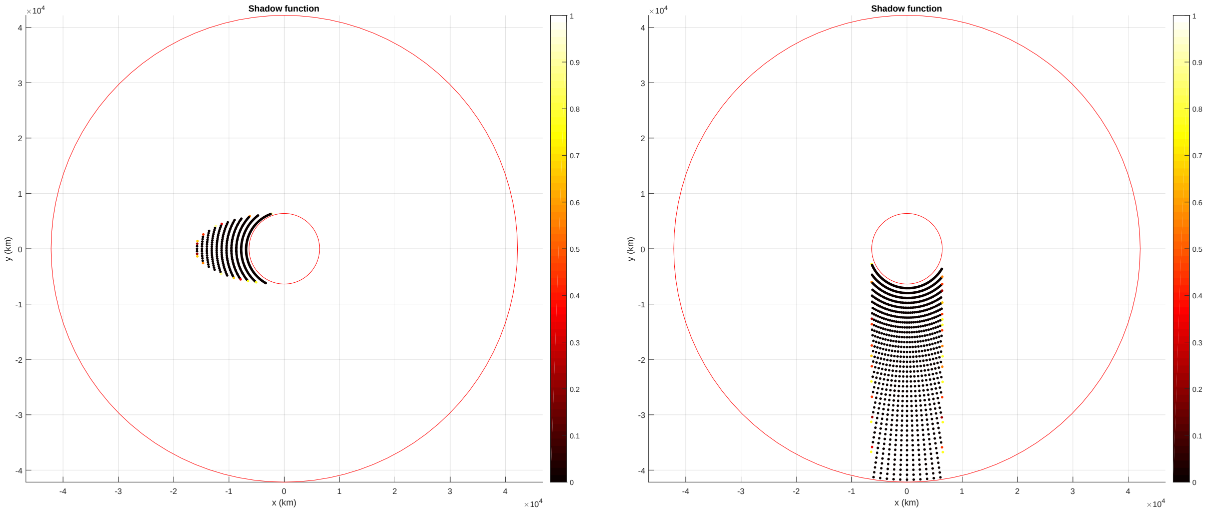

Figure 6 – $\Omega = 0°$ (left) and $\Omega = 90°$ (right)

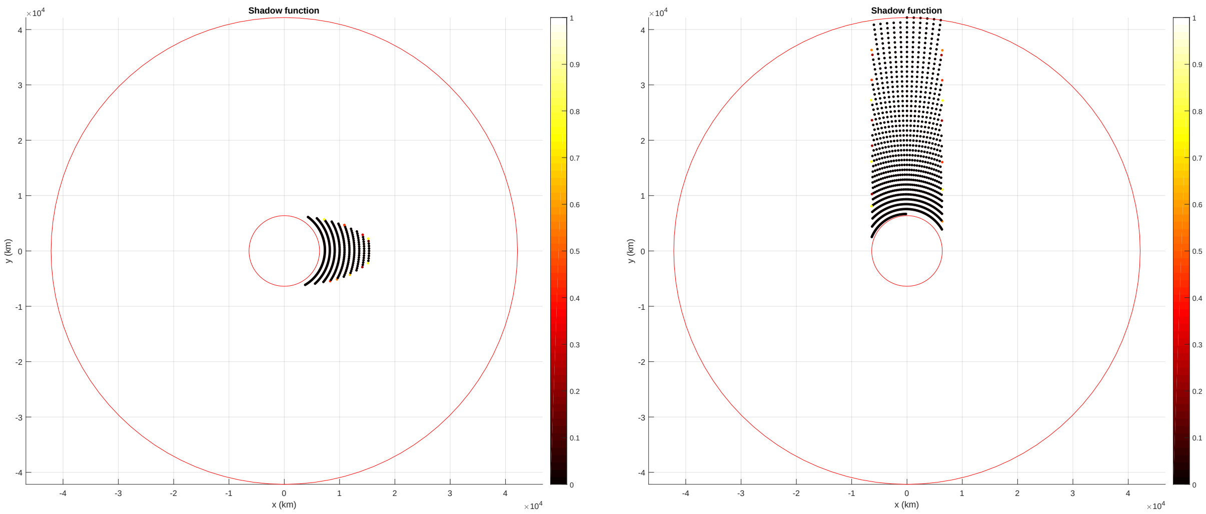

Figure 7 – $\Omega = 180°$ (left) and $\Omega = 270°$ (right)

Orbital perturbations - Solar radiation pressure

The solar radiation pressure applied on a satellite is:

$$\vec{F}_{\text{SRP}} = \nu \, \frac{\Phi}{c} \, C_R \, A \, \vec{u}_{\text{SRP}}$$

with ν, the shadow function (0 = complete shadow, 1 = complete illumination and between 0 and 1 for penumbra),
Φ solar flux is 1 367 W/m^2 at 1 UA,
c speed of light is 299 792 458 m/s,
C_R satellite reflective coefficient, characterises its capacity to absorb or reflect light,
A satellite impact surface,
and $\vec{u}_{\text{SRP}}$ the unit vector from the Sun center to the Satellite position.

To determine the reflective coefficient C_R, we will rely on experimental measurements carried out on 6 different satellites by the Institut de Physique du Globe de Paris. The measured reflective coefficient are between 0.9 and 1.4. To calculate the maximum radiation strength we will take the value 1.4. (Institut Géographique National, Institut de Physique du Globe de Paris, "Estimating daily Solar Radiation Pressure coefficients," 2008.)

The solar radiation pressure for our satellite is almost constant around the Earth and its maximum value is 4.8 mN.

Results with Earth shadow and Solar radiation pressure

The transfer of the 15-tonne satellite from low Earth orbit (300 km) to geostationary orbit with 30 electric thrusters is completed in 191 days (174 days for low thrust transfer and 17 days for circularisation). The ergol mass required is 1913 kg. The electric thrusters used are the 702HP XIPS from Boeing (0.165 N thrust and 3500 s specific impulsion). The satellite impact surface is 750 m^2.

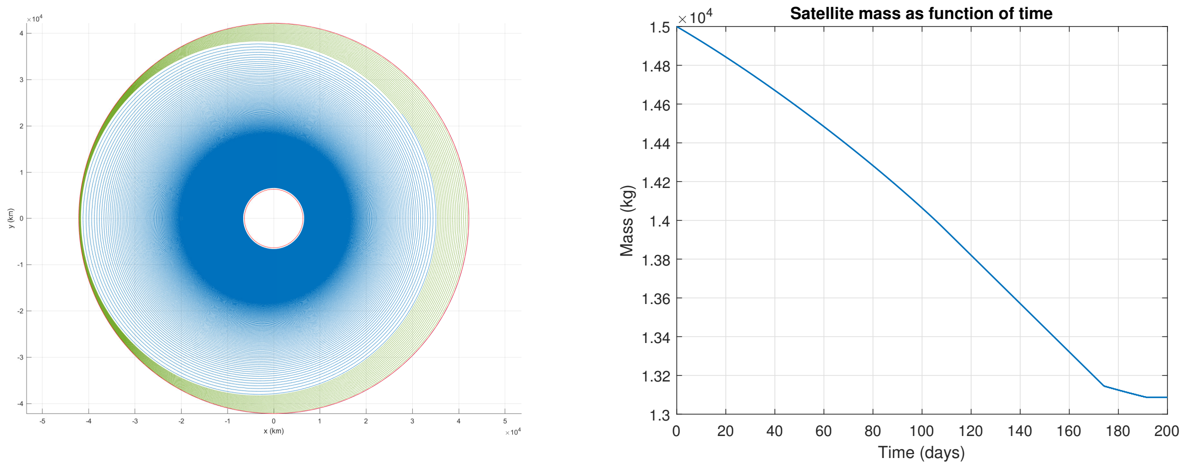

Figure 8 – Satellite trajectory with blue transfer and green circularisation (left) and Satellite mass over time (right)

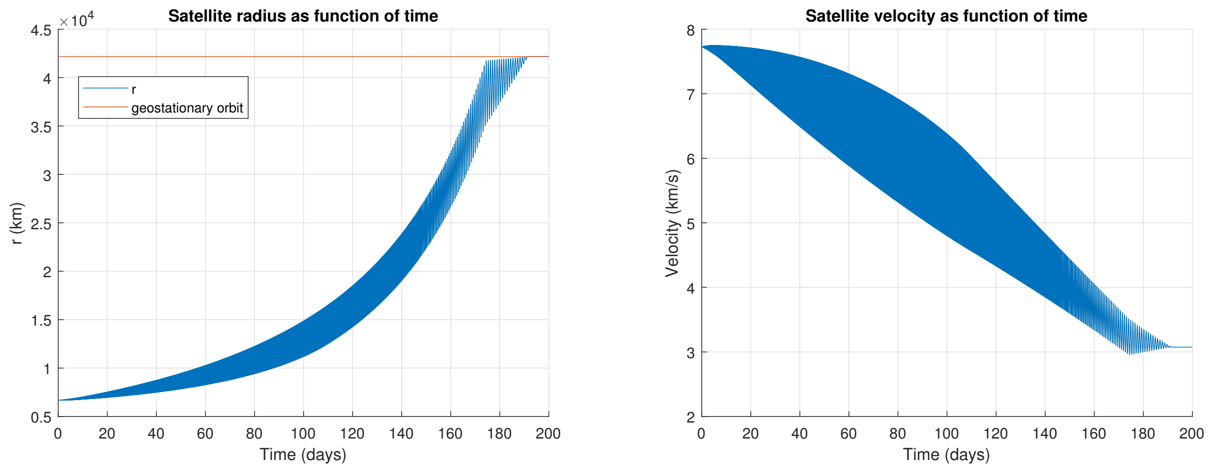

Figure 9 – Satellite radius over time (left) and Satellite velocity over time (right)

In this model, the Sun is considered fixed relative to the Earth ($\Omega = 0°$), whereas the duration of the transfer would require the Sun to move relative to the Earth. The Earth motion around Sun will be added in the future.